Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/101-4.3.1.2-d-sec-^m-a+b-tan-^n

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 1:12pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [700]. This is test number [101].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (700)	0.00 (0)
Mathematica	100.00 (700)	0.00 (0)
Maple	82.86 (580)	17.14 (120)
Fricas	81.86 (573)	18.14 (127)
Maxima	57.86 (405)	42.14 (295)
Mupad	52.71 (369)	47.29 (331)
Giac	37.43 (262)	62.57 (438)
Sympy	17.71 (124)	82.29 (576)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

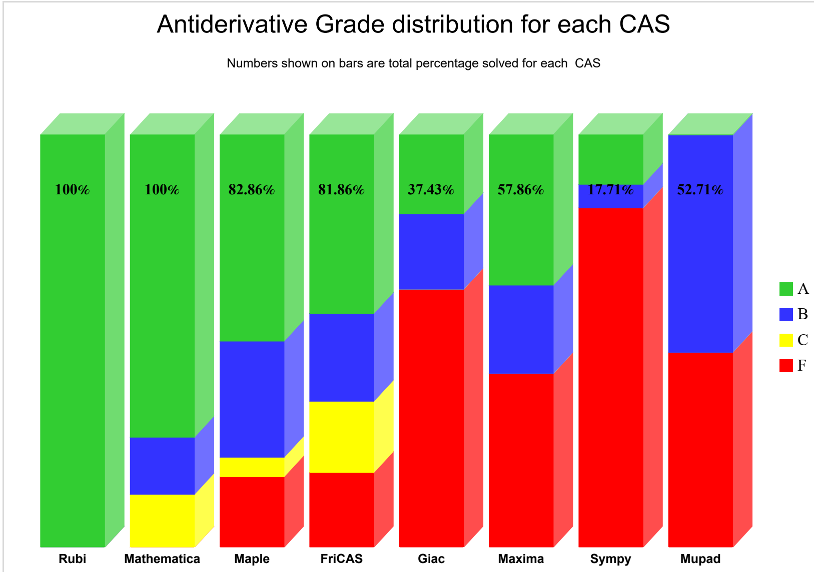
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

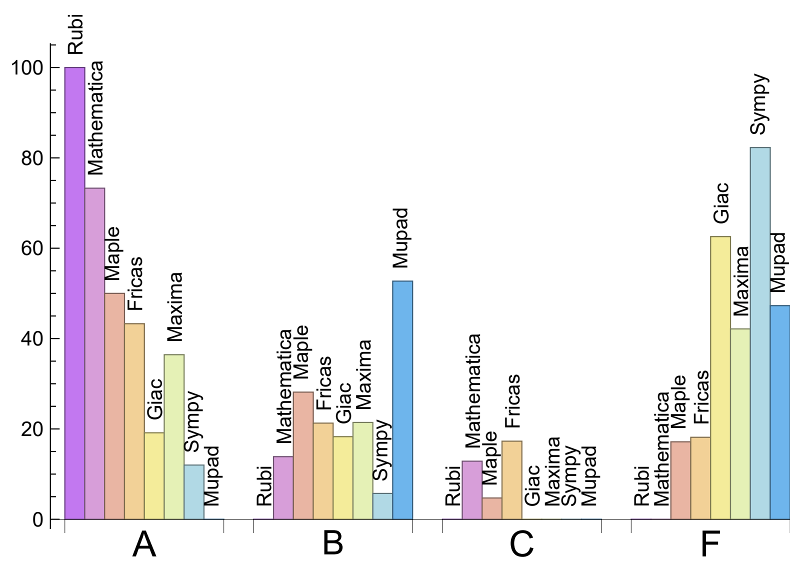
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	73.29	13.86	12.86	0.00
Maple	50.00	28.14	4.71	17.14
Fricas	43.29	21.29	17.29	18.14
Maxima	36.43	21.43	0.00	42.14
Giac	19.14	18.29	0.00	62.57
Sympy	12.00	5.71	0.00	82.29
Mupad	N/A	52.71	0.00	47.29

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	120	100.00 %	0.00 %	0.00 %
Fricas	127	77.95 %	20.47 %	1.57 %
Giac	438	97.26 %	2.05 %	0.68 %
Maxima	295	63.39 %	5.08 %	31.53 %
Sympy	576	72.57 %	15.10 %	12.33 %
Mupad	331	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

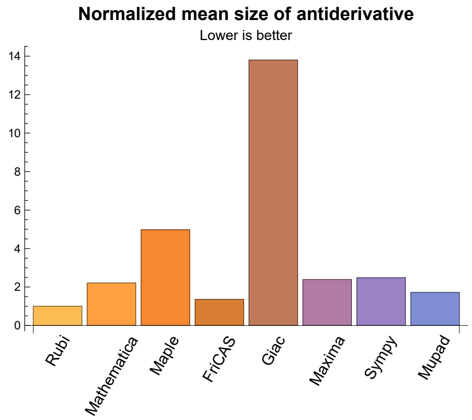
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

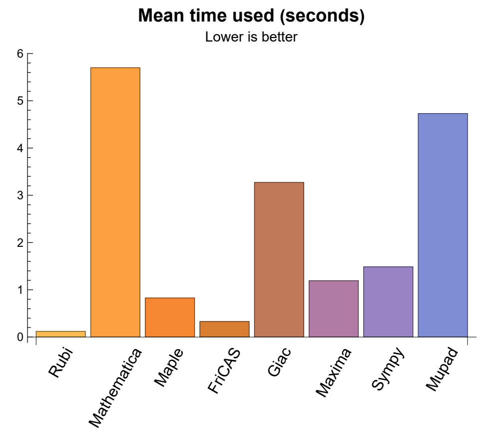
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	144.93	1.00	110.00	1.00
Mathematica	5.70	514.72	2.21	119.00	1.04
Maple	0.83	1784.75	4.98	175.50	1.54
Maxima	1.19	369.32	2.39	130.00	1.22
Fricas	0.33	157.25	1.36	118.00	1.13
Sympy	1.49	212.93	2.49	186.00	1.57
Giac	3.27	1448.48	13.80	151.00	1.77
Mupad	4.73	169.67	1.72	112.00	1.36

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {92, 93, 176, 177, 401, 403, 408, 410, 423, 431, 450, 462, 463, 476, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 626, 630, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 651, 652, 653, 654, 694, 696, 697, 698, 699, 700}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

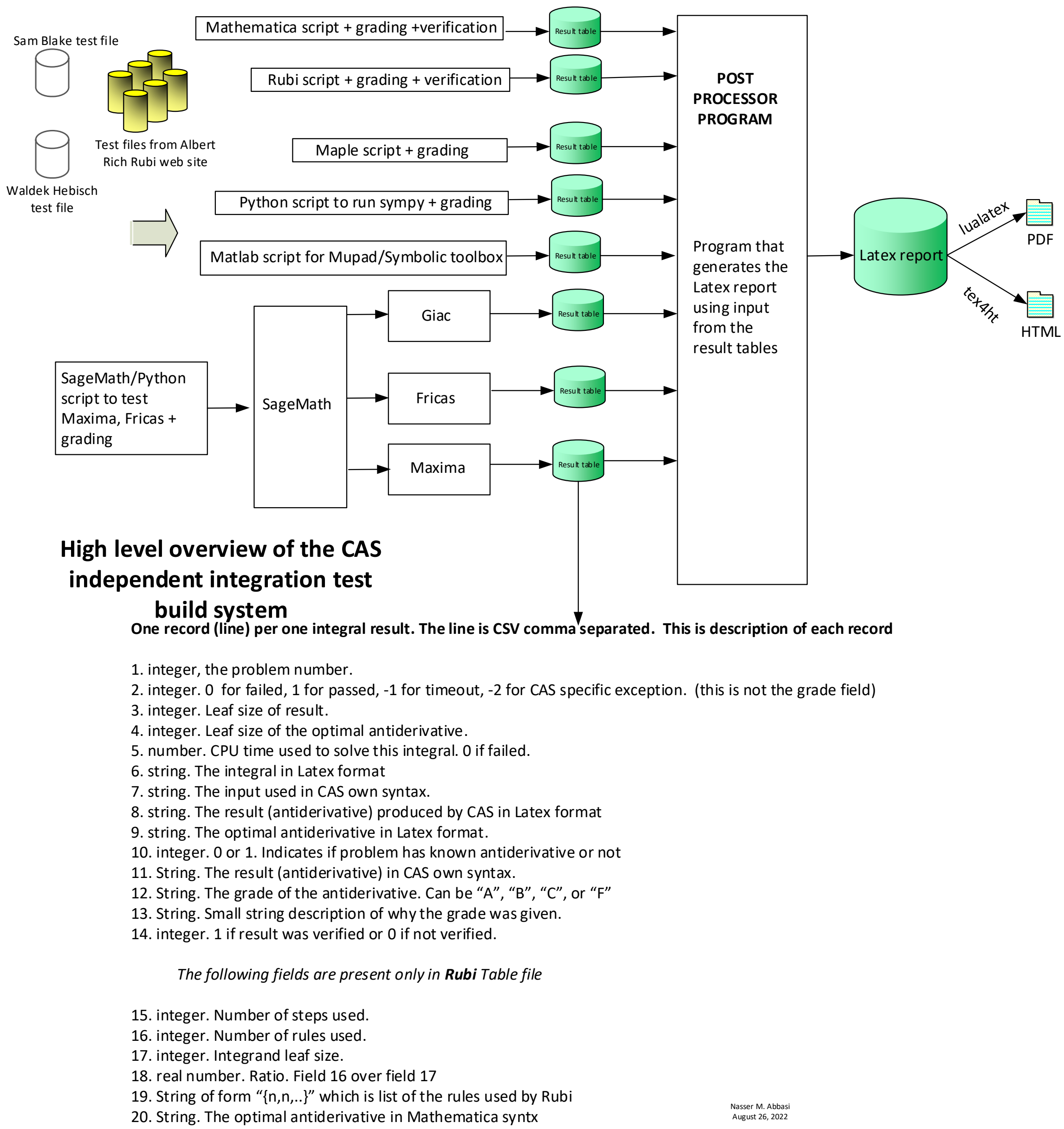
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 85, 86, 87, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 124, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 155, 156, 157, 158, 161, 162, 163, 164, 165, 169, 170, 171, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 402, 404, 405, 406, 407, 409, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 471, 472, 473, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 548, 550, 551, 552, 553, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 624, 625, 626, 627, 628, 629, 631, 640, 647, 648, 649, 650, 655, 657, 661, 663, 665, 667, 669, 671, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 693, 694, 695 }

B grade: { 22, 23, 29, 30, 31, 39, 41, 47, 54, 55, 61, 62, 64, 67, 77, 78, 79, 80, 82, 83, 84, 88, 92, 93, 94, 116, 122, 123, 125, 133, 149, 150, 152, 154, 159, 160, 166, 167, 168, 172, 176, 177, 267, 278, 296, 309, 322, 329, 401, 403, 408, 410, 423, 431, 450, 451, 454, 455, 456, 466, 467, 468, 469, 470, 474, 475, 497, 499, 501, 502, 503, 520, 522, 534, 535, 536, 537, 538, 539, 549, 554, 569, 570, 571, 573, 601, 603, 604, 607, 610, 630, 635, 646, 668, 670, 691, 692 }

C grade: { 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 547, 560, 561, 572, 574, 605, 606, 608, 609, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 636, 637, 638, 639, 641, 642, 643, 644, 645, 651, 652, 653, 654, 656, 658, 659, 660, 662, 664, 666, 672, 696, 697, 698, 699, 700 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 44, 45, 46, 47, 49, 50, 51, 53, 54, 55, 63, 71, 72, 81, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 190, 192, 194, 196, 198, 200, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 296, 300, 301, 302, 303, 306, 309, 313, 314, 315, 319, 322, 326, 327, 328, 333, 334, 335, 336, 339, 340, 341, 342, 347, 348, 349, 350, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 366, 368, 369, 370, 377, 378, 379, 380, 381, 382, 384, 385, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 575, 576, 577, 648, 655, 657, 658, 659, 665, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687 }

B grade: { 22, 24, 36, 37, 38, 39, 41, 42, 43, 48, 52, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 112, 113, 122, 128, 129, 147, 179, 185, 187, 188, 189, 191, 193, 195, 197, 199, 203, 205, 207, 209, 211, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 290, 291, 292, 295, 297, 298, 299, 304, 305, 307, 308, 310, 311, 312, 316, 317, 318, 320, 321, 323, 324, 325, 329, 330, 331, 332, 337, 338, 343, 344, 345, 346, 351, 357, 358, 359, 360, 367, 371, 372, 373, 374, 375, 376, 383, 386, 387, 388, 389, 390, 391, 392, 404, 412, 418, 423, 424, 425, 430, 431, 432, 520, 534, 549, 560, 572, 574, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 656, 660, 661, 662, 663, 664, 666, 684 }

C grade: { 465, 466, 483, 484, 485, 486, 487, 504, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602 }

F grade: { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 71, 72, 75, 76, 80, 81, 82, 83, 84, 89, 90, 99, 100, 101, 102, 103, 110, 114, 115, 116, 117, 118, 126, 130, 131, 132, 134, 135, 136, 143, 144, 148, 149, 151, 152, 154, 160, 161, 162, 166, 167, 168, 172, 178, 179, 180, 181, 182, 279, 280, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 335, 336, 337, 338, 339, 347, 348, 349, 350, 351, 352, 353, 362, 363, 364, 365, 366, 367, 368, 378, 379, 380, 381, 382, 383, 384, 397, 398, 399, 405, 406, 407, 413, 414, 415, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 483, 484, 485, 486, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 552, 554, 555, 556, 557, 558, 566, 567, 568, 569, 646, 647, 648, 673, 674, 675, 681, 682, 683, 685 }

B grade: { 42, 48, 56, 61, 66, 67, 73, 74, 77, 78, 79, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 122, 123, 124, 125, 133, 140, 141, 142, 150, 153, 158, 159, 169, 170, 171, 176, 177, 288, 290, 291, 292, 300, 301, 303, 304, 313, 315, 316, 317, 327, 328, 329, 330, 333, 334, 340, 341, 342, 343, 345, 346, 354, 355, 356, 357, 358, 360, 361, 369, 370, 371, 372, 373, 374, 376, 377, 385, 386, 387, 388, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 416, 417, 418, 423, 424, 425, 430, 431, 432, 443, 444, 445, 446, 447, 448, 449, 487, 491, 493, 495, 504, 505, 549, 550, 553, 559, 560, 561, 562, 563, 564, 565, 570, 571, 572, 573, 574, 575, 576, 577, 676, 677, 678, 679, 680, 684, 686, 687 }

C grade: { }

F grade: { 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 286, 287, 289, 302, 305, 314, 318, 326, 331, 332, 344, 359, 375, 389, 391, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 506, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.5 FriCAS

A grade: { 6, 7, 8, 9, 10, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 117, 118, 119, 120, 121, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 279, 280, 281, 284, 285, 286, 287, 288, 289, 292, 293, 299, 300, 301, 302, 303, 305, 312, 313, 314, 315, 318, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 368, 369, 370, 376, 377, 378, 379, 380, 381, 383, 384, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 483, 484, 485, 486, 491, 493, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 553, 558, 559, 564, 565, 647, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687 }

B grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 67, 70, 77, 78, 79, 80, 82, 88, 107, 108, 109, 114, 115, 116, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 166, 172, 282, 283, 290, 291, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 310, 311, 316, 317, 319, 320, 321, 322, 323, 324, 325, 329, 337, 344, 345, 350, 351, 357, 358, 359, 360, 366, 367, 371, 372, 373, 374, 375, 382, 385, 386, 387, 388, 389, 390, 391, 396, 404, 412, 418, 425, 432, 465, 466, 467, 487, 495, 504, 512, 520, 534, 545, 546, 550, 551, 552, 554, 555, 556, 557, 560, 561, 562, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 646, 648 }

C grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672 }

F grade: { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 23, 24, 25, 26, 27, 31, 32, 40, 41, 43, 44, 47, 48, 49, 50, 54, 55, 56, 57, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 110, 119, 120, 121, 137, 138, 139, 155, 156, 157, 163, 173, 174, 175, 183, 184, 486, 487, 507, 509, 511, 512 }

B grade: { 16, 17, 18, 33, 34, 35, 42, 51, 58, 67, 87, 88, 111, 112, 113, 118, 126, 127, 128, 129, 136, 143, 144, 145, 146, 147, 153, 154, 161, 162, 164, 165, 169, 170, 171, 172, 179, 180, 181, 182 }

C grade: { }

F grade: { 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 70, 77, 78, 79, 80, 99, 100, 101, 102, 103, 107, 108, 109, 114, 115, 116, 117, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 148, 149, 150, 151, 152, 158, 159, 160, 166, 167, 168, 176, 177, 178, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 508, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19, 20, 21, 22, 23, 24, 26, 30, 31, 36, 37, 38, 41, 45, 46, 52, 53, 59, 60, 64, 70, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 507, 509, 511, 517, 518, 519, 531, 532, 533, 544, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 576, 673, 674, 675, 676 }

B grade: { 11, 12, 13, 14, 15, 16, 17, 18, 25, 27, 28, 29, 32, 33, 34, 35, 39, 40, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 61, 62, 63, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 104, 112, 113, 117, 129, 133, 134, 135, 136, 150, 152, 154, 166, 170, 171, 172, 179, 282, 336, 467, 508, 510, 512, 513, 514, 515, 516, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 559, 560, 570, 571, 572, 573, 574, 575, 577 }

C grade: { }

F grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 188, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 316, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 362, 363, 364, 365, 366, 369, 370, 371, 372, 378, 379, 380, 382, 385, 386, 387, 397, 398, 399, 405, 406, 407, 412, 413, 414, 415, 418, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 446, 447, 448, 449, 465, 466, 467, 483, 484, 485, 486, 491, 493, 495, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 581, 648, 659, 673, 674, 675, 681, 682, 683 }

C grade: { }

F grade: { 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 381, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 416, 417, 423, 424, 425, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 676, 677, 678, 679, 680, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	69	114	189	83	114	106
N.S.	1	1.00	0.84	0.73	1.21	2.01	0.88	1.21	1.13
time (sec)	N/A	0.034	0.433	0.316	0.270	0.379	5.644	0.546	3.521

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	63	59	92	153	68	92	149
N.S.	1	1.00	0.84	0.79	1.23	2.04	0.91	1.23	1.99
time (sec)	N/A	0.034	0.131	0.249	0.275	0.353	3.660	0.586	3.269

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	49	70	117	60	70	112
N.S.	1	1.00	0.89	0.79	1.13	1.89	0.97	1.13	1.81
time (sec)	N/A	0.031	0.147	0.249	0.287	0.341	2.461	0.531	3.241

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	39	48	81	48	48	48
N.S.	1	1.00	0.93	0.85	1.04	1.76	1.04	1.04	1.04
time (sec)	N/A	0.030	0.058	0.286	0.280	0.353	1.751	0.582	3.244

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	30	30	26	21	45	37	26	23
N.S.	1	1.11	1.11	0.96	0.78	1.67	1.37	0.96	0.85
time (sec)	N/A	0.027	0.022	0.221	0.277	0.358	1.277	0.477	3.209

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	17	18	24	18	17
N.S.	1	1.00	1.00	1.21	0.89	0.95	1.26	0.95	0.89
time (sec)	N/A	0.008	0.011	0.029	0.277	0.362	0.075	0.470	3.257

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	48	42	38	23	39	23	22
N.S.	1	1.00	1.07	0.93	0.84	0.51	0.87	0.51	0.49
time (sec)	N/A	0.029	0.055	0.240	0.511	0.359	0.072	0.528	3.231

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	46	53	61	56	136	103	64
N.S.	1	1.00	0.69	0.79	0.91	0.84	2.03	1.54	0.96
time (sec)	N/A	0.036	0.057	0.286	0.506	0.351	0.145	0.472	3.336

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	56	63	82	80	211	127	108
N.S.	1	1.00	0.63	0.71	0.92	0.90	2.37	1.43	1.21
time (sec)	N/A	0.045	0.072	0.211	0.493	0.358	0.206	0.610	3.568

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	68	73	103	104	279	151	152
N.S.	1	1.00	0.61	0.66	0.93	0.94	2.51	1.36	1.37
time (sec)	N/A	0.057	0.140	0.219	0.508	0.376	0.290	0.546	4.848

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	61	74	106	372	0	181	247
N.S.	1	1.00	0.62	0.76	1.08	3.80	0.00	1.85	2.52
time (sec)	N/A	0.050	0.365	0.266	0.294	0.386	0.000	0.540	7.374

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	64	86	276	0	139	178
N.S.	1	1.00	0.92	0.84	1.13	3.63	0.00	1.83	2.34
time (sec)	N/A	0.038	0.197	0.222	0.286	0.372	0.000	0.547	6.903

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	61	180	0	97	107
N.S.	1	1.00	1.00	0.94	1.13	3.33	0.00	1.80	1.98
time (sec)	N/A	0.030	0.023	0.297	0.288	0.382	0.000	0.463	5.130

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	32	82	41	52	39
N.S.	1	1.00	1.00	1.26	1.19	3.04	1.52	1.93	1.44
time (sec)	N/A	0.014	0.017	0.082	0.284	0.383	2.440	0.488	3.345

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	24	22	15	26	84	20
N.S.	1	1.00	1.96	0.92	0.85	0.58	1.00	3.23	0.77
time (sec)	N/A	0.017	0.031	0.199	0.284	0.367	0.061	0.443	3.280

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	42	105	196	54
N.S.	1	1.00	1.00	0.80	0.78	0.91	2.28	4.26	1.17
time (sec)	N/A	0.026	0.017	0.266	0.277	0.368	0.145	0.500	3.413

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	47	49	66	184	220	70
N.S.	1	1.00	1.05	0.76	0.79	1.06	2.97	3.55	1.13
time (sec)	N/A	0.029	0.026	0.215	0.275	0.388	0.234	0.554	4.929

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	57	58	90	253	244	93
N.S.	1	1.00	1.07	0.75	0.76	1.18	3.33	3.21	1.22
time (sec)	N/A	0.031	0.060	0.226	0.275	0.390	0.306	0.629	6.073

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	141	108	189	0	108	151
N.S.	1	1.00	0.91	1.29	0.99	1.73	0.00	0.99	1.39
time (sec)	N/A	0.061	0.897	0.244	0.283	0.398	0.000	0.613	3.263

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	90	113	95	151	0	95	132
N.S.	1	1.00	1.10	1.38	1.16	1.84	0.00	1.16	1.61
time (sec)	N/A	0.044	0.570	0.241	0.279	0.355	0.000	0.566	3.244

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	77	85	56	113	0	56	56
N.S.	1	1.00	1.40	1.55	1.02	2.05	0.00	1.02	1.02
time (sec)	N/A	0.034	0.364	0.237	0.282	0.343	0.000	0.565	3.220

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	68	51	21	75	0	42	35
N.S.	1	1.00	2.52	1.89	0.78	2.78	0.00	1.56	1.30
time (sec)	N/A	0.029	0.315	0.267	0.271	0.333	0.000	0.520	3.221

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	100	40	41	56	53	66	29
N.S.	1	1.00	2.63	1.05	1.08	1.47	1.39	1.74	0.76
time (sec)	N/A	0.015	0.610	0.045	0.491	0.354	0.121	0.508	3.230

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	73	32	17	36	17	18
N.S.	1	1.00	1.24	2.92	1.28	0.68	1.44	0.68	0.72
time (sec)	N/A	0.035	0.085	0.238	0.485	0.360	0.097	0.564	3.276

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	86	100	67	41	87	257	50
N.S.	1	1.00	1.37	1.59	1.06	0.65	1.38	4.08	0.79
time (sec)	N/A	0.050	0.335	0.220	0.488	0.363	0.135	0.743	3.275

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	116	121	92	78	185	169	88
N.S.	1	1.00	0.99	1.03	0.79	0.67	1.58	1.44	0.75
time (sec)	N/A	0.065	0.436	0.228	0.490	0.362	0.214	0.757	3.373

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	138	141	115	106	270	342	144
N.S.	1	1.00	0.81	0.82	0.67	0.62	1.58	2.00	0.84
time (sec)	N/A	0.094	0.451	0.230	0.495	0.380	0.286	0.754	4.027

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	159	152	181	364	0	237	290
N.S.	1	1.00	1.35	1.29	1.53	3.08	0.00	2.01	2.46
time (sec)	N/A	0.075	0.865	0.256	0.272	0.362	0.000	0.641	7.153

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	215	121	130	256	0	173	198
N.S.	1	1.00	2.29	1.29	1.38	2.72	0.00	1.84	2.11
time (sec)	N/A	0.065	1.186	0.245	0.278	0.359	0.000	0.606	6.783

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	146	86	83	148	0	107	104
N.S.	1	1.00	2.15	1.26	1.22	2.18	0.00	1.57	1.53
time (sec)	N/A	0.032	0.825	0.143	0.278	0.378	0.000	0.534	3.819

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	180	56	61	52	68	56	41
N.S.	1	1.00	3.91	1.22	1.33	1.13	1.48	1.22	0.89
time (sec)	N/A	0.027	0.212	0.215	0.274	0.355	0.170	0.566	3.372

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	54	52	34	75	531	78
N.S.	1	1.00	0.98	1.06	1.02	0.67	1.47	10.41	1.53
time (sec)	N/A	0.032	0.183	0.225	0.286	0.341	0.141	0.636	3.357

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	91	79	62	153	613	71
N.S.	1	1.00	1.04	1.32	1.14	0.90	2.22	8.88	1.03
time (sec)	N/A	0.037	0.371	0.230	0.278	0.362	0.224	0.716	4.175

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	111	111	98	90	238	641	256
N.S.	1	1.00	1.28	1.28	1.13	1.03	2.74	7.37	2.94
time (sec)	N/A	0.039	0.384	0.225	0.283	0.364	0.347	0.773	3.663

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	133	131	119	118	314	669	330
N.S.	1	1.00	1.27	1.25	1.13	1.12	2.99	6.37	3.14
time (sec)	N/A	0.043	0.712	0.246	0.286	0.361	0.417	0.885	5.080

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	117	220	108	215	0	108	151
N.S.	1	1.00	1.07	2.02	0.99	1.97	0.00	0.99	1.39
time (sec)	N/A	0.051	1.173	0.269	0.284	0.366	0.000	0.655	3.312

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	174	108	177	0	108	151
N.S.	1	1.00	1.29	2.12	1.32	2.16	0.00	1.32	1.84
time (sec)	N/A	0.045	0.772	0.259	0.292	0.356	0.000	0.645	3.286

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	97	128	82	139	0	82	114
N.S.	1	1.00	1.76	2.33	1.49	2.53	0.00	1.49	2.07
time (sec)	N/A	0.035	0.663	0.237	0.273	0.347	0.000	0.637	3.264

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	84	73	21	101	0	56	56
N.S.	1	1.00	3.11	2.70	0.78	3.74	0.00	2.07	2.07
time (sec)	N/A	0.030	0.481	0.228	0.283	0.357	0.000	0.578	3.268

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	119	51	76	97	94	118	41
N.S.	1	1.00	1.89	0.81	1.21	1.54	1.49	1.87	0.65
time (sec)	N/A	0.027	0.798	0.038	0.492	0.376	0.176	0.462	3.276

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	99	99	62	36	61	36	39
N.S.	1	1.00	2.02	2.02	1.27	0.73	1.24	0.73	0.80
time (sec)	N/A	0.037	0.280	0.206	0.493	0.353	0.196	0.641	3.291

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	50	114	57	34	80	135	36
N.S.	1	1.00	1.85	4.22	2.11	1.26	2.96	5.00	1.33
time (sec)	N/A	0.029	0.203	0.204	0.496	0.351	0.178	0.734	3.335

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	109	156	105	55	131	457	77
N.S.	1	1.00	1.21	1.73	1.17	0.61	1.46	5.08	0.86
time (sec)	N/A	0.055	0.432	0.211	0.503	0.355	0.238	0.805	3.362

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	131	176	128	92	226	514	125
N.S.	1	1.00	0.91	1.22	0.89	0.64	1.57	3.57	0.87
time (sec)	N/A	0.074	0.528	0.227	0.496	0.372	0.341	0.909	3.651

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	200	155	310	0	189	228
N.S.	1	1.00	0.80	1.57	1.22	2.44	0.00	1.49	1.80
time (sec)	N/A	0.096	0.643	0.298	0.289	0.360	0.000	0.838	6.983

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	147	109	202	0	125	136
N.S.	1	1.00	0.94	1.48	1.10	2.04	0.00	1.26	1.37
time (sec)	N/A	0.058	0.451	0.151	0.282	0.367	0.000	0.662	5.230

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	123	97	82	107	107	234	102
N.S.	1	1.00	2.02	1.59	1.34	1.75	1.75	3.84	1.67
time (sec)	N/A	0.040	0.572	0.213	0.280	0.376	0.205	0.783	3.640

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	76	75	17	36	901	66
N.S.	1	1.00	0.97	2.38	2.34	0.53	1.12	28.16	2.06
time (sec)	N/A	0.027	0.107	0.193	0.273	0.364	0.135	0.903	3.331

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	126	105	48	116	929	130
N.S.	1	1.00	0.62	1.43	1.19	0.55	1.32	10.56	1.48
time (sec)	N/A	0.059	0.332	0.212	0.279	0.359	0.226	1.003	3.568

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	146	123	76	190	465	134
N.S.	1	1.00	0.73	1.38	1.16	0.72	1.79	4.39	1.26
time (sec)	N/A	0.065	0.548	0.225	0.274	0.375	0.346	0.995	4.544

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	166	145	104	275	1039	330
N.S.	1	1.00	0.94	1.34	1.17	0.84	2.22	8.38	2.66
time (sec)	N/A	0.064	0.617	0.241	0.275	0.376	0.459	0.874	4.676

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	171	293	246	364	0	237	290
N.S.	1	1.00	1.05	1.80	1.51	2.23	0.00	1.45	1.78
time (sec)	N/A	0.131	1.299	0.258	0.285	0.373	0.000	1.197	7.163

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	237	222	180	256	0	173	198
N.S.	1	1.00	1.78	1.67	1.35	1.92	0.00	1.30	1.49
time (sec)	N/A	0.077	1.230	0.184	0.286	0.382	0.000	0.987	6.774

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	906	154	137	162	153	372	159
N.S.	1	1.00	9.34	1.59	1.41	1.67	1.58	3.84	1.64
time (sec)	N/A	0.062	6.272	0.258	0.280	0.378	0.265	0.872	5.300

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	246	113	121	68	109	1299	88
N.S.	1	1.00	3.15	1.45	1.55	0.87	1.40	16.65	1.13
time (sec)	N/A	0.057	0.494	0.219	0.296	0.366	0.271	1.639	3.563

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	139	118	34	80	915	130
N.S.	1	1.00	0.76	2.11	1.79	0.52	1.21	13.86	1.97
time (sec)	N/A	0.054	0.267	0.242	0.280	0.357	0.244	1.210	3.547

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	203	149	62	156	1327	186
N.S.	1	1.00	0.72	1.99	1.46	0.61	1.53	13.01	1.82
time (sec)	N/A	0.070	0.496	0.247	0.282	0.365	0.360	0.914	4.266

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	233	181	90	228	1409	145
N.S.	1	1.00	0.92	1.94	1.51	0.75	1.90	11.74	1.21
time (sec)	N/A	0.072	0.583	0.251	0.287	0.363	0.423	0.917	4.693

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	167	377	160	267	0	160	146
N.S.	1	1.00	1.53	3.46	1.47	2.45	0.00	1.47	1.34
time (sec)	N/A	0.059	1.793	0.282	0.272	0.358	0.000	0.981	3.708

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	154	295	108	229	0	108	151
N.S.	1	1.00	1.88	3.60	1.32	2.79	0.00	1.32	1.84
time (sec)	N/A	0.049	1.330	0.272	0.277	0.357	0.000	0.906	3.326

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	143	213	108	191	0	108	151
N.S.	1	1.00	2.60	3.87	1.96	3.47	0.00	1.96	2.75
time (sec)	N/A	0.037	0.948	0.248	0.279	0.377	0.000	0.871	3.295

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	134	115	21	153	0	82	114
N.S.	1	1.00	4.96	4.26	0.78	5.67	0.00	3.04	4.22
time (sec)	N/A	0.028	0.885	0.239	0.278	0.339	0.000	0.858	3.258

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	228	72	165	177	178	222	73
N.S.	1	1.00	1.95	0.62	1.41	1.51	1.52	1.90	0.62
time (sec)	N/A	0.053	2.503	0.050	0.510	0.361	0.255	0.561	3.272

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	649	205	86	125	131	146	70
N.S.	1	1.00	7.82	2.47	1.04	1.51	1.58	1.76	0.84
time (sec)	N/A	0.051	6.308	0.263	0.494	0.376	0.258	0.955	3.302

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	110	188	88	51	102	450	64
N.S.	1	1.00	1.51	2.58	1.21	0.70	1.40	6.16	0.88
time (sec)	N/A	0.048	0.538	0.217	0.493	0.369	0.281	0.811	3.312

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	231	93	34	80	187	53
N.S.	1	1.00	0.91	4.20	1.69	0.62	1.45	3.40	0.96
time (sec)	N/A	0.037	0.340	0.235	0.492	0.353	0.289	0.832	3.281

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	73	301	103	62	162	267	63
N.S.	1	1.00	2.70	11.15	3.81	2.30	6.00	9.89	2.33
time (sec)	N/A	0.032	0.624	0.244	0.509	0.396	0.369	0.822	3.301

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	137	331	164	83	209	857	122
N.S.	1	1.00	0.95	2.30	1.14	0.58	1.45	5.95	0.85
time (sec)	N/A	0.071	0.875	0.298	0.510	0.402	0.443	0.942	3.637

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	159	361	187	120	301	914	171
N.S.	1	1.00	0.80	1.82	0.94	0.61	1.52	4.62	0.86
time (sec)	N/A	0.100	1.523	0.293	0.519	0.393	0.562	0.963	4.905

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	115	312	215	310	0	189	228
N.S.	1	1.00	0.69	1.87	1.29	1.86	0.00	1.13	1.37
time (sec)	N/A	0.104	0.845	0.296	0.285	0.352	0.000	0.858	7.043

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	151	226	173	216	197	510	222
N.S.	1	1.00	1.16	1.74	1.33	1.66	1.52	3.92	1.71
time (sec)	N/A	0.090	1.403	0.256	0.283	0.394	0.306	0.957	7.135

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	130	165	154	122	148	1683	162
N.S.	1	1.00	1.33	1.68	1.57	1.24	1.51	17.17	1.65
time (sec)	N/A	0.071	1.377	0.196	0.288	0.369	0.318	1.221	5.431

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	170	152	17	36	1669	104
N.S.	1	1.00	0.97	5.31	4.75	0.53	1.12	52.16	3.25
time (sec)	N/A	0.027	0.185	0.218	0.295	0.370	0.217	0.901	3.448

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	257	187	48	121	1697	186
N.S.	1	1.00	0.54	2.54	1.85	0.48	1.20	16.80	1.84
time (sec)	N/A	0.088	0.491	0.256	0.294	0.388	0.371	1.018	4.215

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	94	287	217	76	192	1725	79
N.S.	1	1.00	0.67	2.04	1.54	0.54	1.36	12.23	0.56
time (sec)	N/A	0.095	0.624	0.251	0.285	0.356	0.441	1.011	3.937

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	118	317	246	104	265	1807	139
N.S.	1	1.00	0.74	1.99	1.55	0.65	1.67	11.36	0.87
time (sec)	N/A	0.101	1.007	0.297	0.291	0.409	0.573	1.117	4.605

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	245	611	186	345	0	186	153
N.S.	1	1.00	2.25	5.61	1.71	3.17	0.00	1.71	1.40
time (sec)	N/A	0.067	3.576	0.323	0.285	0.376	0.000	1.285	4.828

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	234	475	173	307	0	173	190
N.S.	1	1.00	2.85	5.79	2.11	3.74	0.00	2.11	2.32
time (sec)	N/A	0.060	2.572	0.303	0.280	0.371	0.000	1.153	5.344

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	223	339	134	269	0	134	107
N.S.	1	1.00	4.05	6.16	2.44	4.89	0.00	2.44	1.95
time (sec)	N/A	0.037	1.975	0.297	0.295	0.358	0.000	1.133	4.223

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	212	180	21	231	0	120	83
N.S.	1	1.00	7.85	6.67	0.78	8.56	0.00	4.44	3.07
time (sec)	N/A	0.029	1.547	0.265	0.283	0.343	0.000	1.097	3.828

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	383	103	121	297	301	378	113
N.S.	1	1.00	1.92	0.52	0.60	1.48	1.50	1.89	0.56
time (sec)	N/A	0.116	2.515	0.063	0.506	0.353	0.409	0.703	3.402

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	321	482	124	245	257	302	102
N.S.	1	1.00	2.41	3.62	0.93	1.84	1.93	2.27	0.77
time (sec)	N/A	0.065	6.206	0.311	0.505	0.384	0.440	0.951	3.354

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	566	409	135	179	216	785	111
N.S.	1	1.00	4.56	3.30	1.09	1.44	1.74	6.33	0.90
time (sec)	N/A	0.066	1.966	0.220	0.502	0.362	0.447	1.121	3.375

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	414	398	146	113	172	799	103
N.S.	1	1.00	3.63	3.49	1.28	0.99	1.51	7.01	0.90
time (sec)	N/A	0.058	1.747	0.199	0.496	0.377	0.507	1.177	3.405

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	451	136	17	36	381	66
N.S.	1	1.00	0.72	10.49	3.16	0.40	0.84	8.86	1.53
time (sec)	N/A	0.033	0.350	0.209	0.508	0.371	0.437	1.168	3.482

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	55	588	152	48	121	409	82
N.S.	1	1.00	0.69	7.35	1.90	0.60	1.51	5.11	1.02
time (sec)	N/A	0.048	0.606	0.252	0.505	0.393	0.577	1.238	3.482

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	77	639	162	76	197	437	82
N.S.	1	1.00	1.40	11.62	2.95	1.38	3.58	7.95	1.49
time (sec)	N/A	0.040	0.799	0.249	0.533	0.398	0.667	2.054	3.396

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	116	689	171	104	279	465	105
N.S.	1	1.00	4.30	25.52	6.33	3.85	10.33	17.22	3.89
time (sec)	N/A	0.031	1.241	0.286	0.508	0.415	0.783	2.428	3.380

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	166	739	246	125	323	1457	195
N.S.	1	1.00	0.74	3.28	1.09	0.56	1.44	6.48	0.87
time (sec)	N/A	0.097	2.687	0.312	0.512	0.434	0.891	1.626	4.849

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	188	789	269	162	413	1514	231
N.S.	1	1.00	0.67	2.83	0.96	0.58	1.48	5.43	0.83
time (sec)	N/A	0.135	3.977	0.318	0.527	0.467	1.050	1.519	5.225

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	205	522	396	378	320	924	399
N.S.	1	1.00	0.87	2.22	1.69	1.61	1.36	3.93	1.70
time (sec)	N/A	0.173	1.702	0.270	0.296	0.422	0.509	1.535	8.298

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	1540	409	352	284	277	2835	343
N.S.	1	1.00	7.51	2.00	1.72	1.39	1.35	13.83	1.67
time (sec)	N/A	0.150	6.631	0.245	0.299	0.382	0.514	2.177	7.818

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	1162	357	326	190	235	2849	281
N.S.	1	1.00	6.72	2.06	1.88	1.10	1.36	16.47	1.62
time (sec)	N/A	0.134	6.511	0.238	0.290	0.371	0.513	1.454	7.389

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	305	369	309	96	187	2863	207
N.S.	1	1.00	2.01	2.43	2.03	0.63	1.23	18.84	1.36
time (sec)	N/A	0.124	2.049	0.231	0.293	0.394	0.568	1.555	6.694

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	447	302	34	80	2451	37
N.S.	1	1.00	0.76	6.77	4.58	0.52	1.21	37.14	0.56
time (sec)	N/A	0.058	0.395	0.240	0.301	0.365	0.554	1.606	3.622

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	73	567	355	62	162	2863	65
N.S.	1	1.00	0.54	4.17	2.61	0.46	1.19	21.05	0.48
time (sec)	N/A	0.122	0.793	0.229	0.298	0.388	0.663	1.798	3.802

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	111	617	405	90	240	2891	93
N.S.	1	1.00	0.53	2.92	1.92	0.43	1.14	13.70	0.44
time (sec)	N/A	0.204	0.991	0.287	0.296	0.433	0.794	1.917	4.080

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	133	667	453	118	313	2919	222
N.S.	1	1.00	0.63	3.15	2.14	0.56	1.48	13.77	1.05
time (sec)	N/A	0.157	1.814	0.303	0.301	0.425	0.935	1.806	5.259

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	71	93	87	146	0	87	92
N.S.	1	1.00	0.66	0.87	0.81	1.36	0.00	0.81	0.86
time (sec)	N/A	0.056	0.527	0.303	0.287	0.378	0.000	0.521	3.530

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	72	67	109	0	67	114
N.S.	1	1.00	0.75	0.90	0.84	1.36	0.00	0.84	1.42
time (sec)	N/A	0.049	0.399	0.250	0.280	0.368	0.000	0.575	3.320

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	51	47	72	0	47	77
N.S.	1	1.00	0.89	0.93	0.85	1.31	0.00	0.85	1.40
time (sec)	N/A	0.044	0.242	0.248	0.278	0.392	0.000	0.526	3.342

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	30	27	33	0	27	25
N.S.	1	1.00	1.03	0.88	0.79	0.97	0.00	0.79	0.74
time (sec)	N/A	0.033	0.236	0.722	0.291	0.355	0.000	0.495	3.331

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	23	20	26	0	57	19
N.S.	1	1.00	1.35	1.00	0.87	1.13	0.00	2.48	0.83
time (sec)	N/A	0.032	0.140	0.355	0.284	0.361	0.000	0.517	3.354

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	48	0	32	60	60	29
N.S.	1	1.00	1.36	1.45	0.00	0.97	1.82	1.82	0.88
time (sec)	N/A	0.011	0.134	0.088	0.000	0.357	0.082	0.467	3.363

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	75	0	54	151	99	60
N.S.	1	1.00	0.95	0.91	0.00	0.66	1.84	1.21	0.73
time (sec)	N/A	0.057	0.330	0.418	0.000	0.346	0.160	0.541	3.445

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	109	102	0	76	219	116	123
N.S.	1	1.00	0.81	0.76	0.00	0.57	1.63	0.87	0.92
time (sec)	N/A	0.075	0.292	0.730	0.000	0.355	0.221	0.511	3.722

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	206	288	266	0	138	193
N.S.	1	1.00	0.71	2.45	3.43	3.17	0.00	1.64	2.30
time (sec)	N/A	0.057	0.434	0.305	0.303	0.398	0.000	0.576	7.005

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	138	186	174	0	99	116
N.S.	1	1.00	0.83	2.30	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.044	0.279	0.296	0.289	0.355	0.000	0.545	5.377

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	70	83	80	0	58	43
N.S.	1	1.00	1.10	2.26	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.036	0.186	0.532	0.296	0.370	0.000	0.559	3.441

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	23	29	17	34	21	25
N.S.	1	1.00	0.89	0.82	1.04	0.61	1.21	0.75	0.89
time (sec)	N/A	0.019	0.036	0.163	0.287	0.430	0.349	0.538	3.351

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	75	0	41	126	67	78
N.S.	1	1.00	1.06	1.60	0.00	0.87	2.68	1.43	1.66
time (sec)	N/A	0.031	0.154	0.348	0.000	0.343	0.151	0.492	3.549

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	72	141	0	63	196	119	134
N.S.	1	1.00	1.07	2.10	0.00	0.94	2.93	1.78	2.00
time (sec)	N/A	0.042	0.212	0.600	0.000	0.349	0.239	0.515	4.789

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	94	207	0	85	264	171	188
N.S.	1	1.00	1.11	2.44	0.00	1.00	3.11	2.01	2.21
time (sec)	N/A	0.045	0.250	0.295	0.000	0.361	0.333	0.504	7.097

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	90	78	77	138	0	77	93
N.S.	1	1.00	1.10	0.95	0.94	1.68	0.00	0.94	1.13
time (sec)	N/A	0.051	0.595	0.289	0.285	0.354	0.000	0.572	3.461

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	77	47	47	97	0	47	77
N.S.	1	1.00	1.40	0.85	0.85	1.76	0.00	0.85	1.40
time (sec)	N/A	0.040	0.453	0.263	0.279	0.369	0.000	0.568	3.360

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	68	20	35	54	0	35	33
N.S.	1	1.00	2.52	0.74	1.30	2.00	0.00	1.30	1.22
time (sec)	N/A	0.032	0.286	0.253	0.282	0.359	0.000	0.575	3.341

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	71	30	32	70	0	100	28
N.S.	1	1.00	1.87	0.79	0.84	1.84	0.00	2.63	0.74
time (sec)	N/A	0.037	0.487	0.272	0.280	0.381	0.000	0.587	3.350

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	32	24	21	17	65	30	22
N.S.	1	1.00	1.23	0.92	0.81	0.65	2.50	1.15	0.85
time (sec)	N/A	0.032	0.065	0.612	0.285	0.371	0.554	0.555	3.335

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	68	62	0	43	117	72	39
N.S.	1	1.00	1.11	1.02	0.00	0.70	1.92	1.18	0.64
time (sec)	N/A	0.022	0.247	0.096	0.000	0.374	0.131	0.471	3.401

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	95	88	0	65	189	103	71
N.S.	1	1.00	0.83	0.77	0.00	0.57	1.66	0.90	0.62
time (sec)	N/A	0.066	0.337	0.820	0.000	0.374	0.212	0.569	3.467

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	120	116	0	87	258	127	149
N.S.	1	1.00	0.73	0.70	0.00	0.53	1.56	0.77	0.90
time (sec)	N/A	0.083	0.372	0.267	0.000	0.355	0.270	0.587	4.183

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	294	238	421	326	0	203	191
N.S.	1	1.00	2.37	1.92	3.40	2.63	0.00	1.64	1.54
time (sec)	N/A	0.068	2.296	0.270	0.309	0.429	0.000	0.594	6.309

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	215	170	295	230	0	151	136
N.S.	1	1.00	2.15	1.70	2.95	2.30	0.00	1.51	1.36
time (sec)	N/A	0.056	1.191	0.259	0.299	0.377	0.000	0.845	5.924

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	146	102	167	134	0	95	104
N.S.	1	1.00	1.97	1.38	2.26	1.81	0.00	1.28	1.41
time (sec)	N/A	0.045	0.524	0.245	0.325	0.374	0.000	0.603	3.927

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	184	54	117	64	0	57	44
N.S.	1	1.00	3.83	1.12	2.44	1.33	0.00	1.19	0.92
time (sec)	N/A	0.038	0.223	0.263	0.517	0.385	0.000	0.624	3.498

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	57	45	30	112	47	79
N.S.	1	1.00	0.58	0.88	0.69	0.46	1.72	0.72	1.22
time (sec)	N/A	0.038	0.127	0.365	0.297	0.351	0.556	0.632	3.507

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	68	108	0	52	163	93	90
N.S.	1	1.00	0.96	1.52	0.00	0.73	2.30	1.31	1.27
time (sec)	N/A	0.036	0.375	0.468	0.000	0.356	0.224	0.679	3.892

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	95	174	0	74	231	145	161
N.S.	1	1.00	1.07	1.96	0.00	0.83	2.60	1.63	1.81
time (sec)	N/A	0.048	0.277	0.260	0.000	0.383	0.319	0.638	6.873

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	240	0	96	299	197	216
N.S.	1	1.00	1.09	2.24	0.00	0.90	2.79	1.84	2.02
time (sec)	N/A	0.049	0.533	0.267	0.000	0.355	0.411	0.778	5.650

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	117	91	87	194	0	87	119
N.S.	1	1.00	1.07	0.83	0.80	1.78	0.00	0.80	1.09
time (sec)	N/A	0.061	1.075	0.301	0.300	0.371	0.000	0.719	3.605

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	93	87	153	0	87	103
N.S.	1	1.00	1.29	1.13	1.06	1.87	0.00	1.06	1.26
time (sec)	N/A	0.051	0.711	0.300	0.285	0.371	0.000	0.813	3.487

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	97	72	67	112	0	67	114
N.S.	1	1.00	1.76	1.31	1.22	2.04	0.00	1.22	2.07
time (sec)	N/A	0.044	0.656	0.291	0.286	0.382	0.000	1.008	3.342

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	84	21	47	69	0	47	77
N.S.	1	1.00	3.11	0.78	1.74	2.56	0.00	1.74	2.85
time (sec)	N/A	0.031	0.520	0.280	0.293	0.357	0.000	0.747	3.340

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	113	41	45	113	0	128	41
N.S.	1	1.00	1.95	0.71	0.78	1.95	0.00	2.21	0.71
time (sec)	N/A	0.038	0.530	0.296	0.292	0.366	0.000	0.870	3.371

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	88	35	66	55	0	100	42
N.S.	1	1.00	1.76	0.70	1.32	1.10	0.00	2.00	0.84
time (sec)	N/A	0.039	0.292	0.296	0.290	0.371	0.000	0.968	3.407

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	24	21	30	153	57	20
N.S.	1	1.00	1.56	0.89	0.78	1.11	5.67	2.11	0.74
time (sec)	N/A	0.033	0.119	0.186	0.287	0.348	0.837	0.914	3.350

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	75	0	54	155	80	50
N.S.	1	1.00	1.06	0.85	0.00	0.61	1.76	0.91	0.57
time (sec)	N/A	0.038	0.304	0.120	0.000	0.368	0.164	0.671	3.490

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	115	102	0	76	224	119	124
N.S.	1	1.00	0.82	0.72	0.00	0.54	1.59	0.84	0.88
time (sec)	N/A	0.073	0.340	0.289	0.000	0.362	0.281	0.807	3.678

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	137	129	0	98	292	136	173
N.S.	1	1.00	0.70	0.66	0.00	0.50	1.50	0.70	0.89
time (sec)	N/A	0.100	0.595	0.269	0.000	0.367	0.315	0.741	5.040

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	206	341	278	0	164	150
N.S.	1	1.00	0.95	1.73	2.87	2.34	0.00	1.38	1.26
time (sec)	N/A	0.090	0.537	0.297	0.311	0.372	0.000	0.742	6.079

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	63	138	215	182	0	112	135
N.S.	1	1.00	0.68	1.48	2.31	1.96	0.00	1.20	1.45
time (sec)	N/A	0.077	0.491	0.284	0.302	0.379	0.000	0.724	5.443

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	108	86	319	112	0	110	105
N.S.	1	1.00	1.66	1.32	4.91	1.72	0.00	1.69	1.62
time (sec)	N/A	0.064	0.384	0.283	0.516	0.408	0.000	0.732	3.770

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	57	29	17	80	36	68
N.S.	1	1.00	1.00	1.78	0.91	0.53	2.50	1.12	2.12
time (sec)	N/A	0.030	0.074	0.306	0.302	0.364	0.898	0.660	3.444

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	54	90	69	41	219	73	133
N.S.	1	1.00	0.55	0.92	0.70	0.42	2.23	0.74	1.36
time (sec)	N/A	0.064	0.229	0.151	0.309	0.379	0.912	0.590	3.680

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	141	0	63	197	119	134
N.S.	1	1.00	0.75	1.40	0.00	0.62	1.95	1.18	1.33
time (sec)	N/A	0.066	0.261	0.307	0.000	0.364	0.306	0.772	5.899

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	98	207	0	85	265	171	188
N.S.	1	1.00	0.81	1.71	0.00	0.70	2.19	1.41	1.55
time (sec)	N/A	0.084	0.329	0.293	0.000	0.369	0.384	0.750	6.609

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	120	273	0	107	333	223	136
N.S.	1	1.00	0.86	1.96	0.00	0.77	2.40	1.60	0.98
time (sec)	N/A	0.093	0.760	0.296	0.000	0.372	0.532	0.806	5.517

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	136	99	97	168	0	97	120
N.S.	1	1.00	1.66	1.21	1.18	2.05	0.00	1.18	1.46
time (sec)	N/A	0.052	0.870	0.301	0.299	0.393	0.000	0.695	3.517

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	127	67	67	127	0	67	113
N.S.	1	1.00	2.31	1.22	1.22	2.31	0.00	1.22	2.05
time (sec)	N/A	0.038	0.564	0.277	0.295	0.369	0.000	0.897	3.336

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	116	20	55	84	0	55	93
N.S.	1	1.00	4.30	0.74	2.04	3.11	0.00	2.04	3.44
time (sec)	N/A	0.030	0.494	0.273	0.297	0.372	0.000	0.876	3.389

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	168	51	53	156	0	154	60
N.S.	1	1.00	1.87	0.57	0.59	1.73	0.00	1.71	0.67
time (sec)	N/A	0.043	0.842	0.293	0.299	0.392	0.000	0.832	3.385

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	214	41	95	102	0	146	55
N.S.	1	1.00	3.40	0.65	1.51	1.62	0.00	2.32	0.87
time (sec)	N/A	0.041	0.792	0.288	0.296	0.399	0.000	0.775	3.357

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	32	36	66	17	95	44	25
N.S.	1	1.00	1.10	1.24	2.28	0.59	3.28	1.52	0.86
time (sec)	N/A	0.031	0.070	0.311	0.291	0.378	1.369	0.766	3.388

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	56	24	21	41	272	85	19
N.S.	1	1.00	2.07	0.89	0.78	1.52	10.07	3.15	0.70
time (sec)	N/A	0.035	0.225	0.215	0.292	0.413	1.547	0.733	3.398

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	89	0	65	189	92	60
N.S.	1	1.00	0.84	0.77	0.00	0.56	1.63	0.79	0.52
time (sec)	N/A	0.052	0.273	0.163	0.000	0.381	0.235	0.517	3.502

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	120	115	0	87	258	123	90
N.S.	1	1.00	0.71	0.68	0.00	0.51	1.53	0.73	0.53
time (sec)	N/A	0.081	0.376	0.288	0.000	0.421	0.327	0.855	4.132

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	142	143	0	109	326	147	197
N.S.	1	1.00	0.63	0.64	0.00	0.49	1.46	0.66	0.88
time (sec)	N/A	0.106	0.805	0.295	0.000	0.388	0.423	0.853	4.939

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	237	170	295	230	0	151	197
N.S.	1	1.00	1.78	1.28	2.22	1.73	0.00	1.14	1.48
time (sec)	N/A	0.096	1.312	0.298	0.313	0.372	0.000	0.838	6.874

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	988	118	457	160	0	113	162
N.S.	1	1.00	9.23	1.10	4.27	1.50	0.00	1.06	1.51
time (sec)	N/A	0.083	6.233	0.285	0.539	0.387	0.000	0.776	5.518

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	247	71	141	76	0	71	88
N.S.	1	1.00	3.01	0.87	1.72	0.93	0.00	0.87	1.07
time (sec)	N/A	0.070	0.361	0.312	0.533	0.428	0.000	0.729	3.675

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	90	53	30	182	73	133
N.S.	1	1.00	0.59	1.32	0.78	0.44	2.68	1.07	1.96
time (sec)	N/A	0.063	0.123	0.309	0.304	0.399	1.378	0.733	3.658

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	73	123	91	52	354	99	64
N.S.	1	1.00	0.55	0.93	0.69	0.39	2.68	0.75	0.48
time (sec)	N/A	0.089	0.276	0.148	0.309	0.374	1.402	0.665	3.740

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	95	174	0	74	231	145	161
N.S.	1	1.00	0.71	1.30	0.00	0.55	1.72	1.08	1.20
time (sec)	N/A	0.096	0.280	0.306	0.000	0.393	0.348	0.818	7.413

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	117	240	0	96	299	197	216
N.S.	1	1.00	0.75	1.54	0.00	0.62	1.92	1.26	1.38
time (sec)	N/A	0.117	0.549	0.299	0.000	0.371	0.457	0.863	5.611

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	139	306	0	118	367	249	262
N.S.	1	1.00	0.80	1.76	0.00	0.68	2.11	1.43	1.51
time (sec)	N/A	0.120	0.968	0.315	0.000	0.376	0.554	0.859	6.967

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	599	85	229	273	0	250	105
N.S.	1	1.00	4.47	0.63	1.71	2.04	0.00	1.87	0.78
time (sec)	N/A	0.069	3.055	0.371	0.311	0.482	0.000	1.162	3.449

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	537	78	212	199	0	224	114
N.S.	1	1.00	4.26	0.62	1.68	1.58	0.00	1.78	0.90
time (sec)	N/A	0.063	1.655	0.338	0.317	0.399	0.000	1.844	3.440

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	397	68	191	124	0	199	104
N.S.	1	1.00	3.42	0.59	1.65	1.07	0.00	1.72	0.90
time (sec)	N/A	0.055	1.123	0.327	0.310	0.400	0.000	1.674	3.498

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	63	158	17	160	70	73
N.S.	1	1.00	0.74	1.47	3.67	0.40	3.72	1.63	1.70
time (sec)	N/A	0.036	0.092	0.337	0.306	0.387	10.732	1.616	3.507

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	49	141	41	466	137	85
N.S.	1	1.00	0.69	0.60	1.74	0.51	5.75	1.69	1.05
time (sec)	N/A	0.045	0.308	0.330	0.295	0.362	10.851	1.460	3.499

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	78	36	121	63	774	163	85
N.S.	1	1.00	1.42	0.65	2.20	1.15	14.07	2.96	1.55
time (sec)	N/A	0.041	0.303	0.341	0.288	0.365	10.901	1.374	3.503

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	100	24	21	85	1081	189	19
N.S.	1	1.00	3.70	0.89	0.78	3.15	40.04	7.00	0.70
time (sec)	N/A	0.030	0.330	0.212	0.299	0.379	11.013	1.245	3.562

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	148	143	0	109	325	132	198
N.S.	1	1.00	0.65	0.62	0.00	0.48	1.42	0.58	0.86
time (sec)	N/A	0.129	0.781	0.324	0.000	0.424	0.447	0.877	4.956

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	170	169	0	131	394	163	235
N.S.	1	1.00	0.61	0.61	0.00	0.47	1.42	0.59	0.85
time (sec)	N/A	0.129	1.280	0.293	0.000	0.364	0.493	1.281	5.292

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	192	197	0	153	462	188	294
N.S.	1	1.00	0.58	0.59	0.00	0.46	1.39	0.56	0.88
time (sec)	N/A	0.154	1.799	0.303	0.000	0.380	0.632	1.333	5.704

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	1704	219	786	267	0	195	344
N.S.	1	1.00	8.31	1.07	3.83	1.30	0.00	0.95	1.68
time (sec)	N/A	0.179	6.476	0.368	0.640	0.428	0.000	2.175	7.566

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	1244	184	531	182	0	165	284
N.S.	1	1.00	6.80	1.01	2.90	0.99	0.00	0.90	1.55
time (sec)	N/A	0.157	6.305	0.341	0.573	0.410	0.000	2.361	7.438

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	304	137	185	98	0	123	207
N.S.	1	1.00	1.95	0.88	1.19	0.63	0.00	0.79	1.33
time (sec)	N/A	0.143	1.130	0.346	0.560	0.384	0.000	1.594	6.936

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	156	53	30	311	125	37
N.S.	1	1.00	0.59	2.29	0.78	0.44	4.57	1.84	0.54
time (sec)	N/A	0.062	0.153	0.339	0.325	0.376	11.310	1.414	3.745

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	73	189	97	52	620	151	64
N.S.	1	1.00	0.53	1.37	0.70	0.38	4.49	1.09	0.46
time (sec)	N/A	0.131	0.371	0.344	0.320	0.358	11.004	1.434	3.915

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	95	222	141	74	928	177	159
N.S.	1	1.00	0.45	1.04	0.66	0.35	4.36	0.83	0.75
time (sec)	N/A	0.208	0.364	0.344	0.327	0.383	10.910	1.276	4.222

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	117	255	179	96	1221	203	224
N.S.	1	1.00	0.43	0.95	0.67	0.36	4.54	0.75	0.83
time (sec)	N/A	0.207	0.640	0.153	0.320	0.399	11.212	1.238	5.360

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	139	306	0	118	367	249	262
N.S.	1	1.00	0.51	1.13	0.00	0.44	1.35	0.92	0.97
time (sec)	N/A	0.250	1.197	0.307	0.000	0.366	0.602	1.251	6.665

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	161	372	0	140	435	301	308
N.S.	1	1.00	0.53	1.24	0.00	0.47	1.45	1.00	1.02
time (sec)	N/A	0.281	1.595	0.305	0.000	0.356	0.719	1.230	9.524

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	156	365	0	195	0	0	-1
N.S.	1	1.00	1.27	2.97	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.075	2.161	6.696	0.000	0.113	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	57	192	0	143	0	0	-1
N.S.	1	1.00	0.61	2.04	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.459	0.463	0.000	0.088	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	102	351	0	113	0	0	-1
N.S.	1	1.00	1.13	3.90	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.816	0.523	0.000	0.087	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	164	0	55	0	0	40
N.S.	1	1.00	0.73	2.73	0.00	0.92	0.00	0.00	0.67
time (sec)	N/A	0.037	0.215	0.924	0.000	0.108	0.000	0.000	3.784

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	73	910	0	26	0	0	-1
N.S.	1	1.00	1.22	15.17	0.00	0.43	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.335	0.586	0.000	0.098	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	62	170	0	68	0	0	-1
N.S.	1	1.00	0.65	1.77	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.398	0.476	0.000	0.096	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	99	339	0	100	0	0	-1
N.S.	1	1.00	1.03	3.53	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.783	0.465	0.000	0.111	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	187	0	109	0	0	-1
N.S.	1	1.00	0.97	1.50	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.742	0.484	0.000	0.098	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	267	374	0	166	0	0	-1
N.S.	1	1.00	1.93	2.71	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.095	2.570	0.489	0.000	0.121	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	67	201	0	110	0	0	-1
N.S.	1	1.00	0.63	1.90	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.561	0.487	0.000	0.108	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	132	1099	0	61	0	0	-1
N.S.	1	1.00	1.23	10.27	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.063	1.007	0.509	0.000	0.097	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	173	0	74	0	0	-1
N.S.	1	1.00	1.34	2.04	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.432	0.471	0.000	0.101	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	343	0	86	0	0	-1
N.S.	1	1.00	1.34	4.04	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.965	0.449	0.000	0.142	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	188	0	88	0	0	-1
N.S.	1	1.00	1.15	1.62	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.813	0.897	0.000	0.094	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	351	0	122	0	0	-1
N.S.	1	1.00	1.15	3.03	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.073	1.759	0.532	0.000	0.100	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	155	205	0	133	0	0	-1
N.S.	1	1.00	1.05	1.39	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.208	0.582	0.000	0.103	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	442	402	0	301	0	0	-1
N.S.	1	1.00	2.19	1.99	0.00	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.155	6.919	0.608	0.000	0.093	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	89	229	0	245	0	0	-1
N.S.	1	1.00	0.51	1.31	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.140	1.217	0.549	0.000	0.099	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	129	392	0	211	0	0	-1
N.S.	1	1.00	0.74	2.24	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.312	0.540	0.000	0.096	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	79	213	0	155	0	0	-1
N.S.	1	1.00	0.57	1.53	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.084	0.574	0.000	0.105	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	146	101	1564	0	120	0	0	-1
N.S.	1	1.18	0.81	12.61	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.106	1.567	0.558	0.000	0.104	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	123	175	0	74	0	0	-1
N.S.	1	1.00	1.11	1.58	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.552	0.530	0.000	0.102	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	108	1086	0	86	0	0	-1
N.S.	1	1.00	0.97	9.78	0.00	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.280	0.547	0.000	0.103	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	133	199	0	88	0	0	-1
N.S.	1	1.00	1.07	1.60	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.823	0.496	0.000	0.095	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	118	370	0	100	0	0	-1
N.S.	1	1.00	0.95	2.98	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.105	2.489	0.559	0.000	0.126	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	148	216	0	102	0	0	-1
N.S.	1	1.00	0.95	1.39	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.147	0.976	0.000	0.098	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	155	380	0	136	0	0	-1
N.S.	1	1.00	1.00	2.45	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.113	6.499	0.681	0.000	0.104	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	170	232	0	147	0	0	-1
N.S.	1	1.00	0.91	1.25	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.148	1.708	0.696	0.000	0.114	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	429	401	0	256	0	0	-1
N.S.	1	1.00	2.00	1.87	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.188	6.886	0.562	0.000	0.121	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	101	230	0	200	0	0	-1
N.S.	1	1.00	0.55	1.26	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.148	1.018	0.575	0.000	0.108	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	123	1618	0	164	0	0	-1
N.S.	1	1.00	0.69	9.09	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.133	3.991	0.587	0.000	0.102	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	198	0	122	0	0	-1
N.S.	1	1.00	0.89	1.36	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.961	0.551	0.000	0.094	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	110	3762	0	86	0	0	-1
N.S.	1	1.00	0.71	24.12	0.00	0.55	0.00	0.00	-0.01
time (sec)	N/A	0.106	2.556	0.681	0.000	0.096	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	133	200	0	88	0	0	-1
N.S.	1	1.00	1.06	1.60	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.937	0.533	0.000	0.096	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	108	370	0	100	0	0	-1
N.S.	1	1.00	0.86	2.96	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.097	3.786	0.991	0.000	0.094	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	148	215	0	102	0	0	-1
N.S.	1	1.00	0.95	1.38	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.247	0.587	0.000	0.096	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	450	380	0	114	0	0	-1
N.S.	1	1.00	2.88	2.44	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.115	6.887	0.645	0.000	0.095	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	155	232	0	116	0	0	-1
N.S.	1	1.00	0.83	1.24	0.00	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.142	2.072	0.669	0.000	0.103	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	375	0	192	0	0	-1
N.S.	1	1.00	0.94	2.76	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.575	0.676	0.000	0.101	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	202	0	141	0	0	-1
N.S.	1	1.00	0.59	1.92	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.640	0.648	0.000	0.097	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	361	0	112	0	0	-1
N.S.	1	1.00	1.01	3.57	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.837	0.639	0.000	0.100	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	174	0	56	0	0	-1
N.S.	1	1.00	0.70	2.49	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.343	0.644	0.000	0.089	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	347	0	91	0	0	-1
N.S.	1	1.00	1.06	4.96	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.384	4.614	0.000	0.100	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	164	0	88	0	0	-1
N.S.	1	1.00	1.04	2.05	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.309	0.579	0.000	0.090	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	109	358	0	98	0	0	-1
N.S.	1	1.00	1.36	4.48	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.935	3.332	0.000	0.151	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	125	218	0	106	0	0	-1
N.S.	1	1.00	1.10	1.91	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.588	0.767	0.000	0.098	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	134	376	0	120	0	0	-1
N.S.	1	1.00	1.18	3.30	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.162	1.126	0.000	0.106	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	142	236	0	128	0	0	-1
N.S.	1	1.00	0.98	1.63	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.851	1.444	0.000	0.116	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	302	384	0	242	0	0	-1
N.S.	1	1.00	1.65	2.10	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.112	2.729	0.732	0.000	0.131	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	85	219	0	189	0	0	-1
N.S.	1	1.00	0.56	1.44	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.646	0.686	0.000	0.114	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	374	0	158	0	0	-1
N.S.	1	1.00	0.81	2.46	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.091	1.174	0.704	0.000	0.100	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	67	201	0	105	0	0	-1
N.S.	1	1.00	0.56	1.69	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.432	0.665	0.000	0.108	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	352	0	91	0	0	-1
N.S.	1	1.00	0.70	3.06	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.593	0.703	0.000	0.104	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	101	201	0	88	0	0	-1
N.S.	1	1.00	1.12	2.23	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.465	0.707	0.000	0.089	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	102	357	0	103	0	0	-1
N.S.	1	1.00	1.13	3.97	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.641	0.694	0.000	0.096	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	180	0	100	0	0	-1
N.S.	1	1.00	0.97	1.55	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.515	0.983	0.000	0.106	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	368	0	109	0	0	-1
N.S.	1	1.00	1.06	3.17	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.070	1.579	0.875	0.000	0.116	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	134	234	0	117	0	0	-1
N.S.	1	1.00	0.89	1.56	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.670	0.900	0.000	0.107	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	149	386	0	131	0	0	-1
N.S.	1	1.00	0.99	2.57	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.088	2.456	1.122	0.000	0.110	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	151	252	0	139	0	0	-1
N.S.	1	1.00	0.83	1.39	0.00	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.106	1.089	1.269	0.000	0.126	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	128	392	0	200	0	0	-1
N.S.	1	1.00	0.72	2.20	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.145	1.866	0.775	0.000	0.099	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	74	213	0	147	0	0	-1
N.S.	1	1.00	0.52	1.51	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.749	0.761	0.000	0.105	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	93	388	0	137	0	0	-1
N.S.	1	1.00	0.66	2.75	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.123	1.144	0.811	0.000	0.099	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	125	203	0	88	0	0	-1
N.S.	1	1.00	1.08	1.75	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.500	0.781	0.000	0.103	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	117	378	0	103	0	0	-1
N.S.	1	1.00	1.01	3.26	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.712	0.804	0.000	0.103	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	104	228	0	100	0	0	-1
N.S.	1	1.00	0.79	1.73	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.692	0.747	0.000	0.103	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	140	388	0	115	0	0	-1
N.S.	1	1.00	1.06	2.94	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.928	0.806	0.000	0.132	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	129	208	0	112	0	0	-1
N.S.	1	1.00	0.85	1.37	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.545	1.119	0.000	0.099	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	145	395	0	120	0	0	-1
N.S.	1	1.00	0.95	2.60	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.106	1.613	1.094	0.000	0.106	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	151	261	0	128	0	0	-1
N.S.	1	1.00	0.81	1.40	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.795	1.061	0.000	0.107	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	124	401	0	179	0	0	-1
N.S.	1	1.00	0.65	2.09	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.131	1.586	0.881	0.000	0.103	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	134	226	0	134	0	0	-1
N.S.	1	1.00	0.85	1.44	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.641	0.882	0.000	0.105	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	106	379	0	103	0	0	-1
N.S.	1	1.00	0.65	2.33	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.688	0.843	0.000	0.098	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	137	228	0	100	0	0	-1
N.S.	1	1.00	1.04	1.73	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.595	0.788	0.000	0.106	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	149	390	0	115	0	0	-1
N.S.	1	1.00	1.13	2.95	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.923	0.830	0.000	0.087	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	144	244	0	112	0	0	-1
N.S.	1	1.00	0.88	1.50	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.698	0.781	0.000	0.094	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	142	398	0	127	0	0	-1
N.S.	1	1.00	0.87	2.44	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.690	1.030	0.000	0.092	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	137	224	0	124	0	0	-1
N.S.	1	1.00	0.72	1.17	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.684	0.762	0.000	0.097	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	104	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.255	0.293	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	92	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.556	0.292	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	98	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.445	0.256	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	106	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.447	0.242	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	267	0	0	0	0	0	-1
N.S.	1	1.00	3.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	2.715	0.293	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	128	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.903	0.275	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	132	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.167	0.270	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	105	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.575	0.260	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	84	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.448	0.466	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	103	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.447	0.431	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	112	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.974	0.444	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	119	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.914	0.447	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	128	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.825	0.515	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	121	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.678	0.501	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	141	0	0	0	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	1.577	0.818	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	143	0	0	0	0	0	-1
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.975	0.814	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	141	76	154	0	0	474
N.S.	1	1.00	0.81	1.21	0.65	1.32	0.00	0.00	4.05
time (sec)	N/A	0.061	0.952	4.785	0.278	0.416	0.000	0.000	12.118

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	114	58	119	0	0	352
N.S.	1	1.00	0.88	1.30	0.66	1.35	0.00	0.00	4.00
time (sec)	N/A	0.053	0.483	0.973	0.269	0.439	0.000	0.000	7.015

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	87	40	84	0	0	230
N.S.	1	1.00	0.98	1.47	0.68	1.42	0.00	0.00	3.90
time (sec)	N/A	0.048	0.403	1.020	0.277	0.376	0.000	0.000	6.253

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	34	24	21	46	0	55	82
N.S.	1	1.00	1.17	0.83	0.72	1.59	0.00	1.90	2.83
time (sec)	N/A	0.042	0.216	0.299	0.286	0.354	0.000	0.615	0.575

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	105	397	122	253	0	0	-1
N.S.	1	1.00	0.88	3.31	1.02	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.548	3.658	0.501	0.387	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	133	741	176	275	0	0	-1
N.S.	1	1.00	0.69	3.84	0.91	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.506	0.977	0.506	0.370	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	159	1085	230	297	0	0	-1
N.S.	1	1.00	0.60	4.08	0.86	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.703	1.004	0.532	0.377	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	95	141	0	132	0	0	289
N.S.	1	1.00	0.65	0.96	0.00	0.90	0.00	0.00	1.97
time (sec)	N/A	0.189	0.716	1.135	0.000	0.431	0.000	0.000	8.354

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	77	114	0	97	0	0	102
N.S.	1	1.00	0.70	1.04	0.00	0.88	0.00	0.00	0.93
time (sec)	N/A	0.139	0.500	0.838	0.000	0.364	0.000	0.000	6.078

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	87	222	62	0	0	88
N.S.	1	1.00	0.86	1.19	3.04	0.85	0.00	0.00	1.21
time (sec)	N/A	0.083	0.335	0.805	19.523	0.385	0.000	0.000	5.945

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	50	0	25	0	0	61
N.S.	1	1.00	1.26	1.61	0.00	0.81	0.00	0.00	1.97
time (sec)	N/A	0.024	0.207	0.583	0.000	0.371	0.000	0.000	0.352

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	217	774	184	0	0	-1
N.S.	1	1.00	1.05	2.61	9.33	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.641	0.756	0.609	0.365	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	126	569	935	245	0	0	-1
N.S.	1	1.00	0.82	3.69	6.07	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.603	0.864	0.632	0.392	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	152	913	2220	267	0	0	-1
N.S.	1	1.00	0.68	4.09	9.96	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.710	0.839	0.790	0.372	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	111	152	76	170	0	0	544
N.S.	1	1.00	0.95	1.30	0.65	1.45	0.00	0.00	4.65
time (sec)	N/A	0.064	1.306	4.119	0.294	0.383	0.000	0.000	16.023

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	125	58	134	0	0	420
N.S.	1	1.00	1.06	1.42	0.66	1.52	0.00	0.00	4.77
time (sec)	N/A	0.058	0.674	0.785	0.295	0.401	0.000	0.000	7.306

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	98	40	98	0	0	296
N.S.	1	1.00	1.37	1.66	0.68	1.66	0.00	0.00	5.02
time (sec)	N/A	0.053	0.669	0.744	0.277	0.364	0.000	0.000	6.098

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	24	21	59	0	0	153
N.S.	1	1.00	2.38	0.83	0.72	2.03	0.00	0.00	5.28
time (sec)	N/A	0.044	0.399	0.208	0.281	0.409	0.000	0.000	1.365

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	97	396	98	244	0	0	-1
N.S.	1	1.00	1.04	4.26	1.05	2.62	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.715	1.483	0.506	0.407	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	143	742	158	287	0	0	-1
N.S.	1	1.00	0.86	4.47	0.95	1.73	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.910	1.098	0.499	0.389	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	169	1086	212	311	0	0	-1
N.S.	1	1.00	0.71	4.54	0.89	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.100	1.179	1.054	0.508	0.377	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	109	125	994	125	0	0	293
N.S.	1	1.00	0.74	0.85	6.76	0.85	0.00	0.00	1.99
time (sec)	N/A	0.186	1.130	0.762	10.555	0.384	0.000	0.000	7.249

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	91	98	580	89	0	0	103
N.S.	1	1.00	0.83	0.89	5.27	0.81	0.00	0.00	0.94
time (sec)	N/A	0.126	0.579	0.730	0.834	0.481	0.000	0.000	6.022

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	71	0	53	0	0	98
N.S.	1	1.00	0.83	1.03	0.00	0.77	0.00	0.00	1.42
time (sec)	N/A	0.048	0.368	0.832	0.000	0.359	0.000	0.000	4.488

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	201	40	0	0	60
N.S.	1	1.00	1.00	1.35	6.48	1.29	0.00	0.00	1.94
time (sec)	N/A	0.040	0.175	0.730	0.573	0.355	0.000	0.000	0.231

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	101	570	884	222	0	0	-1
N.S.	1	1.00	0.83	4.67	7.25	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.124	1.087	0.859	0.625	0.380	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	160	914	0	274	0	0	-1
N.S.	1	1.00	0.83	4.76	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.208	1.632	0.817	0.000	0.398	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	113	171	76	190	0	0	626
N.S.	1	1.00	0.97	1.46	0.65	1.62	0.00	0.00	5.35
time (sec)	N/A	0.063	1.606	11.268	0.281	0.473	0.000	0.000	15.904

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	97	144	58	152	0	0	498
N.S.	1	1.00	1.10	1.64	0.66	1.73	0.00	0.00	5.66
time (sec)	N/A	0.060	0.847	0.937	0.291	0.395	0.000	0.000	11.316

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	85	117	40	114	0	0	370
N.S.	1	1.00	1.44	1.98	0.68	1.93	0.00	0.00	6.27
time (sec)	N/A	0.051	0.645	0.756	0.291	0.375	0.000	0.000	6.421

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	73	24	21	73	0	0	242
N.S.	1	1.00	2.52	0.83	0.72	2.52	0.00	0.00	8.34
time (sec)	N/A	0.055	0.491	0.196	0.282	0.391	0.000	0.000	6.354

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	116	398	98	236	0	0	-1
N.S.	1	1.00	1.30	4.47	1.10	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.738	3.004	0.502	0.378	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	116	744	140	263	0	0	-1
N.S.	1	1.00	0.85	5.43	1.02	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.871	0.985	0.502	0.384	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	142	1088	194	309	0	0	-1
N.S.	1	1.00	0.68	5.18	0.92	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.102	1.013	1.072	0.499	0.371	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	103	117	624	121	0	0	301
N.S.	1	1.00	0.70	0.80	4.24	0.82	0.00	0.00	2.05
time (sec)	N/A	0.189	0.938	0.760	280.243	0.389	0.000	0.000	7.775

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	83	0	0	105
N.S.	1	1.00	0.89	0.87	0.00	0.80	0.00	0.00	1.01
time (sec)	N/A	0.078	0.469	0.609	0.000	0.380	0.000	0.000	6.083

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	53	331	45	0	0	64
N.S.	1	1.00	0.71	0.82	5.09	0.69	0.00	0.00	0.98
time (sec)	N/A	0.074	0.348	0.905	0.613	0.418	0.000	0.000	0.393

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	69	63	328	59	0	0	89
N.S.	1	1.00	1.97	1.80	9.37	1.69	0.00	0.00	2.54
time (sec)	N/A	0.045	0.385	0.871	0.631	0.357	0.000	0.000	0.908

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	118	916	1076	244	0	0	-1
N.S.	1	1.00	0.74	5.76	6.77	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.170	1.231	0.836	0.695	0.415	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	155	1260	0	300	0	0	-1
N.S.	1	1.00	0.67	5.45	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.252	1.283	1.171	0.000	0.393	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	113	181	76	202	0	0	690
N.S.	1	1.00	0.97	1.55	0.65	1.73	0.00	0.00	5.90
time (sec)	N/A	0.070	1.971	32.618	0.269	0.411	0.000	0.000	15.040

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	97	154	58	164	0	0	562
N.S.	1	1.00	1.10	1.75	0.66	1.86	0.00	0.00	6.39
time (sec)	N/A	0.056	1.045	0.998	0.268	0.506	0.000	0.000	15.661

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	85	127	40	126	0	0	434
N.S.	1	1.00	1.44	2.15	0.68	2.14	0.00	0.00	7.36
time (sec)	N/A	0.050	0.873	0.808	0.273	0.395	0.000	0.000	7.566

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	73	24	21	85	0	0	306
N.S.	1	1.00	2.52	0.83	0.72	2.93	0.00	0.00	10.55
time (sec)	N/A	0.046	0.539	0.212	0.268	0.366	0.000	0.000	6.283

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	412	117	235	0	0	-1
N.S.	1	1.00	1.18	3.55	1.01	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.068	1.550	1.013	0.497	0.442	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	152	742	138	261	0	0	-1
N.S.	1	1.00	1.11	5.42	1.01	1.91	0.00	0.00	-0.01
time (sec)	N/A	0.070	1.600	0.998	0.490	0.371	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	129	1088	176	277	0	0	-1
N.S.	1	1.00	0.71	6.01	0.97	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.226	1.096	0.487	0.391	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	109	100	0	109	0	0	286
N.S.	1	1.00	0.78	0.72	0.00	0.78	0.00	0.00	2.06
time (sec)	N/A	0.114	0.868	0.603	0.000	0.394	0.000	0.000	6.004

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	59	73	418	71	0	0	102
N.S.	1	1.00	0.57	0.70	4.02	0.68	0.00	0.00	0.98
time (sec)	N/A	0.115	0.498	0.960	0.576	0.367	0.000	0.000	4.718

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	71	504	59	0	0	85
N.S.	1	1.00	1.21	1.00	7.10	0.83	0.00	0.00	1.20
time (sec)	N/A	0.093	0.466	0.769	0.599	0.388	0.000	0.000	0.853

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	73	73	454	73	0	0	112
N.S.	1	1.00	2.09	2.09	12.97	2.09	0.00	0.00	3.20
time (sec)	N/A	0.046	0.647	0.884	0.594	0.361	0.000	0.000	5.255

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	131	1260	1253	258	0	0	-1
N.S.	1	1.00	0.67	6.43	6.39	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.221	2.073	2.839	0.639	0.399	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	188	1604	0	314	0	0	-1
N.S.	1	1.00	0.70	5.99	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.332	3.496	2.092	0.000	0.574	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	194	1948	0	342	0	0	-1
N.S.	1	1.00	0.57	5.70	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	6.981	1.804	0.000	0.444	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	127	297	150	0	0	434
N.S.	1	1.00	0.81	1.09	2.54	1.28	0.00	0.00	3.71
time (sec)	N/A	0.059	0.683	0.965	0.285	0.388	0.000	0.000	8.939

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	100	169	113	0	0	306
N.S.	1	1.00	0.88	1.14	1.92	1.28	0.00	0.00	3.48
time (sec)	N/A	0.070	0.409	0.859	0.288	0.426	0.000	0.000	6.378

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	65	73	79	76	0	0	155
N.S.	1	1.00	1.10	1.24	1.34	1.29	0.00	0.00	2.63
time (sec)	N/A	0.047	0.281	0.816	0.288	0.390	0.000	0.000	1.314

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	24	21	37	0	55	47
N.S.	1	1.00	1.19	0.89	0.78	1.37	0.00	2.04	1.74
time (sec)	N/A	0.041	0.209	0.263	0.273	0.364	0.000	0.799	0.163

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	126	335	138	271	0	0	-1
N.S.	1	1.00	0.86	2.29	0.95	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.736	5.149	0.497	0.371	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	152	362	192	293	0	0	-1
N.S.	1	1.00	0.69	1.65	0.88	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.105	0.866	0.985	0.502	0.392	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	178	389	246	315	0	0	-1
N.S.	1	1.00	0.61	1.33	0.84	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.122	1.203	1.049	0.492	0.394	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	95	154	608	153	0	0	301
N.S.	1	1.00	0.65	1.05	4.14	1.04	0.00	0.00	2.05
time (sec)	N/A	0.187	0.742	1.480	0.467	0.503	0.000	0.000	8.976

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	77	127	474	116	0	0	105
N.S.	1	1.00	0.70	1.15	4.31	1.05	0.00	0.00	0.95
time (sec)	N/A	0.130	0.488	0.937	0.413	0.427	0.000	0.000	6.156

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	100	340	79	0	0	91
N.S.	1	1.00	0.89	1.37	4.66	1.08	0.00	0.00	1.25
time (sec)	N/A	0.083	0.331	0.797	0.391	0.440	0.000	0.000	9.264

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	40	73	206	40	0	0	98
N.S.	1	1.00	1.14	2.09	5.89	1.14	0.00	0.00	2.80
time (sec)	N/A	0.040	0.199	0.815	0.377	0.370	0.000	0.000	1.019

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	70	137	0	149	0	0	-1
N.S.	1	1.00	1.35	2.63	0.00	2.87	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.441	0.842	0.000	0.437	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	96	319	837	245	0	0	-1
N.S.	1	1.00	0.79	2.61	6.86	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.609	1.191	0.585	0.380	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	117	346	1938	267	0	0	-1
N.S.	1	1.00	0.61	1.79	10.04	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.747	0.938	0.690	0.384	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	110	117	76	149	0	0	370
N.S.	1	1.00	0.94	1.00	0.65	1.27	0.00	0.00	3.16
time (sec)	N/A	0.062	0.849	0.822	0.279	0.374	0.000	0.000	7.643

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	90	58	108	0	0	242
N.S.	1	1.00	1.05	1.02	0.66	1.23	0.00	0.00	2.75
time (sec)	N/A	0.067	0.395	0.776	0.282	0.382	0.000	0.000	6.676

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	61	38	67	0	0	85
N.S.	1	1.00	1.40	1.07	0.67	1.18	0.00	0.00	1.49
time (sec)	N/A	0.053	0.291	0.728	0.278	0.360	0.000	0.000	3.658

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	49	0	0	67
N.S.	1	1.00	1.00	0.89	0.78	1.81	0.00	0.00	2.48
time (sec)	N/A	0.048	0.197	0.236	0.273	0.386	0.000	0.000	0.244

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	142	362	153	294	0	0	-1
N.S.	1	1.00	0.81	2.07	0.87	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.092	1.106	0.842	0.497	0.369	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	168	389	207	316	0	0	-1
N.S.	1	1.00	0.68	1.57	0.83	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.114	1.380	1.010	0.501	0.394	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	203	416	261	338	0	0	-1
N.S.	1	1.00	0.63	1.30	0.81	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.136	2.256	1.125	0.499	0.399	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	108	171	764	184	0	0	301
N.S.	1	1.00	0.73	1.16	5.20	1.25	0.00	0.00	2.05
time (sec)	N/A	0.188	1.044	3.521	0.525	0.440	0.000	0.000	9.862

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	92	144	626	143	0	0	105
N.S.	1	1.00	0.84	1.31	5.69	1.30	0.00	0.00	0.95
time (sec)	N/A	0.143	0.635	1.000	0.475	0.434	0.000	0.000	8.297

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	117	488	102	0	0	91
N.S.	1	1.00	1.10	1.60	6.68	1.40	0.00	0.00	1.25
time (sec)	N/A	0.094	0.507	0.832	0.433	0.422	0.000	0.000	6.437

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	90	350	59	0	0	139
N.S.	1	1.00	1.69	2.57	10.00	1.69	0.00	0.00	3.97
time (sec)	N/A	0.046	0.332	0.769	0.391	0.364	0.000	0.000	1.699

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	157	813	196	0	0	-1
N.S.	1	1.00	1.17	1.83	9.45	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.925	0.809	0.605	0.367	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	95	318	0	246	0	0	-1
N.S.	1	1.00	1.09	3.66	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.761	0.730	0.000	0.390	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	120	346	1821	270	0	0	-1
N.S.	1	1.00	0.76	2.20	11.60	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.150	1.225	0.792	0.651	0.371	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	145	373	2632	292	0	0	-1
N.S.	1	1.00	0.62	1.60	11.30	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.254	1.873	0.842	0.706	0.382	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	116	127	94	175	0	0	434
N.S.	1	1.00	0.79	0.87	0.64	1.20	0.00	0.00	2.97
time (sec)	N/A	0.069	0.948	0.836	0.291	0.450	0.000	0.000	8.762

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	108	100	76	134	0	0	306
N.S.	1	1.00	0.92	0.85	0.65	1.15	0.00	0.00	2.62
time (sec)	N/A	0.063	0.727	0.794	0.279	0.377	0.000	0.000	6.679

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	94	73	58	93	0	0	155
N.S.	1	1.00	1.09	0.85	0.67	1.08	0.00	0.00	1.80
time (sec)	N/A	0.057	0.487	0.756	0.278	0.370	0.000	0.000	1.228

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	65	44	50	0	0	72
N.S.	1	1.00	0.65	1.18	0.80	0.91	0.00	0.00	1.31
time (sec)	N/A	0.055	0.342	0.808	0.279	0.421	0.000	0.000	0.275

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	39	24	21	61	0	0	23
N.S.	1	1.00	1.34	0.83	0.72	2.10	0.00	0.00	0.79
time (sec)	N/A	0.052	0.284	0.226	0.277	0.375	0.000	0.000	3.584

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	163	389	175	305	0	0	-1
N.S.	1	1.00	0.80	1.91	0.86	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.097	1.370	0.862	0.496	0.377	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	189	416	229	327	0	0	-1
N.S.	1	1.00	0.68	1.50	0.83	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.128	1.826	1.038	0.510	0.425	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	112	181	902	199	0	0	301
N.S.	1	1.00	0.76	1.23	6.14	1.35	0.00	0.00	2.05
time (sec)	N/A	0.192	1.106	10.808	0.611	0.578	0.000	0.000	11.589

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	154	764	158	0	0	105
N.S.	1	1.00	0.85	1.40	6.95	1.44	0.00	0.00	0.95
time (sec)	N/A	0.139	0.876	0.999	0.534	0.439	0.000	0.000	8.751

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	127	626	117	0	0	91
N.S.	1	1.00	1.10	1.74	8.58	1.60	0.00	0.00	1.25
time (sec)	N/A	0.102	0.630	0.807	0.477	0.398	0.000	0.000	6.384

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	57	100	488	74	0	0	50
N.S.	1	1.00	1.63	2.86	13.94	2.11	0.00	0.00	1.43
time (sec)	N/A	0.045	0.508	0.775	0.431	0.375	0.000	0.000	2.016

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	82	281	1074	270	0	0	-1
N.S.	1	1.00	0.67	2.28	8.73	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.969	0.854	0.641	0.399	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	149	443	827	245	0	0	-1
N.S.	1	1.00	1.73	5.15	9.62	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.110	1.155	0.908	0.607	0.367	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	346	0	267	0	0	-1
N.S.	1	1.00	0.99	2.84	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.960	0.782	0.000	0.359	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	143	373	2297	289	0	0	-1
N.S.	1	1.00	0.74	1.94	11.96	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.202	1.524	0.902	0.653	0.441	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	165	400	3789	311	0	0	-1
N.S.	1	1.00	0.61	1.48	14.03	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.464	0.866	0.736	0.391	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	114	117	94	160	0	0	370
N.S.	1	1.00	0.78	0.80	0.64	1.10	0.00	0.00	2.53
time (sec)	N/A	0.072	0.991	0.827	0.266	0.439	0.000	0.000	7.626

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	110	90	76	119	0	0	242
N.S.	1	1.00	0.97	0.80	0.67	1.05	0.00	0.00	2.14
time (sec)	N/A	0.064	0.709	0.790	0.277	0.369	0.000	0.000	6.736

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	88	62	77	0	0	110
N.S.	1	1.00	0.73	1.05	0.74	0.92	0.00	0.00	1.31
time (sec)	N/A	0.061	0.534	0.884	0.288	0.391	0.000	0.000	0.742

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	88	32	61	0	0	-1
N.S.	1	1.00	1.40	1.54	0.56	1.07	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.278	0.878	0.277	0.369	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	39	24	21	72	0	0	23
N.S.	1	1.00	1.34	0.83	0.72	2.48	0.00	0.00	0.79
time (sec)	N/A	0.045	0.345	0.249	0.268	0.380	0.000	0.000	3.639

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	176	416	195	316	0	0	-1
N.S.	1	1.00	0.76	1.79	0.84	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.106	1.851	1.069	0.499	0.383	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	202	443	249	338	0	0	-1
N.S.	1	1.00	0.66	1.45	0.81	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.139	2.709	1.181	0.492	0.399	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	92	171	902	173	0	0	105
N.S.	1	1.00	0.84	1.55	8.20	1.57	0.00	0.00	0.95
time (sec)	N/A	0.138	1.142	1.431	0.602	0.489	0.000	0.000	9.276

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	82	144	764	132	0	0	91
N.S.	1	1.00	1.12	1.97	10.47	1.81	0.00	0.00	1.25
time (sec)	N/A	0.097	0.840	0.861	0.532	0.442	0.000	0.000	6.624

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	115	626	89	0	0	50
N.S.	1	1.00	1.69	3.29	17.89	2.54	0.00	0.00	1.43
time (sec)	N/A	0.045	0.655	0.766	0.466	0.395	0.000	0.000	6.490

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	399	1164	326	0	0	-1
N.S.	1	1.00	0.81	2.49	7.28	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.521	1.068	0.667	0.401	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	126	318	0	245	0	0	-1
N.S.	1	1.00	1.04	2.63	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.159	1.115	0.820	0.000	0.398	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	120	346	977	267	0	0	-1
N.S.	1	1.00	0.96	2.77	7.82	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.138	1.049	0.915	0.626	0.397	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	119	373	0	278	0	0	-1
N.S.	1	1.00	0.76	2.38	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.125	1.663	0.711	0.000	0.375	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	141	400	2779	300	0	0	-1
N.S.	1	1.00	0.62	1.76	12.24	1.32	0.00	0.00	-0.00
time (sec)	N/A	0.252	2.489	0.856	0.673	0.369	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	175	427	5821	322	0	0	-1
N.S.	1	1.00	0.57	1.39	18.96	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.360	2.833	0.931	0.810	0.452	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	524	373	309	1825	386	0	0	-1
N.S.	1	1.00	0.71	0.59	3.48	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.327	1.946	3.762	0.658	0.436	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	277	230	1399	335	0	0	-1
N.S.	1	1.00	0.86	0.71	4.33	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.145	1.701	1.033	0.674	0.451	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	56	75	58	0	0	-1
N.S.	1	1.00	1.00	1.56	2.08	1.61	0.00	0.00	-0.03
time (sec)	N/A	0.045	0.080	0.889	0.530	0.378	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	48	75	53	69	0	0	86
N.S.	1	1.00	0.59	0.93	0.65	0.85	0.00	0.00	1.06
time (sec)	N/A	0.096	0.219	0.852	0.575	0.383	0.000	0.000	4.630

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	63	85	129	80	0	0	101
N.S.	1	1.00	0.52	0.70	1.06	0.66	0.00	0.00	0.83
time (sec)	N/A	0.161	0.260	0.890	0.588	0.410	0.000	0.000	4.681

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	80	102	177	91	0	0	109
N.S.	1	1.00	0.49	0.62	1.08	0.55	0.00	0.00	0.66
time (sec)	N/A	0.204	0.333	0.889	0.582	0.349	0.000	0.000	5.159

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	376	414	2734	594	0	0	-1
N.S.	1	1.00	0.83	0.91	6.04	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.359	4.572	0.966	0.831	0.420	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	11319	363	2302	511	0	0	-1
N.S.	1	1.00	19.82	0.64	4.03	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.382	58.461	0.963	0.731	0.450	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	338	304	1870	433	0	0	-1
N.S.	1	1.00	0.93	0.84	5.14	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.209	3.573	0.985	0.668	0.390	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	11314	286	1461	439	0	0	-1
N.S.	1	1.00	21.76	0.55	2.81	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.303	6.197	0.934	0.726	0.376	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	76	75	70	0	0	-1
N.S.	1	1.00	1.00	2.00	1.97	1.84	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.090	0.784	0.522	0.385	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	86	58	73	0	0	102
N.S.	1	1.00	1.04	1.06	0.72	0.90	0.00	0.00	1.26
time (sec)	N/A	0.103	0.428	0.792	0.552	0.381	0.000	0.000	4.705

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	98	103	83	85	0	0	110
N.S.	1	1.00	0.78	0.82	0.66	0.68	0.00	0.00	0.88
time (sec)	N/A	0.153	0.494	0.813	0.552	0.375	0.000	0.000	4.923

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	113	113	159	97	0	0	125
N.S.	1	1.00	0.68	0.68	0.95	0.58	0.00	0.00	0.75
time (sec)	N/A	0.197	0.651	0.844	0.559	0.390	0.000	0.000	5.584

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	11411	424	2914	608	0	0	-1
N.S.	1	1.00	18.65	0.69	4.76	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.482	58.709	1.002	0.812	0.403	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	387	371	2430	539	0	0	-1
N.S.	1	1.00	0.94	0.90	5.91	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.282	4.513	1.171	0.707	0.396	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	11357	347	2008	463	0	0	-1
N.S.	1	1.00	20.17	0.62	3.57	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.388	6.361	1.141	0.663	0.434	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	343	323	1491	468	0	0	-1
N.S.	1	1.00	0.95	0.89	4.12	1.29	0.00	0.00	-0.00
time (sec)	N/A	0.212	3.867	0.957	0.630	0.431	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	88	75	74	0	0	104
N.S.	1	1.00	1.00	2.32	1.97	1.95	0.00	0.00	2.74
time (sec)	N/A	0.053	0.149	0.792	0.495	0.373	0.000	0.000	4.554

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	105	93	88	0	0	112
N.S.	1	1.00	1.14	1.30	1.15	1.09	0.00	0.00	1.38
time (sec)	N/A	0.108	0.507	0.813	0.548	0.379	0.000	0.000	4.992

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	115	95	93	0	0	127
N.S.	1	1.00	0.83	0.92	0.76	0.74	0.00	0.00	1.02
time (sec)	N/A	0.156	0.536	0.815	0.565	0.384	0.000	0.000	5.523

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	121	132	123	93	0	0	133
N.S.	1	1.00	0.72	0.78	0.73	0.55	0.00	0.00	0.79
time (sec)	N/A	0.199	0.808	0.942	0.552	0.394	0.000	0.000	6.067

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	350	316	2137	411	0	0	-1
N.S.	1	1.00	0.95	0.86	5.79	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.226	3.927	3.922	0.633	0.424	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	302	232	713	357	0	0	-1
N.S.	1	1.00	0.63	0.48	1.48	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.240	1.452	1.115	0.585	0.381	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	74	75	64	0	0	40
N.S.	1	1.00	1.00	2.06	2.08	1.78	0.00	0.00	1.11
time (sec)	N/A	0.043	0.074	0.885	0.495	0.355	0.000	0.000	4.307

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	85	79	72	0	0	78
N.S.	1	1.00	0.60	1.06	0.99	0.90	0.00	0.00	0.98
time (sec)	N/A	0.095	0.142	0.897	0.544	0.353	0.000	0.000	0.777

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	68	105	129	83	0	0	86
N.S.	1	1.00	0.56	0.87	1.07	0.69	0.00	0.00	0.71
time (sec)	N/A	0.154	0.309	0.869	0.555	0.353	0.000	0.000	4.030

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	79	115	177	94	0	0	101
N.S.	1	1.00	0.48	0.70	1.07	0.57	0.00	0.00	0.61
time (sec)	N/A	0.208	0.450	0.889	0.560	0.391	0.000	0.000	4.210

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	87	132	225	105	0	0	109
N.S.	1	1.00	0.42	0.64	1.09	0.51	0.00	0.00	0.53
time (sec)	N/A	0.254	0.474	1.135	0.560	0.360	0.000	0.000	4.312

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	11282	1022	1700	433	0	0	-1
N.S.	1	1.00	21.33	1.93	3.21	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.388	57.147	1.053	0.619	0.392	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	338	957	735	488	0	0	-1
N.S.	1	1.00	0.93	2.62	2.01	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.219	4.259	1.122	0.596	0.396	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	85	75	64	0	0	-1
N.S.	1	1.00	1.00	2.24	1.97	1.68	0.00	0.00	-0.03
time (sec)	N/A	0.051	0.120	0.807	0.508	0.373	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	101	79	76	0	0	84
N.S.	1	1.00	0.79	1.26	0.99	0.95	0.00	0.00	1.05
time (sec)	N/A	0.094	0.212	0.865	0.558	0.381	0.000	0.000	3.945

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	83	106	129	83	0	0	104
N.S.	1	1.00	0.69	0.88	1.07	0.69	0.00	0.00	0.86
time (sec)	N/A	0.151	0.371	1.174	0.563	0.372	0.000	0.000	4.160

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	100	132	177	94	0	0	112
N.S.	1	1.00	0.61	0.80	1.07	0.57	0.00	0.00	0.68
time (sec)	N/A	0.204	0.565	0.891	0.555	0.364	0.000	0.000	4.199

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	100	142	225	105	0	0	127
N.S.	1	1.00	0.48	0.68	1.08	0.50	0.00	0.00	0.61
time (sec)	N/A	0.260	0.610	0.896	0.572	0.356	0.000	0.000	4.570

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	370	1439	2281	490	0	0	-1
N.S.	1	1.00	0.90	3.50	5.55	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.303	6.255	1.024	0.643	0.422	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	11295	1324	1423	492	0	0	-1
N.S.	1	1.00	21.43	2.51	2.70	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.303	30.194	0.848	0.645	0.399	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	103	75	64	0	0	-1
N.S.	1	1.00	1.00	2.71	1.97	1.68	0.00	0.00	-0.03
time (sec)	N/A	0.057	0.284	0.918	0.502	0.412	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	112	81	76	0	0	102
N.S.	1	1.00	0.79	1.40	1.01	0.95	0.00	0.00	1.28
time (sec)	N/A	0.107	0.264	1.072	0.549	0.398	0.000	0.000	4.226

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	85	128	129	88	0	0	109
N.S.	1	1.00	0.70	1.06	1.07	0.73	0.00	0.00	0.90
time (sec)	N/A	0.158	0.381	0.896	0.543	0.384	0.000	0.000	4.217

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	102	140	177	83	0	0	118
N.S.	1	1.00	0.63	0.86	1.09	0.51	0.00	0.00	0.73
time (sec)	N/A	0.195	0.500	0.878	0.556	0.380	0.000	0.000	4.515

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	107	159	225	105	0	0	135
N.S.	1	1.00	0.52	0.77	1.09	0.51	0.00	0.00	0.66
time (sec)	N/A	0.274	0.668	0.892	0.562	0.415	0.000	0.000	4.723

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	118	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.793	1.228	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	116	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.668	0.989	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	116	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.521	0.862	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.613	0.863	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.855	0.890	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	112	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.776	0.941	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	240	0	4194	556	0	0	-1
N.S.	1	1.00	0.55	0.00	9.60	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.992	0.328	0.702	0.458	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	220	0	2045	542	0	0	-1
N.S.	1	1.00	0.58	0.00	5.41	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.250	1.390	0.330	0.599	0.408	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	161	0	1881	385	0	0	-1
N.S.	1	1.00	0.47	0.00	5.53	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.690	0.336	0.591	0.392	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	0	114	58	0	0	81
N.S.	1	1.00	1.27	0.00	3.08	1.57	0.00	0.00	2.19
time (sec)	N/A	0.057	0.388	0.373	0.540	0.356	0.000	0.000	4.814

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	336	61	0	0	90
N.S.	1	1.00	0.86	0.00	4.15	0.75	0.00	0.00	1.11
time (sec)	N/A	0.118	0.591	0.368	0.558	0.380	0.000	0.000	4.130

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	426	114	0	0	122
N.S.	1	1.00	0.82	0.00	3.49	0.93	0.00	0.00	1.00
time (sec)	N/A	0.166	0.691	0.373	0.544	0.405	0.000	0.000	5.269

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	116	0	1041	142	0	0	303
N.S.	1	1.00	0.71	0.00	6.39	0.87	0.00	0.00	1.86
time (sec)	N/A	0.232	1.270	0.370	0.560	0.389	0.000	0.000	7.879

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	1214	0	0	0	0	0	-1
N.S.	1	1.00	14.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	12.262	0.458	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	438	0	0	0	0	0	-1
N.S.	1	1.00	5.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	6.434	0.395	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	156	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.007	0.307	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	147	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.699	0.350	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	212	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.025	0.943	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	279	0	0	0	0	0	-1
N.S.	1	1.00	3.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	16.725	0.732	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	347	0	0	0	0	0	-1
N.S.	1	1.00	4.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	10.136	0.610	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	2.954	0.983	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	3.704	0.946	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	186	0	0	0	0	0	-1
N.S.	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.418	0.926	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	174	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.802	1.135	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	174	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	1.300	0.928	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.857	0.910	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	3.635	0.884	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	165	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	9.420	0.687	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	171	3316	0	247	0	0	168
N.S.	1	1.00	1.76	34.19	0.00	2.55	0.00	0.00	1.73
time (sec)	N/A	0.058	14.434	0.853	0.000	0.367	0.000	0.000	8.127

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	143	1668	0	142	0	0	216
N.S.	1	1.00	2.20	25.66	0.00	2.18	0.00	0.00	3.32
time (sec)	N/A	0.044	13.759	0.539	0.000	0.394	0.000	0.000	2.436

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	111	31	28	60	0	62	104
N.S.	1	1.00	3.47	0.97	0.88	1.88	0.00	1.94	3.25
time (sec)	N/A	0.034	13.199	0.103	0.287	0.381	0.000	0.725	0.416

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	256	0	0	0	0	0	-1
N.S.	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	14.456	0.968	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	143	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	8.335	1.076	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	143	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	10.843	1.058	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	153	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	14.335	0.338	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	153	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	13.539	0.332	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	146	0	0	0	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	8.821	0.256	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	195	0	0	0	0	0	-1
N.S.	1	1.00	2.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	13.556	0.332	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	321	0	0	0	0	0	-1
N.S.	1	1.00	3.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	14.752	0.686	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	10.803	0.990	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	181	0	0	0	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	9.628	0.439	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	170	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	9.364	0.426	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	170	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	8.824	0.435	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	143	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	10.087	0.444	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	143	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	10.841	0.422	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	157	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	11.148	0.451	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	165	5823	430	334	0	0	511
N.S.	1	1.00	0.61	21.65	1.60	1.24	0.00	0.00	1.90
time (sec)	N/A	0.292	0.726	1.433	0.597	0.369	0.000	0.000	9.901

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	119	4994	341	264	0	0	425
N.S.	1	1.00	0.58	24.36	1.66	1.29	0.00	0.00	2.07
time (sec)	N/A	0.232	0.748	1.284	0.600	0.389	0.000	0.000	9.238

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	3327	173	177	0	0	227
N.S.	1	1.00	0.55	22.48	1.17	1.20	0.00	0.00	1.53
time (sec)	N/A	0.149	0.297	1.019	0.590	0.393	0.000	0.000	9.304

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	58	2490	111	128	296	0	121
N.S.	1	1.00	0.62	26.49	1.18	1.36	3.15	0.00	1.29
time (sec)	N/A	0.089	0.252	1.055	0.572	0.395	16.384	0.000	1.894

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	874	84	83	51	0	-1
N.S.	1	1.00	1.00	23.62	2.27	2.24	1.38	0.00	-0.03
time (sec)	N/A	0.035	0.059	0.607	0.516	0.367	11.010	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	5.140	0.839	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	12.450	0.732	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	116	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	7.720	0.763	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	122	0	865	165	0	0	318
N.S.	1	1.00	0.78	0.00	5.54	1.06	0.00	0.00	2.04
time (sec)	N/A	0.170	2.353	0.841	1.735	0.426	0.000	0.000	10.330

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	14.210	0.746	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	91	0	505	133	0	0	174
N.S.	1	1.00	0.93	0.00	5.15	1.36	0.00	0.00	1.78
time (sec)	N/A	0.098	1.361	0.767	1.513	0.387	0.000	0.000	5.934

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	12.375	0.763	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	59	0	222	108	0	0	106
N.S.	1	1.00	1.28	0.00	4.83	2.35	0.00	0.00	2.30
time (sec)	N/A	0.041	0.720	0.740	0.528	0.375	0.000	0.000	4.240

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	154	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	8.564	0.740	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	146	0	0	0	0	0	-1
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	1.829	0.432	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	157	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	12.973	0.767	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	151	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	13.241	0.801	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	13.588	0.780	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0	-1
N.S.	1	1.00	2.50	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.140	120.185	1.260	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0	-1
N.S.	1	1.00	2.50	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.152	14.643	1.258	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	150	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	1.164	0.750	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	1291	147	125	0	0	62
N.S.	1	1.00	1.00	32.28	3.68	3.12	0.00	0.00	1.55
time (sec)	N/A	0.043	0.476	0.958	0.519	0.394	0.000	0.000	4.591

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	61	0	319	150	0	0	260
N.S.	1	1.00	0.66	0.00	3.47	1.63	0.00	0.00	2.83
time (sec)	N/A	0.098	1.203	0.723	0.615	0.443	0.000	0.000	7.823

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	129	0	0	184	0	0	321
N.S.	1	1.00	0.87	0.00	0.00	1.24	0.00	0.00	2.17
time (sec)	N/A	0.154	2.049	0.844	0.000	0.374	0.000	0.000	12.468

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	70	57	56	70	68
N.S.	1	1.00	0.88	0.80	1.17	0.95	0.93	1.17	1.13
time (sec)	N/A	0.030	0.191	0.204	0.265	0.364	2.585	0.546	3.675

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	63	86	88	0	141	175
N.S.	1	1.00	0.92	0.85	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.042	0.240	0.211	0.267	0.379	0.000	0.552	7.072

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	48	45	44	48	46
N.S.	1	1.00	0.93	0.86	1.09	1.02	1.00	1.09	1.05
time (sec)	N/A	0.026	0.102	0.210	0.268	0.412	1.825	0.505	3.586

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	50	61	74	0	99	105
N.S.	1	1.00	1.00	0.96	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.029	0.024	0.209	0.285	0.392	0.000	0.534	5.414

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	20	30	34	25	23
N.S.	1	1.00	1.00	0.89	0.71	1.07	1.21	0.89	0.82
time (sec)	N/A	0.023	0.023	0.171	0.267	0.372	1.336	0.520	3.691

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	31	54	37	54	38
N.S.	1	1.00	1.00	1.33	1.29	2.25	1.54	2.25	1.58
time (sec)	N/A	0.012	0.017	0.054	0.280	0.407	2.593	0.550	3.749

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	23	23	0	129	38
N.S.	1	1.00	1.92	0.96	0.96	0.96	0.00	5.38	1.58
time (sec)	N/A	0.016	0.028	0.158	0.282	0.373	0.000	0.504	3.718

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	38	35	0	146	31
N.S.	1	1.00	1.07	0.95	0.88	0.81	0.00	3.40	0.72
time (sec)	N/A	0.023	0.066	0.199	0.518	0.386	0.000	0.507	3.684

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	0	11886	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	0.00	270.14	1.07
time (sec)	N/A	0.025	0.017	0.187	0.262	0.407	0.000	2.518	3.757

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	61	51	0	426	41
N.S.	1	1.00	0.95	0.80	0.94	0.78	0.00	6.55	0.63
time (sec)	N/A	0.032	0.109	0.192	0.474	0.382	0.000	0.641	3.709

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	133	138	133	122	0	156	132
N.S.	1	1.00	1.12	1.16	1.12	1.03	0.00	1.31	1.11
time (sec)	N/A	0.078	0.630	0.278	0.265	0.395	0.000	0.643	3.676

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	104	110	104	100	0	118	102
N.S.	1	1.00	1.07	1.13	1.07	1.03	0.00	1.22	1.05
time (sec)	N/A	0.065	0.735	0.217	0.264	0.385	0.000	0.656	3.567

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	54	82	71	79	0	80	71
N.S.	1	1.00	0.72	1.09	0.95	1.05	0.00	1.07	0.95
time (sec)	N/A	0.059	0.210	0.218	0.271	0.366	0.000	0.649	3.548

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	56	48	20	55	0	41	39
N.S.	1	1.00	2.55	2.18	0.91	2.50	0.00	1.86	1.77
time (sec)	N/A	0.030	0.758	0.235	0.273	0.375	0.000	0.586	3.561

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	70	55	52	0	245	50
N.S.	1	1.00	1.06	1.43	1.12	1.06	0.00	5.00	1.02
time (sec)	N/A	0.039	0.136	0.209	0.500	0.378	0.000	0.607	3.583

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	216	97	85	75	0	2286	83
N.S.	1	1.00	2.45	1.10	0.97	0.85	0.00	25.98	0.94
time (sec)	N/A	0.059	3.239	0.215	0.488	0.373	0.000	2.668	3.655

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	131	177	220	163	0	437	432
N.S.	1	1.00	0.80	1.09	1.35	1.00	0.00	2.68	2.65
time (sec)	N/A	0.102	0.851	0.274	0.264	0.390	0.000	0.629	7.099

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	104	149	180	142	0	343	328
N.S.	1	1.00	0.79	1.14	1.37	1.08	0.00	2.62	2.50
time (sec)	N/A	0.097	0.590	0.241	0.273	0.383	0.000	0.684	6.378

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	120	118	129	120	0	249	216
N.S.	1	1.00	1.21	1.19	1.30	1.21	0.00	2.52	2.18
time (sec)	N/A	0.076	0.072	0.235	0.266	0.385	0.000	0.594	6.242

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	83	82	96	0	122	106
N.S.	1	1.00	1.03	1.28	1.26	1.48	0.00	1.88	1.63
time (sec)	N/A	0.043	0.046	0.124	0.272	0.378	0.000	0.650	4.169

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	84	53	60	62	0	1316	66
N.S.	1	1.00	1.79	1.13	1.28	1.32	0.00	28.00	1.40
time (sec)	N/A	0.026	0.158	0.199	0.265	0.384	0.000	0.808	3.894

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	64	52	52	53	0	11162	77
N.S.	1	1.00	0.71	0.58	0.58	0.59	0.00	124.02	0.86
time (sec)	N/A	0.071	0.509	0.207	0.269	0.411	0.000	18.114	3.760

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	116	88	77	74	0	28204	115
N.S.	1	1.00	1.02	0.77	0.68	0.65	0.00	247.40	1.01
time (sec)	N/A	0.091	0.243	0.204	0.276	0.366	0.000	50.381	3.769

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	154	108	98	94	0	52002	176
N.S.	1	1.00	1.12	0.78	0.71	0.68	0.00	376.83	1.28
time (sec)	N/A	0.084	0.482	0.202	0.269	0.361	0.000	97.441	3.841

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	177	219	176	150	0	220	175
N.S.	1	1.00	0.91	1.13	0.91	0.77	0.00	1.13	0.90
time (sec)	N/A	0.120	2.145	0.258	0.267	0.441	0.000	0.919	3.677

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	115	173	142	128	0	166	139
N.S.	1	1.00	0.83	1.25	1.03	0.93	0.00	1.20	1.01
time (sec)	N/A	0.101	0.638	0.260	0.270	0.387	0.000	0.837	3.605

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	54	127	98	105	0	112	97
N.S.	1	1.00	0.72	1.69	1.31	1.40	0.00	1.49	1.29
time (sec)	N/A	0.056	0.398	0.240	0.267	0.381	0.000	0.929	3.573

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	79	72	20	78	0	57	55
N.S.	1	1.00	3.59	3.27	0.91	3.55	0.00	2.59	2.50
time (sec)	N/A	0.026	0.645	0.222	0.265	0.366	0.000	0.873	3.569

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	401	98	81	79	0	601	141
N.S.	1	1.00	4.66	1.14	0.94	0.92	0.00	6.99	1.64
time (sec)	N/A	0.071	0.818	0.194	0.486	0.468	0.000	0.948	3.662

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	257	114	110	100	0	2496	109
N.S.	1	1.00	3.06	1.36	1.31	1.19	0.00	29.71	1.30
time (sec)	N/A	0.055	3.808	0.215	0.483	0.437	0.000	12.205	3.797

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	177	637	248	208	170	0	465	423
N.S.	1	1.11	4.01	1.56	1.31	1.07	0.00	2.92	2.66
time (sec)	N/A	0.105	2.324	0.296	0.291	0.394	0.000	0.903	7.340

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	144	464	198	157	147	0	333	293
N.S.	1	1.14	3.68	1.57	1.25	1.17	0.00	2.64	2.33
time (sec)	N/A	0.091	1.383	0.249	0.294	0.449	0.000	0.845	7.325

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	109	293	146	111	123	0	171	160
N.S.	1	1.20	3.22	1.60	1.22	1.35	0.00	1.88	1.76
time (sec)	N/A	0.067	1.683	0.124	0.278	0.388	0.000	0.899	5.535

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	102	131	96	84	109	0	4741	116
N.S.	1	1.21	1.56	1.14	1.00	1.30	0.00	56.44	1.38
time (sec)	N/A	0.059	1.135	0.179	0.266	0.380	0.000	2.837	4.251

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	81	75	77	77	0	24430	104
N.S.	1	1.00	1.16	1.07	1.10	1.10	0.00	349.00	1.49
time (sec)	N/A	0.048	0.413	0.184	0.286	0.422	0.000	126.456	3.706

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	150	125	107	102	0	56572	147
N.S.	1	1.00	1.43	1.19	1.02	0.97	0.00	538.78	1.40
time (sec)	N/A	0.070	0.756	0.275	0.264	0.407	0.000	255.257	3.810

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	204	145	126	123	0	101962	214
N.S.	1	1.00	1.44	1.02	0.89	0.87	0.00	718.04	1.51
time (sec)	N/A	0.102	1.147	0.239	0.265	0.383	0.000	82.295	3.944

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	119	106	108	183	0	120	119
N.S.	1	1.00	1.03	0.91	0.93	1.58	0.00	1.03	1.03
time (sec)	N/A	0.076	1.277	0.383	0.272	0.407	0.000	0.508	3.730

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	53	53	117	0	54	57
N.S.	1	1.00	1.20	0.90	0.90	1.98	0.00	0.92	0.97
time (sec)	N/A	0.051	0.321	0.370	0.275	0.394	0.000	0.494	3.590

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	59	0	19	18
N.S.	1	1.00	1.00	1.06	1.00	3.28	0.00	1.06	1.00
time (sec)	N/A	0.031	0.020	0.235	0.266	0.372	0.000	0.482	3.585

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	143	120	141	119	0	182	156
N.S.	1	1.00	1.54	1.29	1.52	1.28	0.00	1.96	1.68
time (sec)	N/A	0.098	0.247	0.358	0.488	0.402	0.000	0.512	3.901

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	218	197	271	208	0	322	318
N.S.	1	1.00	1.43	1.30	1.78	1.37	0.00	2.12	2.09
time (sec)	N/A	0.152	0.454	0.421	0.508	0.418	0.000	0.502	4.199

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	152	321	269	361	259	0	278	724
N.S.	1	1.09	2.29	1.92	2.58	1.85	0.00	1.99	5.17
time (sec)	N/A	0.154	2.206	0.447	0.508	0.467	0.000	0.563	5.401

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	109	129	163	191	0	136	310
N.S.	1	1.00	1.38	1.63	2.06	2.42	0.00	1.72	3.92
time (sec)	N/A	0.075	0.164	0.417	0.483	0.416	0.000	0.538	3.872

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	80	131	0	74	39
N.S.	1	1.00	0.98	0.93	1.74	2.85	0.00	1.61	0.85
time (sec)	N/A	0.025	0.055	0.164	0.490	0.411	0.000	0.514	3.752

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	90	142	187	0	118	110
N.S.	1	1.00	0.88	1.00	1.58	2.08	0.00	1.31	1.22
time (sec)	N/A	0.075	0.353	0.404	0.476	0.397	0.000	0.540	3.837

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	137	221	379	262	0	286	342
N.S.	1	1.00	0.83	1.34	2.30	1.59	0.00	1.73	2.07
time (sec)	N/A	0.153	1.359	0.446	0.514	0.399	0.000	0.550	6.339

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	373	201	186	386	0	253	258
N.S.	1	1.00	2.10	1.13	1.04	2.17	0.00	1.42	1.45
time (sec)	N/A	0.114	1.977	0.361	0.279	0.422	0.000	0.579	3.633

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	207	114	115	281	0	149	130
N.S.	1	1.00	1.78	0.98	0.99	2.42	0.00	1.28	1.12
time (sec)	N/A	0.076	1.622	0.303	0.270	0.487	0.000	0.534	3.714

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	121	57	60	178	0	71	67
N.S.	1	1.00	1.98	0.93	0.98	2.92	0.00	1.16	1.10
time (sec)	N/A	0.051	0.367	0.306	0.264	0.424	0.000	0.519	3.715

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	20	57	0	20	20
N.S.	1	1.00	1.60	1.05	1.00	2.85	0.00	1.00	1.00
time (sec)	N/A	0.030	0.055	0.266	0.272	0.374	0.000	0.550	3.653

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	304	154	282	279	0	250	246
N.S.	1	1.00	2.00	1.01	1.86	1.84	0.00	1.64	1.62
time (sec)	N/A	0.124	4.213	0.418	0.503	0.429	0.000	0.597	3.928

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	416	248	502	424	0	464	463
N.S.	1	1.00	1.77	1.06	2.14	1.80	0.00	1.97	1.97
time (sec)	N/A	0.202	3.469	0.404	0.493	0.450	0.000	0.564	4.770

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	1152	454	827	472	0	530	2500
N.S.	1	1.00	4.90	1.93	3.52	2.01	0.00	2.26	10.64
time (sec)	N/A	0.194	6.235	0.515	0.503	0.589	0.000	0.655	6.545

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	709	259	471	355	0	280	585
N.S.	1	1.00	4.03	1.47	2.68	2.02	0.00	1.59	3.32
time (sec)	N/A	0.118	6.152	0.427	0.494	0.450	0.000	0.593	5.013

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	132	120	135	212	293	0	166	383
N.S.	1	1.45	1.32	1.48	2.33	3.22	0.00	1.82	4.21
time (sec)	N/A	0.083	0.877	0.400	0.501	0.449	0.000	0.615	4.188

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	105	78	118	182	215	0	138	136
N.S.	1	1.28	0.95	1.44	2.22	2.62	0.00	1.68	1.66
time (sec)	N/A	0.053	0.418	0.223	0.493	0.390	0.000	0.553	3.984

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	153	172	348	302	0	286	286
N.S.	1	1.00	0.97	1.10	2.22	1.92	0.00	1.82	1.82
time (sec)	N/A	0.098	0.609	0.453	0.511	0.418	0.000	0.604	5.952

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	249	320	772	418	0	438	674
N.S.	1	1.00	1.03	1.33	3.20	1.73	0.00	1.82	2.80
time (sec)	N/A	0.188	1.284	0.470	0.516	0.424	0.000	0.596	7.108

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	302	195	200	476	0	243	234
N.S.	1	1.00	1.63	1.05	1.08	2.57	0.00	1.31	1.26
time (sec)	N/A	0.126	5.385	0.411	0.274	0.467	0.000	0.688	3.677

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	191	115	128	354	0	140	143
N.S.	1	1.00	1.58	0.95	1.06	2.93	0.00	1.16	1.18
time (sec)	N/A	0.080	2.433	0.409	0.274	0.417	0.000	0.683	3.731

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	115	63	78	284	0	62	80
N.S.	1	1.00	1.67	0.91	1.13	4.12	0.00	0.90	1.16
time (sec)	N/A	0.051	0.664	0.346	0.278	0.414	0.000	0.637	3.747

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	58	21	20	142	0	20	39
N.S.	1	1.00	2.64	0.95	0.91	6.45	0.00	0.91	1.77
time (sec)	N/A	0.029	0.211	0.273	0.273	0.382	0.000	0.638	3.663

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	458	219	458	503	0	439	419
N.S.	1	1.00	2.27	1.08	2.27	2.49	0.00	2.17	2.07
time (sec)	N/A	0.183	6.323	0.508	0.516	0.411	0.000	0.695	4.564

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	596	312	738	671	0	587	715
N.S.	1	1.00	2.02	1.06	2.50	2.27	0.00	1.99	2.42
time (sec)	N/A	0.270	6.232	0.553	0.505	0.491	0.000	0.719	5.434

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	688	444	902	564	0	510	1203
N.S.	1	1.00	2.88	1.86	3.77	2.36	0.00	2.13	5.03
time (sec)	N/A	0.185	2.608	0.529	0.519	0.511	0.000	0.735	6.937

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	189	396	269	518	513	0	314	1311
N.S.	1	1.28	2.68	1.82	3.50	3.47	0.00	2.12	8.86
time (sec)	N/A	0.117	2.607	0.479	0.505	0.471	0.000	0.692	5.680

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	118	132	191	326	294	0	221	260
N.S.	1	1.24	1.39	2.01	3.43	3.09	0.00	2.33	2.74
time (sec)	N/A	0.066	0.340	0.403	0.484	0.368	0.000	0.690	5.880

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	110	280	412	352	0	293	443
N.S.	1	1.00	0.71	1.81	2.66	2.27	0.00	1.89	2.86
time (sec)	N/A	0.093	1.058	0.322	0.499	0.408	0.000	0.662	4.822

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	183	283	658	480	0	399	610
N.S.	1	1.00	0.83	1.28	2.98	2.17	0.00	1.81	2.76
time (sec)	N/A	0.164	2.074	0.655	0.531	0.416	0.000	0.743	7.324

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	371	457	1229	619	0	640	1128
N.S.	1	1.00	1.20	1.47	3.96	2.00	0.00	2.06	3.64
time (sec)	N/A	0.285	2.143	0.663	0.674	0.468	0.000	0.783	8.827

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	69	371	0	156	0	0	-1
N.S.	1	1.00	0.57	3.07	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.683	8.409	0.000	0.138	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	58	195	0	133	0	0	-1
N.S.	1	1.00	0.63	2.12	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.467	0.473	0.000	0.111	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	58	356	0	130	0	0	-1
N.S.	1	1.00	0.66	4.05	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.323	0.522	0.000	0.111	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	42	168	0	79	0	0	39
N.S.	1	1.00	0.72	2.90	0.00	1.36	0.00	0.00	0.67
time (sec)	N/A	0.036	0.229	0.513	0.000	0.115	0.000	0.000	0.363

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	916	0	95	0	0	-1
N.S.	1	1.00	0.93	15.79	0.00	1.64	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.319	0.610	0.000	0.119	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	69	172	0	111	0	0	-1
N.S.	1	1.00	0.73	1.83	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.218	0.895	0.000	0.122	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	345	0	119	0	0	-1
N.S.	1	1.00	0.79	3.67	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.681	0.443	0.000	0.138	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	94	190	0	127	0	0	-1
N.S.	1	1.00	0.76	1.54	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.406	0.460	0.000	0.129	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	127	382	0	183	0	0	-1
N.S.	1	1.00	0.89	2.67	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.862	0.528	0.000	0.123	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	712	0	182	0	0	-1
N.S.	1	1.00	0.88	4.98	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.719	0.509	0.000	0.122	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	339	0	143	0	0	-1
N.S.	1	1.00	0.84	3.29	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.699	0.753	0.000	0.128	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	64	2565	0	129	0	0	-1
N.S.	1	1.00	0.67	27.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.098	1.008	0.540	0.000	0.128	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	101	320	0	139	0	0	-1
N.S.	1	1.00	0.73	2.30	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.593	0.517	0.000	0.126	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	92	670	0	147	0	0	-1
N.S.	1	1.00	0.63	4.62	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.024	0.574	0.000	0.144	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	127	359	0	163	0	0	-1
N.S.	1	1.00	0.69	1.95	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.152	2.464	0.543	0.000	0.145	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	126	697	0	173	0	0	-1
N.S.	1	1.00	0.68	3.79	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.143	3.085	0.580	0.000	0.153	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	157	414	0	214	0	0	-1
N.S.	1	1.00	0.79	2.09	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.111	1.875	0.619	0.000	0.127	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	155	759	0	212	0	0	-1
N.S.	1	1.00	0.88	4.31	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.102	1.959	0.602	0.000	0.133	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	132	373	0	175	0	0	-1
N.S.	1	1.00	1.02	2.89	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.077	2.207	1.118	0.000	0.125	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	130	3065	0	180	0	0	-1
N.S.	1	1.00	0.73	17.22	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.061	0.624	0.000	0.130	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	117	342	0	156	0	0	-1
N.S.	1	1.00	0.80	2.34	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.088	1.458	0.746	0.000	0.130	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	150	1923	0	173	0	0	-1
N.S.	1	1.00	0.74	9.43	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.110	1.584	0.579	0.000	0.114	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	150	391	0	191	0	0	-1
N.S.	1	1.00	0.88	2.30	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.102	2.816	0.560	0.000	0.145	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	372	745	0	201	0	0	-1
N.S.	1	1.00	2.11	4.23	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.099	6.426	0.675	0.000	0.157	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	296	430	0	217	0	0	-1
N.S.	1	1.00	1.36	1.97	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	6.504	0.640	0.000	0.160	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	8039	26371	0	0	0	0	-1
N.S.	1	1.00	17.63	57.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	92.791	1.027	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	2853	10704	0	0	0	0	-1
N.S.	1	1.00	7.20	27.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	67.626	0.803	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	6301	3737	0	0	0	0	-1
N.S.	1	1.00	18.87	11.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	42.927	0.801	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	4648	3131	0	0	0	0	-1
N.S.	1	1.00	14.35	9.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	44.541	0.810	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	4497	8825	0	0	0	0	-1
N.S.	1	1.00	9.97	19.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	69.386	1.370	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	9313	6252	0	0	0	0	-1
N.S.	1	1.00	22.07	14.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	67.528	0.859	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	5312	14547	0	0	0	0	-1
N.S.	1	1.00	9.35	25.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	90.739	0.982	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	4735	44463	0	0	0	0	-1
N.S.	1	1.00	9.86	92.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	69.233	1.555	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	3091	5337	0	0	0	0	-1
N.S.	1	1.00	7.02	12.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	42.400	0.957	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	4487	25422	0	0	0	0	-1
N.S.	1	1.00	9.41	53.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	69.392	1.090	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	8876	14301	0	0	0	0	-1
N.S.	1	1.00	20.64	33.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	67.199	1.009	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	5250	38627	0	0	0	0	-1
N.S.	1	1.00	9.46	69.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	90.802	1.337	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	11962	15455	0	0	0	0	-1
N.S.	1	1.00	23.00	29.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	68.059	1.645	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	5832	44329	0	0	0	0	-1
N.S.	1	1.00	8.33	63.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	91.437	2.034	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	8652	101372	0	0	0	0	-1
N.S.	1	1.00	14.84	173.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	69.827	2.525	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	4925	45973	0	0	0	0	-1
N.S.	1	1.00	9.26	86.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	68.912	1.525	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	8774	80250	0	0	0	0	-1
N.S.	1	1.00	15.50	141.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	71.624	2.233	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	4857	82052	0	0	0	0	-1
N.S.	1	1.00	9.43	159.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	69.088	2.094	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	8633	100394	0	0	0	0	-1
N.S.	1	1.00	13.00	151.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	92.402	3.235	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	5131	82289	0	0	0	0	-1
N.S.	1	1.00	8.28	132.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.558	69.795	3.243	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	814	814	9297	114407	0	0	0	0	-1
N.S.	1	1.00	11.42	140.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.691	92.924	3.842	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.549	0.283	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.204	0.283	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	119	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	11.061	0.237	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	94	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.490	0.252	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	108	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	1.888	0.322	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.525	0.287	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	4138	0	0	0	0	0	-1
N.S.	1	1.00	34.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	31.291	0.280	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.571	0.277	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	276	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.623	25.225	0.975	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	280	0	0	0	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	23.662	1.002	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	285	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.588	79.071	0.938	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	6862	0	0	0	0	0	-1
N.S.	1	1.00	11.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	129.531	0.905	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	3398	0	0	0	0	0	-1
N.S.	1	1.00	4.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.682	115.754	1.066	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	4485	0	0	0	0	0	-1
N.S.	1	1.00	6.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.622	73.633	1.061	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	9626	0	0	0	0	0	-1
N.S.	1	1.00	13.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	148.107	1.061	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	5235	0	0	0	0	0	-1
N.S.	1	1.00	7.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.692	74.783	1.059	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	167	334	0	0	0	0	0	-1
N.S.	1	0.97	1.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	6.488	0.373	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	11095	0	0	0	0	0	-1
N.S.	1	1.00	75.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	26.489	0.277	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	3302	0	0	0	0	0	-1
N.S.	1	1.00	35.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	17.050	0.296	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	1158	0	0	0	0	0	-1
N.S.	1	1.00	8.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	14.766	1.015	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	2453	0	0	0	0	0	-1
N.S.	1	1.00	10.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	18.157	1.089	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	187	699	0	0	0	0	0	-1
N.S.	1	1.03	3.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	9.035	0.371	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	377	0	286	420	0	0	-1
N.S.	1	1.00	2.34	0.00	1.78	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.096	2.041	0.387	0.307	0.423	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	116	176	0	0	-1
N.S.	1	1.00	1.15	0.00	1.32	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.668	0.291	0.285	0.385	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	64	0	0	51
N.S.	1	1.00	1.00	1.04	1.00	2.46	0.00	0.00	1.96
time (sec)	N/A	0.032	0.194	0.140	0.287	0.339	0.000	0.000	4.307

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	225	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	1.388	0.398	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	360	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	4.911	0.531	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	306	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	4.219	0.312	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	340	0	0	0	0	0	-1
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	4.300	0.185	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	5.036	0.234	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	5.905	0.826	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	133	241	0	101	0	0	-1
N.S.	1	1.00	1.07	1.94	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.659	1.416	0.000	0.088	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	387	205	0	74	0	0	-1
N.S.	1	1.00	4.30	2.28	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.076	12.656	1.285	0.000	0.085	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	100	168	0	55	0	0	-1
N.S.	1	1.00	1.11	1.87	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.337	1.178	0.000	0.082	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	244	108	0	26	0	0	-1
N.S.	1	1.00	4.07	1.80	0.00	0.43	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.685	0.824	0.000	0.080	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	143	94	0	85	0	0	74
N.S.	1	1.00	2.38	1.57	0.00	1.42	0.00	0.00	1.23
time (sec)	N/A	0.058	1.038	1.230	0.000	0.082	0.000	0.000	0.558

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	369	214	0	141	0	0	-1
N.S.	1	1.03	4.15	2.40	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.079	4.070	1.659	0.000	0.081	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	57	283	0	178	0	0	-1
N.S.	1	1.00	0.59	2.95	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.383	2.135	0.000	0.085	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	596	396	0	221	0	0	-1
N.S.	1	1.00	4.58	3.05	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.094	6.229	3.674	0.000	0.085	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	156	387	0	136	0	0	-1
N.S.	1	1.00	0.82	2.04	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.167	1.361	1.914	0.000	0.102	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	471	351	0	127	0	0	-1
N.S.	1	1.00	3.06	2.28	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.146	3.295	1.755	0.000	0.097	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	131	315	0	112	0	0	-1
N.S.	1	1.00	0.85	2.05	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.758	1.699	0.000	0.087	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	420	277	0	103	0	0	-1
N.S.	1	1.05	3.50	2.31	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.107	1.883	1.412	0.000	0.088	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	158	239	0	84	0	0	-1
N.S.	1	1.05	1.32	1.99	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.698	1.431	0.000	0.081	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	244	206	0	87	0	0	-1
N.S.	1	1.00	2.65	2.24	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.734	1.379	0.000	0.080	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	116	171	0	72	0	0	-1
N.S.	1	1.00	1.26	1.86	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.438	1.257	0.000	0.081	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	255	135	0	105	0	0	-1
N.S.	1	1.00	2.09	1.11	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.061	1.006	0.000	0.083	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	67	208	0	142	0	0	-1
N.S.	1	1.00	0.53	1.65	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.421	1.674	0.000	0.083	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	406	321	0	186	0	0	-1
N.S.	1	1.00	2.48	1.96	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.139	6.102	2.562	0.000	0.086	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	80	97	177	91	0	64	96
N.S.	1	1.00	0.45	0.54	0.99	0.51	0.00	0.36	0.54
time (sec)	N/A	0.265	0.649	6.245	0.622	0.330	0.000	0.747	5.173

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	63	80	129	79	0	50	84
N.S.	1	1.00	0.48	0.61	0.98	0.60	0.00	0.38	0.64
time (sec)	N/A	0.189	0.385	0.944	0.606	0.354	0.000	0.717	0.772

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	56	70	53	67	0	36	88
N.S.	1	1.01	0.66	0.82	0.62	0.79	0.00	0.42	1.04
time (sec)	N/A	0.144	0.240	1.081	0.606	0.357	0.000	0.698	0.533

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	75	51	0	18	-1
N.S.	1	1.00	1.00	1.25	2.08	1.42	0.00	0.50	-0.03
time (sec)	N/A	0.086	0.189	1.079	0.559	0.339	0.000	0.691	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	125	225	1399	301	0	0	-1
N.S.	1	1.00	0.37	0.67	4.18	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.958	1.475	0.672	0.356	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	620	274	309	1825	433	0	0	-1
N.S.	1	1.18	0.52	0.59	3.48	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.411	3.945	1.163	0.826	0.345	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	227	366	2232	522	0	0	-1
N.S.	1	1.00	0.44	0.71	4.36	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.375	2.594	1.135	0.729	0.346	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	719	305	417	2638	600	0	0	-1
N.S.	1	1.00	0.42	0.58	3.67	0.83	0.00	0.00	-0.00
time (sec)	N/A	0.576	2.962	1.101	0.848	0.340	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	80	110	177	94	0	0	110
N.S.	1	1.00	0.46	0.63	1.01	0.54	0.00	0.00	0.63
time (sec)	N/A	0.252	0.708	1.048	0.601	0.323	0.000	0.000	5.239

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	63	100	129	82	0	0	100
N.S.	1	1.00	0.50	0.79	1.02	0.65	0.00	0.00	0.79
time (sec)	N/A	0.209	0.423	0.918	0.604	0.325	0.000	0.000	1.129

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	74	79	70	0	0	82
N.S.	1	1.00	0.60	0.92	0.99	0.88	0.00	0.00	1.02
time (sec)	N/A	0.140	0.229	1.027	0.617	0.328	0.000	0.000	0.731

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	67	75	54	0	0	-1
N.S.	1	1.00	1.00	1.86	2.08	1.50	0.00	0.00	-0.03
time (sec)	N/A	0.095	0.203	0.930	0.558	0.318	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	209	230	713	321	0	0	-1
N.S.	1	1.00	0.42	0.46	1.44	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.238	10.550	4.864	0.629	0.360	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	250	314	2137	465	0	0	-1
N.S.	1	1.00	0.53	0.67	4.55	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.299	1.932	0.988	0.703	0.361	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	245	371	2238	557	0	0	-1
N.S.	1	1.00	0.36	0.54	3.28	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.492	1.629	1.053	0.736	0.367	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	202	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	13.494	0.758	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	144	0	0	0	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	1.565	0.400	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	131	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	6.595	0.419	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	201	0	0	0	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	59.195	0.970	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	264	0	0	0	0	0	-1
N.S.	1	1.00	3.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	67.570	0.934	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.974	1.110	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	143	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	14.868	1.010	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	175	212	0	0	0	0	0	-1
N.S.	1	0.96	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	3.369	0.843	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	330	0	0	0	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	4.132	0.385	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	203	0	0	0	0	0	-1
N.S.	1	1.01	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	1.141	0.342	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	1132	0	0	0	0	0	-1
N.S.	1	1.00	8.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	13.965	0.994	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	2502	0	0	0	0	0	-1
N.S.	1	1.00	11.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	17.259	1.287	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	365	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	22.250	0.360	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [501] had the largest ratio of [32]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	3	2	1.00	22	0.091
3	A	3	2	1.00	22	0.091
4	A	3	2	1.00	22	0.091
5	A	3	3	1.11	22	0.136
6	A	2	1	1.00	13	0.077
7	A	3	3	1.00	22	0.136
8	A	4	3	1.00	22	0.136
9	A	5	3	1.00	22	0.136
10	A	6	3	1.00	22	0.136
11	A	5	3	1.00	22	0.136
12	A	4	3	1.00	22	0.136
13	A	3	3	1.00	22	0.136
14	A	2	2	1.00	20	0.100
15	A	2	2	1.00	20	0.100
16	A	3	2	1.00	22	0.091
17	A	3	2	1.00	22	0.091
18	A	3	2	1.00	22	0.091
19	A	3	2	1.00	24	0.083
20	A	3	2	1.00	24	0.083
21	A	3	2	1.00	24	0.083
22	A	2	2	1.00	24	0.083
23	A	2	2	1.00	15	0.133
24	A	2	2	1.00	24	0.083
25	A	4	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	24	0.125
27	A	4	3	1.00	24	0.125
28	A	5	4	1.00	24	0.167
29	A	4	4	1.00	24	0.167
30	A	3	3	1.00	22	0.136
31	A	2	2	1.00	22	0.091
32	A	2	2	1.00	24	0.083
33	A	3	2	1.00	24	0.083
34	A	3	2	1.00	24	0.083
35	A	3	2	1.00	24	0.083
36	A	3	2	1.00	24	0.083
37	A	3	2	1.00	24	0.083
38	A	3	2	1.00	24	0.083
39	A	2	2	1.00	24	0.083
40	A	3	3	1.00	15	0.200
41	A	3	2	1.00	24	0.083
42	A	2	2	1.00	24	0.083
43	A	4	3	1.00	24	0.125
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	22	0.136
47	A	3	3	1.00	22	0.136
48	A	1	1	1.00	24	0.042
49	A	4	3	1.00	24	0.125
50	A	4	3	1.00	24	0.125
51	A	4	3	1.00	24	0.125
52	A	6	4	1.00	24	0.167
53	A	5	3	1.00	22	0.136
54	A	4	4	1.00	22	0.182
55	A	3	2	1.00	24	0.083
56	A	2	2	1.00	24	0.083
57	A	4	2	1.00	24	0.083
58	A	4	2	1.00	24	0.083
59	A	3	2	1.00	24	0.083
60	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	24	0.083
62	A	2	2	1.00	24	0.083
63	A	5	3	1.00	15	0.200
64	A	3	2	1.00	24	0.083
65	A	3	2	1.00	24	0.083
66	A	3	2	1.00	24	0.083
67	A	2	2	1.00	24	0.083
68	A	4	3	1.00	24	0.125
69	A	4	3	1.00	24	0.125
70	A	6	3	1.00	22	0.136
71	A	5	4	1.00	22	0.182
72	A	4	3	1.00	24	0.125
73	A	1	1	1.00	24	0.042
74	A	3	2	1.00	24	0.083
75	A	5	3	1.00	24	0.125
76	A	5	3	1.00	24	0.125
77	A	3	2	1.00	24	0.083
78	A	3	2	1.00	24	0.083
79	A	3	2	1.00	24	0.083
80	A	2	2	1.00	24	0.083
81	A	8	3	1.00	15	0.200
82	A	3	2	1.00	24	0.083
83	A	3	2	1.00	24	0.083
84	A	3	2	1.00	24	0.083
85	A	2	2	1.00	24	0.083
86	A	3	2	1.00	24	0.083
87	A	3	2	1.00	24	0.083
88	A	2	2	1.00	24	0.083
89	A	4	3	1.00	24	0.125
90	A	4	3	1.00	24	0.125
91	A	8	4	1.00	22	0.182
92	A	7	4	1.00	24	0.167
93	A	6	4	1.00	24	0.167
94	A	5	2	1.00	24	0.083
95	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	2	1.00	24	0.083
97	A	6	2	1.00	24	0.083
98	A	6	2	1.00	24	0.083
99	A	3	2	1.00	24	0.083
100	A	3	2	1.00	24	0.083
101	A	3	2	1.00	24	0.083
102	A	2	1	1.00	24	0.042
103	A	2	2	1.00	24	0.083
104	A	2	2	1.00	15	0.133
105	A	4	3	1.00	24	0.125
106	A	4	3	1.00	24	0.125
107	A	4	3	1.00	24	0.125
108	A	3	3	1.00	24	0.125
109	A	2	2	1.00	24	0.083
110	A	1	1	1.00	22	0.045
111	A	2	2	1.00	22	0.091
112	A	3	2	1.00	24	0.083
113	A	3	2	1.00	24	0.083
114	A	3	2	1.00	24	0.083
115	A	3	2	1.00	24	0.083
116	A	2	2	1.00	24	0.083
117	A	3	2	1.00	24	0.083
118	A	2	2	1.00	24	0.083
119	A	3	2	1.00	15	0.133
120	A	4	3	1.00	24	0.125
121	A	4	3	1.00	24	0.125
122	A	5	3	1.00	24	0.125
123	A	4	3	1.00	24	0.125
124	A	3	3	1.00	24	0.125
125	A	2	2	1.00	24	0.083
126	A	2	2	1.00	22	0.091
127	A	3	2	1.00	22	0.091
128	A	3	2	1.00	24	0.083
129	A	3	2	1.00	24	0.083
130	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	2	1.00	24	0.083
132	A	3	2	1.00	24	0.083
133	A	2	2	1.00	24	0.083
134	A	3	2	1.00	24	0.083
135	A	3	2	1.00	24	0.083
136	A	2	2	1.00	24	0.083
137	A	4	2	1.00	15	0.133
138	A	4	3	1.00	24	0.125
139	A	4	3	1.00	24	0.125
140	A	5	4	1.00	24	0.167
141	A	4	4	1.00	24	0.167
142	A	3	3	1.00	24	0.125
143	A	1	1	1.00	24	0.042
144	A	3	2	1.00	22	0.091
145	A	4	3	1.00	22	0.136
146	A	4	3	1.00	24	0.125
147	A	4	3	1.00	24	0.125
148	A	3	2	1.00	24	0.083
149	A	3	2	1.00	24	0.083
150	A	2	2	1.00	24	0.083
151	A	3	2	1.00	24	0.083
152	A	3	2	1.00	24	0.083
153	A	2	2	1.00	24	0.083
154	A	2	2	1.00	24	0.083
155	A	5	2	1.00	15	0.133
156	A	4	3	1.00	24	0.125
157	A	4	3	1.00	24	0.125
158	A	5	3	1.00	24	0.125
159	A	4	3	1.00	24	0.125
160	A	3	2	1.00	24	0.083
161	A	2	2	1.00	24	0.083
162	A	4	2	1.00	22	0.091
163	A	5	3	1.00	22	0.136
164	A	5	3	1.00	24	0.125
165	A	5	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	24	0.083
168	A	3	2	1.00	24	0.083
169	A	2	2	1.00	24	0.083
170	A	3	2	1.00	24	0.083
171	A	3	2	1.00	24	0.083
172	A	2	2	1.00	24	0.083
173	A	9	2	1.00	15	0.133
174	A	4	3	1.00	24	0.125
175	A	4	3	1.00	24	0.125
176	A	7	3	1.00	24	0.125
177	A	6	3	1.00	24	0.125
178	A	5	2	1.00	24	0.083
179	A	2	2	1.00	24	0.083
180	A	4	2	1.00	24	0.083
181	A	6	2	1.00	24	0.083
182	A	8	2	1.00	22	0.091
183	A	9	3	1.00	22	0.136
184	A	9	3	1.00	24	0.125
185	A	5	4	1.00	26	0.154
186	A	4	4	1.00	26	0.154
187	A	4	4	1.00	26	0.154
188	A	3	3	1.00	26	0.115
189	A	3	3	1.00	26	0.115
190	A	4	4	1.00	26	0.154
191	A	4	4	1.00	26	0.154
192	A	5	4	1.00	26	0.154
193	A	5	5	1.00	28	0.179
194	A	4	4	1.00	28	0.143
195	A	4	4	1.00	28	0.143
196	A	3	3	1.00	28	0.107
197	A	3	3	1.00	28	0.107
198	A	4	4	1.00	28	0.143
199	A	4	4	1.00	28	0.143
200	A	5	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	7	5	1.00	28	0.179
202	A	6	5	1.00	28	0.179
203	A	6	5	1.00	28	0.179
204	A	5	4	1.00	28	0.143
205	A	5	5	1.18	28	0.179
206	A	4	4	1.00	28	0.143
207	A	4	4	1.00	28	0.143
208	A	4	4	1.00	28	0.143
209	A	4	4	1.00	28	0.143
210	A	5	5	1.00	28	0.179
211	A	5	5	1.00	28	0.179
212	A	6	5	1.00	28	0.179
213	A	7	5	1.00	28	0.179
214	A	6	4	1.00	28	0.143
215	A	6	6	1.00	28	0.214
216	A	5	5	1.00	28	0.179
217	A	5	4	1.00	28	0.143
218	A	4	3	1.00	28	0.107
219	A	4	3	1.00	28	0.107
220	A	5	4	1.00	28	0.143
221	A	5	4	1.00	28	0.143
222	A	6	4	1.00	28	0.143
223	A	5	4	1.00	28	0.143
224	A	4	4	1.00	28	0.143
225	A	4	4	1.00	28	0.143
226	A	3	3	1.00	28	0.107
227	A	3	3	1.00	28	0.107
228	A	3	3	1.00	28	0.107
229	A	3	3	1.00	28	0.107
230	A	4	4	1.00	28	0.143
231	A	4	4	1.00	28	0.143
232	A	5	4	1.00	28	0.143
233	A	6	4	1.00	28	0.143
234	A	5	4	1.00	28	0.143
235	A	5	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	28	0.143
237	A	4	4	1.00	28	0.143
238	A	3	3	1.00	28	0.107
239	A	3	3	1.00	28	0.107
240	A	4	4	1.00	28	0.143
241	A	4	4	1.00	28	0.143
242	A	5	4	1.00	28	0.143
243	A	5	4	1.00	28	0.143
244	A	6	4	1.00	28	0.143
245	A	6	5	1.00	28	0.179
246	A	5	5	1.00	28	0.179
247	A	5	5	1.00	28	0.179
248	A	4	4	1.00	28	0.143
249	A	4	4	1.00	28	0.143
250	A	4	4	1.00	28	0.143
251	A	4	4	1.00	28	0.143
252	A	5	5	1.00	28	0.179
253	A	5	5	1.00	28	0.179
254	A	6	5	1.00	28	0.179
255	A	6	4	1.00	28	0.143
256	A	5	4	1.00	28	0.143
257	A	5	4	1.00	28	0.143
258	A	4	3	1.00	28	0.107
259	A	4	3	1.00	28	0.107
260	A	5	4	1.00	28	0.143
261	A	5	4	1.00	28	0.143
262	A	6	5	1.00	28	0.179
263	A	4	4	1.00	26	0.154
264	A	4	4	1.00	26	0.154
265	A	4	4	1.00	26	0.154
266	A	4	4	1.00	26	0.154
267	A	4	4	1.00	28	0.143
268	A	4	4	1.00	28	0.143
269	A	4	4	1.00	28	0.143
270	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	4	1.00	28	0.143
272	A	4	4	1.00	28	0.143
273	A	4	4	1.00	28	0.143
274	A	4	4	1.00	28	0.143
275	A	4	4	1.00	28	0.143
276	A	4	4	1.00	28	0.143
277	A	4	4	1.00	28	0.143
278	A	4	4	1.00	28	0.143
279	A	3	2	1.00	26	0.077
280	A	3	2	1.00	26	0.077
281	A	3	2	1.00	26	0.077
282	A	2	2	1.00	26	0.077
283	A	5	5	1.00	26	0.192
284	A	7	5	1.00	26	0.192
285	A	9	5	1.00	26	0.192
286	A	4	2	1.00	26	0.077
287	A	3	2	1.00	26	0.077
288	A	2	2	1.00	26	0.077
289	A	1	1	1.00	24	0.042
290	A	3	3	1.00	24	0.125
291	A	5	5	1.00	26	0.192
292	A	7	5	1.00	26	0.192
293	A	3	2	1.00	26	0.077
294	A	3	2	1.00	26	0.077
295	A	3	2	1.00	26	0.077
296	A	2	2	1.00	26	0.077
297	A	4	4	1.00	26	0.154
298	A	6	5	1.00	26	0.192
299	A	8	5	1.00	26	0.192
300	A	4	2	1.00	26	0.077
301	A	3	2	1.00	26	0.077
302	A	2	2	1.00	24	0.083
303	A	1	1	1.00	24	0.042
304	A	4	3	1.00	26	0.115
305	A	6	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	2	1.00	26	0.077
307	A	3	2	1.00	26	0.077
308	A	3	2	1.00	26	0.077
309	A	2	2	1.00	26	0.077
310	A	4	4	1.00	26	0.154
311	A	5	4	1.00	26	0.154
312	A	7	5	1.00	26	0.192
313	A	4	2	1.00	26	0.077
314	A	3	2	1.00	24	0.083
315	A	2	2	1.00	24	0.083
316	A	1	1	1.00	26	0.038
317	A	5	3	1.00	26	0.115
318	A	7	5	1.00	26	0.192
319	A	3	2	1.00	26	0.077
320	A	3	2	1.00	26	0.077
321	A	3	2	1.00	26	0.077
322	A	2	2	1.00	26	0.077
323	A	5	5	1.00	26	0.192
324	A	5	5	1.00	26	0.192
325	A	6	4	1.00	26	0.154
326	A	4	2	1.00	24	0.083
327	A	3	2	1.00	24	0.083
328	A	2	2	1.00	26	0.077
329	A	1	1	1.00	26	0.038
330	A	6	3	1.00	26	0.115
331	A	8	5	1.00	26	0.192
332	A	10	5	1.00	26	0.192
333	A	3	2	1.00	26	0.077
334	A	3	2	1.00	26	0.077
335	A	3	2	1.00	26	0.077
336	A	2	2	1.00	26	0.077
337	A	6	5	1.00	26	0.192
338	A	8	5	1.00	26	0.192
339	A	10	5	1.00	26	0.192
340	A	4	2	1.00	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	3	2	1.00	26	0.077
342	A	2	2	1.00	26	0.077
343	A	1	1	1.00	26	0.038
344	A	2	2	1.00	24	0.083
345	A	4	4	1.00	24	0.167
346	A	6	5	1.00	26	0.192
347	A	3	2	1.00	26	0.077
348	A	3	2	1.00	26	0.077
349	A	3	2	1.00	26	0.077
350	A	2	2	1.00	26	0.077
351	A	7	5	1.00	26	0.192
352	A	9	5	1.00	26	0.192
353	A	11	5	1.00	26	0.192
354	A	4	2	1.00	26	0.077
355	A	3	2	1.00	26	0.077
356	A	2	2	1.00	26	0.077
357	A	1	1	1.00	26	0.038
358	A	3	3	1.00	26	0.115
359	A	3	3	1.00	24	0.125
360	A	5	4	1.00	24	0.167
361	A	7	5	1.00	26	0.192
362	A	3	2	1.00	26	0.077
363	A	3	2	1.00	26	0.077
364	A	3	2	1.00	26	0.077
365	A	3	2	1.00	26	0.077
366	A	2	2	1.00	26	0.077
367	A	8	5	1.00	26	0.192
368	A	10	5	1.00	26	0.192
369	A	4	2	1.00	26	0.077
370	A	3	2	1.00	26	0.077
371	A	2	2	1.00	26	0.077
372	A	1	1	1.00	26	0.038
373	A	4	3	1.00	26	0.115
374	A	4	4	1.00	26	0.154
375	A	4	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	6	4	1.00	24	0.167
377	A	8	5	1.00	26	0.192
378	A	3	2	1.00	26	0.077
379	A	3	2	1.00	26	0.077
380	A	3	2	1.00	26	0.077
381	A	3	2	1.00	26	0.077
382	A	2	2	1.00	26	0.077
383	A	9	5	1.00	26	0.192
384	A	11	5	1.00	26	0.192
385	A	3	2	1.00	26	0.077
386	A	2	2	1.00	26	0.077
387	A	1	1	1.00	26	0.038
388	A	5	3	1.00	26	0.115
389	A	5	4	1.00	26	0.154
390	A	5	4	1.00	26	0.154
391	A	5	3	1.00	24	0.125
392	A	7	4	1.00	24	0.167
393	A	9	5	1.00	26	0.192
394	A	12	9	1.00	30	0.300
395	A	10	7	1.00	30	0.233
396	A	1	1	1.00	30	0.033
397	A	2	2	1.00	30	0.067
398	A	3	3	1.00	30	0.100
399	A	4	3	1.00	30	0.100
400	A	13	9	1.00	30	0.300
401	A	13	9	1.00	30	0.300
402	A	11	8	1.00	30	0.267
403	A	12	9	1.00	30	0.300
404	A	1	1	1.00	30	0.033
405	A	2	2	1.00	30	0.067
406	A	3	2	1.00	30	0.067
407	A	4	3	1.00	30	0.100
408	A	14	9	1.00	30	0.300
409	A	12	8	1.00	30	0.267
410	A	13	10	1.00	30	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	11	8	1.00	30	0.267
412	A	1	1	1.00	30	0.033
413	A	2	2	1.00	30	0.067
414	A	3	2	1.00	30	0.067
415	A	4	2	1.00	30	0.067
416	A	11	8	1.00	30	0.267
417	A	11	8	1.00	30	0.267
418	A	1	1	1.00	30	0.033
419	A	2	2	1.00	30	0.067
420	A	3	3	1.00	30	0.100
421	A	4	3	1.00	30	0.100
422	A	5	3	1.00	30	0.100
423	A	13	10	1.00	30	0.333
424	A	11	8	1.00	30	0.267
425	A	1	1	1.00	30	0.033
426	A	2	2	1.00	30	0.067
427	A	3	2	1.00	30	0.067
428	A	4	3	1.00	30	0.100
429	A	5	3	1.00	30	0.100
430	A	12	9	1.00	30	0.300
431	A	12	9	1.00	30	0.300
432	A	1	1	1.00	30	0.033
433	A	2	2	1.00	30	0.067
434	A	3	2	1.00	30	0.067
435	A	4	2	1.00	30	0.067
436	A	5	3	1.00	30	0.100
437	A	4	4	1.00	30	0.133
438	A	4	4	1.00	30	0.133
439	A	4	4	1.00	30	0.133
440	A	4	4	1.00	30	0.133
441	A	4	4	1.00	30	0.133
442	A	4	4	1.00	30	0.133
443	A	9	8	1.00	30	0.267
444	A	8	8	1.00	30	0.267
445	A	6	6	1.00	30	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	1	1	1.00	30	0.033
447	A	2	2	1.00	30	0.067
448	A	3	2	1.00	30	0.067
449	A	4	2	1.00	30	0.067
450	A	4	4	1.00	26	0.154
451	A	4	4	1.00	26	0.154
452	A	4	4	1.00	26	0.154
453	A	4	4	1.00	24	0.167
454	A	4	4	1.00	26	0.154
455	A	4	4	1.00	26	0.154
456	A	4	4	1.00	26	0.154
457	A	4	4	1.00	28	0.143
458	A	4	4	1.00	28	0.143
459	A	4	4	1.00	28	0.143
460	A	4	4	1.00	28	0.143
461	A	4	4	1.00	28	0.143
462	A	4	4	1.00	28	0.143
463	A	4	4	1.00	28	0.143
464	A	4	4	1.00	26	0.154
465	A	3	2	1.00	24	0.083
466	A	3	2	1.00	24	0.083
467	A	2	2	1.00	24	0.083
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	2	2	1.00	24	0.083
471	A	4	4	1.00	24	0.167
472	A	4	4	1.00	24	0.167
473	A	4	4	1.00	22	0.182
474	A	4	4	1.00	22	0.182
475	A	4	4	1.00	24	0.167
476	A	4	4	1.00	24	0.167
477	A	4	4	1.00	28	0.143
478	A	4	4	1.00	28	0.143
479	A	4	4	1.00	28	0.143
480	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	4	4	1.00	28	0.143
482	A	4	4	1.00	28	0.143
483	A	5	2	1.00	30	0.067
484	A	4	2	1.00	30	0.067
485	A	3	2	1.00	30	0.067
486	A	2	2	1.00	30	0.067
487	A	1	1	1.00	28	0.036
488	A	4	4	1.00	30	0.133
489	A	4	4	1.00	30	0.133
490	A	4	4	1.00	30	0.133
491	A	3	2	1.00	30	0.067
492	A	5	5	1.00	30	0.167
493	A	2	2	1.00	30	0.067
494	A	5	5	1.00	30	0.167
495	A	1	1	1.00	30	0.033
496	A	5	5	1.00	30	0.167
497	A	3	3	1.00	28	0.107
498	A	5	5	1.00	30	0.167
499	A	4	4	1.00	30	0.133
500	A	5	5	1.00	30	0.167
501	A	4	4	1.00	32	0.125
502	A	4	4	1.00	32	0.125
503	A	3	3	1.00	30	0.100
504	A	1	1	1.00	32	0.031
505	A	2	2	1.00	32	0.062
506	A	3	2	1.00	32	0.062
507	A	3	2	1.00	19	0.105
508	A	4	3	1.00	19	0.158
509	A	3	2	1.00	19	0.105
510	A	3	3	1.00	19	0.158
511	A	3	3	1.00	19	0.158
512	A	2	2	1.00	17	0.118
513	A	2	2	1.00	17	0.118
514	A	3	3	1.00	19	0.158
515	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	3	1.00	19	0.158
517	A	4	3	1.00	21	0.143
518	A	4	3	1.00	21	0.143
519	A	3	2	1.00	21	0.095
520	A	2	2	1.00	21	0.095
521	A	3	3	1.00	21	0.143
522	A	4	4	1.00	21	0.190
523	A	6	4	1.00	21	0.190
524	A	5	4	1.00	21	0.190
525	A	4	4	1.00	21	0.190
526	A	3	3	1.00	19	0.158
527	A	1	1	1.00	19	0.053
528	A	4	3	1.00	21	0.143
529	A	4	3	1.00	21	0.143
530	A	4	3	1.00	21	0.143
531	A	4	3	1.00	21	0.143
532	A	3	2	1.00	21	0.095
533	A	3	2	1.00	21	0.095
534	A	2	2	1.00	21	0.095
535	A	6	6	1.00	21	0.286
536	A	4	4	1.00	21	0.190
537	A	6	5	1.11	21	0.238
538	A	5	5	1.14	21	0.238
539	A	4	4	1.20	19	0.210
540	A	4	4	1.21	19	0.210
541	A	3	3	1.00	21	0.143
542	A	4	4	1.00	21	0.190
543	A	5	5	1.00	21	0.238
544	A	3	2	1.00	21	0.095
545	A	3	2	1.00	21	0.095
546	A	2	2	1.00	21	0.095
547	A	7	6	1.00	21	0.286
548	A	8	7	1.00	21	0.333
549	A	9	6	1.09	21	0.286
550	A	5	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	2	2	1.00	19	0.105
552	A	5	5	1.00	19	0.263
553	A	9	6	1.00	21	0.286
554	A	3	2	1.00	21	0.095
555	A	3	2	1.00	21	0.095
556	A	3	2	1.00	21	0.095
557	A	2	2	1.00	21	0.095
558	A	7	6	1.00	21	0.286
559	A	8	7	1.00	21	0.333
560	A	8	7	1.00	21	0.333
561	A	7	7	1.00	21	0.333
562	A	6	6	1.45	21	0.286
563	A	4	4	1.28	19	0.210
564	A	5	5	1.00	19	0.263
565	A	6	6	1.00	21	0.286
566	A	3	2	1.00	21	0.095
567	A	3	2	1.00	21	0.095
568	A	3	2	1.00	21	0.095
569	A	2	2	1.00	21	0.095
570	A	7	6	1.00	21	0.286
571	A	8	7	1.00	21	0.333
572	A	8	8	1.00	21	0.381
573	A	7	7	1.28	21	0.333
574	A	4	4	1.24	21	0.190
575	A	5	5	1.00	19	0.263
576	A	6	6	1.00	19	0.316
577	A	7	7	1.00	21	0.333
578	A	5	4	1.00	23	0.174
579	A	4	4	1.00	23	0.174
580	A	4	4	1.00	23	0.174
581	A	3	3	1.00	23	0.130
582	A	3	3	1.00	23	0.130
583	A	4	4	1.00	23	0.174
584	A	4	4	1.00	23	0.174
585	A	5	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	5	5	1.00	25	0.200
587	A	5	5	1.00	25	0.200
588	A	4	4	1.00	25	0.160
589	A	4	4	1.00	25	0.160
590	A	5	5	1.00	25	0.200
591	A	5	5	1.00	25	0.200
592	A	6	5	1.00	25	0.200
593	A	6	5	1.00	25	0.200
594	A	5	5	1.00	25	0.200
595	A	5	5	1.00	25	0.200
596	A	4	4	1.00	25	0.160
597	A	5	5	1.00	25	0.200
598	A	4	4	1.00	25	0.160
599	A	5	5	1.00	25	0.200
600	A	4	4	1.00	25	0.160
601	A	4	4	1.00	25	0.160
602	A	5	5	1.00	25	0.200
603	A	17	15	1.00	25	0.600
604	A	17	15	1.00	25	0.600
605	A	13	11	1.00	25	0.440
606	A	14	12	1.00	25	0.480
607	A	17	15	1.00	25	0.600
608	A	17	15	1.00	25	0.600
609	A	18	16	1.00	25	0.640
610	A	17	15	1.00	25	0.600
611	A	17	15	1.00	25	0.600
612	A	17	15	1.00	25	0.600
613	A	17	15	1.00	25	0.600
614	A	18	16	1.00	25	0.640
615	A	18	16	1.00	25	0.640
616	A	19	17	1.00	25	0.680
617	A	18	16	1.00	25	0.640
618	A	18	16	1.00	25	0.640
619	A	18	16	1.00	25	0.640
620	A	18	16	1.00	25	0.640

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	19	16	1.00	25	0.640
622	A	19	16	1.00	25	0.640
623	A	20	17	1.00	25	0.680
624	A	3	3	1.00	23	0.130
625	A	3	3	1.00	23	0.130
626	A	3	3	1.00	23	0.130
627	A	3	3	1.00	23	0.130
628	A	4	4	1.00	25	0.160
629	A	4	4	1.00	25	0.160
630	A	4	4	1.00	25	0.160
631	A	4	4	1.00	25	0.160
632	A	16	11	1.00	25	0.440
633	A	16	11	1.00	25	0.440
634	A	17	12	1.00	25	0.480
635	A	17	12	1.00	25	0.480
636	A	18	13	1.00	25	0.520
637	A	18	13	1.00	25	0.520
638	A	19	14	1.00	25	0.560
639	A	19	14	1.00	25	0.560
640	A	4	4	0.97	23	0.174
641	A	4	4	1.00	23	0.174
642	A	3	3	1.00	21	0.143
643	A	6	5	1.00	23	0.217
644	A	7	6	1.00	23	0.261
645	A	3	3	1.03	23	0.130
646	A	3	2	1.00	21	0.095
647	A	3	2	1.00	21	0.095
648	A	2	2	1.00	21	0.095
649	A	6	4	1.00	21	0.190
650	A	7	5	1.00	21	0.238
651	A	3	3	1.00	21	0.143
652	A	3	3	1.00	19	0.158
653	A	3	3	1.00	19	0.158
654	A	3	3	1.00	21	0.143
655	A	6	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	5	5	1.00	26	0.192
657	A	5	5	1.00	26	0.192
658	A	4	4	1.00	26	0.154
659	A	4	4	1.00	26	0.154
660	A	5	5	1.03	26	0.192
661	A	5	5	1.00	26	0.192
662	A	6	5	1.00	26	0.192
663	A	7	5	1.00	28	0.179
664	A	6	5	1.00	28	0.179
665	A	6	5	1.00	28	0.179
666	A	5	5	1.05	28	0.179
667	A	5	5	1.05	28	0.179
668	A	4	4	1.00	28	0.143
669	A	4	4	1.00	28	0.143
670	A	5	5	1.00	28	0.179
671	A	5	5	1.00	28	0.179
672	A	6	5	1.00	28	0.179
673	A	5	4	1.00	30	0.133
674	A	4	4	1.00	30	0.133
675	A	3	3	1.01	30	0.100
676	A	2	2	1.00	30	0.067
677	A	10	7	1.00	30	0.233
678	A	13	10	1.18	30	0.333
679	A	13	10	1.00	30	0.333
680	A	15	11	1.00	30	0.367
681	A	5	4	1.00	30	0.133
682	A	4	4	1.00	30	0.133
683	A	3	3	1.00	30	0.100
684	A	2	2	1.00	30	0.067
685	A	11	8	1.00	30	0.267
686	A	12	9	1.00	30	0.300
687	A	14	11	1.00	30	0.367
688	A	5	5	1.00	26	0.192
689	A	5	5	1.00	26	0.192
690	A	5	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	5	5	1.00	26	0.192
692	A	5	5	1.00	26	0.192
693	A	5	5	1.00	28	0.179
694	A	5	5	1.00	28	0.179
695	A	5	5	0.96	23	0.217
696	A	5	5	1.00	23	0.217
697	A	4	4	1.01	21	0.190
698	A	7	6	1.00	23	0.261
699	A	8	7	1.00	23	0.304
700	A	4	4	1.00	23	0.174

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\tan(c+dx)}{d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{a\tan^9(c+dx)}{9d}$$

```
[Out] 1/10*I*a*sec(d*x+c)^10/d+a*tan(d*x+c)/d+4/3*a*tan(d*x+c)^3/d+6/5*a*tan(d*x+
c)^5/d+4/7*a*tan(d*x+c)^7/d+1/9*a*tan(d*x+c)^9/d
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3567, 3852}

$$\frac{a\tan^9(c+dx)}{9d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*Tan[c + d*x])/d + (4*a*Tan[c + d*x]^3)/(3
*d) + (6*a*Tan[c + d*x]^5)/(5*d) + (4*a*Tan[c + d*x]^7)/(7*d) + (a*Tan[c +
d*x]^9)/(9*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^{10}(c+dx)}{10d}+a\int \sec^{10}(c+dx)\,dx\\ &= \frac{ia\sec^{10}(c+dx)}{10d}-\frac{a\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-\text{t}\right.}{d}\\ &= \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\tan(c+dx)}{d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{6a\tan^5(c+}{5d}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 79, normalized size = 0.84

$$\frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\left(\tan(c+dx)+\frac{4}{3}\tan^3(c+dx)+\frac{6}{5}\tan^5(c+dx)+\frac{4}{7}\tan^7(c+dx)+\frac{1}{9}\tan^9(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*(Tan[c + d*x] + (4*Tan[c + d*x]^3)/3 + (6
*Tan[c + d*x]^5)/5 + (4*Tan[c + d*x]^7)/7 + Tan[c + d*x]^9/9))/d
```

Maple [A]
time = 0.32, size = 69, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\frac{ia}{10\cos(dx+c)^{10}}-a\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)}{d}$	69
default	$\frac{\frac{ia}{10\cos(dx+c)^{10}}-a\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)}{d}$	69
risch	$\frac{256ia\left(252\,e^{10i(dx+c)}+210\,e^{8i(dx+c)}+120\,e^{6i(dx+c)}+45\,e^{4i(dx+c)}+10\,e^{2i(dx+c)}+1\right)}{315d\left(e^{2i(dx+c)}+1\right)^{10}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/10*I*a/cos(d*x+c)^10-a*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-
16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 114, normalized size = 1.21

$$\frac{63i\,a\tan(dx+c)^{10}+70\,a\tan(dx+c)^9+315i\,a\tan(dx+c)^8+360\,a\tan(dx+c)^7+630i\,a\tan(dx+c)^6+756\,a\tan(dx+c)^5+630i\,a\tan(dx+c)^4+840\,a\tan(dx+c)^3+315i\,a\tan(dx+c)^2+630\,a\tan(dx+c)}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/630*(63*I*a*tan(d*x + c)^10 + 70*a*tan(d*x + c)^9 + 315*I*a*tan(d*x + c)^
8 + 360*a*tan(d*x + c)^7 + 630*I*a*tan(d*x + c)^6 + 756*a*tan(d*x + c)^5 +
630*I*a*tan(d*x + c)^4 + 840*a*tan(d*x + c)^3 + 315*I*a*tan(d*x + c)^2 + 63
0*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(82) = 164$.
time = 0.38, size = 189, normalized size = 2.01

$$-\frac{256\left(-252i\,ae^{(10i\,dx+10i\,c)}-210i\,ae^{(8i\,dx+8i\,c)}-120i\,ae^{(6i\,dx+6i\,c)}-45i\,ae^{(4i\,dx+4i\,c)}-10i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{315\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+210\,de^{(8i\,dx+8i\,c)}+120\,de^{(6i\,dx+6i\,c)}+45\,de^{(4i\,dx+4i\,c)}+10\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -256/315*(-252*I*a*e^(10*I*d*x + 10*I*c) - 210*I*a*e^(8*I*d*x + 8*I*c) - 12
0*I*a*e^(6*I*d*x + 6*I*c) - 45*I*a*e^(4*I*d*x + 4*I*c) - 10*I*a*e^(2*I*d*x
+ 2*I*c) - I*a)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*
d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x +
 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*
e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [A]
time = 5.64, size = 83, normalized size = 0.88

$$\begin{cases} \frac{a\left(\frac{\tan^{9}{(c+dx)}}{9}+\frac{4\tan^{7}{(c+dx)}}{7}+\frac{6\tan^{5}{(c+dx)}}{5}+\frac{4\tan^{3}{(c+dx)}}{3}+\tan{(c+dx)}\right)+\frac{ia\sec^{10}{(c+dx)}}{10}}{d} & \text{for } d \neq 0 \\ x(ia\tan{(c)}+a)\sec^{10}{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**9/9 + 4*tan(c + d*x)**7/7 + 6*tan(c + d*x)**5/
5 + 4*tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**10/10)/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c)**10, True))
```

Giac [A]
time = 0.55, size = 114, normalized size = 1.21

$$-\frac{-63i\,a\tan{(dx+c)}^{10}-70\,a\tan{(dx+c)}^{9}-315i\,a\tan{(dx+c)}^{8}-360\,a\tan{(dx+c)}^{7}-630i\,a\tan{(dx+c)}^{6}-756\,a\tan{(dx+c)}^{5}-630i\,a\tan{(dx+c)}^{4}-840\,a\tan{(dx+c)}^{3}-315i\,a\tan{(dx+c)}^{2}-630\,a\tan{(dx+c)}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/630*(-63*I*a*tan(d*x + c)^10 - 70*a*tan(d*x + c)^9 - 315*I*a*tan(d*x + c
)^8 - 360*a*tan(d*x + c)^7 - 630*I*a*tan(d*x + c)^6 - 756*a*tan(d*x + c)^5
- 630*I*a*tan(d*x + c)^4 - 840*a*tan(d*x + c)^3 - 315*I*a*tan(d*x + c)^2 -
630*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.52, size = 106, normalized size = 1.13

$$\frac{a\left(-\cos{(c+d\,x)}^{10}\,63\mathrm{i}+256\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^{9}+128\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^{7}+96\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^{5}+80\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^{3}+70\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}+63\mathrm{i}\right)}{630\,d\,\cos{(c+d\,x)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^10,x)
```

```
[Out] (a*(70*cos(c + d*x)*sin(c + d*x) + 80*cos(c + d*x)^3*sin(c + d*x) + 96*cos(
c + d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + 256*cos(c + d*x
)^9*sin(c + d*x) - cos(c + d*x)^10*63i + 63i))/(630*d*cos(c + d*x)^10)
```

3.2 $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^7(c+dx)}{7d}$$

```
[Out] 1/8*I*a*sec(d*x+c)^8/d+a*tan(d*x+c)/d+a*tan(d*x+c)^3/d+3/5*a*tan(d*x+c)^5/d
+1/7*a*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3567, 3852}

$$\frac{a\tan^7(c+dx)}{7d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^3(c+dx)}{d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/d + (3
*a*Tan[c + d*x]^5)/(5*d) + (a*Tan[c + d*x]^7)/(7*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^8(c+dx)}{8d}+a\int \sec^8(c+dx)\,dx$$
$$=\frac{ia\sec^8(c+dx)}{8d}-\frac{a\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(c+$$
$$=\frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+d$$

Mathematica [A]
time = 0.13, size = 63, normalized size = 0.84

$$\frac{ia\sec^8(c+dx)}{8d}+\frac{a\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c +
 d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A]
time = 0.25, size = 59, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{ia}{8\cos(dx+c)^8}-a\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)}{d}$	59
default	$\frac{\frac{ia}{8\cos(dx+c)^8}-a\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)}{d}$	59
risch	$\frac{32ia\left(70\,e^{8i(dx+c)}+56\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{35d\left(e^{2i(dx+c)}+1\right)^8}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*I*a/cos(d*x+c)^8-a*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35
*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.23

$$\frac{35i\,a\tan(dx+c)^8+40\,a\tan(dx+c)^7+140i\,a\tan(dx+c)^6+168\,a\tan(dx+c)^5+210i\,a\tan(dx+c)^4+280\,a\tan(dx+c)^3+140i\,a\tan(dx+c)^2+280\,a\tan(dx+c)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(35*I*a*tan(d*x + c)^8 + 40*a*tan(d*x + c)^7 + 140*I*a*tan(d*x + c)^6
 + 168*a*tan(d*x + c)^5 + 210*I*a*tan(d*x + c)^4 + 280*a*tan(d*x + c)^3 + 1
40*I*a*tan(d*x + c)^2 + 280*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(67) = 134.
time = 0.35, size = 153, normalized size = 2.04

$$-\frac{32\left(-70i\,ae^{(8i\,dx+8i\,c)}-56i\,ae^{(6i\,dx+6i\,c)}-28i\,ae^{(4i\,dx+4i\,c)}-8i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{35\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c)}+8\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -32/35*(-70*I*a*e^(8*I*d*x + 8*I*c) - 56*I*a*e^(6*I*d*x + 6*I*c) - 28*I*a*e
^(4*I*d*x + 4*I*c) - 8*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(16*I*d*x + 16*I
*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I
*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d
*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 3.66, size = 68, normalized size = 0.91

$$\begin{cases} \frac{a\left(\frac{\tan^7(c+dx)}{7}+\frac{3\tan^5(c+dx)}{5}+\tan^3(c+dx)+\tan(c+dx)\right)+\frac{ia\sec^8(c+dx)}{8}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^8(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**7/7 + 3*tan(c + d*x)**5/5 + tan(c + d*x)**3 +
tan(c + d*x)) + I*a*sec(c + d*x)**8/8)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*se
c(c)**8, True))
```

Giac [A]
time = 0.59, size = 92, normalized size = 1.23

$$-\frac{-35ia\tan(dx+c)^8-40a\tan(dx+c)^7-140ia\tan(dx+c)^6-168a\tan(dx+c)^5-210ia\tan(dx+c)^4-280a\tan(dx+c)^3-140ia\tan(dx+c)^2-280a\tan(dx+c)}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(-35*I*a*tan(d*x + c)^8 - 40*a*tan(d*x + c)^7 - 140*I*a*tan(d*x + c)
^6 - 168*a*tan(d*x + c)^5 - 210*I*a*tan(d*x + c)^4 - 280*a*tan(d*x + c)^3 -
 140*I*a*tan(d*x + c)^2 - 280*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.27, size = 149, normalized size = 1.99

$$\frac{a\sin(c+dx)\left(280\cos(c+dx)^7+\cos(c+dx)^6\sin(c+dx)\,140\mathrm{i}+280\cos(c+dx)^5\sin(c+dx)^2+\cos(c+dx)^4\sin(c+dx)^3\,210\mathrm{i}+168\cos(c+dx)^3\sin(c+dx)^4+\cos(c+dx)^2\sin(c+dx)^5\,140\mathrm{i}+40\cos(c+dx)\sin(c+dx)^6+\sin(c+dx)^7\,35\mathrm{i}\right)}{280d\cos(c+dx)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^8,x)
```

```
[Out] (a*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c + d*
x)*140i + 280*cos(c + d*x)^7 + sin(c + d*x)^7*35i + cos(c + d*x)^2*sin(c +
d*x)^5*140i + 168*cos(c + d*x)^3*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)^3*210i + 280*cos(c + d*x)^5*sin(c + d*x)^2))/(280*d*cos(c + d*x)^8)
```

3.3 $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{ia\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/6*I*a*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)
^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3567, 3852}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d
) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^6(c+dx)}{6d}+a\int \sec^6(c+dx)\,dx\\ &= \frac{ia\sec^6(c+dx)}{6d}-\frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx}{5d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 55, normalized size = 0.89

$$\frac{ia\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[
c + d*x]^5/5))/d
```

Maple [A]
time = 0.25, size = 49, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{ia}{6\cos(dx+c)^6}-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$	49
default	$\frac{\frac{ia}{6\cos(dx+c)^6}-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$	49
risch	$\frac{16ia\left(20\,e^{6i(dx+c)}+15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*I*a/cos(d*x+c)^6-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(
d*x+c))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.13

$$\frac{5i\,a\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15i\,a\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15i\,a\tan\left(dx+c\right)^2+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*a*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*I*a*tan(d*x + c)^4 + 2
0*a*tan(d*x + c)^3 + 15*I*a*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(54) = 108.
time = 0.34, size = 117, normalized size = 1.89

$$-\frac{16\left(-20i\,ae^{(6i\,dx+6i\,c)}-15i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -16/15*(-20*I*a*e^(6*I*d*x + 6*I*c) - 15*I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^
(2*I*d*x + 2*I*c) - I*a)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*
c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x
+ 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 2.46, size = 60, normalized size = 0.97

$$\begin{cases} \frac{a\left(\frac{\tan^{5}(c+dx)}{5}+\frac{2\tan^{3}(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^{6}(c+dx)}{6}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + I*
a*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**6, True))
```

Giac [A]
time = 0.53, size = 70, normalized size = 1.13

$$-\frac{-5i\,a\tan(dx+c)^{6}-6\,a\tan(dx+c)^{5}-15i\,a\tan(dx+c)^{4}-20\,a\tan(dx+c)^{3}-15i\,a\tan(dx+c)^{2}-30\,a\tan(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(-5*I*a*tan(d*x + c)^6 - 6*a*tan(d*x + c)^5 - 15*I*a*tan(d*x + c)^4 -
 20*a*tan(d*x + c)^3 - 15*I*a*tan(d*x + c)^2 - 30*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.24, size = 112, normalized size = 1.81

$$\frac{a\sin(c+d\,x)\,\left(30\cos(c+d\,x)^{5}+\cos(c+d\,x)^{4}\sin(c+d\,x)\,15\mathrm{i}+20\cos(c+d\,x)^{3}\sin(c+d\,x)^{2}+\cos(c+d\,x)^{2}\sin(c+d\,x)^{3}\,15\mathrm{i}+6\cos(c+d\,x)\,\sin(c+d\,x)^{4}+\sin(c+d\,x)^{5}\,5\mathrm{i}\right)}{30\,d\cos(c+d\,x)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^6,x)
```

```
[Out] (a*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*x
)*15i + 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.4 $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=46

$$\frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/4*I*a*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3567, 3852}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^4(c+dx)}{4d}+a\int \sec^4(c+dx)\,dx\\ &= \frac{ia\sec^4(c+dx)}{4d}-\frac{a\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 0.93

$$\frac{ia\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.29, size = 39, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{ia}{4\cos(dx+c)^4}-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	39
default	$\frac{\frac{ia}{4\cos(dx+c)^4}-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	39
risch	$\frac{4ia\left(6\,e^{4i(dx+c)}+4\,e^{2i(dx+c)}+1\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*I*a/cos(d*x+c)^4-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.04

$$\frac{3i\,a\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6i\,a\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(3*I*a*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 + 12
*a*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(40) = 80.
time = 0.35, size = 81, normalized size = 1.76

$$-\frac{4\left(-6i\,ae^{(4i\,dx+4i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(-6*I*a*e^(4*I*d*x + 4*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(8
*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e
^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 1.75, size = 48, normalized size = 1.04

$$\begin{cases} \frac{a\left(\frac{\tan^3{(c+dx)}}{3}+\tan{(c+dx)}\right)+\frac{ia\sec^4{(c+dx)}}{4}}{d} & \text{for } d \neq 0 \\ x(ia\tan{(c)}+a)\sec^4{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**4/4)/d
, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**4, True))
```

Giac [A]
time = 0.58, size = 48, normalized size = 1.04

$$-\frac{-3i\,a\tan{(dx+c)}^4-4\,a\tan{(dx+c)}^3-6i\,a\tan{(dx+c)}^2-12\,a\tan{(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 - 6*I*a*tan(d*x + c)^2 -
12*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.24, size = 48, normalized size = 1.04

$$\frac{\frac{1\mathrm{i}\,a\,\tan(c+d\,x)^4}{4}+\frac{a\,\tan(c+d\,x)^3}{3}+\frac{1\mathrm{i}\,a\,\tan(c+d\,x)^2}{2}+a\,\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^4,x)
```

```
[Out] (a*tan(c + d*x) + (a*tan(c + d*x)^2*1i)/2 + (a*tan(c + d*x)^3)/3 + (a*tan(c
 + d*x)^4*1i)/4)/d
```

3.5 $\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^2}{2ad}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^2/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^2(c+dx)}{2d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{ia\sec^2(c+dx)}{2d}-\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.11

$$\frac{ia\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.22, size = 26, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{ia}{2\cos(dx+c)^2}+a\tan(dx+c)}{d}$	26
default	$\frac{\frac{ia}{2\cos(dx+c)^2}+a\tan(dx+c)}{d}$	26
risch	$\frac{2ia(2\,\mathrm{e}^{2i(dx+c)}+1)}{d(\mathrm{e}^{2i(dx+c)}+1)^2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*I*a/cos(d*x+c)^2+a*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.78

$$-\frac{i\,(i\,a\tan\left(dx+c\right)+a)^2}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*I*(I*a*tan(d*x + c) + a)^2/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.
time = 0.36, size = 45, normalized size = 1.67

$$-\frac{2\left(-2i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*(-2*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [A]
time = 1.28, size = 37, normalized size = 1.37

$$\begin{cases} \frac{\frac{ia\tan^2{(c+dx)}}{2}+a\tan{(c+dx)}}{d} & \text{for } d\neq 0 \\ x(ia\tan{(c)}+a)\sec^2{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((I*a*tan(c + d*x)**2/2 + a*tan(c + d*x))/d, Ne(d, 0)), (x*(I*a*t
an(c) + a)*sec(c)**2, True))
```

Giac [A]
time = 0.48, size = 26, normalized size = 0.96

$$-\frac{-i\,a\tan{(dx+c)}^2-2\,a\tan{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(-I*a*tan(d*x + c)^2 - 2*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.21, size = 23, normalized size = 0.85

$$\frac{a\tan(c+d\,x)\,\left(2+\tan(c+d\,x)\,1{}\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^2,x)
```

```
[Out] (a*tan(c + d*x)*(tan(c + d*x)*1i + 2))/(2*d)
```

3.6 $\int (a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=19

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

```
[Out] a*x-I*a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3556}

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + I*a*Tan[c + d*x],x]
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))\,dx &= ax+(ia)\int \tan(c+dx)\,dx\\ &= ax-\frac{ia\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*a*Tan[c + d*x],x]
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.03, size = 23, normalized size = 1.21

method	result	size
default	$ax+\frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	23
norman	$ax+\frac{ia\ln\left(1+\tan^2(dx+c)\right)}{2d}$	23
derivativedivides	$\frac{a\left(\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}+\arctan(\tan(dx+c))\right)}{d}$	28
risch	$-\frac{ia\ln(e^{2i(dx+c)}+1)}{d}-\frac{2ac}{d}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+I*a*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.89

$$ax+\frac{i\,a\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + I*a*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.95

$$-\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

Sympy [A]
time = 0.07, size = 24, normalized size = 1.26

$$-\frac{ia\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x)
```

```
[Out] -I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [A]
time = 0.47, size = 18, normalized size = 0.95

$$ax - \frac{i\,a\log\left(\left|\cos\left(dx + c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
[Out] a*x - I*a*log(abs(cos(d*x + c)))/d
```

Mupad [B]
time = 3.26, size = 17, normalized size = 0.89

$$\frac{a\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + a*tan(c + d*x)*1i,x)
[Out] (a*log(tan(c + d*x) + 1i)*1i)/d
```

3.7 $\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

[Out] 1/2*a*x-1/2*I*a*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 2715, 8}

$$-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]

[Out] (a*x)/2 - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^2(c+dx)}{2d}+a\int \cos^2(c+dx)\,dx\\
&= -\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\
&= \frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.07

$$\frac{a(c+dx)}{2d}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A]
time = 0.24, size = 42, normalized size = 0.93

method	result	size
risch	$\frac{ax}{2}-\frac{ia\,\mathrm{e}^{2i(dx+c)}}{4d}$	22
derivativedivides	$\frac{-\frac{ia\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42
default	$\frac{-\frac{ia\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2*I*a*cos(d*x+c)^2+a*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.51, size = 38, normalized size = 0.84

$$\frac{(dx+c)a+\frac{a\tan(dx+c)-i\,a}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - I*a)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 23, normalized size = 0.51

$$\frac{2\,adx-i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A]
time = 0.07, size = 39, normalized size = 0.87

$$\frac{ax}{2}+\begin{cases}-\frac{iae^{2ic}e^{2idx}}{4d} & \text{for } d\neq 0\\ \frac{axe^{2ic}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*x/2 + Piecewise((-I*a*exp(2*I*c)*exp(2*I*d*x)/(4*d), Ne(d, 0)), (a*x*exp(
2*I*c)/2, True))
```

Giac [A]
time = 0.53, size = 23, normalized size = 0.51

$$\frac{2\,adx-i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Mupad [B]
time = 3.23, size = 22, normalized size = 0.49

$$\frac{a\,x}{2}+\frac{a}{2\,d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (a*x)/2 + a/(2*d*(tan(c + d*x) + 1i))
```

3.8 $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=67

$$\frac{3ax}{8}-\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*I*a*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+
c)^3*sin(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {3567, 2715, 8}

$$-\frac{ia\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - ((I/4)*a*Cos[c + d*x]^4)/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^4(c+dx)}{4d} + a\int \cos^4(c+dx)\,dx$$
$$= -\frac{ia\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+$$
$$= -\frac{ia\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin}{4d}$$
$$= \frac{3ax}{8} - \frac{ia\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+d}{4}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.69

$$\frac{a(12c+12dx-8i\cos^4(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(12*c + 12*d*x - (8*I)*Cos[c + d*x]^4 + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(32*d)
```

Maple [A]
time = 0.29, size = 53, normalized size = 0.79

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
default	$\frac{-\frac{ia\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
risch	$\frac{3ax}{8}-\frac{ia\,e^{4i(dx+c)}}{32d}-\frac{ia\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*I*a*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.51, size = 61, normalized size = 0.91

$$\frac{3\,(dx+c)a+\frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2i\,a}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*I*a)/(tan(d
*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.84

$$\frac{\left(12\,adxe^{(2i\,dx+2i\,c)}-i\,ae^{(6i\,dx+6i\,c)}-6i\,ae^{(4i\,dx+4i\,c)}+2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) - I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I
*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A]
time = 0.15, size = 136, normalized size = 2.03

$$\frac{3ax}{8}+\begin{cases}\frac{\left(-256iad^2e^{6ic}e^{4idx}-1536iad^2e^{4ic}e^{2idx}+512iad^2e^{-2idx}\right)e^{-2ic}}{8192d^3} & \text{for } d^3e^{2ic}\neq 0\\ x\left(-\frac{3a}{8}+\frac{\left(ae^{6ic}+3ae^{4ic}+3ae^{2ic}+a\right)e^{-2ic}}{8}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 3*a*x/8 + Piecewise(((-256*I*a*d**2*exp(6*I*c)*exp(4*I*d*x) - 1536*I*a*d**2
*exp(4*I*c)*exp(2*I*d*x) + 512*I*a*d**2*exp(-2*I*d*x))*exp(-2*I*c)/(8192*d*
*3), Ne(d**3*exp(2*I*c), 0)), (x*(-3*a/8 + (a*exp(6*I*c) + 3*a*exp(4*I*c) +
 3*a*exp(2*I*c) + a)*exp(-2*I*c)/8), True))
```

Giac [A]
time = 0.47, size = 103, normalized size = 1.54

$$\frac{\left(12\,adxe^{(2i\,dx+2i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-i\,ae^{(6i\,dx+6i\,c)}-6i\,ae^{(4i\,dx+4i\,c)}+2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) + I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*
a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*
I*c)/d
```

Mupad [B]
time = 3.34, size = 64, normalized size = 0.96

$$\frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\tan(c+d\,x)^2}{8}+\frac{3\mathrm{i}\,a\,\tan(c+d\,x)}{8}+\frac{a}{4}}{d\,\left(\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,1\mathrm{i}+\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (3*a*x)/8 + (a/4 + (a*tan(c + d*x)*3i)/8 + (3*a*tan(c + d*x)^2)/8)/(d*(tan(
c + d*x) + tan(c + d*x)^2*1i + tan(c + d*x)^3 + 1i))
```

3.9 $\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{5ax}{16}-\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 5/16*a*x-1/6*I*a*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d
*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 2715, 8}

$$-\frac{ia\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - ((I/6)*a*Cos[c + d*x]^6)/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(
16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
+ d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^6(c+dx)}{6d} + a\int \cos^6(c+dx)\,dx$$
$$= -\frac{ia\cos^6(c+dx)}{6d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+$$
$$= -\frac{ia\cos^6(c+dx)}{6d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin}{6d}$$
$$= -\frac{ia\cos^6(c+dx)}{6d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin}{24d}$$
$$= \frac{5ax}{16} - \frac{ia\cos^6(c+dx)}{6d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+}{2}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 0.63

$$\frac{a(60c+60dx-32i\cos^6(c+dx)+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(60*c + 60*d*x - (32*I)*Cos[c + d*x]^6 + 45*Sin[2*(c + d*x)] + 9*Sin[4*(
c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^6(dx+c)\right)}{6}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
default	$\frac{-\frac{ia\left(\cos^6(dx+c)\right)}{6}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
risch	$\frac{5ax}{16}-\frac{ia\,e^{6i(dx+c)}}{192d}-\frac{ia\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5ia\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2c)}{64d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*I*a*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d
*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 0.92

$$\frac{15\,(dx+c)a+\frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8i\,a}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*ta
n(d*x + c) - 8*I*a)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 +
 1))/d
```

Fricas [A]
time = 0.36, size = 80, normalized size = 0.90

$$\frac{\left(120\,adxe^{(4i\,dx+4i\,c)}-2i\,ae^{(10i\,dx+10i\,c)}-15i\,ae^{(8i\,dx+8i\,c)}-60i\,ae^{(6i\,dx+6i\,c)}+30i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-4i\,dx-4i\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 4*I*c) - 2*I*a*e^(10*I*d*x + 10*I*c) - 15*I*a
*e^(8*I*d*x + 8*I*c) - 60*I*a*e^(6*I*d*x + 6*I*c) + 30*I*a*e^(2*I*d*x + 2*I
*c) + 3*I*a)*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [A]
time = 0.21, size = 211, normalized size = 2.37

$$\frac{5ax}{16}+\begin{cases}\frac{\left(-33554432iad^4e^{12ic}e^{6idx}-251658240iad^4e^{10ic}e^{4idx}-1006632960iad^4e^{8ic}e^{2idx}+503316480iad^4e^{4ic}e^{-2idx}+50331648iad^4e^{2ic}e^{-4idx}\right)e^{-6ic}}{6442450944d^5} & \text{for } d^5e^{6ic}\neq 0\\ x\left(-\frac{5a}{16}+\frac{\left(ae^{10ic}+5ae^{8ic}+10ae^{6ic}+10ae^{4ic}+5ae^{2ic}+a\right)e^{-4ic}}{32}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 5*a*x/16 + Piecewise(((-33554432*I*a*d**4*exp(12*I*c)*exp(6*I*d*x) - 251658
240*I*a*d**4*exp(10*I*c)*exp(4*I*d*x) - 1006632960*I*a*d**4*exp(8*I*c)*exp(
2*I*d*x) + 503316480*I*a*d**4*exp(4*I*c)*exp(-2*I*d*x) + 50331648*I*a*d**4*
exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(6442450944*d**5), Ne(d**5*exp(6*I*c)
, 0)), (x*(-5*a/16 + (a*exp(10*I*c) + 5*a*exp(8*I*c) + 10*a*exp(6*I*c) + 10
*a*exp(4*I*c) + 5*a*exp(2*I*c) + a)*exp(-4*I*c)/32), True))
```

Giac [A]
time = 0.61, size = 127, normalized size = 1.43

$$\frac{\left(120\,adxe^{(4i\,dx+2i\,c)}+12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-2i\,ae^{(10i\,dx+8i\,c)}-15i\,ae^{(8i\,dx+6i\,c)}-60i\,ae^{(6i\,dx+4i\,c)}+30i\,ae^{(2i\,dx)}+3i\,ae^{(-2i\,c)}\right)e^{(-4i\,dx-2i\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 2*I*c) + 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) - 2*I*a*e^(10*I*d*x + 8*I*c) - 15*I*a*e^(8*I*d*x + 6*I*c) - 60*I*a*e^(6
*I*d*x + 4*I*c) + 30*I*a*e^(2*I*d*x) + 3*I*a*e^(-2*I*c))*e^(-4*I*d*x - 2*I*
c)/d
```

Mupad [B]
time = 3.57, size = 108, normalized size = 1.21

$$\frac{5\,a\,x}{16}+\frac{\frac{5\,a\,{\mathrm{tan}\left(c+d\,x\right)}^4}{16}+\frac{5{}\mathrm{i}\,a\,{\mathrm{tan}\left(c+d\,x\right)}^3}{16}+\frac{25\,a\,{\mathrm{tan}\left(c+d\,x\right)}^2}{48}+\frac{25{}\mathrm{i}\,a\,\mathrm{tan}\left(c+d\,x\right)}{48}+\frac{a}{6}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^5+{\mathrm{tan}\left(c+d\,x\right)}^4\,1{}\mathrm{i}+2\,{\mathrm{tan}\left(c+d\,x\right)}^3+{\mathrm{tan}\left(c+d\,x\right)}^2\,2{}\mathrm{i}+\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (5*a*x)/16 + (a/6 + (a*tan(c + d*x)*25i)/48 + (25*a*tan(c + d*x)^2)/48 + (a
*tan(c + d*x)^3*5i)/16 + (5*a*tan(c + d*x)^4)/16)/(d*(tan(c + d*x) + tan(c
+ d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4*1i + tan(c + d*x)^5 + 1i))
```

3.10 $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{35ax}{128}-\frac{ia\cos^8(c+dx)}{8d}+\frac{35a\cos(c+dx)\sin(c+dx)}{128d}+\frac{35a\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{7a\cos^5(c+dx)\sin(c+}{48d}$$

```
[Out] 35/128*a*x-1/8*I*a*cos(d*x+c)^8/d+35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/192*a
*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a*cos(d*x+c
)^7*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$,
Rules used = {3567, 2715, 8}

$$-\frac{ia\cos^8(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a\sin(c+dx)\cos(c+dx)}{128d}+\frac{35ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (35*a*x)/128 - ((I/8)*a*Cos[c + d*x]^8)/d + (35*a*Cos[c + d*x]*Sin[c + d*x]
)/(128*d) + (35*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*a*Cos[c + d*x]^
5*Sin[c + d*x])/(48*d) + (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^8(c+dx)}{8d} + a\int \cos^8(c+dx)\,dx$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(7a)\int \cos^6(c+$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin}{8d}$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7a\cos^5(c+dx)}{48d}$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)}{192d}$$
$$= \frac{35ax}{128} - \frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c}{}$$

Mathematica [A]
time = 0.14, size = 68, normalized size = 0.61

$$\frac{a(840c+840dx-384i\cos^8(c+dx)+672\sin(2(c+dx))+168\sin(4(c+dx))+32\sin(6(c+dx))+3\sin(8(c+dx)))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(840*c + 840*d*x - (384*I)*Cos[c + d*x]^8 + 672*Sin[2*(c + d*x)] + 168*S
in[4*(c + d*x)] + 32*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)]))/(3072*d)
```

Maple [A]
time = 0.22, size = 73, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{ia\left(\cos^8(dx+c)\right)}{8}+a\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
default	$\frac{-\frac{ia\left(\cos^8(dx+c)\right)}{8}+a\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
risch	$\frac{35ax}{128}-\frac{ia\,e^{8i(dx+c)}}{1024d}-\frac{ia\cos(6dx+6c)}{128d}+\frac{a\sin(6dx+6c)}{96d}-\frac{7ia\cos(4dx+4c)}{256d}+\frac{7a\sin(4dx+4c)}{128d}-\frac{7ia\cos(2dx+2c}{128d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*I*a*cos(d*x+c)^8+a*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(
d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.51, size = 103, normalized size = 0.93

$$\frac{105\,(dx+c)a+\frac{105\,a\tan(dx+c)^7+385\,a\tan(dx+c)^5+511\,a\tan(dx+c)^3+279\,a\tan(dx+c)-48i\,a}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/384*(105*(d*x + c)*a + (105*a*tan(d*x + c)^7 + 385*a*tan(d*x + c)^5 + 511
*a*tan(d*x + c)^3 + 279*a*tan(d*x + c) - 48*I*a)/(tan(d*x + c)^8 + 4*tan(d*
x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 104, normalized size = 0.94

$$\frac{\left(840\,adxe^{(6i\,dx+6i\,c)}-3i\,ae^{(14i\,dx+14i\,c)}-28i\,ae^{(12i\,dx+12i\,c)}-126i\,ae^{(10i\,dx+10i\,c)}-420i\,ae^{(8i\,dx+8i\,c)}+252i\,ae^{(4i\,dx+4i\,c)}+42i\,ae^{(2i\,dx+2i\,c)}+4i\,a\right)e^{(-6i\,dx-6i\,c)}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 6*I*c) - 3*I*a*e^(14*I*d*x + 14*I*c) - 28*I*
a*e^(12*I*d*x + 12*I*c) - 126*I*a*e^(10*I*d*x + 10*I*c) - 420*I*a*e^(8*I*d*
x + 8*I*c) + 252*I*a*e^(4*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x + 2*I*c) + 4*I
*a)*e^(-6*I*d*x - 6*I*c)/d
```

Sympy [A]
time = 0.29, size = 279, normalized size = 2.51

$$\frac{35ax}{128}+\begin{cases}\frac{\left(-10133099161583616iad^6e^{20ic}e^{8idx}-94575592174780416iad^6e^{18ic}e^{6idx}-425590164786511872iad^6e^{16ic}e^{4idx}-1418633882621706240iad^6e^{14ic}e^{2idx}+851180329573023744iad^6e^{10ic}e^{-2idx}+141863388262170624iad^6e^{8ic}e^{-4idx}+13510798882111488iad^6e^{6ic}e^{-6idx}\right)e^{-12ic}}{10376293541461622784d^7} & \text{for}\ d^7e^{12ic}\neq 0\\ x\left(-\frac{35a}{128}+\frac{\left(ae^{14ic}+7ae^{12ic}+21ae^{10ic}+35ae^{8ic}+35ae^{6ic}+21ae^{4ic}+7ae^{2ic}+a\right)e^{-6ic}}{128}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 35*a*x/128 + Piecewise(((-10133099161583616*I*a*d**6*exp(20*I*c)*exp(8*I*d*
x) - 94575592174780416*I*a*d**6*exp(18*I*c)*exp(6*I*d*x) - 4255901647865118
72*I*a*d**6*exp(16*I*c)*exp(4*I*d*x) - 1418633882621706240*I*a*d**6*exp(14*
I*c)*exp(2*I*d*x) + 851180329573023744*I*a*d**6*exp(10*I*c)*exp(-2*I*d*x) +
 141863388262170624*I*a*d**6*exp(8*I*c)*exp(-4*I*d*x) + 13510798882111488*I
*a*d**6*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(10376293541461622784*d**7),
 Ne(d**7*exp(12*I*c), 0)), (x*(-35*a/128 + (a*exp(14*I*c) + 7*a*exp(12*I*c)
 + 21*a*exp(10*I*c) + 35*a*exp(8*I*c) + 35*a*exp(6*I*c) + 21*a*exp(4*I*c) +
 7*a*exp(2*I*c) + a)*exp(-6*I*c)/128), True))
```

Giac [A]
time = 0.55, size = 151, normalized size = 1.36

$$\frac{\left(840\,adxe^{(6i\,dx+2i\,c)}+84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-3i\,ae^{(14i\,dx+10i\,c)}-28i\,ae^{(12i\,dx+8i\,c)}-126i\,ae^{(10i\,dx+6i\,c)}-420i\,ae^{(8i\,dx+4i\,c)}+42i\,ae^{(2i\,dx-2i\,c)}+252i\,ae^{(4i\,dx)}+4i\,ae^{(-4i\,c)}\right)e^{(-6i\,dx-2i\,c)}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 2*I*c) + 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I
*c)) - 3*I*a*e^(14*I*d*x + 10*I*c) - 28*I*a*e^(12*I*d*x + 8*I*c) - 126*I*a*
e^(10*I*d*x + 6*I*c) - 420*I*a*e^(8*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x - 2*
I*c) + 252*I*a*e^(4*I*d*x) + 4*I*a*e^(-4*I*c))*e^(-6*I*d*x - 2*I*c)/d
```

Mupad [B]
time = 4.85, size = 152, normalized size = 1.37

$$\frac{35\,a\,x}{128}+\frac{\frac{35\,a\,\tan(c+d\,x)^6}{128}+\frac{35\mathrm{i}\,a\,\tan(c+d\,x)^5}{128}+\frac{35\,a\,\tan(c+d\,x)^4}{48}+\frac{35\mathrm{i}\,a\,\tan(c+d\,x)^3}{48}+\frac{77\,a\,\tan(c+d\,x)^2}{128}+\frac{77\mathrm{i}\,a\,\tan(c+d\,x)}{128}+\frac{a}{8}}{d\,\left(\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,1\mathrm{i}+3\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,3\mathrm{i}+3\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,3\mathrm{i}+\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (35*a*x)/128 + (a/8 + (a*tan(c + d*x)*77i)/128 + (77*a*tan(c + d*x)^2)/128
+ (a*tan(c + d*x)^3*35i)/48 + (35*a*tan(c + d*x)^4)/48 + (a*tan(c + d*x)^5*
35i)/128 + (35*a*tan(c + d*x)^6)/128)/(d*(tan(c + d*x) + tan(c + d*x)^2*3i
+ 3*tan(c + d*x)^3 + tan(c + d*x)^4*3i + 3*tan(c + d*x)^5 + tan(c + d*x)^6*
1i + tan(c + d*x)^7 + 1i))
```

3.11 $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=98

$$\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{ia\sec^7(c+dx)}{7d}+\frac{5a\sec(c+dx)\tan(c+dx)}{16d}+\frac{5a\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a\sec^5(\cdots}{\cdots}$$

```
[Out] 5/16*a*arctanh(sin(d*x+c))/d+1/7*I*a*sec(d*x+c)^7/d+5/16*a*sec(d*x+c)*tan(d
*x+c)/d+5/24*a*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3853, 3855}

$$\frac{ia\sec^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) +
(a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^7(c+dx)}{7d} + a\int \sec^7(c+dx)\,dx$$

$$= \frac{ia\sec^7(c+dx)}{7d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}(5a)\int \sec^5(c+dx$$

$$= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(}{6d}$$

$$= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(}{24d}$$

$$= \frac{5a\tanh^{-1}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+}{16d}$$

Mathematica [A]
time = 0.36, size = 61, normalized size = 0.62

$$\frac{a(3360\tanh^{-1}(\sin(c+dx)) + \sec^7(c+dx)(1536i + 1981\sin(2(c+dx)) + 700\sin(4(c+dx)) + 105\sin(6(c+dx))))}{10752d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(3360*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]^7*(1536*I + 1981*Sin[2*(c + d
*x)] + 700*Sin[4*(c + d*x)] + 105*Sin[6*(c + d*x)])))/(10752*d)
```

Maple [A]
time = 0.27, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
default	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
risch	$-\frac{ia\left(105\,e^{13i(dx+c)}+700\,e^{11i(dx+c)}+1981\,e^{9i(dx+c)}-3072\,e^{7i(dx+c)}-1981\,e^{5i(dx+c)}-700\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{168d\left(e^{2i(dx+c)}+1\right)^7} - 5a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*I*a/cos(d*x+c)^7+a*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec
(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 106, normalized size = 1.08

$$-\frac{7\,a\left(\frac{2\,\left(15\,\sin\left(dx+c\right)^{5}-40\,\sin\left(dx+c\right)^{3}+33\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{6}-3\,\sin\left(dx+c\right)^{4}+3\,\sin\left(dx+c\right)^{2}-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{96i\,a}{\cos\left(dx+c\right)^{7}}}{672\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/672*(7*a*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
 c) + 1) + 15*log(sin(d*x + c) - 1)) - 96*I*a/cos(d*x + c)^7)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(86) = 172.
time = 0.39, size = 372, normalized size = 3.80

$$\frac{-210i\,ae^{(13i\,dx+13ic)}-1400i\,ae^{(11i\,dx+11ic)}-3962i\,ae^{(9i\,dx+9ic)}+6144i\,ae^{(7i\,dx+7ic)}+3962i\,ae^{(5i\,dx+5ic)}+1400i\,ae^{(3i\,dx+3ic)}+210i\,ae^{(i\,dx+ic)}+105\left(ae^{(14i\,dx+14ic)}+7\,ae^{(12i\,dx+12ic)}+21\,ae^{(10i\,dx+10ic)}+35\,ae^{(8i\,dx+8ic)}+35\,ae^{(6i\,dx+6ic)}+21\,ae^{(4i\,dx+4ic)}+7\,ae^{(2i\,dx+2ic)}+a\right)\log\left(e^{(i\,dx+ic)}+i\right)-105\left(ae^{(14i\,dx+14ic)}+7\,ae^{(12i\,dx+12ic)}+21\,ae^{(10i\,dx+10ic)}+35\,ae^{(8i\,dx+8ic)}+35\,ae^{(6i\,dx+6ic)}+21\,ae^{(4i\,dx+4ic)}+7\,ae^{(2i\,dx+2ic)}+a\right)\log\left(e^{(i\,dx+ic)}-i\right)}{336\,\left(de^{(14i\,dx+14ic)}+7\,de^{(12i\,dx+12ic)}+21\,de^{(10i\,dx+10ic)}+35\,de^{(8i\,dx+8ic)}+35\,de^{(6i\,dx+6ic)}+21\,de^{(4i\,dx+4ic)}+7\,de^{(2i\,dx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/336*(-210*I*a*e^(13*I*d*x + 13*I*c) - 1400*I*a*e^(11*I*d*x + 11*I*c) - 39
62*I*a*e^(9*I*d*x + 9*I*c) + 6144*I*a*e^(7*I*d*x + 7*I*c) + 3962*I*a*e^(5*I
*d*x + 5*I*c) + 1400*I*a*e^(3*I*d*x + 3*I*c) + 210*I*a*e^(I*d*x + I*c) + 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*
e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I
*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x
 + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sec^{7}\left(c+dx\right)\right)\,dx+\int\tan\left(c+dx\right)\sec^{7}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sec(c + d*x)**7, x) + Integral(tan(c + d*x)*sec(c + d*x)**
7, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(86) = 172.
time = 0.54, size = 181, normalized size = 1.85

$$\frac{105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-336i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-196\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+595\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1680i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-595\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1008i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+196\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48i\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{7}}}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/336*(105*a*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a*log(tan(1/2*d*x + 1/2*c)
 - 1) + 2*(231*a*tan(1/2*d*x + 1/2*c)^13 - 336*I*a*tan(1/2*d*x + 1/2*c)^12
- 196*a*tan(1/2*d*x + 1/2*c)^11 + 595*a*tan(1/2*d*x + 1/2*c)^9 - 1680*I*a*t
an(1/2*d*x + 1/2*c)^8 - 595*a*tan(1/2*d*x + 1/2*c)^5 - 1008*I*a*tan(1/2*d*x
 + 1/2*c)^4 + 196*a*tan(1/2*d*x + 1/2*c)^3 - 231*a*tan(1/2*d*x + 1/2*c) - 4
8*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 7.37, size = 247, normalized size = 2.52

$$\frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+10\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{85\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{24}+6\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{a\,2\mathrm{i}}{7}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^7,x)
```

```
[Out] (5*a*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a*2i)/7 + (11*a*tan(c/2 + (d*x)/2
))/8 - (7*a*tan(c/2 + (d*x)/2)^3)/6 + a*tan(c/2 + (d*x)/2)^4*6i + (85*a*tan
(c/2 + (d*x)/2)^5)/24 + a*tan(c/2 + (d*x)/2)^8*10i - (85*a*tan(c/2 + (d*x)/
2)^9)/24 + (7*a*tan(c/2 + (d*x)/2)^11)/6 + a*tan(c/2 + (d*x)/2)^12*2i - (11
*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x
)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (
d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.12 $\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*I*a*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x
+c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3853, 3855}

$$\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^5(c+dx)}{5d} + a\int \sec^5(c+dx)\,dx$$
$$= \frac{ia\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx$$
$$= \frac{ia\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{ia\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+}{8d}$$

Mathematica [A]
time = 0.20, size = 70, normalized size = 0.92

$$\frac{ia\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/5)*a*Sec[c + d*x]^5)/d + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(
ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.22, size = 64, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{ia\left(15\,e^{9i(dx+c)}+70\,e^{7i(dx+c)}-128\,e^{5i(dx+c)}-70\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{20d\left(e^{2i(dx+c)}+1\right)^5}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*I*a/cos(d*x+c)^5+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)
+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.13

$$-\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{16i\,a}{\cos(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*I*
a/cos(d*x + c)^5)/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(66) = 132$.
time = 0.37, size = 276, normalized size = 3.63

$$\frac{-30i\, ae^{(9i\, dx+9i\, c)} - 140i\, ae^{(7i\, dx+7i\, c)} + 256i\, ae^{(5i\, dx+5i\, c)} + 140i\, ae^{(3i\, dx+3i\, c)} + 30i\, ae^{(i\, dx+i\, c)} + 15\left(ae^{(10i\, dx+10i\, c)} + 5\, ae^{(8i\, dx+8i\, c)} + 10\, ae^{(6i\, dx+6i\, c)} + 10\, ae^{(4i\, dx+4i\, c)} + 5\, ae^{(2i\, dx+2i\, c)} + a\right)\log\left(e^{(i\, dx+i\, c)} + i\right) - 15\left(ae^{(10i\, dx+10i\, c)} + 5\, ae^{(8i\, dx+8i\, c)} + 10\, ae^{(6i\, dx+6i\, c)} + 10\, ae^{(4i\, dx+4i\, c)} + 5\, ae^{(2i\, dx+2i\, c)} + a\right)\log\left(e^{(i\, dx+i\, c)} - i\right)}{40\left(de^{(10i\, dx+10i\, c)} + 5\, de^{(8i\, dx+8i\, c)} + 10\, de^{(6i\, dx+6i\, c)} + 10\, de^{(4i\, dx+4i\, c)} + 5\, de^{(2i\, dx+2i\, c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(-30*I*a*e^(9*I*d*x + 9*I*c) - 140*I*a*e^(7*I*d*x + 7*I*c) + 256*I*a*e
^(5*I*d*x + 5*I*c) + 140*I*a*e^(3*I*d*x + 3*I*c) + 30*I*a*e^(I*d*x + I*c) +
 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x +
6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d
*x + I*c) + I) - 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10
*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c)
 + a)*log(e^(I*d*x + I*c) - I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sec^{5}\left(c+dx\right)\right)\, dx + \int\tan\left(c+dx\right)\sec^{5}\left(c+dx\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sec(c + d*x)**5, x) + Integral(tan(c + d*x)*sec(c + d*x)**
5, x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(66) = 132$.
time = 0.55, size = 139, normalized size = 1.83

$$\frac{15\, a\log\left(\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) + 1\right) - 15\, a\log\left(\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) - 1\right) + \frac{2\left(25\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{9} - 40i\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{8} - 10\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{7} - 80i\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{4} + 10\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{3} - 25\, a\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right) - 8i\, a\right)}{\left(\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2} - 1\right)^{5}}}{40\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a*log(tan(1/2*d*x + 1/2*c) -
1) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*I*a*tan(1/2*d*x + 1/2*c)^8 - 10*a*
tan(1/2*d*x + 1/2*c)^7 - 80*I*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x +
 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

Mupad [B]
time = 6.90, size = 178, normalized size = 2.34

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\mathrm{i}\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\mathrm{i}\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a\,2\mathrm{i}}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*2i)/5 + (5*a*tan(c/2 + (d*x)/2)
)/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + a*tan(c/2 + (d*x)/2)^4*4i + (a*tan(c/2 +
 (d*x)/2)^7)/2 + a*tan(c/2 + (d*x)/2)^8*2i - (5*a*tan(c/2 + (d*x)/2)^9)/4)/
(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2
)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.13 $\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ia\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*I*a*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x
+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3853, 3855}

$$\frac{ia\sec^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^3(c+dx)}{3d} + a\int \sec^3(c+dx)\,dx$$
$$= \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.30, size = 51, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
default	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
risch	$-\frac{ia\left(3\,e^{5i(dx+c)}-8\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*I*a/cos(d*x+c)^3+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c))))
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.13

$$-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4i\,a}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*I*a/cos(d*x + c)^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.
time = 0.38, size = 180, normalized size = 3.33

$$\frac{-6i\,ae^{(5i\,dx+5i\,c)}+16i\,ae^{(3i\,dx+3i\,c)}+6i\,ae^{(i\,dx+i\,c)}+3\left(ae^{(6i\,dx+6i\,c)}+3\,ae^{(4i\,dx+4i\,c)}+3\,ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(ae^{(6i\,dx+6i\,c)}+3\,ae^{(4i\,dx+4i\,c)}+3\,ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*a*e^(5*I*d*x + 5*I*c) + 16*I*a*e^(3*I*d*x + 3*I*c) + 6*I*a*e^(I*d
*x + I*c) + 3*(a*e^(6*I*d*x + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I
*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 3*(a*e^(6*I*d*x + 6*I*c) + 3*
a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) -
I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sec^{3}\left(c+dx\right)\right)\,dx+\int\tan\left(c+dx\right)\sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sec(c + d*x)**3, x) + Integral(tan(c + d*x)*sec(c + d*x)**
3, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(46) = 92$.
time = 0.46, size = 97, normalized size = 1.80

$$\frac{3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2i\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a*log(tan(1/2*d*x + 1/2*c) - 1)
+ 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*I*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(1/
2*d*x + 1/2*c) - 2*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 5.13, size = 107, normalized size = 1.98

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,2\mathrm{i}}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((a*2i)/3 + a*tan(c/2 + (d*x)/2) + a*tan(
c/2 + (d*x)/2)^4*2i - a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.14 $\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=27

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

[Out] a*arctanh(sin(d*x+c))/d+I*a*sec(d*x+c)/d

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3567, 3855}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]

[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec(c+dx)}{d}+a\int \sec(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Maple [A]
time = 0.08, size = 34, normalized size = 1.26

method	result	size
derivativedivides	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
default	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
risch	$\frac{2ia\,\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.19

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{i\,a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + I*a/cos(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.
time = 0.38, size = 82, normalized size = 3.04

$$\frac{2i\,ae^{(i\,dx+i\,c)}+\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*I*a*e^(I*d*x + I*c) + (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) +
I) - (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [A]
time = 2.44, size = 41, normalized size = 1.52

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+ia\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + I*a*sec(c + d*x))/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.
time = 0.49, size = 52, normalized size = 1.93

$$\frac{a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(tan(1/2*d*x + 1/2*c) + 1) - a*log(tan(1/2*d*x + 1/2*c) - 1) - 2*I*a/
(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 3.35, size = 39, normalized size = 1.44

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,2\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a*2i)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3567, 2717}

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos(c+dx)}{d}+a\int \cos(c+dx)\,dx\\ &= -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.96

$$-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{ia\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.20, size = 24, normalized size = 0.92

method	result	size
risch	$-\frac{i a \,{\mathrm e}^{i \left(d x +c \right)}}{d}$	17
derivativedivides	$\frac{-i a \cos \left(d x +c \right)+a \sin \left(d x +c \right)}{d}$	24
default	$\frac{-i a \cos \left(d x +c \right)+a \sin \left(d x +c \right)}{d}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a*cos(d*x+c)+a*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.85

$$\frac{-i\,a\cos\left(dx+c\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (-I*a*cos(d*x + c) + a*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 15, normalized size = 0.58

$$-\frac{i\,ae^{\left(i\,dx+i\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -I*a*e^(I*d*x + I*c)/d
```

Sympy [A]
time = 0.06, size = 26, normalized size = 1.00

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for}\: d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(24) = 48$.
time = 0.44, size = 84, normalized size = 3.23

$$-\frac{4i\,ae^{(i\,dx+i\,c)}+a\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+a\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*a*e^(I*d*x + I*c) + a*log(I*e^(I*d*x + I*c) + 1) + a*log(I*e^(I*d
*x + I*c) - 1) - a*log(-I*e^(I*d*x + I*c) + 1) - a*log(-I*e^(I*d*x + I*c) -
 1))/d
```

Mupad [B]
time = 3.28, size = 20, normalized size = 0.77

$$\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

3.16 $\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=46

$$-\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/3*I*a*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3567, 2713}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*
d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^3(c+dx)}{3d}+a\int \cos^3(c+dx)\,dx\\
&= -\frac{ia\cos^3(c+dx)}{3d}-\frac{a\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= -\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$-\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*
d)
```

Maple [A]
time = 0.27, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	37
default	$\frac{-\frac{ia\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	37
risch	$-\frac{ia\,\mathrm{e}^{3i(dx+c)}}{12d}-\frac{ia\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*I*a*cos(d*x+c)^3+1/3*a*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.78

$$-\frac{i\,a\cos{(dx+c)}^3+\left(\sin{(dx+c)}^3-3\,\sin{(dx+c)}\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(I*a*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.91

$$\frac{\left(-i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-i\,dx-i\,c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) + 3*I*a)*e^(-I*d
*x - I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(37) = 74$.
time = 0.14, size = 105, normalized size = 2.28

$$\begin{cases} \frac{(-8iad^2e^{4ic}e^{3idx}-48iad^2e^{2ic}e^{idx}+24iad^2e^{-idx})e^{-ic}}{96d^3} & \text{for } d^3e^{ic} \neq 0 \\ \frac{x(ae^{4ic}+2ae^{2ic}+a)e^{-ic}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-8*I*a*d**2*exp(4*I*c)*exp(3*I*d*x) - 48*I*a*d**2*exp(2*I*c)*ex
p(I*d*x) + 24*I*a*d**2*exp(-I*d*x))*exp(-I*c)/(96*d**3), Ne(d**3*exp(I*c),
0)), (x*(a*exp(4*I*c) + 2*a*exp(2*I*c) + a)*exp(-I*c)/4, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(40) = 80$.
time = 0.50, size = 196, normalized size = 4.26

$$-\frac{\left(9\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+6\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-9\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-6\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-3\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+3\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+4i\,ae^{(4i\,dx+4i\,c)}+24i\,ae^{(2i\,dx+2i\,c)}-12i\,a\right)e^{(-i\,dx-i\,c)}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(9*a*e^(I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 6*a*e^(I*d*x + I*c)
*log(I*e^(I*d*x + I*c) - 1) - 9*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) +
1) - 6*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a*e^(I*d*x + I*c)*
log(I*e^(I*d*x) + e^(-I*c)) + 3*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*
c)) + 4*I*a*e^(4*I*d*x + 4*I*c) + 24*I*a*e^(2*I*d*x + 2*I*c) - 12*I*a)*e^(-
I*d*x - I*c)/d
```

Mupad [B]
time = 3.41, size = 54, normalized size = 1.17

$$\frac{2\,a\,\left(-\frac{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}}{4}-\frac{{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}^2\,1{}\mathrm{i}}{4}+\frac{9\,\sin\left(c+d\,x\right)}{8}+\frac{\sin\left(3\,c+3\,d\,x\right)}{8}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (2*a*((9*sin(c + d*x))/8 + sin(3*c + 3*d*x)/8 - (cos(c/2 + (d*x)/2)^2*3i)/4
 - (cos((3*c)/2 + (3*d*x)/2)^2*1i)/4))/(3*d)
```

3.17 $\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=62

$$-\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] -1/5*I*a*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c
)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3567, 2713}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/5*I)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(
3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^5(c+dx)}{5d}+a\int \cos^5(c+dx)\,dx\\ &= -\frac{ia\cos^5(c+dx)}{5d}-\frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 1.05

$$-\frac{ia\cos^5(c+dx)}{5d}+\frac{5a\sin(c+dx)}{8d}+\frac{5a\sin(3(c+dx))}{48d}+\frac{a\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/5*I)*a*Cos[c + d*x]^5)/d + (5*a*Sin[c + d*x])/(8*d) + (5*a*Sin[3*(c +
d*x)])/(48*d) + (a*Sin[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.22, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^5(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
default	$\frac{-\frac{ia\left(\cos^5(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
risch	$-\frac{ia\,e^{5i(dx+c)}}{80d}-\frac{ia\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{ia\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*I*a*cos(d*x+c)^5+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.79

$$-\frac{3i\,a\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*I*a*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a)/d
```

Fricas [A]
time = 0.39, size = 66, normalized size = 1.06

$$\frac{\left(-3i\,ae^{(8i\,dx+8i\,c)}-20i\,ae^{(6i\,dx+6i\,c)}-90i\,ae^{(4i\,dx+4i\,c)}+60i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{(-3i\,dx-3i\,c)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*a*e^(8*I*d*x + 8*I*c) - 20*I*a*e^(6*I*d*x + 6*I*c) - 90*I*a*e^(
4*I*d*x + 4*I*c) + 60*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*e^(-3*I*d*x - 3*I*c)
/d
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(53) = 106$.
time = 0.23, size = 184, normalized size = 2.97

$$\begin{cases} \frac{(-18432iad^4e^{9ic}e^{5idx}-122880iad^4e^{7ic}e^{3idx}-552960iad^4e^{5ic}e^{idx}+368640iad^4e^{3ic}e^{-idx}+30720iad^4e^{ic}e^{-3idx})e^{-4ic}}{1474560d^5} & \text{for } d^5e^{4ic}\neq 0 \\ \frac{x(ae^{8ic}+4ae^{6ic}+6ae^{4ic}+4ae^{2ic}+a)e^{-3ic}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-18432*I*a*d**4*exp(9*I*c)*exp(5*I*d*x) - 122880*I*a*d**4*exp(7
*I*c)*exp(3*I*d*x) - 552960*I*a*d**4*exp(5*I*c)*exp(I*d*x) + 368640*I*a*d**
4*exp(3*I*c)*exp(-I*d*x) + 30720*I*a*d**4*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*
c)/(1474560*d**5), Ne(d**5*exp(4*I*c), 0)), (x*(a*exp(8*I*c) + 4*a*exp(6*I*
c) + 6*a*exp(4*I*c) + 4*a*exp(2*I*c) + a)*exp(-3*I*c)/16, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(54) = 108$.
time = 0.55, size = 220, normalized size = 3.55

$$-\frac{\left(135\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+90\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-135\,ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-90\,ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-45\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+45\,ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+12i\,ae^{(8i\,dx+6i\,c)}+80i\,ae^{(6i\,dx+4i\,c)}+360i\,ae^{(4i\,dx+2i\,c)}-240i\,ae^{(2i\,dx)}-20i\,ae^{(-2i\,c)}\right)e^{(-3i\,dx-i\,c)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(135*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a*e^(3*I*d*
x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 135*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 90*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 45*a*
e^(3*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 45*a*e^(3*I*d*x + I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 12*I*a*e^(8*I*d*x + 6*I*c) + 80*I*a*e^(6*I*d*x +
 4*I*c) + 360*I*a*e^(4*I*d*x + 2*I*c) - 240*I*a*e^(2*I*d*x) - 20*I*a*e^(-2*
I*c))*e^(-3*I*d*x - I*c)/d
```

Mupad [**B**]
time = 4.93, size = 70, normalized size = 1.13

$$-\frac{2\,a\,\left(-\frac{75\,\sin\left(c+d\,x\right)}{16}-\frac{25\,\sin\left(3\,c+3\,d\,x\right)}{32}-\frac{3\,\sin\left(5\,c+5\,d\,x\right)}{32}+\frac{\cos\left(c+d\,x\right)\,15{}\mathrm{i}}{16}+\frac{\cos\left(3\,c+3\,d\,x\right)\,15{}\mathrm{i}}{32}+\frac{\cos\left(5\,c+5\,d\,x\right)\,3{}\mathrm{i}}{32}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -(2*a*((cos(c + d*x)*15i)/16 - (75*sin(c + d*x))/16 + (cos(3*c + 3*d*x)*15i
)/32 + (cos(5*c + 5*d*x)*3i)/32 - (25*sin(3*c + 3*d*x))/32 - (3*sin(5*c + 5
*d*x))/32))/(15*d)
```

3.18 $\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] -1/7*I*a*cos(d*x+c)^7/d+a*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/
d-1/7*a*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3567, 2713}

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/7*I)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d +
 (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^7(c+dx)}{7d}+a\int \cos^7(c+dx)\,dx\\ &= -\frac{ia\cos^7(c+dx)}{7d}-\frac{a\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+\right.}{d}\\ &= -\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+d}{5d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 81, normalized size = 1.07

$$-\frac{ia\cos^7(c+dx)}{7d}+\frac{35a\sin(c+dx)}{64d}+\frac{7a\sin(3(c+dx))}{64d}+\frac{7a\sin(5(c+dx))}{320d}+\frac{a\sin(7(c+dx))}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/7*I)*a*Cos[c + d*x]^7)/d + (35*a*Sin[c + d*x])/(64*d) + (7*a*Sin[3*(c
+ d*x)])/(64*d) + (7*a*Sin[5*(c + d*x)])/(320*d) + (a*Sin[7*(c + d*x)])/(44
8*d)
```

Maple [A]
time = 0.23, size = 57, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{ia\left(\cos^7(dx+c)\right)}{7}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
default	$\frac{-\frac{ia\left(\cos^7(dx+c)\right)}{7}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
risch	$-\frac{ia\,e^{7i(dx+c)}}{448d}-\frac{5ia\cos(dx+c)}{64d}+\frac{35a\sin(dx+c)}{64d}-\frac{ia\cos(5dx+5c)}{64d}+\frac{7a\sin(5dx+5c)}{320d}-\frac{3ia\cos(3dx+3c)}{64d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*I*a*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.76

$$-\frac{5i\,a\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/35*(5*I*a*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*si
n(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

Fricas [A]
time = 0.39, size = 90, normalized size = 1.18

$$\frac{\left(-5i\,ae^{(12i\,dx+12i\,c)}-42i\,ae^{(10i\,dx+10i\,c)}-175i\,ae^{(8i\,dx+8i\,c)}-700i\,ae^{(6i\,dx+6i\,c)}+525i\,ae^{(4i\,dx+4i\,c)}+70i\,ae^{(2i\,dx+2i\,c)}+7i\,a\right)e^{(-5i\,dx-5i\,c)}}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2240*(-5*I*a*e^(12*I*d*x + 12*I*c) - 42*I*a*e^(10*I*d*x + 10*I*c) - 175*I
*a*e^(8*I*d*x + 8*I*c) - 700*I*a*e^(6*I*d*x + 6*I*c) + 525*I*a*e^(4*I*d*x +
 4*I*c) + 70*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(65) = 130$.
time = 0.31, size = 253, normalized size = 3.33

$$\begin{cases} \frac{(-107374182400iad^6e^{16ic}e^{7idx}-901943132160iad^6e^{14ic}e^{5idx}-3758096384000iad^6e^{12ic}e^{3idx}-15032385536000iad^6e^{10ic}e^{idx}+11274289152000iad^6e^{8ic}e^{-idx}+1503238553600iad^6e^{6ic}e^{-3idx}+150323855360iad^6e^{4ic}e^{-5idx})e^{-9ic}}{48103633715200d^7} & \text{for } d^7e^{9ic} \neq 0 \\ \frac{x(ae^{12ic}+6ae^{10ic}+15ae^{8ic}+20ae^{6ic}+15ae^{4ic}+6ae^{2ic}+a)e^{-5ic}}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-107374182400*I*a*d**6*exp(16*I*c)*exp(7*I*d*x) - 901943132160*
I*a*d**6*exp(14*I*c)*exp(5*I*d*x) - 3758096384000*I*a*d**6*exp(12*I*c)*exp(
3*I*d*x) - 15032385536000*I*a*d**6*exp(10*I*c)*exp(I*d*x) + 11274289152000*
I*a*d**6*exp(8*I*c)*exp(-I*d*x) + 1503238553600*I*a*d**6*exp(6*I*c)*exp(-3*
I*d*x) + 150323855360*I*a*d**6*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(48103
633715200*d**7), Ne(d**7*exp(9*I*c), 0)), (x*(a*exp(12*I*c) + 6*a*exp(10*I*
c) + 15*a*exp(8*I*c) + 20*a*exp(6*I*c) + 15*a*exp(4*I*c) + 6*a*exp(2*I*c) +
 a)*exp(-5*I*c)/64, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(68) = 136$.
time = 0.63, size = 244, normalized size = 3.21

$$-\frac{\left(1015\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+700\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-1015\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-700\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-315\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+315\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+20i\,ae^{(12i\,dx+8i\,c)}+168i\,ae^{(10i\,dx+6i\,c)}+700i\,ae^{(8i\,dx+4i\,c)}+2800i\,ae^{(6i\,dx+2i\,c)}-280i\,ae^{(2i\,dx-2i\,c)}-2100i\,ae^{(4i\,dx)}-28i\,ae^{(-4i\,c)}\right)e^{(-5i\,dx-i\,c)}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8960*(1015*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 700*a*e^(5*I
*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 1015*a*e^(5*I*d*x + I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 700*a*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) -
315*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 315*a*e^(5*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 20*I*a*e^(12*I*d*x + 8*I*c) + 168*I*a*e^(
10*I*d*x + 6*I*c) + 700*I*a*e^(8*I*d*x + 4*I*c) + 2800*I*a*e^(6*I*d*x + 2*I
*c) - 280*I*a*e^(2*I*d*x - 2*I*c) - 2100*I*a*e^(4*I*d*x) - 28*I*a*e^(-4*I*c
))*e^(-5*I*d*x - I*c)/d
```

Mupad [B]
time = 6.07, size = 93, normalized size = 1.22

$$-\frac{2\,a\,\left(-\frac{1225\,\sin\left(c+d\,x\right)}{128}-\frac{245\,\sin\left(3\,c+3\,d\,x\right)}{128}-\frac{49\,\sin\left(5\,c+5\,d\,x\right)}{128}-\frac{5\,\sin\left(7\,c+7\,d\,x\right)}{128}+\frac{\cos\left(c+d\,x\right)\,175{}\mathrm{i}}{128}+\frac{\cos\left(3\,c+3\,d\,x\right)\,105{}\mathrm{i}}{128}+\frac{\cos\left(5\,c+5\,d\,x\right)\,35{}\mathrm{i}}{128}+\frac{\cos\left(7\,c+7\,d\,x\right)\,5{}\mathrm{i}}{128}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -(2*a*((cos(c + d*x)*175i)/128 - (1225*sin(c + d*x))/128 + (cos(3*c + 3*d*x
)*105i)/128 + (cos(5*c + 5*d*x)*35i)/128 + (cos(7*c + 7*d*x)*5i)/128 - (245
*sin(3*c + 3*d*x))/128 - (49*sin(5*c + 5*d*x))/128 - (5*sin(7*c + 7*d*x))/1
28))/(35*d)
```

3.19 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=109

$$-\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\frac{i(a+ia\tan(c+dx))^9}{9a^7d}$$

```
[Out] -4/3*I*(a+I*a*tan(d*x+c))^6/a^4/d+12/7*I*(a+I*a*tan(d*x+c))^7/a^5/d-3/4*I*(
a+I*a*tan(d*x+c))^8/a^6/d+1/9*I*(a+I*a*tan(d*x+c))^9/a^7/d
```

Rubi [A]

time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^9}{9a^7d}-\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^4*d) + (((12*I)/7)*(a + I*a*Tan[c
+ d*x])^7)/(a^5*d) - (((3*I)/4)*(a + I*a*Tan[c + d*x])^8)/(a^6*d) + ((I/9)*
(a + I*a*Tan[c + d*x])^9)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^5\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^5-12a^2(a+x)^6+6a(a+x)^7-(a+x)^8\right)\right.}{a^7d}\\ &= -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{3i(a+ia\,\mathrm{t}}{4}\end{aligned}$$

Mathematica [A]
time = 0.90, size = 99, normalized size = 0.91

$$\frac{a^2\sec(c)\sec^9(c+dx)(63i\cos(dx)+63i\cos(2c+dx)+63\sin(dx)-63\sin(2c+dx)+84\sin(2c+3dx)+36\sin(4c+5dx)+9\sin(6c+7dx)+\sin(8c+9dx))}{504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^9*((63*I)*Cos[d*x] + (63*I)*Cos[2*c + d*x] + 63*Si
n[d*x] - 63*Sin[2*c + d*x] + 84*Sin[2*c + 3*d*x] + 36*Sin[4*c + 5*d*x] + 9*
Sin[6*c + 7*d*x] + Sin[8*c + 9*d*x]))/(504*d)
```

Maple [A]
time = 0.24, size = 141, normalized size = 1.29

method	result
risch	$\frac{64ia^2\left(126\,e^{10i(dx+c)}+126\,e^{8i(dx+c)}+84\,e^{6i(dx+c)}+36\,e^{4i(dx+c)}+9\,e^{2i(dx+c)}+1\right)}{63d\left(e^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\frac{16\left(\sin^3(dx+c)\right)}{315\cos(dx+c)^3}\right)+\frac{ia^2}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx\right.}{35}\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\frac{16\left(\sin^3(dx+c)\right)}{315\cos(dx+c)^3}\right)+\frac{ia^2}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx\right.}{35}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/1
05*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+1/4*I*a^2/co
s(d*x+c)^8-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 0.99

$$-\frac{28\,a^2\tan(dx+c)^9-63i\,a^2\tan(dx+c)^8+72\,a^2\tan(dx+c)^7-252i\,a^2\tan(dx+c)^6-378i\,a^2\tan(dx+c)^4-168\,a^2\tan(dx+c)^3-252i\,a^2\tan(dx+c)^2-252\,a^2\tan(dx+c)}{252\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(d*
x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(85) = 170$.
time = 0.40, size = 189, normalized size = 1.73

$$-\frac{64\left(-126i\,a^{2}e^{(10i\,dx+10i\,c)}-126i\,a^{2}e^{(8i\,dx+8i\,c)}-84i\,a^{2}e^{(6i\,dx+6i\,c)}-36i\,a^{2}e^{(4i\,dx+4i\,c)}-9i\,a^{2}e^{(2i\,dx+2i\,c)}-i\,a^{2}\right)}{63\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i\,dx+8i\,c)}+84\,de^{(6i\,dx+6i\,c)}+36\,de^{(4i\,dx+4i\,c)}+9\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -64/63*(-126*I*a^2*e^(10*I*d*x + 10*I*c) - 126*I*a^2*e^(8*I*d*x + 8*I*c) -
84*I*a^2*e^(6*I*d*x + 6*I*c) - 36*I*a^2*e^(4*I*d*x + 4*I*c) - 9*I*a^2*e^(2*
I*d*x + 2*I*c) - I*a^2)/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*I*c
) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(10*I
*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c) + 36*
d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int \tan^{2}(c+dx)\sec^{8}(c+dx)\,dx+\int\left(-2i\tan(c+dx)\sec^{8}(c+dx)\right)\,dx+\int\left(-\sec^{8}(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**8, x) + Integral(-sec(c + d*x)**8, x))
```

Giac [A]
time = 0.61, size = 108, normalized size = 0.99

$$-\frac{28\,a^{2}\tan(dx+c)^{9}-63i\,a^{2}\tan(dx+c)^{8}+72\,a^{2}\tan(dx+c)^{7}-252i\,a^{2}\tan(dx+c)^{6}-378i\,a^{2}\tan(dx+c)^{4}-168\,a^{2}\tan(dx+c)^{3}-252i\,a^{2}\tan(dx+c)^{2}-252\,a^{2}\tan(dx+c)}{252\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(d*
x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.26, size = 151, normalized size = 1.39

$$\frac{a^{2}\sin(c+d\,x)\,\left(252\cos(c+d\,x)^{8}+\cos(c+d\,x)^{7}\sin(c+d\,x)\,252\mathrm{i}+168\cos(c+d\,x)^{6}\sin(c+d\,x)^{2}+\cos(c+d\,x)^{5}\sin(c+d\,x)^{3}\,378\mathrm{i}+\cos(c+d\,x)^{3}\sin(c+d\,x)^{5}\,252\mathrm{i}-72\cos(c+d\,x)^{2}\sin(c+d\,x)^{6}+\cos(c+d\,x)\,\sin(c+d\,x)^{7}\,63\mathrm{i}-28\sin(c+d\,x)^{8}\right)}{252\,d\cos(c+d\,x)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^8,x)

[Out] (a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^7*63i + cos(c + d*x)^7*sin(c +
 d*x)*252i + 252*cos(c + d*x)^8 - 28*sin(c + d*x)^8 - 72*cos(c + d*x)^2*sin
(c + d*x)^6 + cos(c + d*x)^3*sin(c + d*x)^5*252i + cos(c + d*x)^5*sin(c + d
*x)^3*378i + 168*cos(c + d*x)^6*sin(c + d*x)^2))/(252*d*cos(c + d*x)^9)
```

3.20 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}$$

```
[Out] -4/5*I*(a+I*a*tan(d*x+c))^5/a^3/d+2/3*I*(a+I*a*tan(d*x+c))^6/a^4/d-1/7*I*(a
+I*a*tan(d*x+c))^7/a^5/d
```

Rubi [A]

time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^3*d) + (((2*I)/3)*(a + I*a*Tan[c +
 d*x])^6)/(a^4*d) - ((I/7)*(a + I*a*Tan[c + d*x])^7)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)dx, x, ia\tan(c+\right.}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{i(a+ia\tan}{7a^5}\end{aligned}$$

Mathematica [A]
time = 0.57, size = 90, normalized size = 1.10

$$\frac{a^2\sec(c)\sec^7(c+dx)(35i\cos(dx)+35i\cos(2c+dx)+35\sin(dx)-35\sin(2c+dx)+42\sin(2c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx))}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^7*((35*I)*Cos[d*x] + (35*I)*Cos[2*c + d*x] + 35*Si
n[d*x] - 35*Sin[2*c + d*x] + 42*Sin[2*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*
Sin[6*c + 7*d*x]))/(210*d)
```

Maple [A]
time = 0.24, size = 113, normalized size = 1.38

method	result
risch	$\frac{128ia^2\left(35\,e^{8i(dx+c)}+35\,e^{6i(dx+c)}+21\,e^{4i(dx+c)}+7\,e^{2i(dx+c)}+1\right)}{105d\left(e^{2i(dx+c)}+1\right)^7}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/1
05*sin(d*x+c)^3/cos(d*x+c)^3)+1/3*I*a^2/cos(d*x+c)^6-a^2*(-8/15-1/5*sec(d*x
+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 1.16

$$-\frac{15\,a^2\tan(dx+c)^7-35i\,a^2\tan(dx+c)^6+21\,a^2\tan(dx+c)^5-105i\,a^2\tan(dx+c)^4-35\,a^2\tan(dx+c)^3-105i\,a^2\tan(dx+c)^2-105\,a^2\tan(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.
time = 0.35, size = 151, normalized size = 1.84

$$-\frac{128\left(-35i\,a^2e^{(8i\,dx+8i\,c)}-35i\,a^2e^{(6i\,dx+6i\,c)}-21i\,a^2e^{(4i\,dx+4i\,c)}-7i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{105\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -128/105*(-35*I*a^2*e^(8*I*d*x + 8*I*c) - 35*I*a^2*e^(6*I*d*x + 6*I*c) - 21
*I*a^2*e^(4*I*d*x + 4*I*c) - 7*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(14*
I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) +
35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I
*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int \tan^{2}(c+dx)\sec^{6}(c+dx)\,dx+\int \left(-2i\tan(c+dx)\sec^{6}(c+dx)\right)\,dx+\int \left(-\sec^{6}(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**6, x) + Integral(-sec(c + d*x)**6, x))
```

Giac [A]
time = 0.57, size = 95, normalized size = 1.16

$$-\frac{15\,a^{2}\tan(dx+c)^{7}-35i\,a^{2}\tan(dx+c)^{6}+21\,a^{2}\tan(dx+c)^{5}-105i\,a^{2}\tan(dx+c)^{4}-35\,a^{2}\tan(dx+c)^{3}-105i\,a^{2}\tan(dx+c)^{2}-105\,a^{2}\tan(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.24, size = 132, normalized size = 1.61

$$\frac{a^{2}\sin(c+d\,x)\,\left(105\cos(c+d\,x)^{6}+\cos(c+d\,x)^{5}\sin(c+d\,x)\,105\mathrm{i}+35\cos(c+d\,x)^{4}\sin(c+d\,x)^{2}+\cos(c+d\,x)^{3}\sin(c+d\,x)^{3}\,105\mathrm{i}-21\cos(c+d\,x)^{2}\sin(c+d\,x)^{4}+\cos(c+d\,x)\,\sin(c+d\,x)^{5}\,35\mathrm{i}-15\sin(c+d\,x)^{6}\right)}{105\,d\cos(c+d\,x)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*35i + cos(c + d*x)^5*sin(c +
 d*x)*105i + 105*cos(c + d*x)^6 - 15*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin
(c + d*x)^4 + cos(c + d*x)^3*sin(c + d*x)^3*105i + 35*cos(c + d*x)^4*sin(c
+ d*x)^2))/(105*d*cos(c + d*x)^7)
```

3.21 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^4/a^2/d+1/5*I*(a+I*a*tan(d*x+c))^5/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^5}{5a^3d}-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*(a + I*a*Tan[c + d*x])^4)/(a^2*d) + ((I/5)*(a + I*a*Tan[c + d*x])
^5)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^3\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 77, normalized size = 1.40

$$\frac{a^2 \sec(c) \sec^5(c+dx)(5i\cos(dx)+5i\cos(2c+dx)+5\sin(dx)-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^5*((5*I)*Cos[d*x] + (5*I)*Cos[2*c + d*x] + 5*Sin[d
*x] - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*d)
```

Maple [A]
time = 0.24, size = 85, normalized size = 1.55

method	result	size
risch	$\frac{8ia^2\left(10\,e^{6i(dx+c)}+10\,e^{4i(dx+c)}+5\,e^{2i(dx+c)}+1\right)}{5d\left(e^{2i(dx+c)}+1\right)^5}$	58
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{ia^2}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	85
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{ia^2}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+1/
2*I*a^2/cos(d*x+c)^4-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.02

$$-\frac{2\,a^2\tan\left(dx+c\right)^5-5i\,a^2\tan\left(dx+c\right)^4-10i\,a^2\tan\left(dx+c\right)^2-10\,a^2\tan\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x + c
)^2 - 10*a^2*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(43) = 86.
time = 0.34, size = 113, normalized size = 2.05

$$-\frac{8\left(-10i\,a^2e^{(6i\,dx+6i\,c)}-10i\,a^2e^{(4i\,dx+4i\,c)}-5i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -8/5*(-10*I*a^2*e^(6*I*d*x + 6*I*c) - 10*I*a^2*e^(4*I*d*x + 4*I*c) - 5*I*a^
2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int \tan^2\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int \left(-2i\tan\left(c+dx\right)\sec^4\left(c+dx\right)\right)\,dx+\int \left(-\sec^4\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**4, x) + Integral(-sec(c + d*x)**4, x))
```

Giac [A]
time = 0.56, size = 56, normalized size = 1.02

$$-\frac{2\,a^2\tan\left(dx+c\right)^5-5i\,a^2\tan\left(dx+c\right)^4-10i\,a^2\tan\left(dx+c\right)^2-10\,a^2\tan\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x + c
)^2 - 10*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.22, size = 56, normalized size = 1.02

$$\frac{-\frac{a^2\tan(c+d\,x)^5}{5}+\frac{a^2\tan(c+d\,x)^4\,1\mathrm{i}}{2}+a^2\tan(c+d\,x)^2\,1\mathrm{i}+a^2\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c + d*x) + a^2*tan(c + d*x)^2*1i + (a^2*tan(c + d*x)^4*1i)/2 - (a^
2*tan(c + d*x)^5)/5)/d
```

3.22 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

```
[Out] -1/3*I*(a+I*a*tan(d*x+c))^3/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/3*I)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^2\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 0.32, size = 68, normalized size = 2.52

$$\frac{a^2\sec(c)\sec^3(c+dx)(3i\cos(dx)+3i\cos(2c+dx)+3\sin(dx)-3\sin(2c+dx)+2\sin(2c+3dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^3*((3*I)*Cos[d*x] + (3*I)*Cos[2*c + d*x] + 3*Sin[d
*x] - 3*Sin[2*c + d*x] + 2*Sin[2*c + 3*d*x]))/(6*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.
time = 0.27, size = 51, normalized size = 1.89

method	result	size
risch	$\frac{8ia^2\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	47
derivativedivides	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51
default	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+I*a^2/cos(d*x+c)^2+a^2*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.78

$$-\frac{i\,(i\,a\tan\left(dx+c\right)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*I*(I*a*tan(d*x + c) + a)^3/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(21) = 42$.
time = 0.33, size = 75, normalized size = 2.78

$$-\frac{8\left(-3i\,a^2e^{(4i\,dx+4i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -8/3*(-3*I*a^2*e^(4*I*d*x + 4*I*c) - 3*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(
d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^2(c+dx)\right)\,dx + \int \left(-\sec^2(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**2, x) + Integral(-sec(c + d*x)**2, x))
```

Giac [A]
time = 0.52, size = 42, normalized size = 1.56

$$-\frac{a^2\tan(dx+c)^3 - 3i\,a^2\tan(dx+c)^2 - 3\,a^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.22, size = 35, normalized size = 1.30

$$\frac{a^2\tan(c+d\,x)\,\left(-\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}+3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^2,x)
```

```
[Out] (a^2*tan(c + d*x)*(tan(c + d*x)*3i - tan(c + d*x)^2 + 3))/(3*d)
```

3.23 $\int (a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=38

$$2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3558, 3556}

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^2\,dx &= 2a^2x-\frac{a^2\tan(c+dx)}{d}+\left(2ia^2\right)\int \tan(c+dx)\,dx\\ &= 2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(38) = 76.
time = 0.61, size = 100, normalized size = 2.63

$$-\frac{a^2\sec(c)\sec(c+dx)\left(4\text{ArcTan}(\tan(3c+dx))\cos(c)\cos(c+dx)-4dx\cos(2c+dx)+\cos(dx)\left(-4dx+i\log\left(\cos^2(c+dx)\right)\right)+i\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2\sin(dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Sec[c]*Sec[c + d*x]*(4*ArcTan[Tan[3*c + d*x]]*Cos[c]*Cos[c + d*x]
 - 4*d*x*Cos[2*c + d*x] + Cos[d*x]*(-4*d*x + I*Log[Cos[c + d*x]^2]) + I*Cos
[2*c + d*x]*Log[Cos[c + d*x]^2] + 2*Sin[d*x]))/d
```

Maple [A]
time = 0.04, size = 40, normalized size = 1.05

method	result	size
derivativedivides	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
default	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
norman	$2a^2x-\frac{a^2\tan(dx+c)}{d}+\frac{ia^2\ln(1+\tan^2(dx+c))}{d}$	42
risch	$-\frac{4a^2c}{d}-\frac{2ia^2}{d(e^{2i(dx+c)}+1)}-\frac{2ia^2\ln(e^{2i(dx+c)}+1)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(-tan(d*x+c)+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 1.08

$$a^2x+\frac{(dx+c-\tan(dx+c))a^2}{d}+\frac{2i\,a^2\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 56, normalized size = 1.47

$$-\frac{2\left(i\,a^2+\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(I*a^2 + (I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.12, size = 53, normalized size = 1.39

$$-\frac{2ia^2}{de^{2ic}e^{2idx}+d}-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) + d) - 2*I*a**2*log(exp(2*I*d*x) + exp
(-2*I*c))/d
```

Giac [A]
time = 0.51, size = 66, normalized size = 1.74

$$-\frac{2\left(i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^2*log(e^(2
*I*d*x + 2*I*c) + 1) + I*a^2)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.23, size = 29, normalized size = 0.76

$$\frac{a^2\left(-\tan(c+d\,x)+\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*(log(tan(c + d*x) + 1i)*2i - tan(c + d*x)))/d
```

3.24 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=25

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

```
[Out] -I*a^3/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^3}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 31, normalized size = 1.24

$$-\frac{ia^2(\cos(c+dx)+i\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*a^2*(Cos[c + d*x] + I*Sin[c + d*x])^2)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(23) = 46$.
time = 0.24, size = 73, normalized size = 2.92

method	result	size
risch	$-\frac{i a^2 e^{2 i (d x+c)}}{2 d}$	19
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-i a^2\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-i a^2\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-I*a^2*cos(d*x+c)^2+a^2
*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 32, normalized size = 1.28

$$\frac{a^2 \tan(dx+c) - i\,a^2}{\left(\tan(dx+c)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a^2*tan(d*x + c) - I*a^2)/((tan(d*x + c)^2 + 1)*d)
```

Fricas [A]
time = 0.36, size = 17, normalized size = 0.68

$$-\frac{i\,a^2 e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A]
time = 0.10, size = 36, normalized size = 1.44

$$\begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2*
I*c), True))
```

Giac [A]
time = 0.56, size = 17, normalized size = 0.68

$$-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Mupad [B]
time = 3.28, size = 18, normalized size = 0.72

$$\frac{a^2}{d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] a^2/(d*(tan(c + d*x) + 1i))
```

3.25 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{a^2x}{4}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}$$

```
[Out] 1/4*a^2*x-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2-1/4*I*a^3/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}+\frac{a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/4)*a^3)/(d*(a -
I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^3} + \frac{1}{4a^2(a-x)^2} + \frac{1}{4a^2(a^2-x^2)}\right)\,dx, x, ia\tan(c-\right.}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2} - \frac{ia^3}{4d(a-ia\tan(c+dx))} - \frac{(ia^3)\,\text{Subst}}{}$$

$$= \frac{a^2x}{4} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2} - \frac{ia^3}{4d(a-ia\tan(c+dx))}$$

Mathematica [A]
time = 0.33, size = 86, normalized size = 1.37

$$\frac{a^2(-4i+(-i+4dx)\cos(2(c+dx))+(1-4idx)\sin(2(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{16d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-4*I + (-I + 4*d*x)*Cos[2*(c + d*x)] + (1 - (4*I)*d*x)*Sin[2*(c + d*x
)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(16*d*(Cos[d*x] + I*Sin[d*
x])^2)
```

Maple [A]
time = 0.22, size = 100, normalized size = 1.59

method	result
risch	$\frac{a^2x}{4} - \frac{ia^2 e^{4i(dx+c)}}{16d} - \frac{ia^2 e^{2i(dx+c)}}{4d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ia^2\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right.}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ia^2\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1
/8*c)-1/2*I*a^2*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.06

$$\frac{(dx+c)a^2 + \frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)-2i\,a^2}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*x + c)*a^2 + (a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 2*I*a^2)/(t
an(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 41, normalized size = 0.65

$$\frac{4\,a^2dx - i\,a^2e^{(4i\,dx+4i\,c)} - 4i\,a^2e^{(2i\,dx+2i\,c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*d*x - I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [A]
time = 0.13, size = 87, normalized size = 1.38

$$\frac{a^2x}{4} + \begin{cases} \frac{-4ia^2de^{4ic}e^{4idx}-16ia^2de^{2ic}e^{2idx}}{64d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{4ic}}{4} + \frac{a^2e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*x/4 + Piecewise(((-4*I*a**2*d*exp(4*I*c)*exp(4*I*d*x) - 16*I*a**2*d*ex
p(2*I*c)*exp(2*I*d*x))/(64*d**2), Ne(d**2, 0)), (x*(a**2*exp(4*I*c)/4 + a**
2*exp(2*I*c)/2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(49) = 98$.
time = 0.74, size = 257, normalized size = 4.08

$$\frac{8\,a^2dxe^{(4i\,dx+2i\,c)} + 16\,a^2dxe^{(2i\,dx)} + 8\,a^2dxe^{(-2i\,c)} - i\,a^2e^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 2i\,a^2e^{(2i\,dx)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - i\,a^2e^{(-2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + i\,a^2e^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) + 2i\,a^2e^{(2i\,dx)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) + i\,a^2e^{(-2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - 2i\,a^2e^{(8i\,dx+6i\,c)} - 12i\,a^2e^{(6i\,dx+4i\,c)} - 18i\,a^2e^{(4i\,dx+2i\,c)} - 8i\,a^2e^{(2i\,dx)}}{32\left(de^{(4i\,dx+2i\,c)} + 2\,de^{(2i\,dx)} + de^{(-2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/32*(8*a^2*d*x*e^(4*I*d*x + 2*I*c) + 16*a^2*d*x*e^(2*I*d*x) + 8*a^2*d*x*e^
(-2*I*c) - I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2*I*a^2
*e^(2*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - I*a^2*e^(-2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) + I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
2*I*a^2*e^(2*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + I*a^2*e^(-2*I*c)*log(e^
(2*I*d*x) + e^(-2*I*c)) - 2*I*a^2*e^(8*I*d*x + 6*I*c) - 12*I*a^2*e^(6*I*d*x
 + 4*I*c) - 18*I*a^2*e^(4*I*d*x + 2*I*c) - 8*I*a^2*e^(2*I*d*x))/(d*e^(4*I*d
*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

Mupad [B]
time = 3.27, size = 50, normalized size = 0.79

$$\frac{a^2\,x}{4}+\frac{\frac{a^2\,\tan\left(c+d\,x\right)}{4}+\frac{a^2\,1\mathrm{i}}{2}}{d\,\left({\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*x)/4 + ((a^2*tan(c + d*x))/4 + (a^2*1i)/2)/(d*(tan(c + d*x)*2i + tan(c
 + d*x)^2 - 1))
```

3.26 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=117

$$\frac{a^2x}{4}-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{3ia^3}{16d(a-ia\tan(c+dx))}+\frac{ia^3}{16d(a+ia\tan(c+dx))}$$

```
[Out] 1/4*a^2*x-1/12*I*a^5/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^4/d/(a-I*a*tan(d*x+c))^
2-3/16*I*a^3/d/(a-I*a*tan(d*x+c))+1/16*I*a^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{3ia^3}{16d(a-ia\tan(c+dx))}+\frac{ia^3}{16d(a+ia\tan(c+dx))}+\frac{a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/12)*a^5)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^4)/(d*(a -
 I*a*Tan[c + d*x])^2) - (((3*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/1
6)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^7)\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4}+\frac{1}{4a^3(a-x)^3}+\frac{3}{16a^4(a-x)^2}+\frac{1}{16a^4(a+x)^2}+\ldots\right.\right.}{d}$$
$$= -\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{\ldots}{16d(a-\ldots}$$
$$= \frac{a^2x}{4}-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{\ldots}{16d(\ldots}$$

Mathematica [A]
time = 0.44, size = 116, normalized size = 0.99

$$\frac{a^2(-9i+3(-i+4dx)\cos(2(c+dx))+i\cos(4(c+dx))+3\sin(2(c+dx))-12idx\sin(2(c+dx))+2\sin(4(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{48d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-9*I + 3*(-I + 4*d*x)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 3*Sin[2
*(c + d*x)] - (12*I)*d*x*Sin[2*(c + d*x)] + 2*Sin[4*(c + d*x)])*(Cos[2*(c +
 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(48*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.23, size = 121, normalized size = 1.03

method	result
risch	$\frac{a^2x}{4}-\frac{ia^2e^{6i(dx+c)}}{96d}-\frac{ia^2e^{4i(dx+c)}}{16d}-\frac{5ia^2\cos(2dx+2c)}{32d}+\frac{7a^2\sin(2dx+2c)}{32d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5(\ldots}{}\ldots\right.}{}\ldots\right.}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5(\ldots}{}\ldots\right.}{}\ldots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+1/16*d*x+1/16*c)-1/3*I*a^2*cos(d*x+c)^6+a^2*(1/6*(cos(d*x+c)^5+5
/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.49, size = 92, normalized size = 0.79

$$\frac{3\,(dx+c)a^2 + \frac{3\,a^2\tan(dx+c)^5+8\,a^2\tan(dx+c)^3+9\,a^2\tan(dx+c)-4i\,a^2}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(d*x + c)*a^2 + (3*a^2*tan(d*x + c)^5 + 8*a^2*tan(d*x + c)^3 + 9*a^
2*tan(d*x + c) - 4*I*a^2)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x +
c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.67

$$\frac{\left(24\,a^2dxe^{(2i\,dx+2i\,c)}-i\,a^2e^{(8i\,dx+8i\,c)}-6i\,a^2e^{(6i\,dx+6i\,c)}-18i\,a^2e^{(4i\,dx+4i\,c)}+3i\,a^2\right)e^{(-2i\,dx-2i\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*a^2*d*x*e^(2*I*d*x + 2*I*c) - I*a^2*e^(8*I*d*x + 8*I*c) - 6*I*a^2*
e^(6*I*d*x + 6*I*c) - 18*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2)*e^(-2*I*d*x -
 2*I*c)/d
```

Sympy [A]
time = 0.21, size = 185, normalized size = 1.58

$$\frac{a^2x}{4}+\begin{cases}\frac{(-8192ia^2d^3e^{8ic}e^{6idx}-49152ia^2d^3e^{6ic}e^{4idx}-147456ia^2d^3e^{4ic}e^{2idx}+24576ia^2d^3e^{-2idx})e^{-2ic}}{786432d^4} & \text{for } d^4e^{2ic}\neq 0\\ x\left(-\frac{a^2}{4}+\frac{(a^2e^{8ic}+4a^2e^{6ic}+6a^2e^{4ic}+4a^2e^{2ic}+a^2)e^{-2ic}}{16}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*x/4 + Piecewise(((-8192*I*a**2*d**3*exp(8*I*c)*exp(6*I*d*x) - 49152*I*
a**2*d**3*exp(6*I*c)*exp(4*I*d*x) - 147456*I*a**2*d**3*exp(4*I*c)*exp(2*I*d
*x) + 24576*I*a**2*d**3*exp(-2*I*d*x))*exp(-2*I*c)/(786432*d**4), Ne(d**4*e
xp(2*I*c), 0)), (x*(-a**2/4 + (a**2*exp(8*I*c) + 4*a**2*exp(6*I*c) + 6*a**2
*exp(4*I*c) + 4*a**2*exp(2*I*c) + a**2)*exp(-2*I*c)/16), True))
```

Giac [A]
time = 0.76, size = 169, normalized size = 1.44

$$\frac{24\,a^2dxe^{(6i\,dx+4i\,c)}+48\,a^2dxe^{(4i\,dx+2i\,c)}+24\,a^2dxe^{(2i\,dx)}-i\,a^2e^{(12i\,dx+10i\,c)}-8i\,a^2e^{(10i\,dx+8i\,c)}-31i\,a^2e^{(8i\,dx+6i\,c)}-42i\,a^2e^{(6i\,dx+4i\,c)}-15i\,a^2e^{(4i\,dx+2i\,c)}+6i\,a^2e^{(2i\,dx)}+3i\,a^2e^{(-2i\,c)}}{96\,\left(de^{(6i\,dx+4i\,c)}+2\,de^{(4i\,dx+2i\,c)}+de^{(2i\,dx)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(24*a^2*d*x*e^(6*I*d*x + 4*I*c) + 48*a^2*d*x*e^(4*I*d*x + 2*I*c) + 24*
a^2*d*x*e^(2*I*d*x) - I*a^2*e^(12*I*d*x + 10*I*c) - 8*I*a^2*e^(10*I*d*x + 8
*I*c) - 31*I*a^2*e^(8*I*d*x + 6*I*c) - 42*I*a^2*e^(6*I*d*x + 4*I*c) - 15*I*
a^2*e^(4*I*d*x + 2*I*c) + 6*I*a^2*e^(2*I*d*x) + 3*I*a^2*e^(-2*I*c))/(d*e^(6
*I*d*x + 4*I*c) + 2*d*e^(4*I*d*x + 2*I*c) + d*e^(2*I*d*x))
```

Mupad [B]
time = 3.37, size = 88, normalized size = 0.75

$$\frac{a^2\,x}{4}+\frac{\frac{a^2\,\tan(c+d\,x)^3}{4}+\frac{a^2\,\tan(c+d\,x)^2\,1\mathrm{i}}{2}-\frac{a^2\,\tan(c+d\,x)}{12}+\frac{a^2\,1\mathrm{i}}{3}}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,2\mathrm{i}+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*x)/4 + ((a^2*1i)/3 - (a^2*tan(c + d*x))/12 + (a^2*tan(c + d*x)^2*1i)/2
 + (a^2*tan(c + d*x)^3)/4)/(d*(tan(c + d*x)*2i + tan(c + d*x)^3*2i + tan(c
+ d*x)^4 - 1))
```

3.27 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=171

$$\frac{15a^2x}{64}-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-\frac{3ia^4}{32d(a-ia\tan(c+dx))^2}-\frac{5ia^3}{32d(a-ia\tan(c+dx))}$$

```
[Out] 15/64*a^2*x-1/32*I*a^6/d/(a-I*a*tan(d*x+c))^4-1/16*I*a^5/d/(a-I*a*tan(d*x+c
))^3-3/32*I*a^4/d/(a-I*a*tan(d*x+c))^2-5/32*I*a^3/d/(a-I*a*tan(d*x+c))+1/64
*I*a^4/d/(a+I*a*tan(d*x+c))^2+5/64*I*a^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-\frac{3ia^4}{32d(a-ia\tan(c+dx))^2}+\frac{ia^4}{64d(a+ia\tan(c+dx))^2}-\frac{5ia^3}{32d(a-ia\tan(c+dx))}+\frac{5ia^3}{64d(a+ia\tan(c+dx))}+\frac{15a^2x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (15*a^2*x)/64 - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/16)*a^5)/(d
*(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^4)/(d*(a - I*a*Tan[c + d*x])^2)
- (((5*I)/32)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/64)*a^4)/(d*(a + I*a*Ta
n[c + d*x])^2) + (((5*I)/64)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{(ia^9)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^9)\,\mathrm{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^5}+\frac{3}{16a^4(a-x)^4}+\frac{3}{16a^5(a-x)^3}+\frac{5}{32a^6(a-x)^2}+\right.\right.}{d}$$

$$= -\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-\frac{}{32d(a-}$$

$$= \frac{15a^2x}{64}-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-$$

Mathematica [A]
time = 0.45, size = 138, normalized size = 0.81

$$\frac{a^2(-80i+30(-i+4dx)\cos(2(c+dx))+16i\cos(4(c+dx))+i\cos(6(c+dx))+30\sin(2(c+dx))-120idx\sin(2(c+dx))+32\sin(4(c+dx))+3\sin(6(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{512d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-80*I + 30*(-I + 4*d*x)*Cos[2*(c + d*x)] + (16*I)*Cos[4*(c + d*x)] +
I*Cos[6*(c + d*x)] + 30*Sin[2*(c + d*x)] - (120*I)*d*x*Sin[2*(c + d*x)] + 3
2*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c +
 2*d*x)]))/(512*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.23, size = 141, normalized size = 0.82

method	result
risch	$\frac{15a^2x}{64}-\frac{ia^2 e^{8i(dx+c)}}{512d}-\frac{ia^2 e^{6i(dx+c)}}{64d}-\frac{7ia^2\cos(4dx+4c)}{128d}+\frac{a^2\sin(4dx+4c)}{16d}-\frac{7ia^2\cos(2dx+2c)}{64d}+\frac{13a^2\sin(2d}{64d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\left(\cos^8(dx+c)\right)}{4}+a^2}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\left(\cos^8(dx+c)\right)}{4}+a^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3
+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/4*I*a^2*cos(d*x+c)^8+a^2*
(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*si
n(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.50, size = 115, normalized size = 0.67

$$\frac{15\,(dx+c)a^2+\frac{15\,a^2\tan(dx+c)^7+55\,a^2\tan(dx+c)^5+73\,a^2\tan(dx+c)^3+49\,a^2\tan(dx+c)-16i\,a^2}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/64*(15*(d*x + c)*a^2 + (15*a^2*tan(d*x + c)^7 + 55*a^2*tan(d*x + c)^5 + 7
3*a^2*tan(d*x + c)^3 + 49*a^2*tan(d*x + c) - 16*I*a^2)/(tan(d*x + c)^8 + 4*
tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 106, normalized size = 0.62

$$\frac{\left(120\,a^2dxe^{(4i\,dx+4i\,c)}-i\,a^2e^{(12i\,dx+12i\,c)}-8i\,a^2e^{(10i\,dx+10i\,c)}-30i\,a^2e^{(8i\,dx+8i\,c)}-80i\,a^2e^{(6i\,dx+6i\,c)}+24i\,a^2e^{(2i\,dx+2i\,c)}+2i\,a^2\right)e^{(-4i\,dx-4i\,c)}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*a^2*d*x*e^(4*I*d*x + 4*I*c) - I*a^2*e^(12*I*d*x + 12*I*c) - 8*I*
a^2*e^(10*I*d*x + 10*I*c) - 30*I*a^2*e^(8*I*d*x + 8*I*c) - 80*I*a^2*e^(6*I*
d*x + 6*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*e^(-4*I*d*x - 4*I*c)
/d
```

Sympy [A]
time = 0.29, size = 270, normalized size = 1.58

$$\frac{15a^2x}{64}+\begin{cases}\frac{\left(-8589934592ia^2d^5e^{14ic}e^{8idx}-68719476736ia^2d^5e^{12ic}e^{6idx}-257698037760ia^2d^5e^{10ic}e^{4idx}-687194767360ia^2d^5e^{8ic}e^{2idx}+206158430208ia^2d^5e^{4ic}e^{-2idx}+17179869184ia^2d^5e^{2ic}e^{-4idx}\right)e^{-6ic}}{4398046511104d^6} & \text{for } d^6e^{6ic}\neq 0\\ x\left(-\frac{15a^2}{64}+\frac{\left(a^2e^{12ic}+6a^2e^{10ic}+15a^2e^{8ic}+20a^2e^{6ic}+15a^2e^{4ic}+6a^2e^{2ic}+a^2\right)e^{-4ic}}{64}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 15*a**2*x/64 + Piecewise(((-8589934592*I*a**2*d**5*exp(14*I*c)*exp(8*I*d*x)
 - 68719476736*I*a**2*d**5*exp(12*I*c)*exp(6*I*d*x) - 257698037760*I*a**2*d
**5*exp(10*I*c)*exp(4*I*d*x) - 687194767360*I*a**2*d**5*exp(8*I*c)*exp(2*I*
d*x) + 206158430208*I*a**2*d**5*exp(4*I*c)*exp(-2*I*d*x) + 17179869184*I*a*
*2*d**5*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(4398046511104*d**6), Ne(d**6
*exp(6*I*c), 0)), (x*(-15*a**2/64 + (a**2*exp(12*I*c) + 6*a**2*exp(10*I*c)
```

```
+ 15*a**2*exp(8*I*c) + 20*a**2*exp(6*I*c) + 15*a**2*exp(4*I*c) + 6*a**2*exp
(2*I*c) + a**2)*exp(-4*I*c)/64), True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(133) = 266$.
time = 0.75, size = 342, normalized size = 2.00

$$\frac{120\,a^2dxe^{(8i\,dx+4i\,c)}+240\,a^2dxe^{(6i\,dx+2i\,c)}+120\,a^2dxe^{(4i\,dx)}+8i\,a^2e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^2e^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+8i\,a^2e^{(4i\,dx)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a^2e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-16i\,a^2e^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-8i\,a^2e^{(4i\,dx)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-i\,a^2e^{(16i\,dx+12i\,c)}-10i\,a^2e^{(14i\,dx+10i\,c)}-47i\,a^2e^{(12i\,dx+8i\,c)}-148i\,a^2e^{(10i\,dx+6i\,c)}-190i\,a^2e^{(8i\,dx+4i\,c)}-56i\,a^2e^{(6i\,dx+2i\,c)}+28i\,a^2e^{(2i\,dx-2i\,c)}+50i\,a^2e^{(4i\,dx)}+2i\,a^2e^{(-4i\,c)}}{512\left(de^{(8i\,dx+4i\,c)}+2\,de^{(6i\,dx+2i\,c)}+de^{(4i\,dx)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(120*a^2*d*x*e^(8*I*d*x + 4*I*c) + 240*a^2*d*x*e^(6*I*d*x + 2*I*c) +
120*a^2*d*x*e^(4*I*d*x) + 8*I*a^2*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) + 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8*I*a
^2*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^2*e^(8*I*d*x + 4*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) - 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) - 8*I*a^2*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*a^2*
e^(16*I*d*x + 12*I*c) - 10*I*a^2*e^(14*I*d*x + 10*I*c) - 47*I*a^2*e^(12*I*d
*x + 8*I*c) - 148*I*a^2*e^(10*I*d*x + 6*I*c) - 190*I*a^2*e^(8*I*d*x + 4*I*c
) - 56*I*a^2*e^(6*I*d*x + 2*I*c) + 28*I*a^2*e^(2*I*d*x - 2*I*c) + 50*I*a^2*
e^(4*I*d*x) + 2*I*a^2*e^(-4*I*c))/(d*e^(8*I*d*x + 4*I*c) + 2*d*e^(6*I*d*x +
 2*I*c) + d*e^(4*I*d*x))
```

Mupad [**B**]
time = 4.03, size = 144, normalized size = 0.84

$$\frac{15\,a^2\,x}{64}+\frac{\frac{15\,a^2\,\tan(c+d\,x)^5}{64}+\frac{a^2\,\tan(c+d\,x)^4\,15\mathrm{i}}{32}+\frac{5\,a^2\,\tan(c+d\,x)^3}{32}+\frac{a^2\,\tan(c+d\,x)^2\,25\mathrm{i}}{32}-\frac{17\,a^2\,\tan(c+d\,x)}{64}+\frac{a^2\,1\mathrm{i}}{4}}{d\,\left(\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,2\mathrm{i}+\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4\mathrm{i}-\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (15*a^2*x)/64 + ((a^2*1i)/4 - (17*a^2*tan(c + d*x))/64 + (a^2*tan(c + d*x)^
2*25i)/32 + (5*a^2*tan(c + d*x)^3)/32 + (a^2*tan(c + d*x)^4*15i)/32 + (15*a
^2*tan(c + d*x)^5)/64)/(d*(tan(c + d*x)*2i - tan(c + d*x)^2 + tan(c + d*x)^
3*4i + tan(c + d*x)^4 + tan(c + d*x)^5*2i + tan(c + d*x)^6 - 1))
```

3.28 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=118

$$\frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{i s}{}$$

```
[Out] 7/16*a^2*arctanh(sin(d*x+c))/d+7/30*I*a^2*sec(d*x+c)^5/d+7/16*a^2*sec(d*x+c
)*tan(d*x+c)/d+7/24*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*I*sec(d*x+c)^5*(a^2+I
*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3579, 3567, 3853, 3855}

$$\frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{7a^2\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/30)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d*
x])/(24*d) + ((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{1}{6}(7a)\int \sec^5(c+dx)(a+i\\ &= \frac{7ia^2\sec^5(c+dx)}{30d}+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{1}{6}\left(7a^2\right)\\ &= \frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{i\sec^5(c+dx)\left(\right.}{}\\ &= \frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{7a^2\sec^3(c+dx}{24d}\\ &= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\sec(c+dx)\tan}{16d}\end{aligned}$$

Mathematica [A]
time = 0.86, size = 159, normalized size = 1.35

$$\frac{a^2\sec^4(c+dx)(\cos(2c)-i\sin(2c))\left(-1536i\cos(c+dx)+1680\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+150\sin(c+dx)-35(17\sin(3(c+dx))+3\sin(5(c+dx)))\right)(-i+\tan(c+dx))^2}{3840d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*(Cos[2*c] - I*Sin[2*c])*((-1536*I)*Cos[c + d*x] + 1680*
Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]) + 150*Sin[c + d*x] - 35*(17*Sin[3*(c + d*x)] + 3*
Sin[5*(c + d*x)]))*(-I + Tan[c + d*x])^2)/(3840*d*(Cos[d*x] + I*Sin[d*x])^2
)
```

Maple [A]
time = 0.26, size = 152, normalized size = 1.29

method	result
risch	$-\frac{ia^2\left(105\,e^{11i(dx+c)}+595\,e^{9i(dx+c)}-1686\,e^{7i(dx+c)}-1386\,e^{5i(dx+c)}-595\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{120d\left(e^{2i(dx+c)}+1\right)^6}+\frac{7a^2\ln\left(e^{i(dx+c)}+i\right.}{16d}$

derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\left(\sec^3(d\right.}{4}\right.\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\left(\sec^3(d\right.}{4}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16
*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+
2/5*I*a^2/cos(d*x+c)^5+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+
3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 181, normalized size = 1.53

$$-\frac{5\,a^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+30\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{192i\,a^2}{\cos(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
 + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) +
3*log(sin(d*x + c) - 1)) - 192*I*a^2/cos(d*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(102) = 204$.
time = 0.36, size = 364, normalized size = 3.08

$$\frac{-210i\,a^2e^{(11i\,dx+11i\,c)}-1190i\,a^2e^{(9i\,dx+9i\,c)}+3372i\,a^2e^{(7i\,dx+7i\,c)}+2772i\,a^2e^{(5i\,dx+5i\,c)}+1190i\,a^2e^{(3i\,dx+3i\,c)}+210i\,a^2e^{(i\,dx+i\,c)}+105\left(a^2e^{(12i\,dx+12i\,c)}+6\,a^2e^{(10i\,dx+10i\,c)}+15\,a^2e^{(8i\,dx+8i\,c)}+20\,a^2e^{(6i\,dx+6i\,c)}+15\,a^2e^{(4i\,dx+4i\,c)}+6\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(a^2e^{(12i\,dx+12i\,c)}+6\,a^2e^{(10i\,dx+10i\,c)}+15\,a^2e^{(8i\,dx+8i\,c)}+20\,a^2e^{(6i\,dx+6i\,c)}+15\,a^2e^{(4i\,dx+4i\,c)}+6\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{240\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(-210*I*a^2*e^(11*I*d*x + 11*I*c) - 1190*I*a^2*e^(9*I*d*x + 9*I*c) +
3372*I*a^2*e^(7*I*d*x + 7*I*c) + 2772*I*a^2*e^(5*I*d*x + 5*I*c) + 1190*I*a^
2*e^(3*I*d*x + 3*I*c) + 210*I*a^2*e^(I*d*x + I*c) + 105*(a^2*e^(12*I*d*x +
12*I*c) + 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2
*e^(6*I*d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*
c) + a^2)*log(e^(I*d*x + I*c) + I) - 105*(a^2*e^(12*I*d*x + 12*I*c) + 6*a^2
*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2*e^(6*I*d*x + 6
```

```
*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e
^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) +
 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*
I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int \tan^2(c+dx)\sec^5(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^5(c+dx)\right)\,dx + \int \left(-\sec^5(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**5, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**5, x) + Integral(-sec(c + d*x)**5, x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(102) = 204.
time = 0.64, size = 237, normalized size = 2.01

$$\frac{105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-960i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+960i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-445\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-96i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96i\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^2*log(tan(1/2*d*x + 1/
2*c) - 1) + 2*(135*a^2*tan(1/2*d*x + 1/2*c)^11 - 480*I*a^2*tan(1/2*d*x + 1/
2*c)^10 - 445*a^2*tan(1/2*d*x + 1/2*c)^9 + 480*I*a^2*tan(1/2*d*x + 1/2*c)^8
 - 330*a^2*tan(1/2*d*x + 1/2*c)^7 - 960*I*a^2*tan(1/2*d*x + 1/2*c)^6 - 330*
a^2*tan(1/2*d*x + 1/2*c)^5 + 960*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 445*a^2*tan
(1/2*d*x + 1/2*c)^3 - 96*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 135*a^2*tan(1/2*d*x
 + 1/2*c) + 96*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [**B**]
time = 7.15, size = 290, normalized size = 2.46

$$\frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}8\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}8\mathrm{i}+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}4\mathrm{i}}{5}-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{a^2\,4\mathrm{i}}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^5,x)
```

```
[Out] (7*a^2*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/5
+ (89*a^2*tan(c/2 + (d*x)/2)^3)/24 - a^2*tan(c/2 + (d*x)/2)^4*8i + (11*a^2*
tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*8i + (11*a^2*tan(c/2 + (
d*x)/2)^7)/4 - a^2*tan(c/2 + (d*x)/2)^8*4i + (89*a^2*tan(c/2 + (d*x)/2)^9)/
24 + a^2*tan(c/2 + (d*x)/2)^10*4i - (9*a^2*tan(c/2 + (d*x)/2)^11)/8 - (a^2*
4i)/5 - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c
/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=94

$$\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

```
[Out] 5/8*a^2*arctanh(sin(d*x+c))/d+5/12*I*a^2*sec(d*x+c)^3/d+5/8*a^2*sec(d*x+c)*
tan(d*x+c)/d+1/4*I*sec(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3579, 3567, 3853, 3855}

$$\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((5*I)/12)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2
*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{1}{4}(5a)\int \sec^3(c+dx)(a+i$$
$$= \frac{5ia^2\sec^3(c+dx)}{12d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2\right)$$
$$= \frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{i\sec^3(c+dx)\,($$
$$= \frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan}{8d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.
time = 1.19, size = 215, normalized size = 2.29

$$\frac{a^2\sec^4(c+dx)\left(128i\cos(c+dx)-45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-60\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-15\cos(4(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-18\sin(c+dx)+30\sin(3(c+dx))\right)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*((128*I)*Cos[c + d*x] - 45*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 15*Cos[4*(c + d*x)]*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*Sin[c + d*x] + 30*Si
n[3*(c + d*x)]))/(192*d)
```

Maple [A]
time = 0.24, size = 121, normalized size = 1.29

method	result
risch	$-\frac{ia^2\left(15\,e^{7i(dx+c)}-73\,e^{5i(dx+c)}-55\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}+\frac{5a^2\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{5a^2\ln\left(e^{i(dx+c)}-i\right)}{8d}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ia^2}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+}{}\right.}{d}$

default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ia^2}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(d}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*
sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+2/3*I*a^2/cos(d*x+c)^3+a^2*(1/2*s
ec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 130, normalized size = 1.38

$$-\frac{3\,a^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{32i\,a^2}{\cos(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 32*I*a^2/cos(d*x + c)^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(80) = 160.
time = 0.36, size = 256, normalized size = 2.72

$$\frac{-30i\,a^2e^{(7i\,dx+7i\,c)}+146i\,a^2e^{(5i\,dx+5i\,c)}+110i\,a^2e^{(3i\,dx+3i\,c)}+30i\,a^2e^{(i\,dx+i\,c)}+15\left(a^2e^{(8i\,dx+8i\,c)}+4\,a^2e^{(6i\,dx+6i\,c)}+6\,a^2e^{(4i\,dx+4i\,c)}+4\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^2e^{(8i\,dx+8i\,c)}+4\,a^2e^{(6i\,dx+6i\,c)}+6\,a^2e^{(4i\,dx+4i\,c)}+4\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{24\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-30*I*a^2*e^(7*I*d*x + 7*I*c) + 146*I*a^2*e^(5*I*d*x + 5*I*c) + 110*I
*a^2*e^(3*I*d*x + 3*I*c) + 30*I*a^2*e^(I*d*x + I*c) + 15*(a^2*e^(8*I*d*x +
8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2
*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 15*(a^2*e^(8*I*d*x + 8*I*
c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2*I*d
*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\left(-2i\tan\left(c+dx\right)\sec^3\left(c+dx\right)\right)dx+\int\left(-\sec^3\left(c+dx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**3, x) + Integral(-sec(c + d*x)**3, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(80) = 160$.
time = 0.61, size = 173, normalized size = 1.84

$$\frac{15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16i\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2*log(tan(1/2*d*x + 1/2*c
) - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*I*a^2*tan(1/2*d*x + 1/2*c)^6
- 33*a^2*tan(1/2*d*x + 1/2*c)^5 + 48*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 33*a^2*
tan(1/2*d*x + 1/2*c)^3 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/2*d*
x + 1/2*c) + 16*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 6.78, size = 198, normalized size = 2.11

$$\frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{3}-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{a^2\,4\mathrm{i}}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^3,x)
```

```
[Out] (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/3
+ (11*a^2*tan(c/2 + (d*x)/2)^3)/4 - a^2*tan(c/2 + (d*x)/2)^4*4i + (11*a^2*t
an(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*4i - (3*a^2*tan(c/2 + (d*
x)/2)^7)/4 - (a^2*4i)/3 - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*
x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)
/2)^8 + 1))
```

3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

```
[Out] 3/2*a^2*arctanh(sin(d*x+c))/d+3/2*I*a^2*sec(d*x+c)/d+1/2*I*sec(d*x+c)*(a^2+
I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3579, 3567, 3855}

$$\frac{3ia^2\sec(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (((3*I)/2)*a^2*Sec[c + d*x])/d + ((I/
2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}(3a)\int \sec(c+dx)(a+ia\,\mathrm{t}\\ &= \frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}\left(3a^2\right)\int\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia\right.}{2d}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.
time = 0.82, size = 146, normalized size = 2.15

$$-\frac{a^2\sec^2(c+dx)\left(-8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Sec[c + d*x]^2*((-8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 2*Sin[c + d*x]))/d
```

Maple [**A**]
time = 0.14, size = 86, normalized size = 1.26

method	result	size
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^2}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^2}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
risch	$\frac{ia^2\left(5\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{3a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{3a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+2*I*a^2/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [**A**]
time = 0.28, size = 83, normalized size = 1.22

$$\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{8i\,a^2}{\cos(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 4*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*I*a^2/cos(
d*x + c))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(56) = 112$.
time = 0.38, size = 148, normalized size = 2.18

$$\frac{10i\,a^2e^{(3i\,dx+3i\,c)}+6i\,a^2e^{(i\,dx+i\,c)}+3\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*I*a^2*e^(3*I*d*x + 3*I*c) + 6*I*a^2*e^(I*d*x + I*c) + 3*(a^2*e^(4*I
*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) -
 3*(a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x
 + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\tan^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\left(-2i\tan\left(c+dx\right)\sec\left(c+dx\right)\right)dx+\int\left(-\sec\left(c+dx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(-2*I*tan(c + d*
x)*sec(c + d*x), x) + Integral(-sec(c + d*x), x))
```

Giac [**A**]
time = 0.53, size = 107, normalized size = 1.57

$$\frac{3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4i\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a^2*log(tan(1/2*d*x + 1/2*c) -
 1) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 + 4*I*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2*
tan(1/2*d*x + 1/2*c) - 4*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 3.82, size = 104, normalized size = 1.53

$$\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4{}\mathrm{i}+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2\,4{}\mathrm{i}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x),x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2*4i + a^2*ta
n(c/2 + (d*x)/2)^3 - a^2*4i + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2
)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

```
[Out] -a^2*arctanh(sin(d*x+c))/d-2*I*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3577, 3855}

$$-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c +
 d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}-a^2\int \sec(c+dx)\,dx\\ &= -\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.
time = 0.21, size = 180, normalized size = 3.91

$$\frac{a^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(2-i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+5dx)\right)+i\sin\left(\frac{1}{2}(c+5dx)\right)\right)}{d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2]*(-2*I + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 - I*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[(c + d*x)
/2])*(Cos[(c + 5*d*x)/2] + I*Sin[(c + 5*d*x)/2]))/(d*(Cos[d*x] + I*Sin[d*x]
)^2)
```

Maple [A]
time = 0.22, size = 56, normalized size = 1.22

method	result	size
derivativedivides	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
default	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
risch	$-\frac{2ia^2e^{i(dx+c)}}{d}+\frac{a^2\ln(e^{i(dx+c)}-i)}{d}-\frac{a^2\ln(e^{i(dx+c)}+i)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*I*a^2*cos(d*x+c)+a^2*si
n(d*x+c))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.33

$$-\frac{a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))+4i\,a^2\cos(dx+c)-2\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
+ 4*I*a^2*cos(d*x + c) - 2*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 52, normalized size = 1.13

$$\frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(e^{(i\,dx+i\,c)}+i\right)+a^2\log\left(e^{(i\,dx+i\,c)}-i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(e^(I*d*x + I*c) + I) + a^2*log(e^(I*d*x
 + I*c) - I))/d
```

Sympy [A]
time = 0.17, size = 68, normalized size = 1.48

$$\frac{a^2\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}-\frac{2ia^2e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 2a^2xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Pi
ecewise((-2*I*a**2*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (2*a**2*x*exp(I*c), Tr
ue))
```

Giac [A]
time = 0.57, size = 56, normalized size = 1.22

$$\frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+a^2\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(I*e^(I*d*x + I*c) - 1) + a^2*log(-I*e^(
I*d*x + I*c) - 1))/d
```

Mupad [B]
time = 3.37, size = 41, normalized size = 0.89

$$-\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (4*a^2)/(d*(tan(c/2 + (d*x)/2) + 1i)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

```
[Out] 1/3*a^2*sin(d*x+c)/d-2/3*I*cos(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3577, 2717}

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d
*x]))/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx&=-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}+\frac{1}{3}a^2\int\cos(c+dx)\,dx\\&=\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 50, normalized size = 0.98

$$\frac{a^2(2\cos(c+dx)-i\sin(c+dx))(-i\cos(2(c+dx))+\sin(2(c+dx)))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(2*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c +
d*x)]))/(3*d)
```

Maple [A]
time = 0.22, size = 54, normalized size = 1.06

method	result	size
risch	$-\frac{i a^2 e^{3 i(dx+c)}}{6d}-\frac{i a^2 e^{i(dx+c)}}{2d}$	38
derivativedivides	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ia^2\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	54
default	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ia^2\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*sin(d*x+c)^3-2/3*I*a^2*cos(d*x+c)^3+1/3*a^2*(cos(d*x+c)^2+2)*
sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 52, normalized size = 1.02

$$-\frac{2i\,a^2\cos\left(dx+c\right)^3+a^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*I*a^2*cos(d*x + c)^3 + a^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin
(d*x + c))*a^2)/d
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.67

$$\frac{-i\,a^2e^{(3i\,dx+3i\,c)}-3i\,a^2e^{(i\,dx+i\,c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(-I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))/d
```

Sympy [A]
time = 0.14, size = 75, normalized size = 1.47

$$\begin{cases} \frac{-2ia^2de^{3ic}e^{3idx}-6ia^2de^{ic}e^{idx}}{12d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{3ic}}{2} + \frac{a^2e^{ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((-2*I*a**2*d*exp(3*I*c)*exp(3*I*d*x) - 6*I*a**2*d*exp(I*c)*exp(I
*d*x))/(12*d**2), Ne(d**2, 0)), (x*(a**2*exp(3*I*c)/2 + a**2*exp(I*c)/2), T
rue))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(43) = 86.
time = 0.64, size = 531, normalized size = 10.41

$$-\frac{24\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+48\,a^{2}e^{(2i\,dx)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24\,a^{2}e^{(-2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+27\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+54\,a^{2}e^{(2i\,dx)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+27\,a^{2}e^{(-2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-24\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-48\,a^{2}e^{(2i\,dx)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-24\,a^{2}e^{(-2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-27\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-54\,a^{2}e^{(2i\,dx)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-27\,a^{2}e^{(-2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)+3\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+6\,a^{2}e^{(2i\,dx)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+3\,a^{2}e^{(-2i\,c)}\log\left(i\,e^{(i\,dx)}+e^{(-i\,c)}\right)-3\,a^{2}e^{(4i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)-6\,a^{2}e^{(2i\,dx)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)-3\,a^{2}e^{(-2i\,c)}\log\left(-i\,e^{(i\,dx)}+e^{(-i\,c)}\right)+16i\,a^{2}e^{(7i\,dx+5i\,c)}+80i\,a^{2}e^{(5i\,dx+3i\,c)}+112i\,a^{2}e^{(3i\,dx+i\,c)}+48i\,a^{2}e^{(i\,dx-i\,c)}}{96\left(de^{(4i\,dx+2i\,c)}+2\,de^{(2i\,dx)}+de^{(-2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(24*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 48*a^2*e^(2*
I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 24*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 27*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 54*a^2*e^(2
*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 27*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c
) - 1) - 24*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 48*a^2*e^
(2*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 24*a^2*e^(-2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 27*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54*a^
2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 27*a^2*e^(-2*I*c)*log(-I*e^(I*d
*x + I*c) - 1) + 3*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6*
a^2*e^(2*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^2*e^(-2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 3*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
6*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(-2*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 16*I*a^2*e^(7*I*d*x + 5*I*c) + 80*I*a^2*e^(5*I*d*x +
3*I*c) + 112*I*a^2*e^(3*I*d*x + I*c) + 48*I*a^2*e^(I*d*x - I*c))/(d*e^(4*I*
d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

Mupad [B]
time = 3.36, size = 78, normalized size = 1.53

$$-\frac{2\,a^2\,\left(3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3{}\mathrm{i}-2\right)}{3\,d\,\left(-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}+3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
[Out] -(2*a^2*(tan(c/2 + (d*x)/2)*3i + 3*tan(c/2 + (d*x)/2)^2 - 2))/(3*d*(3*tan(c
/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=69

$$\frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

```
[Out] 3/5*a^2*sin(d*x+c)/d-1/5*a^2*sin(d*x+c)^3/d-2/5*I*cos(d*x+c)^5*(a^2+I*a^2*t
an(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3577, 2713}

$$-\frac{a^2\sin^3(c+dx)}{5d}+\frac{3a^2\sin(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*Sin[c + d*x])/(5*d) - (a^2*Sin[c + d*x]^3)/(5*d) - (((2*I)/5)*Cos[c
+ d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}\left(3a^2\right)\int \cos^3(c+dx \\ &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}-\frac{\left(3a^2\right)\text{Subst}\left(\int (1-x^2\right.}{5} \\ &= \frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan\right.}{5d}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 72, normalized size = 1.04

$$\frac{a^2(-i\cos(2(c+dx))+\sin(2(c+dx)))(10\cos(c+dx)-2\cos(3(c+dx))-5i\sin(c+dx)+3i\sin(3(c+dx)))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(10*Cos[c + d*x] - 2*Cos[3*
(c + d*x)] - (5*I)*Sin[c + d*x] + (3*I)*Sin[3*(c + d*x)]))/(20*d)
```

Maple [A]
time = 0.23, size = 91, normalized size = 1.32

method	result
risch	$-\frac{i a^2 e^{5i(dx+c)}}{40d}-\frac{i a^2 e^{3i(dx+c)}}{8d}-\frac{i a^2\cos(dx+c)}{4d}+\frac{a^2\sin(dx+c)}{2d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{2ia^2\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(d}{5}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{2ia^2\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(d}{5}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-2
/5*I*a^2*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
))
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.14

$$-\frac{6i\,a^2\cos\left(dx+c\right)^5-\left(3\sin\left(dx+c\right)^5-5\sin\left(dx+c\right)^3\right)a^2-\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)a^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*I*a^2*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2 -
 (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.90

$$\frac{\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}-15i\,a^2e^{(2i\,dx+2i\,c)}+5i\,a^2\right)e^{(-i\,dx-i\,c)}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) - 15*I*a^2*e
^(2*I*d*x + 2*I*c) + 5*I*a^2)*e^(-I*d*x - I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.
time = 0.22, size = 153, normalized size = 2.22

$$\begin{cases}\frac{(-512ia^2d^3e^{6ic}e^{5idx}-2560ia^2d^3e^{4ic}e^{3idx}-7680ia^2d^3e^{2ic}e^{idx}+2560ia^2d^3e^{-idx})e^{-ic}}{20480d^4} & \text{for } d^4e^{ic}\neq 0\\ \frac{x(a^2e^{6ic}+3a^2e^{4ic}+3a^2e^{2ic}+a^2)e^{-ic}}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-512*I*a**2*d**3*exp(6*I*c)*exp(5*I*d*x) - 2560*I*a**2*d**3*exp
(4*I*c)*exp(3*I*d*x) - 7680*I*a**2*d**3*exp(2*I*c)*exp(I*d*x) + 2560*I*a**2
*d**3*exp(-I*d*x))*exp(-I*c)/(20480*d**4), Ne(d**4*exp(I*c), 0)), (x*(a**2*
exp(6*I*c) + 3*a**2*exp(4*I*c) + 3*a**2*exp(2*I*c) + a**2)*exp(-I*c)/8, Tru
e))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(59) = 118$.
time = 0.72, size = 613, normalized size = 8.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(45*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a^2*e^(3
*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 45*a^2*e^(I*d*x - I*c)*log(I*e^(
```

```
I*d*x + I*c) + 1) + 40*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 80*a^2*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 40*a^2*e^(I*d*x - I*
c)*log(I*e^(I*d*x + I*c) - 1) - 45*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 90*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 45*a^2
*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 40*a^2*e^(5*I*d*x + 3*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 80*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c
) - 1) - 40*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5*a^2*e^(5*I*
d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10*a^2*e^(3*I*d*x + I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 5*a^2*e^(I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
5*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10*a^2*e^(3*I*d*x
+ I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 4*I*a^2*e^(10*I*d*x + 8*I*c) + 28*I*a^2*e^(8*I*d*x + 6*I*c)
 + 104*I*a^2*e^(6*I*d*x + 4*I*c) + 120*I*a^2*e^(4*I*d*x + 2*I*c) + 20*I*a^2
*e^(2*I*d*x) - 20*I*a^2*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x
 + I*c) + d*e^(I*d*x - I*c))
```

Mupad [B]
time = 4.17, size = 71, normalized size = 1.03

$$\frac{2\,a^2\,\left(\frac{5\,\sin\left(3\,c+3\,d\,x\right)}{16}-\frac{\cos\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}}{16}-\frac{\cos\left(3\,c+3\,d\,x\right)\,5{}\mathrm{i}}{16}+\frac{\sin\left(5\,c+5\,d\,x\right)}{16}+\frac{5\,\sqrt{3}\,\sin\left(c+d\,x-\frac{\ln\left(3\right)\,1{}\mathrm{i}}{2}\right)}{8}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*((5*sin(3*c + 3*d*x))/16 - (cos(5*c + 5*d*x)*1i)/16 - (cos(3*c + 3*d
*x)*5i)/16 + sin(5*c + 5*d*x)/16 + (5*3^(1/2)*sin(c - (log(3)*1i)/2 + d*x))
/8))/(5*d)
```

3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

```
[Out] 5/7*a^2*sin(d*x+c)/d-10/21*a^2*sin(d*x+c)^3/d+1/7*a^2*sin(d*x+c)^5/d-2/7*I*
cos(d*x+c)^7*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 2713}

$$\frac{a^2\sin^5(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{5a^2\sin(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*Sin[c + d*x])/(7*d) - (10*a^2*Sin[c + d*x]^3)/(21*d) + (a^2*Sin[c +
d*x]^5)/(7*d) - (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int \cos^5(c+dx$$

$$= -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\mathrm{Subst}\left(\int (1-2x\right.}{}$$

$$= \frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(}{}$$

Mathematica [A]
time = 0.38, size = 111, normalized size = 1.28

$$\frac{a^2(-140i\cos(c+dx)+42i\cos(3(c+dx))+2i\cos(5(c+dx))-70\sin(c+dx)+63\sin(3(c+dx))+5\sin(5(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{336d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-140*I)*Cos[c + d*x] + (42*I)*Cos[3*(c + d*x)] + (2*I)*Cos[5*(c + d*
x)] - 70*Sin[c + d*x] + 63*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)])*(Cos[2*(c
 + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(336*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.22, size = 111, normalized size = 1.28

method	result
risch	$-\frac{ia^2 e^{7i(dx+c)}}{224d}-\frac{ia^2 e^{5i(dx+c)}}{32d}-\frac{5ia^2\cos(dx+c)}{32d}+\frac{15a^2\sin(dx+c)}{32d}-\frac{3ia^2\cos(3dx+3c)}{32d}+\frac{11a^2\sin(3dx+3c)}{96d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6}{}\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6}{}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))-2/7*I*a^2*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos
(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.13

$$-\frac{30i\,a^2\cos(dx+c)^7+\left(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+c)^3\right)a^2+3\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^2}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*I*a^2*cos(d*x + c)^7 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 +
35*sin(d*x + c)^3)*a^2 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d
*x + c)^3 - 35*sin(d*x + c))*a^2)/d
```

Fricas [A]
time = 0.36, size = 90, normalized size = 1.03

$$\frac{\left(-3i\, a^2 e^{(10i\, dx+10i\, c)} - 21i\, a^2 e^{(8i\, dx+8i\, c)} - 70i\, a^2 e^{(6i\, dx+6i\, c)} - 210i\, a^2 e^{(4i\, dx+4i\, c)} + 105i\, a^2 e^{(2i\, dx+2i\, c)} + 7i\, a^2\right) e^{(-3i\, dx-3i\, c)}}{672\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-3*I*a^2*e^(10*I*d*x + 10*I*c) - 21*I*a^2*e^(8*I*d*x + 8*I*c) - 70*I
*a^2*e^(6*I*d*x + 6*I*c) - 210*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I
*d*x + 2*I*c) + 7*I*a^2)*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(76) = 152.
time = 0.35, size = 238, normalized size = 2.74

$$\begin{cases} \frac{\left(-75497472ia^2d^5e^{11ic}e^{7idx} - 528482304ia^2d^5e^{9ic}e^{5idx} - 1761607680ia^2d^5e^{7ic}e^{3idx} - 5284823040ia^2d^5e^{5ic}e^{idx} + 2642411520ia^2d^5e^{3ic}e^{-idx} + 176160768ia^2d^5e^{ic}e^{-3idx}\right)e^{-4ic}}{16911433728d^6} & \text{for}\ d^6e^{4ic} \neq 0 \\ \frac{x\left(a^2e^{10ic} + 5a^2e^{8ic} + 10a^2e^{6ic} + 10a^2e^{4ic} + 5a^2e^{2ic} + a^2\right)e^{-3ic}}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-75497472*I*a**2*d**5*exp(11*I*c)*exp(7*I*d*x) - 528482304*I*a*
*2*d**5*exp(9*I*c)*exp(5*I*d*x) - 1761607680*I*a**2*d**5*exp(7*I*c)*exp(3*I
*d*x) - 5284823040*I*a**2*d**5*exp(5*I*c)*exp(I*d*x) + 2642411520*I*a**2*d*
*5*exp(3*I*c)*exp(-I*d*x) + 176160768*I*a**2*d**5*exp(I*c)*exp(-3*I*d*x))*e
xp(-4*I*c)/(16911433728*d**6), Ne(d**6*exp(4*I*c), 0)), (x*(a**2*exp(10*I*c
) + 5*a**2*exp(8*I*c) + 10*a**2*exp(6*I*c) + 10*a**2*exp(4*I*c) + 5*a**2*ex
p(2*I*c) + a**2)*exp(-3*I*c)/32, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(75) = 150.
time = 0.77, size = 641, normalized size = 7.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10752*(2583*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5166*a^
2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 2583*a^2*e^(3*I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 2121*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 4242*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 2121*a
^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 2583*a^2*e^(7*I*d*x + 3*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 5166*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 2583*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 21
21*a^2*e^(7*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4242*a^2*e^(5*I*d*
x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2121*a^2*e^(3*I*d*x - I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 462*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) - 924*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 462*a^2*e^(3*I*
d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 462*a^2*e^(7*I*d*x + 3*I*c)*log(-I
*e^(I*d*x) + e^(-I*c)) + 924*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 462*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 48*I*a^2*e^
(14*I*d*x + 10*I*c) + 432*I*a^2*e^(12*I*d*x + 8*I*c) + 1840*I*a^2*e^(10*I*d
*x + 6*I*c) + 5936*I*a^2*e^(8*I*d*x + 4*I*c) + 6160*I*a^2*e^(6*I*d*x + 2*I*
c) - 1904*I*a^2*e^(2*I*d*x - 2*I*c) - 112*I*a^2*e^(4*I*d*x) - 112*I*a^2*e^(
-4*I*c))/(d*e^(7*I*d*x + 3*I*c) + 2*d*e^(5*I*d*x + I*c) + d*e^(3*I*d*x - I*
c))
```

Mupad [B]

time = 3.66, size = 256, normalized size = 2.94

$$\frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{256\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}-\frac{8\,a^2\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}-\frac{128\,a^2\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}+\frac{16\,a^2\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}-\frac{32\,a^2\,\left(22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}+\frac{32\,a^2\,\left(31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (256*a^2
*(tan(c/2 + (d*x)/2) - 1i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (8*a^2*(4*
tan(c/2 + (d*x)/2) - 9i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (128*a^2*(6*
tan(c/2 + (d*x)/2) - 7i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (16*a^2*(8*t
an(c/2 + (d*x)/2) - 15i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (32*a^2*(22*
tan(c/2 + (d*x)/2) - 35i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (32*a^2*(31
*tan(c/2 + (d*x)/2) - 42i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=105

$$\frac{7a^2\sin(c+dx)}{9d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{a^2\sin^7(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

```
[Out] 7/9*a^2*sin(d*x+c)/d-7/9*a^2*sin(d*x+c)^3/d+7/15*a^2*sin(d*x+c)^5/d-1/9*a^2
*sin(d*x+c)^7/d-2/9*I*cos(d*x+c)^9*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 2713}

$$-\frac{a^2\sin^7(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*Sin[c + d*x])/(9*d) - (7*a^2*Sin[c + d*x]^3)/(9*d) + (7*a^2*Sin[c +
d*x]^5)/(15*d) - (a^2*Sin[c + d*x]^7)/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^
2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}+\frac{1}{9}\left(7a^2\right)\int \cos^7(c+dx)$$
$$= -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}-\frac{\left(7a^2\right)\text{Subst}\left(\int\left(1-3x\right.\right.}{}$$
$$= \frac{7a^2\sin(c+dx)}{9d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{a^2\sin^7(}{9}$$

Mathematica [A]
time = 0.71, size = 133, normalized size = 1.27

$$\frac{a^2(-1050i\cos(c+dx)+378i\cos(3(c+dx))+30i\cos(5(c+dx))+2i\cos(7(c+dx))-525\sin(c+dx)+567\sin(3(c+dx))+75\sin(5(c+dx))+7\sin(7(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{2880d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-1050*I)*Cos[c + d*x] + (378*I)*Cos[3*(c + d*x)] + (30*I)*Cos[5*(c +
 d*x)] + (2*I)*Cos[7*(c + d*x)] - 525*Sin[c + d*x] + 567*Sin[3*(c + d*x)] +
 75*Sin[5*(c + d*x)] + 7*Sin[7*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c
 + 2*d*x)]))/(2880*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.25, size = 131, normalized size = 1.25

method	result
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\left(\cos^9(dx+c)\right)}{9}+\frac{a^2\left(\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\left(\cos^9(dx+c)\right)}{9}+\frac{a^2\left(\right.}{}}{d}$
risch	$-\frac{ia^2e^{9i(dx+c)}}{1152d}-\frac{ia^2e^{7i(dx+c)}}{128d}-\frac{7ia^2\cos(dx+c)}{64d}+\frac{7a^2\sin(dx+c)}{16d}-\frac{ia^2\cos(5dx+5c)}{32d}+\frac{11a^2\sin(5dx+5c)}{320d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-2/9*I*a^2*cos(d*x+c)^9+1/9*a^2*(128/35+
cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.13

$$-\frac{70i\,a^2\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-135\,\sin\left(dx+c\right)^7+189\,\sin\left(dx+c\right)^5-105\,\sin\left(dx+c\right)^3\right)a^2-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3+315\,\sin\left(dx+c\right)\right)a^2}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(70*I*a^2*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 +
 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^2 - (35*sin(d*x + c)^9 - 180*si
n(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*
a^2)/d
```

Fricas [A]
time = 0.36, size = 118, normalized size = 1.12

$$\frac{\left(-5i\,a^2e^{\left(14i\,dx+14i\,c\right)}-45i\,a^2e^{\left(12i\,dx+12i\,c\right)}-189i\,a^2e^{\left(10i\,dx+10i\,c\right)}-525i\,a^2e^{\left(8i\,dx+8i\,c\right)}-1575i\,a^2e^{\left(6i\,dx+6i\,c\right)}+945i\,a^2e^{\left(4i\,dx+4i\,c\right)}+105i\,a^2e^{\left(2i\,dx+2i\,c\right)}+9i\,a^2\right)e^{\left(-5i\,dx-5i\,c\right)}}{5760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-5*I*a^2*e^(14*I*d*x + 14*I*c) - 45*I*a^2*e^(12*I*d*x + 12*I*c) - 1
89*I*a^2*e^(10*I*d*x + 10*I*c) - 525*I*a^2*e^(8*I*d*x + 8*I*c) - 1575*I*a^2
*e^(6*I*d*x + 6*I*c) + 945*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I*d*x
 + 2*I*c) + 9*I*a^2)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(94) = 188$.
time = 0.42, size = 314, normalized size = 2.99

$$\begin{cases} \frac{\left(-126663739519795200ia^2d^7e^{18ic}e^{9idx}-1139973655678156800ia^2d^7e^{16ic}e^{7idx}-4787889353848258560ia^2d^7e^{14ic}e^{5idx}-13299692649578496000ia^2d^7e^{12ic}e^{3idx}-39899077948735488000ia^2d^7e^{10ic}e^{idx}+23939446769241292800ia^2d^7e^{8ic}e^{-idx}+2659938529915699200ia^2d^7e^{6ic}e^{-3idx}+227994731135631360ia^2d^7e^{4ic}e^{-5idx}\right)e^{-9ic}}{145916627926804070400d^8} & \text{for}\: d^8e^{9ic} \neq 0 \\ \frac{x\left(a^2e^{14ic}+7a^2e^{12ic}+21a^2e^{10ic}+35a^2e^{8ic}+35a^2e^{6ic}+21a^2e^{4ic}+7a^2e^{2ic}+a^2\right)e^{-5ic}}{128} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-126663739519795200*I*a**2*d**7*exp(18*I*c)*exp(9*I*d*x) - 1139
973655678156800*I*a**2*d**7*exp(16*I*c)*exp(7*I*d*x) - 4787889353848258560*
I*a**2*d**7*exp(14*I*c)*exp(5*I*d*x) - 13299692649578496000*I*a**2*d**7*exp
(12*I*c)*exp(3*I*d*x) - 39899077948735488000*I*a**2*d**7*exp(10*I*c)*exp(I*
d*x) + 23939446769241292800*I*a**2*d**7*exp(8*I*c)*exp(-I*d*x) + 2659938529
915699200*I*a**2*d**7*exp(6*I*c)*exp(-3*I*d*x) + 227994731135631360*I*a**2*
d**7*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(145916627926804070400*d**8), Ne
(d**8*exp(9*I*c), 0)), (x*(a**2*exp(14*I*c) + 7*a**2*exp(12*I*c) + 21*a**2*
exp(10*I*c) + 35*a**2*exp(8*I*c) + 35*a**2*exp(6*I*c) + 21*a**2*exp(4*I*c)
+ 7*a**2*exp(2*I*c) + a**2)*exp(-5*I*c)/128, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(91) = 182$.
time = 0.89, size = 669, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/92160*(18585*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 37170*
a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 18585*a^2*e^(5*I*d*x - I
*c)*log(I*e^(I*d*x + I*c) + 1) + 14625*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 29250*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
4625*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 18585*a^2*e^(9*I*d*
x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 37170*a^2*e^(7*I*d*x + I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 18585*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 14625*a^2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29250*a^
2*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14625*a^2*e^(5*I*d*x - I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 3960*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*
x) + e^(-I*c)) - 7920*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3
960*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(9*I*d*x
 + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 7920*a^2*e^(7*I*d*x + I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 80*I*a^2*e^(18*I*d*x + 12*I*c) + 880*I*a^2*e^(16*I*d*x + 10*I*c) + 4
544*I*a^2*e^(14*I*d*x + 8*I*c) + 15168*I*a^2*e^(12*I*d*x + 6*I*c) + 45024*I
*a^2*e^(10*I*d*x + 4*I*c) + 43680*I*a^2*e^(8*I*d*x + 2*I*c) - 18624*I*a^2*e
^(4*I*d*x - 2*I*c) - 1968*I*a^2*e^(2*I*d*x - 4*I*c) - 6720*I*a^2*e^(6*I*d*x
) - 144*I*a^2*e^(-6*I*c))/(d*e^(9*I*d*x + 3*I*c) + 2*d*e^(7*I*d*x + I*c) +
d*e^(5*I*d*x - I*c))
```

Mupad [**B**]
time = 5.08, size = 330, normalized size = 3.14

$$\frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{1024\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9}-\frac{8\,a^2\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}-\frac{512\,a^2\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^8}+\frac{128\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}-\frac{64\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35{}\mathrm{i}\right)}{5\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}+\frac{56\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-40{}\mathrm{i}\right)}{15\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}-\frac{128\,a^2\,\left(59\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-84{}\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}+\frac{32\,a^2\,\left(781\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1260{}\mathrm{i}\right)}{45\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (1024*a^
2*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (8*a^2*(5
*tan(c/2 + (d*x)/2) - 12i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (512*a^2*(
8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) + (128*a^2*(
19*tan(c/2 + (d*x)/2) - 24i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (64*a^2*
(19*tan(c/2 + (d*x)/2) - 35i))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (56*a^2
*(19*tan(c/2 + (d*x)/2) - 40i))/(15*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (128*
a^2*(59*tan(c/2 + (d*x)/2) - 84i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (32
*a^2*(781*tan(c/2 + (d*x)/2) - 1260i))/(45*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=109

$$-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}$$

```
[Out] -8/7*I*(a+I*a*tan(d*x+c))^7/a^4/d+3/2*I*(a+I*a*tan(d*x+c))^8/a^5/d-2/3*I*(a
+I*a*tan(d*x+c))^9/a^6/d+1/10*I*(a+I*a*tan(d*x+c))^10/a^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^4*d) + (((3*I)/2)*(a + I*a*Tan[c +
 d*x])^8)/(a^5*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^9)/(a^6*d) + ((I/10)*
(a + I*a*Tan[c + d*x])^10)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^6\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^6-12a^2(a+x)^7+6a(a+x)^8-(a+x)^9\right)\right.}{a^7d}\\ &= -\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{2i(a+ia\,\text{ta}}{3a}\end{aligned}$$

Mathematica [A]
time = 1.17, size = 117, normalized size = 1.07

$$\frac{a^3 \sec(c) \sec^{10}(c+dx)(126i\cos(c)+105i\cos(c+2dx)+105i\cos(3c+2dx)-126\sin(c)+105\sin(c+2dx)-105\sin(3c+2dx)+120\sin(3c+4dx)+45\sin(5c+6dx)+10\sin(7c+8dx)+\sin(9c+10dx))}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^10*((126*I)*Cos[c] + (105*I)*Cos[c + 2*d*x] + (105
*I)*Cos[3*c + 2*d*x] - 126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*
x] + 120*Sin[3*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin
[9*c + 10*d*x]))/(840*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(93) = 186.
time = 0.27, size = 220, normalized size = 2.02

method	result
risch	$\frac{128ia^3\left(210\,e^{12i(dx+c)}+252\,e^{10i(dx+c)}+210\,e^{8i(dx+c)}+120\,e^{6i(dx+c)}+45\,e^{4i(dx+c)}+10\,e^{2i(dx+c)}+1\right)}{105d\left(e^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^4(dx+c)\right)}{40\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{20\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{40\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right.}{105\cos(dx+c)}\right.}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^4(dx+c)\right)}{40\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{20\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{40\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right.}{105\cos(dx+c)}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8
+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-3*a^3*(1/9*
sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3
/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*I*a^3/cos(d*x+c)^8-a^3*
(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 0.99

$$-\frac{21i\,a^3\tan(dx+c)^{10}+70\,a^3\tan(dx+c)^9+240\,a^3\tan(dx+c)^7-210i\,a^3\tan(dx+c)^6+252\,a^3\tan(dx+c)^5-420i\,a^3\tan(dx+c)^4-315i\,a^3\tan(dx+c)^2-210\,a^3\tan(dx+c)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
+ c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*tan(
d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(85) = 170$.
time = 0.37, size = 215, normalized size = 1.97

$$-\frac{128\left(-210 i a^{3} e^{(12 i d x+12 i c)}-252 i a^{3} e^{(10 i d x+10 i c)}-210 i a^{3} e^{(8 i d x+8 i c)}-120 i a^{3} e^{(6 i d x+6 i c)}-45 i a^{3} e^{(4 i d x+4 i c)}-10 i a^{3} e^{(2 i d x+2 i c)}-i a^{3}\right)}{105\left(d e^{(20 i d x+20 i c)}+10 d e^{(18 i d x+18 i c)}+45 d e^{(16 i d x+16 i c)}+120 d e^{(14 i d x+14 i c)}+210 d e^{(12 i d x+12 i c)}+252 d e^{(10 i d x+10 i c)}+210 d e^{(8 i d x+8 i c)}+120 d e^{(6 i d x+6 i c)}+45 d e^{(4 i d x+4 i c)}+10 d e^{(2 i d x+2 i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -128/105*(-210*I*a^3*e^(12*I*d*x + 12*I*c) - 252*I*a^3*e^(10*I*d*x + 10*I*c
) - 210*I*a^3*e^(8*I*d*x + 8*I*c) - 120*I*a^3*e^(6*I*d*x + 6*I*c) - 45*I*a^
3*e^(4*I*d*x + 4*I*c) - 10*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(20*I*d*
x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120
*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x
+ 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^
(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-i a^{3}\left(\int i \sec^{8}(c+d x)\,dx+\int\left(-3 \tan(c+d x) \sec^{8}(c+d x)\right)\,dx+\int \tan^{3}(c+d x) \sec^{8}(c+d x)\,dx+\int\left(-3 i \tan^{2}(c+d x) \sec^{8}(c+d x)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**8, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**8, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**8, x))
```

Giac [A]
time = 0.65, size = 108, normalized size = 0.99

$$-\frac{21 i a^{3} \tan(d x+c)^{10}+70 a^{3} \tan(d x+c)^{9}+240 a^{3} \tan(d x+c)^{7}-210 i a^{3} \tan(d x+c)^{6}+252 a^{3} \tan(d x+c)^{5}-420 i a^{3} \tan(d x+c)^{4}-315 i a^{3} \tan(d x+c)^{2}-210 a^{3} \tan(d x+c)}{210 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
+ c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*tan(
d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.31, size = 151, normalized size = 1.39

$$-\frac{a^{3} \sin(c+d x)\left(-210 \cos(c+d x)^{9}-\cos(c+d x)^{8} \sin(c+d x)\, 315\mathrm{i}-\cos(c+d x)^{6} \sin(c+d x)^{3}\, 420\mathrm{i}+252 \cos(c+d x)^{5} \sin(c+d x)^{4}-\cos(c+d x)^{4} \sin(c+d x)^{5}\, 210\mathrm{i}+240 \cos(c+d x)^{3} \sin(c+d x)^{6}+70 \cos(c+d x) \sin(c+d x)^{8}+\sin(c+d x)^{9}\, 21\mathrm{i}\right)}{210 d \cos(c+d x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^8,x)
```

```
[Out] -(a^3*sin(c + d*x)*(70*cos(c + d*x)*sin(c + d*x)^8 - cos(c + d*x)^8*sin(c +
 d*x)*315i - 210*cos(c + d*x)^9 + sin(c + d*x)^9*21i + 240*cos(c + d*x)^3*s
in(c + d*x)^6 - cos(c + d*x)^4*sin(c + d*x)^5*210i + 252*cos(c + d*x)^5*sin
(c + d*x)^4 - cos(c + d*x)^6*sin(c + d*x)^3*420i))/(210*d*cos(c + d*x)^10)
```

3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=82

$$-\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^6/a^3/d+4/7*I*(a+I*a*tan(d*x+c))^7/a^4/d-1/8*I*(a
+I*a*tan(d*x+c))^8/a^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^3*d) + (((4*I)/7)*(a + I*a*Tan[c +
 d*x])^7)/(a^4*d) - ((I/8)*(a + I*a*Tan[c + d*x])^8)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^5-4a(a+x)^6+(a+x)^7\right)\,dx,x,ia\tan(c+\right.}{a^5d}\\ &= -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{i(a+ia\tan}{8a^5}\end{aligned}$$

Mathematica [A]
time = 0.77, size = 106, normalized size = 1.29

$$\frac{a^3 \sec(c) \sec^8(c+dx)(35i\cos(c)+28i\cos(c+2dx)+28i\cos(3c+2dx)-35\sin(c)+28\sin(c+2dx)-28\sin(3c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx))}{168d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^8*((35*I)*Cos[c] + (28*I)*Cos[c + 2*d*x] + (28*I)*
Cos[3*c + 2*d*x] - 35*Sin[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 28
*Sin[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(168*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(70) = 140.
time = 0.26, size = 174, normalized size = 2.12

method	result
risch	$\frac{32ia^3\left(56\,e^{10i(dx+c)}+70\,e^{8i(dx+c)}+56\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{21d\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}-}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}-}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1
/24*sin(d*x+c)^4/cos(d*x+c)^4)-3*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*si
n(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+1/2*I*a^3/cos(d*x+
c)^6-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 1.32

$$-\frac{21i\,a^3\tan(dx+c)^8+72\,a^3\tan(dx+c)^7-28i\,a^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5-210i\,a^3\tan(dx+c)^4+56\,a^3\tan(dx+c)^3-252i\,a^3\tan(dx+c)^2-168\,a^3\tan(dx+c)}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
+ c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d*x
 + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.
time = 0.36, size = 177, normalized size = 2.16

$$-\frac{32\left(-56i\,a^3e^{(10i\,dx+10i\,c)}-70i\,a^3e^{(8i\,dx+8i\,c)}-56i\,a^3e^{(6i\,dx+6i\,c)}-28i\,a^3e^{(4i\,dx+4i\,c)}-8i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{21\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c)}+8\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/21*(-56*I*a^3*e^(10*I*d*x + 10*I*c) - 70*I*a^3*e^(8*I*d*x + 8*I*c) - 56
*I*a^3*e^(6*I*d*x + 6*I*c) - 28*I*a^3*e^(4*I*d*x + 4*I*c) - 8*I*a^3*e^(2*I*
d*x + 2*I*c) - I*a^3)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c)
+ 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x
 + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sec^6\left(c+dx\right)dx+\int\left(-3\tan\left(c+dx\right)\sec^6\left(c+dx\right)\right)\,dx+\int\tan^3\left(c+dx\right)\sec^6\left(c+dx\right)dx+\int\left(-3i\tan^2\left(c+dx\right)\sec^6\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**6, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**6, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.65, size = 108, normalized size = 1.32

$$-\frac{21i\,a^3\tan\left(dx+c\right)^8+72\,a^3\tan\left(dx+c\right)^7-28i\,a^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(dx+c\right)^5-210i\,a^3\tan\left(dx+c\right)^4+56\,a^3\tan\left(dx+c\right)^3-252i\,a^3\tan\left(dx+c\right)^2-168\,a^3\tan\left(dx+c\right)}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
+ c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d*x
 + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.29, size = 151, normalized size = 1.84

$$-\frac{a^3\sin\left(c+d\,x\right)\,\left(-168\cos\left(c+d\,x\right)^7-\cos\left(c+d\,x\right)^6\sin\left(c+d\,x\right)\,252\mathrm{i}+56\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^4\sin\left(c+d\,x\right)^3\,210\mathrm{i}+168\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^4-\cos\left(c+d\,x\right)^2\sin\left(c+d\,x\right)^5\,28\mathrm{i}+72\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^6+\sin\left(c+d\,x\right)^7\,21\mathrm{i}\right)}{168\,d\cos\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^6,x)
```

```
[Out] -(a^3*sin(c + d*x)*(72*cos(c + d*x)*sin(c + d*x)^6 - cos(c + d*x)^6*sin(c +
 d*x)*252i - 168*cos(c + d*x)^7 + sin(c + d*x)^7*21i - cos(c + d*x)^2*sin(c
 + d*x)^5*28i + 168*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c +
d*x)^3*210i + 56*cos(c + d*x)^5*sin(c + d*x)^2))/(168*d*cos(c + d*x)^8)
```

3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=55

$$-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^5/a^2/d+1/6*I*(a+I*a*tan(d*x+c))^6/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^6}{6a^3d}-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^2*d) + ((I/6)*(a + I*a*Tan[c + d*x
])^6)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^4\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 97, normalized size = 1.76

$$\frac{a^3 \sec(c) \sec^6(c+dx)(20i\cos(c)+15i\cos(c+2dx)+15i\cos(3c+2dx)-20\sin(c)+15\sin(c+2dx)-15\sin(3c+2dx)+12\sin(3c+4dx)+2\sin(5c+6dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^6*((20*I)*Cos[c] + (15*I)*Cos[c + 2*d*x] + (15*I)*
Cos[3*c + 2*d*x] - 20*Sin[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 12
*Sin[3*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(60*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(47) = 94.
time = 0.24, size = 128, normalized size = 2.33

method	result
risch	$\frac{32ia^3\left(15\,e^{8i(dx+c)}+20\,e^{6i(dx+c)}+15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)-
3*a^3*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/4*I*
a^3/cos(d*x+c)^4-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.49

$$-\frac{5i\,a^3\tan\left(dx+c\right)^6+18\,a^3\tan\left(dx+c\right)^5-15i\,a^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3-45i\,a^3\tan\left(dx+c\right)^2-30\,a^3\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x + c
))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(43) = 86$.
time = 0.35, size = 139, normalized size = 2.53

$$-\frac{32 \left(-15 i a^3 e^{(8 i dx+8 i c)}-20 i a^3 e^{(6 i dx+6 i c)}-15 i a^3 e^{(4 i dx+4 i c)}-6 i a^3 e^{(2 i dx+2 i c)}-i a^3\right)}{15 \left(d e^{(12 i dx+12 i c)}+6 d e^{(10 i dx+10 i c)}+15 d e^{(8 i dx+8 i c)}+20 d e^{(6 i dx+6 i c)}+15 d e^{(4 i dx+4 i c)}+6 d e^{(2 i dx+2 i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/15*(-15*I*a^3*e^(8*I*d*x + 8*I*c) - 20*I*a^3*e^(6*I*d*x + 6*I*c) - 15*I
*a^3*e^(4*I*d*x + 4*I*c) - 6*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(12*I*
d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d
*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-i a^3 \left(\int i \sec^4(c+dx)\, dx + \int \left(-3 \tan(c+dx) \sec^4(c+dx)\right) dx + \int \tan^3(c+dx) \sec^4(c+dx)\, dx + \int \left(-3 i \tan^2(c+dx) \sec^4(c+dx)\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**4, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**4, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.64, size = 82, normalized size = 1.49

$$-\frac{5 i a^3 \tan(dx+c)^6 + 18 a^3 \tan(dx+c)^5 - 15 i a^3 \tan(dx+c)^4 + 20 a^3 \tan(dx+c)^3 - 45 i a^3 \tan(dx+c)^2 - 30 a^3 \tan(dx+c)}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x + c
))/d
```

Mupad [B]
time = 3.26, size = 114, normalized size = 2.07

$$-\frac{a^3 \sin(c+dx) \left(-30 \cos(c+dx)^5 - \cos(c+dx)^4 \sin(c+dx)\, 45\mathrm{i} + 20 \cos(c+dx)^3 \sin(c+dx)^2 - \cos(c+dx)^2 \sin(c+dx)^3\, 15\mathrm{i} + 18 \cos(c+dx) \sin(c+dx)^4 + \sin(c+dx)^5\, 5\mathrm{i}\right)}{30 d \cos(c+dx)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^4,x)
```

```
[Out] -(a^3*sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c +
 d*x)*45i - 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c +
d*x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

```
[Out] -1/4*I*(a+I*a*tan(d*x+c))^4/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/4*I)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^3\,dx,x,ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^4}{4ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.
time = 0.48, size = 84, normalized size = 3.11

$$\frac{a^3\sec(c)\sec^4(c+dx)(3i\cos(c)+2i\cos(c+2dx)+2i\cos(3c+2dx)-3\sin(c)+2\sin(c+2dx)-2\sin(3c+2dx)+\sin(3c+4dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^4*((3*I)*Cos[c] + (2*I)*Cos[c + 2*d*x] + (2*I)*Cos
[3*c + 2*d*x] - 3*Sin[c] + 2*Sin[c + 2*d*x] - 2*Sin[3*c + 2*d*x] + Sin[3*c
+ 4*d*x]))/(4*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(23) = 46$.
time = 0.23, size = 73, normalized size = 2.70

method	result	size
risch	$\frac{4 i a^3 \left(4\,{\mathrm e}^{6 i (d x +c)}+6\,{\mathrm e}^{4 i (d x +c)}+4\,{\mathrm e}^{2 i (d x +c)}+1\right)}{d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{4}}$	58
derivativedivides	$\frac{-\frac{i a^3 \left(\sin^{4}(d x +c)\right)}{4 \cos \left(d x +c \right)^{4}}-\frac{a^3 \left(\sin^{3}(d x +c)\right)}{\cos \left(d x +c \right)^{3}}+\frac{3 i a^3}{2 \cos \left(d x +c \right)^{2}}+a^3 \tan \left(d x +c \right)}{d}$	73
default	$\frac{-\frac{i a^3 \left(\sin^{4}(d x +c)\right)}{4 \cos \left(d x +c \right)^{4}}-\frac{a^3 \left(\sin^{3}(d x +c)\right)}{\cos \left(d x +c \right)^{3}}+\frac{3 i a^3}{2 \cos \left(d x +c \right)^{2}}+a^3 \tan \left(d x +c \right)}{d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*I*a^3*sin(d*x+c)^4/cos(d*x+c)^4-a^3*sin(d*x+c)^3/cos(d*x+c)^3+3/2
*I*a^3/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.78

$$-\frac{i\,{\left(i\,a \tan\left(dx + c\right) + a\right)}^{4}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*(I*a*tan(d*x + c) + a)^4/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(21) = 42$.
time = 0.36, size = 101, normalized size = 3.74

$$-\frac{4\,{\left(-4 i\,a^{3} e^{\left(6 i\,dx + 6 i\,c\right)} - 6 i\,a^{3} e^{\left(4 i\,dx + 4 i\,c\right)} - 4 i\,a^{3} e^{\left(2 i\,dx + 2 i\,c\right)} - i\,a^{3}\right)}}{de^{\left(8 i\,dx + 8 i\,c\right)} + 4\,de^{\left(6 i\,dx + 6 i\,c\right)} + 6\,de^{\left(4 i\,dx + 4 i\,c\right)} + 4\,de^{\left(2 i\,dx + 2 i\,c\right)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4*(-4*I*a^3*e^(6*I*d*x + 6*I*c) - 6*I*a^3*e^(4*I*d*x + 4*I*c) - 4*I*a^3*e^
(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c)
 + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sec^2(c+dx)\,dx+\int(-3\tan(c+dx)\sec^2(c+dx))\,dx+\int\tan^3(c+dx)\sec^2(c+dx)\,dx+\int(-3i\tan^2(c+dx)\sec^2(c+dx))\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**2, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**2, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(21) = 42$.
time = 0.58, size = 56, normalized size = 2.07

$$-\frac{i\,a^3\tan(dx+c)^4+4\,a^3\tan(dx+c)^3-6i\,a^3\tan(dx+c)^2-4\,a^3\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 6*I*a^3*tan(d*x + c)^2
- 4*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.27, size = 56, normalized size = 2.07

$$\frac{-\frac{a^3\tan(c+d\,x)^4\,1\mathrm{i}}{4}-a^3\tan(c+d\,x)^3+\frac{a^3\tan(c+d\,x)^2\,3\mathrm{i}}{2}+a^3\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^2,x)
```

```
[Out] (a^3*tan(c + d*x) + (a^3*tan(c + d*x)^2*3i)/2 - a^3*tan(c + d*x)^3 - (a^3*t
an(c + d*x)^4*1i)/4)/d
```

3.40 $\int (a+ia\tan(c+dx))^3 \, dx$

Optimal. Leaf size=63

$$4a^3x-\frac{4ia^3\log(\cos(c+dx))}{d}-\frac{2a^3\tan(c+dx)}{d}+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] 4*a^3*x-4*I*a^3*ln(cos(d*x+c))/d-2*a^3*tan(d*x+c)/d+1/2*I*a*(a+I*a*tan(d*x+
c))^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3559, 3558, 3556}

$$-\frac{2a^3\tan(c+dx)}{d}-\frac{4ia^3\log(\cos(c+dx))}{d}+4a^3x+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] 4*a^3*x - ((4*I)*a^3*Log[Cos[c + d*x]])/d - (2*a^3*Tan[c + d*x])/d + ((I/2)
*a*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(c+dx))^3\,dx &= \frac{ia(a+ia\tan(c+dx))^2}{2d} + (2a)\int (a+ia\tan(c+dx))^2\,dx \\
&= 4a^3x - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d} + \left(4ia^3\right)\int \tan(c+dx)\,dx \\
&= 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.80, size = 119, normalized size = 1.89

$$
\frac{a^3\sec(c)\sec^2(c+dx)\left(2dx\cos(3c+2dx)+\cos(c+2dx)\left(2dx-i\log\left(\cos^2(c+dx)\right)\right)+\cos(c)\left(-i+4dx-2i\log\left(\cos^2(c+dx)\right)\right)-i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)+3\sin(c)-3\sin(c+2dx)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^2*(2*d*x*Cos[3*c + 2*d*x] + Cos[c + 2*d*x]*(2*d*x
- I*Log[Cos[c + d*x]^2]) + Cos[c]*(-I + 4*d*x - (2*I)*Log[Cos[c + d*x]^2])
- I*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] + 3*Sin[c] - 3*Sin[c + 2*d*x]))/(2
*d)
```

Maple [A]
time = 0.04, size = 51, normalized size = 0.81

method	result	size
derivativedivides	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$	51
default	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$	51
norman	$4a^3x-\frac{3a^3\tan(dx+c)}{d}-\frac{ia^3\left(\tan^2(dx+c)\right)}{2d}+\frac{2ia^3\ln\left(1+\tan^2(dx+c)\right)}{d}$	59
risch	$-\frac{8a^3c}{d}-\frac{2ia^3\left(4\,e^{2i(dx+c)}+3\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{4ia^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(-3*tan(d*x+c)-1/2*I*tan(d*x+c)^2+2*I*ln(1+tan(d*x+c)^2)+4*arctan(t
an(d*x+c)))
```

Maxima [A]
time = 0.49, size = 76, normalized size = 1.21

$$
a^3x + \frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^3}{d} + \frac{i\,a^3\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d} + \frac{3i\,a^3\log\left(\sec\left(dx+c\right)\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 97, normalized size = 1.54

$$-\frac{2\left(4i\,a^{3}e^{(2i\,dx+2i\,c)}+3i\,a^{3}+2\left(i\,a^{3}e^{(4i\,dx+4i\,c)}+2i\,a^{3}e^{(2i\,dx+2i\,c)}+i\,a^{3}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*I*a^3*e^(2*I*d*x + 2*I*c) + 3*I*a^3 + 2*(I*a^3*e^(4*I*d*x + 4*I*c) +
2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*
I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.18, size = 94, normalized size = 1.49

$$-\frac{4ia^{3}\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-8ia^{3}e^{2ic}e^{2idx}-6ia^{3}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(2*I*c)*exp(2*I
*d*x) - 6*I*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x)
+ d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.
time = 0.46, size = 118, normalized size = 1.87

$$-\frac{2\left(2i\,a^{3}e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^{3}e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^{3}e^{(2i\,dx+2i\,c)}+2i\,a^{3}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+3i\,a^{3}\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*I*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2
*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2*I*d*x + 2*I*c)
+ 2*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3)/(d*e^(4*I*d*x + 4*I*c) +
2*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.28, size = 41, normalized size = 0.65

$$-\frac{a^3\,\left(6\,\mathrm{tan}\left(c+d\,x\right)-\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,8{}\mathrm{i}+{\mathrm{tan}\left(c+d\,x\right)}^2\,1{}\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(6*tan(c + d*x) - log(tan(c + d*x) + 1i)*8i + tan(c + d*x)^2*1i))/(2*
d)
```

3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=49

$$-a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}$$

```
[Out] -a^3*x+I*a^3*ln(cos(d*x+c))/d-2*I*a^4/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$-\frac{2ia^4}{d(a-ia\tan(c+dx))}+\frac{ia^3\log(\cos(c+dx))}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (I*a^3*Log[Cos[c + d*x]])/d - ((2*I)*a^4)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.
time = 0.28, size = 99, normalized size = 2.02

$$-\frac{a^3(\cos(c+dx)\,(2i+2dx-i\log\left(\cos^2(c+dx)\right))+(-2-2idx-\log\left(\cos^2(c+dx)\right))\sin(c+dx))\,(\cos(c+4dx)+i\sin(c+4dx))}{2d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*(Cos[c + d*x]*(2*I + 2*d*x - I*Log[Cos[c + d*x]^2]) + (-2 - (2*I)
*d*x - Log[Cos[c + d*x]^2])*Sin[c + d*x])*(Cos[c + 4*d*x] + I*Sin[c + 4*d*x
]))/(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.21, size = 99, normalized size = 2.02

method	result
risch	$-\frac{i a^{3} {\mathrm e}^{2 i (d x +c)}}{d}+\frac{2 a^{3} c}{d}+\frac{i a^{3} \ln \left({\mathrm e}^{2 i (d x +c)}+1\right)}{d}$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\left(\sin^{2}(d x +c)\right)}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \left(\cos^{2}(d x +c)\right)}{2}+a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}\right.}{d}$
default	$\frac{-i a^{3} \left(-\frac{\left(\sin^{2}(d x +c)\right)}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \left(\cos^{2}(d x +c)\right)}{2}+a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-3*a^3*(-1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-3/2*I*a^3*cos(d*x+c)^2+a^3*(1/2*sin(d*x+c)*cos(d*x+c)+
1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 62, normalized size = 1.27

$$-\frac{2\,{\left(dx+c\right)}a^{3}+i\,a^{3}\log\left(\tan\left(dx+c\right)^{2}+1\right)-\frac{4\,{\left(a^{3}\tan\left(dx+c\right)-i\,a^{3}\right)}}{\tan\left(dx+c\right)^{2}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a^3 + I*a^3*log(tan(d*x + c)^2 + 1) - 4*(a^3*tan(d*x + c)
 - I*a^3)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.73

$$\frac{-i\,a^3 e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A]
time = 0.20, size = 61, normalized size = 1.24

$$\frac{ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\begin{cases}-\frac{ia^3e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0\\ 2a^3xe^{2ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
[Out] I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise((-I*a**3*exp(2*I*c)*ex
p(2*I*d*x)/d, Ne(d, 0)), (2*a**3*x*exp(2*I*c), True))
```

Giac [A]
time = 0.64, size = 36, normalized size = 0.73

$$\frac{-i\,a^3 e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Mupad [B]
time = 3.29, size = 39, normalized size = 0.80

$$\frac{2\,a^3}{d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}-\frac{a^3\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
[Out] (2*a^3)/(d*(tan(c + d*x) + 1i)) - (a^3*log(tan(c + d*x) + 1i)*1i)/d
```

3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

```
[Out] -1/2*I*a^5/d/(a-I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/2*I)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{(ia^5)\operatorname{Subst}\left(\int \frac{1}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^5}{2d(a-ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 50, normalized size = 1.85

$$\frac{a^3(3\cos(c+dx)-i\sin(c+dx))(-i\cos(3(c+dx))+\sin(3(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(3*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(8*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(23) = 46.
time = 0.20, size = 114, normalized size = 4.22

method	result
risch	$-\frac{i a^3 e^{4i(dx+c)}}{8d} - \frac{i a^3 e^{2i(dx+c)}}{4d}$
derivativedivides	$\frac{-\frac{i a^3 \left(\sin^4(dx+c)\right)}{4} - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) - \frac{3i a^3\left(\cos^4(dx+c)\right)}{4} + a^3\left(\frac{\left(\cos^3(dx+c) + \frac{3c}{}\right.}{}\right.}{d}$
default	$\frac{-\frac{i a^3 \left(\sin^4(dx+c)\right)}{4} - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) - \frac{3i a^3\left(\cos^4(dx+c)\right)}{4} + a^3\left(\frac{\left(\cos^3(dx+c) + \frac{3c}{}\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*I*a^3*sin(d*x+c)^4-3*a^3*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/4*I*a^3*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.
time = 0.50, size = 57, normalized size = 2.11

$$-\frac{-i\,a^3\tan\left(dx+c\right)^2 - 2\,a^3\tan\left(dx+c\right) + i\,a^3}{2\left(\tan\left(dx+c\right)^4 + 2\,\tan\left(dx+c\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^3*tan(d*x + c)^2 - 2*a^3*tan(d*x + c) + I*a^3)/((tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1)*d)
```

Fricas [A]
time = 0.35, size = 34, normalized size = 1.26

$$\frac{-i\,a^3 e^{(4i\,dx+4i\,c)} - 2i\,a^3 e^{(2i\,dx+2i\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*a^3*e^(4*I*d*x + 4*I*c) - 2*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(22) = 44$.
time = 0.18, size = 80, normalized size = 2.96

$$\begin{cases} \frac{-4ia^3de^{4ic}e^{4idx}-8ia^3de^{2ic}e^{2idx}}{32d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^3e^{4ic}}{2}+\frac{a^3e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-4*I*a**3*d*exp(4*I*c)*exp(4*I*d*x) - 8*I*a**3*d*exp(2*I*c)*exp
(2*I*d*x))/(32*d**2), Ne(d**2, 0)), (x*(a**3*exp(4*I*c)/2 + a**3*exp(2*I*c)
/2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(21) = 42$.
time = 0.73, size = 135, normalized size = 5.00

$$-\frac{i\,a^3e^{(12i\,dx+8i\,c)}+6i\,a^3e^{(10i\,dx+6i\,c)}+14i\,a^3e^{(8i\,dx+4i\,c)}+16i\,a^3e^{(6i\,dx+2i\,c)}+2i\,a^3e^{(2i\,dx-2i\,c)}+9i\,a^3e^{(4i\,dx)}}{8\left(de^{(8i\,dx+4i\,c)}+4\,de^{(6i\,dx+2i\,c)}+4\,de^{(2i\,dx-2i\,c)}+6\,de^{(4i\,dx)}+de^{(-4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(I*a^3*e^(12*I*d*x + 8*I*c) + 6*I*a^3*e^(10*I*d*x + 6*I*c) + 14*I*a^3*
e^(8*I*d*x + 4*I*c) + 16*I*a^3*e^(6*I*d*x + 2*I*c) + 2*I*a^3*e^(2*I*d*x - 2
*I*c) + 9*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*
c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B]
time = 3.34, size = 36, normalized size = 1.33

$$-\frac{a^3\left(\frac{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}}{4}\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(exp(c*2i + d*x*2i)/2 + exp(c*4i + d*x*4i)/4)*1i)/(2*d)
```

3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=90

$$\frac{a^3x}{8}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}$$

```
[Out] 1/8*a^3*x-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^5/d/(a-I*a*tan(d*x+c))^2
-1/8*I*a^4/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*x)/8 - ((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^5)/(d*(a -
I*a*Tan[c + d*x])^2) - ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^7)\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^4} + \frac{1}{4a^2(a-x)^3} + \frac{1}{8a^3(a-x)^2} + \frac{1}{8a^3(a^2-x^2)}\right)\,dx\right.}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3} - \frac{ia^5}{8d(a-ia\tan(c+dx))^2} - \frac{}{8d(a-i}$$

$$= \frac{a^3x}{8} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3} - \frac{ia^5}{8d(a-ia\tan(c+dx))^2} - \frac{}{8d(}$$

Mathematica [A]
time = 0.43, size = 109, normalized size = 1.21

$$\frac{a^3(-27i\cos(c+dx)+2(-i+6dx)\cos(3(c+dx))-9\sin(c+dx)+2\sin(3(c+dx))-12idx\sin(3(c+dx)))(\cos(3(c+2dx))+i\sin(3(c+2dx)))}{96d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-27*I)*Cos[c + d*x] + 2*(-I + 6*d*x)*Cos[3*(c + d*x)] - 9*Sin[c + d*
x] + 2*Sin[3*(c + d*x)] - (12*I)*d*x*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x)]
+ I*Sin[3*(c + 2*d*x)]))/(96*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(76) = 152.
time = 0.21, size = 156, normalized size = 1.73

method	result
risch	$\frac{a^3x}{8} - \frac{ia^3 e^{6i(dx+c)}}{48d} - \frac{3ia^3 e^{4i(dx+c)}}{32d} - \frac{3ia^3 e^{2i(dx+c)}}{16d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6} - \frac{\left(\cos^4(dx+c)\right)}{12}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6} + \frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{24}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6} - \frac{\left(\cos^4(dx+c)\right)}{12}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6} + \frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{24}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-3*a^3*(-1/6*
sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*
d*x+1/16*c)-1/2*I*a^3*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.50, size = 105, normalized size = 1.17

$$\frac{3\,(dx+c)a^3+\frac{3\,a^3\tan(dx+c)^5+8\,a^3\tan(dx+c)^3+6i\,a^3\tan(dx+c)^2+21\,a^3\tan(dx+c)-10i\,a^3}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(d*x + c)*a^3 + (3*a^3*tan(d*x + c)^5 + 8*a^3*tan(d*x + c)^3 + 6*I*
a^3*tan(d*x + c)^2 + 21*a^3*tan(d*x + c) - 10*I*a^3)/(tan(d*x + c)^6 + 3*ta
n(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 55, normalized size = 0.61

$$\frac{12\,a^3dx-2i\,a^3e^{(6i\,dx+6i\,c)}-9i\,a^3e^{(4i\,dx+4i\,c)}-18i\,a^3e^{(2i\,dx+2i\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*a^3*d*x - 2*I*a^3*e^(6*I*d*x + 6*I*c) - 9*I*a^3*e^(4*I*d*x + 4*I*c
) - 18*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A]
time = 0.24, size = 131, normalized size = 1.46

$$\frac{a^3x}{8}+\begin{cases}\frac{-512ia^3d^2e^{6ic}e^{6idx}-2304ia^3d^2e^{4ic}e^{4idx}-4608ia^3d^2e^{2ic}e^{2idx}}{24576d^3} & \text{for } d^3\neq 0\\ x\left(\frac{a^3e^{6ic}}{8}+\frac{3a^3e^{4ic}}{8}+\frac{3a^3e^{2ic}}{8}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*x/8 + Piecewise(((-512*I*a**3*d**2*exp(6*I*c)*exp(6*I*d*x) - 2304*I*a*
*3*d**2*exp(4*I*c)*exp(4*I*d*x) - 4608*I*a**3*d**2*exp(2*I*c)*exp(2*I*d*x))
/(24576*d**3), Ne(d**3, 0)), (x*(a**3*exp(6*I*c)/8 + 3*a**3*exp(4*I*c)/8 +
3*a**3*exp(2*I*c)/8), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(70) = 140$.
time = 0.81, size = 457, normalized size = 5.08

$$\frac{12\,a^3dxe^{(8i\,dx+4i\,c)}+48\,a^3dxe^{(6i\,dx+2i\,c)}+48\,a^3dxe^{(2i\,dx-2i\,c)}+72\,a^3dxe^{(4i\,dx)}+12\,a^3dxe^{(-4i\,c)}-3i\,a^3e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^3e^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^3e^{(2i\,dx-2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-18i\,a^3e^{(4i\,dx)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3i\,a^3e^{(-4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+3i\,a^3e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)+12i\,a^3e^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)+12i\,a^3e^{(2i\,dx-2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)+18i\,a^3e^{(4i\,dx)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)+3i\,a^3e^{(-4i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-2i\,a^3e^{(14i\,dx+10i\,c)}-17i\,a^3e^{(12i\,dx+8i\,c)}-66i\,a^3e^{(10i\,dx+6i\,c)}-134i\,a^3e^{(8i\,dx+4i\,c)}-146i\,a^3e^{(6i\,dx+2i\,c)}-18i\,a^3e^{(2i\,dx-2i\,c)}-81i\,a^3e^{(4i\,dx)}}{96\left(de^{(8i\,dx+4i\,c)}+4\,de^{(6i\,dx+2i\,c)}+4\,de^{(2i\,dx-2i\,c)}+6\,de^{(4i\,dx)}+de^{(-4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(12*a^3*d*x*e^(8*I*d*x + 4*I*c) + 48*a^3*d*x*e^(6*I*d*x + 2*I*c) + 48*
a^3*d*x*e^(2*I*d*x - 2*I*c) + 72*a^3*d*x*e^(4*I*d*x) + 12*a^3*d*x*e^(-4*I*c
) - 3*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(
6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(2*I*d*x - 2*I*c
)*log(e^(2*I*d*x + 2*I*c) + 1) - 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*
c) + 1) - 3*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3*e^(8*I*
d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(6*I*d*x + 2*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(2*I*d*x - 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) + 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + 3*I*a
^3*e^(-4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 2*I*a^3*e^(14*I*d*x + 10*I*c)
 - 17*I*a^3*e^(12*I*d*x + 8*I*c) - 66*I*a^3*e^(10*I*d*x + 6*I*c) - 134*I*a^
3*e^(8*I*d*x + 4*I*c) - 146*I*a^3*e^(6*I*d*x + 2*I*c) - 18*I*a^3*e^(2*I*d*x
 - 2*I*c) - 81*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x +
 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B]
time = 3.36, size = 77, normalized size = 0.86

$$\frac{a^3\,x}{8}-\frac{\frac{a^3\,\tan\left(c+d\,x\right)^2}{8}+\frac{a^3\,\tan\left(c+d\,x\right)\,3{}\mathrm{i}}{8}-\frac{5\,a^3}{12}}{d\,\left(-{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,3{}\mathrm{i}+3\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (a^3*x)/8 - ((a^3*tan(c + d*x)*3i)/8 - (5*a^3)/12 + (a^3*tan(c + d*x)^2/8)
/(d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=144

$$\frac{5a^3x}{32}-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}$$

```
[Out] 5/32*a^3*x-1/16*I*a^7/d/(a-I*a*tan(d*x+c))^4-1/12*I*a^6/d/(a-I*a*tan(d*x+c)
)^3-3/32*I*a^5/d/(a-I*a*tan(d*x+c))^2-1/8*I*a^4/d/(a-I*a*tan(d*x+c))+1/32*I
*a^4/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{ia^4}{32d(a+ia\tan(c+dx))}+\frac{5a^3x}{32}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/32 - ((I/16)*a^7)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/12)*a^6)/(d*
(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^5)/(d*(a - I*a*Tan[c + d*x])^2) -
 ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x])) + ((I/32)*a^4)/(d*(a + I*a*Tan[c +
d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^9)\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^5} + \frac{1}{4a^3(a-x)^4} + \frac{3}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \right.\right.}{d}$$
$$= -\frac{ia^7}{16d(a-ia\tan(c+dx))^4} - \frac{ia^6}{12d(a-ia\tan(c+dx))^3} - \frac{}{32d(a}$$
$$= \frac{5a^3x}{32} - \frac{ia^7}{16d(a-ia\tan(c+dx))^4} - \frac{ia^6}{12d(a-ia\tan(c+dx))^3} -$$

Mathematica [A]
time = 0.53, size = 131, normalized size = 0.91

$$\frac{a^3(-180i\cos(c+dx)+20(-i+6dx)\cos(3(c+dx))+9i\cos(5(c+dx))-60\sin(c+dx)+20\sin(3(c+dx))-120idx\sin(3(c+dx))+15\sin(5(c+dx)))(\cos(3(c+2dx))+i\sin(3(c+2dx)))}{768d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-180*I)*Cos[c + d*x] + 20*(-I + 6*d*x)*Cos[3*(c + d*x)] + (9*I)*Cos[
5*(c + d*x)] - 60*Sin[c + d*x] + 20*Sin[3*(c + d*x)] - (120*I)*d*x*Sin[3*(c
 + d*x)] + 15*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*x)])
)/(768*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.23, size = 176, normalized size = 1.22

method	result
risch	$\frac{5a^3x}{32} - \frac{ia^3e^{8i(dx+c)}}{256d} - \frac{5ia^3e^{6i(dx+c)}}{192d} - \frac{5ia^3e^{4i(dx+c)}}{64d} - \frac{9ia^3\cos(2dx+2c)}{64d} + \frac{11a^3\sin(2dx+2c)}{64d}$
derivativedivides	$\frac{-ia^3\left(-\frac{(\sin^2(dx+c))(\cos^6(dx+c))}{8} - \frac{(\cos^6(dx+c))}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8} + \frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos}{}\right.}{48}\right.}{}$
default	$\frac{-ia^3\left(-\frac{(\sin^2(dx+c))(\cos^6(dx+c))}{8} - \frac{(\cos^6(dx+c))}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8} + \frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos}{}\right.}{48}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-3*a^3*(-1/8*
sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c)
)*sin(d*x+c)+5/128*d*x+5/128*c)-3/8*I*a^3*cos(d*x+c)^8+a^3*(1/8*(cos(d*x+c)
^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*
d*x+35/128*c))
```

Maxima [A]
time = 0.50, size = 128, normalized size = 0.89

$$\frac{15\,(dx+c)a^3+\frac{15\,a^3\tan(dx+c)^7+55\,a^3\tan(dx+c)^5+73\,a^3\tan(dx+c)^3+16i\,a^3\tan(dx+c)^2+81\,a^3\tan(dx+c)-32i\,a^3}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(15*(d*x + c)*a^3 + (15*a^3*tan(d*x + c)^7 + 55*a^3*tan(d*x + c)^5 + 7
3*a^3*tan(d*x + c)^3 + 16*I*a^3*tan(d*x + c)^2 + 81*a^3*tan(d*x + c) - 32*I
*a^3)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c
)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 92, normalized size = 0.64

$$\frac{\left(120\,a^3dxe^{(2i\,dx+2i\,c)}-3i\,a^3e^{(10i\,dx+10i\,c)}-20i\,a^3e^{(8i\,dx+8i\,c)}-60i\,a^3e^{(6i\,dx+6i\,c)}-120i\,a^3e^{(4i\,dx+4i\,c)}+12i\,a^3\right)e^{(-2i\,dx-2i\,c)}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*a^3*d*x*e^(2*I*d*x + 2*I*c) - 3*I*a^3*e^(10*I*d*x + 10*I*c) - 20
*I*a^3*e^(8*I*d*x + 8*I*c) - 60*I*a^3*e^(6*I*d*x + 6*I*c) - 120*I*a^3*e^(4*
I*d*x + 4*I*c) + 12*I*a^3)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A]
time = 0.34, size = 226, normalized size = 1.57

$$\frac{5a^3x}{32}+\begin{cases}\frac{\left(-25165824ia^3d^4e^{10ic}e^{8idx}-167772160ia^3d^4e^{8ic}e^{6idx}-503316480ia^3d^4e^{6ic}e^{4idx}-1006632960ia^3d^4e^{4ic}e^{2idx}+100663296ia^3d^4e^{-2idx}\right)e^{-2ic}}{6442450944d^5} & \text{for } d^5e^{2ic}\neq 0\\ x\left(-\frac{5a^3}{32}+\frac{\left(a^3e^{10ic}+5a^3e^{8ic}+10a^3e^{6ic}+10a^3e^{4ic}+5a^3e^{2ic}+a^3\right)e^{-2ic}}{32}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 5*a**3*x/32 + Piecewise(((-25165824*I*a**3*d**4*exp(10*I*c)*exp(8*I*d*x) -
167772160*I*a**3*d**4*exp(8*I*c)*exp(6*I*d*x) - 503316480*I*a**3*d**4*exp(6
*I*c)*exp(4*I*d*x) - 1006632960*I*a**3*d**4*exp(4*I*c)*exp(2*I*d*x) + 10066
3296*I*a**3*d**4*exp(-2*I*d*x))*exp(-2*I*c)/(6442450944*d**5), Ne(d**5*exp(
```

```
2*I*c), 0)), (x*(-5*a**3/32 + (a**3*exp(10*I*c) + 5*a**3*exp(8*I*c) + 10*a*
*3*exp(6*I*c) + 10*a**3*exp(4*I*c) + 5*a**3*exp(2*I*c) + a**3)*exp(-2*I*c)/
32), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(112) = 224$.
time = 0.91, size = 514, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1536*(240*a^3*d*x*e^(10*I*d*x + 6*I*c) + 960*a^3*d*x*e^(8*I*d*x + 4*I*c)
+ 1440*a^3*d*x*e^(6*I*d*x + 2*I*c) + 240*a^3*d*x*e^(2*I*d*x - 2*I*c) + 960*
a^3*d*x*e^(4*I*d*x) - 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 198*I*
a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 33*I*a^3*e^(2*I*d*x
- 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 132*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*
x + 2*I*c) + 1) + 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c
)) + 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 198*I*a^
3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 33*I*a^3*e^(2*I*d*x -
 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 132*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*
x) + e^(-2*I*c)) - 6*I*a^3*e^(18*I*d*x + 14*I*c) - 64*I*a^3*e^(16*I*d*x + 1
2*I*c) - 316*I*a^3*e^(14*I*d*x + 10*I*c) - 984*I*a^3*e^(12*I*d*x + 8*I*c) -
 1846*I*a^3*e^(10*I*d*x + 6*I*c) - 1936*I*a^3*e^(8*I*d*x + 4*I*c) - 984*I*a
^3*e^(6*I*d*x + 2*I*c) + 96*I*a^3*e^(2*I*d*x - 2*I*c) - 96*I*a^3*e^(4*I*d*x
) + 24*I*a^3*e^(-4*I*c))/(d*e^(10*I*d*x + 6*I*c) + 4*d*e^(8*I*d*x + 4*I*c)
+ 6*d*e^(6*I*d*x + 2*I*c) + d*e^(2*I*d*x - 2*I*c) + 4*d*e^(4*I*d*x))
```

Mupad [B]
time = 3.65, size = 125, normalized size = 0.87

$$\frac{5\,a^3\,x}{32}-\frac{\frac{5\,a^3\,\mathrm{tan}\left(c+d\,x\right)^4}{32}+\frac{a^3\,\mathrm{tan}\left(c+d\,x\right)^3\,15{}\mathrm{i}}{32}-\frac{35\,a^3\,\mathrm{tan}\left(c+d\,x\right)^2}{96}+\frac{a^3\,\mathrm{tan}\left(c+d\,x\right)\,5{}\mathrm{i}}{32}-\frac{a^3}{3}}{d\,\left(-{\mathrm{tan}\left(c+d\,x\right)}^5-{\mathrm{tan}\left(c+d\,x\right)}^4\,3{}\mathrm{i}+2\,{\mathrm{tan}\left(c+d\,x\right)}^3-{\mathrm{tan}\left(c+d\,x\right)}^2\,2{}\mathrm{i}+3\,\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (5*a^3*x)/32 - ((a^3*tan(c + d*x)*5i)/32 - a^3/3 - (35*a^3*tan(c + d*x)^2)/
96 + (a^3*tan(c + d*x)^3*15i)/32 + (5*a^3*tan(c + d*x)^4)/32)/(d*(3*tan(c +
 d*x) - tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 - tan(c + d*x)^4*3i - tan(c +
d*x)^5 + 1i))
```

3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=127

$$\frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx)}{5d}$$

```
[Out] 7/8*a^3*arctanh(sin(d*x+c))/d+7/12*I*a^3*sec(d*x+c)^3/d+7/8*a^3*sec(d*x+c)*
tan(d*x+c)/d+1/5*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d+7/20*I*sec(d*x+c)^
3*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3579, 3567, 3853, 3855}

$$\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7i\sec^3(c+dx)\,(a^3+ia^3\tan(c+dx))}{20d}+\frac{7a^3\tan(c+dx)\sec(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (((7*I)/12)*a^3*Sec[c + d*x]^3)/d + (
7*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/5)*a*Sec[c + d*x]^3*(a + I*a*T
an[c + d*x])^2)/d + (((7*I)/20)*Sec[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/
d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{1}{5}(7a)\int \sec^3(c+dx)(a+$$
$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\mathrm{t}\right.}{20d}$$
$$= \frac{7ia^3\sec^3(c+dx)}{12d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{7i\sec}{}$$
$$= \frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{ia\sec^3(c+dx}{}$$
$$= \frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\sec(c+dx)\,\mathrm{ta}}{8d}$$

Mathematica [A]
time = 0.64, size = 102, normalized size = 0.80

$$\frac{a^3(\cos(3dx)+i\sin(3dx))\left(1680\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(448i+640i\cos(2(c+dx))-150\sin(2(c+dx))+105\sin(4(c+dx)))\right)}{960d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(1680*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + Sec[c + d*x]^5*(448*I + (640*I)*Cos[2*(c + d*x)] - 150*Sin[2*(c + d*x)]
 + 105*Sin[4*(c + d*x)])))/(960*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.30, size = 200, normalized size = 1.57

method	result
risch	$-\frac{ia^3\left(105\,\mathrm{e}^{9i(dx+c)}-790\,\mathrm{e}^{7i(dx+c)}-896\,\mathrm{e}^{5i(dx+c)}-490\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{60d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{7a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}-\frac{7a^3\ln(\mathrm{e}^{i(d}}{8d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+}{8}\right.}{d}$

default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1
/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))-3*a^3*(1/4*si
n(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln
(sec(d*x+c)+tan(d*x+c)))+I*a^3/cos(d*x+c)^3+a^3*(1/2*sec(d*x+c)*tan(d*x+c)+
1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 155, normalized size = 1.22

$$-\frac{45\,a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{240i\,a^3}{\cos(dx+c)^3}-\frac{16i\left(5\,\cos(dx+c)^2-3\right)a^3}{\cos(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(45*a^3*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) - 240*I*a^3/cos(d*x + c)^3 - 16*I*(5*cos(d*x + c)^2 - 3)*a^3/cos(d
*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.
time = 0.36, size = 310, normalized size = 2.44

$$\frac{-210i\,a^3e^{(9i\,dx+9i\,c)}+1580i\,a^3e^{(7i\,dx+7i\,c)}+1792i\,a^3e^{(5i\,dx+5i\,c)}+980i\,a^3e^{(3i\,dx+3i\,c)}+210i\,a^3e^{(i\,dx+i\,c)}+105\left(a^3e^{(10i\,dx+10i\,c)}+5\,a^3e^{(8i\,dx+8i\,c)}+10\,a^3e^{(6i\,dx+6i\,c)}+10\,a^3e^{(4i\,dx+4i\,c)}+5\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(a^3e^{(10i\,dx+10i\,c)}+5\,a^3e^{(8i\,dx+8i\,c)}+10\,a^3e^{(6i\,dx+6i\,c)}+10\,a^3e^{(4i\,dx+4i\,c)}+5\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{120\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(-210*I*a^3*e^(9*I*d*x + 9*I*c) + 1580*I*a^3*e^(7*I*d*x + 7*I*c) + 17
92*I*a^3*e^(5*I*d*x + 5*I*c) + 980*I*a^3*e^(3*I*d*x + 3*I*c) + 210*I*a^3*e^
(I*d*x + I*c) + 105*(a^3*e^(10*I*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c)
+ 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*
x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 105*(a^3*e^(10*I*d*x + 10*I*c)
 + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d
*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sec^3(c+dx)\,dx+\int\left(-3\tan(c+dx)\sec^3(c+dx)\right)\,dx+\int\tan^3(c+dx)\sec^3(c+dx)\,dx+\int\left(-3i\tan^2(c+dx)\sec^3(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
[Out] -I*a**3*(Integral(I*sec(c + d*x)**3, x) + Integral(-3*tan(c + d*x)*sec(c + 
d*x)**3, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**3, x))
```

Giac [A]
time = 0.84, size = 189, normalized size = 1.49

$$\frac{105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+960i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-400i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+390\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+320i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-136i\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/120*(105*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^3*log(tan(1/2*d*x + 1/
2*c) - 1) + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*I*a^3*tan(1/2*d*x + 1/2*
c)^8 - 390*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*I*a^3*tan(1/2*d*x + 1/2*c)^6 -
400*I*a^3*tan(1/2*d*x + 1/2*c)^4 + 390*a^3*tan(1/2*d*x + 1/2*c)^3 + 320*I*a
^3*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 136*I*a^3)/(tan(1
/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 6.98, size = 228, normalized size = 1.80

$$\frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,6\mathrm{i}+\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,16\mathrm{i}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,20\mathrm{i}}{3}-\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{3}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a^3\,34\mathrm{i}}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^3,x)
[Out] (7*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^3*tan(c/2 + (d*x)/2)^4*20i)/3
 - (13*a^3*tan(c/2 + (d*x)/2)^3)/2 - (a^3*tan(c/2 + (d*x)/2)^2*16i)/3 - a^3
*tan(c/2 + (d*x)/2)^6*16i + (13*a^3*tan(c/2 + (d*x)/2)^7)/2 + a^3*tan(c/2 +
 (d*x)/2)^8*6i - (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*34i)/15 + (a^3*tan(c/2
 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*t
an(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ia^3\sec(c+dx)}{2d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan\right.}{6d}$$

```
[Out] 5/2*a^3*arctanh(sin(d*x+c))/d+5/2*I*a^3*sec(d*x+c)/d+1/3*I*a*sec(d*x+c)*(a+
I*a*tan(d*x+c))^2/d+5/6*I*sec(d*x+c)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3579, 3567, 3855}

$$\frac{5ia^3\sec(c+dx)}{2d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (((5*I)/2)*a^3*Sec[c + d*x])/d + ((I/
3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (((5*I)/6)*Sec[c + d*x]*(a^
3 + I*a^3*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{1}{3}(5a)\int \sec(c+dx)(a+ia$$
$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan\right.}{6d}$$
$$= \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c}{}$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+}{3d}$$

Mathematica [A]
time = 0.45, size = 93, normalized size = 0.94

$$\frac{a^3(\cos(3dx)+i\sin(3dx))\left(60\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^3(c+dx)(20+24\cos(2(c+dx))+9i\sin(2(c+dx)))\right)}{12d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]
+ I*Sec[c + d*x]^3*(20 + 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(1
2*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.15, size = 147, normalized size = 1.48

method	result
risch	$\frac{ia^3\left(33\,e^{5i(dx+c)}+40\,e^{3i(dx+c)}+15\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{5a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{5a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)-3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)-3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*
(2+sin(d*x+c)^2)*cos(d*x+c))-3*a^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d
*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*I*a^3/cos(d*x+c)+a^3*ln(sec(d*x+c)+t
an(d*x+c)))
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.10

$$\frac{9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{36i\,a^3}{\cos(dx+c)}+\frac{4i\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 12*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*I*a^3
/cos(d*x + c) + 4*I*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(81) = 162.
time = 0.37, size = 202, normalized size = 2.04

$$\frac{66i\,a^3e^{(5i\,dx+5i\,c)}+80i\,a^3e^{(3i\,dx+3i\,c)}+30i\,a^3e^{(i\,dx+i\,c)}+15\left(a^3e^{(6i\,dx+6i\,c)}+3\,a^3e^{(4i\,dx+4i\,c)}+3\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^3e^{(6i\,dx+6i\,c)}+3\,a^3e^{(4i\,dx+4i\,c)}+3\,a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(66*I*a^3*e^(5*I*d*x + 5*I*c) + 80*I*a^3*e^(3*I*d*x + 3*I*c) + 30*I*a^3
*e^(I*d*x + I*c) + 15*(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c)
+ 3*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 15*(a^3*e^(6*
I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*
c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sec\left(c+dx\right)dx+\int\left(-3\tan\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx+\int\tan^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int\left(-3i\tan^2\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x), x) + Integral(-3*tan(c + d*x)*sec(c + d*x
), x) + Integral(tan(c + d*x)**3*sec(c + d*x), x) + Integral(-3*I*tan(c + d
*x)**2*sec(c + d*x), x))
```

Giac [A]
time = 0.66, size = 125, normalized size = 1.26

$$\frac{15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-48i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+22i\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^3*log(tan(1/2*d*x + 1/2*c)
 - 1) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 + 18*I*a^3*tan(1/2*d*x + 1/2*c)^4 -
 48*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) + 22*I*a^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 5.23, size = 136, normalized size = 1.37

$$\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}-a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}-3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3\,22\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x),x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^4*6i - a^3*ta
n(c/2 + (d*x)/2)^2*16i + 3*a^3*tan(c/2 + (d*x)/2)^5 + (a^3*22i)/3 - 3*a^3*t
an(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 - 1))
```

3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=61

$$-\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

```
[Out] -3*a^3*arctanh(sin(d*x+c))/d-3*I*a^3*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a+I*a*t
an(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3577, 3567, 3855}

$$-\frac{3ia^3\sec(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Sin[c + d*x]])/d - ((3*I)*a^3*Sec[c + d*x])/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d} - (3a^2)\int \sec(c+dx)(a+$$
$$= -\frac{3ia^3\sec(c+dx)}{d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d} - (3a^3$$
$$= -\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{3ia^3\sec(c+dx)}{d} - \frac{2ia\cos(c+dx)(a$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 123 vs. $2(61) = 122$.
time = 0.57, size = 123, normalized size = 2.02

$$\frac{a^3\cos^2(c+dx)\left(6\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(3c)+\sin(3c))+(-\cos(2c-dx)+i\sin(2c-dx))(5\cos(c+dx)-i\sin(c+dx))\right)(-i+\tan(c+dx))^3}{d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]^2*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*(
I*Cos[3*c] + Sin[3*c]) + (-Cos[2*c - d*x] + I*Sin[2*c - d*x])*(5*Cos[c + d*
x] - I*Sin[c + d*x]))*(-I + Tan[c + d*x])^3)/(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.21, size = 97, normalized size = 1.59

method	result
risch	$-\frac{4ia^3 e^{i(dx+c)}}{d} - \frac{2ie^{i(dx+c)}a^3}{d\left(e^{2i(dx+c)}+1\right)} + \frac{3a^3\ln\left(e^{i(dx+c)}-i\right)}{d} - \frac{3a^3\ln\left(e^{i(dx+c)}+i\right)}{d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)-3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3ia^3\cos(dx+c)+\sin(}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)-3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3ia^3\cos(dx+c)+\sin(}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))-3*a^3*(-s
in(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-3*I*a^3*cos(d*x+c)+sin(d*x+c)*a^3)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.34

$$-\frac{2i\,a^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))+6i\,a^3\cos(dx+c)-2\,a^3\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*a^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a^3*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 6*I*a^3*cos(d*x + c) - 2*a^3*
sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 107, normalized size = 1.75

$$\frac{-4i\,a^3e^{(3i\,dx+3i\,c)} - 6i\,a^3e^{(i\,dx+i\,c)} - 3\left(a^3e^{(2i\,dx+2i\,c)} + a^3\right)\log\left(e^{(i\,dx+i\,c)} + i\right) + 3\left(a^3e^{(2i\,dx+2i\,c)} + a^3\right)\log\left(e^{(i\,dx+i\,c)} - i\right)}{de^{(2i\,dx+2i\,c)} + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-4*I*a^3*e^(3*I*d*x + 3*I*c) - 6*I*a^3*e^(I*d*x + I*c) - 3*(a^3*e^(2*I*d*x
 + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) + 3*(a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.20, size = 107, normalized size = 1.75

$$-\frac{2ia^3e^{ic}e^{idx}}{de^{2ic}e^{2idx} + d} + \frac{3a^3\left(\log\left(e^{idx} - ie^{-ic}\right) - \log\left(e^{idx} + ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{4ia^3e^{ic}e^{idx}}{d} & \text{for}\ d \neq 0 \\ 4a^3xe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -2*I*a**3*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 3*a**3*(log
(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise((
-4*I*a**3*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (4*a**3*x*exp(I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(55) = 110.
time = 0.78, size = 234, normalized size = 3.84

$$\frac{63\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)} + 1\right) - 33\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)} - 1\right) - 63\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)} + 1\right) + 33\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)} - 1\right) - 128i\,a^3e^{(3i\,dx+3i\,c)} - 192i\,a^3e^{(i\,dx+i\,c)} + 63\,a^3\log\left(i\,e^{(i\,dx+i\,c)} + 1\right) - 33\,a^3\log\left(i\,e^{(i\,dx+i\,c)} - 1\right) - 63\,a^3\log\left(-i\,e^{(i\,dx+i\,c)} + 1\right) + 33\,a^3\log\left(-i\,e^{(i\,dx+i\,c)} - 1\right)}{32\left(de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/32*(63*a^3*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*e^(2*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*e^(2*I*d*x + 2*I*c)*log(-
I*e^(I*d*x + I*c) + 1) + 33*a^3*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 128*I*a^3*e^(3*I*d*x + 3*I*c) - 192*I*a^3*e^(I*d*x + I*c) + 63*a^3*l
```

```
og(I*e^(I*d*x + I*c) + 1) - 33*a^3*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*log(
-I*e^(I*d*x + I*c) + 1) + 33*a^3*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(2*I*d*x
 + 2*I*c) + d)
```

Mupad [B]
time = 3.64, size = 102, normalized size = 1.67

$$-\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2\mathrm{i}-10\,a^3}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] - (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (8*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^
3 + a^3*tan(c/2 + (d*x)/2)*2i)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^
2*1i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=32

$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[Out] -1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3569\}$

$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Mathematica [A]
time = 0.11, size = 31, normalized size = 0.97

$$-\frac{ia^3(\cos(c+dx)+i\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*a^3*(Cos[c + d*x] + I*Sin[c + d*x])^3)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.19, size = 76, normalized size = 2.38

method	result	size
risch	$-\frac{i a^{3} {\mathrm e}^{3 i \left(d x +c \right)}}{3 d}$	19
derivativedivides	$\frac{\frac{i a^{3} \left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)}{3}-a^{3} \left(\sin^{3}\left(d x +c \right)\right)-i a^{3} \left(\cos^{3}\left(d x +c \right)\right)+\frac{a^{3} \left(\cos^{2}\left(d x +c \right)+2\right) \sin \left(d x +c \right)}{3}}{d}$	76
default	$\frac{\frac{i a^{3} \left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)}{3}-a^{3} \left(\sin^{3}\left(d x +c \right)\right)-i a^{3} \left(\cos^{3}\left(d x +c \right)\right)+\frac{a^{3} \left(\cos^{2}\left(d x +c \right)+2\right) \sin \left(d x +c \right)}{3}}{d}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*I*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)-a^3*sin(d*x+c)^3-I*a^3*cos(d*x+c
)^3+1/3*a^3*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.
time = 0.27, size = 75, normalized size = 2.34

$$-\frac{3i\,a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\sin\left(dx+c\right)^{3}+i\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{3}+\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/3*(3*I*a^3*cos(d*x + c)^3 + 3*a^3*sin(d*x + c)^3 + I*(cos(d*x + c)^3 - 3
*cos(d*x + c))*a^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3)/d
```

Fricas [A]
time = 0.36, size = 17, normalized size = 0.53

$$-\frac{i\,a^{3}e^{\left(3i\,dx+3i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A]
time = 0.14, size = 36, normalized size = 1.12

$$\begin{cases} -\frac{ia^{3}e^{3ic}e^{3idx}}{3d} & \text{for}\: d \neq 0 \\ a^{3}xe^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3*
I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(26) = 52$.
time = 0.90, size = 901, normalized size = 28.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/384*(108*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^
(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^(2*I*d*x - 2*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 648*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) + 1)
 + 108*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 111*a^3*e^(8*I*d*x + 4*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 444*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 444*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 666
*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 111*a^3*e^(-4*I*c)*log(I*e^(I
*d*x + I*c) - 1) - 108*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 432*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 432*a^3*e^(2*I*
d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 648*a^3*e^(4*I*d*x)*log(-I*e^(I*
d*x + I*c) + 1) - 108*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 111*a^3*
e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(6*I*d*x + 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 666*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 111*a^3*e
^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 3*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) + 12*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 + 12*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18*a^3*e^(4*I*d
*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^3*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 3*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(6*
I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(2*I*d*x - 2*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 18*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)
) - 3*a^3*e^(-4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 128*I*a^3*e^(11*I*d*x +
 7*I*c) + 512*I*a^3*e^(9*I*d*x + 5*I*c) + 768*I*a^3*e^(7*I*d*x + 3*I*c) + 5
12*I*a^3*e^(5*I*d*x + I*c) + 128*I*a^3*e^(3*I*d*x - I*c))/(d*e^(8*I*d*x + 4
*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x)
 + d*e^(-4*I*c))
```

Mupad [B]

time = 3.33, size = 66, normalized size = 2.06

$$-\frac{2\,a^3\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
+ (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{ia^3\cos^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

```
[Out] -1/15*I*a^3*cos(d*x+c)^3/d+1/5*a^3*sin(d*x+c)/d-1/15*a^3*sin(d*x+c)^3/d-2/5
*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3577, 3567, 2713}

$$-\frac{a^3\sin^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/15*I)*a^3*Cos[c + d*x]^3)/d + (a^3*Sin[c + d*x])/(5*d) - (a^3*Sin[c +
d*x]^3)/(15*d) - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}a^2\int \cos^3(c+dx)(a$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}a$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} - \frac{a^3 S}{}$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} + \frac{a^3\sin(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{15d} - \frac{2ia\cos^5(}{}$$

Mathematica [A]
time = 0.33, size = 55, normalized size = 0.62

$$\frac{a^3(5+9\cos(2(c+dx))-6i\sin(2(c+dx)))(-i\cos(3(c+dx))+\sin(3(c+dx)))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(5 + 9*Cos[2*(c + d*x)] - (6*I)*Sin[2*(c + d*x)])*((-I)*Cos[3*(c + d*x
)] + Sin[3*(c + d*x)]))/(30*d)
```

Maple [A]
time = 0.21, size = 126, normalized size = 1.43

method	result
risch	$-\frac{ia^3 e^{5i(dx+c)}}{20d} - \frac{ia^3 e^{3i(dx+c)}}{6d} - \frac{ia^3 e^{i(dx+c)}}{4d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5} - \frac{2\left(\cos^3(dx+c)\right)}{15}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5} + \frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right) - \frac{3ia^3(}{}}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5} - \frac{2\left(\cos^3(dx+c)\right)}{15}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5} + \frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right) - \frac{3ia^3(}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)-3*a^3*(-1/5*
sin(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-3/5*I*a^3*cos(d*x
+c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.19

$$-\frac{9i a^3 \cos(dx+c)^5 + i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^3 - 3\left(3\sin(dx+c)^5 - 5\sin(dx+c)^3\right)a^3 - \left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^3}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*I*a^3*cos(d*x + c)^5 + I*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 - 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^3 - (3*sin(d*x + c)^5 - 10*sin
(d*x + c)^3 + 15*sin(d*x + c))*a^3)/d
```

Fricas [A]
time = 0.36, size = 48, normalized size = 0.55

$$\frac{-3i a^3 e^{(5i dx+5i c)} - 10i a^3 e^{(3i dx+3i c)} - 15i a^3 e^{(i dx+i c)}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(-3*I*a^3*e^(5*I*d*x + 5*I*c) - 10*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a^
3*e^(I*d*x + I*c))/d
```

Sympy [A]
time = 0.23, size = 116, normalized size = 1.32

$$\begin{cases} \frac{-24ia^3d^2e^{5ic}e^{5idx} - 80ia^3d^2e^{3ic}e^{3idx} - 120ia^3d^2e^{ic}e^{idx}}{480d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^3e^{5ic}}{4} + \frac{a^3e^{3ic}}{2} + \frac{a^3e^{ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-24*I*a**3*d**2*exp(5*I*c)*exp(5*I*d*x) - 80*I*a**3*d**2*exp(3*
I*c)*exp(3*I*d*x) - 120*I*a**3*d**2*exp(I*c)*exp(I*d*x))/(480*d**3), Ne(d**
3, 0)), (x*(a**3*exp(5*I*c)/4 + a**3*exp(3*I*c)/2 + a**3*exp(I*c)/4), True)
)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(74) = 148.
time = 1.00, size = 929, normalized size = 10.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/7680*(1785*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*
e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*e^(2*I*d*x - 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 10710*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c)
 + 1) + 1785*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1530*a^3*e^(8*I*d*
x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(6*I*d*x + 2*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 6120*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 9180*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1530*a^3*e^(-4*I*c)
*log(I*e^(I*d*x + I*c) - 1) - 1785*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 7140*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 71
40*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10710*a^3*e^(4*I*d
*x)*log(-I*e^(I*d*x + I*c) + 1) - 1785*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 1530*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^
3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^3*e^(2*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9180*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x +
I*c) - 1) - 1530*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 255*a^3*e^(8*
I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(6*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) - 1530*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 255*a^3*e^(
-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 1020*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 1020*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1530*a^3
*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 384*I*a^3*e^(13*I*d*x + 9*I*c) - 2816*I*a^3*e^(11*I*d*x
+ 7*I*c) - 9344*I*a^3*e^(9*I*d*x + 5*I*c) - 16896*I*a^3*e^(7*I*d*x + 3*I*c)
 - 17024*I*a^3*e^(5*I*d*x + I*c) - 8960*I*a^3*e^(3*I*d*x - I*c) - 1920*I*a^
3*e^(I*d*x - 3*I*c))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d
*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B]
time = 3.57, size = 130, normalized size = 1.48

$$\frac{2\,a^3\,\left(15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,30\mathrm{i}-40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,20\mathrm{i}+7\right)}{15\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5\mathrm{i}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (2*a^3*(tan(c/2 + (d*x)/2)^3*30i - 40*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)*20i + 15*tan(c/2 + (d*x)/2)^4 + 7))/(15*d*(5*tan(c/2 + (d*x)/2) - tan(
c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i +
tan(c/2 + (d*x)/2)^5 + 1i))
```

3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=106

$$-\frac{3ia^3\cos^5(c+dx)}{35d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+d}{7d}$$

```
[Out] -3/35*I*a^3*cos(d*x+c)^5/d+3/7*a^3*sin(d*x+c)/d-2/7*a^3*sin(d*x+c)^3/d+3/35
*a^3*sin(d*x+c)^5/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3577, 3567, 2713}

$$\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{3ia^3\cos^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/35)*a^3*Cos[c + d*x]^5)/d + (3*a^3*Sin[c + d*x])/(7*d) - (2*a^3*Si
n[c + d*x]^3)/(7*d) + (3*a^3*Sin[c + d*x]^5)/(35*d) - (((2*I)/7)*a*Cos[c +
d*x]^7*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
```

```
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}(3a^2)\int \cos^5(c+dx)$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} - (3$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} + \frac{3a^3\sin(c+dx)}{7d} - \frac{2a^3\sin^3(c+dx)}{7d} + \frac{3a^3 s}{}$$

Mathematica [A]
time = 0.55, size = 77, normalized size = 0.73

$$\frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))(35+84\cos(2(c+dx))-15\cos(4(c+dx))-56i\sin(2(c+dx))+20i\sin(4(c+dx)))}{280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35 + 84*Cos[2*(c + d*x)] -
 15*Cos[4*(c + d*x)] - (56*I)*Sin[2*(c + d*x)] + (20*I)*Sin[4*(c + d*x)]))/
(280*d)
```

Maple [A]
time = 0.22, size = 146, normalized size = 1.38

method	result
risch	$-\frac{ia^3 e^{7i(dx+c)}}{112d} - \frac{ia^3 e^{5i(dx+c)}}{20d} - \frac{ia^3 e^{3i(dx+c)}}{8d} - \frac{3ia^3\cos(dx+c)}{16d} + \frac{5a^3\sin(dx+c)}{16d}$
derivativedivides	$\frac{-ia^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7} - \frac{2(\cos^5(dx+c))}{35}\right) - 3a^3\left(-\frac{\sin(dx+c)(\cos^6(dx+c))}{7} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)s}{35}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7} - \frac{2(\cos^5(dx+c))}{35}\right) - 3a^3\left(-\frac{\sin(dx+c)(\cos^6(dx+c))}{7} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)s}{35}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)-3*a^3*(-1/7*
sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)-3/7*I*a^3*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.16

$$-\frac{15i\,a^3\cos\left(dx+c\right)^7+i\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^3+\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3\right)a^3+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*I*a^3*cos(d*x + c)^7 + I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^
3 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3 + (5*si
n(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3
)/d
```

Fricas [A]
time = 0.37, size = 76, normalized size = 0.72

$$\frac{\left(-5i\,a^3e^{(8i\,dx+8i\,c)}-28i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-140i\,a^3e^{(2i\,dx+2i\,c)}+35i\,a^3\right)e^{(-i\,dx-i\,c)}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-5*I*a^3*e^(8*I*d*x + 8*I*c) - 28*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a
^3*e^(4*I*d*x + 4*I*c) - 140*I*a^3*e^(2*I*d*x + 2*I*c) + 35*I*a^3)*e^(-I*d*
x - I*c)/d
```

Sympy [A]
time = 0.35, size = 190, normalized size = 1.79

$$\begin{cases} \frac{\left(-10240ia^3d^4e^{8ic}e^{7idx}-57344ia^3d^4e^{6ic}e^{5idx}-143360ia^3d^4e^{4ic}e^{3idx}-286720ia^3d^4e^{2ic}e^{idx}+71680ia^3d^4e^{-idx}\right)e^{-ic}}{1146880d^5} & \text{for}\: d^5e^{ic}\neq 0 \\ \frac{x\left(a^3e^{8ic}+4a^3e^{6ic}+6a^3e^{4ic}+4a^3e^{2ic}+a^3\right)e^{-ic}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-10240*I*a**3*d**4*exp(8*I*c)*exp(7*I*d*x) - 57344*I*a**3*d**4*
exp(6*I*c)*exp(5*I*d*x) - 143360*I*a**3*d**4*exp(4*I*c)*exp(3*I*d*x) - 2867
20*I*a**3*d**4*exp(2*I*c)*exp(I*d*x) + 71680*I*a**3*d**4*exp(-I*d*x))*exp(-
I*c)/(1146880*d**5), Ne(d**5*exp(I*c), 0)), (x*(a**3*exp(8*I*c) + 4*a**3*ex
p(6*I*c) + 6*a**3*exp(4*I*c) + 4*a**3*exp(2*I*c) + a**3)*exp(-I*c)/16, True
))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(90) = 180$.
time = 0.99, size = 465, normalized size = 4.39

$$\frac{19635\,a^{3}e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+39270\,a^{3}e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^{3}e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^{3}e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+39270\,a^{3}e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+19635\,a^{3}e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-19635\,a^{3}e^{(5i\,dx+3i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-39270\,a^{3}e^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-19635\,a^{3}e^{(i\,dx-i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-19635\,a^{3}e^{(5i\,dx+3i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-39270\,a^{3}e^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-19635\,a^{3}e^{(i\,dx-i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-640i\,a^{3}e^{(12i\,dx+10i\,c)}-4864i\,a^{3}e^{(10i\,dx+8i\,c)}-16768i\,a^{3}e^{(8i\,dx+6i\,c)}-39424i\,a^{3}e^{(6i\,dx+4i\,c)}-40320i\,a^{3}e^{(4i\,dx+2i\,c)}-8960i\,a^{3}e^{(2i\,dx)}+4480i\,a^{3}e^{(-2i\,c)}}{71680\left(de^{(5i\,dx+3i\,c)}+2\,de^{(3i\,dx+i\,c)}+de^{(i\,dx-i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 39270*a
^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 39270*a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1963
5*a^3*e^(I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(5*I*d*x + 3
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 39270*a^3*e^(3*I*d*x + I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) -
19635*a^3*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 39270*a^3*e^(3*
I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I
*e^(I*d*x + I*c) - 1) - 640*I*a^3*e^(12*I*d*x + 10*I*c) - 4864*I*a^3*e^(10*
I*d*x + 8*I*c) - 16768*I*a^3*e^(8*I*d*x + 6*I*c) - 39424*I*a^3*e^(6*I*d*x +
 4*I*c) - 40320*I*a^3*e^(4*I*d*x + 2*I*c) - 8960*I*a^3*e^(2*I*d*x) + 4480*I
*a^3*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x + I*c) + d*e^(I*d*
x - I*c))
```

Mupad [B]
time = 4.54, size = 134, normalized size = 1.26

$$-\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{17\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{17\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}+\frac{31\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}-\frac{5\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}+\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,35{}\mathrm{i}}{8}-\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,35{}\mathrm{i}}{8}+\frac{\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,119{}\mathrm{i}}{8}-\frac{\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,15{}\mathrm{i}}{8}\right)}{35\,d\,\left(\cos\left(3\,c+3\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)*((cos(c/2 + (d*x)/2)*35i)/8 - (cos((3*c)/2 + (3*
d*x)/2)*35i)/8 + (cos((5*c)/2 + (5*d*x)/2)*119i)/8 - (cos((7*c)/2 + (7*d*x)
/2)*15i)/8 + (17*sin(c/2 + (d*x)/2))/2 - (17*sin((3*c)/2 + (3*d*x)/2))/2 +
(31*sin((5*c)/2 + (5*d*x)/2))/2 - (5*sin((7*c)/2 + (7*d*x)/2))/2))/(35*d*(c
os(3*c + 3*d*x) - sin(3*c + 3*d*x)*1i))
```

3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=124

$$-\frac{5ia^3\cos^7(c+dx)}{63d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(}{}$$

```
[Out] -5/63*I*a^3*cos(d*x+c)^7/d+5/9*a^3*sin(d*x+c)/d-5/9*a^3*sin(d*x+c)^3/d+1/3*
a^3*sin(d*x+c)^5/d-5/63*a^3*sin(d*x+c)^7/d-2/9*I*a*cos(d*x+c)^9*(a+I*a*tan(
d*x+c))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3577, 3567, 2713}

$$-\frac{5a^3\sin^7(c+dx)}{63d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-5*I)/63)*a^3*Cos[c + d*x]^7)/d + (5*a^3*Sin[c + d*x])/(9*d) - (5*a^3*Si
n[c + d*x]^3)/(9*d) + (a^3*Sin[c + d*x]^5)/(3*d) - (5*a^3*Sin[c + d*x]^7)/(
63*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
```

```
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}+\frac{1}{9}(5a^2)\int \cos^7(c+dx$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}+\frac{1}{9}$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}-\frac{(5}{}$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{a^3\sin}{}$$

Mathematica [A]
time = 0.62, size = 116, normalized size = 0.94

$$\frac{a^3(210+567\cos(2(c+dx))-162\cos(4(c+dx))-7\cos(6(c+dx))-378i\sin(2(c+dx))+216i\sin(4(c+dx))+14i\sin(6(c+dx)))(-i\cos(3(c+2dx))+\sin(3(c+2dx)))}{2016d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
[Out] (a^3*(210 + 567*Cos[2*(c + d*x)] - 162*Cos[4*(c + d*x)] - 7*Cos[6*(c + d*x)
] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14*I)*Sin[6*(c +
 d*x)])*((-I)*Cos[3*(c + 2*d*x)] + Sin[3*(c + 2*d*x)]))/(2016*d*(Cos[d*x] +
 I*Sin[d*x])^3)
```

Maple [A]
time = 0.24, size = 166, normalized size = 1.34

method	result
risch	$-\frac{ia^3 e^{9i(dx+c)}}{576d}-\frac{3ia^3 e^{7i(dx+c)}}{224d}-\frac{3ia^3 e^{5i(dx+c)}}{64d}-\frac{9ia^3\cos(dx+c)}{64d}+\frac{21a^3\sin(dx+c)}{64d}-\frac{19ia^3\cos(3dx+3c)}{192d}+$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\right.}{63}\right.}{}$
default	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\right.}{63}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-I*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-3*a^3*(-1/9*
sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*
x+c)^2)*sin(d*x+c))-1/3*I*a^3*cos(d*x+c)^9+1/9*a^3*(128/35+cos(d*x+c)^8+8/7
*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.17

$$-\frac{105i\,a^3\cos\left(dx+c\right)^9+5i\left(7\cos\left(dx+c\right)^9-9\cos\left(dx+c\right)^7\right)a^3-3\left(35\sin\left(dx+c\right)^9-135\sin\left(dx+c\right)^7+189\sin\left(dx+c\right)^5-105\sin\left(dx+c\right)^3\right)a^3-\left(35\sin\left(dx+c\right)^9-180\sin\left(dx+c\right)^7+378\sin\left(dx+c\right)^5-420\sin\left(dx+c\right)^3+315\sin\left(dx+c\right)\right)a^3}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*(105*I*a^3*cos(d*x + c)^9 + 5*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7
)*a^3 - 3*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 10
5*sin(d*x + c)^3)*a^3 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d
*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3)/d
```

Fricas [A]
time = 0.38, size = 104, normalized size = 0.84

$$\frac{\left(-7i\,a^3e^{(12i\,dx+12i\,c)}-54i\,a^3e^{(10i\,dx+10i\,c)}-189i\,a^3e^{(8i\,dx+8i\,c)}-420i\,a^3e^{(6i\,dx+6i\,c)}-945i\,a^3e^{(4i\,dx+4i\,c)}+378i\,a^3e^{(2i\,dx+2i\,c)}+21i\,a^3\right)e^{(-3i\,dx-3i\,c)}}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4032*(-7*I*a^3*e^(12*I*d*x + 12*I*c) - 54*I*a^3*e^(10*I*d*x + 10*I*c) - 1
89*I*a^3*e^(8*I*d*x + 8*I*c) - 420*I*a^3*e^(6*I*d*x + 6*I*c) - 945*I*a^3*e^
(4*I*d*x + 4*I*c) + 378*I*a^3*e^(2*I*d*x + 2*I*c) + 21*I*a^3)*e^(-3*I*d*x -
 3*I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(112) = 224.
time = 0.46, size = 275, normalized size = 2.22

$$\begin{cases}\frac{\left(-270582939648ia^3d^6e^{13ic}e^{9idx}-2087354105856ia^3d^6e^{11ic}e^{7idx}-7305739370496ia^3d^6e^{9ic}e^{5idx}-16234976378880ia^3d^6e^{7ic}e^{3idx}-36528696852480ia^3d^6e^{5ic}e^{idx}+14611478740992ia^3d^6e^{3ic}e^{-idx}+811748818944ia^3d^6e^{ic}e^{-3idx}\right)e^{-4ic}}{155855773237248d^7} & \text{for}\ d^7e^{4ic}\neq 0\\ \frac{x\left(a^3e^{12ic}+6a^3e^{10ic}+15a^3e^{8ic}+20a^3e^{6ic}+15a^3e^{4ic}+6a^3e^{2ic}+a^3\right)e^{-3ic}}{64} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-270582939648*I*a**3*d**6*exp(13*I*c)*exp(9*I*d*x) - 2087354105
856*I*a**3*d**6*exp(11*I*c)*exp(7*I*d*x) - 7305739370496*I*a**3*d**6*exp(9*
I*c)*exp(5*I*d*x) - 16234976378880*I*a**3*d**6*exp(7*I*c)*exp(3*I*d*x) - 36
528696852480*I*a**3*d**6*exp(5*I*c)*exp(I*d*x) + 14611478740992*I*a**3*d**6
```

```
*exp(3*I*c)*exp(-I*d*x) + 811748818944*I*a**3*d**6*exp(I*c)*exp(-3*I*d*x))*
exp(-4*I*c)/(155855773237248*d**7), Ne(d**7*exp(4*I*c), 0)), (x*(a**3*exp(1
2*I*c) + 6*a**3*exp(10*I*c) + 15*a**3*exp(8*I*c) + 20*a**3*exp(6*I*c) + 15*
a**3*exp(4*I*c) + 6*a**3*exp(2*I*c) + a**3)*exp(-3*I*c)/64, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1039 vs. 2(106) = 212.
time = 0.87, size = 1039, normalized size = 8.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/516096*(119511*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4780
44*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 717066*a^3*e^(7*I*d
*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 478044*a^3*e^(5*I*d*x - I*c)*log(I*e
^(I*d*x + I*c) + 1) + 119511*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 128898*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 773388*a^3*e^(7*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*a^3*e^(5*I*d*x - I*c)*log(I*e^(I
*d*x + I*c) - 1) + 128898*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1
) - 119511*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^
3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 717066*a^3*e^(7*I*d*x +
 I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^3*e^(5*I*d*x - I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 119511*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) +
 1) - 128898*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 773388*a^3*e^(7*I*d*x
 + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*a^3*e^(5*I*d*x - I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 128898*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) + 9387*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a
^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 56322*a^3*e^(7*I*d*x +
 I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a^3*e^(5*I*d*x - I*c)*log(I*e^(I*
d*x) + e^(-I*c)) + 9387*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 - 9387*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e
^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 56322*a^3*e^(7*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e^(5*I*d*x - I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) - 9387*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
 - 896*I*a^3*e^(20*I*d*x + 14*I*c) - 10496*I*a^3*e^(18*I*d*x + 12*I*c) - 57
216*I*a^3*e^(16*I*d*x + 10*I*c) - 195584*I*a^3*e^(14*I*d*x + 8*I*c) - 50969
6*I*a^3*e^(12*I*d*x + 6*I*c) - 861696*I*a^3*e^(10*I*d*x + 4*I*c) - 768768*I
*a^3*e^(8*I*d*x + 2*I*c) + 88704*I*a^3*e^(4*I*d*x - 2*I*c) + 59136*I*a^3*e^
(2*I*d*x - 4*I*c) - 236544*I*a^3*e^(6*I*d*x) + 2688*I*a^3*e^(-6*I*c))/(d*e^
(11*I*d*x + 5*I*c) + 4*d*e^(9*I*d*x + 3*I*c) + 6*d*e^(7*I*d*x + I*c) + 4*d*
e^(5*I*d*x - I*c) + d*e^(3*I*d*x - 3*I*c))
```

Mupad [B]
time = 4.68, size = 330, normalized size = 2.66

$$\frac{2\,a^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2048\,a^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9}-\frac{1024\,a^3\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^8}-\frac{4\,a^3\,\left(14\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-39{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}+\frac{8\,a^3\,\left(43\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-97{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}-\frac{16\,a^3\,\left(188\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-357{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}+\frac{128\,a^3\,\left(263\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-333{}\mathrm{i}\right)}{21\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}-\frac{64\,a^3\,\left(1598\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2289{}\mathrm{i}\right)}{63\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}+\frac{32\,a^3\,\left(2041\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3339{}\mathrm{i}\right)}{63\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (2*a^3*(tan(c/2 + (d*x)/2) - 3i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2048*a^
3*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (1024*a^3
*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) - (4*a^3*(
14*tan(c/2 + (d*x)/2) - 39i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) + (8*a^3*(
43*tan(c/2 + (d*x)/2) - 97i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (16*a^3*
(188*tan(c/2 + (d*x)/2) - 357i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (128*
a^3*(263*tan(c/2 + (d*x)/2) - 333i))/(21*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) -
(64*a^3*(1598*tan(c/2 + (d*x)/2) - 2289i))/(63*d*(tan(c/2 + (d*x)/2)^2 + 1)
^6) + (32*a^3*(2041*tan(c/2 + (d*x)/2) - 3339i))/(63*d*(tan(c/2 + (d*x)/2)^
2 + 1)^5)
```

3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=163

$$\frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+$$

```
[Out] 21/16*a^4*arctanh(sin(d*x+c))/d+7/8*I*a^4*sec(d*x+c)^3/d+21/16*a^4*sec(d*x+
c)*tan(d*x+c)/d+1/6*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+3/10*I*sec(d*x+
c)^3*(a^2+I*a^2*tan(d*x+c))^2/d+21/40*I*sec(d*x+c)^3*(a^4+I*a^4*tan(d*x+c))
/d
```

Rubi [A]
time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3579, 3567, 3853, 3855}

$$\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}+\frac{21a^4\tan(c+dx)\sec(c+dx)}{16d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (21*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/8)*a^4*Sec[c + d*x]^3)/d +
(21*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((I/6)*a*Sec[c + d*x]^3*(a + I*
a*Tan[c + d*x])^3)/d + (((3*I)/10)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]
)^2)/d + (((21*I)/40)*Sec[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx &= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{1}{2}(3a)\int \sec^3(c+dx)(a+i\\ &= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan\right.}{10d}\\ &= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan\right.}{10d}\\ &= \frac{7ia^4\sec^3(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{3i\sec^3(}{}\\ &= \frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{ia\sec^3(c+dx)}{}\\ &= \frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\sec(c+dx)\,\mathrm{t}}{16d}\end{aligned}$$

Mathematica [A]
time = 1.30, size = 171, normalized size = 1.05

$$-\frac{a^4\sec^2(c+dx)(\cos(4c)-i\sin(4c))\left(-4608i\cos(c+dx)+5040\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+5(-512i\cos(3(c+dx))+90\sin(c+dx)+155\sin(3(c+dx))-63\sin(5(c+dx)))\right)(-i+\tan(c+dx))^4}{3840d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/3840*(a^4*Sec[c + d*x]^2*(Cos[4*c] - I*Sin[4*c])*((-4608*I)*Cos[c + d*x]
 + 5040*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-512*I)*Cos[3*(c + d*x)] + 90*Sin[c
 + d*x] + 155*Sin[3*(c + d*x)] - 63*Sin[5*(c + d*x)]))*(-I + Tan[c + d*x])^
4)/(d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(144) = 288.
time = 0.26, size = 293, normalized size = 1.80

method	result
risch	$-\frac{i a^4 \left(315\, e^{11 i (dx+c)}-3335\, e^{9 i (dx+c)}-5058\, e^{7 i (dx+c)}-4158\, e^{5 i (dx+c)}-1785\, e^{3 i (dx+c)}-315\, e^{i (dx+c)}\right)}{120 d \left(e^{2 i (dx+c)}+1\right)^6}-\frac{21 a^4 \ln(e^{i(dx}}{16d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin^4(dx+c}{5\cos(dx+c)}\right.}{}$
default	$\frac{a^4\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin^4(dx+c}{5\cos(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48
*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*
x+c)+tan(d*x+c)))-4*I*a^4*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/
cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))
-6*a^4*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin
(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+4/3*I*a^4/cos(d*x+c)^3+a^4*(1/2*sec(
d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 246, normalized size = 1.51

$$-\frac{5\,a^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+180\,a^4\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+120\,a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{640i\,a^4}{\cos(dx+c)^3}-\frac{128i\left(5\,\cos(dx+c)^2-3\right)a^4}{\cos(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 180*a^4*(2*(sin(d*x + c)^3 + sin(d*x +
c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*I*a^4/cos(d*x + c)^3 - 128*I*(
5*cos(d*x + c)^2 - 3)*a^4/cos(d*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(137) = 274$.
time = 0.37, size = 364, normalized size = 2.23

$$\frac{-630i\,a^4e^{(11i\,dx+11i\,c)}+6670i\,a^4e^{(9i\,dx+9i\,c)}+10116i\,a^4e^{(7i\,dx+7i\,c)}+8316i\,a^4e^{(5i\,dx+5i\,c)}+3570i\,a^4e^{(3i\,dx+3i\,c)}+630i\,a^4e^{(i\,dx+i\,c)}+315\left(a^4e^{(12i\,dx+12i\,c)}+6\,a^4e^{(10i\,dx+10i\,c)}+15\,a^4e^{(8i\,dx+8i\,c)}+20\,a^4e^{(6i\,dx+6i\,c)}+15\,a^4e^{(4i\,dx+4i\,c)}+6\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(a^4e^{(12i\,dx+12i\,c)}+6\,a^4e^{(10i\,dx+10i\,c)}+15\,a^4e^{(8i\,dx+8i\,c)}+20\,a^4e^{(6i\,dx+6i\,c)}+15\,a^4e^{(4i\,dx+4i\,c)}+6\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{240\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(-630*I*a^4*e^(11*I*d*x + 11*I*c) + 6670*I*a^4*e^(9*I*d*x + 9*I*c) +
10116*I*a^4*e^(7*I*d*x + 7*I*c) + 8316*I*a^4*e^(5*I*d*x + 5*I*c) + 3570*I*a
^4*e^(3*I*d*x + 3*I*c) + 630*I*a^4*e^(I*d*x + I*c) + 315*(a^4*e^(12*I*d*x +
 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^
4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I
*c) + a^4)*log(e^(I*d*x + I*c) + I) - 315*(a^4*e^(12*I*d*x + 12*I*c) + 6*a^
4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I*d*x +
6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(
e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c)
+ 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4
*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\left(-6\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)\right)\,dx+\int\tan^4\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int 4i\tan\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\left(-4i\tan^3\left(c+dx\right)\sec^3\left(c+dx\right)\right)\,dx+\int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(tan(c + d*
x)**4*sec(c + d*x)**3, x) + Integral(4*I*tan(c + d*x)*sec(c + d*x)**3, x) +
 Integral(-4*I*tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)*
*3, x))
```

Giac [A]
time = 1.20, size = 237, normalized size = 1.45

$$\frac{315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+960i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1175\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-4800i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1890\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4480i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1890\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1920i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1175\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1728i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-448i\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(315*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^4*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(75*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*I*a^4*tan(1/2*d*x + 1/2
*c)^10 + 1175*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*I*a^4*tan(1/2*d*x + 1/2*c)^
8 - 1890*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*I*a^4*tan(1/2*d*x + 1/2*c)^6 - 1
890*a^4*tan(1/2*d*x + 1/2*c)^5 - 1920*I*a^4*tan(1/2*d*x + 1/2*c)^4 + 1175*a
^4*tan(1/2*d*x + 1/2*c)^3 + 1728*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 75*a^4*tan(
1/2*d*x + 1/2*c) - 448*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 7.16, size = 290, normalized size = 1.78

$$\frac{21\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\frac{5\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}+a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,8{}\mathrm{i}+\frac{235\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{24}-a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,40{}\mathrm{i}-\frac{63\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+\frac{a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,112{}\mathrm{i}}{3}-\frac{63\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}-a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,16{}\mathrm{i}+\frac{235\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24}+\frac{a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,72{}\mathrm{i}}{5}+\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{a^4\,56{}\mathrm{i}}{15}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x)^3,x)
```

```
[Out] (21*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^4*tan(c/2 + (d*x)/2)^2*72i)/
5 + (235*a^4*tan(c/2 + (d*x)/2)^3)/24 - a^4*tan(c/2 + (d*x)/2)^4*16i - (63*
a^4*tan(c/2 + (d*x)/2)^5)/4 + (a^4*tan(c/2 + (d*x)/2)^6*112i)/3 - (63*a^4*t
an(c/2 + (d*x)/2)^7)/4 - a^4*tan(c/2 + (d*x)/2)^8*40i + (235*a^4*tan(c/2 +
(d*x)/2)^9)/24 + a^4*tan(c/2 + (d*x)/2)^10*8i + (5*a^4*tan(c/2 + (d*x)/2)^1
1)/8 - (a^4*56i)/15 + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/
2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x
)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=133

$$\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{35ia^4\sec(c+dx)}{8d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\,\mathrm{t}\right.}{12d}$$

```
[Out] 35/8*a^4*arctanh(sin(d*x+c))/d+35/8*I*a^4*sec(d*x+c)/d+1/4*I*a*sec(d*x+c)*(
a+I*a*tan(d*x+c))^3/d+7/12*I*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+35/24*I*
sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]

time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {3579, 3567, 3855}

$$\frac{35ia^4\sec(c+dx)}{8d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (((35*I)/8)*a^4*Sec[c + d*x])/d + ((
I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (((7*I)/12)*Sec[c + d*x]*
(a^2 + I*a^2*Tan[c + d*x])^2)/d + (((35*I)/24)*Sec[c + d*x]*(a^4 + I*a^4*Ta
n[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{1}{4}(7a)\int \sec(c+dx)(a+ia$$

$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(\right.}{12d}$$

$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(\right.}{12d}$$

$$= \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c}{}$$

$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+}{4}$$

Mathematica [A]
time = 1.23, size = 237, normalized size = 1.78

$$-\frac{a^4\sec^4(c+dx)\left(-896i\cos(c+dx)+3\left(-128i\cos(3(c+dx))+105\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+35\cos(4(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+140\cos(2(c+dx))\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)-105\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-35\cos(4(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+42\sin(c+dx)+58\sin(3(c+dx))\right)\right)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/192*(a^4*Sec[c + d*x]^4*((-896*I)*Cos[c + d*x] + 3*((-128*I)*Cos[3*(c +
d*x)] + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
- 105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])
))/d
```

Maple [A]
time = 0.18, size = 222, normalized size = 1.67

method	result
risch	$\frac{ia^4\left(279\,e^{7i(dx+c)}+511\,e^{5i(dx+c)}+385\,e^{3i(dx+c)}+105\,e^{i(dx+c)}\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{35a^4\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{35a^4\ln\left(e^{i(dx+c)}+i\right)}{8d}$
derivativedivides	$a^4\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c}{3\cos(dx+c}\right.$
default	$a^4\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c}{3\cos(dx+c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*I*a^4*(1/3*sin(
d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(
d*x+c))-6*a^4*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+
c)+tan(d*x+c)))+4*I*a^4/cos(d*x+c)+a^4*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 180, normalized size = 1.35

$$\frac{3\,a^4\left(\frac{2\,\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+72\,a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+48\,a^4\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{192i\,a^4}{\cos(dx+c)}+\frac{64i\left(3\,\cos(dx+c)^2-1\right)a^4}{\cos(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 72*a
^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 48*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*I*a^4/cos(d*x
 + c) + 64*I*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(109) = 218.
time = 0.38, size = 256, normalized size = 1.92

$$\frac{558i\,a^4e^{(7i\,dx+7i\,c)}+1022i\,a^4e^{(5i\,dx+5i\,c)}+770i\,a^4e^{(3i\,dx+3i\,c)}+210i\,a^4e^{(i\,dx+i\,c)}+105\left(a^4e^{(8i\,dx+8i\,c)}+4\,a^4e^{(6i\,dx+6i\,c)}+6\,a^4e^{(4i\,dx+4i\,c)}+4\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(a^4e^{(8i\,dx+8i\,c)}+4\,a^4e^{(6i\,dx+6i\,c)}+6\,a^4e^{(4i\,dx+4i\,c)}+4\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{24\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(558*I*a^4*e^(7*I*d*x + 7*I*c) + 1022*I*a^4*e^(5*I*d*x + 5*I*c) + 770*
I*a^4*e^(3*I*d*x + 3*I*c) + 210*I*a^4*e^(I*d*x + I*c) + 105*(a^4*e^(8*I*d*x
 + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e
^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 105*(a^4*e^(8*I*d*x +
8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
+ d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\left(-6\tan^2\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx+\int\tan^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int4i\tan\left(c+dx\right)\sec\left(c+dx\right)dx+\int\left(-4i\tan^3\left(c+dx\right)\sec\left(c+dx\right)\right)\,dx+\int\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)*
*4*sec(c + d*x), x) + Integral(4*I*tan(c + d*x)*sec(c + d*x), x) + Integral
(-4*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [A]
time = 0.99, size = 173, normalized size = 1.30

$$\frac{105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-480i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+544i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-160i\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^4*log(tan(1/2*d*x + 1/2
*c) - 1) - 2*(81*a^4*tan(1/2*d*x + 1/2*c)^7 + 96*I*a^4*tan(1/2*d*x + 1/2*c)
^6 - 105*a^4*tan(1/2*d*x + 1/2*c)^5 - 480*I*a^4*tan(1/2*d*x + 1/2*c)^4 - 10
5*a^4*tan(1/2*d*x + 1/2*c)^3 + 544*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 81*a^4*ta
n(1/2*d*x + 1/2*c) - 160*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 6.77, size = 198, normalized size = 1.49

$$\frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,8\mathrm{i}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,40\mathrm{i}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,136\mathrm{i}}{3}+\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{a^4\,40\mathrm{i}}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x),x)
```

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^4*tan(c/2 + (d*x)/2)^2*136i)
/3 - (35*a^4*tan(c/2 + (d*x)/2)^3)/4 - a^4*tan(c/2 + (d*x)/2)^4*40i - (35*a
^4*tan(c/2 + (d*x)/2)^5)/4 + a^4*tan(c/2 + (d*x)/2)^6*8i + (27*a^4*tan(c/2
+ (d*x)/2)^7)/4 - (a^4*40i)/3 + (27*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1))
```

3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=97

$$-\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{15ia^4\sec(c+dx)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}-\frac{5i\sec(c+dx)\left(a^4+i\right.}{2d}$$

```
[Out] -15/2*a^4*arctanh(sin(d*x+c))/d-15/2*I*a^4*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a
+I*a*tan(d*x+c))^3/d-5/2*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3577, 3579, 3567, 3855}

$$-\frac{15ia^4\sec(c+dx)}{2d}-\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (((15*I)/2)*a^4*Sec[c + d*x])/d - (
(2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - (((5*I)/2)*Sec[c + d*x]*
(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
```

```
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \left(5a^2\right)\int \sec(c+dx)(a+\\ &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+ia^4\,\mathrm{t}\right.}{2d}\\ &= -\frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\,\mathrm{s}}{}\\ &= -\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx}{}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 906 vs. 2(97) = 194.
time = 6.27, size = 906, normalized size = 9.34

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) - (15*Cos[4*c]*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[d*x]*Cos[c + d*x]^4*((-8*I)*C
os[3*c] - 8*Sin[3*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^
4) + (Cos[c + d*x]^4*Sec[c]*((-4*I)*Cos[4*c] - 4*Sin[4*c])*(a + I*a*Tan[c +
 d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) - (((15*I)/2)*Cos[c + d*x]^4*Log[Co
s[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(
d*(Cos[d*x] + I*Sin[d*x])^4) + (((15*I)/2)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x]
 + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(8*Cos[3*c] - (8*I)*Sin[3*c])*Sin[d*x]*
(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(
```

```
Cos[4*c]/4 - (I/4)*Sin[4*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin
[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (I*Cos[c + d*x]^4*(
4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos
[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])) + (Cos[c + d*x]^4*(-1/4*Cos[4*c] + (I/4)*Sin[4*c])*(a + I*a*Tan
[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^2) + (I*Cos[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]
*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])
^4*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.26, size = 154, normalized size = 1.59

method	result
risch	$-\frac{8 i a^{4} {\mathrm e}^{i (d x +c)}}{d}-\frac{i a^{4} \left(9 \,{\mathrm e}^{3 i (d x +c)}+7 \,{\mathrm e}^{i (d x +c)}\right)}{d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{2}}-\frac{15 a^{4} \ln \left({\mathrm e}^{i (d x +c)}+i\right)}{2 d}+\frac{15 a^{4} \ln \left({\mathrm e}^{i (d x +c)}-i\right)}{2 d}$
derivativedivides	$\frac{a^{4} \left(\frac{\sin^{5}\left(d x +c \right)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{3}\left(d x +c \right)\right)}{2}+\frac{3 \sin \left(d x +c \right)}{2}-\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-4 i a^{4} \left(\frac{\sin^{4}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)\right.}{d}$
default	$\frac{a^{4} \left(\frac{\sin^{5}\left(d x +c \right)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{3}\left(d x +c \right)\right)}{2}+\frac{3 \sin \left(d x +c \right)}{2}-\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-4 i a^{4} \left(\frac{\sin^{4}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2
*ln(sec(d*x+c)+tan(d*x+c)))-4*I*a^4*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))-6*a^4*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-4*I*a^4*cos(d*
x+c)+a^4*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.41

$$-\frac{a^{4} {\left(\frac{2 \, \sin\left(d x + c\right)}{\sin\left(d x + c\right)^{2} - 1} + 3 \, \log\left(\sin\left(d x + c\right) + 1\right) - 3 \, \log\left(\sin\left(d x + c\right) - 1\right) - 4 \, \sin\left(d x + c\right)\right)} + 16 i \, a^{4} {\left(\frac{1}{\cos\left(d x + c\right)} + \cos\left(d x + c\right)\right)} + 12 \, a^{4} {\left(\log\left(\sin\left(d x + c\right) + 1\right) - \log\left(\sin\left(d x + c\right) - 1\right) - 2 \, \sin\left(d x + c\right)\right)} + 16 i \, a^{4} \cos\left(d x + c\right) - 4 \, a^{4} \sin\left(d x + c\right)}{4 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 16*I*a^4*(1/cos(d*x + c) + cos(
d*x + c)) + 12*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d
*x + c)) + 16*I*a^4*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 162, normalized size = 1.67

$$\frac{-16 i \, a^{4} e^{\left(5 i \, d x + 5 i \, c\right)} - 50 i \, a^{4} e^{\left(3 i \, d x + 3 i \, c\right)} - 30 i \, a^{4} e^{\left(i \, d x + i \, c\right)} - 15 \, {\left(a^{4} e^{\left(4 i \, d x + 4 i \, c\right)} + 2 \, a^{4} e^{\left(2 i \, d x + 2 i \, c\right)} + a^{4}\right)} \log\left(e^{\left(i \, d x + i \, c\right)} + i\right) + 15 \, {\left(a^{4} e^{\left(4 i \, d x + 4 i \, c\right)} + 2 \, a^{4} e^{\left(2 i \, d x + 2 i \, c\right)} + a^{4}\right)} \log\left(e^{\left(i \, d x + i \, c\right)} - i\right)}{2 \, {\left(d e^{\left(4 i \, d x + 4 i \, c\right)} + 2 \, d e^{\left(2 i \, d x + 2 i \, c\right)} + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(-16*I*a^4*e^(5*I*d*x + 5*I*c) - 50*I*a^4*e^(3*I*d*x + 3*I*c) - 30*I*a^
4*e^(I*d*x + I*c) - 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2*I*d*x + 2*I*c)
 + a^4)*log(e^(I*d*x + I*c) + I) + 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2
*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.26, size = 153, normalized size = 1.58

$$\frac{15a^4\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-9ia^4e^{3ic}e^{3idx}-7ia^4e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}+\begin{cases}-\frac{8ia^4e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 8a^4xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 15*a**4*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-9*I*a**4*exp(3*I*c)*exp(3*I*d*x) - 7*I*a**4*exp(I*c)*exp(I*d*x))/(d*
exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise((-8*
I*a**4*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (8*a**4*x*exp(I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(81) = 162.
time = 0.87, size = 372, normalized size = 3.84

$$\frac{235\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+470\,a^4e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-10\,a^4e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-235\,a^4e^{(4i\,dx+4i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-470\,a^4e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)+5\,a^4e^{(4i\,dx+4i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)+10\,a^4e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)-256i\,a^4e^{(5i\,dx+5i\,c)}-800i\,a^4e^{(3i\,dx+3i\,c)}-480i\,a^4e^{(i\,dx+i\,c)}+235\,a^4\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^4\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-235\,a^4\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)+5\,a^4\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{32\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32*(235*a^4*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 470*a^4*e^(2
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*e^(4*I*d*x + 4*I*c)*log(
I*e^(I*d*x + I*c) - 1) - 10*a^4*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 235*a^4*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 470*a^4*e^(
2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*e^(4*I*d*x + 4*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) + 10*a^4*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 256*I*a^4*e^(5*I*d*x + 5*I*c) - 800*I*a^4*e^(3*I*d*x + 3*I*c) - 4
80*I*a^4*e^(I*d*x + I*c) + 235*a^4*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*log(I
*e^(I*d*x + I*c) - 1) - 235*a^4*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*log(-I*
e^(I*d*x + I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 5.30, size = 159, normalized size = 1.64

$$-\frac{15\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,9{}\mathrm{i}-39\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,7{}\mathrm{i}+24\,a^4}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,1{}\mathrm{i}-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,2{}\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^3*9i - 39*a^4*tan(c/2 + (d*x)/2)^2 + 17*a^4*tan(c/2
 + (d*x)/2)^4 + 24*a^4 - a^4*tan(c/2 + (d*x)/2)*7i)/(d*(tan(c/2 + (d*x)/2)
- tan(c/2 + (d*x)/2)^2*2i - 2*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*1
i + tan(c/2 + (d*x)/2)^5 + 1i)) - (15*a^4*atanh(tan(c/2 + (d*x)/2)))/d
```

3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=78

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}$$

```
[Out] a^4*arctanh(sin(d*x+c))/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+2*I*c
os(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 3855}

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (((2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c
+ d*x])^3)/d + ((2*I)*Cos[c + d*x]*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} - a^2\int \cos(c+dx)(a+ia$$
$$= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2i\cos(c+dx)\left(a^4+ia^4\right.}{d}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. $2(78) = 156$.
time = 0.49, size = 246, normalized size = 3.15

$$\frac{a^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\cos(3dx)\sin(c)+6\cos(dx)\sin(3c)+3i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4c)-3i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4c)+\cos(3c)(6i\cos(dx)-6\sin(dx))+6i\sin(3c)\sin(dx)-2i\sin(c)\sin(3dx)+2\cos(c)(-i\cos(3dx)+\sin(3dx))\right)(\cos(c+dx)+i\sin(c+dx))^4}{3d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Si
n[3*c] + (3*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*c] - (3*I)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*c] + Cos[3*c]*((6*I)*Cos[d*x]
- 6*Sin[d*x]) + (6*I)*Sin[3*c]*Sin[d*x] - (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c
]*((-I)*Cos[3*d*x] + Sin[3*d*x]))*(Cos[c + d*x] + I*Sin[c + d*x])^4)/(3*d*(
Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A]
time = 0.22, size = 113, normalized size = 1.45

method	result
risch	$-\frac{2ia^4 e^{3i(dx+c)}}{3d}+\frac{2ia^4 e^{i(dx+c)}}{d}+\frac{a^4\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4\left(\cos\right.}{}}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4\left(\cos\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+4/3*I*a^4
*(2+sin(d*x+c)^2)*cos(d*x+c)-2*a^4*sin(d*x+c)^3-4/3*I*a^4*cos(d*x+c)^3+1/3*
a^4*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.30, size = 121, normalized size = 1.55

$$-\frac{8i\,a^4\cos\left(dx+c\right)^3+12\,a^4\sin\left(dx+c\right)^3+8i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^4+\left(2\,\sin\left(dx+c\right)^3-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)a^4+2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*I*a^4*cos(d*x + c)^3 + 12*a^4*sin(d*x + c)^3 + 8*I*(cos(d*x + c)^3
- 3*cos(d*x + c))*a^4 + (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log
(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^4 + 2*(sin(d*x + c)^3 - 3*sin(d*x +
c))*a^4)/d
```

Fricas [A]
time = 0.37, size = 68, normalized size = 0.87

$$\frac{-2i\,a^4e^{(3i\,dx+3i\,c)}+6i\,a^4e^{(i\,dx+i\,c)}+3\,a^4\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,a^4\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*a^4*e^(3*I*d*x + 3*I*c) + 6*I*a^4*e^(I*d*x + I*c) + 3*a^4*log(e^(
I*d*x + I*c) + I) - 3*a^4*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A]
time = 0.27, size = 109, normalized size = 1.40

$$\frac{a^4\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}\frac{-2ia^4de^{3ic}e^{3idx}+6ia^4de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x\left(2a^4e^{3ic}-2a^4e^{ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-2*I*a**4*d*exp(3*I*c)*exp(3*I*d*x) + 6*I*a**4*d*exp(I*c)*exp(I*
d*x))/(3*d**2), Ne(d**2, 0)), (x*(2*a**4*exp(3*I*c) - 2*a**4*exp(I*c)), Tru
e))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(68) = 136.
time = 1.64, size = 1299, normalized size = 16.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/768*(1110*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 22200*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1110*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1875*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 37500*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1875*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1110*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 16650*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 16650*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22200*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1110*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1875*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 37500*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1875*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 18*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 18*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 60*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3*a^4*e^(-6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 18*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 45*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 45*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 18*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 60*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 3*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 512*I*a^4*e^(15*I*d*x + 9*I*c) - 1536*I*a^4*e^(13*I*d*x + 7*I*c) + 1536*I*a^4*e^(11*I*d*x + 5*I*c) + 12800*I*a^4*e^(9*I*d*x + 3*I*c) + 23040*I*a^4*e^(7*I*d*x + I*c) + 19968*I*a^4*e^(5*I*d*x - I*c) + 8704*I*a^4*e^(3*I*d*x - 3*I*c) + 1536*I*a^4*e^(I*d*x - 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))

Mupad [B]
time = 3.56, size = 88, normalized size = 1.13

$$\frac{2\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\frac{8\,a^4}{3}-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,8{}\mathrm{i}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((8*a^4)/3 - a^4*tan(c/2 + (d*x)/2)*8
i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^
3 + 1i))
```

3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=66

$$-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

```
[Out] -1/15*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan
(d*x+c))^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3578, 3569}

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*
x]^5*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}+\frac{1}{5}a\int \cos^3(c+dx)(a+ia\,\text{t}\\ &= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}-\frac{i\cos^5(c+dx)(a+ia\tan}{5d}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 50, normalized size = 0.76

$$\frac{a^4(4\cos(c+dx)-i\sin(c+dx))(-i\cos(4(c+dx))+\sin(4(c+dx)))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(4*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[4*(c + d*x)] + Sin[4*(c +
d*x)]))/(15*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.
time = 0.24, size = 139, normalized size = 2.11

method	result
risch	$-\frac{i a^4 e^{5i(dx+c)}}{10d}-\frac{i a^4 e^{3i(dx+c)}}{6d}$
derivativedivides	$\frac{\frac{a^4\left(\sin^5(dx+c)\right)}{5}-4ia^4\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-6a^4\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin}{15}\right.}{d}$
default	$\frac{\frac{a^4\left(\sin^5(dx+c)\right)}{5}-4ia^4\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-6a^4\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin}{15}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a^4*sin(d*x+c)^5-4*I*a^4*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(
d*x+c)^3)-6*a^4*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x
+c))-4/5*I*a^4*cos(d*x+c)^5+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(54) = 108.
time = 0.28, size = 118, normalized size = 1.79

$$-\frac{12i\,a^4\cos(dx+c)^5-3\,a^4\sin(dx+c)^5+4i\,\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^4-6\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)a^4-\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*I*a^4*cos(d*x + c)^5 - 3*a^4*sin(d*x + c)^5 + 4*I*(3*cos(d*x + c)
^5 - 5*cos(d*x + c)^3)*a^4 - 6*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^4 -
(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4)/d
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.52

$$\frac{-3i\,a^4e^{(5i\,dx+5i\,c)} - 5i\,a^4e^{(3i\,dx+3i\,c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(-3*I*a^4*e^(5*I*d*x + 5*I*c) - 5*I*a^4*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A]
time = 0.24, size = 80, normalized size = 1.21

$$\begin{cases} \frac{-6ia^4de^{5ic}e^{5idx}-10ia^4de^{3ic}e^{3idx}}{60d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^4e^{5ic}}{2}+\frac{a^4e^{3ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-6*I*a**4*d*exp(5*I*c)*exp(5*I*d*x) - 10*I*a**4*d*exp(3*I*c)*ex
p(3*I*d*x))/(60*d**2), Ne(d**2, 0)), (x*(a**4*exp(5*I*c)/2 + a**4*exp(3*I*c
)/2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(54) = 108.
time = 1.21, size = 915, normalized size = 13.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/7680*(9075*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4
*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4*e^(2*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 54450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*
c) + 1) + 9075*a^4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9000*a^4*e^(8*I*
d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(6*I*d*x + 2*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 54000*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 9000*a^4*e^(-4
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 9075*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 36300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 36300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 54450*a^4*e
^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 9075*a^4*e^(-4*I*c)*log(-I*e^(I*d*
```

```
x + I*c) + 1) - 9000*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
36000*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 36000*a^4*e^(2*
I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54000*a^4*e^(4*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 9000*a^4*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 75*
a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(6*I*d*x +
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) - 450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 75*a^
4*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(8*I*d*x + 4*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 450*a^
4*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 768*I*a^4*e^(13*I*d*x + 9*I*c) - 4352*I*a^4*e^(11*I*d*x
+ 7*I*c) - 9728*I*a^4*e^(9*I*d*x + 5*I*c) - 10752*I*a^4*e^(7*I*d*x + 3*I*c)
 - 5888*I*a^4*e^(5*I*d*x + I*c) - 1280*I*a^4*e^(3*I*d*x - I*c))/(d*e^(8*I*d
*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*
I*d*x) + d*e^(-4*I*c))
```

Mupad [B]
time = 3.55, size = 130, normalized size = 1.97

$$\frac{2\,a^4\,\left(15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,15{}\mathrm{i}-25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5{}\mathrm{i}+4\right)}{15\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5{}\mathrm{i}-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10{}\mathrm{i}+5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*(tan(c/2 + (d*x)/2)^3*15i - 25*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)*5i + 15*tan(c/2 + (d*x)/2)^4 + 4))/(15*d*(5*tan(c/2 + (d*x)/2) - tan(c
/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i + t
an(c/2 + (d*x)/2)^5 + 1i))
```

3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{3a^4\sin(c+dx)}{35d}-\frac{a^4\sin^3(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+d\right.}{35d}$$

```
[Out] 3/35*a^4*sin(d*x+c)/d-1/35*a^4*sin(d*x+c)^3/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*t
an(d*x+c))^3/d-2/35*I*cos(d*x+c)^5*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 2713}

$$-\frac{a^4\sin^3(c+dx)}{35d}+\frac{3a^4\sin(c+dx)}{35d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*a^4*Sin[c + d*x])/(35*d) - (a^4*Sin[c + d*x]^3)/(35*d) - (((2*I)/7)*a*Co
s[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*Cos[c + d*x]^5*(a^4
+ I*a^4*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d} + \frac{1}{7}a^2\int \cos^5(c+dx)(a$$
$$= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d} - \frac{2i\cos^5(c+dx)\left(a^4+i\right.}{35d}$$
$$= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d} - \frac{2i\cos^5(c+dx)\left(a^4+i\right.}{35d}$$
$$= \frac{3a^4\sin(c+dx)}{35d} - \frac{a^4\sin^3(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(}{7d}$$

Mathematica [A]
time = 0.50, size = 73, normalized size = 0.72

$$\frac{a^4(28\cos(c+dx)+20\cos(3(c+dx))-i(7\sin(c+dx)+15\sin(3(c+dx))))(-i\cos(4(c+dx))+\sin(4(c+dx)))}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
[Out] (a^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - I*(7*Sin[c + d*x] + 15*Sin[3*
(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]))/(140*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(90) = 180.
time = 0.25, size = 203, normalized size = 1.99

method	result
risch	$-\frac{ia^4 e^{7i(dx+c)}}{56d} - \frac{3ia^4 e^{5i(dx+c)}}{40d} - \frac{ia^4 e^{3i(dx+c)}}{8d} - \frac{ia^4 e^{i(dx+c)}}{8d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7} - \frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35} + \frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{35}\right) - 4ia^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c\right.}{7}\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7} - \frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35} + \frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{35}\right) - 4ia^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c\right.}{7}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(cos(d*x+c)^2+2)*sin(d*x+c))-4*I*a^4*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*c
```

```
os(d*x+c)^5)-6*a^4*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))-4/7*I*a^4*cos(d*x+c)^7+1/7*a^4*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 149, normalized size = 1.46

$$-\frac{20i\,a^4\cos\left(dx+c\right)^7+4i\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^4+2\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3\right)a^4+\left(5\,\sin\left(dx+c\right)^7-7\,\sin\left(dx+c\right)^5\right)a^4+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^4}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*I*a^4*cos(d*x + c)^7 + 4*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
a^4 + 2*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^4 + (
5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^4 + (5*sin(d*x + c)^7 - 21*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.61

$$\frac{-5i\,a^4e^{(7i\,dx+7i\,c)}-21i\,a^4e^{(5i\,dx+5i\,c)}-35i\,a^4e^{(3i\,dx+3i\,c)}-35i\,a^4e^{(i\,dx+i\,c)}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(-5*I*a^4*e^(7*I*d*x + 7*I*c) - 21*I*a^4*e^(5*I*d*x + 5*I*c) - 35*I*a
^4*e^(3*I*d*x + 3*I*c) - 35*I*a^4*e^(I*d*x + I*c))/d
```

Sympy [A]
time = 0.36, size = 156, normalized size = 1.53

$$\begin{cases} \frac{-2560ia^4d^3e^{7ic}e^{7idx}-10752ia^4d^3e^{5ic}e^{5idx}-17920ia^4d^3e^{3ic}e^{3idx}-17920ia^4d^3e^{ic}e^{idx}}{143360d^4} & \text{for}\ d^4\neq 0 \\ x\left(\frac{a^4e^{7ic}}{8}+\frac{3a^4e^{5ic}}{8}+\frac{3a^4e^{3ic}}{8}+\frac{a^4e^{ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-2560*I*a**4*d**3*exp(7*I*c)*exp(7*I*d*x) - 10752*I*a**4*d**3*e
xp(5*I*c)*exp(5*I*d*x) - 17920*I*a**4*d**3*exp(3*I*c)*exp(3*I*d*x) - 17920*
I*a**4*d**3*exp(I*c)*exp(I*d*x))/(143360*d**4), Ne(d**4, 0)), (x*(a**4*exp(
7*I*c)/8 + 3*a**4*exp(5*I*c)/8 + 3*a**4*exp(3*I*c)/8 + a**4*exp(I*c)/8), Tr
ue))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(86) = 172$.
time = 0.91, size = 1327, normalized size = 13.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/143360*(89950*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 53970
0*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(8*I*
d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(4*I*d*x - 2*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 539700*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 1799000*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 89950*a^4
*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 86065*a^4*e^(12*I*d*x + 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 12
90975*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(2*
I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1721300*a^4*e^(6*I*d*x)*log(I*e
^(I*d*x + I*c) - 1) + 86065*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 899
50*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 539700*a^4*e^(10*
I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(8*I*d*x + 2*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 539700*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 1799000*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 89950*a^4*e^(-6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 86065*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 516390*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) - 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 12909
75*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516390*a^4*e^(2*I*
d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1721300*a^4*e^(6*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 86065*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 38
85*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23310*a^4*e^(10*I
*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(8*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 23310*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7
7700*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3885*a^4*e^(-6*I*c)*log(
I*e^(I*d*x) + e^(-I*c)) + 3885*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x) +
e^(-I*c)) + 23310*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5
8275*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 58275*a^4*e^(4*
I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 23310*a^4*e^(2*I*d*x - 4*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 77700*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^
(-I*c)) + 3885*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2560*I*a^4*e^(
19*I*d*x + 13*I*c) - 26112*I*a^4*e^(17*I*d*x + 11*I*c) - 120832*I*a^4*e^(15
*I*d*x + 9*I*c) - 337920*I*a^4*e^(13*I*d*x + 7*I*c) - 629760*I*a^4*e^(11*I*
d*x + 5*I*c) - 803840*I*a^4*e^(9*I*d*x + 3*I*c) - 694272*I*a^4*e^(7*I*d*x +
```

```
 I*c) - 387072*I*a^4*e^(5*I*d*x - I*c) - 125440*I*a^4*e^(3*I*d*x - 3*I*c) -
 17920*I*a^4*e^(I*d*x - 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x +
 4*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*
d*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

Mupad [B]
time = 4.27, size = 186, normalized size = 1.82

$$-\frac{2\,a^4\,\left(35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,105{}\mathrm{i}-210\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,210{}\mathrm{i}+147\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,49{}\mathrm{i}-12\right)}{35\,d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,7{}\mathrm{i}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,35{}\mathrm{i}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,21{}\mathrm{i}+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(2*a^4*(tan(c/2 + (d*x)/2)*49i + 147*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)^3*210i - 210*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*105i + 35*tan
(c/2 + (d*x)/2)^6 - 12))/(35*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d
*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=120

$$\frac{5a^4\sin(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{a^4\sin^5(c+dx)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx}{}$$

```
[Out] 5/21*a^4*sin(d*x+c)/d-10/63*a^4*sin(d*x+c)^3/d+1/21*a^4*sin(d*x+c)^5/d-2/9*
I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3/d-2/21*I*cos(d*x+c)^7*(a^4+I*a^4*tan(
d*x+c))/d
```

Rubi [A]

time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 2713}

$$\frac{a^4\sin^5(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{5a^4\sin(c+dx)}{21d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (5*a^4*Sin[c + d*x])/(21*d) - (10*a^4*Sin[c + d*x]^3)/(63*d) + (a^4*Sin[c +
 d*x]^5)/(21*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d -
 (((2*I)/21)*Cos[c + d*x]^7*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d} + \frac{1}{3}a^2\int \cos^7(c+dx)(a+$$
$$= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d} - \frac{2i\cos^7(c+dx)\left(a^4+ia^4\right.}{21d}$$
$$= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d} - \frac{2i\cos^7(c+dx)\left(a^4+ia^4\right.}{21d}$$
$$= \frac{5a^4\sin(c+dx)}{21d} - \frac{10a^4\sin^3(c+dx)}{63d} + \frac{a^4\sin^5(c+dx)}{21d} - \frac{2ia\cos^9(\ldots}{}$$

Mathematica [A]
time = 0.58, size = 111, normalized size = 0.92

$$\frac{a^4(-168i\cos(c+dx)-180i\cos(3(c+dx))+28i\cos(5(c+dx))-42\sin(c+dx)-135\sin(3(c+dx))+35\sin(5(c+dx)))(\cos(4(c+2dx))+i\sin(4(c+2dx)))}{1008d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*((-168*I)*Cos[c + d*x] - (180*I)*Cos[3*(c + d*x)] + (28*I)*Cos[5*(c +
d*x)] - 42*Sin[c + d*x] - 135*Sin[3*(c + d*x)] + 35*Sin[5*(c + d*x)])*(Cos[
4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(1008*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(106) = 212.
time = 0.25, size = 233, normalized size = 1.94

method	result
risch	$-\frac{ia^4e^{9i(dx+c)}}{288d} - \frac{5ia^4e^{7i(dx+c)}}{224d} - \frac{ia^4e^{5i(dx+c)}}{16d} - \frac{5ia^4e^{3i(dx+c)}}{48d} - \frac{ia^4\cos(dx+c)}{8d} + \frac{3a^4\sin(dx+c)}{16d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9} - \frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right) - 4ia^4\left(-\frac{\left(\sin^2(dx+\right.}{}\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9} - \frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right) - 4ia^4\left(-\frac{\left(\sin^2(dx+\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-4*I*a^4*(-1/9*sin(d*x+c)^2
```

```
*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-6*a^4*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(
16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*I*a^4*
cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 181, normalized size = 1.51

$$-\frac{140i\,a^4\cos\left(dx+c\right)^9+20i\left(7\cos\left(dx+c\right)^9-9\cos\left(dx+c\right)^7\right)a^4-\left(35\sin\left(dx+c\right)^9-90\sin\left(dx+c\right)^7+63\sin\left(dx+c\right)^5\right)a^4-6\left(35\sin\left(dx+c\right)^9-135\sin\left(dx+c\right)^7+189\sin\left(dx+c\right)^5-105\sin\left(dx+c\right)^3\right)a^4-\left(35\sin\left(dx+c\right)^9-180\sin\left(dx+c\right)^7+378\sin\left(dx+c\right)^5-420\sin\left(dx+c\right)^3+315\sin\left(dx+c\right)\right)a^4}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*I*a^4*cos(d*x + c)^9 + 20*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^4 - (35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^4 -
6*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*
x + c)^3)*a^4 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^
5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4)/d
```

Fricas [A]
time = 0.36, size = 90, normalized size = 0.75

$$\frac{\left(-7i\,a^4e^{\left(10i\,dx+10i\,c\right)}-45i\,a^4e^{\left(8i\,dx+8i\,c\right)}-126i\,a^4e^{\left(6i\,dx+6i\,c\right)}-210i\,a^4e^{\left(4i\,dx+4i\,c\right)}-315i\,a^4e^{\left(2i\,dx+2i\,c\right)}+63i\,a^4\right)e^{\left(-i\,dx-i\,c\right)}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-7*I*a^4*e^(10*I*d*x + 10*I*c) - 45*I*a^4*e^(8*I*d*x + 8*I*c) - 126
*I*a^4*e^(6*I*d*x + 6*I*c) - 210*I*a^4*e^(4*I*d*x + 4*I*c) - 315*I*a^4*e^(2
*I*d*x + 2*I*c) + 63*I*a^4)*e^(-I*d*x - I*c)/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(107) = 214$.
time = 0.42, size = 228, normalized size = 1.90

$$\begin{cases} \frac{\left(-176160768ia^4d^5e^{10ic}e^{9idx}-1132462080ia^4d^5e^{8ic}e^{7idx}-3170893824ia^4d^5e^{6ic}e^{5idx}-5284823040ia^4d^5e^{4ic}e^{3idx}-7927234560ia^4d^5e^{2ic}e^{idx}+1585446912ia^4d^5e^{-idx}\right)e^{-ic}}{50734301184d^6} & \text{for}\ d^6e^{ic}\neq 0\\ \frac{x\left(a^4e^{10ic}+5a^4e^{8ic}+10a^4e^{6ic}+10a^4e^{4ic}+5a^4e^{2ic}+a^4\right)e^{-ic}}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-176160768*I*a**4*d**5*exp(10*I*c)*exp(9*I*d*x) - 1132462080*I*
a**4*d**5*exp(8*I*c)*exp(7*I*d*x) - 3170893824*I*a**4*d**5*exp(6*I*c)*exp(5
*I*d*x) - 5284823040*I*a**4*d**5*exp(4*I*c)*exp(3*I*d*x) - 7927234560*I*a**
4*d**5*exp(2*I*c)*exp(I*d*x) + 1585446912*I*a**4*d**5*exp(-I*d*x))*exp(-I*c
)/(50734301184*d**6), Ne(d**6*exp(I*c), 0)), (x*(a**4*exp(10*I*c) + 5*a**4*
```

```
exp(8*I*c) + 10*a**4*exp(6*I*c) + 10*a**4*exp(4*I*c) + 5*a**4*exp(2*I*c) +
a**4)*exp(-I*c)/32, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(102) = 204$.
time = 0.92, size = 1409, normalized size = 11.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/516096*(435267*a^4*e^(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2611
602*a^4*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(9*
I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8705340*a^4*e^(7*I*d*x + I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x +
I*c) + 1) + 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 43
5267*a^4*e^(I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 427896*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(11*I*d*x + 5*I*c)*
log(I*e^(I*d*x + I*c) - 1) + 6418440*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 8557920*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 6
418440*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(3*
I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 427896*a^4*e^(I*d*x - 5*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) - 435267*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 2611602*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 6529005*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8705340*a^4
*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6529005*a^4*e^(5*I*d*x - I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 435267*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 427896*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2567376*
a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(9*I*d
*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8557920*a^4*e^(7*I*d*x + I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 2567376*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4
27896*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7371*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(11*I*d*x + 5*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 110565*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 147420*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1
10565*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(3*I*
d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7371*a^4*e^(I*d*x - 5*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 7371*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 44226*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11
0565*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 147420*a^4*e^(7
*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 110565*a^4*e^(5*I*d*x - I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) + 44226*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x
```

```
) + e^(-I*c)) + 7371*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1
792*I*a^4*e^(22*I*d*x + 16*I*c) - 22272*I*a^4*e^(20*I*d*x + 14*I*c) - 12825
6*I*a^4*e^(18*I*d*x + 12*I*c) - 455936*I*a^4*e^(16*I*d*x + 10*I*c) - 114432
0*I*a^4*e^(14*I*d*x + 8*I*c) - 2102784*I*a^4*e^(12*I*d*x + 6*I*c) - 2742784
*I*a^4*e^(10*I*d*x + 4*I*c) - 2382336*I*a^4*e^(8*I*d*x + 2*I*c) - 295680*I*
a^4*e^(4*I*d*x - 2*I*c) + 16128*I*a^4*e^(2*I*d*x - 4*I*c) - 1241856*I*a^4*e
^(6*I*d*x) + 16128*I*a^4*e^(-6*I*c))/(d*e^(13*I*d*x + 7*I*c) + 6*d*e^(11*I*
d*x + 5*I*c) + 15*d*e^(9*I*d*x + 3*I*c) + 20*d*e^(7*I*d*x + I*c) + 15*d*e^(
5*I*d*x - I*c) + 6*d*e^(3*I*d*x - 3*I*c) + d*e^(I*d*x - 5*I*c))
```

Mupad [B]
time = 4.69, size = 145, normalized size = 1.21

$$\frac{2\,a^4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{89\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{55\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}+\frac{55\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{355\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{35\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,21{}\mathrm{i}}{2}+\frac{\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,21{}\mathrm{i}}{2}-\frac{\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,87{}\mathrm{i}}{4}+\frac{\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\,7{}\mathrm{i}}{4}\right)}{63\,d\,\left(\cos\left(4\,c+4\,d\,x\right)-\sin\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*cos(c/2 + (d*x)/2)*((cos((5*c)/2 + (5*d*x)/2)*21i)/2 - (cos((3*c)/2
+ (3*d*x)/2)*21i)/2 - (cos((7*c)/2 + (7*d*x)/2)*87i)/4 + (cos((9*c)/2 + (9*
d*x)/2)*7i)/4 + (89*sin(c/2 + (d*x)/2))/8 - (55*sin((3*c)/2 + (3*d*x)/2))/4
 + (55*sin((5*c)/2 + (5*d*x)/2))/4 - (355*sin((7*c)/2 + (7*d*x)/2))/16 + (3
5*sin((9*c)/2 + (9*d*x)/2))/16))/(63*d*(cos(4*c + 4*d*x) - sin(4*c + 4*d*x)
*1i))
```

3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=109

$$-\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}$$

```
[Out] -8/9*I*(a+I*a*tan(d*x+c))^9/a^4/d+6/5*I*(a+I*a*tan(d*x+c))^10/a^5/d-6/11*I*
(a+I*a*tan(d*x+c))^11/a^6/d+1/12*I*(a+I*a*tan(d*x+c))^12/a^7/d
```

Rubi [A]

time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^9)/(a^4*d) + (((6*I)/5)*(a + I*a*Tan[c +
 d*x])^10)/(a^5*d) - (((6*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^6*d) + ((I/1
2)*(a + I*a*Tan[c + d*x])^12)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^8\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^8-12a^2(a+x)^9+6a(a+x)^{10}-(a+x)^{11}\right.\right.}{a^7d}\\ &= -\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{6i(a+ia\,\mathrm{t}}{1}\end{aligned}$$

Mathematica [A]
time = 1.79, size = 167, normalized size = 1.53

$$\frac{a^5\sec(c)\sec^{12}(c+dx)(924i\cos(c)+792i\cos(c+2dx)+792i\cos(3c+2dx)+495i\cos(3c+4dx)+495i\cos(5c+4dx)-924\sin(c)+792\sin(c+2dx)-792\sin(3c+2dx)+495\sin(3c+4dx)-495\sin(5c+4dx)+440\sin(5c+6dx)+132\sin(7c+8dx)+24\sin(9c+10dx)+2\sin(11c+12dx))}{3960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^12*((924*I)*Cos[c] + (792*I)*Cos[c + 2*d*x] + (792
*I)*Cos[3*c + 2*d*x] + (495*I)*Cos[3*c + 4*d*x] + (495*I)*Cos[5*c + 4*d*x]
- 924*Sin[c] + 792*Sin[c + 2*d*x] - 792*Sin[3*c + 2*d*x] + 495*Sin[3*c + 4*
d*x] - 495*Sin[5*c + 4*d*x] + 440*Sin[5*c + 6*d*x] + 132*Sin[7*c + 8*d*x] +
 24*Sin[9*c + 10*d*x] + 2*Sin[11*c + 12*d*x]))/(3960*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(93) = 186$.
time = 0.28, size = 377, normalized size = 3.46

method	result
risch	$\frac{1024ia^5\left(495\,e^{16i(dx+c)}+792\,e^{14i(dx+c)}+924\,e^{12i(dx+c)}+792\,e^{10i(dx+c)}+495\,e^{8i(dx+c)}+220\,e^{6i(dx+c)}+66\,e^{4i(dx+c)}+12\right.}{495d\left(e^{2i(dx+c)}+1\right)^{12}}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{40\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{120\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^5(dx+c)\right)}{33\cos(dx+c)^9}+\frac{8\left(\sin^5(dx+c\right.}{231\cos(dx+c}\right.$
default	$ia^5\left(\frac{\sin^6(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{40\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{120\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^5(dx+c)\right)}{33\cos(dx+c)^9}+\frac{8\left(\sin^5(dx+c\right.}{231\cos(dx+c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10
+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1/1
1*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c
)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/10*sin(d*x+
c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x
+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/9*sin(d*x+c)^3/cos(d*x+c)^9
+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(
d*x+c)^3/cos(d*x+c)^3)+5/8*I*a^5/cos(d*x+c)^8-a^5*(-16/35-1/7*sec(d*x+c)^6-
6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 160, normalized size = 1.47

$$-\frac{-165i\,a^5\tan(dx+c)^{12}-900\,a^5\tan(dx+c)^{11}+1386i\,a^5\tan(dx+c)^{10}-1100\,a^5\tan(dx+c)^9+5445i\,a^5\tan(dx+c)^8+3960\,a^5\tan(dx+c)^7+4620i\,a^5\tan(dx+c)^6+8712\,a^5\tan(dx+c)^5-2475i\,a^5\tan(dx+c)^4+4620\,a^5\tan(dx+c)^3-4950i\,a^5\tan(dx+c)^2-1980\,a^5\tan(dx+c)}{1980\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5*
tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 396
0*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)^5
- 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(d*x
+ c)^2 - 1980*a^5*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.
time = 0.36, size = 267, normalized size = 2.45

$$-\frac{1024 \left(-495 i a^{5} e^{(16 i dx+16 i c)}-792 i a^{5} e^{(14 i dx+14 i c)}-924 i a^{5} e^{(12 i dx+12 i c)}-792 i a^{5} e^{(10 i dx+10 i c)}-495 i a^{5} e^{(8 i dx+8 i c)}-220 i a^{5} e^{(6 i dx+6 i c)}-66 i a^{5} e^{(4 i dx+4 i c)}-12 i a^{5} e^{(2 i dx+2 i c)}-i a^{5}\right)}{495 \left(d e^{(24 i dx+24 i c)}+12 d e^{(22 i dx+22 i c)}+66 d e^{(20 i dx+20 i c)}+220 d e^{(18 i dx+18 i c)}+495 d e^{(16 i dx+16 i c)}+792 d e^{(14 i dx+14 i c)}+924 d e^{(12 i dx+12 i c)}+792 d e^{(10 i dx+10 i c)}+495 d e^{(8 i dx+8 i c)}+220 d e^{(6 i dx+6 i c)}+66 d e^{(4 i dx+4 i c)}+12 d e^{(2 i dx+2 i c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1024/495*(-495*I*a^5*e^(16*I*d*x + 16*I*c) - 792*I*a^5*e^(14*I*d*x + 14*I*
c) - 924*I*a^5*e^(12*I*d*x + 12*I*c) - 792*I*a^5*e^(10*I*d*x + 10*I*c) - 49
5*I*a^5*e^(8*I*d*x + 8*I*c) - 220*I*a^5*e^(6*I*d*x + 6*I*c) - 66*I*a^5*e^(4
*I*d*x + 4*I*c) - 12*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(24*I*d*x + 24
*I*c) + 12*d*e^(22*I*d*x + 22*I*c) + 66*d*e^(20*I*d*x + 20*I*c) + 220*d*e^(
18*I*d*x + 18*I*c) + 495*d*e^(16*I*d*x + 16*I*c) + 792*d*e^(14*I*d*x + 14*I
*c) + 924*d*e^(12*I*d*x + 12*I*c) + 792*d*e^(10*I*d*x + 10*I*c) + 495*d*e^(
8*I*d*x + 8*I*c) + 220*d*e^(6*I*d*x + 6*I*c) + 66*d*e^(4*I*d*x + 4*I*c) + 1
2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$i a^{5} \left(\int \left(-i \sec^{8}{\left(c+dx \right)}\right)\, dx+\int 5 \tan{\left(c+dx \right)} \sec^{8}{\left(c+dx \right)}\, dx+\int \left(-10 \tan^{3}{\left(c+dx \right)} \sec^{8}{\left(c+dx \right)}\right)\, dx+\int \tan^{5}{\left(c+dx \right)} \sec^{8}{\left(c+dx \right)}\, dx+\int 10 i \tan^{2}{\left(c+dx \right)} \sec^{8}{\left(c+dx \right)}\, dx+\int \left(-5 i \tan^{4}{\left(c+dx \right)} \sec^{8}{\left(c+dx \right)}\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**8, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**8, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**8, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**8, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**8, x))
```

Giac [A]
time = 0.98, size = 160, normalized size = 1.47

$$-\frac{-165 i a^{5} \tan \left(dx+c\right)^{12}-900 a^{5} \tan \left(dx+c\right)^{11}+1386 i a^{5} \tan \left(dx+c\right)^{10}-1100 a^{5} \tan \left(dx+c\right)^{9}+5445 i a^{5} \tan \left(dx+c\right)^{8}+3960 a^{5} \tan \left(dx+c\right)^{7}+4620 i a^{5} \tan \left(dx+c\right)^{6}+8712 a^{5} \tan \left(dx+c\right)^{5}-2475 i a^{5} \tan \left(dx+c\right)^{4}+4620 a^{5} \tan \left(dx+c\right)^{3}-4950 i a^{5} \tan \left(dx+c\right)^{2}-1980 a^{5} \tan \left(dx+c\right)}{1980 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5*
tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 396
0*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)^5
- 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(d*x
+ c)^2 - 1980*a^5*tan(d*x + c))/d
```

Mupad [B]
time = 3.71, size = 146, normalized size = 1.34

$$\frac{a^5\left(-\cos\left(c+d\,x\right)^{12}\,1749{}\mathrm{i}+2048\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{11}+1024\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^9+768\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+640\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^5+\cos\left(c+d\,x\right)^4\,3960{}\mathrm{i}-3400\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^3-\cos\left(c+d\,x\right)^2\,2376{}\mathrm{i}+900\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)+165{}\mathrm{i}\right)}{1980\,d\,\cos\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^8,x)
```

```
[Out] (a^5*(900*cos(c + d*x)*sin(c + d*x) - 3400*cos(c + d*x)^3*sin(c + d*x) + 64
0*cos(c + d*x)^5*sin(c + d*x) + 768*cos(c + d*x)^7*sin(c + d*x) + 1024*cos(
c + d*x)^9*sin(c + d*x) + 2048*cos(c + d*x)^11*sin(c + d*x) - cos(c + d*x)^
2*2376i + cos(c + d*x)^4*3960i - cos(c + d*x)^12*1749i + 165i))/(1980*d*cos
(c + d*x)^12)
```

3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=82

$$-\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^8/a^3/d+4/9*I*(a+I*a*tan(d*x+c))^9/a^4/d-1/10*I*(
a+I*a*tan(d*x+c))^10/a^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^8}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/2*I)*(a + I*a*Tan[c + d*x])^8)/(a^3*d) + (((4*I)/9)*(a + I*a*Tan[c + d
*x])^9)/(a^4*d) - ((I/10)*(a + I*a*Tan[c + d*x])^10)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^7\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^7-4a(a+x)^8+(a+x)^9\right)dx, x, ia\tan(c+\right.}{a^5d}\\ &= -\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(}{10a^5}\end{aligned}$$

Mathematica [**A**]
time = 1.33, size = 154, normalized size = 1.88

$$\frac{a^5\sec(c)\sec^{10}(c+dx)(126i\cos(c)+105i\cos(c+2dx)+105i\cos(3c+2dx)+60i\cos(3c+4dx)+60i\cos(5c+4dx)-126\sin(c)+105\sin(c+2dx)-105\sin(3c+2dx)+60\sin(3c+4dx)-60\sin(5c+4dx)+45\sin(5c+6dx)+10\sin(7c+8dx)+\sin(9c+10dx))}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^10*((126*I)*Cos[c] + (105*I)*Cos[c + 2*d*x] + (105
*I)*Cos[3*c + 2*d*x] + (60*I)*Cos[3*c + 4*d*x] + (60*I)*Cos[5*c + 4*d*x] -
126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*x] + 60*Sin[3*c + 4*d*x
] - 60*Sin[5*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin[9
*c + 10*d*x]))/(360*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(70) = 140$.
time = 0.27, size = 295, normalized size = 3.60

method	result
risch	$\frac{128ia^5\left(120\,e^{14i(dx+c)}+210\,e^{12i(dx+c)}+252\,e^{10i(dx+c)}+210\,e^{8i(dx+c)}+120\,e^{6i(dx+c)}+45\,e^{4i(dx+c)}+10\,e^{2i(dx+c)}+1\right)}{45d\left(e^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4\left(\sin^5(dx+c)\right)}{63\cos(dx+c)^7}+\frac{8\left(\sin^5(dx+c)\right)}{315\cos(dx+c)^5}\right)-10ia^5\Big(\frac{\sin^4(}{8\cos(}$
default	$ia^5\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4\left(\sin^5(dx+c)\right)}{63\cos(dx+c)^7}+\frac{8\left(\sin^5(dx+c)\right)}{315\cos(dx+c)^5}\right)-10ia^5\Big(\frac{\sin^4(}{8\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x+c)^6/cos(d*x+c)^8+
1/60*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*s
in(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/8*sin
(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos
(d*x+c)^4)-10*a^5*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+
c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+5/6*I*a^5/cos(d*x+c)^6-a^5*(-8/15-1/5
*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 108, normalized size = 1.32

$$-\frac{-9i\,a^5\tan\left(dx+c\right)^{10}-50\,a^5\tan\left(dx+c\right)^9+90i\,a^5\tan\left(dx+c\right)^8+210i\,a^5\tan\left(dx+c\right)^6+252\,a^5\tan\left(dx+c\right)^5+240\,a^5\tan\left(dx+c\right)^3-225i\,a^5\tan\left(dx+c\right)^2-90\,a^5\tan\left(dx+c\right)}{90\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
+ c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(d*
x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(64) = 128$.
time = 0.36, size = 229, normalized size = 2.79

$$-\frac{128\left(-120i\,a^5e^{(14i\,dx+14i\,c)}-210i\,a^5e^{(12i\,dx+12i\,c)}-252i\,a^5e^{(10i\,dx+10i\,c)}-210i\,a^5e^{(8i\,dx+8i\,c)}-120i\,a^5e^{(6i\,dx+6i\,c)}-45i\,a^5e^{(4i\,dx+4i\,c)}-10i\,a^5e^{(2i\,dx+2i\,c)}-i\,a^5\right)}{45\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+210\,de^{(8i\,dx+8i\,c)}+120\,de^{(6i\,dx+6i\,c)}+45\,de^{(4i\,dx+4i\,c)}+10\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -128/45*(-120*I*a^5*e^(14*I*d*x + 14*I*c) - 210*I*a^5*e^(12*I*d*x + 12*I*c)
 - 252*I*a^5*e^(10*I*d*x + 10*I*c) - 210*I*a^5*e^(8*I*d*x + 8*I*c) - 120*I*
a^5*e^(6*I*d*x + 6*I*c) - 45*I*a^5*e^(4*I*d*x + 4*I*c) - 10*I*a^5*e^(2*I*d*
x + 2*I*c) - I*a^5)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) +
 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d
*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 12
0*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia^5\left(\int\left(-i\sec^6(c+dx)\right)dx+\int 5\tan(c+dx)\sec^6(c+dx)\,dx+\int\left(-10\tan^3(c+dx)\sec^6(c+dx)\right)dx+\int\tan^5(c+dx)\sec^6(c+dx)\,dx+\int 10i\tan^2(c+dx)\sec^6(c+dx)\,dx+\int\left(-5i\tan^4(c+dx)\sec^6(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**6, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**6, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**6, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**6, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.91, size = 108, normalized size = 1.32

$$-\frac{-9i\,a^5\tan(dx+c)^{10}-50\,a^5\tan(dx+c)^9+90i\,a^5\tan(dx+c)^8+210i\,a^5\tan(dx+c)^6+252\,a^5\tan(dx+c)^5+240\,a^5\tan(dx+c)^3-225i\,a^5\tan(dx+c)^2-90\,a^5\tan(dx+c)}{90\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```


```
[Out] -1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
+ c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(d*
x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Mupad [B]
time = 3.33, size = 151, normalized size = 1.84

$$\frac{a^5\,\sin\left(c+d\,x\right)\,\left(90\cos\left(c+d\,x\right)^9+\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,225\mathrm{i}-240\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)^2-252\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^4-\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^5\,210\mathrm{i}-\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^7\,90\mathrm{i}+50\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^8+\sin\left(c+d\,x\right)^9\,9\mathrm{i}\right)}{90\,d\,\cos\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^6,x)
```

```
[Out] (a^5*sin(c + d*x)*(50*cos(c + d*x)*sin(c + d*x)^8 + cos(c + d*x)^8*sin(c +
d*x)*225i + 90*cos(c + d*x)^9 + sin(c + d*x)^9*9i - cos(c + d*x)^2*sin(c +
d*x)^7*90i - cos(c + d*x)^4*sin(c + d*x)^5*210i - 252*cos(c + d*x)^5*sin(c
+ d*x)^4 - 240*cos(c + d*x)^7*sin(c + d*x)^2))/(90*d*cos(c + d*x)^10)
```

3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=55

$$-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}$$

```
[Out] -2/7*I*(a+I*a*tan(d*x+c))^7/a^2/d+1/8*I*(a+I*a*tan(d*x+c))^8/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^8}{8a^3d}-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^2*d) + ((I/8)*(a + I*a*Tan[c + d*x
])^8)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^6\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^6-(a+x)^7\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(55) = 110.
time = 0.95, size = 143, normalized size = 2.60

$$\frac{a^5\sec(c)\sec^8(c+dx)(35i\cos(c)+28i\cos(c+2dx)+28i\cos(3c+2dx)+14i\cos(3c+4dx)+14i\cos(5c+4dx)-35\sin(c)+28\sin(c+2dx)-28\sin(3c+2dx)+14\sin(3c+4dx)-14\sin(5c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx))}{56d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^8*((35*I)*Cos[c] + (28*I)*Cos[c + 2*d*x] + (28*I)*
Cos[3*c + 2*d*x] + (14*I)*Cos[3*c + 4*d*x] + (14*I)*Cos[5*c + 4*d*x] - 35*S
in[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 14*Sin[3*c + 4*d*x] - 14*
Sin[5*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(56*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(47) = 94.
time = 0.25, size = 213, normalized size = 3.87

method	result
risch	$\frac{32ia^5\left(28\,e^{12i(dx+c)}+56\,e^{10i(dx+c)}+70\,e^{8i(dx+c)}+56\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{7d\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\frac{}{5}\right.}{d}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\frac{}{5}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/8*sin(d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6)+5
*a^5*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^
5*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/
5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+5/4*I*a^5/cos(d
*x+c)^4-a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.
time = 0.28, size = 108, normalized size = 1.96

$$-\frac{-7i\,a^5\tan\left(dx+c\right)^8-40\,a^5\tan\left(dx+c\right)^7+84i\,a^5\tan\left(dx+c\right)^6+56\,a^5\tan\left(dx+c\right)^5+70i\,a^5\tan\left(dx+c\right)^4+168\,a^5\tan\left(dx+c\right)^3-140i\,a^5\tan\left(dx+c\right)^2-56\,a^5\tan\left(dx+c\right)}{56\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x +
 c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x +
 c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(43) = 86.
time = 0.38, size = 191, normalized size = 3.47

$$-\frac{32\left(-28i\,a^5e^{(12i\,dx+12i\,c)}-56i\,a^5e^{(10i\,dx+10i\,c)}-70i\,a^5e^{(8i\,dx+8i\,c)}-56i\,a^5e^{(6i\,dx+6i\,c)}-28i\,a^5e^{(4i\,dx+4i\,c)}-8i\,a^5e^{(2i\,dx+2i\,c)}-i\,a^5\right)}{7\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c)}+8\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -32/7*(-28*I*a^5*e^(12*I*d*x + 12*I*c) - 56*I*a^5*e^(10*I*d*x + 10*I*c) - 7
0*I*a^5*e^(8*I*d*x + 8*I*c) - 56*I*a^5*e^(6*I*d*x + 6*I*c) - 28*I*a^5*e^(4*
I*d*x + 4*I*c) - 8*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(16*I*d*x + 16*I
*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I
*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d
*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia^5\left(\int\left(-i\sec^4\left(c+dx\right)\right)dx+\int 5\tan\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\left(-10\tan^3\left(c+dx\right)\sec^4\left(c+dx\right)\right)dx+\int\tan^5\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int 10i\tan^2\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\left(-5i\tan^4\left(c+dx\right)\sec^4\left(c+dx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**4, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**4, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**4, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**4, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**4, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.
time = 0.87, size = 108, normalized size = 1.96

$$-\frac{-7i\,a^5\tan\left(dx+c\right)^8-40\,a^5\tan\left(dx+c\right)^7+84i\,a^5\tan\left(dx+c\right)^6+56\,a^5\tan\left(dx+c\right)^5+70i\,a^5\tan\left(dx+c\right)^4+168\,a^5\tan\left(dx+c\right)^3-140i\,a^5\tan\left(dx+c\right)^2-56\,a^5\tan\left(dx+c\right)}{56\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```


```
[Out] -1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x +
 c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x +
 c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Mupad [B]
time = 3.30, size = 151, normalized size = 2.75

$$\frac{a^5\,\sin\left(c+d\,x\right)\,\left(56\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}-168\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^4\sin\left(c+d\,x\right)^3\,70\mathrm{i}-56\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^4-\cos\left(c+d\,x\right)^2\sin\left(c+d\,x\right)^5\,84\mathrm{i}+40\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^6+\sin\left(c+d\,x\right)^7\,7\mathrm{i}\right)}{56\,d\cos\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^4,x)
```

```
[Out] (a^5*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c +
d*x)*140i + 56*cos(c + d*x)^7 + sin(c + d*x)^7*7i - cos(c + d*x)^2*sin(c +
d*x)^5*84i - 56*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)
^3*70i - 168*cos(c + d*x)^5*sin(c + d*x)^2))/(56*d*cos(c + d*x)^8)
```

3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

```
[Out] -1/6*I*(a+I*a*tan(d*x+c))^6/a/d
```

Rubi [A]

time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/6*I)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^5\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^6}{6ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(27) = 54.
time = 0.89, size = 134, normalized size = 4.96

$$\frac{a^5\sec(c)\sec^6(c+dx)(20i\cos(c)+15i\cos(c+2dx)+15i\cos(3c+2dx)+6i\cos(3c+4dx)+6i\cos(5c+4dx)-20\sin(c)+15\sin(c+2dx)-15\sin(3c+2dx)+6\sin(3c+4dx)-6\sin(5c+4dx)+2\sin(5c+6dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^6*((20*I)*Cos[c] + (15*I)*Cos[c + 2*d*x] + (15*I)*
Cos[3*c + 2*d*x] + (6*I)*Cos[3*c + 4*d*x] + (6*I)*Cos[5*c + 4*d*x] - 20*Sin
[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 6*Sin[3*c + 4*d*x] - 6*Sin[
5*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(12*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(23) = 46$.
time = 0.24, size = 115, normalized size = 4.26

method	result	size
risch	$\frac{32 i a^{5} \left(6 \,{\mathrm e}^{10 i \left(d x +c \right)}+15 \,{\mathrm e}^{8 i \left(d x +c \right)}+20 \,{\mathrm e}^{6 i \left(d x +c \right)}+15 \,{\mathrm e}^{4 i \left(d x +c \right)}+6 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{3 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{6}}$	80
derivativedivides	$\frac{\frac{i a^{5} \left(\sin^{6}\left(d x +c \right)\right)}{6 \cos \left(d x +c \right)^{6}}+\frac{a^{5} \left(\sin^{5}\left(d x +c \right)\right)}{\cos \left(d x +c \right)^{5}}-\frac{5 i a^{5} \left(\sin^{4}\left(d x +c \right)\right)}{2 \cos \left(d x +c \right)^{4}}-\frac{10 a^{5} \left(\sin^{3}\left(d x +c \right)\right)}{3 \cos \left(d x +c \right)^{3}}+\frac{5 i a^{5}}{2 \cos \left(d x +c \right)^{2}}+a^{5} \tan \left(d x +c \right)}{d}$	115
default	$\frac{\frac{i a^{5} \left(\sin^{6}\left(d x +c \right)\right)}{6 \cos \left(d x +c \right)^{6}}+\frac{a^{5} \left(\sin^{5}\left(d x +c \right)\right)}{\cos \left(d x +c \right)^{5}}-\frac{5 i a^{5} \left(\sin^{4}\left(d x +c \right)\right)}{2 \cos \left(d x +c \right)^{4}}-\frac{10 a^{5} \left(\sin^{3}\left(d x +c \right)\right)}{3 \cos \left(d x +c \right)^{3}}+\frac{5 i a^{5}}{2 \cos \left(d x +c \right)^{2}}+a^{5} \tan \left(d x +c \right)}{d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*I*a^5*sin(d*x+c)^6/cos(d*x+c)^6+a^5*sin(d*x+c)^5/cos(d*x+c)^5-5/2*
I*a^5*sin(d*x+c)^4/cos(d*x+c)^4-10/3*a^5*sin(d*x+c)^3/cos(d*x+c)^3+5/2*I*a^
5/cos(d*x+c)^2+a^5*tan(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 21, normalized size = 0.78

$$-\frac{i \, {\left(i \, a \tan\left(d x + c\right) + a\right)}^{6}}{6 \, a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*I*(I*a*tan(d*x + c) + a)^6/(a*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(21) = 42$.
time = 0.34, size = 153, normalized size = 5.67

$$-\frac{32 \, {\left(-6 i \, a^{5} e^{\left(10 i \, d x + 10 i \, c\right)} - 15 i \, a^{5} e^{\left(8 i \, d x + 8 i \, c\right)} - 20 i \, a^{5} e^{\left(6 i \, d x + 6 i \, c\right)} - 15 i \, a^{5} e^{\left(4 i \, d x + 4 i \, c\right)} - 6 i \, a^{5} e^{\left(2 i \, d x + 2 i \, c\right)} - i \, a^{5}\right)}}{3 \, {\left(d e^{\left(12 i \, d x + 12 i \, c\right)} + 6 \, d e^{\left(10 i \, d x + 10 i \, c\right)} + 15 \, d e^{\left(8 i \, d x + 8 i \, c\right)} + 20 \, d e^{\left(6 i \, d x + 6 i \, c\right)} + 15 \, d e^{\left(4 i \, d x + 4 i \, c\right)} + 6 \, d e^{\left(2 i \, d x + 2 i \, c\right)} + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -32/3*(-6*I*a^5*e^(10*I*d*x + 10*I*c) - 15*I*a^5*e^(8*I*d*x + 8*I*c) - 20*I
*a^5*e^(6*I*d*x + 6*I*c) - 15*I*a^5*e^(4*I*d*x + 4*I*c) - 6*I*a^5*e^(2*I*d*
x + 2*I*c) - I*a^5)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) +
15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I
*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia^{5}\left(\int \left(-i\sec^{2}(c+dx)\right)\,dx+\int 5\tan(c+dx)\sec^{2}(c+dx)\,dx+\int \left(-10\tan^{3}(c+dx)\sec^{2}(c+dx)\right)\,dx+\int \tan^{5}(c+dx)\sec^{2}(c+dx)\,dx+\int 10i\tan^{2}(c+dx)\sec^{2}(c+dx)\,dx+\int \left(-5i\tan^{4}(c+dx)\sec^{2}(c+dx)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**2, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**2, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**2, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**2, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(21) = 42$.
time = 0.86, size = 82, normalized size = 3.04

$$-\frac{-i\,a^{5}\tan(dx+c)^{6}-6\,a^{5}\tan(dx+c)^{5}+15i\,a^{5}\tan(dx+c)^{4}+20\,a^{5}\tan(dx+c)^{3}-15i\,a^{5}\tan(dx+c)^{2}-6\,a^{5}\tan(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/6*(-I*a^5*tan(d*x + c)^6 - 6*a^5*tan(d*x + c)^5 + 15*I*a^5*tan(d*x + c)^
4 + 20*a^5*tan(d*x + c)^3 - 15*I*a^5*tan(d*x + c)^2 - 6*a^5*tan(d*x + c))/d
```

Mupad [B]
time = 3.26, size = 114, normalized size = 4.22

$$\frac{a^{5}\,\sin(c+d\,x)\,\left(6\cos(c+d\,x)^{5}+\cos(c+d\,x)^{4}\,\sin(c+d\,x)\,15\mathrm{i}-20\cos(c+d\,x)^{3}\sin(c+d\,x)^{2}-\cos(c+d\,x)^{2}\sin(c+d\,x)^{3}\,15\mathrm{i}+6\,\cos(c+d\,x)\,\sin(c+d\,x)^{4}+\sin(c+d\,x)^{5}\,1\mathrm{i}\right)}{6\,d\cos(c+d\,x)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^2,x)
```

```
[Out] (a^5*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d
*x)*15i + 6*cos(c + d*x)^5 + sin(c + d*x)^5*1i - cos(c + d*x)^2*sin(c + d*x
)^3*15i - 20*cos(c + d*x)^3*sin(c + d*x)^2))/(6*d*cos(c + d*x)^6)
```

3.63 $\int (a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=117

$$16a^5x-\frac{16ia^5\log(\cos(c+dx))}{d}-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\frac{2ia(}{}$$

```
[Out] 16*a^5*x-16*I*a^5*ln(cos(d*x+c))/d-8*a^5*tan(d*x+c)/d+2/3*I*a^2*(a+I*a*tan(
d*x+c))^3/d+1/4*I*a*(a+I*a*tan(d*x+c))^4/d+2*I*a*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3559, 3558, 3556}

$$-\frac{8a^5\tan(c+dx)}{d}-\frac{16ia^5\log(\cos(c+dx))}{d}+16a^5x+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{2ia(a^2+ia^2\tan(c+dx))^2}{d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] 16*a^5*x - ((16*I)*a^5*Log[Cos[c + d*x]])/d - (8*a^5*Tan[c + d*x])/d + (((2
*I)/3)*a^2*(a + I*a*Tan[c + d*x])^3)/d + ((I/4)*a*(a + I*a*Tan[c + d*x])^4)
/d + ((2*I)*a*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^5\,dx &= \frac{ia(a+ia\tan(c+dx))^4}{4d}+(2a)\int (a+ia\tan(c+dx))^4\,dx\\ &= \frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\left(4a^2\right)\int (a+ia\tan(c+\\ &= \frac{2ia^3(a+ia\tan(c+dx))^2}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx)}{4d}\\ &= 16a^5x-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^3(a+ia\tan(c+dx))^2}{d}+\frac{2ia^2(a+ia\tan(c+dx}{3d}\\ &= 16a^5x-\frac{16ia^5\log(\cos(c+dx))}{d}-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^3(a+ia\tan(c+dx))^2}{d}\end{aligned}$$

Mathematica [A]

time = 2.50, size = 228, normalized size = 1.95

$$\frac{a^5\sec(c)\sec^4(c+dx)\left(-18i\cos(3c+2dx)+48dx\cos(3c+2dx)+12dx\cos(3c+4dx)+12dx\cos(5c+4dx)+6\cos(c+2dx)\left(-3i+8dx-4i\log\left(\cos^2(c+dx)\right)\right)+\cos(c)\left(-33i+72dx-36i\log\left(\cos^2(c+dx)\right)\right)-24i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)-6i\cos(3c+4dx)\log\left(\cos^2(c+dx)\right)-6i\cos(5c+4dx)\log\left(\cos^2(c+dx)\right)+75\sin(c)-70\sin(c+2dx)+30\sin(3c+2dx)-25\sin(3c+4dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^4*((-18*I)*Cos[3*c + 2*d*x] + 48*d*x*Cos[3*c + 2*d
*x] + 12*d*x*Cos[3*c + 4*d*x] + 12*d*x*Cos[5*c + 4*d*x] + 6*Cos[c + 2*d*x]*
(-3*I + 8*d*x - (4*I)*Log[Cos[c + d*x]^2]) + Cos[c]*(-33*I + 72*d*x - (36*I
)*Log[Cos[c + d*x]^2]) - (24*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] - (6*I
)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x]^2] - (6*I)*Cos[5*c + 4*d*x]*Log[Cos[c +
 d*x]^2] + 75*Sin[c] - 70*Sin[c + 2*d*x] + 30*Sin[3*c + 2*d*x] - 25*Sin[3*c
 + 4*d*x]))/(12*d)
```

Maple [A]

time = 0.05, size = 72, normalized size = 0.62

method	result
derivativedivides	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln\left(1+\tan^2(dx+c)\right)+16\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln\left(1+\tan^2(dx+c)\right)+16\arctan(\tan(dx+c))\right)}{d}$
risch	$-\frac{32a^5c}{d}-\frac{4ia^5\left(48\,e^{6i(dx+c)}+108\,e^{4i(dx+c)}+88\,e^{2i(dx+c)}+25\right)}{3d\left(e^{2i(dx+c)}+1\right)^4}-\frac{16ia^5\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$16a^5x-\frac{15a^5\tan(dx+c)}{d}+\frac{5a^5\left(\tan^3(dx+c)\right)}{3d}-\frac{11ia^5\left(\tan^2(dx+c)\right)}{2d}+\frac{ia^5\left(\tan^4(dx+c)\right)}{4d}+\frac{8ia^5\ln(1+\tan^2(dx+}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^5*(-15*tan(d*x+c)+1/4*I*tan(d*x+c)^4+5/3*tan(d*x+c)^3-11/2*I*tan(d*x+
c)^2+8*I*ln(1+tan(d*x+c)^2)+16*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 165, normalized size = 1.41

$$a^{5}x+\frac{5\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^{5}}{3\,d}+\frac{10\left(dx+c-\tan\left(dx+c\right)\right)a^{5}}{d}+\frac{i\,a^{5}\left(\frac{4\,\sin(dx+c)^{2}-3}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-2\,\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)}{4\,d}+\frac{5i\,a^{5}\left(\frac{1}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)}{d}+\frac{5i\,a^{5}\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^5/d + 10*(d*x
 + c - tan(d*x + c))*a^5/d + 1/4*I*a^5*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*(1/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*log(sec(d*x + c))/
d
```

Fricas [A]
time = 0.36, size = 177, normalized size = 1.51

$$-\frac{4\left(48i\,a^{5}e^{(6i\,dx+6i\,c)}+108i\,a^{5}e^{(4i\,dx+4i\,c)}+88i\,a^{5}e^{(2i\,dx+2i\,c)}+25i\,a^{5}+12\left(i\,a^{5}e^{(8i\,dx+8i\,c)}+4i\,a^{5}e^{(6i\,dx+6i\,c)}+6i\,a^{5}e^{(4i\,dx+4i\,c)}+4i\,a^{5}e^{(2i\,dx+2i\,c)}+i\,a^{5}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -4/3*(48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x + 4*I*c) + 88*I*a
^5*e^(2*I*d*x + 2*I*c) + 25*I*a^5 + 12*(I*a^5*e^(8*I*d*x + 8*I*c) + 4*I*a^5
*e^(6*I*d*x + 6*I*c) + 6*I*a^5*e^(4*I*d*x + 4*I*c) + 4*I*a^5*e^(2*I*d*x + 2
*I*c) + I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.26, size = 178, normalized size = 1.52

$$-\frac{16ia^{5}\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-192ia^{5}e^{6ic}e^{6idx}-432ia^{5}e^{4ic}e^{4idx}-352ia^{5}e^{2ic}e^{2idx}-100ia^{5}}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -16*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-192*I*a**5*exp(6*I*c)*exp(
6*I*d*x) - 432*I*a**5*exp(4*I*c)*exp(4*I*d*x) - 352*I*a**5*exp(2*I*c)*exp(2
*I*d*x) - 100*I*a**5)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*
I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(99) = 198$.
time = 0.56, size = 222, normalized size = 1.90

$$-\frac{4\left(12i\,a^{5}e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+48i\,a^{5}e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+72i\,a^{5}e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+48i\,a^{5}e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+48i\,a^{5}e^{(6i\,dx+6i\,c)}+108i\,a^{5}e^{(4i\,dx+4i\,c)}+88i\,a^{5}e^{(2i\,dx+2i\,c)}+12i\,a^{5}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+25i\,a^{5}\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -4/3*(12*I*a^5*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5*
e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 72*I*a^5*e^(4*I*d*x + 4*
I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5*e^(2*I*d*x + 2*I*c)*log(e^(2*I
*d*x + 2*I*c) + 1) + 48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x +
4*I*c) + 88*I*a^5*e^(2*I*d*x + 2*I*c) + 12*I*a^5*log(e^(2*I*d*x + 2*I*c) +
1) + 25*I*a^5)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*
I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [**B**]
time = 3.27, size = 73, normalized size = 0.62

$$\frac{a^{5}\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,16\mathrm{i}-15\,a^{5}\tan(c+d\,x)-\frac{a^{5}\tan(c+d\,x)^{2}\,11\mathrm{i}}{2}+\frac{5\,a^{5}\tan(c+d\,x)^{3}}{3}+\frac{a^{5}\tan(c+d\,x)^{4}\,1\mathrm{i}}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*log(tan(c + d*x) + 1i)*16i - 15*a^5*tan(c + d*x) - (a^5*tan(c + d*x)^2
*11i)/2 + (5*a^5*tan(c + d*x)^3)/3 + (a^5*tan(c + d*x)^4*1i)/4)/d
```

3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=83

$$-12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}$$

```
[Out] -12*a^5*x+12*I*a^5*ln(cos(d*x+c))/d+5*a^5*tan(d*x+c)/d+1/2*I*a^5*tan(d*x+c)
^2/d-8*I*a^6/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{8ia^6}{d(a-ia\tan(c+dx))}+\frac{ia^5\tan^2(c+dx)}{2d}+\frac{5a^5\tan(c+dx)}{d}+\frac{12ia^5\log(\cos(c+dx))}{d}-12a^5x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] -12*a^5*x + ((12*I)*a^5*Log[Cos[c + d*x]])/d + (5*a^5*Tan[c + d*x])/d + ((I
/2)*a^5*Tan[c + d*x]^2)/d - ((8*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^3}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^3)\,\text{Subst}\left(\int \left(5a+\frac{8a^3}{(a-x)^2}-\frac{12a^2}{a-x}+x\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c}{2d}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 649 vs. $2(83) = 166$.
time = 6.31, size = 649, normalized size = 7.82

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (-12*x*Cos[5*c]*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/(Cos[d*x] + I*Sin[
d*x])^5 + ((6*I)*Cos[5*c]*Cos[c + d*x]^5*Log[Cos[c + d*x]^2]*(a + I*a*Tan[c
 + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + (Cos[2*d*x]*Cos[c + d*x]^5*((-4
*I)*Cos[3*c] - 4*Sin[3*c])*(a + I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d
*x])^5) + (Cos[c + d*x]^3*((I/2)*Cos[5*c] + Sin[5*c]/2)*(a + I*a*Tan[c + d*
x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + ((12*I)*x*Cos[c + d*x]^5*Sin[5*c]*(a
 + I*a*Tan[c + d*x])^5)/(Cos[d*x] + I*Sin[d*x])^5 + (6*Cos[c + d*x]^5*Log[C
os[c + d*x]^2]*Sin[5*c]*(a + I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x]
)^5) + (Cos[c + d*x]^4*(5*Cos[5*c] - (5*I)*Sin[5*c])*Sin[d*x]*(a + I*a*Tan[
c + d*x])^5)/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*S
in[d*x])^5) + (Cos[c + d*x]^5*(4*Cos[3*c] - (4*I)*Sin[3*c])*Sin[2*d*x]*(a +
 I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + (x*Cos[c + d*x]^5*(6*
Cos[c]^3 - 6*Cos[c]^5 - (24*I)*Cos[c]^2*Sin[c] + (36*I)*Cos[c]^4*Sin[c] - 3
6*Cos[c]*Sin[c]^2 + 90*Cos[c]^3*Sin[c]^2 + (24*I)*Sin[c]^3 - (120*I)*Cos[c]
^2*Sin[c]^3 - 90*Cos[c]*Sin[c]^4 + (36*I)*Sin[c]^5 + 6*Sin[c]^3*Tan[c] + 6*
Sin[c]^5*Tan[c] - I*(12*Cos[5*c] - (12*I)*Sin[5*c])*Tan[c])*(a + I*a*Tan[c
+ d*x])^5)/(Cos[d*x] + I*Sin[d*x])^5
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(77) = 154$.
time = 0.26, size = 205, normalized size = 2.47

method	result
risch	$-\frac{4ia^5 e^{2i(dx+c)}}{d}+\frac{24a^5c}{d}+\frac{2ia^5\left(6\,e^{2i(dx+c)}+5\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{12ia^5\ln\left(e^{2i(dx+c)}+1\right)}{d}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+5a^5\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c\right.$
default	$ia^5\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+5a^5\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*l
n(cos(d*x+c)))+5*a^5*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))
```

```
*cos(d*x+c)-3/2*d*x-3/2*c)-10*I*a^5*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-10*a
^5*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-5/2*I*a^5*cos(d*x+c)^2+a^5*(1
/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 86, normalized size = 1.04

$$-\frac{-i\,a^5\tan\left(dx+c\right)^2+24\,(dx+c)a^5+12i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right)-10\,a^5\tan\left(dx+c\right)-\frac{16\,\left(a^5\tan(dx+c)-i\,a^5\right)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^5*tan(d*x + c)^2 + 24*(d*x + c)*a^5 + 12*I*a^5*log(tan(d*x + c)^
2 + 1) - 10*a^5*tan(d*x + c) - 16*(a^5*tan(d*x + c) - I*a^5)/(tan(d*x + c)^
2 + 1))/d
```

Fricas [A]
time = 0.38, size = 125, normalized size = 1.51

$$-\frac{2\left(2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}-5i\,a^5+6\left(-i\,a^5e^{(4i\,dx+4i\,c)}-2i\,a^5e^{(2i\,dx+2i\,c)}-i\,a^5\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -2*(2*I*a^5*e^(6*I*d*x + 6*I*c) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(
2*I*d*x + 2*I*c) - 5*I*a^5 + 6*(-I*a^5*e^(4*I*d*x + 4*I*c) - 2*I*a^5*e^(2*I
*d*x + 2*I*c) - I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.26, size = 131, normalized size = 1.58

$$\frac{12ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{12ia^5e^{2ic}e^{2idx}+10ia^5}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}+\begin{cases}-\frac{4ia^5e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0\\ 8a^5xe^{2ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 12*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (12*I*a**5*exp(2*I*c)*exp(2*I
*d*x) + 10*I*a**5)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x)
 + d) + Piecewise((-4*I*a**5*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (8*a**5*
x*exp(2*I*c), True))
```

Giac [A]
time = 0.96, size = 146, normalized size = 1.76

$$-\frac{2\left(-6i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^5e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}-6i\,a^5\log\left(e^{(2i\,dx+2i\,c)}+1\right)-5i\,a^5\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -2*(-6*I*a^5*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^5*e^
(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(6*I*d*x + 6*I*c
) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c) - 6*I*a^5*log
(e^(2*I*d*x + 2*I*c) + 1) - 5*I*a^5)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*
x + 2*I*c) + d)
```

Mupad [B]
time = 3.30, size = 70, normalized size = 0.84

$$\frac{8\,a^5}{d\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}-\frac{a^5\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,12{}\mathrm{i}}{d}+\frac{5\,a^5\,\tan(c+d\,x)}{d}+\frac{a^5\,\tan(c+d\,x)^2\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (8*a^5)/(d*(tan(c + d*x) + 1i)) - (a^5*log(tan(c + d*x) + 1i)*12i)/d + (5*a
^5*tan(c + d*x))/d + (a^5*tan(c + d*x)^2*1i)/(2*d)
```

3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=73

$$a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}$$

```
[Out] a^5*x-I*a^5*ln(cos(d*x+c))/d-2*I*a^7/d/(a-I*a*tan(d*x+c))^2+4*I*a^6/d/(a-I*
a*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}-\frac{ia^5\log(\cos(c+dx))}{d}+a^5x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] a^5*x - (I*a^5*Log[Cos[c + d*x]])/d - ((2*I)*a^7)/(d*(a - I*a*Tan[c + d*x])
^2) + ((4*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{(a+x)^2}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{4a^2}{(a-x)^3}-\frac{4a}{(a-x)^2}+\frac{1}{a-x}\right)\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4}{d(a-ia\tan}\end{aligned}$$

Mathematica [A]
time = 0.54, size = 110, normalized size = 1.51

$$\frac{a^5(2i+\cos(2(c+dx))\left(-i+2dx-i\log\left(\cos^2(c+dx)\right)\right)+\left(1-2idx-\log\left(\cos^2(c+dx)\right)\right)\sin(2(c+dx)))\left(\cos(2c+7dx)+i\sin(2c+7dx)\right)}{2d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(2*I + Cos[2*(c + d*x)]*(-I + 2*d*x - I*Log[Cos[c + d*x]^2]) + (1 - (2
*I)*d*x - Log[Cos[c + d*x]^2])*Sin[2*(c + d*x)])*(Cos[2*c + 7*d*x] + I*Sin[
2*c + 7*d*x]))/(2*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(68) = 136.
time = 0.22, size = 188, normalized size = 2.58

method	result
risch	$-\frac{i a^5 e^{4i(dx+c)}}{2d}+\frac{i a^5 e^{2i(dx+c)}}{d}-\frac{2a^5 c}{d}-\frac{i a^5 \ln\left(e^{2i(dx+c)}+1\right)}{d}$
derivativedivides	$ia^5\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5ia^5\left(\sin^4(\right.}{2}$
default	$ia^5\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5ia^5\left(\sin^4(\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+5*a^5*(-1/4*
(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-5/2*I*a^5*sin(d*x+c
)^4-10*a^5*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+
1/8*c)-5/4*I*a^5*cos(d*x+c)^4+a^5*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 88, normalized size = 1.21

$$\frac{2\,(dx+c)a^5+i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{4\left(2\,a^5\tan(dx+c)^3-3i\,a^5\tan(dx+c)^2-i\,a^5\right)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a^5 + I*a^5*log(tan(d*x + c)^2 + 1) - 4*(2*a^5*tan(d*x + c
)^3 - 3*I*a^5*tan(d*x + c)^2 - I*a^5)/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1))/d
```

Fricas [A]
time = 0.37, size = 51, normalized size = 0.70

$$\frac{-i\,a^5e^{(4i\,dx+4i\,c)}+2i\,a^5e^{(2i\,dx+2i\,c)}-2i\,a^5\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a^5*e^(4*I*d*x + 4*I*c) + 2*I*a^5*e^(2*I*d*x + 2*I*c) - 2*I*a^5*log
(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A]
time = 0.28, size = 102, normalized size = 1.40

$$-\frac{ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\begin{cases}\frac{-ia^5de^{4ic}e^{4idx}+2ia^5de^{2ic}e^{2idx}}{2d^2} & \text{for } d^2\neq 0\\ x(2a^5e^{4ic}-2a^5e^{2ic}) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise(((-I*a**5*d*exp(4*I*c
)*exp(4*I*d*x) + 2*I*a**5*d*exp(2*I*c)*exp(2*I*d*x))/(2*d**2), Ne(d**2, 0))
, (x*(2*a**5*exp(4*I*c) - 2*a**5*exp(2*I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(63) = 126.
time = 0.81, size = 450, normalized size = 6.16

$$-\frac{2i\,a^5e^{(16i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^5e^{(14i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+56i\,a^5e^{(12i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+112i\,a^5e^{(10i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+112i\,a^5e^{(6i\,dx-2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+56i\,a^5e^{(4i\,dx-4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^5e^{(2i\,dx-6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+140i\,a^5e^{(8i\,dx)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2i\,a^5e^{(-8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^5e^{(20i\,dx+12i\,c)}+6i\,a^5e^{(18i\,dx+10i\,c)}+12i\,a^5e^{(16i\,dx+8i\,c)}-42i\,a^5e^{(12i\,dx+4i\,c)}-84i\,a^5e^{(10i\,dx+2i\,c)}-48i\,a^5e^{(6i\,dx-2i\,c)}-15i\,a^5e^{(4i\,dx-4i\,c)}-2i\,a^5e^{(2i\,dx-6i\,c)}-84i\,a^5e^{(8i\,dx)}}{2\left(de^{(16i\,dx+8i\,c)}+8\,de^{(14i\,dx+6i\,c)}+28\,de^{(12i\,dx+4i\,c)}+56\,de^{(10i\,dx+2i\,c)}+56\,de^{(6i\,dx-2i\,c)}+28\,de^{(4i\,dx-4i\,c)}+8\,de^{(2i\,dx-6i\,c)}+70\,de^{(8i\,dx)}+de^{(-8i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/2*(2*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 16*I*a^5*
e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 56*I*a^5*e^(12*I*d*x +
4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) + 56*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 16*I
*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 140*I*a^5*e^(8*I*d*
x)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x + 2*I*c
```

```
) + 1) + I*a^5*e^(20*I*d*x + 12*I*c) + 6*I*a^5*e^(18*I*d*x + 10*I*c) + 12*I
*a^5*e^(16*I*d*x + 8*I*c) - 42*I*a^5*e^(12*I*d*x + 4*I*c) - 84*I*a^5*e^(10*
I*d*x + 2*I*c) - 48*I*a^5*e^(6*I*d*x - 2*I*c) - 15*I*a^5*e^(4*I*d*x - 4*I*c
) - 2*I*a^5*e^(2*I*d*x - 6*I*c) - 84*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x + 8*
I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*
d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^
(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 3.31, size = 64, normalized size = 0.88

$$\frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}-\frac{4\,a^5\,\tan(c+d\,x)+a^5\,2\mathrm{i}}{d\,\left({\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^5,x)
[Out] (a^5*log(tan(c + d*x) + 1i)*1i)/d - (4*a^5*tan(c + d*x) + a^5*2i)/(d*(tan(c
 + d*x)*2i + tan(c + d*x)^2 - 1))
```

3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=55

$$-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}+\frac{ia^7}{2d(a-ia\tan(c+dx))^2}$$

```
[Out] -2/3*I*a^8/d/(a-I*a*tan(d*x+c))^3+1/2*I*a^7/d/(a-I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{ia^7}{2d(a-ia\tan(c+dx))^2}-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/3)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) + ((I/2)*a^7)/(d*(a - I*a*Tan
[c + d*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{a+x}{(a-x)^4}\,dx,x,ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^7)\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^4}-\frac{1}{(a-x)^3}\right)\,dx,x,ia\tan(c+dx)\right)}{d}\\
&= -\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}+\frac{ia^7}{2d(a-ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 50, normalized size = 0.91

$$\frac{a^5(5\cos(c+dx)-i\sin(c+dx))(-i\cos(5(c+dx))+\sin(5(c+dx)))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(5*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[5*(c + d*x)] + Sin[5*(c +
d*x)]))/(24*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(47) = 94.
time = 0.24, size = 231, normalized size = 4.20

method	result
risch	$-\frac{i a^5 e^{6i(dx+c)}}{12d}-\frac{i a^5 e^{4i(dx+c)}}{8d}$
derivativedivides	$\frac{\frac{i a^5\left(\sin^6(dx+c)\right)}{6}+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)\cos(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)-10ia^5\left(-\frac{(\sin}{}\right.}{}$
default	$\frac{\frac{i a^5\left(\sin^6(dx+c)\right)}{6}+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)\cos(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)-10ia^5\left(-\frac{(\sin}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*I*a^5*sin(d*x+c)^6+5*a^5*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d
*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)-10*I*a^5*(-1
/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-10*a^5*(-1/6*sin(d*x+c)*cos
(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-5/
6*I*a^5*cos(d*x+c)^6+a^5*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c
))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.
time = 0.49, size = 93, normalized size = 1.69

$$-\frac{3i\,a^5\tan\left(dx+c\right)^4+10\,a^5\tan\left(dx+c\right)^3-12i\,a^5\tan\left(dx+c\right)^2-6\,a^5\tan\left(dx+c\right)+i\,a^5}{6\left(\tan\left(dx+c\right)^6+3\,\tan\left(dx+c\right)^4+3\,\tan\left(dx+c\right)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*I*a^5*tan(d*x + c)^4 + 10*a^5*tan(d*x + c)^3 - 12*I*a^5*tan(d*x + c
)^2 - 6*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*t
an(d*x + c)^2 + 1)*d)
```

Fricas [A]
time = 0.35, size = 34, normalized size = 0.62

$$\frac{-2i\,a^5e^{(6i\,dx+6i\,c)}-3i\,a^5e^{(4i\,dx+4i\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*a^5*e^(6*I*d*x + 6*I*c) - 3*I*a^5*e^(4*I*d*x + 4*I*c))/d
```

Sympy [A]
time = 0.29, size = 80, normalized size = 1.45

$$\begin{cases} \frac{-8ia^5de^{6ic}e^{6idx}-12ia^5de^{4ic}e^{4idx}}{96d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^5e^{6ic}}{2}+\frac{a^5e^{4ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-8*I*a**5*d*exp(6*I*c)*exp(6*I*d*x) - 12*I*a**5*d*exp(4*I*c)*ex
p(4*I*d*x))/(96*d**2), Ne(d**2, 0)), (x*(a**5*exp(6*I*c)/2 + a**5*exp(4*I*c
)/2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(43) = 86.
time = 0.83, size = 187, normalized size = 3.40

$$-\frac{2i\,a^5e^{(18i\,dx+12i\,c)}+15i\,a^5e^{(16i\,dx+10i\,c)}+48i\,a^5e^{(14i\,dx+8i\,c)}+85i\,a^5e^{(12i\,dx+6i\,c)}+90i\,a^5e^{(10i\,dx+4i\,c)}+57i\,a^5e^{(8i\,dx+2i\,c)}+3i\,a^5e^{(4i\,dx-2i\,c)}+20i\,a^5e^{(6i\,dx)}}{24\,\left(de^{(12i\,dx+6i\,c)}+6\,de^{(10i\,dx+4i\,c)}+15\,de^{(8i\,dx+2i\,c)}+15\,de^{(4i\,dx-2i\,c)}+6\,de^{(2i\,dx-4i\,c)}+20\,de^{(6i\,dx)}+de^{(-6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/24*(2*I*a^5*e^(18*I*d*x + 12*I*c) + 15*I*a^5*e^(16*I*d*x + 10*I*c) + 48*
I*a^5*e^(14*I*d*x + 8*I*c) + 85*I*a^5*e^(12*I*d*x + 6*I*c) + 90*I*a^5*e^(10
*I*d*x + 4*I*c) + 57*I*a^5*e^(8*I*d*x + 2*I*c) + 3*I*a^5*e^(4*I*d*x - 2*I*c
) + 20*I*a^5*e^(6*I*d*x))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4*I*c
) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d*x -
4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

Mupad [B]
time = 3.28, size = 53, normalized size = 0.96

$$\frac{a^5\,\left(1+\tan(c+d\,x)\,3{}\mathrm{i}\right)}{6\,d\,\left(-{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,3{}\mathrm{i}+3\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^5,x)
[Out] (a^5*(tan(c + d*x)*3i + 1))/(6*d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(
c + d*x)^3 + 1i))
```

3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=27

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

```
[Out] -1/4*I*a^9/d/(a-I*a*tan(d*x+c))^4
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/4*I)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(27) = 54.
time = 0.62, size = 73, normalized size = 2.70

$$\frac{a^5(10\cos(c+dx)+5\cos(3(c+dx))-i(2\sin(c+dx)+3\sin(3(c+dx))))(-i\cos(5(c+dx))+\sin(5(c+dx)))}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(10*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - I*(2*Sin[c + d*x] + 3*Sin[3*(c
 + d*x)]))*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]))/(64*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(23) = 46$.
time = 0.24, size = 301, normalized size = 11.15

method	result
risch	$-\frac{i a^5 e^{8i(dx+c)}}{64d}-\frac{i a^5 e^{6i(dx+c)}}{16d}-\frac{3i a^5 e^{4i(dx+c)}}{32d}-\frac{i a^5 e^{2i(dx+c)}}{16d}$
derivativedivides	$\frac{i a^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^4(dx+c)\right)}{8}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{12}-\frac{\left(\cos^4(dx+c)\right)}{24}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c}{}\right.}{}$
default	$\frac{i a^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^4(dx+c)\right)}{8}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{12}-\frac{\left(\cos^4(dx+c)\right)}{24}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1
/24*cos(d*x+c)^4)+5*a^5*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos
(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-
10*I*a^5*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-10*a^5*(-1/8*si
n(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*
sin(d*x+c)+5/128*d*x+5/128*c)-5/8*I*a^5*cos(d*x+c)^8+a^5*(1/8*(cos(d*x+c)^7
+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*
x+35/128*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(21) = 42$.
time = 0.51, size = 103, normalized size = 3.81

$$-\frac{i\,a^5\tan\left(dx+c\right)^4+4\,a^5\tan\left(dx+c\right)^3-6i\,a^5\tan\left(dx+c\right)^2-4\,a^5\tan\left(dx+c\right)+i\,a^5}{4\left(\tan\left(dx+c\right)^8+4\,\tan\left(dx+c\right)^6+6\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/4*(I*a^5*tan(d*x + c)^4 + 4*a^5*tan(d*x + c)^3 - 6*I*a^5*tan(d*x + c)^2
- 4*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d
*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(21) = 42.
time = 0.40, size = 62, normalized size = 2.30

$$\frac{-i\,a^5e^{(8i\,dx+8i\,c)}-4i\,a^5e^{(6i\,dx+6i\,c)}-6i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/64*(-I*a^5*e^(8*I*d*x + 8*I*c) - 4*I*a^5*e^(6*I*d*x + 6*I*c) - 6*I*a^5*e^
(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(22) = 44.
time = 0.37, size = 162, normalized size = 6.00

$$\begin{cases} \frac{-8192ia^5d^3e^{8ic}e^{8idx}-32768ia^5d^3e^{6ic}e^{6idx}-49152ia^5d^3e^{4ic}e^{4idx}-32768ia^5d^3e^{2ic}e^{2idx}}{524288d^4} & \text{for } d^4\neq 0 \\ x\left(\frac{a^5e^{8ic}}{8}+\frac{3a^5e^{6ic}}{8}+\frac{3a^5e^{4ic}}{8}+\frac{a^5e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-8192*I*a**5*d**3*exp(8*I*c)*exp(8*I*d*x) - 32768*I*a**5*d**3*e
xp(6*I*c)*exp(6*I*d*x) - 49152*I*a**5*d**3*exp(4*I*c)*exp(4*I*d*x) - 32768*
I*a**5*d**3*exp(2*I*c)*exp(2*I*d*x))/(524288*d**4), Ne(d**4, 0)), (x*(a**5*
exp(8*I*c)/8 + 3*a**5*exp(6*I*c)/8 + 3*a**5*exp(4*I*c)/8 + a**5*exp(2*I*c)/
8), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(21) = 42.
time = 0.82, size = 267, normalized size = 9.89

$$-\frac{i\,a^5e^{(24i\,dx+16i\,c)}+12i\,a^5e^{(22i\,dx+14i\,c)}+66i\,a^5e^{(20i\,dx+12i\,c)}+220i\,a^5e^{(18i\,dx+10i\,c)}+494i\,a^5e^{(16i\,dx+8i\,c)}+784i\,a^5e^{(14i\,dx+6i\,c)}+896i\,a^5e^{(12i\,dx+4i\,c)}+736i\,a^5e^{(10i\,dx+2i\,c)}+164i\,a^5e^{(6i\,dx-2i\,c)}+38i\,a^5e^{(4i\,dx-4i\,c)}+4i\,a^5e^{(2i\,dx-6i\,c)}+425i\,a^5e^{(8i\,dx)}}{64\left(de^{(16i\,dx+8i\,c)}+8\,de^{(14i\,dx+6i\,c)}+28\,de^{(12i\,dx+4i\,c)}+56\,de^{(10i\,dx+2i\,c)}+56\,de^{(6i\,dx-2i\,c)}+28\,de^{(4i\,dx-4i\,c)}+8\,de^{(2i\,dx-6i\,c)}+70\,de^{(8i\,dx)}+de^{(-8i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/64*(I*a^5*e^(24*I*d*x + 16*I*c) + 12*I*a^5*e^(22*I*d*x + 14*I*c) + 66*I*
a^5*e^(20*I*d*x + 12*I*c) + 220*I*a^5*e^(18*I*d*x + 10*I*c) + 494*I*a^5*e^(
16*I*d*x + 8*I*c) + 784*I*a^5*e^(14*I*d*x + 6*I*c) + 896*I*a^5*e^(12*I*d*x
+ 4*I*c) + 736*I*a^5*e^(10*I*d*x + 2*I*c) + 164*I*a^5*e^(6*I*d*x - 2*I*c) +
 38*I*a^5*e^(4*I*d*x - 4*I*c) + 4*I*a^5*e^(2*I*d*x - 6*I*c) + 425*I*a^5*e^(
8*I*d*x))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I
```

```
*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d
*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8
*I*c))
```

Mupad [B]
time = 3.30, size = 63, normalized size = 2.33

$$-\frac{\frac{a^5\cos(c+d\,x)^4\,1\mathrm{i}}{4}+a^5\cos\left(c+d\,x\right)^6\left(\tan(c+d\,x)-2\mathrm{i}\right)-2\,a^5\cos\left(c+d\,x\right)^8\left(\tan(c+d\,x)-\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -((a^5*cos(c + d*x)^4*1i)/4 + a^5*cos(c + d*x)^6*(tan(c + d*x) - 2i) - 2*a^
5*cos(c + d*x)^8*(tan(c + d*x) - 1i))/d
```

3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=144

$$\frac{a^5x}{32}-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+d$$

```
[Out] 1/32*a^5*x-1/10*I*a^10/d/(a-I*a*tan(d*x+c))^5-1/16*I*a^9/d/(a-I*a*tan(d*x+c
))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-1/32*I*a^7/d/(a-I*a*tan(d*x+c))^2-1/
32*I*a^6/d/(a-I*a*tan(d*x+c))
```

Rubi [A]

time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}-\frac{ia^6}{32d(a-ia\tan(c+dx))}+\frac{a^5x}{32}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*x)/32 - ((I/10)*a^10)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/16)*a^9)/(d*(
a - I*a*Tan[c + d*x])^4) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/
32)*a^7)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{(ia^{11})\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^6(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{11})\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a-x)^6}+\frac{1}{4a^2(a-x)^5}+\frac{1}{8a^3(a-x)^4}+\frac{1}{16a^4(a-x)^3}+ \ldots\right.\right.}{d}$$

$$= -\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{\ldots}{24d(a-\ldots}$$

$$= \frac{a^5x}{32}-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{\ldots}{24\ldots}$$

Mathematica [A]
time = 0.88, size = 137, normalized size = 0.95

$$\frac{a^5(-500i\cos(c+dx)-375i\cos(3(c+dx))-12i\cos(5(c+dx))+120dx\cos(5(c+dx))-100\sin(c+dx)-225\sin(3(c+dx))+12\sin(5(c+dx))-120idx\sin(5(c+dx)))(\cos(5(c+2dx))+i\sin(5(c+2dx)))}{3840d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*((-500*I)*Cos[c + d*x] - (375*I)*Cos[3*(c + d*x)] - (12*I)*Cos[5*(c +
d*x)] + 120*d*x*Cos[5*(c + d*x)] - 100*Sin[c + d*x] - 225*Sin[3*(c + d*x)]
+ 12*Sin[5*(c + d*x)] - (120*I)*d*x*Sin[5*(c + d*x)])*(Cos[5*(c + 2*d*x)] +
 I*Sin[5*(c + 2*d*x)]))/(3840*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(122) = 244.
time = 0.30, size = 331, normalized size = 2.30

method	result
risch	$\frac{a^5x}{32}-\frac{ia^5 e^{10i(dx+c)}}{320d}-\frac{5ia^5 e^{8i(dx+c)}}{256d}-\frac{5ia^5 e^{6i(dx+c)}}{96d}-\frac{5ia^5 e^{4i(dx+c)}}{64d}-\frac{5ia^5 e^{2i(dx+c)}}{64d}$
derivativedivides	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\sin(dx\ldots}{}\ldots\right.}{\ldots}$
default	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\sin(dx\ldots}{}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-
1/60*cos(d*x+c)^6)+5*a^5*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)-10*I*a^5*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x
+c)^8)-10*a^5*(-1/10*cos(d*x+c)^9*sin(d*x+c)+1/80*(cos(d*x+c)^7+7/6*cos(d*x
+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256*c)-1/
2*I*a^5*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*
x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+63/256
*c))
```

Maxima [A]
time = 0.51, size = 164, normalized size = 1.14

$$\frac{15\,(dx+c)a^5 + \frac{15\,a^5\tan(dx+c)^9+70\,a^5\tan(dx+c)^7+128\,a^5\tan(dx+c)^5-80i\,a^5\tan(dx+c)^4-230\,a^5\tan(dx+c)^3+560i\,a^5\tan(dx+c)^2+465\,a^5\tan(dx+c)-128i\,a^5}{\tan(dx+c)^{10}+5\,\tan(dx+c)^8+10\,\tan(dx+c)^6+10\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/480*(15*(d*x + c)*a^5 + (15*a^5*tan(d*x + c)^9 + 70*a^5*tan(d*x + c)^7 +
128*a^5*tan(d*x + c)^5 - 80*I*a^5*tan(d*x + c)^4 - 230*a^5*tan(d*x + c)^3 +
 560*I*a^5*tan(d*x + c)^2 + 465*a^5*tan(d*x + c) - 128*I*a^5)/(tan(d*x + c)
^10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x
+ c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 83, normalized size = 0.58

$$\frac{120\,a^5dx - 12i\,a^5e^{(10i\,dx+10i\,c)} - 75i\,a^5e^{(8i\,dx+8i\,c)} - 200i\,a^5e^{(6i\,dx+6i\,c)} - 300i\,a^5e^{(4i\,dx+4i\,c)} - 300i\,a^5e^{(2i\,dx+2i\,c)}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(120*a^5*d*x - 12*I*a^5*e^(10*I*d*x + 10*I*c) - 75*I*a^5*e^(8*I*d*x
+ 8*I*c) - 200*I*a^5*e^(6*I*d*x + 6*I*c) - 300*I*a^5*e^(4*I*d*x + 4*I*c) -
300*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A]
time = 0.44, size = 209, normalized size = 1.45

$$\frac{a^5x}{32} + \begin{cases} \frac{-100663296ia^5d^4e^{10ic}e^{10idx}-629145600ia^5d^4e^{8ic}e^{8idx}-1677721600ia^5d^4e^{6ic}e^{6idx}-2516582400ia^5d^4e^{4ic}e^{4idx}-2516582400ia^5d^4e^{2ic}e^{2idx}}{32212254720d^5} & \text{for } d^5 \neq 0 \\ x\left(\frac{a^5e^{10ic}}{32}+\frac{5a^5e^{8ic}}{32}+\frac{5a^5e^{6ic}}{16}+\frac{5a^5e^{4ic}}{16}+\frac{5a^5e^{2ic}}{32}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*x/32 + Piecewise(((-100663296*I*a**5*d**4*exp(10*I*c)*exp(10*I*d*x) -
629145600*I*a**5*d**4*exp(8*I*c)*exp(8*I*d*x) - 1677721600*I*a**5*d**4*exp(
6*I*c)*exp(6*I*d*x) - 2516582400*I*a**5*d**4*exp(4*I*c)*exp(4*I*d*x) - 2516
582400*I*a**5*d**4*exp(2*I*c)*exp(2*I*d*x))/(32212254720*d**5), Ne(d**5, 0)
), (x*(a**5*exp(10*I*c)/32 + 5*a**5*exp(8*I*c)/32 + 5*a**5*exp(6*I*c)/16 +
5*a**5*exp(4*I*c)/16 + 5*a**5*exp(2*I*c)/32), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(112) = 224.
time = 0.94, size = 857, normalized size = 5.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/15360*(480*a^5*d*x*e^(16*I*d*x + 8*I*c) + 3840*a^5*d*x*e^(14*I*d*x + 6*I*
c) + 13440*a^5*d*x*e^(12*I*d*x + 4*I*c) + 26880*a^5*d*x*e^(10*I*d*x + 2*I*c
) + 26880*a^5*d*x*e^(6*I*d*x - 2*I*c) + 13440*a^5*d*x*e^(4*I*d*x - 4*I*c) +
 3840*a^5*d*x*e^(2*I*d*x - 6*I*c) + 33600*a^5*d*x*e^(8*I*d*x) + 480*a^5*d*x
*e^(-8*I*c) - 195*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5460*I*a^5*
e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(10*I*d*x
 + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(6*I*d*x - 2*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 5460*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) - 1560*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
 - 13650*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 195*I*a^5*e^(-8*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 195*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*
I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^
(-2*I*c)) + 5460*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
 10920*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 10920*I*a
^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 5460*I*a^5*e^(4*I*d*
x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(2*I*d*x - 6*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) + 13650*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^
(-2*I*c)) + 195*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 48*I*a^5*e
^(26*I*d*x + 18*I*c) - 684*I*a^5*e^(24*I*d*x + 16*I*c) - 4544*I*a^5*e^(22*I
*d*x + 14*I*c) - 18688*I*a^5*e^(20*I*d*x + 12*I*c) - 53360*I*a^5*e^(18*I*d*
x + 10*I*c) - 111688*I*a^5*e^(16*I*d*x + 8*I*c) - 174944*I*a^5*e^(14*I*d*x
+ 6*I*c) - 204784*I*a^5*e^(12*I*d*x + 4*I*c) - 176048*I*a^5*e^(10*I*d*x + 2
*I*c) - 44000*I*a^5*e^(6*I*d*x - 2*I*c) - 10800*I*a^5*e^(4*I*d*x - 4*I*c) -
 1200*I*a^5*e^(2*I*d*x - 6*I*c) - 107500*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x
+ 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(1
0*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*
d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 3.64, size = 122, normalized size = 0.85

$$\frac{a^5\,x}{32}+\frac{\frac{a^5\,\tan(c+d\,x)^4}{32}+\frac{a^5\,\tan(c+d\,x)^3\,5\mathrm{i}}{32}-\frac{31\,a^5\,\tan(c+d\,x)^2}{96}-\frac{a^5\,\tan(c+d\,x)\,35\mathrm{i}}{96}+\frac{4\,a^5}{15}}{d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*x)/32 + ((4*a^5)/15 - (a^5*tan(c + d*x)*35i)/96 - (31*a^5*tan(c + d*x)
^2)/96 + (a^5*tan(c + d*x)^3*5i)/32 + (a^5*tan(c + d*x)^4)/32)/(d*(5*tan(c
+ d*x) - tan(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c
 + d*x)^5 + 1i))
```

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=198

$$\frac{7a^5x}{128}-\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6}-\frac{ia^{10}}{20d(a-ia\tan(c+dx))^5}-\frac{3ia^9}{64d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+a$$

```
[Out] 7/128*a^5*x-1/24*I*a^11/d/(a-I*a*tan(d*x+c))^6-1/20*I*a^10/d/(a-I*a*tan(d*x
+c))^5-3/64*I*a^9/d/(a-I*a*tan(d*x+c))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-
5/128*I*a^7/d/(a-I*a*tan(d*x+c))^2-3/64*I*a^6/d/(a-I*a*tan(d*x+c))+1/128*I*
a^6/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6}-\frac{ia^{10}}{20d(a-ia\tan(c+dx))^5}-\frac{3ia^9}{64d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{5ia^7}{128d(a-ia\tan(c+dx))^2}-\frac{3ia^6}{64d(a-ia\tan(c+dx))}+\frac{ia^6}{128d(a+ia\tan(c+dx))}+\frac{7a^5x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (7*a^5*x)/128 - ((I/24)*a^11)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/20)*a^10)/
(d*(a - I*a*Tan[c + d*x])^5) - (((3*I)/64)*a^9)/(d*(a - I*a*Tan[c + d*x])^4
) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - (((5*I)/128)*a^7)/(d*(a - I
*a*Tan[c + d*x])^2) - (((3*I)/64)*a^6)/(d*(a - I*a*Tan[c + d*x])) + ((I/128
)*a^6)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{(ia^{13})\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^7(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{13})\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^7} + \frac{1}{4a^3(a-x)^6} + \frac{3}{16a^4(a-x)^5} + \frac{1}{8a^5(a-x)^4} + \right.\right.}{}$$

$$= -\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{}{64d(a}$$

$$= \frac{7a^5x}{128} - \frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} -$$

Mathematica [A]
time = 1.52, size = 159, normalized size = 0.80

$$\frac{a^5(-1750i\cos(c+dx) - 1575i\cos(3(c+dx)) - 84i\cos(5(c+dx)) + 840dx\cos(5(c+dx)) + 50i\cos(7(c+dx)) - 350\sin(c+dx) - 945\sin(3(c+dx)) + 84\sin(5(c+dx)) - 840idx\sin(5(c+dx)) + 70\sin(7(c+dx)))(\cos(5(c+2dx)) + i\sin(5(c+2dx)))}{15360d(\cos(dx) + i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*((-1750*I)*Cos[c + d*x] - (1575*I)*Cos[3*(c + d*x)] - (84*I)*Cos[5*(c
+ d*x)] + 840*d*x*Cos[5*(c + d*x)] + (50*I)*Cos[7*(c + d*x)] - 350*Sin[c +
d*x] - 945*Sin[3*(c + d*x)] + 84*Sin[5*(c + d*x)] - (840*I)*d*x*Sin[5*(c +
d*x)] + 70*Sin[7*(c + d*x)])*(Cos[5*(c + 2*d*x)] + I*Sin[5*(c + 2*d*x)]))/(
15360*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(168) = 336.
time = 0.29, size = 361, normalized size = 1.82

method	result
risch	$\frac{7a^5x}{128} - \frac{ia^5 e^{12i(dx+c)}}{1536d} - \frac{7ia^5 e^{10i(dx+c)}}{1280d} - \frac{21ia^5 e^{8i(dx+c)}}{1024d} - \frac{35ia^5 e^{6i(dx+c)}}{768d} - \frac{35ia^5 e^{4i(dx+c)}}{512d} - \frac{5ia^5\cos(2dx}{64d}$
derivativedivides	$\frac{ia^5\left(-\frac{(\sin^4(dx+c))(\cos^8(dx+c))}{12} - \frac{(\sin^2(dx+c))(\cos^8(dx+c))}{30} - \frac{(\cos^8(dx+c))}{120}\right) + 5a^5\left(-\frac{(\sin^3(dx+c))(\cos^9(dx+c))}{12} - \frac{(\cos^9(}{}\right.}{}$

default	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^8(dx+c)\right)}{12}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{30}-\frac{\left(\cos^8(dx+c)\right)}{120}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^9(dx+c)\right)}{12}-\frac{\left(\cos^9(d\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-
1/120*cos(d*x+c)^8)+5*a^5*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*cos(d*x+c)^
9*sin(d*x+c)+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*
cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-10*I*a^5*(-1/12*sin(d*x+c)^2*co
s(d*x+c)^10-1/60*cos(d*x+c)^10)-10*a^5*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/12
0*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315
/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-5/12*I*a^5*cos(d*x+c)^12
+a^5*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos
(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+
231/1024*c))
```

Maxima [A]

time = 0.52, size = 187, normalized size = 0.94

$$\frac{105\,(dx+c)a^5+\frac{105\,a^5\tan(dx+c)^{11}+595\,a^5\tan(dx+c)^9+1386\,a^5\tan(dx+c)^7+1686\,a^5\tan(dx+c)^5-240i\,a^5\tan(dx+c)^4+45\,a^5\tan(dx+c)^3+1824i\,a^5\tan(dx+c)^2+1815\,a^5\tan(dx+c)-496i\,a^5}{\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan(dx+c)^4+6\,\tan(dx+c)^2+1}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1920*(105*(d*x + c)*a^5 + (105*a^5*tan(d*x + c)^11 + 595*a^5*tan(d*x + c)
^9 + 1386*a^5*tan(d*x + c)^7 + 1686*a^5*tan(d*x + c)^5 - 240*I*a^5*tan(d*x
+ c)^4 + 45*a^5*tan(d*x + c)^3 + 1824*I*a^5*tan(d*x + c)^2 + 1815*a^5*tan(d
*x + c) - 496*I*a^5)/(tan(d*x + c)^12 + 6*tan(d*x + c)^10 + 15*tan(d*x + c)
^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 1))/d
```

Fricas [A]

time = 0.39, size = 120, normalized size = 0.61

$$\frac{\left(840\,a^5dxe^{(2i\,dx+2i\,c)}-10i\,a^5e^{(14i\,dx+14i\,c)}-84i\,a^5e^{(12i\,dx+12i\,c)}-315i\,a^5e^{(10i\,dx+10i\,c)}-700i\,a^5e^{(8i\,dx+8i\,c)}-1050i\,a^5e^{(6i\,dx+6i\,c)}-1260i\,a^5e^{(4i\,dx+4i\,c)}+60i\,a^5\right)e^{(-2i\,dx-2i\,c)}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/15360*(840*a^5*d*x*e^(2*I*d*x + 2*I*c) - 10*I*a^5*e^(14*I*d*x + 14*I*c) -
 84*I*a^5*e^(12*I*d*x + 12*I*c) - 315*I*a^5*e^(10*I*d*x + 10*I*c) - 700*I*a
^5*e^(8*I*d*x + 8*I*c) - 1050*I*a^5*e^(6*I*d*x + 6*I*c) - 1260*I*a^5*e^(4*I
*d*x + 4*I*c) + 60*I*a^5)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A]
time = 0.56, size = 301, normalized size = 1.52

$$\frac{7a^5x}{128} + \begin{cases} \frac{(-33776997205278720ia^5d^6e^{14ic}e^{12idx} - 283726776524341248ia^5d^6e^{12ic}e^{10idx} - 1063975411966279680ia^5d^6e^{10ic}e^{8idx} - 2364389804369510400ia^5d^6e^{8ic}e^{6idx} - 3546584706554265600ia^5d^6e^{6ic}e^{4idx} - 4255901647865118720ia^5d^6e^{4ic}e^{2idx} + 202661983231672320ia^5d^6e^{-2idx})e^{-2ic}}{51881467707308113920d^7} & \text{for } d^7e^{2ic} \neq 0 \\ x\left(-\frac{7a^5}{128} + \frac{(a^5e^{14ic} + 7a^5e^{12ic} + 21a^5e^{10ic} + 35a^5e^{8ic} + 35a^5e^{6ic} + 21a^5e^{4ic} + 7a^5e^{2ic} + a^5)e^{-2ic}}{128}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 7*a**5*x/128 + Piecewise(((-33776997205278720*I*a**5*d**6*exp(14*I*c)*exp(1
2*I*d*x) - 283726776524341248*I*a**5*d**6*exp(12*I*c)*exp(10*I*d*x) - 10639
75411966279680*I*a**5*d**6*exp(10*I*c)*exp(8*I*d*x) - 2364389804369510400*I
*a**5*d**6*exp(8*I*c)*exp(6*I*d*x) - 3546584706554265600*I*a**5*d**6*exp(6*
I*c)*exp(4*I*d*x) - 4255901647865118720*I*a**5*d**6*exp(4*I*c)*exp(2*I*d*x)
 + 202661983231672320*I*a**5*d**6*exp(-2*I*d*x))*exp(-2*I*c)/(5188146770730
8113920*d**7), Ne(d**7*exp(2*I*c), 0)), (x*(-7*a**5/128 + (a**5*exp(14*I*c)
 + 7*a**5*exp(12*I*c) + 21*a**5*exp(10*I*c) + 35*a**5*exp(8*I*c) + 35*a**5*
exp(6*I*c) + 21*a**5*exp(4*I*c) + 7*a**5*exp(2*I*c) + a**5)*exp(-2*I*c)/128
), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 914 vs. 2(154) = 308.
time = 0.96, size = 914, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/122880*(6720*a^5*d*x*e^(18*I*d*x + 10*I*c) + 53760*a^5*d*x*e^(16*I*d*x +
8*I*c) + 188160*a^5*d*x*e^(14*I*d*x + 6*I*c) + 376320*a^5*d*x*e^(12*I*d*x +
 4*I*c) + 470400*a^5*d*x*e^(10*I*d*x + 2*I*c) + 188160*a^5*d*x*e^(6*I*d*x -
 2*I*c) + 53760*a^5*d*x*e^(4*I*d*x - 4*I*c) + 6720*a^5*d*x*e^(2*I*d*x - 6*I
*c) + 376320*a^5*d*x*e^(8*I*d*x) - 2355*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) - 65940*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
- 131880*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 164850*I
*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 65940*I*a^5*e^(6*I
*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(4*I*d*x - 4*I*c
)*log(e^(2*I*d*x + 2*I*c) + 1) - 2355*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) - 131880*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) +
 2355*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 18840*I*a
^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 65940*I*a^5*e^(14*I
*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880*I*a^5*e^(12*I*d*x + 4*
I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 164850*I*a^5*e^(10*I*d*x + 2*I*c)*log(
e^(2*I*d*x) + e^(-2*I*c)) + 65940*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x)
```

```
 + e^(-2*I*c)) + 18840*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) + 2355*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880
*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 80*I*a^5*e^(30*I*d*x + 2
2*I*c) - 1312*I*a^5*e^(28*I*d*x + 20*I*c) - 10136*I*a^5*e^(26*I*d*x + 18*I*
c) - 49056*I*a^5*e^(24*I*d*x + 16*I*c) - 166992*I*a^5*e^(22*I*d*x + 14*I*c)
 - 426720*I*a^5*e^(20*I*d*x + 12*I*c) - 845712*I*a^5*e^(18*I*d*x + 10*I*c)
- 1304736*I*a^5*e^(16*I*d*x + 8*I*c) - 1538256*I*a^5*e^(14*I*d*x + 6*I*c) -
 1340192*I*a^5*e^(12*I*d*x + 4*I*c) - 820120*I*a^5*e^(10*I*d*x + 2*I*c) - 6
2160*I*a^5*e^(6*I*d*x - 2*I*c) + 3360*I*a^5*e^(4*I*d*x - 4*I*c) + 3840*I*a^
5*e^(2*I*d*x - 6*I*c) - 321440*I*a^5*e^(8*I*d*x) + 480*I*a^5*e^(-8*I*c))/(d
*e^(18*I*d*x + 10*I*c) + 8*d*e^(16*I*d*x + 8*I*c) + 28*d*e^(14*I*d*x + 6*I*
c) + 56*d*e^(12*I*d*x + 4*I*c) + 70*d*e^(10*I*d*x + 2*I*c) + 28*d*e^(6*I*d*
x - 2*I*c) + 8*d*e^(4*I*d*x - 4*I*c) + d*e^(2*I*d*x - 6*I*c) + 56*d*e^(8*I*
d*x))
```

Mupad [B]
time = 4.90, size = 171, normalized size = 0.86

$$\frac{7\,a^5\,x}{128}-\frac{-\frac{7\,a^5\,\tan(c+d\,x)^6}{128}-\frac{a^5\,\tan(c+d\,x)^5\,35\mathrm{i}}{128}+\frac{49\,a^5\,\tan(c+d\,x)^4}{96}+\frac{a^5\,\tan(c+d\,x)^3\,35\mathrm{i}}{96}+\frac{63\,a^5\,\tan(c+d\,x)^2}{640}+\frac{a^5\,\tan(c+d\,x)\,133\mathrm{i}}{384}-\frac{31\,a^5}{120}}{d\,\left(\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,5\mathrm{i}-9\,\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,5\mathrm{i}-5\,\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,9\mathrm{i}+5\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (7*a^5*x)/128 - ((a^5*tan(c + d*x)*133i)/384 - (31*a^5)/120 + (63*a^5*tan(c
 + d*x)^2)/640 + (a^5*tan(c + d*x)^3*35i)/96 + (49*a^5*tan(c + d*x)^4)/96 -
 (a^5*tan(c + d*x)^5*35i)/128 - (7*a^5*tan(c + d*x)^6)/128)/(d*(5*tan(c + d
*x) - tan(c + d*x)^2*9i - 5*tan(c + d*x)^3 - tan(c + d*x)^4*5i - 9*tan(c +
d*x)^5 + tan(c + d*x)^6*5i + tan(c + d*x)^7 + 1i))
```

3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=167

$$\frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{63ia^5\sec(c+dx)}{8d}+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}+\frac{ia\sec(c+dx)(a+ia}{5d}$$

```
[Out] 63/8*a^5*arctanh(sin(d*x+c))/d+63/8*I*a^5*sec(d*x+c)/d+9/20*I*a^2*sec(d*x+c
)*(a+I*a*tan(d*x+c))^3/d+1/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^4/d+21/20*I*
a*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+21/8*I*sec(d*x+c)*(a^5+I*a^5*tan(d*
x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3579, 3567, 3855}

$$\frac{63ia^5\sec(c+dx)}{8d}+\frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d}+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (63*a^5*ArcTanh[Sin[c + d*x]])/(8*d) + (((63*I)/8)*a^5*Sec[c + d*x])/d + ((
(9*I)/20)*a^2*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + ((I/5)*a*Sec[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d + (((21*I)/20)*a*Sec[c + d*x]*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((21*I)/8)*Sec[c + d*x]*(a^5 + I*a^5*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{1}{5}(9a)\int \sec(c+dx)(a+ia\,\text{ta}$$
$$= \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} + \frac{ia\sec(c+dx)(a+ia\tan(c}{5d}$$
$$= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\,\text{ta}}{20d}$$
$$= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\,\text{ta}}{20d}$$
$$= \frac{63ia^5\sec(c+dx)}{8d} + \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\,\text{s}}{}$$
$$= \frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d} + \frac{63ia^5\sec(c+dx)}{8d} + \frac{21ia^3\sec(c+dx)(}{2}$$

Mathematica [A]
time = 0.85, size = 115, normalized size = 0.69

$$\frac{a^5(\cos(5dx)+i\sin(5dx))\left(5040\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^5(c+dx)(1344+1920\cos(2(c+dx))+640\cos(4(c+dx))+450i\sin(2(c+dx))+325i\sin(4(c+dx)))\right)}{320d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(Cos[5*d*x] + I*Sin[5*d*x])*(5040*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + I*Sec[c + d*x]^5*(1344 + 1920*Cos[2*(c + d*x)] + 640*Cos[4*(c + d*x)] +
 (450*I)*Sin[2*(c + d*x)] + (325*I)*Sin[4*(c + d*x)])))/(320*d*(Cos[d*x] +
I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(146) = 292.
time = 0.30, size = 312, normalized size = 1.87

method	result
risch	$\frac{ia^5\left(965\,e^{9i(dx+c)}+2370\,e^{7i(dx+c)}+2688\,e^{5i(dx+c)}+1470\,e^{3i(dx+c)}+315\,e^{i(dx+c)}\right)}{20d\left(e^{2i(dx+c)}+1\right)^5}+\frac{63a^5\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{63a^5\ln(e^{i}}{8}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+5a^5\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx}{8\cos(dx}\right.$

default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^6(dx+c)}{15\cos(dx+c)^3}+\frac{\sin^6(dx+c)}{5\cos(dx+c)}+\frac{\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}\right)+5a^5\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(c}{8\cos(c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/5*sin(d*x+c)^6/cos(d*x+c)^5-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/
5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c
))+5*a^5*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-10*I*a^5*(1/3*sin
(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos
(d*x+c))-10*a^5*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*
x+c)+tan(d*x+c)))+5*I*a^5/cos(d*x+c)+a^5*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 215, normalized size = 1.29

$$\frac{75\,a^5\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+600\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+240\,a^5\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{1200i\,a^5}{\cos(dx+c)}+\frac{800i\left(3\,\cos(dx+c)^2-1\right)a^5}{\cos(dx+c)^3}+\frac{16i\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2+3\right)a^5}{\cos(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a^5*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 60
0*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 240*a^5*log(sec(d*x + c) + tan(d*x + c)) + 1200*I*a^5/co
s(d*x + c) + 800*I*(3*cos(d*x + c)^2 - 1)*a^5/cos(d*x + c)^3 + 16*I*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^5/cos(d*x + c)^5)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(137) = 274.
time = 0.35, size = 310, normalized size = 1.86

$$\frac{1930i\,a^5e^{(9i\,dx+9i\,c)}+4740i\,a^5e^{(7i\,dx+7i\,c)}+5376i\,a^5e^{(5i\,dx+5i\,c)}+2940i\,a^5e^{(3i\,dx+3i\,c)}+630i\,a^5e^{(i\,dx+i\,c)}+315\left(a^5e^{(10i\,dx+10i\,c)}+5\,a^5e^{(8i\,dx+8i\,c)}+10\,a^5e^{(6i\,dx+6i\,c)}+10\,a^5e^{(4i\,dx+4i\,c)}+5\,a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(a^5e^{(10i\,dx+10i\,c)}+5\,a^5e^{(8i\,dx+8i\,c)}+10\,a^5e^{(6i\,dx+6i\,c)}+10\,a^5e^{(4i\,dx+4i\,c)}+5\,a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{40\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/40*(1930*I*a^5*e^(9*I*d*x + 9*I*c) + 4740*I*a^5*e^(7*I*d*x + 7*I*c) + 537
6*I*a^5*e^(5*I*d*x + 5*I*c) + 2940*I*a^5*e^(3*I*d*x + 3*I*c) + 630*I*a^5*e^
(I*d*x + I*c) + 315*(a^5*e^(10*I*d*x + 10*I*c) + 5*a^5*e^(8*I*d*x + 8*I*c)
+ 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d*x + 4*I*c) + 5*a^5*e^(2*I*d*
x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) + I) - 315*(a^5*e^(10*I*d*x + 10*I*c)
```

```
 + 5*a^5*e^(8*I*d*x + 8*I*c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d
*x + 4*I*c) + 5*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia^5\left(\int\left(-i\sec\left(c+dx\right)\right)dx+\int 5\tan\left(c+dx\right)\sec\left(c+dx\right)dx+\int\left(-10\tan^3\left(c+dx\right)\sec\left(c+dx\right)\right)dx+\int\tan^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int 10i\tan^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\left(-5i\tan^4\left(c+dx\right)\sec\left(c+dx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x), x) + Integral(5*tan(c + d*x)*sec(c + d*x)
, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x), x) + Integral(tan(c + d*x
)**5*sec(c + d*x), x) + Integral(10*I*tan(c + d*x)**2*sec(c + d*x), x) + In
tegral(-5*I*tan(c + d*x)**4*sec(c + d*x), x))
```

Giac [A]
time = 0.86, size = 189, normalized size = 1.13

$$\frac{315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+200i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-750\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1600i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3280i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+750\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2240i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+488i\,a^5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^5*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^5*log(tan(1/2*d*x + 1/2
*c) - 1) - 2*(275*a^5*tan(1/2*d*x + 1/2*c)^9 + 200*I*a^5*tan(1/2*d*x + 1/2*
c)^8 - 750*a^5*tan(1/2*d*x + 1/2*c)^7 - 1600*I*a^5*tan(1/2*d*x + 1/2*c)^6 +
 3280*I*a^5*tan(1/2*d*x + 1/2*c)^4 + 750*a^5*tan(1/2*d*x + 1/2*c)^3 - 2240*
I*a^5*tan(1/2*d*x + 1/2*c)^2 - 275*a^5*tan(1/2*d*x + 1/2*c) + 488*I*a^5)/(t
an(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 7.04, size = 228, normalized size = 1.37

$$\frac{63\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{55\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,10{}\mathrm{i}-\frac{75\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,80{}\mathrm{i}+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,164{}\mathrm{i}+\frac{75\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,112{}\mathrm{i}-\frac{55\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a^5\,122{}\mathrm{i}}{5}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x),x)
```

```
[Out] (63*a^5*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((75*a^5*tan(c/2 + (d*x)/2)^3)/2
 - a^5*tan(c/2 + (d*x)/2)^2*112i + a^5*tan(c/2 + (d*x)/2)^4*164i - a^5*tan(
c/2 + (d*x)/2)^6*80i - (75*a^5*tan(c/2 + (d*x)/2)^7)/2 + a^5*tan(c/2 + (d*x
)/2)^8*10i + (55*a^5*tan(c/2 + (d*x)/2)^9)/4 + (a^5*122i)/5 - (55*a^5*tan(c
/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)
)
```

3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=130

$$-\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{35ia^5\sec(c+dx)}{2d}-\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos(c+dx)(a+}{d}$$

```
[Out] -35/2*a^5*arctanh(sin(d*x+c))/d-35/2*I*a^5*sec(d*x+c)/d-7/3*I*a^3*sec(d*x+c
)*(a+I*a*tan(d*x+c))^2/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^4/d-35/6*I*sec
(d*x+c)*(a^5+I*a^5*tan(d*x+c))/d
```

Rubi [A]

time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3577, 3579, 3567, 3855}

$$-\frac{35ia^5\sec(c+dx)}{2d}-\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d}-\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (-35*a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (((35*I)/2)*a^5*Sec[c + d*x])/d - (
((7*I)/3)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d - (((35*I)/6)*Sec[c + d*x]*(a^5 + I*a^5*Tan
[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
```

```
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - (7a^2)\int \sec(c+dx)(a+i\\ &= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\,\mathrm{ta}}{d}\\ &= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\,\mathrm{ta}}{d}\\ &= -\frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\,\mathrm{c}}{}\\ &= -\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(}{}\end{aligned}$$

Mathematica [A]
time = 1.40, size = 151, normalized size = 1.16

$$\frac{a^5\cos^2(c+dx)\left(-840i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^3(c+dx)(\cos(5c)-i\sin(5c))+(\cos(4c-dx)-i\sin(4c-dx))(511\cos(c+dx)+153\cos(3(c+dx))-i(49\sin(c+dx)+57\sin(3(c+dx))))\right)(-i+\tan(c+dx))^5}{24d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^2*((-840*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c +
 d*x]^3*(Cos[5*c] - I*Sin[5*c]) + (Cos[4*c - d*x] - I*Sin[4*c - d*x])*(511*
Cos[c + d*x] + 153*Cos[3*(c + d*x)] - I*(49*Sin[c + d*x] + 57*Sin[3*(c + d*
x)])))*(-I + Tan[c + d*x])^5)/(24*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [A]
time = 0.26, size = 226, normalized size = 1.74

method	result

risch	$-\frac{16ia^5 e^{i(dx+c)}}{d}-\frac{ia^5\left(87\,e^{5i(dx+c)}+136\,e^{3i(dx+c)}+57\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{35a^5\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{35a^5\ln\left(e^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin}{}\right.}{}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+5*a^5*(1/2*sin(d*x+c)^5/cos(d*x+c)^2
+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))-10*I*a^5*(s
in(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))-10*a^5*(-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c)))-5*I*a^5*cos(d*x+c)+a^5*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 173, normalized size = 1.33

$$-\frac{15\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)+120i\,a^5\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+4i\,a^5\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)+60\,a^5(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))+60i\,a^5\cos\left(dx+c\right)-12\,a^5\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 120*I*a^5*(1/cos(d*x + c) +
 cos(d*x + c)) + 4*I*a^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
 + c)) + 60*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x
+ c)) + 60*I*a^5*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 216, normalized size = 1.66

$$\frac{-96i\,a^5e^{(7i\,dx+7i\,c)}-462i\,a^5e^{(5i\,dx+5i\,c)}-560i\,a^5e^{(3i\,dx+3i\,c)}-210i\,a^5e^{(i\,dx+i\,c)}-105\left(a^5e^{(6i\,dx+6i\,c)}+3\,a^5e^{(4i\,dx+4i\,c)}+3\,a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)+105\left(a^5e^{(6i\,dx+6i\,c)}+3\,a^5e^{(4i\,dx+4i\,c)}+3\,a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(-96*I*a^5*e^(7*I*d*x + 7*I*c) - 462*I*a^5*e^(5*I*d*x + 5*I*c) - 560*I*
a^5*e^(3*I*d*x + 3*I*c) - 210*I*a^5*e^(I*d*x + I*c) - 105*(a^5*e^(6*I*d*x +
 6*I*c) + 3*a^5*e^(4*I*d*x + 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(
e^(I*d*x + I*c) + I) + 105*(a^5*e^(6*I*d*x + 6*I*c) + 3*a^5*e^(4*I*d*x + 4*
I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I
*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.31, size = 197, normalized size = 1.52

$$\frac{35a^5\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-87ia^5e^{5ic}e^{5idx}-136ia^5e^{3ic}e^{3idx}-57ia^5e^{ic}e^{idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}+\begin{cases}-\frac{16ia^5e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 16a^5xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 35*a**5*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-87*I*a**5*exp(5*I*c)*exp(5*I*d*x) - 136*I*a**5*exp(3*I*c)*exp(3*I*d*
x) - 57*I*a**5*exp(I*c)*exp(I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*exp(
4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d) + Piecewise((-16*I
*a**5*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (16*a**5*x*exp(I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(108) = 216.
time = 0.96, size = 510, normalized size = 3.92

$$\frac{8295a^5e^{(6idx+6ic)}\log\left(ie^{(idx+ic)}+1\right)+24885a^5e^{(4idx+4ic)}\log\left(ie^{(idx+ic)}+1\right)+24885a^5e^{(2idx+2ic)}\log\left(ie^{(idx+ic)}+1\right)-18585a^5e^{(6idx+6ic)}\log\left(ie^{(idx+ic)}-1\right)-55755a^5e^{(4idx+4ic)}\log\left(ie^{(idx+ic)}-1\right)-55755a^5e^{(2idx+2ic)}\log\left(ie^{(idx+ic)}-1\right)-8295a^5e^{(6idx+6ic)}\log\left(-ie^{(idx+ic)}+1\right)-24885a^5e^{(4idx+4ic)}\log\left(-ie^{(idx+ic)}+1\right)-24885a^5e^{(2idx+2ic)}\log\left(-ie^{(idx+ic)}+1\right)+18585a^5e^{(6idx+6ic)}\log\left(-ie^{(idx+ic)}-1\right)+55755a^5e^{(4idx+4ic)}\log\left(-ie^{(idx+ic)}-1\right)+55755a^5e^{(2idx+2ic)}\log\left(-ie^{(idx+ic)}-1\right)-24576ia^5e^{(7idx+7ic)}-118272ia^5e^{(5idx+5ic)}-143360ia^5e^{(3idx+3ic)}-53760ia^5e^{(idx+ic)}+8295a^5\log\left(ie^{(idx+ic)}+1\right)-18585a^5\log\left(ie^{(idx+ic)}-1\right)-8295a^5\log\left(-ie^{(idx+ic)}+1\right)+18585a^5\log\left(-ie^{(idx+ic)}-1\right)}{1536\left(de^{(6idx+6ic)}+3de^{(4idx+4ic)}+3de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1536*(8295*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5
*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5*e^(2*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) - 18585*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*
d*x + I*c) - 1) - 55755*a^5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 55755*a^5*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*e^(6*
I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(4*I*d*x + 4*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) + 18585*a^5*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) +
55755*a^5*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 55755*a^5*e^(2*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 24576*I*a^5*e^(7*I*d*x + 7*I*c
) - 118272*I*a^5*e^(5*I*d*x + 5*I*c) - 143360*I*a^5*e^(3*I*d*x + 3*I*c) - 5
3760*I*a^5*e^(I*d*x + I*c) + 8295*a^5*log(I*e^(I*d*x + I*c) + 1) - 18585*a^
5*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*log(-I*e^(I*d*x + I*c) + 1) + 18585
*a^5*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 7.14, size = 222, normalized size = 1.71

$$-\frac{35\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{37\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,27{}\mathrm{i}-118\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,48{}\mathrm{i}+139\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,55{}\mathrm{i}}{3}-\frac{166\,a^5}{3}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,1{}\mathrm{i}+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,3{}\mathrm{i}-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^5,x)

[Out] - (35*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (139*a^5*tan(c/2 + (d*x)/2)^2 - a^5*tan(c/2 + (d*x)/2)^3*48i - 118*a^5*tan(c/2 + (d*x)/2)^4 + a^5*tan(c/2 + (d*x)/2)^5*27i + 37*a^5*tan(c/2 + (d*x)/2)^6 - (166*a^5)/3 + (a^5*tan(c/2 + (d*x)/2)*55i)/3)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + 3*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*1i - tan(c/2 + (d*x)/2)^7 + 1i))

3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=98

$$\frac{5a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia}{3d}$$

```
[Out] 5*a^5*arctanh(sin(d*x+c))/d+5*I*a^5*sec(d*x+c)/d+10/3*I*a^3*cos(d*x+c)*(a+I
*a*tan(d*x+c))^2/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3577, 3567, 3855}

$$\frac{5ia^5\sec(c+dx)}{d}+\frac{5a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/d + ((5*I)*a^5*Sec[c + d*x])/d + (((10*I)/3)*
a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/3)*a*Cos[c + d*x]^3*
(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d} - \frac{1}{3}\left(5a^2\right)\int \cos(c+dx)(\\
&= \frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos^3(c+dx)(a+ia}{3d} \\
&= \frac{5ia^5\sec(c+dx)}{d} + \frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia}{} \\
&= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{d} + \frac{5ia^5\sec(c+dx)}{d} + \frac{10ia^3\cos(c+dx)(}{}
\end{aligned}
$$

Mathematica [A]

time = 1.38, size = 130, normalized size = 1.33

$$\frac{a^5\cos^4(c+dx)\left(30\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(5c)+\sin(5c))-(\cos(3c-2dx)-i\sin(3c-2dx))(10+13\cos(2(c+dx))-17i\sin(2(c+dx)))\right)(-i+\tan(c+dx))^5}{3d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^4*(30*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*
(I*Cos[5*c] + Sin[5*c]) - (Cos[3*c - 2*d*x] - I*Sin[3*c - 2*d*x])*(10 + 13*
Cos[2*(c + d*x)] - (17*I)*Sin[2*(c + d*x)]))*(-I + Tan[c + d*x])^5)/(3*d*(C
os[d*x] + I*Sin[d*x])^5)
```

Maple [A]

time = 0.20, size = 165, normalized size = 1.68

method	result
risch	$-\frac{4ia^5 e^{3i(dx+c)}}{3d}+\frac{8ia^5 e^{i(dx+c)}}{d}+\frac{2ia^5 e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{5a^5\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{5a^5\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos
(d*x+c))+5*a^5*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+10/
```

```
3*I*a^5*(2+sin(d*x+c)^2)*cos(d*x+c)-10/3*a^5*sin(d*x+c)^3-5/3*I*a^5*cos(d*x
+c)^3+1/3*a^5*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 154, normalized size = 1.57

$$-\frac{10i\,a^5\cos\left(dx+c\right)^3+20\,a^5\sin\left(dx+c\right)^3+2i\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^5+20i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^5+5\left(2\,\sin\left(dx+c\right)^3-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)a^5+2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^5}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*I*a^5*cos(d*x + c)^3 + 20*a^5*sin(d*x + c)^3 + 2*I*(cos(d*x + c)^3
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^5 + 20*I*(cos(d*x + c)^3 - 3*cos(d*x
+ c))*a^5 + 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
 c) - 1) + 6*sin(d*x + c))*a^5 + 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^5)/d
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.24

$$\frac{-4i\,a^5e^{(5i\,dx+5i\,c)}+20i\,a^5e^{(3i\,dx+3i\,c)}+30i\,a^5e^{(i\,dx+i\,c)}+15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-4*I*a^5*e^(5*I*d*x + 5*I*c) + 20*I*a^5*e^(3*I*d*x + 3*I*c) + 30*I*a^5
*e^(I*d*x + I*c) + 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) +
 I) - 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.32, size = 148, normalized size = 1.51

$$\frac{2ia^5e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d}+\frac{5a^5\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}\frac{-4ia^5de^{3ic}e^{3idx}+24ia^5de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x(4a^5e^{3ic}-8a^5e^{ic}) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 2*I*a**5*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 5*a**5*(-log
(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise((
(-4*I*a**5*d*exp(3*I*c)*exp(3*I*d*x) + 24*I*a**5*d*exp(I*c)*exp(I*d*x))/(3*
d**2), Ne(d**2, 0)), (x*(4*a**5*exp(3*I*c) - 8*a**5*exp(I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1683 vs. 2(84) = 168.
time = 1.22, size = 1683, normalized size = 17.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")

[Out] -1/6144*(39225*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 313800*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 313800*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2745750*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 39225*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8520*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 68160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 68160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 596400*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 8520*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 39225*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 313800*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 313800*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2745750*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 39225*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8520*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 68160*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 238560*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 238560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 68160*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 596400*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 8520*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 15*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 120*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 420*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 420*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 120*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1050*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 15*a^5*e^(-8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 420*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 420*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1050*a^5*e^(8*I*d*x)*l

```
og(-I*e^(I*d*x) + e^(-I*c)) - 15*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 8192*I*a^5*e^(19*I*d*x + 11*I*c) + 16384*I*a^5*e^(17*I*d*x + 9*I*c) - 1
76128*I*a^5*e^(15*I*d*x + 7*I*c) - 1003520*I*a^5*e^(13*I*d*x + 5*I*c) - 243
7120*I*a^5*e^(11*I*d*x + 3*I*c) - 3411968*I*a^5*e^(9*I*d*x + I*c) - 2953216
*I*a^5*e^(7*I*d*x - I*c) - 1568768*I*a^5*e^(5*I*d*x - 3*I*c) - 471040*I*a^5
*e^(3*I*d*x - 5*I*c) - 61440*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*
c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*
x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2
*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 5.43, size = 162, normalized size = 1.65

$$\frac{10\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{8\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,34\mathrm{i}-\frac{82\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,38\mathrm{i}+\frac{46\,a^5}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,3\mathrm{i}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (10*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (a^5*tan(c/2 + (d*x)/2)^3*34i - (82*
a^5*tan(c/2 + (d*x)/2)^2)/3 + 8*a^5*tan(c/2 + (d*x)/2)^4 + (46*a^5)/3 - a^5
*tan(c/2 + (d*x)/2)*38i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*4i
 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + tan(c/2 + (d*x)/2)^5
+ 1i))
```

3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=32

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

```
[Out] -1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3569\}$

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

Mathematica [A]
time = 0.19, size = 31, normalized size = 0.97

$$-\frac{ia^5(\cos(c+dx)+i\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/5*I)*a^5*(Cos[c + d*x] + I*Sin[c + d*x])^5)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(28) = 56.
time = 0.22, size = 170, normalized size = 5.31

method	result
risch	$-\frac{i a^5 e^{5 i (d x+c)}}{5 d}$
derivativedivides	$\frac{-\frac{i a^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10 i a^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-}{}$
default	$\frac{-\frac{i a^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10 i a^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*I*a^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a^5*sin(d*x+
c)^5-10*I*a^5*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)-10*a^5*(-1
/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-I*a^5*cos(d*x+
c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(26) = 52.
time = 0.30, size = 152, normalized size = 4.75

$$-\frac{15 i a^5 \cos(dx+c)^5-15 a^5 \sin(dx+c)^5+10 i\left(3 \cos(dx+c)^5-5 \cos(dx+c)^3\right)a^5+i\left(3 \cos(dx+c)^5-10 \cos(dx+c)^3+15 \cos(dx+c)\right)a^5-10\left(3 \sin(dx+c)^5-5 \sin(dx+c)^3\right)a^5-\left(3 \sin(dx+c)^5-10 \sin(dx+c)^3+15 \sin(dx+c)\right)a^5}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/15*(15*I*a^5*cos(d*x + c)^5 - 15*a^5*sin(d*x + c)^5 + 10*I*(3*cos(d*x +
c)^5 - 5*cos(d*x + c)^3)*a^5 + I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15
*cos(d*x + c))*a^5 - 10*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^5 - (3*sin(
d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5)/d
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.53

$$-\frac{i\,a^5 e^{(5 i\,dx+5 i\,c)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/5*I*a^5*e^(5*I*d*x + 5*I*c)/d
```

Sympy [A]
time = 0.22, size = 36, normalized size = 1.12

$$\begin{cases} -\frac{ia^5e^{5ic}e^{5idx}}{5d} & \text{for } d \neq 0 \\ a^5xe^{5ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise((-I*a**5*exp(5*I*c)*exp(5*I*d*x)/(5*d), Ne(d, 0)), (a**5*x*exp(5*
I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. 2(26) = 52.
time = 0.90, size = 1669, normalized size = 52.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/40960*(11375*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 91000
*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 318500*a^5*e^(12*I*d
*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(10*I*d*x + 2*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 318500*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 910
00*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 796250*a^5*e^(8*I*d
*x)*log(I*e^(I*d*x + I*c) + 1) + 11375*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 11590*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 92720*a
^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 324520*a^5*e^(12*I*d*x
 + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(10*I*d*x + 2*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 324520*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 92720
*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 811300*a^5*e^(8*I*d*x
)*log(I*e^(I*d*x + I*c) - 1) + 11590*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 11375*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 91000*a^
5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 318500*a^5*e^(12*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(10*I*d*x + 2*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 318500*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9
1000*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 796250*a^5*e^(8*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 11375*a^5*e^(-8*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 11590*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9
```

```
2720*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 324520*a^5*e^(1
2*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 649040*a^5*e^(10*I*d*x + 2*I
*c)*log(-I*e^(I*d*x + I*c) - 1) - 649040*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(
I*d*x + I*c) - 1) - 324520*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) -
 1) - 92720*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 811300*a^
5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 11590*a^5*e^(-8*I*c)*log(-I*e^(
I*d*x + I*c) - 1) + 215*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) + 1720*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6020*a^5*e^
(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12040*a^5*e^(10*I*d*x + 2*
I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) + 6020*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) + 1720*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 15050*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 215*a^5*e^(-8*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 215*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1
720*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 6020*a^5*e^(12*
I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12040*a^5*e^(10*I*d*x + 2*I*c
)*log(-I*e^(I*d*x) + e^(-I*c)) - 12040*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 6020*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) - 1720*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 15050*a^5*e
^(8*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 215*a^5*e^(-8*I*c)*log(-I*e^(I*d*
x) + e^(-I*c)) + 8192*I*a^5*e^(21*I*d*x + 13*I*c) + 65536*I*a^5*e^(19*I*d*x
 + 11*I*c) + 229376*I*a^5*e^(17*I*d*x + 9*I*c) + 458752*I*a^5*e^(15*I*d*x +
 7*I*c) + 573440*I*a^5*e^(13*I*d*x + 5*I*c) + 458752*I*a^5*e^(11*I*d*x + 3*
I*c) + 229376*I*a^5*e^(9*I*d*x + I*c) + 65536*I*a^5*e^(7*I*d*x - I*c) + 819
2*I*a^5*e^(5*I*d*x - 3*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*
I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*
d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^
(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 3.45, size = 104, normalized size = 3.25

$$\frac{2\,a^5\,\left(5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{5\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5{}\mathrm{i}-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10{}\mathrm{i}+5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (2*a^5*(5*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 1))/(5*d*(5*tan(
c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c
/2 + (d*x)/2)^4*5i + tan(c/2 + (d*x)/2)^5 + 1i))
```

3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=101

$$-\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}-\frac{i\cos^7(c+dx)(a+ia\tan($$

```
[Out] -2/105*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-2/35*I*a*cos(d*x+c)^5*(a+I
*a*tan(d*x+c))^4/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d
```

Rubi [A]

time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3578, 3569}

$$-\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/105)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*
a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4)/d - ((I/7)*Cos[c + d*x]^7*(a + I
*a*Tan[c + d*x])^5)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}+\frac{1}{7}(2a)\int \cos^5(c+dx)(a-\\ &= -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}-\frac{i\cos^7(c+dx)(a+ia\,\mathrm{t}}{7d}\\ &= -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{2ia\cos^5(c+dx)(a+}{35d}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 55, normalized size = 0.54

$$\frac{a^5(21 + 25\cos(2(c + dx)) - 10i\sin(2(c + dx)))(-i\cos(5(c + dx)) + \sin(5(c + dx)))}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(21 + 25*Cos[2*(c + d*x)] - (10*I)*Sin[2*(c + d*x)])*((-I)*Cos[5*(c +
d*x)] + Sin[5*(c + d*x)]))/(210*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(89) = 178.
time = 0.26, size = 257, normalized size = 2.54

method	result
risch	$-\frac{i a^5 e^{7i(dx+c)}}{28d} - \frac{i a^5 e^{5i(dx+c)}}{10d} - \frac{i a^5 e^{3i(dx+c)}}{12d}$
derivativedivides	$ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{7} - \frac{4\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{35} - \frac{8\left(\cos^3(dx+c)\right)}{105}\right) + 5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7} - \frac{3\sin(d}{}\right.$
default	$ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{7} - \frac{4\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{35} - \frac{8\left(\cos^3(dx+c)\right)}{105}\right) + 5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7} - \frac{3\sin(d}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*cos(d*x+c)^3*sin(d*x+c)^2-8
/105*cos(d*x+c)^3)+5*a^5*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*co
s(d*x+c)^4+1/35*(cos(d*x+c)^2+2)*sin(d*x+c))-10*I*a^5*(-1/7*sin(d*x+c)^2*co
s(d*x+c)^5-2/35*cos(d*x+c)^5)-10*a^5*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/
3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-5/7*I*a^5*cos(d*x+c)^7+1/7*a^5
*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(83) = 166.
time = 0.29, size = 187, normalized size = 1.85

$$-\frac{75i\,a^5\cos(dx+c)^7 + i\left(15\cos(dx+c)^7 - 42\cos(dx+c)^5 + 35\cos(dx+c)^3\right)a^5 + 30i\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)a^5 + 10\left(15\sin(dx+c)^7 - 42\sin(dx+c)^5 + 35\sin(dx+c)^3\right)a^5 + 15\left(5\sin(dx+c)^7 - 7\sin(dx+c)^5\right)a^5 + 3\left(5\sin(dx+c)^7 - 21\sin(dx+c)^5 + 35\sin(dx+c)^3 - 35\sin(dx+c)\right)a^5}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*I*a^5*cos(d*x + c)^7 + I*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5
+ 35*cos(d*x + c)^3)*a^5 + 30*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^5 +
 10*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5 + 15*(5
*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^5 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5)/d
```

Fricas [A]
time = 0.39, size = 48, normalized size = 0.48

$$\frac{-15i\,a^5e^{(7i\,dx+7i\,c)} - 42i\,a^5e^{(5i\,dx+5i\,c)} - 35i\,a^5e^{(3i\,dx+3i\,c)}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/420*(-15*I*a^5*e^(7*I*d*x + 7*I*c) - 42*I*a^5*e^(5*I*d*x + 5*I*c) - 35*I*
a^5*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A]
time = 0.37, size = 121, normalized size = 1.20

$$\begin{cases} \frac{-120ia^5d^2e^{7ic}e^{7idx}-336ia^5d^2e^{5ic}e^{5idx}-280ia^5d^2e^{3ic}e^{3idx}}{3360d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^5e^{7ic}}{4}+\frac{a^5e^{5ic}}{2}+\frac{a^5e^{3ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-120*I*a**5*d**2*exp(7*I*c)*exp(7*I*d*x) - 336*I*a**5*d**2*exp(
5*I*c)*exp(5*I*d*x) - 280*I*a**5*d**2*exp(3*I*c)*exp(3*I*d*x))/(3360*d**3),
 Ne(d**3, 0)), (x*(a**5*exp(7*I*c)/4 + a**5*exp(5*I*c)/2 + a**5*exp(3*I*c)/
4), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(83) = 166$.
time = 1.02, size = 1697, normalized size = 16.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(7357770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5
8862160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a^5
*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(10*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(6*I*d*x - 2*I*c)*l
```

og(I*e^(I*d*x + I*c) + 1) + 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 58862160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 515043900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7357770*a^5*e^(-8
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 206931900*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
13863800*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 413863800*a^
5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 206931900*a^5*e^(4*I*d*x
 - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(2*I*d*x - 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 517329750*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) -
 1) + 7390425*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 7357770*a^5*e^(16
*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 58862160*a^5*e^(14*I*d*x + 6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(12*I*d*x + 4*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 412035120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5886216
0*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 515043900*a^5*e^(8*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 7357770*a^5*e^(-8*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1)
 - 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 20693190
0*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(1
0*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(6*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 206931900*a^5*e^(4*I*d*x - 4*I*c)*log(-
I*e^(I*d*x + I*c) - 1) - 59123400*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 517329750*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 739042
5*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 32655*a^5*e^(16*I*d*x + 8*I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) + 914340*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) + 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18286
80*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 914340*a^5*e^(4*I*
d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(2*I*d*x - 6*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) + 2285850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-
I*c)) + 32655*a^5*e^(-8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(16*
I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(14*I*d*x + 6*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 914340*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) - 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) - 1828680*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 9
14340*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(
2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2285850*a^5*e^(8*I*d*x)*log
(-I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) + 122880*I*a^5*e^(23*I*d*x + 15*I*c) + 1327104*I*a^5*e^(21*I*d*x + 13*I*
c) + 6479872*I*a^5*e^(19*I*d*x + 11*I*c) + 18808832*I*a^5*e^(17*I*d*x + 9*I
*c) + 35897344*I*a^5*e^(15*I*d*x + 7*I*c) + 47022080*I*a^5*e^(13*I*d*x + 5*
I*c) + 42778624*I*a^5*e^(11*I*d*x + 3*I*c) + 26673152*I*a^5*e^(9*I*d*x + I*
c) + 10903552*I*a^5*e^(7*I*d*x - I*c) + 2637824*I*a^5*e^(5*I*d*x - 3*I*c) +

```
286720*I*a^5*e^(3*I*d*x - 5*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*
x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e
^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 7
0*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 4.21, size = 186, normalized size = 1.84

$$-\frac{2\,a^5\,\left(105\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,210{}\mathrm{i}-455\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,350{}\mathrm{i}+273\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,56{}\mathrm{i}-23\right)}{105\,d\,\left(-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,7{}\mathrm{i}+21\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,35{}\mathrm{i}-35\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,21{}\mathrm{i}+7\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -(2*a^5*(tan(c/2 + (d*x)/2)*56i + 273*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)^3*350i - 455*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*210i + 105*ta
n(c/2 + (d*x)/2)^6 - 23))/(105*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)
^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 +
(d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=141

$$-\frac{ia^5\cos^5(c+dx)}{105d}+\frac{a^5\sin(c+dx)}{21d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))}{63d}$$

```
[Out] -1/105*I*a^5*cos(d*x+c)^5/d+1/21*a^5*sin(d*x+c)/d-2/63*a^5*sin(d*x+c)^3/d+1
/105*a^5*sin(d*x+c)^5/d-2/63*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d-2/9*
I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3577, 3567, 2713}

$$\frac{a^5\sin^5(c+dx)}{105d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin(c+dx)}{21d}-\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/105*I)*a^5*Cos[c + d*x]^5)/d + (a^5*Sin[c + d*x])/(21*d) - (2*a^5*Sin[
c + d*x]^3)/(63*d) + (a^5*Sin[c + d*x]^5)/(105*d) - (((2*I)/63)*a^3*Cos[c +
 d*x]^7*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*
Tan[c + d*x])^4)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
```

```
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}+\frac{1}{9}a^2\int \cos^7(c+dx)(a\\ &= -\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia\cos^9(c+dx)(a+}{9d}\\ &= -\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2i}{}\\ &= -\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2i}{}\\ &= -\frac{ia^5\cos^5(c+dx)}{105d}+\frac{a^5\sin(c+dx)}{21d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin^5(}{10}\end{aligned}$$

Mathematica [A]
time = 0.62, size = 94, normalized size = 0.67

$$\frac{a^5(189+300\cos(2(c+dx))+175\cos(4(c+dx))-120i\sin(2(c+dx))-140i\sin(4(c+dx)))(-i\cos(5(c+2dx))+\sin(5(c+2dx)))}{2520d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(189 + 300*Cos[2*(c + d*x)] + 175*Cos[4*(c + d*x)] - (120*I)*Sin[2*(c
+ d*x)] - (140*I)*Sin[4*(c + d*x)])*((-I)*Cos[5*(c + 2*d*x)] + Sin[5*(c + 2
*d*x)]))/(2520*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(124) = 248.
time = 0.25, size = 287, normalized size = 2.04

method	result
risch	$-\frac{ia^5e^{9i(dx+c)}}{144d}-\frac{ia^5e^{7i(dx+c)}}{28d}-\frac{3ia^5e^{5i(dx+c)}}{40d}-\frac{ia^5e^{3i(dx+c)}}{12d}-\frac{ia^5e^{i(dx+c)}}{16d}$
derivativedivides	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^5(dx+c)\right)}{9}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{63}-\frac{8\left(\cos^5(dx+c)\right)}{315}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(}{}\right.}{}$

default	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^5(dx+c)\right)}{9}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{63}-\frac{8\left(\cos^5(dx+c)\right)}{315}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8
/315*cos(d*x+c)^5)+5*a^5*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*co
s(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-10*I*a^5*(
-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-10*a^5*(-1/9*sin(d*x+c)*c
os(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(
d*x+c))-5/9*I*a^5*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^
6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 217, normalized size = 1.54

$$-\frac{175i\,a^5\cos(dx+c)^9+i\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^5+50i\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^5-5\left(35\sin(dx+c)^9-90\sin(dx+c)^7+63\sin(dx+c)^5\right)a^5-10\left(35\sin(dx+c)^9-135\sin(dx+c)^7+189\sin(dx+c)^5-105\sin(dx+c)^3\right)a^5-\left(35\sin(dx+c)^9-180\sin(dx+c)^7+378\sin(dx+c)^5-420\sin(dx+c)^3+315\sin(dx+c)\right)a^5}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*I*a^5*cos(d*x + c)^9 + I*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7
 + 63*cos(d*x + c)^5)*a^5 + 50*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^5
- 5*(35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^5 - 10*(3
5*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
c)^3)*a^5 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 -
420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5)/d
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.54

$$\frac{-35i\,a^5e^{(9i\,dx+9i\,c)}-180i\,a^5e^{(7i\,dx+7i\,c)}-378i\,a^5e^{(5i\,dx+5i\,c)}-420i\,a^5e^{(3i\,dx+3i\,c)}-315i\,a^5e^{(i\,dx+i\,c)}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/5040*(-35*I*a^5*e^(9*I*d*x + 9*I*c) - 180*I*a^5*e^(7*I*d*x + 7*I*c) - 378
*I*a^5*e^(5*I*d*x + 5*I*c) - 420*I*a^5*e^(3*I*d*x + 3*I*c) - 315*I*a^5*e^(I
*d*x + I*c))/d
```

Sympy [A]
time = 0.44, size = 192, normalized size = 1.36

$$\begin{cases} \frac{-215040ia^5d^4e^{9ic}e^{9idx}-1105920ia^5d^4e^{7ic}e^{7idx}-2322432ia^5d^4e^{5ic}e^{5idx}-2580480ia^5d^4e^{3ic}e^{3idx}-1935360ia^5d^4e^{ic}e^{idx}}{30965760d^5} & \text{for } d^5 \neq 0 \\ x\left(\frac{a^5e^{9ic}}{16}+\frac{a^5e^{7ic}}{4}+\frac{3a^5e^{5ic}}{8}+\frac{a^5e^{3ic}}{4}+\frac{a^5e^{ic}}{16}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-215040*I*a**5*d**4*exp(9*I*c)*exp(9*I*d*x) - 1105920*I*a**5*d*
*4*exp(7*I*c)*exp(7*I*d*x) - 2322432*I*a**5*d**4*exp(5*I*c)*exp(5*I*d*x) -
2580480*I*a**5*d**4*exp(3*I*c)*exp(3*I*d*x) - 1935360*I*a**5*d**4*exp(I*c)*
exp(I*d*x))/(30965760*d**5), Ne(d**5, 0)), (x*(a**5*exp(9*I*c)/16 + a**5*ex
p(7*I*c)/4 + 3*a**5*exp(5*I*c)/8 + a**5*exp(3*I*c)/4 + a**5*exp(I*c)/16), T
rue))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(119) = 238.
time = 1.01, size = 1725, normalized size = 12.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/41287680*(69853455*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740
*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(10
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(6*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 4889741850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 69853455
*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 70703325*a^5*e^(16*I*d*x + 8*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 197969310
0*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(2*I
*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4949232750*a^5*e^(8*I*d*x)*log(I
*e^(I*d*x + I*c) - 1) + 70703325*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 69853455*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 558827640
*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(1
2*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(10*I*d*x +
 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(6*I*d*x - 2*I*c)*lo
```

```
g(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 4889741850*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 69853455*a^
5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 70703325*a^5*e^(16*I*d*x + 8*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 197969
3100*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^
(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4949232750*a^5*e^(8*I*d*x)*
log(-I*e^(I*d*x + I*c) - 1) - 70703325*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c
) - 1) + 849870*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6798
960*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(
12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(10*I*d*x +
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(6*I*d*x - 2*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) + 6798960*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
59490900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 849870*a^5*e^(-8*I*c
)*log(I*e^(I*d*x) + e^(-I*c)) - 849870*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) - 6798960*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 23796360*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 4
7592720*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 47592720*a^
5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 23796360*a^5*e^(4*I*d*
x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 6798960*a^5*e^(2*I*d*x - 6*I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) - 59490900*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x) + e
^(-I*c)) - 849870*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 286720*I*a^
5*e^(25*I*d*x + 17*I*c) + 3768320*I*a^5*e^(23*I*d*x + 15*I*c) + 22921216*I*
a^5*e^(21*I*d*x + 13*I*c) + 85557248*I*a^5*e^(19*I*d*x + 11*I*c) + 21945548
8*I*a^5*e^(17*I*d*x + 9*I*c) + 409665536*I*a^5*e^(15*I*d*x + 7*I*c) + 57229
3120*I*a^5*e^(13*I*d*x + 5*I*c) + 602341376*I*a^5*e^(11*I*d*x + 3*I*c) + 47
2096768*I*a^5*e^(9*I*d*x + I*c) + 267091968*I*a^5*e^(7*I*d*x - I*c) + 10287
5136*I*a^5*e^(5*I*d*x - 3*I*c) + 24084480*I*a^5*e^(3*I*d*x - 5*I*c) + 25804
80*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I
*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d
*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(
8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B]
time = 3.94, size = 79, normalized size = 0.56

$$-\frac{a^5\,\left(\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{16}+\frac{{\mathrm{e}}^{c\,3{}\mathrm{i}+d\,x\,3{}\mathrm{i}}\,1{}\mathrm{i}}{12}+\frac{{\mathrm{e}}^{c\,5{}\mathrm{i}+d\,x\,5{}\mathrm{i}}\,3{}\mathrm{i}}{40}+\frac{{\mathrm{e}}^{c\,7{}\mathrm{i}+d\,x\,7{}\mathrm{i}}\,1{}\mathrm{i}}{28}+\frac{{\mathrm{e}}^{c\,9{}\mathrm{i}+d\,x\,9{}\mathrm{i}}\,1{}\mathrm{i}}{144}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -(a^5*((exp(c*1i + d*x*1i)*1i)/16 + (exp(c*3i + d*x*3i)*1i)/12 + (exp(c*5i
+ d*x*5i)*3i)/40 + (exp(c*7i + d*x*7i)*1i)/28 + (exp(c*9i + d*x*9i)*1i)/144
))/d
```

3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=159

$$-\frac{5ia^5\cos^7(c+dx)}{231d}+\frac{5a^5\sin(c+dx)}{33d}-\frac{5a^5\sin^3(c+dx)}{33d}+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^7(c+dx)}{231d}-\frac{2ia^3\cos^9(c+dx}{}$$

```
[Out] -5/231*I*a^5*cos(d*x+c)^7/d+5/33*a^5*sin(d*x+c)/d-5/33*a^5*sin(d*x+c)^3/d+1
/11*a^5*sin(d*x+c)^5/d-5/231*a^5*sin(d*x+c)^7/d-2/33*I*a^3*cos(d*x+c)^9*(a+
I*a*tan(d*x+c))^2/d-2/11*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3577, 3567, 2713}

$$-\frac{5a^5\sin^7(c+dx)}{231d}+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^3(c+dx)}{33d}+\frac{5a^5\sin(c+dx)}{33d}-\frac{5ia^5\cos^7(c+dx)}{231d}-\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d}-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-5*I)/231)*a^5*Cos[c + d*x]^7)/d + (5*a^5*Sin[c + d*x])/(33*d) - (5*a^5*
Sin[c + d*x]^3)/(33*d) + (a^5*Sin[c + d*x]^5)/(11*d) - (5*a^5*Sin[c + d*x]^
7)/(231*d) - (((2*I)/33)*a^3*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d - (
((2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
```

```
LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} + \frac{1}{11}(3a^2)\int \cos^9(c+$$
$$= -\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a-}{11d}$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} -$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} -$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} + \frac{5a^5\sin(c+dx)}{33d} - \frac{5a^5\sin^3(c+dx)}{33d} + \frac{a^5 \mathrm{s}}{}$$

Mathematica [A]
time = 1.01, size = 118, normalized size = 0.74

$$\frac{ia^5(-462-825\cos(2(c+dx))-770\cos(4(c+dx))+105\cos(6(c+dx))+330i\sin(2(c+dx))+616i\sin(4(c+dx))-126i\sin(6(c+dx)))(\cos(5(c+2dx))+i\sin(5(c+2dx)))}{7392d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((I/7392)*a^5*(-462 - 825*Cos[2*(c + d*x)] - 770*Cos[4*(c + d*x)] + 105*Cos
[6*(c + d*x)] + (330*I)*Sin[2*(c + d*x)] + (616*I)*Sin[4*(c + d*x)] - (126*
I)*Sin[6*(c + d*x)])*(Cos[5*(c + 2*d*x)] + I*Sin[5*(c + 2*d*x)]))/(d*(Cos[d
*x] + I*Sin[d*x])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(140) = 280.
time = 0.30, size = 317, normalized size = 1.99

method	result
risch	$-\frac{ia^5 e^{11i(dx+c)}}{704d} - \frac{ia^5 e^{9i(dx+c)}}{96d} - \frac{15ia^5 e^{7i(dx+c)}}{448d} - \frac{ia^5 e^{5i(dx+c)}}{16d} - \frac{5ia^5 e^{3i(dx+c)}}{64d} - \frac{5ia^5\cos(dx+c)}{64d} + \frac{7a^5 \sin}{6}$
derivativedivides	$\frac{ia^5\left(-\frac{(\sin^4(dx+c))(\cos^7(dx+c))}{11} - \frac{4(\sin^2(dx+c))(\cos^7(dx+c))}{99} - \frac{8(\cos^7(dx+c))}{693}\right) + 5a^5\left(-\frac{(\sin^3(dx+c))(\cos^8(dx+c))}{11} - \frac{\sin(}{}\right.}{}$

default	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99}-\frac{8\left(\cos^7(dx+c)\right)}{693}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{11}-\frac{\sin(dx}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*a^5*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+5*a^5*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*
cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))-10*I*a^5*(-1/11*sin(d*x+c)^2*cos(d*x+c)^9-2/99*cos(d*x+c)^9)-10*a
^5*(-1/11*cos(d*x+c)^10*sin(d*x+c)+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)
^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-5/11*I*a^5*cos(d*x+c)
^11+1/11*a^5*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/
21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 246, normalized size = 1.55

$$-\frac{315i\,a^5\cos(dx+c)^{11}+i\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^5+70i\left(9\cos(dx+c)^{11}-11\cos(dx+c)^9\right)a^5+2\left(315\sin(dx+c)^{11}-1540\sin(dx+c)^9+2970\sin(dx+c)^7-2772\sin(dx+c)^5+1155\sin(dx+c)^3\right)a^5+3\left(105\sin(dx+c)^{11}-385\sin(dx+c)^9+495\sin(dx+c)^7-231\sin(dx+c)^5\right)a^5+\left(63\sin(dx+c)^{11}-385\sin(dx+c)^9+990\sin(dx+c)^7-1386\sin(dx+c)^5+1155\sin(dx+c)^3-693\sin(dx+c)\right)a^5}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/693*(315*I*a^5*cos(d*x + c)^11 + I*(63*cos(d*x + c)^11 - 154*cos(d*x + c
)^9 + 99*cos(d*x + c)^7)*a^5 + 70*I*(9*cos(d*x + c)^11 - 11*cos(d*x + c)^9)
*a^5 + 2*(315*sin(d*x + c)^11 - 1540*sin(d*x + c)^9 + 2970*sin(d*x + c)^7 -
 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a^5 + 3*(105*sin(d*x + c)^11 -
385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 231*sin(d*x + c)^5)*a^5 + (63*sin
(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(d*x + c)^7 - 1386*sin(d*x + c)^
5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*a^5)/d
```

Fricas [A]
time = 0.41, size = 104, normalized size = 0.65

$$\frac{\left(-21i\,a^5e^{(12i\,dx+12i\,c)}-154i\,a^5e^{(10i\,dx+10i\,c)}-495i\,a^5e^{(8i\,dx+8i\,c)}-924i\,a^5e^{(6i\,dx+6i\,c)}-1155i\,a^5e^{(4i\,dx+4i\,c)}-1386i\,a^5e^{(2i\,dx+2i\,c)}+231i\,a^5\right)e^{(-i\,dx-i\,c)}}{14784\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/14784*(-21*I*a^5*e^(12*I*d*x + 12*I*c) - 154*I*a^5*e^(10*I*d*x + 10*I*c)
- 495*I*a^5*e^(8*I*d*x + 8*I*c) - 924*I*a^5*e^(6*I*d*x + 6*I*c) - 1155*I*a^
5*e^(4*I*d*x + 4*I*c) - 1386*I*a^5*e^(2*I*d*x + 2*I*c) + 231*I*a^5)*e^(-I*d
*x - I*c)/d
```

Sympy [**A**]
time = 0.57, size = 265, normalized size = 1.67

$$\begin{cases} \frac{(-90194313216ia^5d^6e^{12ic}e^{11idx}-661424963584ia^5d^6e^{10ic}e^{9idx}-2126008811520ia^5d^6e^{8ic}e^{7idx}-3968549781504ia^5d^6e^{6ic}e^{5idx}-4960687226880ia^5d^6e^{4ic}e^{3idx}-5952824672256ia^5d^6e^{2ic}e^{idx}+992137445376ia^5d^6e^{-idx})e^{-ic}}{63496796504064d^7} & \text{for } d^7e^{ic} \neq 0 \\ \frac{x(a^5e^{12ic}+6a^5e^{10ic}+15a^5e^{8ic}+20a^5e^{6ic}+15a^5e^{4ic}+6a^5e^{2ic}+a^5)e^{-ic}}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-90194313216*I*a**5*d**6*exp(12*I*c)*exp(11*I*d*x) - 6614249635
84*I*a**5*d**6*exp(10*I*c)*exp(9*I*d*x) - 2126008811520*I*a**5*d**6*exp(8*I
*c)*exp(7*I*d*x) - 3968549781504*I*a**5*d**6*exp(6*I*c)*exp(5*I*d*x) - 4960
687226880*I*a**5*d**6*exp(4*I*c)*exp(3*I*d*x) - 5952824672256*I*a**5*d**6*e
xp(2*I*c)*exp(I*d*x) + 992137445376*I*a**5*d**6*exp(-I*d*x))*exp(-I*c)/(634
96796504064*d**7), Ne(d**7*exp(I*c), 0)), (x*(a**5*exp(12*I*c) + 6*a**5*exp
(10*I*c) + 15*a**5*exp(8*I*c) + 20*a**5*exp(6*I*c) + 15*a**5*exp(4*I*c) + 6
*a**5*exp(2*I*c) + a**5)*exp(-I*c)/64, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1807 vs. 2(135) = 270.
time = 1.12, size = 1807, normalized size = 11.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/121110528*(168111405*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4707119
340*a^5*e^(13*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^
(11*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11767798350*a^5*e^(9*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^(7*I*d*x - I*c)*log(I*
e^(I*d*x + I*c) + 1) + 4707119340*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 1344891240*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 168111405*a^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 170251620*a^5
*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*e^(15*I*d
*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767045360*a^5*e^(13*I*d*x + 5*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 9534090720*a^5*e^(11*I*d*x + 3*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 11917613400*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*x + I*c
) - 1) + 9534090720*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767
045360*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*
e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 170251620*a^5*e^(I*d*x - 7
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 168111405*a^5*e^(17*I*d*x + 9*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 4707119340*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 9414238680*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11
```

```
767798350*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9414238680*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4707119340*a^5*e^(5*I*d*x
 - 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(3*I*d*x - 5*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 168111405*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 170251620*a^5*e^(17*I*d*x + 9*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 1362012960*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 47
67045360*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*
a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11917613400*a^5*e^(9
*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*a^5*e^(7*I*d*x - I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 4767045360*a^5*e^(5*I*d*x - 3*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1362012960*a^5*e^(3*I*d*x - 5*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 170251620*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 2
140215*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720*a^5*
e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 59926020*a^5*e^(13*I*d*x
 + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 119852040*a^5*e^(11*I*d*x + 3*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) + 149815050*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*
x) + e^(-I*c)) + 119852040*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)
) + 59926020*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720
*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 2140215*a^5*e^(I*d*x
 - 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(17*I*d*x + 9*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 59926020*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 119852040*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
149815050*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 119852040*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 59926020*a^5*e^(5*I*d*x
- 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(3*I*d*x - 5*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x)
 + e^(-I*c)) + 172032*I*a^5*e^(28*I*d*x + 20*I*c) + 2637824*I*a^5*e^(26*I*d
*x + 18*I*c) + 18964480*I*a^5*e^(24*I*d*x + 16*I*c) + 84967424*I*a^5*e^(22*
I*d*x + 14*I*c) + 266248192*I*a^5*e^(20*I*d*x + 12*I*c) + 624017408*I*a^5*e
^(18*I*d*x + 10*I*c) + 1137074176*I*a^5*e^(16*I*d*x + 8*I*c) + 1626275840*I
*a^5*e^(14*I*d*x + 6*I*c) + 1792860160*I*a^5*e^(12*I*d*x + 4*I*c) + 1464320
000*I*a^5*e^(10*I*d*x + 2*I*c) + 295206912*I*a^5*e^(6*I*d*x - 2*I*c) + 4730
8800*I*a^5*e^(4*I*d*x - 4*I*c) - 3784704*I*a^5*e^(2*I*d*x - 6*I*c) + 832905
216*I*a^5*e^(8*I*d*x) - 1892352*I*a^5*e^(-8*I*c))/(d*e^(17*I*d*x + 9*I*c) +
 8*d*e^(15*I*d*x + 7*I*c) + 28*d*e^(13*I*d*x + 5*I*c) + 56*d*e^(11*I*d*x +
3*I*c) + 70*d*e^(9*I*d*x + I*c) + 56*d*e^(7*I*d*x - I*c) + 28*d*e^(5*I*d*x
- 3*I*c) + 8*d*e^(3*I*d*x - 5*I*c) + d*e^(I*d*x - 7*I*c))
```

Mupad [B]
time = 4.60, size = 139, normalized size = 0.87

$$\frac{a^5\left(\frac{5\sin(3c+3dx)}{64}-\frac{\cos(5c+5dx)\,1i}{16}-\frac{\cos(7c+7dx)\,15i}{448}-\frac{\cos(9c+9dx)\,1i}{96}-\frac{\cos(11c+11dx)\,1i}{704}-\frac{\cos(3c+3dx)\,5i}{64}+\frac{\sin(5c+5dx)}{16}+\frac{15\sin(7c+7dx)}{448}+\frac{\sin(9c+9dx)}{96}+\frac{\sin(11c+11dx)}{704}+\frac{\sqrt{24}\,\cos\left(c+dx-\operatorname{atanh}\left(\frac{7}{5}\right)1i\right)}{64}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*((5*sin(3*c + 3*d*x))/64 - (cos(5*c + 5*d*x)*1i)/16 - (cos(7*c + 7*d*x
)*15i)/448 - (cos(9*c + 9*d*x)*1i)/96 - (cos(11*c + 11*d*x)*1i)/704 - (cos(
3*c + 3*d*x)*5i)/64 + sin(5*c + 5*d*x)/16 + (15*sin(7*c + 7*d*x))/448 + sin
(9*c + 9*d*x)/96 + sin(11*c + 11*d*x)/704 + (24^(1/2)*cos(c - atanh(7/5)*1i
 + d*x))/64))/d
```

3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=109

$$-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^12/a^4/d+12/13*I*(a+I*a*tan(d*x+c))^13/a^5/d-3/7*
I*(a+I*a*tan(d*x+c))^14/a^6/d+1/15*I*(a+I*a*tan(d*x+c))^15/a^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^12)/(a^4*d) + (((12*I)/13)*(a + I*a*Tan[
c + d*x])^13)/(a^5*d) - (((3*I)/7)*(a + I*a*Tan[c + d*x])^14)/(a^6*d) + ((I
/15)*(a + I*a*Tan[c + d*x])^15)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{11}\,dx,x,ia\tan(c+dx)\right)}{a^7d}$$

$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{11}-12a^2(a+x)^{12}+6a(a+x)^{13}-(a+x)\right.\right.}{a^7d}$$

$$= -\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{3i(a+ia}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 245 vs. $2(109) = 218$.
time = 3.58, size = 245, normalized size = 2.25

$$\frac{a^8 \sec(c)\sec^{15}(c+dx)(6435i\cos(dx)+6435i\cos(2c+dx)+5005i\cos(2c+3dx)+5005i\cos(4c+3dx)+3003i\cos(4c+5dx)+3003i\cos(6c+5dx)+1365i\cos(6c+7dx)+1365i\cos(8c+7dx)+6435\sin(dx)-6435\sin(2c+dx)+5005\sin(2c+3dx)-5005\sin(4c+3dx)+3003\sin(4c+5dx)-3003\sin(6c+5dx)+1365\sin(6c+7dx)-1365\sin(8c+7dx)+910\sin(8c+9dx)+210\sin(10c+11dx)+30\sin(12c+13dx)+2\sin(14c+15dx))}{10920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^15*((6435*I)*Cos[d*x] + (6435*I)*Cos[2*c + d*x] +
(5005*I)*Cos[2*c + 3*d*x] + (5005*I)*Cos[4*c + 3*d*x] + (3003*I)*Cos[4*c +
5*d*x] + (3003*I)*Cos[6*c + 5*d*x] + (1365*I)*Cos[6*c + 7*d*x] + (1365*I)*C
os[8*c + 7*d*x] + 6435*Sin[d*x] - 6435*Sin[2*c + d*x] + 5005*Sin[2*c + 3*d*
x] - 5005*Sin[4*c + 3*d*x] + 3003*Sin[4*c + 5*d*x] - 3003*Sin[6*c + 5*d*x]
+ 1365*Sin[6*c + 7*d*x] - 1365*Sin[8*c + 7*d*x] + 910*Sin[8*c + 9*d*x] + 21
0*Sin[10*c + 11*d*x] + 30*Sin[12*c + 13*d*x] + 2*Sin[14*c + 15*d*x]))/(1092
0*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(93) = 186$.
time = 0.32, size = 611, normalized size = 5.61 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/15*sin(d*x+c)^9/cos(d*x+c)^15+2/65*sin(d*x+c)^9/cos(d*x+c)^13+8
/715*sin(d*x+c)^9/cos(d*x+c)^11+16/6435*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a^8
*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10+1/40*sin(
d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(1/13*sin(d*x
+c)^7/cos(d*x+c)^13+6/143*sin(d*x+c)^7/cos(d*x+c)^11+8/429*sin(d*x+c)^7/cos
(d*x+c)^9+16/3003*sin(d*x+c)^7/cos(d*x+c)^7)-56*I*a^8*(1/10*sin(d*x+c)^4/co
s(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1
/40*sin(d*x+c)^4/cos(d*x+c)^4)+70*a^8*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33
*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+
c)^5/cos(d*x+c)^5)-8*I*a^8*(1/14*sin(d*x+c)^8/cos(d*x+c)^14+1/28*sin(d*x+c)
^8/cos(d*x+c)^12+1/70*sin(d*x+c)^8/cos(d*x+c)^10+1/280*sin(d*x+c)^8/cos(d*x
+c)^8)-28*a^8*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7
+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+I*a^8/co
s(d*x+c)^8-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.
time = 0.29, size = 186, normalized size = 1.71

$$\frac{91\,a^8\tan(dx+c)^{15}-780i\,a^8\tan(dx+c)^{14}-2625\,a^8\tan(dx+c)^{13}+3640i\,a^8\tan(dx+c)^{12}-1365\,a^8\tan(dx+c)^{11}+12012i\,a^8\tan(dx+c)^{10}+15015\,a^8\tan(dx+c)^{9}+19305\,a^8\tan(dx+c)^{7}-20020i\,a^8\tan(dx+c)^{6}-3003\,a^8\tan(dx+c)^{5}-10920i\,a^8\tan(dx+c)^{4}-11375\,a^8\tan(dx+c)^{3}+5460i\,a^8\tan(dx+c)^{2}+1365\,a^8\tan(dx+c)}{1365\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(d
*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 12012*
I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c)^7
 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*tan(d
*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 1365*a^8
*tan(d*x + c))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(85) = 170$.
time = 0.38, size = 345, normalized size = 3.17

$$-\frac{8192 \left(-1365 i a^{8} e^{(22 i dx+22 i c)} - 3003 i a^{8} e^{(20 i dx+20 i c)} - 5005 i a^{8} e^{(18 i dx+18 i c)} - 6435 i a^{8} e^{(16 i dx+16 i c)} - 6435 i a^{8} e^{(14 i dx+14 i c)} - 5005 i a^{8} e^{(12 i dx+12 i c)} - 3003 i a^{8} e^{(10 i dx+10 i c)} - 1365 i a^{8} e^{(8 i dx+8 i c)} - 455 i a^{8} e^{(6 i dx+6 i c)} - 105 i a^{8} e^{(4 i dx+4 i c)} - 15 i a^{8} e^{(2 i dx+2 i c)} - i a^{8}\right)}{1365 \left(de^{(30 i dx+30 i c)} + 15 de^{(28 i dx+28 i c)} + 105 de^{(26 i dx+26 i c)} + 455 de^{(24 i dx+24 i c)} + 1365 de^{(22 i dx+22 i c)} + 3003 de^{(20 i dx+20 i c)} + 5005 de^{(18 i dx+18 i c)} + 6435 de^{(16 i dx+16 i c)} + 6435 de^{(14 i dx+14 i c)} + 5005 de^{(12 i dx+12 i c)} + 3003 de^{(10 i dx+10 i c)} + 1365 de^{(8 i dx+8 i c)} + 455 de^{(6 i dx+6 i c)} + 105 de^{(4 i dx+4 i c)} + 15 de^{(2 i dx+2 i c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -8192/1365*(-1365*I*a^8*e^(22*I*d*x + 22*I*c) - 3003*I*a^8*e^(20*I*d*x + 20
*I*c) - 5005*I*a^8*e^(18*I*d*x + 18*I*c) - 6435*I*a^8*e^(16*I*d*x + 16*I*c)
 - 6435*I*a^8*e^(14*I*d*x + 14*I*c) - 5005*I*a^8*e^(12*I*d*x + 12*I*c) - 30
03*I*a^8*e^(10*I*d*x + 10*I*c) - 1365*I*a^8*e^(8*I*d*x + 8*I*c) - 455*I*a^8
*e^(6*I*d*x + 6*I*c) - 105*I*a^8*e^(4*I*d*x + 4*I*c) - 15*I*a^8*e^(2*I*d*x
+ 2*I*c) - I*a^8)/(d*e^(30*I*d*x + 30*I*c) + 15*d*e^(28*I*d*x + 28*I*c) + 1
05*d*e^(26*I*d*x + 26*I*c) + 455*d*e^(24*I*d*x + 24*I*c) + 1365*d*e^(22*I*d
*x + 22*I*c) + 3003*d*e^(20*I*d*x + 20*I*c) + 5005*d*e^(18*I*d*x + 18*I*c)
+ 6435*d*e^(16*I*d*x + 16*I*c) + 6435*d*e^(14*I*d*x + 14*I*c) + 5005*d*e^(1
2*I*d*x + 12*I*c) + 3003*d*e^(10*I*d*x + 10*I*c) + 1365*d*e^(8*I*d*x + 8*I*
c) + 455*d*e^(6*I*d*x + 6*I*c) + 105*d*e^(4*I*d*x + 4*I*c) + 15*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{8} \left(\int \left(-28 \tan^{2}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\right)\, dx + \int 70 \tan^{4}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\, dx + \int \left(-28 \tan^{6}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\right)\, dx + \int \tan^{8}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\, dx + \int 8 i \tan{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\, dx + \int \left(-56 i \tan^{3}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\right)\, dx + \int 56 i \tan^{5}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\, dx + \int \left(-8 i \tan^{7}{\left(c + dx \right)} \sec^{8}{\left(c + dx \right)}\right)\, dx + \int \sec^{8}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**8, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
8, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**8, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**8, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**8,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**8, x) + Integral(sec(c + d*x)**8, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(85) = 170.
time = 1.28, size = 186, normalized size = 1.71

$$\frac{91\,a^{8}\tan\left(dx+c\right)^{15}-780i\,a^{8}\tan\left(dx+c\right)^{14}-2625\,a^{8}\tan\left(dx+c\right)^{13}+3640i\,a^{8}\tan\left(dx+c\right)^{12}-1365\,a^{8}\tan\left(dx+c\right)^{11}+12012i\,a^{8}\tan\left(dx+c\right)^{10}+15015\,a^{8}\tan\left(dx+c\right)^{9}+19305\,a^{8}\tan\left(dx+c\right)^{7}-20020i\,a^{8}\tan\left(dx+c\right)^{6}-3003\,a^{8}\tan\left(dx+c\right)^{5}-10920i\,a^{8}\tan\left(dx+c\right)^{4}-11375\,a^{8}\tan\left(dx+c\right)^{3}+5460i\,a^{8}\tan\left(dx+c\right)^{2}+1365\,a^{8}\tan\left(dx+c\right)}{1365\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(d
*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 12012*
I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c)^7
 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*tan(d
*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 1365*a^8
*tan(d*x + c))/d
```

Mupad [B]
time = 4.83, size = 153, normalized size = 1.40

$$\frac{a^{8}\left(\frac{\sin(9\,c+9\,d\,x)}{12}+\frac{\sin(11\,c+11\,d\,x)}{52}+\frac{\sin(13\,c+13\,d\,x)}{364}+\frac{\sin(15\,c+15\,d\,x)}{5460}+\frac{\cos(c+d\,x)\,297\mathrm{i}}{7168}+\frac{\cos(3\,c+3\,d\,x)\,33\mathrm{i}}{1024}+\frac{\cos(5\,c+5\,d\,x)\,99\mathrm{i}}{5120}+\frac{\cos(7\,c+7\,d\,x)\,9\mathrm{i}}{1024}-\frac{\cos(9\,c+9\,d\,x)\,247\mathrm{i}}{3072}-\frac{\cos(11\,c+11\,d\,x)\,19\mathrm{i}}{1024}-\frac{\cos(13\,c+13\,d\,x)\,19\mathrm{i}}{7168}-\frac{\cos(15\,c+15\,d\,x)\,19\mathrm{i}}{107520}\right)}{d\cos\left(c+d\,x\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^8,x)
```

```
[Out] (a^8*((cos(c + d*x)*297i)/7168 + (cos(3*c + 3*d*x)*33i)/1024 + (cos(5*c + 5
*d*x)*99i)/5120 + (cos(7*c + 7*d*x)*9i)/1024 - (cos(9*c + 9*d*x)*247i)/3072
 - (cos(11*c + 11*d*x)*19i)/1024 - (cos(13*c + 13*d*x)*19i)/7168 - (cos(15*
c + 15*d*x)*19i)/107520 + sin(9*c + 9*d*x)/12 + sin(11*c + 11*d*x)/52 + sin
(13*c + 13*d*x)/364 + sin(15*c + 15*d*x)/5460))/(d*cos(c + d*x)^15)
```

3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=82

$$-\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}$$

```
[Out] -4/11*I*(a+I*a*tan(d*x+c))^11/a^3/d+1/3*I*(a+I*a*tan(d*x+c))^12/a^4/d-1/13*
I*(a+I*a*tan(d*x+c))^13/a^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^3*d) + ((I/3)*(a + I*a*Tan[c + d
*x])^12)/(a^4*d) - ((I/13)*(a + I*a*Tan[c + d*x])^13)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)\,dx,x,ia\tan(\right.}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{i(a+ia\tan}{13}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(82) = 164.
time = 2.57, size = 234, normalized size = 2.85

$$\frac{a^8 \sec(c)\sec^{13}(c+dx)(1716i\cos(dx)+1716i\cos(2c+dx)+1287i\cos(2c+3dx)+1287i\cos(4c+3dx)+715i\cos(4c+5dx)+715i\cos(6c+5dx)+286i\cos(6c+7dx)+286i\cos(8c+7dx)+1716\sin(dx)-1716\sin(2c+dx)+1287\sin(2c+3dx)-1287\sin(4c+3dx)+715\sin(4c+5dx)-715\sin(6c+5dx)+286\sin(6c+7dx)-286\sin(8c+7dx)+156\sin(8c+9dx)+26\sin(10c+11dx)+2\sin(12c+13dx))}{1716d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^13*((1716*I)*Cos[d*x] + (1716*I)*Cos[2*c + d*x] +
(1287*I)*Cos[2*c + 3*d*x] + (1287*I)*Cos[4*c + 3*d*x] + (715*I)*Cos[4*c + 5
*d*x] + (715*I)*Cos[6*c + 5*d*x] + (286*I)*Cos[6*c + 7*d*x] + (286*I)*Cos[8
*c + 7*d*x] + 1716*Sin[d*x] - 1716*Sin[2*c + d*x] + 1287*Sin[2*c + 3*d*x] -
 1287*Sin[4*c + 3*d*x] + 715*Sin[4*c + 5*d*x] - 715*Sin[6*c + 5*d*x] + 286*
Sin[6*c + 7*d*x] - 286*Sin[8*c + 7*d*x] + 156*Sin[8*c + 9*d*x] + 26*Sin[10*
c + 11*d*x] + 2*Sin[12*c + 13*d*x]))/(1716*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(70) = 140.
time = 0.30, size = 475, normalized size = 5.79

method	result
risch	$\frac{4096 i a^8 \left(286\,e^{20 i (dx+c)}+715\,e^{18 i (dx+c)}+1287\,e^{16 i (dx+c)}+1716\,e^{14 i (dx+c)}+1716\,e^{12 i (dx+c)}+1287\,e^{10 i (dx+c)}+715\,e^{8 i (dx\right.}{429 d \left(e^{2 i (dx+c)}+1\right)^{13}}$
derivativedivides	$a^8\left(\frac{\sin^9(dx+c)}{13\cos(dx+c)^{13}}+\frac{4\left(\sin^9(dx+c)\right)}{143\cos(dx+c)^{11}}+\frac{8\left(\sin^9(dx+c)\right)}{1287\cos(dx+c)^9}\right)+\frac{4ia^8}{3\cos(dx+c)^6}-28a^8\left(\frac{\sin^7(dx+c)}{11\cos(dx+c)^{11}}+\frac{4\left(\sin^7(dx+c)\right)}{99\cos(dx+c)^9}+\frac{8\left(\sin^7(dx\right.}{693\cos(dx}\right.$
default	$a^8\left(\frac{\sin^9(dx+c)}{13\cos(dx+c)^{13}}+\frac{4\left(\sin^9(dx+c)\right)}{143\cos(dx+c)^{11}}+\frac{8\left(\sin^9(dx+c)\right)}{1287\cos(dx+c)^9}\right)+\frac{4ia^8}{3\cos(dx+c)^6}-28a^8\left(\frac{\sin^7(dx+c)}{11\cos(dx+c)^{11}}+\frac{4\left(\sin^7(dx+c)\right)}{99\cos(dx+c)^9}+\frac{8\left(\sin^7(dx\right.}{693\cos(dx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/13*sin(d*x+c)^9/cos(d*x+c)^13+4/143*sin(d*x+c)^9/cos(d*x+c)^11+
8/1287*sin(d*x+c)^9/cos(d*x+c)^9)+4/3*I*a^8/cos(d*x+c)^6-28*a^8*(1/11*sin(d
*x+c)^7/cos(d*x+c)^11+4/99*sin(d*x+c)^7/cos(d*x+c)^9+8/693*sin(d*x+c)^7/cos
(d*x+c)^7)+56*I*a^8*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x+c)^6/cos(
d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6)+70*a^8*(1/9*sin(d*x+c)^5/cos(d*x+c
)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)-56*I*a^
8*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*
x+c)^4/cos(d*x+c)^4)-28*a^8*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^
3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-8*I*a^8*(1/12*sin(d*x+c)^8/
cos(d*x+c)^12+1/30*sin(d*x+c)^8/cos(d*x+c)^10+1/120*sin(d*x+c)^8/cos(d*x+c)
^8)-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.
time = 0.28, size = 173, normalized size = 2.11

$$\frac{33 a^{8} \tan \left(dx+c\right)^{13}-286 i a^{8} \tan \left(dx+c\right)^{12}-1014 a^{8} \tan \left(dx+c\right)^{11}+1716 i a^{8} \tan \left(dx+c\right)^{10}+715 a^{8} \tan \left(dx+c\right)^{9}+2574 i a^{8} \tan \left(dx+c\right)^{8}+5148 a^{8} \tan \left(dx+c\right)^{7}-3432 i a^{8} \tan \left(dx+c\right)^{6}+1287 a^{8} \tan \left(dx+c\right)^{5}-4290 i a^{8} \tan \left(dx+c\right)^{4}-3718 a^{8} \tan \left(dx+c\right)^{3}+1716 i a^{8} \tan \left(dx+c\right)^{2}+429 a^{8} \tan \left(dx+c\right)}{429 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d*
x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*a^
8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 + 12
87*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x + c)^3
 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(64) = 128$.
time = 0.37, size = 307, normalized size = 3.74

$$-\frac{4096 \left(-286 i a^{8} e^{\left(20 i dx+20 i c\right)}-715 i a^{8} e^{\left(18 i dx+18 i c\right)}-1287 i a^{8} e^{\left(16 i dx+16 i c\right)}-1716 i a^{8} e^{\left(14 i dx+14 i c\right)}-1716 i a^{8} e^{\left(12 i dx+12 i c\right)}-1287 i a^{8} e^{\left(10 i dx+10 i c\right)}-715 i a^{8} e^{\left(8 i dx+8 i c\right)}-286 i a^{8} e^{\left(6 i dx+6 i c\right)}-78 i a^{8} e^{\left(4 i dx+4 i c\right)}-13 i a^{8} e^{\left(2 i dx+2 i c\right)}-i a^{8}\right)}{429 \left(d e^{\left(26 i dx+26 i c\right)}+13 d e^{\left(24 i dx+24 i c\right)}+78 d e^{\left(22 i dx+22 i c\right)}+286 d e^{\left(20 i dx+20 i c\right)}+715 d e^{\left(18 i dx+18 i c\right)}+1287 d e^{\left(16 i dx+16 i c\right)}+1716 d e^{\left(14 i dx+14 i c\right)}+1716 d e^{\left(12 i dx+12 i c\right)}+1287 d e^{\left(10 i dx+10 i c\right)}+715 d e^{\left(8 i dx+8 i c\right)}+286 d e^{\left(6 i dx+6 i c\right)}+78 d e^{\left(4 i dx+4 i c\right)}+13 d e^{\left(2 i dx+2 i c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -4096/429*(-286*I*a^8*e^(20*I*d*x + 20*I*c) - 715*I*a^8*e^(18*I*d*x + 18*I*
c) - 1287*I*a^8*e^(16*I*d*x + 16*I*c) - 1716*I*a^8*e^(14*I*d*x + 14*I*c) -
1716*I*a^8*e^(12*I*d*x + 12*I*c) - 1287*I*a^8*e^(10*I*d*x + 10*I*c) - 715*I
*a^8*e^(8*I*d*x + 8*I*c) - 286*I*a^8*e^(6*I*d*x + 6*I*c) - 78*I*a^8*e^(4*I*
d*x + 4*I*c) - 13*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(26*I*d*x + 26*I*
c) + 13*d*e^(24*I*d*x + 24*I*c) + 78*d*e^(22*I*d*x + 22*I*c) + 286*d*e^(20*
I*d*x + 20*I*c) + 715*d*e^(18*I*d*x + 18*I*c) + 1287*d*e^(16*I*d*x + 16*I*c
) + 1716*d*e^(14*I*d*x + 14*I*c) + 1716*d*e^(12*I*d*x + 12*I*c) + 1287*d*e^
(10*I*d*x + 10*I*c) + 715*d*e^(8*I*d*x + 8*I*c) + 286*d*e^(6*I*d*x + 6*I*c)
 + 78*d*e^(4*I*d*x + 4*I*c) + 13*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{8} \left(\int \left(-28 \tan^{2}\left(c+dx\right) \sec^{6}\left(c+dx\right)\right)\, dx + \int 70 \tan^{4}\left(c+dx\right) \sec^{6}\left(c+dx\right)\, dx + \int \left(-28 \tan^{6}\left(c+dx\right) \sec^{6}\left(c+dx\right)\right)\, dx + \int \tan^{8}\left(c+dx\right) \sec^{6}\left(c+dx\right)\, dx + \int 8 i \tan\left(c+dx\right) \sec^{6}\left(c+dx\right)\, dx + \int \left(-56 i \tan^{3}\left(c+dx\right) \sec^{6}\left(c+dx\right)\right)\, dx + \int 56 i \tan^{5}\left(c+dx\right) \sec^{6}\left(c+dx\right)\, dx + \int \left(-8 i \tan^{7}\left(c+dx\right) \sec^{6}\left(c+dx\right)\right)\, dx + \int \sec^{6}\left(c+dx\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**6, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
```

```
6, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**6, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**6, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**6,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.
time = 1.15, size = 173, normalized size = 2.11

$$\frac{33\,a^8\tan\left(dx+c\right)^{13}-286i\,a^8\tan\left(dx+c\right)^{12}-1014\,a^8\tan\left(dx+c\right)^{11}+1716i\,a^8\tan\left(dx+c\right)^{10}+715\,a^8\tan\left(dx+c\right)^{9}+2574i\,a^8\tan\left(dx+c\right)^{8}+5148\,a^8\tan\left(dx+c\right)^{7}-3432i\,a^8\tan\left(dx+c\right)^{6}+1287\,a^8\tan\left(dx+c\right)^{5}-4290i\,a^8\tan\left(dx+c\right)^{4}-3718\,a^8\tan\left(dx+c\right)^{3}+1716i\,a^8\tan\left(dx+c\right)^{2}+429\,a^8\tan\left(dx+c\right)}{429\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d*
x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*a^
8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 + 12
87*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x + c)^3
 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Mupad [B]
time = 5.34, size = 190, normalized size = 2.32

$$\frac{a^8\,\sin\left(c+d\,x\right)\,\left(2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(-184\,\sin\left(c+d\,x\right)^2-184\,\sin\left(2\,c+2\,d\,x\right)^2+\frac{\sin(2\,c+2\,d\,x)\,9867\mathrm{i}}{256}-184\,\sin\left(3\,c+3\,d\,x\right)^2-184\,\sin\left(4\,c+4\,d\,x\right)^2+\frac{\sin(4\,c+4\,d\,x)\,69069\mathrm{i}}{1024}-28\,\sin\left(5\,c+5\,d\,x\right)^2-2\,\sin\left(6\,c+6\,d\,x\right)^2+\frac{\sin(6\,c+6\,d\,x)\,42757\mathrm{i}}{512}+\frac{\sin(8\,c+8\,d\,x)\,23023\mathrm{i}}{256}+\frac{\sin(10\,c+10\,d\,x)\,7007\mathrm{i}}{512}+\frac{\sin(12\,c+12\,d\,x)\,1001\mathrm{i}}{1024}+429\right)}{429\,d\,\left(\sin\left(c+d\,x\right)^2-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^6,x)
```

```
[Out] (a^8*sin(c + d*x)*(2*sin(c/2 + (d*x)/2)^2 - 1)*((sin(2*c + 2*d*x)*9867i)/25
6 + (sin(4*c + 4*d*x)*69069i)/1024 + (sin(6*c + 6*d*x)*42757i)/512 + (sin(8
*c + 8*d*x)*23023i)/256 + (sin(10*c + 10*d*x)*7007i)/512 + (sin(12*c + 12*d
*x)*1001i)/1024 - 184*sin(2*c + 2*d*x)^2 - 184*sin(3*c + 3*d*x)^2 - 184*sin
(4*c + 4*d*x)^2 - 28*sin(5*c + 5*d*x)^2 - 2*sin(6*c + 6*d*x)^2 - 184*sin(c
+ d*x)^2 + 429))/(429*d*(sin(c + d*x)^2 - 1)^7)
```

3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=55

$$-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

```
[Out] -1/5*I*(a+I*a*tan(d*x+c))^10/a^2/d+1/11*I*(a+I*a*tan(d*x+c))^11/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/5*I)*(a + I*a*Tan[c + d*x])^10)/(a^2*d) + ((I/11)*(a + I*a*Tan[c + d*x
])^11)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^9\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 223 vs. $2(55) = 110$.
time = 1.97, size = 223, normalized size = 4.05

$$\frac{a^8 \sec(c) \sec^{11}(c+dx)(462i\cos(dx)+462i\cos(2c+dx)+330i\cos(2c+3dx)+330i\cos(4c+3dx)+165i\cos(4c+5dx)+165i\cos(6c+5dx)+55i\cos(6c+7dx)+55i\cos(8c+7dx)+462\sin(dx)-462\sin(2c+dx)+330\sin(2c+3dx)-330\sin(4c+3dx)+165\sin(4c+5dx)-165\sin(6c+5dx)+55\sin(6c+7dx)-55\sin(8c+7dx)+22\sin(8c+9dx)+2\sin(10c+11dx))}{220d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^11*((462*I)*Cos[d*x] + (462*I)*Cos[2*c + d*x] + (3
30*I)*Cos[2*c + 3*d*x] + (330*I)*Cos[4*c + 3*d*x] + (165*I)*Cos[4*c + 5*d*x
] + (165*I)*Cos[6*c + 5*d*x] + (55*I)*Cos[6*c + 7*d*x] + (55*I)*Cos[8*c + 7
*d*x] + 462*Sin[d*x] - 462*Sin[2*c + d*x] + 330*Sin[2*c + 3*d*x] - 330*Sin[
4*c + 3*d*x] + 165*Sin[4*c + 5*d*x] - 165*Sin[6*c + 5*d*x] + 55*Sin[6*c + 7
*d*x] - 55*Sin[8*c + 7*d*x] + 22*Sin[8*c + 9*d*x] + 2*Sin[10*c + 11*d*x]))/
(220*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(47) = 94$.
time = 0.30, size = 339, normalized size = 6.16

method	result
risch	$\frac{1024 i a^8 (55\,e^{18 i(dx+c)}+165\,e^{16 i(dx+c)}+330\,e^{14 i(dx+c)}+462\,e^{12 i(dx+c)}+462\,e^{10 i(dx+c)}+330\,e^{8 i(dx+c)}+165\,e^{6 i(dx+c)}+5}{55 d (e^{2 i(dx+c)}+1)^{11}}$
derivativedivides	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-8ia^8\left(\frac{\sin^8(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^8(dx+c)}{40\cos(dx+c)^8}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+\frac{}{\cos(}$
default	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-8ia^8\left(\frac{\sin^8(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^8(dx+c)}{40\cos(dx+c)^8}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+\frac{}{\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/11*sin(d*x+c)^9/cos(d*x+c)^11+2/99*sin(d*x+c)^9/cos(d*x+c)^9)-8
*I*a^8*(1/10*sin(d*x+c)^8/cos(d*x+c)^10+1/40*sin(d*x+c)^8/cos(d*x+c)^8)-28*
a^8*(1/9*sin(d*x+c)^7/cos(d*x+c)^9+2/63*sin(d*x+c)^7/cos(d*x+c)^7)+2*I*a^8/
cos(d*x+c)^4+70*a^8*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*
x+c)^5)-56*I*a^8*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c
)^4)-28*a^8*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+
56*I*a^8*(1/8*sin(d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6)-a^8
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.
time = 0.30, size = 134, normalized size = 2.44

$$\frac{5\,a^8\tan(dx+c)^{11}-44i\,a^8\tan(dx+c)^{10}-165\,a^8\tan(dx+c)^9+330i\,a^8\tan(dx+c)^8+330\,a^8\tan(dx+c)^7+462\,a^8\tan(dx+c)^5-660i\,a^8\tan(dx+c)^4-495\,a^8\tan(dx+c)^3+220i\,a^8\tan(dx+c)^2+55\,a^8\tan(dx+c)}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*x
+ c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*tan(
d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(43) = 86.
time = 0.36, size = 269, normalized size = 4.89

$$-\frac{1024\left(-55i\,a^8e^{(18i\,dx+18i\,c)}-165i\,a^8e^{(16i\,dx+16i\,c)}-330i\,a^8e^{(14i\,dx+14i\,c)}-462i\,a^8e^{(12i\,dx+12i\,c)}-462i\,a^8e^{(10i\,dx+10i\,c)}-330i\,a^8e^{(8i\,dx+8i\,c)}-165i\,a^8e^{(6i\,dx+6i\,c)}-55i\,a^8e^{(4i\,dx+4i\,c)}-11i\,a^8e^{(2i\,dx+2i\,c)}-i\,a^8\right)}{55\left(de^{(22i\,dx+22i\,c)}+11\,de^{(20i\,dx+20i\,c)}+55\,de^{(18i\,dx+18i\,c)}+165\,de^{(16i\,dx+16i\,c)}+330\,de^{(14i\,dx+14i\,c)}+462\,de^{(12i\,dx+12i\,c)}+462\,de^{(10i\,dx+10i\,c)}+330\,de^{(8i\,dx+8i\,c)}+165\,de^{(6i\,dx+6i\,c)}+55\,de^{(4i\,dx+4i\,c)}+11\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1024/55*(-55*I*a^8*e^(18*I*d*x + 18*I*c) - 165*I*a^8*e^(16*I*d*x + 16*I*c)
 - 330*I*a^8*e^(14*I*d*x + 14*I*c) - 462*I*a^8*e^(12*I*d*x + 12*I*c) - 462*
I*a^8*e^(10*I*d*x + 10*I*c) - 330*I*a^8*e^(8*I*d*x + 8*I*c) - 165*I*a^8*e^(
6*I*d*x + 6*I*c) - 55*I*a^8*e^(4*I*d*x + 4*I*c) - 11*I*a^8*e^(2*I*d*x + 2*I
*c) - I*a^8)/(d*e^(22*I*d*x + 22*I*c) + 11*d*e^(20*I*d*x + 20*I*c) + 55*d*e
^(18*I*d*x + 18*I*c) + 165*d*e^(16*I*d*x + 16*I*c) + 330*d*e^(14*I*d*x + 14
*I*c) + 462*d*e^(12*I*d*x + 12*I*c) + 462*d*e^(10*I*d*x + 10*I*c) + 330*d*e
^(8*I*d*x + 8*I*c) + 165*d*e^(6*I*d*x + 6*I*c) + 55*d*e^(4*I*d*x + 4*I*c) +
 11*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^8\left(\int\left(-28\tan^2(c+dx)\sec^4(c+dx)\right)dx+\int 70\tan^4(c+dx)\sec^4(c+dx)\,dx+\int\left(-28\tan^6(c+dx)\sec^4(c+dx)\right)dx+\int\tan^8(c+dx)\sec^4(c+dx)\,dx+\int 8i\tan(c+dx)\sec^4(c+dx)\,dx+\int\left(-56i\tan^3(c+dx)\sec^4(c+dx)\right)dx+\int 56i\tan^5(c+dx)\sec^4(c+dx)\,dx+\int\left(-8i\tan^7(c+dx)\sec^4(c+dx)\right)dx+\int\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**4, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
4, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**4, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**4, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**4,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(43) = 86.
time = 1.13, size = 134, normalized size = 2.44

$$\frac{5\,a^8\tan(dx+c)^{11}-44i\,a^8\tan(dx+c)^{10}-165\,a^8\tan(dx+c)^9+330i\,a^8\tan(dx+c)^8+330\,a^8\tan(dx+c)^7+462\,a^8\tan(dx+c)^5-660i\,a^8\tan(dx+c)^4-495\,a^8\tan(dx+c)^3+220i\,a^8\tan(dx+c)^2+55\,a^8\tan(dx+c)}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")

[Out] 1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x + c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*x + c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*tan(d*x + c)^2 + 55*a^8*tan(d*x + c))/d

Mupad [B]
time = 4.22, size = 107, normalized size = 1.95

$$\frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{10}+\frac{\sin(11\,c+11\,d\,x)}{110}+\frac{\cos(c+d\,x)\,63\mathrm{i}}{1280}+\frac{\cos(3\,c+3\,d\,x)\,9\mathrm{i}}{256}+\frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{512}+\frac{\cos(7\,c+7\,d\,x)\,3\mathrm{i}}{512}-\frac{\cos(9\,c+9\,d\,x)\,253\mathrm{i}}{2560}-\frac{\cos(11\,c+11\,d\,x)\,23\mathrm{i}}{2560}\right)}{d\cos\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^4,x)

[Out] (a^8*((cos(c + d*x)*63i)/1280 + (cos(3*c + 3*d*x)*9i)/256 + (cos(5*c + 5*d*x)*9i)/512 + (cos(7*c + 7*d*x)*3i)/512 - (cos(9*c + 9*d*x)*253i)/2560 - (cos(11*c + 11*d*x)*23i)/2560 + sin(9*c + 9*d*x)/10 + sin(11*c + 11*d*x)/110))/(d*cos(c + d*x)^11)

3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

[Out] -1/9*I*(a+I*a*tan(d*x+c))^9/a/d

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]

[Out] ((-1/9*I)*(a + I*a*Tan[c + d*x])^9)/(a*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^8\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^9}{9ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 212 vs. 2(27) = 54.
time = 1.55, size = 212, normalized size = 7.85

$$\frac{a^8\sec(c)\sec^9(c+dx)(126i\cos(dx)+126i\cos(2c+dx)+84i\cos(2c+3dx)+84i\cos(4c+3dx)+36i\cos(4c+5dx)+36i\cos(6c+5dx)+9i\cos(6c+7dx)+9i\cos(8c+7dx)+126\sin(dx)-126\sin(2c+dx)+84\sin(2c+3dx)-84\sin(4c+3dx)+36\sin(4c+5dx)-36\sin(6c+5dx)+9\sin(6c+7dx)-9\sin(8c+7dx)+2\sin(8c+9dx))}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^9*((126*I)*Cos[d*x] + (126*I)*Cos[2*c + d*x] + (84
*I)*Cos[2*c + 3*d*x] + (84*I)*Cos[4*c + 3*d*x] + (36*I)*Cos[4*c + 5*d*x] +
(36*I)*Cos[6*c + 5*d*x] + (9*I)*Cos[6*c + 7*d*x] + (9*I)*Cos[8*c + 7*d*x] +
 126*Sin[d*x] - 126*Sin[2*c + d*x] + 84*Sin[2*c + 3*d*x] - 84*Sin[4*c + 3*d
*x] + 36*Sin[4*c + 5*d*x] - 36*Sin[6*c + 5*d*x] + 9*Sin[6*c + 7*d*x] - 9*Si
n[8*c + 7*d*x] + 2*Sin[8*c + 9*d*x]))/(18*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(23) = 46$.
time = 0.26, size = 180, normalized size = 6.67

method	result
risch	$\frac{512ia^8\left(9\,\mathrm{e}^{16i(dx+c)}+36\,\mathrm{e}^{14i(dx+c)}+84\,\mathrm{e}^{12i(dx+c)}+126\,\mathrm{e}^{10i(dx+c)}+126\,\mathrm{e}^{8i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+36\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{2i(dx+}}{9d\left(\mathrm{e}^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9\cos(dx+c)^9}+\frac{4ia^8}{\cos(dx+c)^2}-\frac{4a^8\left(\sin^7(dx+c)\right)}{\cos(dx+c)^7}-\frac{14ia^8\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}+\frac{14a^8\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}+\frac{28ia^8\left(\sin^6(dx+c)\right)}{3\cos(dx+c)^6}-\frac{28a^8\left(\sin^3(dx}{3\cos(dx+c}}{d}$
default	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9\cos(dx+c)^9}+\frac{4ia^8}{\cos(dx+c)^2}-\frac{4a^8\left(\sin^7(dx+c)\right)}{\cos(dx+c)^7}-\frac{14ia^8\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}+\frac{14a^8\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}+\frac{28ia^8\left(\sin^6(dx+c)\right)}{3\cos(dx+c)^6}-\frac{28a^8\left(\sin^3(dx}{3\cos(dx+c}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/9*a^8*sin(d*x+c)^9/cos(d*x+c)^9+4*I*a^8/cos(d*x+c)^2-4*a^8*sin(d*x+c
)^7/cos(d*x+c)^7-14*I*a^8*sin(d*x+c)^4/cos(d*x+c)^4+14*a^8*sin(d*x+c)^5/cos
(d*x+c)^5+28/3*I*a^8*sin(d*x+c)^6/cos(d*x+c)^6-28/3*a^8*sin(d*x+c)^3/cos(d*
x+c)^3-I*a^8*sin(d*x+c)^8/cos(d*x+c)^8+a^8*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.78

$$-\frac{i\,(i\,a\tan\left(dx+c\right)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9*I*(I*a*tan(d*x + c) + a)^9/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(21) = 42$.
time = 0.34, size = 231, normalized size = 8.56

$$-\frac{512\left(-9i\,a^8e^{(16i\,dx+16i\,c)}-36i\,a^8e^{(14i\,dx+14i\,c)}-84i\,a^8e^{(12i\,dx+12i\,c)}-126i\,a^8e^{(10i\,dx+10i\,c)}-126i\,a^8e^{(8i\,dx+8i\,c)}-84i\,a^8e^{(6i\,dx+6i\,c)}-36i\,a^8e^{(4i\,dx+4i\,c)}-9i\,a^8e^{(2i\,dx+2i\,c)}-i\,a^8\right)}{9\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i\,dx+8i\,c)}+84\,de^{(6i\,dx+6i\,c)}+36\,de^{(4i\,dx+4i\,c)}+9\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -512/9*(-9*I*a^8*e^(16*I*d*x + 16*I*c) - 36*I*a^8*e^(14*I*d*x + 14*I*c) - 8
4*I*a^8*e^(12*I*d*x + 12*I*c) - 126*I*a^8*e^(10*I*d*x + 10*I*c) - 126*I*a^8
*e^(8*I*d*x + 8*I*c) - 84*I*a^8*e^(6*I*d*x + 6*I*c) - 36*I*a^8*e^(4*I*d*x +
 4*I*c) - 9*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(18*I*d*x + 18*I*c) + 9
*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x +
12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^
(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^8\left(\int\left(-28\tan^2\left(c+dx\right)\sec^2\left(c+dx\right)\right)\,dx+\int 70\tan^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\left(-28\tan^6\left(c+dx\right)\sec^2\left(c+dx\right)\right)\,dx+\int\tan^8\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 8i\tan\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\left(-56i\tan^3\left(c+dx\right)\sec^2\left(c+dx\right)\right)\,dx+\int 56i\tan^5\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\left(-8i\tan^7\left(c+dx\right)\sec^2\left(c+dx\right)\right)\,dx+\int\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**2, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
2, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**2, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**2, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**2,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.
time = 1.10, size = 120, normalized size = 4.44

$$\frac{a^8\tan\left(dx+c\right)^9-9i\,a^8\tan\left(dx+c\right)^8-36\,a^8\tan\left(dx+c\right)^7+84i\,a^8\tan\left(dx+c\right)^6+126\,a^8\tan\left(dx+c\right)^5-126i\,a^8\tan\left(dx+c\right)^4-84\,a^8\tan\left(dx+c\right)^3+36i\,a^8\tan\left(dx+c\right)^2+9\,a^8\tan\left(dx+c\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a^8*tan(d*x + c)^9 - 9*I*a^8*tan(d*x + c)^8 - 36*a^8*tan(d*x + c)^7 +
84*I*a^8*tan(d*x + c)^6 + 126*a^8*tan(d*x + c)^5 - 126*I*a^8*tan(d*x + c)^4
 - 84*a^8*tan(d*x + c)^3 + 36*I*a^8*tan(d*x + c)^2 + 9*a^8*tan(d*x + c))/d
```

Mupad [B]
time = 3.83, size = 83, normalized size = 3.07

$$\frac{a^8\left(\sin\left(9\,c+9\,d\,x\right)+\frac{\cos\left(c+d\,x\right)\,63{}\mathrm{i}}{128}+\frac{\cos\left(3\,c+3\,d\,x\right)\,21{}\mathrm{i}}{64}+\frac{\cos\left(5\,c+5\,d\,x\right)\,9{}\mathrm{i}}{64}+\frac{\cos\left(7\,c+7\,d\,x\right)\,9{}\mathrm{i}}{256}-\frac{\cos\left(9\,c+9\,d\,x\right)\,255{}\mathrm{i}}{256}\right)}{9\,d\,{\cos\left(c+d\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^2,x)
```

```
[Out] (a^8*((cos(c + d*x)*63i)/128 + (cos(3*c + 3*d*x)*21i)/64 + (cos(5*c + 5*d*x
)*9i)/64 + (cos(7*c + 7*d*x)*9i)/256 - (cos(9*c + 9*d*x)*255i)/256 + sin(9*
c + 9*d*x)))/(9*d*cos(c + d*x)^9)
```

3.81 $\int (a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=200

$$128a^8x-\frac{128ia^8\log(\cos(c+dx))}{d}-\frac{64a^8\tan(c+dx)}{d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+$$

```
[Out] 128*a^8*x-128*I*a^8*ln(cos(d*x+c))/d-64*a^8*tan(d*x+c)/d+4/5*I*a^3*(a+I*a*t
an(d*x+c))^5/d+1/3*I*a^2*(a+I*a*tan(d*x+c))^6/d+1/7*I*a*(a+I*a*tan(d*x+c))^
7/d+16/3*I*a^2*(a^2+I*a^2*tan(d*x+c))^3/d+2*I*(a^2+I*a^2*tan(d*x+c))^4/d+16
*I*(a^4+I*a^4*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3559, 3558, 3556}

$$-\frac{64a^8\tan(c+dx)}{d}-\frac{128ia^8\log(\cos(c+dx))}{d}+128a^8x+\frac{16i(a^4+ia^4\tan(c+dx))^2}{d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{16ia^2(a^2+ia^2\tan(c+dx))^3}{3d}+\frac{2i(a^2+ia^2\tan(c+dx))^4}{d}+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] 128*a^8*x - ((128*I)*a^8*Log[Cos[c + d*x]])/d - (64*a^8*Tan[c + d*x])/d + (
((4*I)/5)*a^3*(a + I*a*Tan[c + d*x])^5)/d + ((I/3)*a^2*(a + I*a*Tan[c + d*x
])^6)/d + ((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + (((16*I)/3)*a^2*(a^2 + I*a
^2*Tan[c + d*x])^3)/d + ((2*I)*(a^2 + I*a^2*Tan[c + d*x])^4)/d + ((16*I)*(a
^4 + I*a^4*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$
\begin{aligned}
\int (a+ia\tan(c+dx))^8\,dx &= \frac{ia(a+ia\tan(c+dx))^7}{7d}+(2a)\int (a+ia\tan(c+dx))^7\,dx \\
&= \frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{ia(a+ia\tan(c+dx))^7}{7d}+\left(4a^2\right)\int (a+ia\tan(c+ \\
&= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{ia(a+ia\tan(c+dx)}{7d} \\
&= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{ia(a+ia\tan(c+dx)}{7d} \\
&= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+}{3d} \\
&= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+}{3d} \\
&= 128a^8x-\frac{64a^8\tan(c+dx)}{d}+\frac{16ia^5(a+ia\tan(c+dx))^3}{3d}+\frac{4ia^3(a+ia\tan(c}{5d} \\
&= 128a^8x-\frac{128ia^8\log(\cos(c+dx))}{d}-\frac{64a^8\tan(c+dx)}{d}+\frac{16ia^5(a+ia\tan(c+}{3d}
\end{aligned}
$$

Mathematica [A]
time = 2.51, size = 383, normalized size = 1.92

$$\frac{a^8\sec(c)\sec^7(c+dx)\left(70\cos(dx)\left(-139i+210dx-105i\log\left(\cos^2(c+dx)\right)\right)+70\cos(2c+dx)\left(-139i+210dx-105i\log\left(\cos^2(c+dx)\right)\right)+3\left(-420i\cos(4c+5dx)+980dx\cos(4c+5dx)-420i\cos(6c+5dx)+980dx\cos(6c+5dx)+140dx\cos(6c+7dx)+140dx\cos(8c+7dx)+70\cos(2c+3dx)\left(-25i+42dx-21i\log\left(\cos^2(c+dx)\right)\right)+70\cos(4c+3dx)\left(-25i+42dx-21i\log\left(\cos^2(c+dx)\right)\right)-490i\cos(4c+5dx)\log\left(\cos^2(c+dx)\right)-490i\cos(6c+5dx)\log\left(\cos^2(c+dx)\right)-70i\cos(6c+7dx)\log\left(\cos^2(c+dx)\right)-70i\cos(8c+7dx)\log\left(\cos^2(c+dx)\right)-6965\sin(dx)+5740\sin(2c+dx)-4963\sin(2c+3dx)+2660\sin(4c+3dx)-1981\sin(4c+5dx)+560\sin(6c+5dx)-363\sin(6c+7dx)\right)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^7*(70*Cos[d*x]*(-139*I + 210*d*x - (105*I)*Log[Cos
[c + d*x]^2]) + 70*Cos[2*c + d*x]*(-139*I + 210*d*x - (105*I)*Log[Cos[c + d
*x]^2]) + 3*((-420*I)*Cos[4*c + 5*d*x] + 980*d*x*Cos[4*c + 5*d*x] - (420*I)
*Cos[6*c + 5*d*x] + 980*d*x*Cos[6*c + 5*d*x] + 140*d*x*Cos[6*c + 7*d*x] + 1
40*d*x*Cos[8*c + 7*d*x] + 70*Cos[2*c + 3*d*x]*(-25*I + 42*d*x - (21*I)*Log[
Cos[c + d*x]^2]) + 70*Cos[4*c + 3*d*x]*(-25*I + 42*d*x - (21*I)*Log[Cos[c +
 d*x]^2]) - (490*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]^2] - (490*I)*Cos[6*c
+ 5*d*x]*Log[Cos[c + d*x]^2] - (70*I)*Cos[6*c + 7*d*x]*Log[Cos[c + d*x]^2]
- (70*I)*Cos[8*c + 7*d*x]*Log[Cos[c + d*x]^2] - 6965*Sin[d*x] + 5740*Sin[2*
c + d*x] - 4963*Sin[2*c + 3*d*x] + 2660*Sin[4*c + 3*d*x] - 1981*Sin[4*c + 5
*d*x] + 560*Sin[6*c + 5*d*x] - 363*Sin[6*c + 7*d*x])))/(420*d)
```

Maple [A]
time = 0.06, size = 103, normalized size = 0.52

method	result
derivativedivides	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(dx\right.\right.}{d}$
default	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(dx\right.\right.}{d}$
risch	$-\frac{256a^8c}{d}-\frac{32ia^8\left(2940\,e^{12i(dx+c)}+13230\,e^{10i(dx+c)}+26950\,e^{8i(dx+c)}+30625\,e^{6i(dx+c)}+20139\,e^{4i(dx+c)}+7203\,e^{2i(dx+\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$
norman	$128a^8x-\frac{127a^8\tan(dx+c)}{d}+\frac{33a^8\left(\tan^3(dx+c)\right)}{d}-\frac{29a^8\left(\tan^5(dx+c)\right)}{5d}+\frac{a^8\left(\tan^7(dx+c)\right)}{7d}-\frac{60ia^8\left(\tan^2(dx+c\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^8*(-127*tan(d*x+c)+1/7*tan(d*x+c)^7-4/3*I*tan(d*x+c)^6-29/5*tan(d*x+c
)^5+16*I*tan(d*x+c)^4+33*tan(d*x+c)^3-60*I*tan(d*x+c)^2+64*I*ln(1+tan(d*x+c
)^2)+128*arctan(tan(d*x+c)))
```

Maxima [A]

time = 0.51, size = 121, normalized size = 0.60

$$\frac{15\,a^8\tan\left(dx+c\right)^7-140i\,a^8\tan\left(dx+c\right)^6-609\,a^8\tan\left(dx+c\right)^5+1680i\,a^8\tan\left(dx+c\right)^4+3465\,a^8\tan\left(dx+c\right)^3-6300i\,a^8\tan\left(dx+c\right)^2+13440\left(dx+c\right)a^8+6720i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right)-13335\,a^8\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^8*tan(d*x + c)^7 - 140*I*a^8*tan(d*x + c)^6 - 609*a^8*tan(d*x +
 c)^5 + 1680*I*a^8*tan(d*x + c)^4 + 3465*a^8*tan(d*x + c)^3 - 6300*I*a^8*ta
n(d*x + c)^2 + 13440*(d*x + c)*a^8 + 6720*I*a^8*log(tan(d*x + c)^2 + 1) - 1
3335*a^8*tan(d*x + c))/d
```

Fricas [A]

time = 0.35, size = 297, normalized size = 1.48

$$-\frac{32\left(2940i\,a^8e^{(12i\,dx+12i\,c)}+13230i\,a^8e^{(10i\,dx+10i\,c)}+26950i\,a^8e^{(8i\,dx+8i\,c)}+30625i\,a^8e^{(6i\,dx+6i\,c)}+20139i\,a^8e^{(4i\,dx+4i\,c)}+7203i\,a^8e^{(2i\,dx+2i\,c)}+1089i\,a^8+420\left(i\,a^8e^{(14i\,dx+14i\,c)}+7i\,a^8e^{(12i\,dx+12i\,c)}+21i\,a^8e^{(10i\,dx+10i\,c)}+35i\,a^8e^{(8i\,dx+8i\,c)}+35i\,a^8e^{(6i\,dx+6i\,c)}+21i\,a^8e^{(4i\,dx+4i\,c)}+7i\,a^8e^{(2i\,dx+2i\,c)}+i\,a^8\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{105\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -32/105*(2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I*
c) + 26950*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) + 20
139*I*a^8*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 1089*I*a^8
 + 420*(I*a^8*e^(14*I*d*x + 14*I*c) + 7*I*a^8*e^(12*I*d*x + 12*I*c) + 21*I*
a^8*e^(10*I*d*x + 10*I*c) + 35*I*a^8*e^(8*I*d*x + 8*I*c) + 35*I*a^8*e^(6*I*
```

```
d*x + 6*I*c) + 21*I*a^8*e^(4*I*d*x + 4*I*c) + 7*I*a^8*e^(2*I*d*x + 2*I*c) +
 I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*
I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 3
5*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c
) + d)
```

Sympy [A]
time = 0.41, size = 301, normalized size = 1.50

$$-\frac{128ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-94080ia^8e^{12ic}e^{12idx}-423360ia^8e^{10ic}e^{10idx}-862400ia^8e^{8ic}e^{8idx}-980000ia^8e^{6ic}e^{6idx}-644448ia^8e^{4ic}e^{4idx}-230496ia^8e^{2ic}e^{2idx}-34848ia^8}{105de^{14ic}e^{14idx}+735de^{12ic}e^{12idx}+2205de^{10ic}e^{10idx}+3675de^{8ic}e^{8idx}+3675de^{6ic}e^{6idx}+2205de^{4ic}e^{4idx}+735de^{2ic}e^{2idx}+105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -128*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-94080*I*a**8*exp(12*I*c)*
exp(12*I*d*x) - 423360*I*a**8*exp(10*I*c)*exp(10*I*d*x) - 862400*I*a**8*exp
(8*I*c)*exp(8*I*d*x) - 980000*I*a**8*exp(6*I*c)*exp(6*I*d*x) - 644448*I*a**
8*exp(4*I*c)*exp(4*I*d*x) - 230496*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 34848*I
*a**8)/(105*d*exp(14*I*c)*exp(14*I*d*x) + 735*d*exp(12*I*c)*exp(12*I*d*x) +
 2205*d*exp(10*I*c)*exp(10*I*d*x) + 3675*d*exp(8*I*c)*exp(8*I*d*x) + 3675*d
*exp(6*I*c)*exp(6*I*d*x) + 2205*d*exp(4*I*c)*exp(4*I*d*x) + 735*d*exp(2*I*c
)*exp(2*I*d*x) + 105*d)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(166) = 332.
time = 0.70, size = 378, normalized size = 1.89

$$-\frac{32\left(420ia^8e^{(14idx+14ic)}\log\left(e^{(2idx+2ic)}+1\right)+2940ia^8e^{(12idx+12ic)}\log\left(e^{(2idx+2ic)}+1\right)+8820ia^8e^{(10idx+10ic)}\log\left(e^{(2idx+2ic)}+1\right)+14700ia^8e^{(8idx+8ic)}\log\left(e^{(2idx+2ic)}+1\right)+14700ia^8e^{(6idx+6ic)}\log\left(e^{(2idx+2ic)}+1\right)+8820ia^8e^{(4idx+4ic)}\log\left(e^{(2idx+2ic)}+1\right)+2940ia^8e^{(2idx+2ic)}\log\left(e^{(2idx+2ic)}+1\right)+2940ia^8e^{(12idx+12ic)}+13230ia^8e^{(10idx+10ic)}+26950ia^8e^{(8idx+8ic)}+30625ia^8e^{(6idx+6ic)}+20139ia^8e^{(4idx+4ic)}+7203ia^8e^{(2idx+2ic)}+420ia^8\log\left(e^{(2idx+2ic)}+1\right)+1089ia^8\right)}{105\left(de^{(14idx+14ic)}+7de^{(12idx+12ic)}+21de^{(10idx+10ic)}+35de^{(8idx+8ic)}+35de^{(6idx+6ic)}+21de^{(4idx+4ic)}+7de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -32/105*(420*I*a^8*e^(14*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 294
0*I*a^8*e^(12*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e^(
10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(8*I*d*x +
8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(6*I*d*x + 6*I*c)*log(e
^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) + 2940*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) +
2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I*c) + 2695
0*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) + 20139*I*a^8
*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 420*I*a^8*log(e^(2*
I*d*x + 2*I*c) + 1) + 1089*I*a^8)/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*
x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*
e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) +
d)
```

Mupad [B]
time = 3.40, size = 113, normalized size = 0.56

$$\frac{33\,a^8\tan(c+d\,x)^3-127\,a^8\tan(c+d\,x)-\frac{29\,a^8\tan(c+d\,x)^5}{5}+\frac{a^8\tan(c+d\,x)^7}{7}+a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,128\mathrm{i}-a^8\tan(c+d\,x)^2\,60\mathrm{i}+a^8\tan(c+d\,x)^4\,16\mathrm{i}-\frac{a^8\tan(c+d\,x)^6\,4\mathrm{i}}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*log(tan(c + d*x) + 1i)*128i - 127*a^8*tan(c + d*x) - a^8*tan(c + d*x)^
2*60i + 33*a^8*tan(c + d*x)^3 + a^8*tan(c + d*x)^4*16i - (29*a^8*tan(c + d*
x)^5)/5 - (a^8*tan(c + d*x)^6*4i)/3 + (a^8*tan(c + d*x)^7)/7)/d
```

3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=133

$$-192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}-\frac{2ia^8\tan^4(}{d}$$

```
[Out] -192*a^8*x+192*I*a^8*ln(cos(d*x+c))/d+129*a^8*tan(d*x+c)/d+36*I*a^8*tan(d*x
+c)^2/d-10*a^8*tan(d*x+c)^3/d-2*I*a^8*tan(d*x+c)^4/d+1/5*a^8*tan(d*x+c)^5/d
-64*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{64ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan^5(c+dx)}{5d}-\frac{2ia^8\tan^4(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{192ia^8\log(\cos(c+dx))}{d}-192a^8x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -192*a^8*x + ((192*I)*a^8*Log[Cos[c + d*x]])/d + (129*a^8*Tan[c + d*x])/d +
 ((36*I)*a^8*Tan[c + d*x]^2)/d - (10*a^8*Tan[c + d*x]^3)/d - ((2*I)*a^8*Tan
[c + d*x]^4)/d + (a^8*Tan[c + d*x]^5)/(5*d) - ((64*I)*a^9)/(d*(a - I*a*Tan[
c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(129a^4 + \frac{64a^6}{(a-x)^2} - \frac{192a^5}{a-x} + 72a^3x + 30a^2x^2 + 8ax^3\right.\right.}{d}$$

$$= -192a^8x + \frac{192ia^8\log(\cos(c+dx))}{d} + \frac{129a^8\tan(c+dx)}{d} + \frac{36ia^8\,\text{t}}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 321 vs. 2(133) = 266.
time = 6.21, size = 321, normalized size = 2.41

$$\frac{\cos^3(c+dx)\left(-960dx\cos(8c)\cos^5(c+dx) + 480i\cos(8c)\cos^5(c+dx)\log\left(\cos^2(c+dx)\right) - 160i\cos(2dx)\cos^5(c+dx)(\cos(6c)-i\sin(6c)) + 960idx\cos^5(c+dx)\sin(8c) + 480\cos^5(c+dx)\log\left(\cos^2(c+dx)\right)\sin(8c) + \sec(c)(\cos(8c)-i\sin(8c))\sin(dx) - 52\cos^2(c+dx)\sec(c)(\cos(8c)-i\sin(8c))\sin(dx) + 696\cos^4(c+dx)\sec(c)(\cos(8c)-i\sin(8c))\sin(dx) + 160\cos^5(c+dx)(\cos(6c)-i\sin(6c))\sin(2dx) + \cos(c+dx)(\cos(8c)-i\sin(8c))(-10i+\tan(c)) - 4\cos^3(c+dx)(\cos(8c)-i\sin(8c))(-50i+13\tan(c))\right)(a+ia\tan(c+dx))^8}{5d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]^3*(-960*d*x*Cos[8*c]*Cos[c + d*x]^5 + (480*I)*Cos[8*c]*Cos[c
+ d*x]^5*Log[Cos[c + d*x]^2] - (160*I)*Cos[2*d*x]*Cos[c + d*x]^5*(Cos[6*c]
- I*Sin[6*c]) + (960*I)*d*x*Cos[c + d*x]^5*Sin[8*c] + 480*Cos[c + d*x]^5*Lo
g[Cos[c + d*x]^2]*Sin[8*c] + Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] - 52*C
os[c + d*x]^2*Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] + 696*Cos[c + d*x]^4*
Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] + 160*Cos[c + d*x]^5*(Cos[6*c] - I*
Sin[6*c])*Sin[2*d*x] + Cos[c + d*x]*(Cos[8*c] - I*Sin[8*c])*(-10*I + Tan[c]
) - 4*Cos[c + d*x]^3*(Cos[8*c] - I*Sin[8*c])*(-50*I + 13*Tan[c]))*(a + I*a*
Tan[c + d*x])^8)/(5*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(126) = 252.
time = 0.31, size = 482, normalized size = 3.62

method	result
risch	$-\frac{32ia^8 e^{2i(dx+c)}}{d} + \frac{384a^8c}{d} + \frac{16ia^8\left(150\,e^{8i(dx+c)}+500\,e^{6i(dx+c)}+650\,e^{4i(dx+c)}+385\,e^{2i(dx+c)}+87\right)}{5d\left(e^{2i(dx+c)}+1\right)^5} + \frac{192ia^8\ln(e^{2}}{d}$
derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5} - \frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3} + \frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)} + \frac{8\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{5}\right.}{}$

default	$a^8\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5}-\frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3}+\frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)}+\frac{8\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c}{5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*
sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)
^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)-4*I*a^8*cos(d*x+c)^2-28*a^8*
(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^
5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)-8*I*a^8*(1/4*
sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/
4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+70*a^8*(sin(d*x+c)^5/cos(
d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)-56*I*a^8*(-1
/2*sin(d*x+c)^2-ln(cos(d*x+c)))-28*a^8*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+
1/2*c)+56*I*a^8*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^
2+2*ln(cos(d*x+c)))+a^8*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 0.93

$$\frac{a^8\tan(dx+c)^5-10i\,a^8\tan(dx+c)^4-50\,a^8\tan(dx+c)^3+180i\,a^8\tan(dx+c)^2-960\,(dx+c)a^8-480i\,a^8\log\left(\tan(dx+c)^2+1\right)+645\,a^8\tan(dx+c)+\frac{320\,(a^8\tan(dx+c)-i\,a^8)}{\tan(dx+c)^2+1}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*tan(d*x + c)^5 - 10*I*a^8*tan(d*x + c)^4 - 50*a^8*tan(d*x + c)^3 +
 180*I*a^8*tan(d*x + c)^2 - 960*(d*x + c)*a^8 - 480*I*a^8*log(tan(d*x + c)^
2 + 1) + 645*a^8*tan(d*x + c) + 320*(a^8*tan(d*x + c) - I*a^8)/(tan(d*x + c
)^2 + 1))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.
time = 0.38, size = 245, normalized size = 1.84

$$-\frac{16\left(10i\,a^8e^{(12i\,dx+12i\,c)}+50i\,a^8e^{(10i\,dx+10i\,c)}-50i\,a^8e^{(8i\,dx+8i\,c)}-400i\,a^8e^{(6i\,dx+6i\,c)}-600i\,a^8e^{(4i\,dx+4i\,c)}-375i\,a^8e^{(2i\,dx+2i\,c)}-87i\,a^8+60\left(-i\,a^8e^{(10i\,dx+10i\,c)}-5i\,a^8e^{(8i\,dx+8i\,c)}-10i\,a^8e^{(6i\,dx+6i\,c)}-10i\,a^8e^{(4i\,dx+4i\,c)}-5i\,a^8e^{(2i\,dx+2i\,c)}-i\,a^8\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -16/5*(10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*c) - 50
*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a^8*e^(4
*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 87*I*a^8 + 60*(-I*a^8*e^(
```

```
10*I*d*x + 10*I*c) - 5*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I*a^8*e^(6*I*d*x + 6*
I*c) - 10*I*a^8*e^(4*I*d*x + 4*I*c) - 5*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)*
log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8
*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A]
time = 0.44, size = 257, normalized size = 1.93

$$\frac{192ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{2400ia^8e^{8ic}e^{8idx}+8000ia^8e^{6ic}e^{6idx}+10400ia^8e^{4ic}e^{4idx}+6160ia^8e^{2ic}e^{2idx}+1392ia^8}{5de^{10ic}e^{10idx}+25de^{8ic}e^{8idx}+50de^{6ic}e^{6idx}+50de^{4ic}e^{4idx}+25de^{2ic}e^{2idx}+5d}+\begin{cases}-\frac{32ia^8e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0\\ 64a^8xe^{2ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 192*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (2400*I*a**8*exp(8*I*c)*exp(
8*I*d*x) + 8000*I*a**8*exp(6*I*c)*exp(6*I*d*x) + 10400*I*a**8*exp(4*I*c)*ex
p(4*I*d*x) + 6160*I*a**8*exp(2*I*c)*exp(2*I*d*x) + 1392*I*a**8)/(5*d*exp(10
*I*c)*exp(10*I*d*x) + 25*d*exp(8*I*c)*exp(8*I*d*x) + 50*d*exp(6*I*c)*exp(6*
I*d*x) + 50*d*exp(4*I*c)*exp(4*I*d*x) + 25*d*exp(2*I*c)*exp(2*I*d*x) + 5*d)
 + Piecewise((-32*I*a**8*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (64*a**8*x*e
xp(2*I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(121) = 242.
time = 0.95, size = 302, normalized size = 2.27

$$-\frac{16\left(-60i\,a^8e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^8e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-600i\,a^8e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-600i\,a^8e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^8e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+10i\,a^8e^{(12i\,dx+12i\,c)}+50i\,a^8e^{(10i\,dx+10i\,c)}-50i\,a^8e^{(8i\,dx+8i\,c)}-400i\,a^8e^{(6i\,dx+6i\,c)}-600i\,a^8e^{(4i\,dx+4i\,c)}-375i\,a^8e^{(2i\,dx+2i\,c)}-60i\,a^8\log\left(e^{(2i\,dx+2i\,c)}+1\right)-87i\,a^8\right)}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -16/5*(-60*I*a^8*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 300*I
*a^8*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(6*I*d*
x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(4*I*d*x + 4*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 300*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) + 10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*
c) - 50*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a
^8*e^(4*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 60*I*a^8*log(e^(2*
I*d*x + 2*I*c) + 1) - 87*I*a^8)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Mupad [B]
time = 3.35, size = 102, normalized size = 0.77

$$\frac{\frac{64\,a^8}{\tan(c+d\,x)+1{}\mathrm{i}}+129\,a^8\,\tan(c+d\,x)-10\,a^8\,\tan(c+d\,x)^3+\frac{a^8\,\tan(c+d\,x)^5}{5}-a^8\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,192{}\mathrm{i}+a^8\,\tan(c+d\,x)^2\,36{}\mathrm{i}-a^8\,\tan(c+d\,x)^4\,2{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((64*a^8)/(tan(c + d*x) + 1i) - a^8*log(tan(c + d*x) + 1i)*192i + 129*a^8*t
an(c + d*x) + a^8*tan(c + d*x)^2*36i - 10*a^8*tan(c + d*x)^3 - a^8*tan(c +
d*x)^4*2i + (a^8*tan(c + d*x)^5)/5)/d
```

3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=124

$$80a^8x-\frac{80ia^8\log(\cos(c+dx))}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{4ia^8\tan^2(c+dx)}{d}+\frac{a^8\tan^3(c+dx)}{3d}-\frac{16ia^{10}}{d(a-ia\tan(c+dx)}$$

```
[Out] 80*a^8*x-80*I*a^8*ln(cos(d*x+c))/d-31*a^8*tan(d*x+c)/d-4*I*a^8*tan(d*x+c)^2
/d+1/3*a^8*tan(d*x+c)^3/d-16*I*a^10/d/(a-I*a*tan(d*x+c))^2+80*I*a^9/d/(a-I*
a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{3568, 45\}$

$$-\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan^3(c+dx)}{3d}-\frac{4ia^8\tan^2(c+dx)}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{80ia^8\log(\cos(c+dx))}{d}+80a^8x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] 80*a^8*x - ((80*I)*a^8*Log[Cos[c + d*x]])/d - (31*a^8*Tan[c + d*x])/d - ((4
*I)*a^8*Tan[c + d*x]^2)/d + (a^8*Tan[c + d*x]^3)/(3*d) - ((16*I)*a^10)/(d*(
a - I*a*Tan[c + d*x])^2) + ((80*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\mathrm{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right) dx$$

$$= 80a^8x - \frac{80ia^8\log(\cos(c+dx))}{d} - \frac{31a^8\tan(c+dx)}{d} - \frac{4ia^8\tan^2(}{d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 566 vs. $2(124) = 248$.
time = 1.97, size = 566, normalized size = 4.56

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^3*(Cos[2*(c + 5*d*x)] + I*Sin[2*(c + 5*d*x)])*((-6
6*I)*Cos[2*c + 3*d*x] + 180*d*x*Cos[2*c + 3*d*x] + (75*I)*Cos[4*c + 3*d*x]
+ 180*d*x*Cos[4*c + 3*d*x] - (50*I)*Cos[4*c + 5*d*x] + 60*d*x*Cos[4*c + 5*d
*x] - (3*I)*Cos[6*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] + 3*Cos[2*c + d*x]*(
71*I + 80*d*x - (40*I)*Log[Cos[c + d*x]^2]) + Cos[d*x]*(119*I + 240*d*x - (
120*I)*Log[Cos[c + d*x]^2]) - (90*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]^2] -
 (90*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]^2] - (30*I)*Cos[4*c + 5*d*x]*Log[
Cos[c + d*x]^2] - (30*I)*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]^2] - 101*Sin[d*x
] - (120*I)*d*x*Sin[d*x] - 60*Log[Cos[c + d*x]^2]*Sin[d*x] + 87*Sin[2*c + d
*x] - (120*I)*d*x*Sin[2*c + d*x] - 60*Log[Cos[c + d*x]^2]*Sin[2*c + d*x] -
96*Sin[2*c + 3*d*x] - (180*I)*d*x*Sin[2*c + 3*d*x] - 90*Log[Cos[c + d*x]^2]
*Sin[2*c + 3*d*x] + 45*Sin[4*c + 3*d*x] - (180*I)*d*x*Sin[4*c + 3*d*x] - 90
*Log[Cos[c + d*x]^2]*Sin[4*c + 3*d*x] - 44*Sin[4*c + 5*d*x] - (60*I)*d*x*Si
n[4*c + 5*d*x] - 30*Log[Cos[c + d*x]^2]*Sin[4*c + 5*d*x] + 3*Sin[6*c + 5*d*
x] - (60*I)*d*x*Sin[6*c + 5*d*x] - 30*Log[Cos[c + d*x]^2]*Sin[6*c + 5*d*x])
)/(12*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(116) = 232$.
time = 0.22, size = 409, normalized size = 3.30

method	result
risch	$-\frac{4ia^8 e^{4i(dx+c)}}{d}+\frac{32ia^8 e^{2i(dx+c)}}{d}-\frac{160a^8c}{d}-\frac{4ia^8\left(60\,e^{4i(dx+c)}+105\,e^{2i(dx+c)}+47\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{80ia^8\ln\left(e^{2i(dx+c)}+1\right)}{d}$

derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)-14*I*a^8*sin(d*x+c)^4-28*a^8*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)-2*I*a^
8*cos(d*x+c)^4+70*a^8*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*
x+3/8*c)+56*I*a^8*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-28*a^
8*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-8*
I*a^8*(1/2*sin(d*x+c)^8/cos(d*x+c)^2+1/2*sin(d*x+c)^6+3/4*sin(d*x+c)^4+3/2*
sin(d*x+c)^2+3*ln(cos(d*x+c)))+a^8*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.50, size = 135, normalized size = 1.09

$$\frac{a^8\tan(dx+c)^3-12i\,a^8\tan(dx+c)^2+240\,(dx+c)a^8+120i\,a^8\log\left(\tan(dx+c)^2+1\right)-93\,a^8\tan(dx+c)-\frac{48\left(5\,a^8\tan(dx+c)^3-6i\,a^8\tan(dx+c)^2+3\,a^8\tan(dx+c)-4i\,a^8\right)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/3*(a^8*tan(d*x + c)^3 - 12*I*a^8*tan(d*x + c)^2 + 240*(d*x + c)*a^8 + 120
*I*a^8*log(tan(d*x + c)^2 + 1) - 93*a^8*tan(d*x + c) - 48*(5*a^8*tan(d*x +
c)^3 - 6*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 4*I*a^8)/(tan(d*x + c)
^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 179, normalized size = 1.44

$$-\frac{4\left(3i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-63i\,a^8e^{(6i\,dx+6i\,c)}-9i\,a^8e^{(4i\,dx+4i\,c)}+81i\,a^8e^{(2i\,dx+2i\,c)}+47i\,a^8+60\left(i\,a^8e^{(6i\,dx+6i\,c)}+3i\,a^8e^{(4i\,dx+4i\,c)}+3i\,a^8e^{(2i\,dx+2i\,c)}+i\,a^8\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -4/3*(3*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 63*I*a
^8*e^(6*I*d*x + 6*I*c) - 9*I*a^8*e^(4*I*d*x + 4*I*c) + 81*I*a^8*e^(2*I*d*x
+ 2*I*c) + 47*I*a^8 + 60*(I*a^8*e^(6*I*d*x + 6*I*c) + 3*I*a^8*e^(4*I*d*x +
```

```
4*I*c) + 3*I*a^8*e^(2*I*d*x + 2*I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [A]
time = 0.45, size = 216, normalized size = 1.74

$$-\frac{80ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-240ia^8e^{4ic}e^{4idx}-420ia^8e^{2ic}e^{2idx}-188ia^8}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}+\begin{cases}\frac{-4ia^8de^{4ic}e^{4idx}+32ia^8de^{2ic}e^{2idx}}{d^2} & \text{for } d^2\neq 0\\ x\left(16a^8e^{4ic}-64a^8e^{2ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -80*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-240*I*a**8*exp(4*I*c)*exp(
4*I*d*x) - 420*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 188*I*a**8)/(3*d*exp(6*I*c)
*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) +
 3*d) + Piecewise(((-4*I*a**8*d*exp(4*I*c)*exp(4*I*d*x) + 32*I*a**8*d*exp(2
*I*c)*exp(2*I*d*x))/d**2, Ne(d**2, 0)), (x*(16*a**8*exp(4*I*c) - 64*a**8*ex
p(2*I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(110) = 220.
time = 1.12, size = 785, normalized size = 6.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -4/3*(60*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 840*I*a
^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(24*I*
d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(22*I*d*x + 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) + 120120*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) + 180180*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
 + 180180*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 120120*
I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a^8*e^(8*
I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(6*I*d*x - 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) + 840*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) + 205920*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 60*I*a^8*
e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^8*e^(32*I*d*x + 18*I*c) +
18*I*a^8*e^(30*I*d*x + 16*I*c) - 63*I*a^8*e^(28*I*d*x + 14*I*c) - 1032*I*a^
8*e^(26*I*d*x + 12*I*c) - 4968*I*a^8*e^(24*I*d*x + 10*I*c) - 13516*I*a^8*e^
(22*I*d*x + 8*I*c) - 22847*I*a^8*e^(20*I*d*x + 6*I*c) - 22066*I*a^8*e^(18*I
```

```
*d*x + 4*I*c) - 3234*I*a^8*e^(16*I*d*x + 2*I*c) + 44979*I*a^8*e^(12*I*d*x -
 2*I*c) + 43332*I*a^8*e^(10*I*d*x - 4*I*c) + 27672*I*a^8*e^(8*I*d*x - 6*I*c
) + 12048*I*a^8*e^(6*I*d*x - 8*I*c) + 3467*I*a^8*e^(4*I*d*x - 10*I*c) + 598
*I*a^8*e^(2*I*d*x - 12*I*c) + 25674*I*a^8*e^(14*I*d*x) + 47*I*a^8*e^(-14*I*
c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*
x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 20
02*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*
x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364
*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I
*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B]
time = 3.37, size = 111, normalized size = 0.90

$$\frac{a^8\tan(c+d\,x)^3}{3\,d}-\frac{80\,a^8\tan(c+d\,x)+a^8\,64\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}-\frac{31\,a^8\tan(c+d\,x)}{d}+\frac{a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,80\mathrm{i}}{d}-\frac{a^8\tan(c+d\,x)^2\,4\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*log(tan(c + d*x) + 1i)*80i)/d - (31*a^8*tan(c + d*x))/d - (80*a^8*tan(
c + d*x) + a^8*64i)/(d*(tan(c + d*x)*2i + tan(c + d*x)^2 - 1)) - (a^8*tan(c
 + d*x)^2*4i)/d + (a^8*tan(c + d*x)^3)/(3*d)
```

3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=114

$$-8a^8x+\frac{8ia^8\log(\cos(c+dx))}{d}+\frac{a^8\tan(c+dx)}{d}-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{}{d(a-ia}$$

```
[Out] -8*a^8*x+8*I*a^8*ln(cos(d*x+c))/d+a^8*tan(d*x+c)/d-16/3*I*a^11/d/(a-I*a*tan
(d*x+c))^3+16*I*a^10/d/(a-I*a*tan(d*x+c))^2-24*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan(c+dx)}{d}+\frac{8ia^8\log(\cos(c+dx))}{d}-8a^8x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -8*a^8*x + ((8*I)*a^8*Log[Cos[c + d*x]])/d + (a^8*Tan[c + d*x])/d - (((16*I
)/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) + ((16*I)*a^10)/(d*(a - I*a*Tan[c +
 d*x])^2) - ((24*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^7)\operatorname{Subst}\left(\int \frac{(a+x)^4}{(a-x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^7)\operatorname{Subst}\left(\int \left(1+\frac{16a^4}{(a-x)^4}-\frac{32a^3}{(a-x)^3}+\frac{24a^2}{(a-x)^2}-\frac{8a}{a-x}\right)\,dx, x, ia\tan\right.}{d}$$

$$= -8a^8x + \frac{8ia^8\log(\cos(c+dx))}{d} + \frac{a^8\tan(c+dx)}{d} - \frac{16ia^{11}}{3d(a-ia\tan(}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 414 vs. 2(114) = 228.
time = 1.75, size = 414, normalized size = 3.63

$$-\frac{a^8\sec(c)\sec(c+dx)\left(12i\cos(c)+10i\cos(3c+2dx)+12dx\cos(3c+2dx)-2i\cos(3c+4dx)+12dx\cos(3c+4dx)+i\cos(5c+4dx)+12dx\cos(5c+4dx)+\cos(c+2dx)\left(7i+12dx-6i\log\left(\cos^2(c+dx)\right)\right)-6i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)-6i\cos(3c+4dx)\log\left(\cos^2(c+dx)\right)-6i\cos(5c+4dx)\log\left(\cos^2(c+dx)\right)+11\sin(c+2dx)-12idx\sin(c+2dx)-6\log\left(\cos^2(c+dx)\right)\sin(c+2dx)+14\sin(3c+2dx)-12idx\sin(3c+2dx)-6\log\left(\cos^2(c+dx)\right)\sin(3c+2dx)-4\sin(3c+4dx)-12idx\sin(3c+4dx)-6\log\left(\cos^2(c+dx)\right)\sin(3c+4dx)-\sin(5c+4dx)-12idx\sin(5c+4dx)-6\log\left(\cos^2(c+dx)\right)\sin(5c+4dx)\right)\left(\cos(3c+11dx)+i\sin(3c+11dx)\right)}{6d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -1/6*(a^8*Sec[c]*Sec[c + d*x]*((12*I)*Cos[c] + (10*I)*Cos[3*c + 2*d*x] + 12
*d*x*Cos[3*c + 2*d*x] - (2*I)*Cos[3*c + 4*d*x] + 12*d*x*Cos[3*c + 4*d*x] +
I*Cos[5*c + 4*d*x] + 12*d*x*Cos[5*c + 4*d*x] + Cos[c + 2*d*x]*(7*I + 12*d*x
 - (6*I)*Log[Cos[c + d*x]^2]) - (6*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2]
- (6*I)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x]^2] - (6*I)*Cos[5*c + 4*d*x]*Log[C
os[c + d*x]^2] + 11*Sin[c + 2*d*x] - (12*I)*d*x*Sin[c + 2*d*x] - 6*Log[Cos[
c + d*x]^2]*Sin[c + 2*d*x] + 14*Sin[3*c + 2*d*x] - (12*I)*d*x*Sin[3*c + 2*d
*x] - 6*Log[Cos[c + d*x]^2]*Sin[3*c + 2*d*x] - 4*Sin[3*c + 4*d*x] - (12*I)*
d*x*Sin[3*c + 4*d*x] - 6*Log[Cos[c + d*x]^2]*Sin[3*c + 4*d*x] - Sin[5*c + 4
*d*x] - (12*I)*d*x*Sin[5*c + 4*d*x] - 6*Log[Cos[c + d*x]^2]*Sin[5*c + 4*d*x
])*(Cos[3*c + 11*d*x] + I*Sin[3*c + 11*d*x]))/(d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(105) = 210.
time = 0.20, size = 398, normalized size = 3.49

method	result
risch	$-\frac{2ia^8e^{6i(dx+c)}}{3d}+\frac{2ia^8e^{4i(dx+c)}}{d}-\frac{6ia^8e^{2i(dx+c)}}{d}+\frac{16a^8c}{d}+\frac{2ia^8}{d\left(e^{2i(dx+c)}+1\right)}+\frac{8ia^8\ln\left(e^{2i(dx+c)}+1\right)}{d}$
derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+\frac{28ia^8\left(\sin^6(dx+c\right.}{3}}{}$

default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+\frac{28ia^8\left(\sin^6(dx+\right.}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(
d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+28/3*I*a^8*sin(d*x
+c)^6-28*a^8*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+
c)+5/16*d*x+5/16*c)-56*I*a^8*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c
)^4)+70*a^8*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/1
6*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)-4/3*I*a^8*cos(d*x+c)^6-28*a^8*(-1/
6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/1
6*d*x+1/16*c)-8*I*a^8*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-
ln(cos(d*x+c)))+a^8*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*si
n(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.50, size = 146, normalized size = 1.28

$$-\frac{24\,(dx+c)a^8+12i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right)-3\,a^8\tan\left(dx+c\right)-\frac{8\left(9\,a^8\tan(dx+c)^5-15i\,a^8\tan(dx+c)^4+4\,a^8\tan(dx+c)^3-12i\,a^8\tan(dx+c)^2+3\,a^8\tan(dx+c)-5i\,a^8\right)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(24*(d*x + c)*a^8 + 12*I*a^8*log(tan(d*x + c)^2 + 1) - 3*a^8*tan(d*x +
 c) - 8*(9*a^8*tan(d*x + c)^5 - 15*I*a^8*tan(d*x + c)^4 + 4*a^8*tan(d*x + c
)^3 - 12*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 5*I*a^8)/(tan(d*x + c)
^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 113, normalized size = 0.99

$$-\frac{2\left(i\,a^8e^{(8i\,dx+8i\,c)}-2i\,a^8e^{(6i\,dx+6i\,c)}+6i\,a^8e^{(4i\,dx+4i\,c)}+9i\,a^8e^{(2i\,dx+2i\,c)}-3i\,a^8+12\left(-i\,a^8e^{(2i\,dx+2i\,c)}-i\,a^8\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -2/3*(I*a^8*e^(8*I*d*x + 8*I*c) - 2*I*a^8*e^(6*I*d*x + 6*I*c) + 6*I*a^8*e^(
4*I*d*x + 4*I*c) + 9*I*a^8*e^(2*I*d*x + 2*I*c) - 3*I*a^8 + 12*(-I*a^8*e^(2*
I*d*x + 2*I*c) - I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c
) + d)
```

Sympy [A]
time = 0.51, size = 172, normalized size = 1.51

$$\frac{2ia^8}{de^{2ic}e^{2idx}+d}+\frac{8ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\begin{cases}\frac{-2ia^8d^2e^{6ic}e^{6idx}+6ia^8d^2e^{4ic}e^{4idx}-18ia^8d^2e^{2ic}e^{2idx}}{3d^3} & \text{for } d^3\neq 0\\ x(4a^8e^{6ic}-8a^8e^{4ic}+12a^8e^{2ic}) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 2*I*a**8/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 8*I*a**8*log(exp(2*I*d*x) + exp(
-2*I*c))/d + Piecewise(((-2*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x) + 6*I*a**8*
d**2*exp(4*I*c)*exp(4*I*d*x) - 18*I*a**8*d**2*exp(2*I*c)*exp(2*I*d*x))/(3*d
**3), Ne(d**3, 0)), (x*(4*a**8*exp(6*I*c) - 8*a**8*exp(4*I*c) + 12*a**8*exp
(2*I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(98) = 196.
time = 1.18, size = 799, normalized size = 7.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/3*(-12*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 168*I*
a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(24*I
*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(22*I*d*x + 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 24024*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 36036*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
 36036*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 24024*I*a^
8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e^(8*I*d*
x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(6*I*d*x - 8*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 168*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 41184*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^8*e^(-14
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^8*e^(34*I*d*x + 20*I*c) + 11*I*a^8
*e^(32*I*d*x + 18*I*c) + 58*I*a^8*e^(30*I*d*x + 16*I*c) + 217*I*a^8*e^(28*I
*d*x + 14*I*c) + 725*I*a^8*e^(26*I*d*x + 12*I*c) + 2236*I*a^8*e^(24*I*d*x +
 10*I*c) + 5772*I*a^8*e^(22*I*d*x + 8*I*c) + 11583*I*a^8*e^(20*I*d*x + 6*I*
c) + 17589*I*a^8*e^(18*I*d*x + 4*I*c) + 20020*I*a^8*e^(16*I*d*x + 2*I*c) +
10231*I*a^8*e^(12*I*d*x - 2*I*c) + 4147*I*a^8*e^(10*I*d*x - 4*I*c) + 872*I*
a^8*e^(8*I*d*x - 6*I*c) - 80*I*a^8*e^(6*I*d*x - 8*I*c) - 111*I*a^8*e^(4*I*d
*x - 10*I*c) - 30*I*a^8*e^(2*I*d*x - 12*I*c) + 16874*I*a^8*e^(14*I*d*x) - 3
```

```
*I*a^8*e^(-14*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) +
 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d
*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3
003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*
x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e
^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B]
time = 3.41, size = 103, normalized size = 0.90

$$\frac{a^8\tan(c+d\,x)}{d}-\frac{24\,a^8\tan(c+d\,x)^2+a^8\tan(c+d\,x)\;32\mathrm{i}-\frac{40\,a^8}{3}}{d\,\left(-\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,3\mathrm{i}+3\tan\left(c+d\,x\right)+1\mathrm{i}\right)}-\frac{a^8\;\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\;8\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*tan(c + d*x))/d - (a^8*log(tan(c + d*x) + 1i)*8i)/d - (a^8*tan(c + d*x
)*32i - (40*a^8)/3 + 24*a^8*tan(c + d*x)^2)/(d*(3*tan(c + d*x) - tan(c + d*
x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=43

$$-\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

```
[Out] -1/8*I*(a^3+I*a^3*tan(d*x+c))^4/d/(a-I*a*tan(d*x+c))^4
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 37}

$$-\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/8*I)*(a^3 + I*a^3*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{(ia^9)\operatorname{Subst}\left(\int \frac{(a+x)^3}{(a-x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}\end{aligned}$$

Mathematica [**A**]
time = 0.35, size = 31, normalized size = 0.72

$$-\frac{ia^8(\cos(c+dx)+i\sin(c+dx))^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/8*I)*a^8*(Cos[c + d*x] + I*Sin[c + d*x])^8)/d
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(38) = 76.
time = 0.21, size = 451, normalized size = 10.49

method	result
risch	$-\frac{ia^8 e^{8i(dx+c)}}{8d}$
derivativedivides	$a^8\left(-\frac{\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-ia^8\left(\sin^8(dx+c)\right)-28a^8\left(-\cdots\right.$
default	$a^8\left(-\frac{\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-ia^8\left(\sin^8(dx+c)\right)-28a^8\left(\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(
d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c)-I*a^8*sin(d*x+c)^8-28*a^8*(-1/8*cos
(d*x+c)^3*sin(d*x+c)^5-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/64*sin(d*x+c)*cos(d
*x+c)^3+5/128*sin(d*x+c)*cos(d*x+c)+5/128*d*x+5/128*c)+56*I*a^8*(-1/8*sin(d
*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a
^8*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-56*I*a^8*(-1/8*sin(d*
x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-28*a^8*(-1/8*sin(d*x+c)*cos(d*x+c)^7
+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+
5/128*c)-I*a^8*cos(d*x+c)^8+a^8*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*c
os(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(35) = 70.
time = 0.51, size = 136, normalized size = 3.16

$$-\frac{a^8\tan(dx+c)^7-4i\,a^8\tan(dx+c)^6-7\,a^8\tan(dx+c)^5+8i\,a^8\tan(dx+c)^4+7\,a^8\tan(dx+c)^3-4i\,a^8\tan(dx+c)^2-a^8\tan(dx+c)}{\left(\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(a^8*tan(d*x + c)^7 - 4*I*a^8*tan(d*x + c)^6 - 7*a^8*tan(d*x + c)^5 + 8*I*
a^8*tan(d*x + c)^4 + 7*a^8*tan(d*x + c)^3 - 4*I*a^8*tan(d*x + c)^2 - a^8*ta
n(d*x + c))/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(
d*x + c)^2 + 1)*d)
```

Fricas [**A**]
time = 0.37, size = 17, normalized size = 0.40

$$-\frac{i\,a^8e^{(8i\,dx+8i\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/8*I*a^8*e^(8*I*d*x + 8*I*c)/d
```

Sympy [**A**]
time = 0.44, size = 36, normalized size = 0.84

$$\begin{cases} -\frac{ia^8e^{8ic}e^{8idx}}{8d} & \text{for } d \neq 0 \\ a^8xe^{8ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*a**8*exp(8*I*c)*exp(8*I*d*x)/(8*d), Ne(d, 0)), (a**8*x*exp(8*
I*c), True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(35) = 70.
time = 1.17, size = 381, normalized size = 8.86

$$-\frac{i\,a^8e^{(36i\,dx+22i\,c)} + 14i\,a^8e^{(34i\,dx+20i\,c)} + 91i\,a^8e^{(32i\,dx+18i\,c)} + 364i\,a^8e^{(30i\,dx+16i\,c)} + 1001i\,a^8e^{(28i\,dx+14i\,c)} + 2002i\,a^8e^{(26i\,dx+12i\,c)} + 3003i\,a^8e^{(24i\,dx+10i\,c)} + 3432i\,a^8e^{(22i\,dx+8i\,c)} + 3003i\,a^8e^{(20i\,dx+6i\,c)} + 2002i\,a^8e^{(18i\,dx+4i\,c)} + 1001i\,a^8e^{(16i\,dx+2i\,c)} + 91i\,a^8e^{(12i\,dx-2i\,c)} + 14i\,a^8e^{(10i\,dx-4i\,c)} + i\,a^8e^{(8i\,dx-6i\,c)} + 364i\,a^8e^{(14i\,dx)}}{8\left(de^{(28i\,dx+14i\,c)} + 14\,de^{(26i\,dx+12i\,c)} + 91\,de^{(24i\,dx+10i\,c)} + 364\,de^{(22i\,dx+8i\,c)} + 1001\,de^{(20i\,dx+6i\,c)} + 2002\,de^{(18i\,dx+4i\,c)} + 3003\,de^{(16i\,dx+2i\,c)} + 3003\,de^{(12i\,dx-2i\,c)} + 2002\,de^{(10i\,dx-4i\,c)} + 1001\,de^{(8i\,dx-6i\,c)} + 364\,de^{(6i\,dx-8i\,c)} + 91\,de^{(4i\,dx-10i\,c)} + 14\,de^{(2i\,dx-12i\,c)} + 3432\,de^{(14i\,dx)} + de^{(-14i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/8*(I*a^8*e^(36*I*d*x + 22*I*c) + 14*I*a^8*e^(34*I*d*x + 20*I*c) + 91*I*a
^8*e^(32*I*d*x + 18*I*c) + 364*I*a^8*e^(30*I*d*x + 16*I*c) + 1001*I*a^8*e^(
28*I*d*x + 14*I*c) + 2002*I*a^8*e^(26*I*d*x + 12*I*c) + 3003*I*a^8*e^(24*I*
d*x + 10*I*c) + 3432*I*a^8*e^(22*I*d*x + 8*I*c) + 3003*I*a^8*e^(20*I*d*x +
6*I*c) + 2002*I*a^8*e^(18*I*d*x + 4*I*c) + 1001*I*a^8*e^(16*I*d*x + 2*I*c)
```

```
+ 91*I*a^8*e^(12*I*d*x - 2*I*c) + 14*I*a^8*e^(10*I*d*x - 4*I*c) + I*a^8*e^(
8*I*d*x - 6*I*c) + 364*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*d*
e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*
I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e
^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*
I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I
*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14
*I*c))
```

Mupad [B]
time = 3.48, size = 66, normalized size = 1.53

$$-\frac{a^8\tan(c+d\,x)\,\left(\tan(c+d\,x)^2-1\right)}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4{}\mathrm{i}-6\tan\left(c+d\,x\right)^2-\tan\left(c+d\,x\right)\,4{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*tan(c + d*x)*(tan(c + d*x)^2 - 1))/(d*(tan(c + d*x)^3*4i - 6*tan(c +
d*x)^2 - tan(c + d*x)*4i + tan(c + d*x)^4 + 1))
```

3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=80

$$-\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

```
[Out] -4/5*I*a^13/d/(a-I*a*tan(d*x+c))^5+I*a^12/d/(a-I*a*tan(d*x+c))^4-1/3*I*a^11
/d/(a-I*a*tan(d*x+c))^3
```

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/5)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) + (I*a^12)/(d*(a - I*a*Tan[c
 + d*x])^4) - ((I/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{(ia^{11})\,\text{Subst}\left(\int \frac{(a+x)^2}{(a-x)^6}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^{11})\,\text{Subst}\left(\int \left(\frac{4a^2}{(a-x)^6}-\frac{4a}{(a-x)^5}+\frac{1}{(a-x)^4}\right)\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{i}{3d(a-ia\,t}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 55, normalized size = 0.69

$$\frac{a^8(15+16\cos(2(c+dx))-4i\sin(2(c+dx)))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(15 + 16*Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*((-I)*Cos[8*(c + d
*x)] + Sin[8*(c + d*x)]))/(240*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(70) = 140.
time = 0.25, size = 588, normalized size = 7.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*cos(d*x+c)^3*sin(d*x+c)^5-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*sin(d*x+c)
*cos(d*x+c)+7/256*d*x+7/256*c)-8*I*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/4
0*sin(d*x+c)^4*cos(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^
4)-28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1
/32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/256*d*x+3/256*c)+56*I*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c
)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^8*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7
-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)-56*I*a^8*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8)-28*a^8*(-1/10*cos(d*x+c)^9*sin(d*x+c)+1/80*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/2
56*d*x+7/256*c)-4/5*I*a^8*cos(d*x+c)^10+a^8*(1/10*(cos(d*x+c)^9+9/8*cos(d*x
+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)
+63/256*d*x+63/256*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.
time = 0.50, size = 152, normalized size = 1.90

$$-\frac{5\,a^8\tan\left(dx+c\right)^7-30i\,a^8\tan\left(dx+c\right)^6-77\,a^8\tan\left(dx+c\right)^5+110i\,a^8\tan\left(dx+c\right)^4+95\,a^8\tan\left(dx+c\right)^3-50i\,a^8\tan\left(dx+c\right)^2-15\,a^8\tan\left(dx+c\right)+2i\,a^8}{15\left(\tan\left(dx+c\right)^{10}+5\,\tan\left(dx+c\right)^8+10\,\tan\left(dx+c\right)^6+10\,\tan\left(dx+c\right)^4+5\,\tan\left(dx+c\right)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^8*tan(d*x + c)^7 - 30*I*a^8*tan(d*x + c)^6 - 77*a^8*tan(d*x + c)
^5 + 110*I*a^8*tan(d*x + c)^4 + 95*a^8*tan(d*x + c)^3 - 50*I*a^8*tan(d*x +
```

```
c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^10 + 5*tan(d*x + c)^8
+ 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)*d)
```

Fricas [A]
time = 0.39, size = 48, normalized size = 0.60

$$\frac{-6i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-10i\,a^8e^{(6i\,dx+6i\,c)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-6*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I
*a^8*e^(6*I*d*x + 6*I*c))/d
```

Sympy [A]
time = 0.58, size = 121, normalized size = 1.51

$$\begin{cases} \frac{-384ia^8d^2e^{10ic}e^{10idx}-960ia^8d^2e^{8ic}e^{8idx}-640ia^8d^2e^{6ic}e^{6idx}}{15360d^3} & \text{for}\ d^3\neq 0\\ x\left(\frac{a^8e^{10ic}}{4}+\frac{a^8e^{8ic}}{2}+\frac{a^8e^{6ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-384*I*a**8*d**2*exp(10*I*c)*exp(10*I*d*x) - 960*I*a**8*d**2*ex
p(8*I*c)*exp(8*I*d*x) - 640*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x))/(15360*d**
3), Ne(d**3, 0)), (x*(a**8*exp(10*I*c)/4 + a**8*exp(8*I*c)/2 + a**8*exp(6*I
*c)/4), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(64) = 128.
time = 1.24, size = 409, normalized size = 5.11

$$-\frac{6i\,a^8e^{(38i\,dx+24i\,c)}+99i\,a^8e^{(36i\,dx+22i\,c)}+766i\,a^8e^{(34i\,dx+20i\,c)}+3689i\,a^8e^{(32i\,dx+18i\,c)}+12376i\,a^8e^{(30i\,dx+16i\,c)}+30667i\,a^8e^{(28i\,dx+14i\,c)}+58058i\,a^8e^{(26i\,dx+12i\,c)}+85657i\,a^8e^{(24i\,dx+10i\,c)}+99528i\,a^8e^{(22i\,dx+8i\,c)}+91377i\,a^8e^{(20i\,dx+6i\,c)}+66066i\,a^8e^{(18i\,dx+4i\,c)}+37219i\,a^8e^{(16i\,dx+2i\,c)}+5089i\,a^8e^{(12i\,dx-2i\,c)}+1126i\,a^8e^{(10i\,dx-4i\,c)}+155i\,a^8e^{(8i\,dx-6i\,c)}+10i\,a^8e^{(6i\,dx-8i\,c)}+16016i\,a^8e^{(14i\,dx)}}{240\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12i\,c)}+91\,de^{(24i\,dx+10i\,c)}+364\,de^{(22i\,dx+8i\,c)}+1001\,de^{(20i\,dx+6i\,c)}+2002\,de^{(18i\,dx+4i\,c)}+3003\,de^{(16i\,dx+2i\,c)}+3003\,de^{(12i\,dx-2i\,c)}+2002\,de^{(10i\,dx-4i\,c)}+1001\,de^{(8i\,dx-6i\,c)}+364\,de^{(6i\,dx-8i\,c)}+91\,de^{(4i\,dx-10i\,c)}+14\,de^{(2i\,dx-12i\,c)}+3432\,de^{(14i\,dx)}+de^{(-14i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(6*I*a^8*e^(38*I*d*x + 24*I*c) + 99*I*a^8*e^(36*I*d*x + 22*I*c) + 76
6*I*a^8*e^(34*I*d*x + 20*I*c) + 3689*I*a^8*e^(32*I*d*x + 18*I*c) + 12376*I*
a^8*e^(30*I*d*x + 16*I*c) + 30667*I*a^8*e^(28*I*d*x + 14*I*c) + 58058*I*a^8
*e^(26*I*d*x + 12*I*c) + 85657*I*a^8*e^(24*I*d*x + 10*I*c) + 99528*I*a^8*e^
(22*I*d*x + 8*I*c) + 91377*I*a^8*e^(20*I*d*x + 6*I*c) + 66066*I*a^8*e^(18*I
*d*x + 4*I*c) + 37219*I*a^8*e^(16*I*d*x + 2*I*c) + 5089*I*a^8*e^(12*I*d*x -
 2*I*c) + 1126*I*a^8*e^(10*I*d*x - 4*I*c) + 155*I*a^8*e^(8*I*d*x - 6*I*c) +
```

```
10*I*a^8*e^(6*I*d*x - 8*I*c) + 16016*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x +
14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e
^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*
I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e
^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c
) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*
d*x) + d*e^(-14*I*c))
```

Mupad [B]
time = 3.48, size = 82, normalized size = 1.02

$$\frac{a^8\left(-5\tan(c+d\,x)^2+\tan(c+d\,x)\ 5\mathrm{i}+2\right)}{15\,d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4 5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2 10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8,x)
[Out] (a^8*(tan(c + d*x)*5i - 5*tan(c + d*x)^2 + 2))/(15*d*(5*tan(c + d*x) - tan(
c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c + d*x)^5 + 1
i))
```

3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=55

$$-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}+\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}$$

```
[Out] -1/3*I*a^14/d/(a-I*a*tan(d*x+c))^6+1/5*I*a^13/d/(a-I*a*tan(d*x+c))^5
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3*I)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) + ((I/5)*a^13)/(d*(a - I*a*Tan
[c + d*x])^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{(ia^{13})\,\text{Subst}\left(\int \frac{a+x}{(a-x)^7}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^{13})\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^7}-\frac{1}{(a-x)^6}\right)\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}+\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}\end{aligned}$$

Mathematica [**A**]
time = 0.80, size = 77, normalized size = 1.40

$$\frac{a^8(45+64\cos(2(c+dx))+20\cos(4(c+dx))-16i\sin(2(c+dx))-10i\sin(4(c+dx)))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(45 + 64*Cos[2*(c + d*x)] + 20*Cos[4*(c + d*x)] - (16*I)*Sin[2*(c + d*
x)] - (10*I)*Sin[4*(c + d*x)])*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/
(960*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(47) = 94.
time = 0.25, size = 639, normalized size = 11.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*sin(d*x+c)*cos(d*x+c)^5+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-8*I*a^8*(-1/12*sin(d
*x+c)^6*cos(d*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d
*x+c)^6-1/120*cos(d*x+c)^6)-28*a^8*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*si
n(d*x+c)^3*cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+56*I*a^8*(-
1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/120*cos(d*x
+c)^8)+70*a^8*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*cos(d*x+c)^9*sin(d*x+c)
+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*
sin(d*x+c)+7/1024*d*x+7/1024*c)-56*I*a^8*(-1/12*sin(d*x+c)^2*cos(d*x+c)^10-
1/60*cos(d*x+c)^10)-28*a^8*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/120*(cos(d*x+c
)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x
+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-2/3*I*a^8*cos(d*x+c)^12+a^8*(1/12*(c
os(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231
/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+231/1024*c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(43) = 86.
time = 0.53, size = 162, normalized size = 2.95

$$-\frac{3\,a^8\tan(dx+c)^7-20i\,a^8\tan(dx+c)^6-57\,a^8\tan(dx+c)^5+90i\,a^8\tan(dx+c)^4+85\,a^8\tan(dx+c)^3-48i\,a^8\tan(dx+c)^2-15\,a^8\tan(dx+c)+2i\,a^8}{15\left(\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan(dx+c)^4+6\,\tan(dx+c)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a^8*tan(d*x + c)^7 - 20*I*a^8*tan(d*x + c)^6 - 57*a^8*tan(d*x + c)
^5 + 90*I*a^8*tan(d*x + c)^4 + 85*a^8*tan(d*x + c)^3 - 48*I*a^8*tan(d*x + c
)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^12 + 6*tan(d*x + c)^10
+ 15*tan(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c
)^2 + 1)*d)
```

Fricas [A]
time = 0.40, size = 76, normalized size = 1.38

$$\frac{-5i\,a^{8}e^{(12i\,dx+12i\,c)}-24i\,a^{8}e^{(10i\,dx+10i\,c)}-45i\,a^{8}e^{(8i\,dx+8i\,c)}-40i\,a^{8}e^{(6i\,dx+6i\,c)}-15i\,a^{8}e^{(4i\,dx+4i\,c)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(-5*I*a^8*e^(12*I*d*x + 12*I*c) - 24*I*a^8*e^(10*I*d*x + 10*I*c) - 45
*I*a^8*e^(8*I*d*x + 8*I*c) - 40*I*a^8*e^(6*I*d*x + 6*I*c) - 15*I*a^8*e^(4*I
*d*x + 4*I*c))/d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(42) = 84.
time = 0.67, size = 197, normalized size = 3.58

$$\begin{cases}\frac{-3932160ia^{8}d^{4}e^{12ic}e^{12idx}-18874368ia^{8}d^{4}e^{10ic}e^{10idx}-35389440ia^{8}d^{4}e^{8ic}e^{8idx}-31457280ia^{8}d^{4}e^{6ic}e^{6idx}-11796480ia^{8}d^{4}e^{4ic}e^{4idx}}{754974720d^{5}} & \text{for}\: d^{5}\neq 0\\x\left(\frac{a^{8}e^{12ic}}{16}+\frac{a^{8}e^{10ic}}{4}+\frac{3a^{8}e^{8ic}}{8}+\frac{a^{8}e^{6ic}}{4}+\frac{a^{8}e^{4ic}}{16}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-3932160*I*a**8*d**4*exp(12*I*c)*exp(12*I*d*x) - 18874368*I*a**
8*d**4*exp(10*I*c)*exp(10*I*d*x) - 35389440*I*a**8*d**4*exp(8*I*c)*exp(8*I*
d*x) - 31457280*I*a**8*d**4*exp(6*I*c)*exp(6*I*d*x) - 11796480*I*a**8*d**4*
exp(4*I*c)*exp(4*I*d*x))/(754974720*d**5), Ne(d**5, 0)), (x*(a**8*exp(12*I*
c)/16 + a**8*exp(10*I*c)/4 + 3*a**8*exp(8*I*c)/8 + a**8*exp(6*I*c)/4 + a**8
*exp(4*I*c)/16), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(43) = 86.
time = 2.05, size = 437, normalized size = 7.95

$$-\frac{5i\,a^{8}e^{(40i\,dx+26i\,c)}+94i\,a^{8}e^{(38i\,dx+24i\,c)}+836i\,a^{8}e^{(36i\,dx+22i\,c)}+4674i\,a^{8}e^{(34i\,dx+20i\,c)}+18411i\,a^{8}e^{(32i\,dx+18i\,c)}+54264i\,a^{8}e^{(30i\,dx+16i\,c)}+124033i\,a^{8}e^{(28i\,dx+14i\,c)}+224822i\,a^{8}e^{(26i\,dx+12i\,c)}+327613i\,a^{8}e^{(24i\,dx+10i\,c)}+386672i\,a^{8}e^{(22i\,dx+8i\,c)}+370513i\,a^{8}e^{(20i\,dx+6i\,c)}+287534i\,a^{8}e^{(18i\,dx+4i\,c)}+179361i\,a^{8}e^{(16i\,dx+2i\,c)}+34011i\,a^{8}e^{(12i\,dx-2i\,c)}+9754i\,a^{8}e^{(10i\,dx-4i\,c)}+1970i\,a^{8}e^{(8i\,dx-6i\,c)}+250i\,a^{8}e^{(6i\,dx-8i\,c)}+15i\,a^{8}e^{(4i\,dx-10i\,c)}+88704i\,a^{8}e^{(14i\,dx)}}{960\,(d e^{(28i\,dx+14i\,c)}+14\,d e^{(26i\,dx+12i\,c)}+91\,d e^{(24i\,dx+10i\,c)}+364\,d e^{(22i\,dx+8i\,c)}+1001\,d e^{(20i\,dx+6i\,c)}+2002\,d e^{(18i\,dx+4i\,c)}+3003\,d e^{(16i\,dx+2i\,c)}+3003\,d e^{(12i\,dx-2i\,c)}+2002\,d e^{(10i\,dx-4i\,c)}+1001\,d e^{(8i\,dx-6i\,c)}+364\,d e^{(6i\,dx-8i\,c)}+91\,d e^{(4i\,dx-10i\,c)}+14\,d e^{(2i\,dx-12i\,c)}+3432\,d e^{(14i\,dx)}+d e^{(-14i\,c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/960*(5*I*a^8*e^(40*I*d*x + 26*I*c) + 94*I*a^8*e^(38*I*d*x + 24*I*c) + 83
6*I*a^8*e^(36*I*d*x + 22*I*c) + 4674*I*a^8*e^(34*I*d*x + 20*I*c) + 18411*I*
a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8*e^(30*I*d*x + 16*I*c) + 124033*I*a^
8*e^(28*I*d*x + 14*I*c) + 224822*I*a^8*e^(26*I*d*x + 12*I*c) + 327613*I*a^8
*e^(24*I*d*x + 10*I*c) + 386672*I*a^8*e^(22*I*d*x + 8*I*c) + 370513*I*a^8*e
^(20*I*d*x + 6*I*c) + 287534*I*a^8*e^(18*I*d*x + 4*I*c) + 179361*I*a^8*e^(1
6*I*d*x + 2*I*c) + 34011*I*a^8*e^(12*I*d*x - 2*I*c) + 9754*I*a^8*e^(10*I*d*
x - 4*I*c) + 1970*I*a^8*e^(8*I*d*x - 6*I*c) + 250*I*a^8*e^(6*I*d*x - 8*I*c)
 + 15*I*a^8*e^(4*I*d*x - 10*I*c) + 88704*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x
 + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*
d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x +
 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*
d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*
I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14
*I*d*x) + d*e^(-14*I*c))
```

Mupad [B]
time = 3.40, size = 82, normalized size = 1.49

$$-\frac{a^8\,\left(3\,\mathrm{tan}\left(c+d\,x\right)-2{}\mathrm{i}\right)}{15\,d\,\left({\mathrm{tan}\left(c+d\,x\right)}^6+{\mathrm{tan}\left(c+d\,x\right)}^5\,6{}\mathrm{i}-15\,{\mathrm{tan}\left(c+d\,x\right)}^4-{\mathrm{tan}\left(c+d\,x\right)}^3\,20{}\mathrm{i}+15\,{\mathrm{tan}\left(c+d\,x\right)}^2+\mathrm{tan}\left(c+d\,x\right)\,6{}\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*(3*tan(c + d*x) - 2i))/(15*d*(tan(c + d*x)*6i + 15*tan(c + d*x)^2 - t
an(c + d*x)^3*20i - 15*tan(c + d*x)^4 + tan(c + d*x)^5*6i + tan(c + d*x)^6
- 1))
```

3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=27

$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

```
[Out] -1/7*I*a^15/d/(a-I*a*tan(d*x+c))^7
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/7*I)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{(ia^{15})\,\text{Subst}\left(\int \frac{1}{(a-x)^8}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 116 vs. 2(27) = 54.
time = 1.24, size = 116, normalized size = 4.30

$$\frac{a^8(35+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+dx))-14i\sin(2(c+dx))-14i\sin(4(c+dx))-6i\sin(6(c+dx)))(-i\cos(8(c+2dx))+\sin(8(c+2dx)))}{896d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8*Cos[6*(c + d*x)] -
 (14*I)*Sin[2*(c + d*x)] - (14*I)*Sin[4*(c + d*x)] - (6*I)*Sin[6*(c + d*x)]
)*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(896*d*(Cos[d*x] + I*Sin[
d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(23) = 46$.
time = 0.29, size = 689, normalized size = 25.52

method	result
risch	$-\frac{i a^{8} e^{14 i (d x+c)}}{896 d}-\frac{i a^{8} e^{12 i (d x+c)}}{128 d}-\frac{3 i a^{8} e^{10 i (d x+c)}}{128 d}-\frac{5 i a^{8} e^{8 i (d x+c)}}{128 d}-\frac{5 i a^{8} e^{6 i (d x+c)}}{128 d}-\frac{3 i a^{8} e^{4 i (d x+c)}}{128 d}-\frac{i a^{8} e^{2}}{12}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/14*sin(d*x+c)^7*cos(d*x+c)^7-1/24*sin(d*x+c)^5*cos(d*x+c)^7-1/
48*sin(d*x+c)^3*cos(d*x+c)^7-1/128*sin(d*x+c)*cos(d*x+c)^7+1/768*(cos(d*x+c
)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2048*d*x+5/2048*c)-8*I*a
^8*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*sin(d*x+c)^4*cos(d*x+c)^8-1/70*sin
(d*x+c)^2*cos(d*x+c)^8-1/280*cos(d*x+c)^8)-28*a^8*(-1/14*sin(d*x+c)^5*cos(d
*x+c)^9-5/168*sin(d*x+c)^3*cos(d*x+c)^9-1/112*cos(d*x+c)^9*sin(d*x+c)+1/896
*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*
x+c)+5/2048*d*x+5/2048*c)+56*I*a^8*(-1/14*sin(d*x+c)^4*cos(d*x+c)^10-1/42*s
in(d*x+c)^2*cos(d*x+c)^10-1/210*cos(d*x+c)^10)+70*a^8*(-1/14*sin(d*x+c)^3*c
os(d*x+c)^11-1/56*sin(d*x+c)*cos(d*x+c)^11+1/560*(cos(d*x+c)^9+9/8*cos(d*x+
c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+
9/2048*d*x+9/2048*c)-56*I*a^8*(-1/14*sin(d*x+c)^2*cos(d*x+c)^12-1/84*cos(d*
x+c)^12)-28*a^8*(-1/14*cos(d*x+c)^13*sin(d*x+c)+1/168*(cos(d*x+c)^11+11/10*
cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+6
93/256*cos(d*x+c))*sin(d*x+c)+33/2048*d*x+33/2048*c)-4/7*I*a^8*cos(d*x+c)^1
4+a^8*(1/14*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/320
*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(d*x
+c))*sin(d*x+c)+429/2048*d*x+429/2048*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(21) = 42$.
time = 0.51, size = 171, normalized size = 6.33

$$-\frac{a^{8} \tan \left(d x+c\right)^{7}-7 i a^{8} \tan \left(d x+c\right)^{6}-21 a^{8} \tan \left(d x+c\right)^{5}+35 i a^{8} \tan \left(d x+c\right)^{4}+35 a^{8} \tan \left(d x+c\right)^{3}-21 i a^{8} \tan \left(d x+c\right)^{2}-7 a^{8} \tan \left(d x+c\right)+i a^{8}}{7 \left(\tan \left(d x+c\right)^{14}+7 \tan \left(d x+c\right)^{12}+21 \tan \left(d x+c\right)^{10}+35 \tan \left(d x+c\right)^{8}+35 \tan \left(d x+c\right)^{6}+21 \tan \left(d x+c\right)^{4}+7 \tan \left(d x+c\right)^{2}+1\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7*(a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 +
 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2
- 7*a^8*tan(d*x + c) + I*a^8)/((tan(d*x + c)^14 + 7*tan(d*x + c)^12 + 21*ta
n(d*x + c)^10 + 35*tan(d*x + c)^8 + 35*tan(d*x + c)^6 + 21*tan(d*x + c)^4 +
 7*tan(d*x + c)^2 + 1)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(21) = 42$.
time = 0.42, size = 104, normalized size = 3.85

$$\frac{-i\,a^8e^{(14i\,dx+14i\,c)}-7i\,a^8e^{(12i\,dx+12i\,c)}-21i\,a^8e^{(10i\,dx+10i\,c)}-35i\,a^8e^{(8i\,dx+8i\,c)}-35i\,a^8e^{(6i\,dx+6i\,c)}-21i\,a^8e^{(4i\,dx+4i\,c)}-7i\,a^8e^{(2i\,dx+2i\,c)}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(-I*a^8*e^(14*I*d*x + 14*I*c) - 7*I*a^8*e^(12*I*d*x + 12*I*c) - 21*I*
a^8*e^(10*I*d*x + 10*I*c) - 35*I*a^8*e^(8*I*d*x + 8*I*c) - 35*I*a^8*e^(6*I*
d*x + 6*I*c) - 21*I*a^8*e^(4*I*d*x + 4*I*c) - 7*I*a^8*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(22) = 44$.
time = 0.78, size = 279, normalized size = 10.33

$$\begin{cases} \frac{-4398046511104ia^8d^6e^{14ic}e^{14idx}-30786325577728ia^8d^6e^{12ic}e^{12idx}-92358976733184ia^8d^6e^{10ic}e^{10idx}-153931627888640ia^8d^6e^{8ic}e^{8idx}-153931627888640ia^8d^6e^{6ic}e^{6idx}-92358976733184ia^8d^6e^{4ic}e^{4idx}-30786325577728ia^8d^6e^{2ic}e^{2idx}}{3940649673949184d^7} & \text{for}\: d^7\neq 0 \\ x\left(\frac{a^8e^{14ic}}{64}+\frac{3a^8e^{12ic}}{32}+\frac{15a^8e^{10ic}}{64}+\frac{5a^8e^{8ic}}{16}+\frac{15a^8e^{6ic}}{64}+\frac{3a^8e^{4ic}}{32}+\frac{a^8e^{2ic}}{64}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**14*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-4398046511104*I*a**8*d**6*exp(14*I*c)*exp(14*I*d*x) - 30786325
577728*I*a**8*d**6*exp(12*I*c)*exp(12*I*d*x) - 92358976733184*I*a**8*d**6*e
xp(10*I*c)*exp(10*I*d*x) - 153931627888640*I*a**8*d**6*exp(8*I*c)*exp(8*I*d
*x) - 153931627888640*I*a**8*d**6*exp(6*I*c)*exp(6*I*d*x) - 92358976733184*
I*a**8*d**6*exp(4*I*c)*exp(4*I*d*x) - 30786325577728*I*a**8*d**6*exp(2*I*c)
*exp(2*I*d*x))/(3940649673949184*d**7), Ne(d**7, 0)), (x*(a**8*exp(14*I*c)/
64 + 3*a**8*exp(12*I*c)/32 + 15*a**8*exp(10*I*c)/64 + 5*a**8*exp(8*I*c)/16
+ 15*a**8*exp(6*I*c)/64 + 3*a**8*exp(4*I*c)/32 + a**8*exp(2*I*c)/64), True)
)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(21) = 42$.
time = 2.43, size = 465, normalized size = 17.22

$$-\frac{i\,a^8e^{(42i\,dx+28i\,c)}+21i\,a^8e^{(40i\,dx+26i\,c)}+210i\,a^8e^{(38i\,dx+24i\,c)}+1330i\,a^8e^{(36i\,dx+22i\,c)}+5985i\,a^8e^{(34i\,dx+20i\,c)}+20349i\,a^8e^{(32i\,dx+18i\,c)}+54264i\,a^8e^{(30i\,dx+16i\,c)}+116279i\,a^8e^{(28i\,dx+14i\,c)}+203476i\,a^8e^{(26i\,dx+12i\,c)}+293839i\,a^8e^{(24i\,dx+10i\,c)}+352352i\,a^8e^{(22i\,dx+8i\,c)}+351715i\,a^8e^{(20i\,dx+6i\,c)}+291928i\,a^8e^{(18i\,dx+4i\,c)}+200487i\,a^8e^{(16i\,dx+2i\,c)}+51261i\,a^8e^{(12i\,dx-2i\,c)}+18347i\,a^8e^{(10i\,dx-4i\,c)}+4984i\,a^8e^{(8i\,dx-6i\,c)}+966i\,a^8e^{(6i\,dx-8i\,c)}+119i\,a^8e^{(4i\,dx-10i\,c)}+7i\,a^8e^{(2i\,dx-12i\,c)}+112848i\,a^8e^{(14i\,dx)}}{896\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12i\,c)}+91\,de^{(24i\,dx+10i\,c)}+364\,de^{(22i\,dx+8i\,c)}+1001\,de^{(20i\,dx+6i\,c)}+2002\,de^{(18i\,dx+4i\,c)}+3003\,de^{(16i\,dx+2i\,c)}+3003\,de^{(12i\,dx-2i\,c)}+2002\,de^{(10i\,dx-4i\,c)}+1001\,de^{(8i\,dx-6i\,c)}+364\,de^{(6i\,dx-8i\,c)}+91\,de^{(4i\,dx-10i\,c)}+14\,de^{(2i\,dx-12i\,c)}+3432\,de^{(14i\,dx)}+de^{(-14i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/896*(I*a^8*e^(42*I*d*x + 28*I*c) + 21*I*a^8*e^(40*I*d*x + 26*I*c) + 210*
I*a^8*e^(38*I*d*x + 24*I*c) + 1330*I*a^8*e^(36*I*d*x + 22*I*c) + 5985*I*a^8
*e^(34*I*d*x + 20*I*c) + 20349*I*a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8*e^
(30*I*d*x + 16*I*c) + 116279*I*a^8*e^(28*I*d*x + 14*I*c) + 203476*I*a^8*e^(
26*I*d*x + 12*I*c) + 293839*I*a^8*e^(24*I*d*x + 10*I*c) + 352352*I*a^8*e^(2
2*I*d*x + 8*I*c) + 351715*I*a^8*e^(20*I*d*x + 6*I*c) + 291928*I*a^8*e^(18*I
*d*x + 4*I*c) + 200487*I*a^8*e^(16*I*d*x + 2*I*c) + 51261*I*a^8*e^(12*I*d*x
 - 2*I*c) + 18347*I*a^8*e^(10*I*d*x - 4*I*c) + 4984*I*a^8*e^(8*I*d*x - 6*I*
c) + 966*I*a^8*e^(6*I*d*x - 8*I*c) + 119*I*a^8*e^(4*I*d*x - 10*I*c) + 7*I*a
^8*e^(2*I*d*x - 12*I*c) + 112848*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*
c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*
I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c)
+ 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*
I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 9
1*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x)
+ d*e^(-14*I*c))
```

Mupad [B]
time = 3.38, size = 105, normalized size = 3.89

$$-\frac{a^8\cos\left(c+d\,x\right)^8\,\left(\mathrm{tan}\left(c+d\,x\right)-7{}\mathrm{i}\right)}{7\,d}+\frac{64\,a^8\cos\left(c+d\,x\right)^{14}\,\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}{7\,d}+\frac{8\,a^8\cos\left(c+d\,x\right)^{10}\,\left(3\,\mathrm{tan}\left(c+d\,x\right)-7{}\mathrm{i}\right)}{7\,d}-\frac{16\,a^8\cos\left(c+d\,x\right)^{12}\,\left(5\,\mathrm{tan}\left(c+d\,x\right)-7{}\mathrm{i}\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (64*a^8*cos(c + d*x)^14*(tan(c + d*x) - 1i))/(7*d) - (a^8*cos(c + d*x)^8*(t
an(c + d*x) - 7i))/(7*d) + (8*a^8*cos(c + d*x)^10*(3*tan(c + d*x) - 7i))/(7
*d) - (16*a^8*cos(c + d*x)^12*(5*tan(c + d*x) - 7i))/(7*d)
```

3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=225

$$\frac{a^8x}{256}-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{80d(a-ia\tan(c+dx$$

```
[Out] 1/256*a^8*x-1/16*I*a^16/d/(a-I*a*tan(d*x+c))^8-1/28*I*a^15/d/(a-I*a*tan(d*x
+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))^6-1/80*I*a^13/d/(a-I*a*tan(d*x+c))^
5-1/128*I*a^12/d/(a-I*a*tan(d*x+c))^4-1/192*I*a^11/d/(a-I*a*tan(d*x+c))^3-1
/256*I*a^10/d/(a-I*a*tan(d*x+c))^2-1/256*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{80d(a-ia\tan(c+dx))^5}-\frac{ia^{12}}{128d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{192d(a-ia\tan(c+dx))^3}-\frac{ia^{10}}{256d(a-ia\tan(c+dx))^2}-\frac{ia^9}{256d(a-ia\tan(c+dx))}+\frac{a^8x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*x)/256 - ((I/16)*a^16)/(d*(a - I*a*Tan[c + d*x])^8) - ((I/28)*a^15)/(d
*(a - I*a*Tan[c + d*x])^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - (
(I/80)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/128)*a^12)/(d*(a - I*a*Tan[
c + d*x])^4) - ((I/192)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/256)*a^10)
/(d*(a - I*a*Tan[c + d*x])^2) - ((I/256)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^{17})\,\text{Subst}\left(\int \frac{1}{(a-x)^9(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^{17})\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^9} + \frac{1}{4a^2(a-x)^8} + \frac{1}{8a^3(a-x)^7} + \frac{1}{16a^4(a-x)^6} + \right.\right.}{}$$
$$= -\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} - \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} - \frac{}{48d(a}$$
$$= \frac{a^8x}{256} - \frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} - \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} - $$

Mathematica [A]
time = 2.69, size = 166, normalized size = 0.74

$$\frac{a^8(-14700i - 25088i\cos(2(c+dx)) - 15680i\cos(4(c+dx)) - 7680i\cos(6(c+dx)) - 105i\cos(8(c+dx)) + 1680dx\cos(8(c+dx)) - 6272\sin(2(c+dx)) - 7840\sin(4(c+dx)) - 5760\sin(6(c+dx)) + 105\sin(8(c+dx)) - 1680idx\sin(8(c+dx)))(\cos(8(c+2dx)) + i\sin(8(c+2dx)))}{430080d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(-14700*I - (25088*I)*Cos[2*(c + d*x)] - (15680*I)*Cos[4*(c + d*x)] -
(7680*I)*Cos[6*(c + d*x)] - (105*I)*Cos[8*(c + d*x)] + 1680*d*x*Cos[8*(c +
d*x)] - 6272*Sin[2*(c + d*x)] - 7840*Sin[4*(c + d*x)] - 5760*Sin[6*(c + d*x
)] + 105*Sin[8*(c + d*x)] - (1680*I)*d*x*Sin[8*(c + d*x)])*(Cos[8*(c + 2*d*
x)] + I*Sin[8*(c + 2*d*x)]))/(430080*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(191) = 382.
time = 0.31, size = 739, normalized size = 3.28

method	result
risch	$\frac{a^8x}{256} - \frac{ia^8e^{16i(dx+c)}}{4096d} - \frac{ia^8e^{14i(dx+c)}}{448d} - \frac{7ia^8e^{12i(dx+c)}}{768d} - \frac{7ia^8e^{10i(dx+c)}}{320d} - \frac{35ia^8e^{8i(dx+c)}}{1024d} - \frac{7ia^8e^{6i(dx+c)}}{192d} -$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/16*sin(d*x+c)^7*cos(d*x+c)^9-1/32*sin(d*x+c)^5*cos(d*x+c)^9-5/
384*sin(d*x+c)^3*cos(d*x+c)^9-1/256*cos(d*x+c)^9*sin(d*x+c)+1/2048*(cos(d*x
+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/3
2768*d*x+35/32768*c)-8*I*a^8*(-1/16*sin(d*x+c)^6*cos(d*x+c)^10-3/112*sin(d*
x+c)^4*cos(d*x+c)^10-1/112*sin(d*x+c)^2*cos(d*x+c)^10-1/560*cos(d*x+c)^10)-
28*a^8*(-1/16*sin(d*x+c)^5*cos(d*x+c)^11-5/224*sin(d*x+c)^3*cos(d*x+c)^11-5
/896*sin(d*x+c)*cos(d*x+c)^11+1/1792*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*c
os(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+45/32768*d*x
+45/32768*c)+56*I*a^8*(-1/16*sin(d*x+c)^4*cos(d*x+c)^12-1/56*sin(d*x+c)^2*c
os(d*x+c)^12-1/336*cos(d*x+c)^12)+70*a^8*(-1/16*sin(d*x+c)^3*cos(d*x+c)^13-
3/224*cos(d*x+c)^13*sin(d*x+c)+1/896*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/8
0*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c)
)*sin(d*x+c)+99/32768*d*x+99/32768*c)-56*I*a^8*(-1/16*sin(d*x+c)^2*cos(d*x+
c)^14-1/112*cos(d*x+c)^14)-28*a^8*(-1/16*cos(d*x+c)^15*sin(d*x+c)+1/224*(co
s(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/320*cos(d*x+c)^7+1
001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(d*x+c))*sin(d*x+c)
+429/32768*d*x+429/32768*c)-1/2*I*a^8*cos(d*x+c)^16+a^8*(1/16*(cos(d*x+c)^1
5+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d*x+c)^9+1287/896*cos
(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+6435/2048*cos(d*x+c))
*sin(d*x+c)+6435/32768*d*x+6435/32768*c))
```

Maxima [A]
time = 0.51, size = 246, normalized size = 1.09

$$\frac{105\,(dx+c)a^8+\frac{105\,a^8\tan(dx+c)^{15}+805\,a^8\tan(dx+c)^{13}+2681\,a^8\tan(dx+c)^{11}+5053\,a^8\tan(dx+c)^9+2883\,a^8\tan(dx+c)^7+21504i\,a^8\tan(dx+c)^6+70791\,a^8\tan(dx+c)^5-114688i\,a^8\tan(dx+c)^4-117285\,a^8\tan(dx+c)^3+74752i\,a^8\tan(dx+c)^2+26775\,a^8\tan(dx+c)-4096i\,a^8}{\tan(dx+c)^{16}+8\tan(dx+c)^{14}+28\tan(dx+c)^{12}+56\tan(dx+c)^{10}+70\tan(dx+c)^8+56\tan(dx+c)^6+28\tan(dx+c)^4+8\tan(dx+c)^2+1}}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/26880*(105*(d*x + c)*a^8 + (105*a^8*tan(d*x + c)^15 + 805*a^8*tan(d*x + c
)^13 + 2681*a^8*tan(d*x + c)^11 + 5053*a^8*tan(d*x + c)^9 + 2883*a^8*tan(d*
x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 70791*a^8*tan(d*x + c)^5 - 114688*I
*a^8*tan(d*x + c)^4 - 117285*a^8*tan(d*x + c)^3 + 74752*I*a^8*tan(d*x + c)^
2 + 26775*a^8*tan(d*x + c) - 4096*I*a^8)/(tan(d*x + c)^16 + 8*tan(d*x + c)^
14 + 28*tan(d*x + c)^12 + 56*tan(d*x + c)^10 + 70*tan(d*x + c)^8 + 56*tan(d
*x + c)^6 + 28*tan(d*x + c)^4 + 8*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.43, size = 125, normalized size = 0.56

$$\frac{1680\,a^8dx-105i\,a^8e^{(16i\,dx+16i\,c)}-960i\,a^8e^{(14i\,dx+14i\,c)}-3920i\,a^8e^{(12i\,dx+12i\,c)}-9408i\,a^8e^{(10i\,dx+10i\,c)}-14700i\,a^8e^{(8i\,dx+8i\,c)}-15680i\,a^8e^{(6i\,dx+6i\,c)}-11760i\,a^8e^{(4i\,dx+4i\,c)}-6720i\,a^8e^{(2i\,dx+2i\,c)}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(1680*a^8*d*x - 105*I*a^8*e^(16*I*d*x + 16*I*c) - 960*I*a^8*e^(14*
I*d*x + 14*I*c) - 3920*I*a^8*e^(12*I*d*x + 12*I*c) - 9408*I*a^8*e^(10*I*d*x
 + 10*I*c) - 14700*I*a^8*e^(8*I*d*x + 8*I*c) - 15680*I*a^8*e^(6*I*d*x + 6*I
*c) - 11760*I*a^8*e^(4*I*d*x + 4*I*c) - 6720*I*a^8*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A]
time = 0.89, size = 323, normalized size = 1.44

$$\frac{a^8x}{256}+\begin{cases}\frac{-3546584706554265 60ia^8d^7e^{16ic}e^{16idx}-324259173170675712 0ia^8d^7e^{14ic}e^{14idx}-13240582904469258240ia^8d^7e^{12ic}e^{12idx}-31777398970726219776ia^8d^7e^{10ic}e^{10idx}-49652185891759718400ia^8d^7e^{8ic}e^{8idx}-52962331617877032960ia^8d^7e^{6ic}e^{6idx}-39721748713407774720ia^8d^7e^{4ic}e^{4idx}-22698142121947299840ia^8d^7e^{2ic}e^{2idx}}{1452681095804627189760d^8} & \text{for } d^8\neq 0\\ x\left(\frac{a^8e^{16ic}}{256}+\frac{a^8e^{14ic}}{32}+\frac{7a^8e^{12ic}}{64}+\frac{7a^8e^{10ic}}{32}+\frac{35a^8e^{8ic}}{128}+\frac{7a^8e^{6ic}}{32}+\frac{7a^8e^{4ic}}{64}+\frac{a^8e^{2ic}}{32}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**16*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*x/256 + Piecewise(((-354658470655426560*I*a**8*d**7*exp(16*I*c)*exp(16
*I*d*x) - 3242591731706757120*I*a**8*d**7*exp(14*I*c)*exp(14*I*d*x) - 13240
582904469258240*I*a**8*d**7*exp(12*I*c)*exp(12*I*d*x) - 3177739897072621977
6*I*a**8*d**7*exp(10*I*c)*exp(10*I*d*x) - 49652185891759718400*I*a**8*d**7*
exp(8*I*c)*exp(8*I*d*x) - 52962331617877032960*I*a**8*d**7*exp(6*I*c)*exp(6
*I*d*x) - 39721748713407774720*I*a**8*d**7*exp(4*I*c)*exp(4*I*d*x) - 226981
42121947299840*I*a**8*d**7*exp(2*I*c)*exp(2*I*d*x))/(1452681095804627189760
*d**8), Ne(d**8, 0)), (x*(a**8*exp(16*I*c)/256 + a**8*exp(14*I*c)/32 + 7*a*
*8*exp(12*I*c)/64 + 7*a**8*exp(10*I*c)/32 + 35*a**8*exp(8*I*c)/128 + 7*a**8
*exp(6*I*c)/32 + 7*a**8*exp(4*I*c)/64 + a**8*exp(2*I*c)/32), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(175) = 350$.
time = 1.63, size = 1457, normalized size = 6.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/13762560*(53760*a^8*d*x*e^(28*I*d*x + 14*I*c) + 752640*a^8*d*x*e^(26*I*d*
x + 12*I*c) + 4892160*a^8*d*x*e^(24*I*d*x + 10*I*c) + 19568640*a^8*d*x*e^(2
2*I*d*x + 8*I*c) + 53813760*a^8*d*x*e^(20*I*d*x + 6*I*c) + 107627520*a^8*d*
x*e^(18*I*d*x + 4*I*c) + 161441280*a^8*d*x*e^(16*I*d*x + 2*I*c) + 161441280
*a^8*d*x*e^(12*I*d*x - 2*I*c) + 107627520*a^8*d*x*e^(10*I*d*x - 4*I*c) + 53
813760*a^8*d*x*e^(8*I*d*x - 6*I*c) + 19568640*a^8*d*x*e^(6*I*d*x - 8*I*c) +
 4892160*a^8*d*x*e^(4*I*d*x - 10*I*c) + 752640*a^8*d*x*e^(2*I*d*x - 12*I*c)
 + 184504320*a^8*d*x*e^(14*I*d*x) + 53760*a^8*d*x*e^(-14*I*c) - 25935*I*a^8
*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8*e^(26*I*
d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8*e^(24*I*d*x + 10
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(22*I*d*x + 8*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 25960935*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*
```

```
d*x + 2*I*c) + 1) - 51921870*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 77882805*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 77882805*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5192
1870*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 25960935*I*a
^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(6*I*
d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8*e^(4*I*d*x - 10*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) - 89008920*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c
) + 1) - 25935*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 25935*I*a^8
*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 363090*I*a^8*e^(26*I
*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2360085*I*a^8*e^(24*I*d*x +
10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 9440340*I*a^8*e^(22*I*d*x + 8*I*c)*
log(e^(2*I*d*x) + e^(-2*I*c)) + 25960935*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 51921870*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x
) + e^(-2*I*c)) + 77882805*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(
-2*I*c)) + 77882805*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 51921870*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 259
60935*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 9440340*I*a
^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2360085*I*a^8*e^(4*I
*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 363090*I*a^8*e^(2*I*d*x - 12
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 89008920*I*a^8*e^(14*I*d*x)*log(e^(2*
I*d*x) + e^(-2*I*c)) + 25935*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) - 3360*I*a^8*e^(44*I*d*x + 30*I*c) - 77760*I*a^8*e^(42*I*d*x + 28*I*c) -
861280*I*a^8*e^(40*I*d*x + 26*I*c) - 6075776*I*a^8*e^(38*I*d*x + 24*I*c) -
30645664*I*a^8*e^(36*I*d*x + 22*I*c) - 117621056*I*a^8*e^(34*I*d*x + 20*I*c
) - 356948704*I*a^8*e^(32*I*d*x + 18*I*c) - 878640896*I*a^8*e^(30*I*d*x + 1
6*I*c) - 1785698272*I*a^8*e^(28*I*d*x + 14*I*c) - 3034111808*I*a^8*e^(26*I*
d*x + 12*I*c) - 4346890912*I*a^8*e^(24*I*d*x + 10*I*c) - 5277021568*I*a^8*e
^(22*I*d*x + 8*I*c) - 5435017952*I*a^8*e^(20*I*d*x + 6*I*c) - 4735681216*I*
a^8*e^(18*I*d*x + 4*I*c) - 3464933024*I*a^8*e^(16*I*d*x + 2*I*c) - 10369936
64*I*a^8*e^(12*I*d*x - 2*I*c) - 404782336*I*a^8*e^(10*I*d*x - 4*I*c) - 1200
14720*I*a^8*e^(8*I*d*x - 6*I*c) - 25338880*I*a^8*e^(6*I*d*x - 8*I*c) - 3386
880*I*a^8*e^(4*I*d*x - 10*I*c) - 215040*I*a^8*e^(2*I*d*x - 12*I*c) - 210182
8096*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I
*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(2
0*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c
) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8
*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 1
4*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B]

time = 4.85, size = 195, normalized size = 0.87

$$\frac{a^8\,x}{256}-\frac{-\frac{a^8\,\mathrm{tan}\left(c+d\,x\right)^7}{256}-\frac{a^8\,\mathrm{tan}\left(c+d\,x\right)^6\,1{}\mathrm{i}}{32}+\frac{85\,a^8\,\mathrm{tan}\left(c+d\,x\right)^5}{768}+\frac{a^8\,\mathrm{tan}\left(c+d\,x\right)^4\,11{}\mathrm{i}}{48}-\frac{1193\,a^8\,\mathrm{tan}\left(c+d\,x\right)^3}{3840}-\frac{a^8\,\mathrm{tan}\left(c+d\,x\right)^2\,143{}\mathrm{i}}{480}+\frac{5993\,a^8\,\mathrm{tan}\left(c+d\,x\right)}{26880}+\frac{a^8\,16{}\mathrm{i}}{105}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^8+{\mathrm{tan}\left(c+d\,x\right)}^7\,8{}\mathrm{i}-28\,{\mathrm{tan}\left(c+d\,x\right)}^6-{\mathrm{tan}\left(c+d\,x\right)}^5\,56{}\mathrm{i}+70\,{\mathrm{tan}\left(c+d\,x\right)}^4+{\mathrm{tan}\left(c+d\,x\right)}^3\,56{}\mathrm{i}-28\,{\mathrm{tan}\left(c+d\,x\right)}^2-\mathrm{tan}\left(c+d\,x\right)\,8{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^16*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*x)/256 - ((5993*a^8*tan(c + d*x))/26880 + (a^8*16i)/105 - (a^8*tan(c +
 d*x)^2*143i)/480 - (1193*a^8*tan(c + d*x)^3)/3840 + (a^8*tan(c + d*x)^4*11
i)/48 + (85*a^8*tan(c + d*x)^5)/768 - (a^8*tan(c + d*x)^6*1i)/32 - (a^8*tan
(c + d*x)^7)/256)/(d*(tan(c + d*x)^3*56i - 28*tan(c + d*x)^2 - tan(c + d*x)
*8i + 70*tan(c + d*x)^4 - tan(c + d*x)^5*56i - 28*tan(c + d*x)^6 + tan(c +
d*x)^7*8i + tan(c + d*x)^8 + 1))
```

3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=279

$$\frac{5a^8x}{512}-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+$$

```
[Out] 5/512*a^8*x-1/36*I*a^17/d/(a-I*a*tan(d*x+c))^9-1/32*I*a^16/d/(a-I*a*tan(d*x
+c))^8-3/112*I*a^15/d/(a-I*a*tan(d*x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))
^6-1/64*I*a^13/d/(a-I*a*tan(d*x+c))^5-3/256*I*a^12/d/(a-I*a*tan(d*x+c))^4-7
/768*I*a^11/d/(a-I*a*tan(d*x+c))^3-1/128*I*a^10/d/(a-I*a*tan(d*x+c))^2-9/10
24*I*a^9/d/(a-I*a*tan(d*x+c))+1/1024*I*a^9/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{64d(a-ia\tan(c+dx))^5}-\frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4}-\frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3}-\frac{ia^{10}}{128d(a-ia\tan(c+dx))^2}-\frac{9ia^9}{1024d(a-ia\tan(c+dx))}+\frac{ia^9}{1024d(a+ia\tan(c+dx))}+\frac{5a^8x}{512}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (5*a^8*x)/512 - ((I/36)*a^17)/(d*(a - I*a*Tan[c + d*x])^9) - ((I/32)*a^16)/
(d*(a - I*a*Tan[c + d*x])^8) - (((3*I)/112)*a^15)/(d*(a - I*a*Tan[c + d*x])
^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/64)*a^13)/(d*(a - I*
a*Tan[c + d*x])^5) - (((3*I)/256)*a^12)/(d*(a - I*a*Tan[c + d*x])^4) - (((7
*I)/768)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/128)*a^10)/(d*(a - I*a*Ta
n[c + d*x])^2) - (((9*I)/1024)*a^9)/(d*(a - I*a*Tan[c + d*x])) + ((I/1024)*
a^9)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^{19})\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^{10}(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^{19})\,\mathrm{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^{10}} + \frac{1}{4a^3(a-x)^9} + \frac{3}{16a^4(a-x)^8} + \frac{1}{8a^5(a-x)^7}\right.\right.}{}$$
$$= -\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{}{112d(}$$
$$= \frac{5a^8x}{512} - \frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} -$$

Mathematica [A]
time = 3.98, size = 188, normalized size = 0.67

$$\frac{a^8(-15876i - 28224i\cos(2(c+dx)) - 20160i\cos(4(c+dx)) - 12960i\cos(6(c+dx)) - 315i\cos(8(c+dx)) + 5040dx\cos(8(c+dx)) + 224i\cos(10(c+dx)) - 7056\sin(2(c+dx)) - 10080\sin(4(c+dx)) - 9720\sin(6(c+dx)) + 315\sin(8(c+dx)) - 5040idx\sin(8(c+dx)) + 280\sin(10(c+dx)))(\cos(8(c+2dx)) + i\sin(8(c+2dx)))}{516096d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(-15876*I - (28224*I)*Cos[2*(c + d*x)] - (20160*I)*Cos[4*(c + d*x)] -
(12960*I)*Cos[6*(c + d*x)] - (315*I)*Cos[8*(c + d*x)] + 5040*d*x*Cos[8*(c +
 d*x)] + (224*I)*Cos[10*(c + d*x)] - 7056*Sin[2*(c + d*x)] - 10080*Sin[4*(c
 + d*x)] - 9720*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] - (5040*I)*d*x*Sin[
8*(c + d*x)] + 280*Sin[10*(c + d*x)])*(Cos[8*(c + 2*d*x)] + I*Sin[8*(c + 2*
d*x)]))/(516096*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(237) = 474.
time = 0.32, size = 789, normalized size = 2.83

method	result
risch	$\frac{5a^8x}{512} - \frac{ia^8e^{18i(dx+c)}}{18432d} - \frac{5ia^8e^{16i(dx+c)}}{8192d} - \frac{45ia^8e^{14i(dx+c)}}{14336d} - \frac{5ia^8e^{12i(dx+c)}}{512d} - \frac{21ia^8e^{10i(dx+c)}}{1024d} - \frac{63ia^8e^{8i(dx}}{2048d}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/18*sin(d*x+c)^7*cos(d*x+c)^11-7/288*sin(d*x+c)^5*cos(d*x+c)^11
-5/576*sin(d*x+c)^3*cos(d*x+c)^11-5/2304*sin(d*x+c)*cos(d*x+c)^11+1/4608*(c
os(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128
*cos(d*x+c))*sin(d*x+c)+35/65536*d*x+35/65536*c)-8*I*a^8*(-1/18*sin(d*x+c)^
6*cos(d*x+c)^12-1/48*sin(d*x+c)^4*cos(d*x+c)^12-1/168*sin(d*x+c)^2*cos(d*x+
c)^12-1/1008*cos(d*x+c)^12)-28*a^8*(-1/18*sin(d*x+c)^5*cos(d*x+c)^13-5/288*
sin(d*x+c)^3*cos(d*x+c)^13-5/1344*cos(d*x+c)^13*sin(d*x+c)+5/16128*(cos(d*x
+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*c
os(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+55/65536*d*x+55/65536*c)+56*I*a^
8*(-1/18*sin(d*x+c)^4*cos(d*x+c)^14-1/72*sin(d*x+c)^2*cos(d*x+c)^14-1/504*c
os(d*x+c)^14)+70*a^8*(-1/18*sin(d*x+c)^3*cos(d*x+c)^15-1/96*cos(d*x+c)^15*s
in(d*x+c)+1/1344*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+42
9/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*co
s(d*x+c))*sin(d*x+c)+143/65536*d*x+143/65536*c)-56*I*a^8*(-1/18*sin(d*x+c)^
2*cos(d*x+c)^16-1/144*cos(d*x+c)^16)-28*a^8*(-1/18*cos(d*x+c)^17*sin(d*x+c)
+1/288*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d
*x+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+6
435/2048*cos(d*x+c))*sin(d*x+c)+715/65536*d*x+715/65536*c)-4/9*I*a^8*cos(d*
x+c)^18+a^8*(1/18*(cos(d*x+c)^17+17/16*cos(d*x+c)^15+255/224*cos(d*x+c)^13+
1105/896*cos(d*x+c)^11+2431/1792*cos(d*x+c)^9+21879/14336*cos(d*x+c)^7+7293
/4096*cos(d*x+c)^5+36465/16384*cos(d*x+c)^3+109395/32768*cos(d*x+c))*sin(d*
x+c)+12155/65536*d*x+12155/65536*c))
```

Maxima [A]
time = 0.53, size = 269, normalized size = 0.96

$$\frac{315\,(dx+c)a^8+\frac{315\,a^8\tan(dx+c)^{17}+2730\,a^8\tan(dx+c)^{15}+10458\,a^8\tan(dx+c)^{13}+23202\,a^8\tan(dx+c)^{11}+32768\,a^8\tan(dx+c)^{9}+27486\,a^8\tan(dx+c)^{7}+21504i\,a^8\tan(dx+c)^{6}+86310\,a^8\tan(dx+c)^{5}-119808i\,a^8\tan(dx+c)^{4}-121002\,a^8\tan(dx+c)^{3}+82944i\,a^8\tan(dx+c)^{2}+31941\,a^8\tan(dx+c)-5120i\,a^8}{\tan(dx+c)^{18}+9\tan(dx+c)^{16}+36\tan(dx+c)^{14}+84\tan(dx+c)^{12}+126\tan(dx+c)^{10}+126\tan(dx+c)^{8}+84\tan(dx+c)^{6}+36\tan(dx+c)^{4}+9\tan(dx+c)^{2}+1}}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/32256*(315*(d*x + c)*a^8 + (315*a^8*tan(d*x + c)^17 + 2730*a^8*tan(d*x +
c)^15 + 10458*a^8*tan(d*x + c)^13 + 23202*a^8*tan(d*x + c)^11 + 32768*a^8*t
an(d*x + c)^9 + 27486*a^8*tan(d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 863
10*a^8*tan(d*x + c)^5 - 119808*I*a^8*tan(d*x + c)^4 - 121002*a^8*tan(d*x +
c)^3 + 82944*I*a^8*tan(d*x + c)^2 + 31941*a^8*tan(d*x + c) - 5120*I*a^8)/(t
an(d*x + c)^18 + 9*tan(d*x + c)^16 + 36*tan(d*x + c)^14 + 84*tan(d*x + c)^1
2 + 126*tan(d*x + c)^10 + 126*tan(d*x + c)^8 + 84*tan(d*x + c)^6 + 36*tan(d
*x + c)^4 + 9*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.47, size = 162, normalized size = 0.58

$$\frac{\left(5040\, a^{8} d x e^{(2 i dx+2 i c)}-28 i\, a^{8} e^{(20 i dx+20 i c)}-315 i\, a^{8} e^{(18 i dx+18 i c)}-1620 i\, a^{8} e^{(16 i dx+16 i c)}-5040 i\, a^{8} e^{(14 i dx+14 i c)}-10584 i\, a^{8} e^{(12 i dx+12 i c)}-15876 i\, a^{8} e^{(10 i dx+10 i c)}-17640 i\, a^{8} e^{(8 i dx+8 i c)}-15120 i\, a^{8} e^{(6 i dx+6 i c)}-11340 i\, a^{8} e^{(4 i dx+4 i c)}+252 i\, a^{8}\right) e^{(-2 i dx-2 i c)}}{516096\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*a^8*d*x*e^(2*I*d*x + 2*I*c) - 28*I*a^8*e^(20*I*d*x + 20*I*c)
 - 315*I*a^8*e^(18*I*d*x + 18*I*c) - 1620*I*a^8*e^(16*I*d*x + 16*I*c) - 504
0*I*a^8*e^(14*I*d*x + 14*I*c) - 10584*I*a^8*e^(12*I*d*x + 12*I*c) - 15876*I
*a^8*e^(10*I*d*x + 10*I*c) - 17640*I*a^8*e^(8*I*d*x + 8*I*c) - 15120*I*a^8*
e^(6*I*d*x + 6*I*c) - 11340*I*a^8*e^(4*I*d*x + 4*I*c) + 252*I*a^8)*e^(-2*I*
d*x - 2*I*c)/d
```

Sympy [A]
time = 1.05, size = 413, normalized size = 1.48

$$\frac{5a^8x}{512}+\begin{cases}\frac{\left(-277298568799925181577403826176 i a^{8} d^{9} e^{20 i c} e^{18 i d x}-311960889899915829274579304480 i a^{8} d^{9} e^{18 i c} e^{16 i d x}-1604370290913852836269264943040 i a^{8} d^{9} e^{16 i c} e^{14 i d x}-4991374238398653268393268871168 0 i a^{8} d^{9} e^{14 i c} e^{12 i d x}-10481885900637171863625864629 4528 i a^{8} d^{9} e^{12 i c} e^{10 i d x}-15722828850955757795438796944179 2 i a^{8} d^{9} e^{10 i c} e^{8 i d x}-17469809834395286439376441049088 0 i a^{8} d^{9} e^{8 i c} e^{6 i d x}-14974122715195959805179806613504 0 i a^{8} d^{9} e^{6 i c} e^{4 i d x}-11230592036396969853884854960128 0 i a^{8} d^{9} e^{4 i c} e^{2 i d x}+24956871191993266341966344355 84 i a^{8} d^{9} e^{-2 i d x}\right) e^{-2 i c}}{5111167220120220946834707324076032 d^{10}} & \text{for}\: d^{10} e^{2 i c} \neq 0 \\ x \left(-\frac{5a^8}{512}+\frac{\left(a^{8} e^{20 i c}+10 a^{8} e^{18 i c}+45 a^{8} e^{16 i c}+120 a^{8} e^{14 i c}+210 a^{8} e^{12 i c}+252 a^{8} e^{10 i c}+210 a^{8} e^{8 i c}+120 a^{8} e^{6 i c}+45 a^{8} e^{4 i c}+10 a^{8} e^{2 i c}+a^{8}\right) e^{-2 i c}}{1024}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**18*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 5*a**8*x/512 + Piecewise(((-277298568799925181577403826176*I*a**8*d**9*exp(
20*I*c)*exp(18*I*d*x) - 3119608898999158292745793044480*I*a**8*d**9*exp(18*
I*c)*exp(16*I*d*x) - 16043702909138528362692649943040*I*a**8*d**9*exp(16*I*
c)*exp(14*I*d*x) - 49913742383986532683932688711680*I*a**8*d**9*exp(14*I*c)
*exp(12*I*d*x) - 104818859006371718636258646294528*I*a**8*d**9*exp(12*I*c)*
exp(10*I*d*x) - 157228288509557577954387969441792*I*a**8*d**9*exp(10*I*c)*e
xp(8*I*d*x) - 174698098343952864393764410490880*I*a**8*d**9*exp(8*I*c)*exp(
6*I*d*x) - 149741227151959598051798066135040*I*a**8*d**9*exp(6*I*c)*exp(4*I
*d*x) - 112305920363969698538848549601280*I*a**8*d**9*exp(4*I*c)*exp(2*I*d*
x) + 24956871191993266341966344355584*I*a**8*d**9*exp(-2*I*d*x))*exp(-2*I*c)
/(5111167220120220946834707324076032*d**10), Ne(d**10*exp(2*I*c), 0)), (x*(
-5*a**8/512 + (a**8*exp(20*I*c) + 10*a**8*exp(18*I*c) + 45*a**8*exp(16*I*c)
 + 120*a**8*exp(14*I*c) + 210*a**8*exp(12*I*c) + 252*a**8*exp(10*I*c) + 210
*a**8*exp(8*I*c) + 120*a**8*exp(6*I*c) + 45*a**8*exp(4*I*c) + 10*a**8*exp(2
*I*c) + a**8)*exp(-2*I*c)/1024), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1514 vs. 2(217) = 434.
time = 1.52, size = 1514, normalized size = 5.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")

[Out] 1/16515072*(161280*a^8*d*x*e^(30*I*d*x + 16*I*c) + 2257920*a^8*d*x*e^(28*I*
d*x + 14*I*c) + 14676480*a^8*d*x*e^(26*I*d*x + 12*I*c) + 58705920*a^8*d*x*e
^(24*I*d*x + 10*I*c) + 161441280*a^8*d*x*e^(22*I*d*x + 8*I*c) + 322882560*a
^8*d*x*e^(20*I*d*x + 6*I*c) + 484323840*a^8*d*x*e^(18*I*d*x + 4*I*c) + 5535
12960*a^8*d*x*e^(16*I*d*x + 2*I*c) + 322882560*a^8*d*x*e^(12*I*d*x - 2*I*c)
 + 161441280*a^8*d*x*e^(10*I*d*x - 4*I*c) + 58705920*a^8*d*x*e^(8*I*d*x - 6
*I*c) + 14676480*a^8*d*x*e^(6*I*d*x - 8*I*c) + 2257920*a^8*d*x*e^(4*I*d*x -
 10*I*c) + 161280*a^8*d*x*e^(2*I*d*x - 12*I*c) + 484323840*a^8*d*x*e^(14*I*
d*x) - 75789*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 106
1046*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6896799*I*a
^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 27587196*I*a^8*e^(2
4*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75864789*I*a^8*e^(22*I*d*x
 + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(20*I*d*x + 6*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) - 227594367*I*a^8*e^(18*I*d*x + 4*I*c)*log(
e^(2*I*d*x + 2*I*c) + 1) - 260107848*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) - 75864789*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) +
1) - 27587196*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6896
799*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1061046*I*a^8*
e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75789*I*a^8*e^(2*I*d*x
- 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 227594367*I*a^8*e^(14*I*d*x)*log(e
^(2*I*d*x + 2*I*c) + 1) + 75789*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) + 1061046*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-
2*I*c)) + 6896799*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c))
 + 27587196*I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 758
64789*I*a^8*e^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 151729578*
I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 227594367*I*a^8*
e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 260107848*I*a^8*e^(16*
I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 151729578*I*a^8*e^(12*I*d*x
- 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 75864789*I*a^8*e^(10*I*d*x - 4*I*c
)*log(e^(2*I*d*x) + e^(-2*I*c)) + 27587196*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^
(2*I*d*x) + e^(-2*I*c)) + 6896799*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) + 1061046*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2
*I*c)) + 75789*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2
27594367*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 896*I*a^8*e^(48
*I*d*x + 34*I*c) - 22624*I*a^8*e^(46*I*d*x + 32*I*c) - 274496*I*a^8*e^(44*I
*d*x + 30*I*c) - 2130464*I*a^8*e^(42*I*d*x + 28*I*c) - 11880064*I*a^8*e^(40
*I*d*x + 26*I*c) - 50679776*I*a^8*e^(38*I*d*x + 24*I*c) - 171966144*I*a^8*e
^(36*I*d*x + 22*I*c) - 476470176*I*a^8*e^(34*I*d*x + 20*I*c) - 1098297984*I
*a^8*e^(32*I*d*x + 18*I*c) - 2135476640*I*a^8*e^(30*I*d*x + 16*I*c) - 35386
01920*I*a^8*e^(28*I*d*x + 14*I*c) - 5032909280*I*a^8*e^(26*I*d*x + 12*I*c)
- 6165461120*I*a^8*e^(24*I*d*x + 10*I*c) - 6498731680*I*a^8*e^(22*I*d*x + 8
*I*c) - 5857001024*I*a^8*e^(20*I*d*x + 6*I*c) - 4459555296*I*a^8*e^(18*I*d*
```

```
x + 4*I*c) - 2817258624*I*a^8*e^(16*I*d*x + 2*I*c) - 573963264*I*a^8*e^(12*
I*d*x - 2*I*c) - 168384384*I*a^8*e^(10*I*d*x - 4*I*c) - 32288256*I*a^8*e^(8
*I*d*x - 6*I*c) - 2628864*I*a^8*e^(6*I*d*x - 8*I*c) + 370944*I*a^8*e^(4*I*d
*x - 10*I*c) + 112896*I*a^8*e^(2*I*d*x - 12*I*c) - 1439738496*I*a^8*e^(14*I
*d*x) + 8064*I*a^8*e^(-14*I*c))/(d*e^(30*I*d*x + 16*I*c) + 14*d*e^(28*I*d*x
 + 14*I*c) + 91*d*e^(26*I*d*x + 12*I*c) + 364*d*e^(24*I*d*x + 10*I*c) + 100
1*d*e^(22*I*d*x + 8*I*c) + 2002*d*e^(20*I*d*x + 6*I*c) + 3003*d*e^(18*I*d*x
 + 4*I*c) + 3432*d*e^(16*I*d*x + 2*I*c) + 2002*d*e^(12*I*d*x - 2*I*c) + 100
1*d*e^(10*I*d*x - 4*I*c) + 364*d*e^(8*I*d*x - 6*I*c) + 91*d*e^(6*I*d*x - 8*
I*c) + 14*d*e^(4*I*d*x - 10*I*c) + d*e^(2*I*d*x - 12*I*c) + 3003*d*e^(14*I*
d*x))
```

Mupad [B]
time = 5.23, size = 231, normalized size = 0.83

$$\frac{5\,a^8\,x}{512}+\frac{\frac{5\,a^8\,\tan(c+d\,x)^9}{512}+\frac{a^8\,\tan(c+d\,x)^8\,5{}\mathrm{i}}{64}-\frac{205\,a^8\,\tan(c+d\,x)^7}{768}-\frac{a^8\,\tan(c+d\,x)^6\,95{}\mathrm{i}}{192}+\frac{a^8\,\tan(c+d\,x)^5}{2}+\frac{a^8\,\tan(c+d\,x)^4\,11{}\mathrm{i}}{64}+\frac{393\,a^8\,\tan(c+d\,x)^3}{1792}+\frac{a^8\,\tan(c+d\,x)^2\,163{}\mathrm{i}}{448}-\frac{9019\,a^8\,\tan(c+d\,x)}{32256}-\frac{a^8\,10{}\mathrm{i}}{63}}{d\,\left({\tan\left(c+d\,x\right)}^{10}+{\tan\left(c+d\,x\right)}^9\,8{}\mathrm{i}-27\,{\tan\left(c+d\,x\right)}^8-{\tan\left(c+d\,x\right)}^7\,48{}\mathrm{i}+42\,{\tan\left(c+d\,x\right)}^6+42\,{\tan\left(c+d\,x\right)}^4+{\tan\left(c+d\,x\right)}^3\,48{}\mathrm{i}-27\,{\tan\left(c+d\,x\right)}^2-\tan\left(c+d\,x\right)\,8{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^18*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (5*a^8*x)/512 + ((a^8*tan(c + d*x)^2*163i)/448 - (a^8*10i)/63 - (9019*a^8*t
an(c + d*x))/32256 + (393*a^8*tan(c + d*x)^3)/1792 + (a^8*tan(c + d*x)^4*11
i)/64 + (a^8*tan(c + d*x)^5)/2 - (a^8*tan(c + d*x)^6*95i)/192 - (205*a^8*ta
n(c + d*x)^7)/768 + (a^8*tan(c + d*x)^8*5i)/64 + (5*a^8*tan(c + d*x)^9)/512
)/(d*(tan(c + d*x)^3*48i - 27*tan(c + d*x)^2 - tan(c + d*x)*8i + 42*tan(c +
 d*x)^4 + 42*tan(c + d*x)^6 - tan(c + d*x)^7*48i - 27*tan(c + d*x)^8 + tan(
c + d*x)^9*8i + tan(c + d*x)^10 + 1))
```

3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=235

$$-\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d}-\frac{3003ia^8\sec(c+dx)}{16d}-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{2ia\cos(c+dx)}{}$$

```
[Out] -3003/16*a^8*arctanh(sin(d*x+c))/d-3003/16*I*a^8*sec(d*x+c)/d-13/6*I*a^3*se
c(d*x+c)*(a+I*a*tan(d*x+c))^5/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^7/d-429
/40*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-143/30*I*sec(d*x+c)*(a^2+I*
a^2*tan(d*x+c))^4/d-1001/40*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))^2/d-1001/16
*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A]

time = 0.17, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3577, 3579, 3567, 3855}

$$-\frac{3003ia^8\sec(c+dx)}{16d}-\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d}-\frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d}-\frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d}-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{429ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{40d}-\frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-3003*a^8*ArcTanh[Sin[c + d*x]])/(16*d) - (((3003*I)/16)*a^8*Sec[c + d*x])
/d - (((13*I)/6)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - (((429*I)/40)*a^2*Sec[c + d*x]*(a^
2 + I*a^2*Tan[c + d*x])^3)/d - (((143*I)/30)*Sec[c + d*x]*(a^2 + I*a^2*Tan[
c + d*x])^4)/d - (((1001*I)/40)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c + d*x])^2)/
d - (((1001*I)/16)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
```

```
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}-\left(13a^2\right)\int \sec(c+dx)(a+\\ &= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{2ia\cos(c+dx)(a+ia}{d}\\ &= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{2ia\cos(c+dx)(a+ia}{d}\\ &= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d}-\frac{13ia^3\sec(c+dx)(a+}{6d}\\ &= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d}-\frac{13ia^3\sec(c+dx)(a+}{6d}\\ &= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d}-\frac{13ia^3\sec(c+dx)(a+}{6d}\\ &= -\frac{3003ia^8\sec(c+dx)}{16d}-\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d}-\\ &= -\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d}-\frac{3003ia^8\sec(c+dx)}{16d}-\frac{429ia^5\sec}{}\end{aligned}$$

Mathematica [A]
time = 1.70, size = 205, normalized size = 0.87

$$\frac{a^8\cos^2(c+dx)(\cos(8c)-i\sin(8c))\left(-658944i\cos(c+dx)+720720\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+5(-73216i\cos(3(c+dx))-19968i\cos(5(c+dx))-1536i\cos(7(c+dx))+12870\sin(c+dx)+22165\sin(3(c+dx))+10959\sin(5(c+dx))+1536\sin(7(c+dx)))\right)(-i+\tan(c+dx))^8}{3840d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Cos[c + d*x]^2*(Cos[8*c] - I*Sin[8*c])*((-658944*I)*Cos[c + d*x] + 720
720*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-73216*I)*Cos[3*(c + d*x)] - (19968*I)*
Cos[5*(c + d*x)] - (1536*I)*Cos[7*(c + d*x)] + 12870*Sin[c + d*x] + 22165*S
in[3*(c + d*x)] + 10959*Sin[5*(c + d*x)] + 1536*Sin[7*(c + d*x)]))*(-I + Ta
n[c + d*x])^8)/(3840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(208) = 416$.
time = 0.27, size = 522, normalized size = 2.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/6*sin(d*x+c)^9/cos(d*x+c)^6-1/8*sin(d*x+c)^9/cos(d*x+c)^4+5/16*
sin(d*x+c)^9/cos(d*x+c)^2+5/16*sin(d*x+c)^7+7/16*sin(d*x+c)^5+35/48*sin(d*x
+c)^3+35/16*sin(d*x+c)-35/16*ln(sec(d*x+c)+tan(d*x+c)))-8*I*a^8*(1/5*sin(d*
x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(
16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))-28*a^8*(1/
4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-
5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+56*I*a^8*(
1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3
*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*
x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))-56*I*a^8*(sin(d*x+c)^4
/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))-28*a^8*(-sin(d*x+c)+ln(sec(d*x+c)+
tan(d*x+c)))-8*I*a^8*cos(d*x+c)+a^8*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(195) = 390$.
time = 0.30, size = 396, normalized size = 1.69

$$-\frac{5\,a^8\left(\frac{2\left(87\,\sin(dx+c)^5-136\,\sin(dx+c)^3+57\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+105\,\log(\sin(dx+c)+1)-105\,\log(\sin(dx+c)-1)-96\,\sin(dx+c)\right)+840\,a^8\left(\frac{2\left(9\,\sin(dx+c)^3-7\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+15\,\log(\sin(dx+c)+1)-15\,\log(\sin(dx+c)-1)-16\,\sin(dx+c)\right)+8400\,a^8\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log(\sin(dx+c)+1)-3\,\log(\sin(dx+c)-1)-4\,\sin(dx+c)\right)+26880i\,a^8\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+8960i\,a^8\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos(dx+c)\right)+768i\,a^8\left(\frac{15\,\cos(dx+c)^4-5\,\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\,\cos(dx+c)\right)+6720\,a^8(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))+3840i\,a^8\cos(dx+c)-480\,a^8\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^8*(2*(87*sin(d*x + c)^5 - 136*sin(d*x + c)^3 + 57*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 105*log(sin(d
*x + c) + 1) - 105*log(sin(d*x + c) - 1) - 96*sin(d*x + c)) + 840*a^8*(2*(9
*sin(d*x + c)^3 - 7*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) +
 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1) - 16*sin(d*x + c)) + 8
400*a^8*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) - 3*
log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 26880*I*a^8*(1/cos(d*x + c) + cos
(d*x + c)) + 8960*I*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
+ c)) + 768*I*a^8*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^
```

```
5 + 5*cos(d*x + c)) + 6720*a^8*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1) - 2*sin(d*x + c)) + 3840*I*a^8*cos(d*x + c) - 480*a^8*sin(d*x + c))/d
```

Fricas [A]
time = 0.42, size = 378, normalized size = 1.61

$$\frac{-30720i\,a^8e^{(13i\,dx+13i\,c)} - 309270i\,a^8e^{(11i\,dx+11i\,c)} - 953810i\,a^8e^{(9i\,dx+9i\,c)} - 1446588i\,a^8e^{(7i\,dx+7i\,c)} - 1189188i\,a^8e^{(5i\,dx+5i\,c)} - 510510i\,a^8e^{(3i\,dx+3i\,c)} - 90090i\,a^8e^{(i\,dx+i\,c)} - 45045\left(a^8e^{(12i\,dx+12i\,c)} + 6\,a^8e^{(10i\,dx+10i\,c)} + 15\,a^8e^{(8i\,dx+8i\,c)} + 20\,a^8e^{(6i\,dx+6i\,c)} + 15\,a^8e^{(4i\,dx+4i\,c)} + 6\,a^8e^{(2i\,dx+2i\,c)} + a^8\right)\log\left(e^{(i\,dx+i\,c)} + i\right) + 45045\left(a^8e^{(12i\,dx+12i\,c)} + 6\,a^8e^{(10i\,dx+10i\,c)} + 15\,a^8e^{(8i\,dx+8i\,c)} + 20\,a^8e^{(6i\,dx+6i\,c)} + 15\,a^8e^{(4i\,dx+4i\,c)} + 6\,a^8e^{(2i\,dx+2i\,c)} + a^8\right)\log\left(e^{(i\,dx+i\,c)} - i\right)}{240\left(de^{(12i\,dx+12i\,c)} + 6\,de^{(10i\,dx+10i\,c)} + 15\,de^{(8i\,dx+8i\,c)} + 20\,de^{(6i\,dx+6i\,c)} + 15\,de^{(4i\,dx+4i\,c)} + 6\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-30720*I*a^8*e^(13*I*d*x + 13*I*c) - 309270*I*a^8*e^(11*I*d*x + 11*I
*c) - 953810*I*a^8*e^(9*I*d*x + 9*I*c) - 1446588*I*a^8*e^(7*I*d*x + 7*I*c)
- 1189188*I*a^8*e^(5*I*d*x + 5*I*c) - 510510*I*a^8*e^(3*I*d*x + 3*I*c) - 90
090*I*a^8*e^(I*d*x + I*c) - 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*
I*d*x + 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) +
 15*a^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x
 + I*c) + I) + 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*I*d*x + 10*I*
c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) + 15*a^8*e^(4*
I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))
/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8
*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A]
time = 0.51, size = 320, normalized size = 1.36

$$\frac{3003a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{16} - \frac{\log\left(e^{idx}+ie^{-ic}\right)}{16}\right)}{d} + \frac{-62475ia^8e^{11ic}e^{11idx} - 246505ia^8e^{9ic}e^{9idx} - 416094ia^8e^{7ic}e^{7idx} - 364194ia^8e^{5ic}e^{5idx} - 163095ia^8e^{3ic}e^{3idx} - 29685ia^8e^{ic}e^{idx}}{120de^{12ic}e^{12idx} + 720de^{10ic}e^{10idx} + 1800de^{8ic}e^{8idx} + 2400de^{6ic}e^{6idx} + 1800de^{4ic}e^{4idx} + 720de^{2ic}e^{2idx} + 120d} + \begin{cases} -\frac{128ia^8e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 128a^8xe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 3003*a**8*(log(exp(I*d*x) - I*exp(-I*c))/16 - log(exp(I*d*x) + I*exp(-I*c))
/16)/d + (-62475*I*a**8*exp(11*I*c)*exp(11*I*d*x) - 246505*I*a**8*exp(9*I*c
)*exp(9*I*d*x) - 416094*I*a**8*exp(7*I*c)*exp(7*I*d*x) - 364194*I*a**8*exp(
5*I*c)*exp(5*I*d*x) - 163095*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 29685*I*a**8*
exp(I*c)*exp(I*d*x))/(120*d*exp(12*I*c)*exp(12*I*d*x) + 720*d*exp(10*I*c)*e
xp(10*I*d*x) + 1800*d*exp(8*I*c)*exp(8*I*d*x) + 2400*d*exp(6*I*c)*exp(6*I*d
*x) + 1800*d*exp(4*I*c)*exp(4*I*d*x) + 720*d*exp(2*I*c)*exp(2*I*d*x) + 120*
d) + Piecewise((-128*I*a**8*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (128*a**8*x*e
xp(I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(195) = 390.
time = 1.53, size = 924, normalized size = 3.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/61440*(11512215*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69
073290*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8
*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 230244300*a^8*e^(6*I*d*x
+ 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8*e^(4*I*d*x + 4*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) + 1) - 19305*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1
15830*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 386100*a^8*e^(6*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) - 115830*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) -
 11512215*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69073290*
a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(8*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 230244300*a^8*e^(6*I*d*x + 6*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(4*I*d*x + 4*I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I
*c) + 1) + 19305*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 11
5830*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 386100*a^8*e^(6*I*d*x + 6*I*
c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(4*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) - 1) + 115830*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 7864320*I*a^8*e^(13*I*d*x + 13*I*c) - 79173120*I*a^8*e^(11*I*d*x + 11*
I*c) - 244175360*I*a^8*e^(9*I*d*x + 9*I*c) - 370326528*I*a^8*e^(7*I*d*x + 7
*I*c) - 304432128*I*a^8*e^(5*I*d*x + 5*I*c) - 130690560*I*a^8*e^(3*I*d*x +
3*I*c) - 23063040*I*a^8*e^(I*d*x + I*c) + 11512215*a^8*log(I*e^(I*d*x + I*c
) + 1) - 19305*a^8*log(I*e^(I*d*x + I*c) - 1) - 11512215*a^8*log(-I*e^(I*d*
x + I*c) + 1) + 19305*a^8*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(12*I*d*x + 12*
I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d
*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B]
time = 8.30, size = 399, normalized size = 1.70

$$\frac{\frac{3019\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{8}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,2891\mathrm{i}}{8}-\frac{52795\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{24}-\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\,45115\mathrm{i}}{24}+\frac{22415\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{4}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\,43757\mathrm{i}}{12}-\frac{97811\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{12}-\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\,12977\mathrm{i}}{4}+\frac{167237\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{24}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,160729\mathrm{i}}{120}-\frac{127113\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{40}-\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,25499\mathrm{i}}{120}+\frac{8848\,a^8}{15}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,1\mathrm{i}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,6\mathrm{i}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}\,15\mathrm{i}-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}\,20\mathrm{i}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}\,15\mathrm{i}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}\,6\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}-\frac{3003\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((a^8*tan(c/2 + (d*x)/2)^3*160729i)/120 - (127113*a^8*tan(c/2 + (d*x)/2)^2)
/40 + (167237*a^8*tan(c/2 + (d*x)/2)^4)/24 - (a^8*tan(c/2 + (d*x)/2)^5*1297
7i)/4 - (97811*a^8*tan(c/2 + (d*x)/2)^6)/12 + (a^8*tan(c/2 + (d*x)/2)^7*437
57i)/12 + (22415*a^8*tan(c/2 + (d*x)/2)^8)/4 - (a^8*tan(c/2 + (d*x)/2)^9*45
115i)/24 - (52795*a^8*tan(c/2 + (d*x)/2)^10)/24 + (a^8*tan(c/2 + (d*x)/2)^1
```

```
1*2891i)/8 + (3019*a^8*tan(c/2 + (d*x)/2)^12)/8 + (8848*a^8)/15 - (a^8*tan(
c/2 + (d*x)/2)*25499i)/120)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*6
i - 6*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*15i + 15*tan(c/2 + (d*x)/
2)^5 - tan(c/2 + (d*x)/2)^6*20i - 20*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)
/2)^8*15i + 15*tan(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*6i - 6*tan(c/2
+ (d*x)/2)^11 + tan(c/2 + (d*x)/2)^12*1i + tan(c/2 + (d*x)/2)^13 + 1i)) - (
3003*a^8*atanh(tan(c/2 + (d*x)/2)))/(8*d)
```

3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=205

$$\frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d}+\frac{1155ia^8\sec(c+dx)}{8d}+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}-\frac{2ia\cos^3(c+dx)(}{}$$

```
[Out] 1155/8*a^8*arctanh(sin(d*x+c))/d+1155/8*I*a^8*sec(d*x+c)/d+22/3*I*a^3*cos(d
*x+c)*(a+I*a*tan(d*x+c))^5/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^7/d+33
/4*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d+77/4*I*sec(d*x+c)*(a^4+I*a^4
*tan(d*x+c))^2/d+385/8*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3577, 3579, 3567, 3855}

$$\frac{1155ia^8\sec(c+dx)}{8d}+\frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d}+\frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}+\frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d}+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}+\frac{33ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{4d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (1155*a^8*ArcTanh[Sin[c + d*x]])/(8*d) + (((1155*I)/8)*a^8*Sec[c + d*x])/d
+ (((22*I)/3)*a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/3)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d + (((33*I)/4)*a^2*Sec[c + d*x]*(a
^2 + I*a^2*Tan[c + d*x])^3)/d + (((77*I)/4)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c
 + d*x])^2)/d + (((385*I)/8)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} - \frac{1}{3}(11a^2)\int \cos(c+dx$$

$$= \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia}{3d}$$

$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+i}{3d}$$

$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+i}{3d}$$

$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+i}{3d}$$

$$= \frac{1155ia^8\sec(c+dx)}{8d} + \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} +$$

$$= \frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d} + \frac{1155ia^8\sec(c+dx)}{8d} + \frac{33ia^5\sec(c}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1540 vs. 2(205) = 410.
time = 6.63, size = 1540, normalized size = 7.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-1155*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]
*(a + I*a*Tan[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (1155*Cos[8*c]
```

```
*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c
 + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((
(-32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*
x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*x]^8*((160*I)*Cos[7*c] + 160*Sin[
7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((1155*I)/
8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a
+ I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((1155*I)/8)*Cos[c
+ d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan
[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*(((236
*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x]
+ I*Sin[d*x])^8) + (Cos[c + d*x]^8*(-160*Cos[7*c] + (160*I)*Sin[7*c])*Sin[d
*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]
^8*((32*Cos[5*c])/3 - ((32*I)/3)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c + d*x]
)^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/16 - (I/16)*
Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])^4) - (I*Cos[c + d*x]^8*((4*Cos[8*c])/3 - ((
4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin
[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^
3) + (Cos[c + d*x]^8*((-375 - 32*I)*Cos[c/2] + (375 - 32*I)*Sin[c/2])*(Cos[
8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2
])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) +
 (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-1/16*Cos[8*
c] + (I/16)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^
8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (I*Cos[c + d*x]^8*((4*Cos[
8*c])/3 - ((4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Co
s[c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^3) + (Cos[c + d*x]^8*((375 - 32*I)*Cos[c/2] + (375 + 32*I)*Sin[
c/2])*(Cos[8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2
] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^2) - (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin
[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*
Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(180) = 360$.
time = 0.24, size = 409, normalized size = 2.00

method	result
risch	$-\frac{32 i a^{8} e^{3 i (d x+c)}}{3 d}+\frac{160 i a^{8} e^{i (d x+c)}}{d}+\frac{i a^{8} \left(2295 e^{7 i (d x+c)}+5855 e^{5 i (d x+c)}+5153 e^{3 i (d x+c)}+1545 e^{i (d x+c)}\right)}{12 d \left(e^{2 i (d x+c)}+1\right)^{4}}+\frac{1155 a^{8}}{}$

derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{4\cos(dx+c)^4}-\frac{5\left(\sin^9(dx+c)\right)}{8\cos(dx+c)^2}-\frac{5\left(\sin^7(dx+c)\right)}{8}-\frac{7\left(\sin^5(dx+c)\right)}{8}-\frac{35\left(\sin^3(dx+c)\right)}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{4\cos(dx+c)^4}-\frac{5\left(\sin^9(dx+c)\right)}{8\cos(dx+c)^2}-\frac{5\left(\sin^7(dx+c)\right)}{8}-\frac{7\left(\sin^5(dx+c)\right)}{8}-\frac{35\left(\sin^3(dx+c)\right)}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/4*sin(d*x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*s
in(d*x+c)^7-7/8*sin(d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec
(d*x+c)+tan(d*x+c)))-8*I*a^8*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^
8/cos(d*x+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(
d*x+c))-28*a^8*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+
c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))+56/3*I*a^8*(2+sin(d*x+c)
^2)*cos(d*x+c)+70*a^8*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c
)))-8/3*I*a^8*cos(d*x+c)^3-28/3*a^8*sin(d*x+c)^3+56*I*a^8*(sin(d*x+c)^6/cos
(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+1/3*a^8*(cos(d*x+c)
^2+2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(169) = 338$.
time = 0.30, size = 352, normalized size = 1.72

$$-\frac{128i\,a^8\cos(dx+c)^3+448\,a^8\sin(dx+c)^3+896i\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^8+128i\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a^8+896i\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^8+\left(16\sin(dx+c)^3-\frac{6\left(13\sin(dx+c)^3-11\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)+144\sin(dx+c)\right)a^8+112\left(4\sin(dx+c)^3-\frac{6\sin(dx+c)}{\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+24\sin(dx+c)\right)a^8+560\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(dx+c)\right)a^8+16\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^8}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(128*I*a^8*cos(d*x + c)^3 + 448*a^8*sin(d*x + c)^3 + 896*I*(cos(d*x +
 c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 + 128*I*(cos(d*x + c)^3 - (9*c
os(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + 896*I*(cos(d*x +
c)^3 - 3*cos(d*x + c))*a^8 + (16*sin(d*x + c)^3 - 6*(13*sin(d*x + c)^3 - 11
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 105*log(sin(d*x +
c) + 1) + 105*log(sin(d*x + c) - 1) + 144*sin(d*x + c))*a^8 + 112*(4*sin(d*
x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) +
 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^8 + 560*(2*sin(d*x + c)^3 -
3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^8 + 1
6*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^8)/d
```

Fricas [A]
time = 0.38, size = 284, normalized size = 1.39

$$\frac{-256i\,a^8e^{(11i\,dx+11i\,c)}+2816i\,a^8e^{(9i\,dx+9i\,c)}+18414i\,a^8e^{(7i\,dx+7i\,c)}+33726i\,a^8e^{(5i\,dx+5i\,c)}+25410i\,a^8e^{(3i\,dx+3i\,c)}+6930i\,a^8e^{(i\,dx+i\,c)}+3465\left(a^8e^{(8i\,dx+8i\,c)}+4\,a^8e^{(6i\,dx+6i\,c)}+6\,a^8e^{(4i\,dx+4i\,c)}+4\,a^8e^{(2i\,dx+2i\,c)}+a^8\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3465\left(a^8e^{(8i\,dx+8i\,c)}+4\,a^8e^{(6i\,dx+6i\,c)}+6\,a^8e^{(4i\,dx+4i\,c)}+4\,a^8e^{(2i\,dx+2i\,c)}+a^8\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{24\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(-256*I*a^8*e^(11*I*d*x + 11*I*c) + 2816*I*a^8*e^(9*I*d*x + 9*I*c) + 1
8414*I*a^8*e^(7*I*d*x + 7*I*c) + 33726*I*a^8*e^(5*I*d*x + 5*I*c) + 25410*I*
a^8*e^(3*I*d*x + 3*I*c) + 6930*I*a^8*e^(I*d*x + I*c) + 3465*(a^8*e^(8*I*d*x
 + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e
^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) + I) - 3465*(a^8*e^(8*I*d*x +
 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e^(
2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) +
4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [**A**]
time = 0.51, size = 277, normalized size = 1.35

$$\frac{1155a^8\left(-\frac{\log\left(e^{idx}-ie^{-ic}\right)}{8}+\frac{\log\left(e^{idx}+ie^{-ic}\right)}{8}\right)}{d}+\frac{2295ia^8e^{7ic}e^{7idx}+5855ia^8e^{5ic}e^{5idx}+5153ia^8e^{3ic}e^{3idx}+1545ia^8e^{ic}e^{idx}}{12de^{8ic}e^{8idx}+48de^{6ic}e^{6idx}+72de^{4ic}e^{4idx}+48de^{2ic}e^{2idx}+12d}+\begin{cases}\frac{-32ia^8de^{3ic}e^{3idx}+480ia^8de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x\left(32a^8e^{3ic}-160a^8e^{ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 1155*a**8*(-log(exp(I*d*x) - I*exp(-I*c))/8 + log(exp(I*d*x) + I*exp(-I*c))
/8)/d + (2295*I*a**8*exp(7*I*c)*exp(7*I*d*x) + 5855*I*a**8*exp(5*I*c)*exp(5
*I*d*x) + 5153*I*a**8*exp(3*I*c)*exp(3*I*d*x) + 1545*I*a**8*exp(I*c)*exp(I*
d*x))/(12*d*exp(8*I*c)*exp(8*I*d*x) + 48*d*exp(6*I*c)*exp(6*I*d*x) + 72*d*e
xp(4*I*c)*exp(4*I*d*x) + 48*d*exp(2*I*c)*exp(2*I*d*x) + 12*d) + Piecewise((
(-32*I*a**8*d*exp(3*I*c)*exp(3*I*d*x) + 480*I*a**8*d*exp(I*c)*exp(I*d*x))/(
3*d**2), Ne(d**2, 0)), (x*(32*a**8*exp(3*I*c) - 160*a**8*exp(I*c)), True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2835 vs. 2(169) = 338.
time = 2.18, size = 2835, normalized size = 13.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/98304*(763587*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1069
0218*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^
(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(22*I*d*x
+ 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8*e^(20*I*d*x + 6*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1528701174*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 2293051761*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 2293051761*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 152
```

8701174*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8
*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(6*I*d*x
- 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^(4*I*d*x - 10*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 10690218*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x
+ I*c) + 1) + 2620630584*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7635
87*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14956128*a^8*e^(28*I*d*x +
14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 209385792*a^8*e^(26*I*d*x + 12*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 1361007648*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 5444030592*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 14971084128*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2
9942168256*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4491325238
4*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 44913252384*a^8*e^(
12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29942168256*a^8*e^(10*I*d*x
- 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 14971084128*a^8*e^(8*I*d*x - 6*I*c)*l
og(I*e^(I*d*x + I*c) - 1) + 5444030592*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 1361007648*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 209385792*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5132
9431296*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 14956128*a^8*e^(-14*I
*c)*log(I*e^(I*d*x + I*c) - 1) - 763587*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 10690218*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I
*c) + 1) - 69486417*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 277945668*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 764350587
*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1528701174*a^8*e^(1
8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2293051761*a^8*e^(16*I*d*x +
 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2293051761*a^8*e^(12*I*d*x - 2*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 1528701174*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 764350587*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 277945668*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69
486417*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10690218*a^8*
e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2620630584*a^8*e^(14*I*d
*x)*log(-I*e^(I*d*x + I*c) + 1) - 763587*a^8*e^(-14*I*c)*log(-I*e^(I*d*x +
I*c) + 1) - 14956128*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 209385792*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1361007
648*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5444030592*a^8*
e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14971084128*a^8*e^(20*I*
d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29942168256*a^8*e^(18*I*d*x + 4*
I*c)*log(-I*e^(I*d*x + I*c) - 1) - 44913252384*a^8*e^(16*I*d*x + 2*I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 44913252384*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I
*d*x + I*c) - 1) - 29942168256*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I
*c) - 1) - 14971084128*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 5444030592*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 13610076
48*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 209385792*a^8*e^(
2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 51329431296*a^8*e^(14*I*d*x
)*log(-I*e^(I*d*x + I*c) - 1) - 14956128*a^8*e^(-14*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 99*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1386

```
*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 9009*a^8*e^(24*I*d
*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 36036*a^8*e^(22*I*d*x + 8*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 99099*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 198198*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c))
- 297297*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 297297*a^8*
e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 198198*a^8*e^(10*I*d*x -
 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 99099*a^8*e^(8*I*d*x - 6*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 36036*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 9009*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1386*a^
8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 339768*a^8*e^(14*I*d*x
)*log(I*e^(I*d*x) + e^(-I*c)) - 99*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) + 99*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1386*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 9009*a^8*e^(24*I*d*x +
 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 36036*a^8*e^(22*I*d*x + 8*I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 99099*a^8*e^(20*I*d*...
```

Mupad [B]

time = 7.82, size = 343, normalized size = 1.67

$$\frac{\frac{1147\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{4}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,3505\mathrm{i}}{4}-\frac{5639\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,3585\mathrm{i}+\frac{25993\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,33847\mathrm{i}}{6}-4575\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,12041\mathrm{i}}{3}+\frac{27565\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{12}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4293\mathrm{i}}{4}-\frac{1360\,a^8}{3}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,3\mathrm{i}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,13\mathrm{i}-18\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,22\mathrm{i}+22\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,18\mathrm{i}-13\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,7\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}+\frac{1155\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((27565*a^8*tan(c/2 + (d*x)/2)^2)/12 - (a^8*tan(c/2 + (d*x)/2)^3*12041i)/3
- 4575*a^8*tan(c/2 + (d*x)/2)^4 + (a^8*tan(c/2 + (d*x)/2)^5*33847i)/6 + (25
993*a^8*tan(c/2 + (d*x)/2)^6)/6 - a^8*tan(c/2 + (d*x)/2)^7*3585i - (5639*a^
8*tan(c/2 + (d*x)/2)^8)/3 + (a^8*tan(c/2 + (d*x)/2)^9*3505i)/4 + (1147*a^8*
tan(c/2 + (d*x)/2)^10)/4 - (1360*a^8)/3 + (a^8*tan(c/2 + (d*x)/2)*4293i)/4)
/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*7i - 13*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*18i + 22*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2
)^6*22i - 18*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*13i + 7*tan(c/2 +
(d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*3i - tan(c/2 + (d*x)/2)^11 + 1i)) + (115
5*a^8*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=173

$$-\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d}-\frac{63ia^8\sec(c+dx)}{2d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia\cos^5(c+dx)(a}{5}$$

```
[Out] -63/2*a^8*arctanh(sin(d*x+c))/d-63/2*I*a^8*sec(d*x+c)/d+6/5*I*a^3*cos(d*x+c
)^3*(a+I*a*tan(d*x+c))^5/d-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^7/d-42/5
*I*a^2*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-21/2*I*sec(d*x+c)*(a^8+I*a^8*t
an(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3577, 3579, 3567, 3855}

$$-\frac{63ia^8\sec(c+dx)}{2d}-\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d}-\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{42ia^2\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{5d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-63*a^8*ArcTanh[Sin[c + d*x]])/(2*d) - (((63*I)/2)*a^8*Sec[c + d*x])/d + (
((6*I)/5)*a^3*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/5)*a*Cos
[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (((42*I)/5)*a^2*Cos[c + d*x]*(a^2
 + I*a^2*Tan[c + d*x])^3)/d - (((21*I)/2)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c +
 d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} - \frac{1}{5}(9a^2)\int \cos^3(c+dx)($$

$$= \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^5(c+dx)(a+ia\,\mathrm{t}}{5d}$$

$$= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6ia^3\cos^3(c+dx)(a+}{5d}$$

$$= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6ia^3\cos^3(c+dx)(a+}{5d}$$

$$= -\frac{63ia^8\sec(c+dx)}{2d} - \frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6i}{}$$

$$= -\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d} - \frac{63ia^8\sec(c+dx)}{2d} - \frac{42ia^5\cos(c+d}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1162 vs. 2(173) = 346.
time = 6.51, size = 1162, normalized size = 6.72

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) - (63*Cos[8*c]*Cos[
c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[5*d*x]*Cos[c + d*x]^8*(((-8*I
```

```
)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((8*I)*Cos[5*c] + 8*Sin[5*c])*(a
 + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*
x]^8*((-48*I)*Cos[7*c] - 48*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x
] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*((-8*I)*Cos[8*c] - 8*Sin[8*c])*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((63*I)/2)*Cos[c
 + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Ta
n[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((63*I)/2)*Cos[c + d*x]^8*L
og[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan[c + d*x])
^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(48*Cos[7*c] - (48*I)*S
in[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) +
 (Cos[c + d*x]^8*(-8*Cos[5*c] + (8*I)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c +
 d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*((8*Cos[3*c])/5 -
 ((8*I)/5)*Sin[3*c])*Sin[5*d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/4 - (I/4)*Sin[8*c])*(a + I*a*Tan[c
 + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^2) - (I*Cos[c + d*x]^8*(8*Cos[8*c] - (8*I)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-1/4*Cos[8*c
] + (I/4)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (I*Cos[c + d*x]^8*(8*Cos[8*c
] - (8*I)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + S
in[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]
))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(152) = 304.
time = 0.24, size = 357, normalized size = 2.06

method	result
risch	$-\frac{8ia^8 e^{5i(dx+c)}}{5d}+\frac{8ia^8 e^{3i(dx+c)}}{d}-\frac{48ia^8 e^{i(dx+c)}}{d}-\frac{ia^8\left(17\,e^{3i(dx+c)}+15\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{63a^8\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{6}{}$
derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^7(dx+c)\right)}{2}+\frac{7\left(\sin^5(dx+c)\right)}{10}+\frac{7\left(\sin^3(dx+c)\right)}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-\frac{56ia^8\left(\frac{8}{3}+\sin\right.}{}}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^7(dx+c)\right)}{2}+\frac{7\left(\sin^5(dx+c)\right)}{10}+\frac{7\left(\sin^3(dx+c)\right)}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-\frac{56ia^8\left(\frac{8}{3}+\sin\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/2*sin(d*x+c)^9/cos(d*x+c)^2+1/2*sin(d*x+c)^7+7/10*sin(d*x+c)^5+
7/6*sin(d*x+c)^3+7/2*sin(d*x+c)-7/2*ln(sec(d*x+c)+tan(d*x+c)))-56/5*I*a^8*(
```

```
8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-28*a^8*(-1/5*sin(d*x+c)^5-1/3
*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-8/5*I*a^8*cos(d*x+c)^5+
14*a^8*sin(d*x+c)^5-8*I*a^8*(sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5
*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))-28*a^8*(-1/5*sin(d*x+c)*cos(d*x
+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-56*I*a^8*(-1/5*cos(d*x+c)^3*sin(d*x
+c)^2-2/15*cos(d*x+c)^3)+1/5*a^8*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(143) = 286$.
time = 0.29, size = 326, normalized size = 1.88

$$-\frac{96i\,a^8\cos(dx+c)^5 - 840\,a^8\sin(dx+c)^5 + 224i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^8 + 224i\left(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c)\right)a^8 + 96i\left(\cos(dx+c)^5 - 5\cos(dx+c)^3 + \frac{5}{\cos(dx+c)} + 15\cos(dx+c)\right)a^8 - \left(12\sin(dx+c)^5 + 40\sin(dx+c)^3 - \frac{30\sin(dx+c)}{\sin(dx+c)^2-1} - 105\log(\sin(dx+c)+1) + 105\log(\sin(dx+c)-1) + 180\sin(dx+c)\right)a^8 - 56\left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1) + 30\sin(dx+c)\right)a^8 - 112\left(3\sin(dx+c)^5 - 5\sin(dx+c)^3\right)a^8 - 4\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^8}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/60*(96*I*a^8*cos(d*x + c)^5 - 840*a^8*sin(d*x + c)^5 + 224*I*(3*cos(d*x
+ c)^5 - 5*cos(d*x + c)^3)*a^8 + 224*I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^
3 + 15*cos(d*x + c))*a^8 + 96*I*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(
d*x + c) + 15*cos(d*x + c))*a^8 - (12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 -
30*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(
sin(d*x + c) - 1) + 180*sin(d*x + c))*a^8 - 56*(6*sin(d*x + c)^5 + 10*sin(d
*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*
x + c))*a^8 - 112*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^8 - 4*(3*sin(d*x
+ c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^8)/d
```

Fricas [A]
time = 0.37, size = 190, normalized size = 1.10

$$\frac{-16i\,a^8e^{(9i\,dx+9i\,c)} + 48i\,a^8e^{(7i\,dx+7i\,c)} - 336i\,a^8e^{(5i\,dx+5i\,c)} - 1050i\,a^8e^{(3i\,dx+3i\,c)} - 630i\,a^8e^{(i\,dx+i\,c)} - 315\left(a^8e^{(4i\,dx+4i\,c)} + 2\,a^8e^{(2i\,dx+2i\,c)} + a^8\right)\log\left(e^{(i\,dx+i\,c)} + i\right) + 315\left(a^8e^{(4i\,dx+4i\,c)} + 2\,a^8e^{(2i\,dx+2i\,c)} + a^8\right)\log\left(e^{(i\,dx+i\,c)} - i\right)}{10\left(de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/10*(-16*I*a^8*e^(9*I*d*x + 9*I*c) + 48*I*a^8*e^(7*I*d*x + 7*I*c) - 336*I*
a^8*e^(5*I*d*x + 5*I*c) - 1050*I*a^8*e^(3*I*d*x + 3*I*c) - 630*I*a^8*e^(I*d
*x + I*c) - 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x + 2*I*c) + a^8)
*log(e^(I*d*x + I*c) + I) + 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x
 + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [A]
time = 0.51, size = 235, normalized size = 1.36

$$\frac{63a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2} - \frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-17ia^8e^{3ic}e^{3idx} - 15ia^8e^{ic}e^{idx}}{de^{4ic}e^{4idx} + 2de^{2ic}e^{2idx} + d} + \begin{cases} \frac{-8ia^8d^2e^{5ic}e^{5idx} + 40ia^8d^2e^{3ic}e^{3idx} - 240ia^8d^2e^{ic}e^{idx}}{5d^3} & \text{for } d^3 \neq 0 \\ x\left(8a^8e^{5ic} - 24a^8e^{3ic} + 48a^8e^{ic}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 63*a**8*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-17*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 15*I*a**8*exp(I*c)*exp(I*d*x))/(
d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise(((
-8*I*a**8*d**2*exp(5*I*c)*exp(5*I*d*x) + 40*I*a**8*d**2*exp(3*I*c)*exp(3*I*
d*x) - 240*I*a**8*d**2*exp(I*c)*exp(I*d*x))/(5*d**3), Ne(d**3, 0)), (x*(8*a
**8*exp(5*I*c) - 24*a**8*exp(3*I*c) + 48*a**8*exp(I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(143) = 286$.
time = 1.45, size = 2849, normalized size = 16.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/655360*(42021645*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5
88303030*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969695*
a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 15295878780*a^8*e^(2
2*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 42063666645*a^8*e^(20*I*d*x +
 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 84127333290*a^8*e^(18*I*d*x + 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(
I*d*x + I*c) + 1) + 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1529587
8780*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969695*a^8*e^
(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 588303030*a^8*e^(2*I*d*x -
12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 144218285640*a^8*e^(14*I*d*x)*log(I*e^
(I*d*x + I*c) + 1) + 42021645*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
21376575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 299272050*a
^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1945268325*a^8*e^(24*
I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8*e^(22*I*d*x + 8
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 21397951575*a^8*e^(20*I*d*x + 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 42795903150*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 64193854725*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
2795903150*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2139795157
5*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8*e^(6*
I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1945268325*a^8*e^(4*I*d*x - 10*
I*c)*log(I*e^(I*d*x + I*c) - 1) + 299272050*a^8*e^(2*I*d*x - 12*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 73364405400*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) -
 1) + 21376575*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 42021645*a^8*e^
```

(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 588303030*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3823969695*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 15295878780*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 42063666645*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 84127333290*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 15295878780*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3823969695*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 588303030*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 144218285640*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 42021645*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 21376575*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 299272050*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1945268325*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7781073300*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 21397951575*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 42795903150*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 64193854725*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 42795903150*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 21397951575*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7781073300*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1945268325*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 299272050*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 73364405400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 21376575*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1230*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 17220*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 111930*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 447720*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1231230*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2462460*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3693690*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3693690*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2462460*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1231230*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 447720*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 111930*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 17220*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 4221360*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 1230*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1230*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 17220*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 111930*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 447...

Mupad [B]

time = 7.39, size = 281, normalized size = 1.62

$$-\frac{63\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{65\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,309\mathrm{i}-761\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1109\mathrm{i}+\frac{7351\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1223\mathrm{i}-\frac{4407\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}-a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,431\mathrm{i}+\frac{496\,a^8}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,5\mathrm{i}-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,20\mathrm{i}+26\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,26\mathrm{i}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,12\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*tan(c/2 + (d*x)/2)^3*1223i - (4407*a^8*tan(c/2 + (d*x)/2)^2)/5 + (7351
*a^8*tan(c/2 + (d*x)/2)^4)/5 - a^8*tan(c/2 + (d*x)/2)^5*1109i - 761*a^8*tan
(c/2 + (d*x)/2)^6 + a^8*tan(c/2 + (d*x)/2)^7*309i + 65*a^8*tan(c/2 + (d*x)/
2)^8 + (496*a^8)/5 - a^8*tan(c/2 + (d*x)/2)*431i)/(d*(5*tan(c/2 + (d*x)/2)
- tan(c/2 + (d*x)/2)^2*12i - 20*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4
*26i + 26*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 12*tan(c/2 + (d
*x)/2)^7 + tan(c/2 + (d*x)/2)^8*5i + tan(c/2 + (d*x)/2)^9 + 1i)) - (63*a^8*
atanh(tan(c/2 + (d*x)/2)))/d
```

3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=152

$$\frac{a^8\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}-2ia^2$$

```
[Out] a^8*arctanh(sin(d*x+c))/d+2/5*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/7
*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-2/3*I*a^2*cos(d*x+c)^3*(a^2+I*a^2*
tan(d*x+c))^3/d+2*I*cos(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 3855}

$$\frac{a^8\tanh^{-1}(\sin(c+dx))}{d}+\frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d}+\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*ArcTanh[Sin[c + d*x]])/d + (((2*I)/5)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[
c + d*x])^5)/d - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d -
(((2*I)/3)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^3)/d + ((2*I)*Cos[
c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} - a^2\int \cos^5(c+dx)(a+ \\
&= \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia}{7d} \\
&= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+}{5d} \\
&= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+}{5d} \\
&= \frac{a^8\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. $2(152) = 304$.
time = 2.05, size = 305, normalized size = 2.01

$$
\frac{a^8\left(-70i\cos\left(\frac{1}{2}(c+dx)\right)+42i\cos\left(\frac{3}{2}(c+dx)\right)+210i\cos\left(\frac{5}{2}(c+dx)\right)-30i\cos\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+105\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-70\sin\left(\frac{1}{2}(c+dx)\right)-42\sin\left(\frac{3}{2}(c+dx)\right)+210\sin\left(\frac{5}{2}(c+dx)\right)+30\sin\left(\frac{7}{2}(c+dx)\right)+105i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)-105i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(7c+23dx)\right)+i\sin\left(\frac{1}{2}(7c+23dx)\right)\right)}{105d(\cos(dx)+i\sin(dx))^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*((-70*I)*Cos[(c + d*x)/2] + (42*I)*Cos[(3*(c + d*x))/2] + (210*I)*Cos[
(5*(c + d*x))/2] - (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c + d*x))/2]*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2]*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(3*(c + d*x
))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] + (105*I)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] - (105*I)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(7*c + 23*d*x)/2
] + I*Sin[(7*c + 23*d*x)/2]))/(105*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(138) = 276$.
time = 0.23, size = 369, normalized size = 2.43

method	result
risch	$-\frac{2ia^8 e^{7i(dx+c)}}{7d}+\frac{2ia^8 e^{5i(dx+c)}}{5d}-\frac{2ia^8 e^{3i(dx+c)}}{3d}+\frac{2ia^8 e^{i(dx+c)}}{d}+\frac{a^8\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^8\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^8\left(-\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)-\frac{8ia^8\left(\cos^7(dx+c)\right)}{7}-4a^8\left(\sin\right.}{}$

default	$\frac{a^8\left(-\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)-\frac{8ia^8\left(\cos^7(dx+c)\right)}{7}-4a^8(\sin^7(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))-8/7*I*a^8*cos(d*x+c)^7-4*a^8*sin(d*x+c)^7-56*I*a^8
*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+70*a^8*(-1/7*sin(d*x+c)
^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(cos(d*x+c)^2+2)*sin(d*x+
c))+8/7*I*a^8*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)-28*a^8*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+56*I*a^8*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*cos(d*x+c)
^3*sin(d*x+c)^2-8/105*cos(d*x+c)^3)+1/7*a^8*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(130) = 260$.
time = 0.29, size = 309, normalized size = 2.03

$$-\frac{240i\,a^8\cos(dx+c)^7+840\,a^8\sin(dx+c)^7+112i\,(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+c)^3)a^8+336i\,(5\cos(dx+c)^7-7\cos(dx+c)^5)a^8+48i\,(5\cos(dx+c)^7-21\cos(dx+c)^5+35\cos(dx+c)^3-35\cos(dx+c))a^8+(30\sin(dx+c)^7+42\sin(dx+c)^5+70\sin(dx+c)^3-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)+210\sin(dx+c))a^8+56\,(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+c)^3)a^8+420\,(5\sin(dx+c)^7-7\sin(dx+c)^5)a^8+6\,(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c))a^8}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(240*I*a^8*cos(d*x + c)^7 + 840*a^8*sin(d*x + c)^7 + 112*I*(15*cos(d
*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 + 336*I*(5*cos(d*x +
 c)^7 - 7*cos(d*x + c)^5)*a^8 + 48*I*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5
+ 35*cos(d*x + c)^3 - 35*cos(d*x + c))*a^8 + (30*sin(d*x + c)^7 + 42*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x
+ c) - 1) + 210*sin(d*x + c))*a^8 + 56*(15*sin(d*x + c)^7 - 42*sin(d*x + c)
^5 + 35*sin(d*x + c)^3)*a^8 + 420*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^8
 + 6*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x
 + c))*a^8)/d
```

Fricas [A]
time = 0.39, size = 96, normalized size = 0.63

$$\frac{-30i\,a^8e^{(7i\,dx+7i\,c)}+42i\,a^8e^{(5i\,dx+5i\,c)}-70i\,a^8e^{(3i\,dx+3i\,c)}+210i\,a^8e^{(i\,dx+i\,c)}+105\,a^8\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,a^8\log\left(e^{(i\,dx+i\,c)}-i\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(-30*I*a^8*e^(7*I*d*x + 7*I*c) + 42*I*a^8*e^(5*I*d*x + 5*I*c) - 70*I*
a^8*e^(3*I*d*x + 3*I*c) + 210*I*a^8*e^(I*d*x + I*c) + 105*a^8*log(e^(I*d*x
+ I*c) + I) - 105*a^8*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A]
time = 0.57, size = 187, normalized size = 1.23

$$\frac{a^8\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}\frac{-30ia^8d^3e^{7ic}e^{7idx}+42ia^8d^3e^{5ic}e^{5idx}-70ia^8d^3e^{3ic}e^{3idx}+210ia^8d^3e^{ic}e^{idx}}{105d^4} & \text{for } d^4\neq 0\\ x(2a^8e^{7ic}-2a^8e^{5ic}+2a^8e^{3ic}-2a^8e^{ic}) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-30*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) + 42*I*a**8*d**3*exp(5*I
*c)*exp(5*I*d*x) - 70*I*a**8*d**3*exp(3*I*c)*exp(3*I*d*x) + 210*I*a**8*d**3
*exp(I*c)*exp(I*d*x))/(105*d**4), Ne(d**4, 0)), (x*(2*a**8*exp(7*I*c) - 2*a
**8*exp(5*I*c) + 2*a**8*exp(3*I*c) - 2*a**8*exp(I*c)), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. 2(130) = 260.
time = 1.55, size = 2863, normalized size = 18.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/55050240*(1635552135*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14883
5244285*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 595340977140
*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^8*e^
(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(18*I*d
*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(16*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(12*I*d*x - 2*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(
I*d*x + I*c) + 1) + 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 22897
729890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5613214927320*
a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1635552135*a^8*e^(-14*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1690450650*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 16921
```

41100650*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 338428220130
0*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e
^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3384282201300*a^8*e^(10*I*d*x - 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 153831009150*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 23666309100*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 5801626630800*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1690450650*a^8
*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1635552135*a^8*e^(28*I*d*x + 14*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 595340977140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 1637187687135*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 3274375374270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 4911563061405*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4911
563061405*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3274375374
270*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1637187687135*a^
8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 595340977140*a^8*e^(6*I
*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(4*I*d*x - 1
0*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(2*I*d*x - 12*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 5613214927320*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x
 + I*c) + 1) - 1635552135*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 169
0450650*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309100
*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 153831009150*a^8*e
^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 615324036600*a^8*e^(22*I
*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1692141100650*a^8*e^(20*I*d*x +
 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3384282201300*a^8*e^(18*I*d*x + 4*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(16*I*d*x + 2*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(
I*d*x + I*c) - 1) - 3384282201300*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1
53831009150*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309
100*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5801626630800*a^
8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1690450650*a^8*e^(-14*I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 151725*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 2124150*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) - 13806975*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 552279
00*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 151876725*a^8*e^(
20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 303753450*a^8*e^(18*I*d*x +
 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(16*I*d*x + 2*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 303753450*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 151876725*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 552

```
27900*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13806975*a^8*e^
(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2124150*a^8*e^(2*I*d*x - 1
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 520720200*a^8*e^(14*I*d*x)*log(I*e^(I*
d*x) + e^(-I*c)) - 151725*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 151
725*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) ...
```

Mupad [B]
time = 6.69, size = 207, normalized size = 1.36

$$\frac{2\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,16\mathrm{i}-\frac{80\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,224\mathrm{i}}{3}+\frac{224\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,304\mathrm{i}}{15}-\frac{304\,a^8}{105}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,7\mathrm{i}+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,35\mathrm{i}-35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,21\mathrm{i}+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (2*a^8*atanh(tan(c/2 + (d*x)/2)))/d + ((224*a^8*tan(c/2 + (d*x)/2)^2)/5 - (
a^8*tan(c/2 + (d*x)/2)^3*224i)/3 - (80*a^8*tan(c/2 + (d*x)/2)^4)/3 + a^8*ta
n(c/2 + (d*x)/2)^5*16i - (304*a^8)/105 + (a^8*tan(c/2 + (d*x)/2)*304i)/15)/
(d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*21i - 35*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2
)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=66

$$-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

```
[Out] -1/63*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-1/9*I*cos(d*x+c)^9*(a+I*a*tan
(d*x+c))^8/d
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3578, 3569}

$$-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/63*I)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d*
x]^9*(a + I*a*Tan[c + d*x])^8)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}+\frac{1}{9}a\int \cos^7(c+dx)(a+ia\\ &= -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}-\frac{i\cos^9(c+dx)(a+ia\tan}{9d}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 50, normalized size = 0.76

$$\frac{a^8(8\cos(c+dx)-i\sin(c+dx))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(8*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[8*(c + d*x)] + Sin[8*(c +
d*x)]))/(63*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(58) = 116.
time = 0.24, size = 447, normalized size = 6.77

method	result
risch	$-\frac{i a^8 e^{9 i (d x+c)}}{18 d}-\frac{i a^8 e^{7 i (d x+c)}}{14 d}$
derivativedivides	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9}-8ia^8\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^6(dx+c)\right)}{9}-\frac{2\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{21}-\frac{8\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{105}-\frac{16\left(\cos^3(dx\right.}{315}}{}$
default	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9}-8ia^8\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^6(dx+c)\right)}{9}-\frac{2\left(\sin^4(dx+c)\right)\left(\cos^3(dx+c)\right)}{21}-\frac{8\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{105}-\frac{16\left(\cos^3(dx\right.}{315}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/9*a^8*sin(d*x+c)^9-8*I*a^8*(-1/9*cos(d*x+c)^3*sin(d*x+c)^6-2/21*sin(
d*x+c)^4*cos(d*x+c)^3-8/105*cos(d*x+c)^3*sin(d*x+c)^2-16/315*cos(d*x+c)^3)-
28*a^8*(-1/9*cos(d*x+c)^4*sin(d*x+c)^5-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*
sin(d*x+c)*cos(d*x+c)^4+1/63*(cos(d*x+c)^2+2)*sin(d*x+c))+56*I*a^8*(-1/9*si
n(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+
70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/9*sin(d*x+c)^2*
cos(d*x+c)^7-2/63*cos(d*x+c)^7)-28*a^8*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(
16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8/9*I*a^8*
cos(d*x+c)^9+1/9*a^8*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(54) = 108.
time = 0.30, size = 302, normalized size = 4.58

$$-\frac{280i a^8\cos(dx+c)^9-35a^8\sin(dx+c)^9+56i\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^8+8i\left(35\cos(dx+c)^9-135\cos(dx+c)^7+189\cos(dx+c)^5-105\cos(dx+c)^3\right)a^8+280i\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^8-70\left(35\sin(dx+c)^9-90\sin(dx+c)^7+63\sin(dx+c)^5\right)a^8-28\left(35\sin(dx+c)^9-135\sin(dx+c)^7+189\sin(dx+c)^5-105\sin(dx+c)^3\right)a^8-\left(35\sin(dx+c)^9-180\sin(dx+c)^7+378\sin(dx+c)^5-420\sin(dx+c)^3+315\sin(dx+c)\right)a^8-140\left(7\sin(dx+c)^9-9\sin(dx+c)^7\right)a^8}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/315*(280*I*a^8*cos(d*x + c)^9 - 35*a^8*sin(d*x + c)^9 + 56*I*(35*cos(d*x
 + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 + 8*I*(35*cos(d*x + c)
^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 28
0*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^8 - 70*(35*sin(d*x + c)^9 - 90*
sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^8 - 28*(35*sin(d*x + c)^9 - 135*sin(d
*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^8 - (35*sin(d*x + c)
^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin
(d*x + c))*a^8 - 140*(7*sin(d*x + c)^9 - 9*sin(d*x + c)^7)*a^8)/d
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.52

$$\frac{-7i\,a^8e^{(9i\,dx+9i\,c)}-9i\,a^8e^{(7i\,dx+7i\,c)}}{126\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/126*(-7*I*a^8*e^(9*I*d*x + 9*I*c) - 9*I*a^8*e^(7*I*d*x + 7*I*c))/d
```

Sympy [A]
time = 0.55, size = 80, normalized size = 1.21

$$\begin{cases} \frac{-14ia^8de^{9ic}e^{9idx}-18ia^8de^{7ic}e^{7idx}}{252d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^8e^{9ic}}{2}+\frac{a^8e^{7ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-14*I*a**8*d*exp(9*I*c)*exp(9*I*d*x) - 18*I*a**8*d*exp(7*I*c)*e
xp(7*I*d*x))/(252*d**2), Ne(d**2, 0)), (x*(a**8*exp(9*I*c)/2 + a**8*exp(7*I
*c)/2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(54) = 108$.
time = 1.61, size = 2451, normalized size = 37.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/66060288*(1419343317*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 17032119804*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 93676
658922*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 312255529740*a
^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(16
*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(14*I*d*x
+ 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(10*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(8*I*d*x - 4*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 93676658922*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 17032119804*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 131147
3224908*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1419343317*a^8*e^(-12
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1419097050*a^8*e^(24*I*d*x + 12*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 312201351000*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 702453039750*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1123924
863600*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1123924863600*
a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 702453039750*a^8*e^(8
*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 312201351000*a^8*e^(6*I*d*x -
6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*e^(4*I*d*x - 8*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 1311245674200*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) - 1)
+ 1419097050*a^8*e^(-12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1419343317*a^8*e^
(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17032119804*a^8*e^(22*I*d
*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 93676658922*a^8*e^(20*I*d*x + 8*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 312255529740*a^8*e^(18*I*d*x + 6*I*c)*lo
g(-I*e^(I*d*x + I*c) + 1) - 702574941915*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 1124119907064*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 1124119907064*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 702574941915*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9367665
8922*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17032119804*a^8*
e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1311473224908*a^8*e^(12*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1419343317*a^8*e^(-12*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 1419097050*a^8*e^(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I
*c) - 1) - 17029164600*a^8*e^(22*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3122
01351000*a^8*e^(18*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 70245303975
0*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1123924863600*a^8*
e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1123924863600*a^8*e^(10*
I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 702453039750*a^8*e^(8*I*d*x -
4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 312201351000*a^8*e^(6*I*d*x - 6*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 93660405300*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(
I*d*x + I*c) - 1) - 17029164600*a^8*e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 1311245674200*a^8*e^(12*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 14
```

```
19097050*a^8*e^(-12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246267*a^8*e^(24*I*d
*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2955204*a^8*e^(22*I*d*x + 10*I*c
)*log(I*e^(I*d*x) + e^(-I*c)) - 16253622*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) - 54178740*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^
(-I*c)) - 121902165*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
195043464*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 195043464*
a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 121902165*a^8*e^(8*I
*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 54178740*a^8*e^(6*I*d*x - 6*I*c
)*log(I*e^(I*d*x) + e^(-I*c)) - 16253622*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) - 2955204*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 227550708*a^8*e^(12*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 246267*a^8
*e^(-12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 246267*a^8*e^(24*I*d*x + 12*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 2955204*a^8*e^(22*I*d*x + 10*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 16253622*a^8*e^(20*I*d*x + 8*I*c)*log(-I*e^(I*d*x) +
e^(-I*c)) + 54178740*a^8*e^(18*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
+ 121902165*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1950434
64*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 195043464*a^8*e^
(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 121902165*a^8*e^(8*I*d*x
- 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 54178740*a^8*e^(6*I*d*x - 6*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) + 16253622*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) + 2955204*a^8*e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 227550708*a^8*e^(12*I*d*x)*log(-I*e^(I*d...
```

Mupad [B]
time = 3.62, size = 37, normalized size = 0.56

$$-\frac{2\,a^8\,\left(\frac{{\mathrm{e}}^{c\,7{}\mathrm{i}+d\,x\,7{}\mathrm{i}}\,9{}\mathrm{i}}{4}+\frac{{\mathrm{e}}^{c\,9{}\mathrm{i}+d\,x\,9{}\mathrm{i}}\,7{}\mathrm{i}}{4}\right)}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(2*a^8*((exp(c*7i + d*x*7i)*9i)/4 + (exp(c*9i + d*x*9i)*7i)/4))/(63*d)
```

3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=136

$$-\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}-\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}$$

```
[Out] -2/1155*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/231*I*a^2*cos(d*x+c)^7*
(a+I*a*tan(d*x+c))^6/d-1/33*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^7/d-1/11*I*
cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8/d
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3578, 3569}

$$-\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}-\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/1155)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/231
)*a^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^6)/d - ((I/33)*a*Cos[c + d*x]^9
*(a + I*a*Tan[c + d*x])^7)/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x
])^8)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d} + \frac{1}{11}(3a)\int \cos^9(c+dx)(a$$
$$= -\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\,\mathrm{ta}}{11d}$$
$$= -\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{ia\cos^9(c+dx)(a+ia}{33d}$$
$$= -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d} - \frac{2ia^2\cos^7(c+dx)(a+}{231d}$$

Mathematica [**A**]
time = 0.79, size = 73, normalized size = 0.54

$$\frac{a^8(440\cos(c+dx)+168\cos(3(c+dx))-i(55\sin(c+dx)+63\sin(3(c+dx))))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{4620d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] - I*(55*Sin[c + d*x] + 63*Sin
[3*(c + d*x)]))*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/(4620*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(120) = 240$.
time = 0.23, size = 567, normalized size = 4.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*cos(d*x+c)^4*sin(d*x+c)^5-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(cos(d*x+c)^
2+2)*sin(d*x+c))-8*I*a^8*(-1/11*sin(d*x+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4
*cos(d*x+c)^5-8/231*sin(d*x+c)^2*cos(d*x+c)^5-16/1155*cos(d*x+c)^5)-28*a^8*
(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/231*sin(d
*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+56
*I*a^8*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/69
3*cos(d*x+c)^7)+70*a^8*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos
(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d
*x+c))-56*I*a^8*(-1/11*sin(d*x+c)^2*cos(d*x+c)^9-2/99*cos(d*x+c)^9)-28*a^8*
(-1/11*cos(d*x+c)^10*sin(d*x+c)+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+
48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8/11*I*a^8*cos(d*x+c)^11
+1/11*a^8*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*
cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(112) = 224$.
time = 0.30, size = 355, normalized size = 2.61

$$-\frac{2520i\,a^8\cos\left(dx+c\right)^{11}+24i\left(105\cos\left(dx+c\right)^{11}-385\cos\left(dx+c\right)^9+495\cos\left(dx+c\right)^7-231\cos\left(dx+c\right)^5\right)a^8+280i\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^9+99\cos\left(dx+c\right)^7\right)a^8+1960i\left(9\cos\left(dx+c\right)^{11}-11\cos\left(dx+c\right)^9\right)a^8+28\left(315\sin\left(dx+c\right)^{11}-1540\sin\left(dx+c\right)^9+2970\sin\left(dx+c\right)^7-2772\sin\left(dx+c\right)^5+1155\sin\left(dx+c\right)^3\right)a^8+210\left(105\sin\left(dx+c\right)^{11}-385\sin\left(dx+c\right)^9+495\sin\left(dx+c\right)^7-231\sin\left(dx+c\right)^5\right)a^8+140\left(63\sin\left(dx+c\right)^{11}-154\sin\left(dx+c\right)^9+99\sin\left(dx+c\right)^7\right)a^8+5\left(63\sin\left(dx+c\right)^{11}-385\sin\left(dx+c\right)^9+990\sin\left(dx+c\right)^7-1386\sin\left(dx+c\right)^5+1155\sin\left(dx+c\right)^3-693\sin\left(dx+c\right)\right)a^8+35\left(9\sin\left(dx+c\right)^{11}-11\sin\left(dx+c\right)^9\right)a^8}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3465*(2520*I*a^8*cos(d*x + c)^11 + 24*I*(105*cos(d*x + c)^11 - 385*cos(d
*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 280*I*(63*cos(d*
x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a^8 + 1960*I*(9*cos(d*x
 + c)^11 - 11*cos(d*x + c)^9)*a^8 + 28*(315*sin(d*x + c)^11 - 1540*sin(d*x
+ c)^9 + 2970*sin(d*x + c)^7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a
^8 + 210*(105*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 2
31*sin(d*x + c)^5)*a^8 + 140*(63*sin(d*x + c)^11 - 154*sin(d*x + c)^9 + 99*
sin(d*x + c)^7)*a^8 + 5*(63*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(
d*x + c)^7 - 1386*sin(d*x + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*
a^8 + 35*(9*sin(d*x + c)^11 - 11*sin(d*x + c)^9)*a^8)/d
```

Fricas [A]
time = 0.39, size = 62, normalized size = 0.46

$$\frac{-105i\,a^8e^{(11i\,dx+11i\,c)}-385i\,a^8e^{(9i\,dx+9i\,c)}-495i\,a^8e^{(7i\,dx+7i\,c)}-231i\,a^8e^{(5i\,dx+5i\,c)}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(-105*I*a^8*e^(11*I*d*x + 11*I*c) - 385*I*a^8*e^(9*I*d*x + 9*I*c) -
495*I*a^8*e^(7*I*d*x + 7*I*c) - 231*I*a^8*e^(5*I*d*x + 5*I*c))/d
```

Sympy [A]
time = 0.66, size = 162, normalized size = 1.19

$$\begin{cases}\frac{-53760ia^8d^3e^{11ic}e^{11idx}-197120ia^8d^3e^{9ic}e^{9idx}-253440ia^8d^3e^{7ic}e^{7idx}-118272ia^8d^3e^{5ic}e^{5idx}}{4730880d^4} & \text{for } d^4 \neq 0\\ x\left(\frac{a^8e^{11ic}}{8}+\frac{3a^8e^{9ic}}{8}+\frac{3a^8e^{7ic}}{8}+\frac{a^8e^{5ic}}{8}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-53760*I*a**8*d**3*exp(11*I*c)*exp(11*I*d*x) - 197120*I*a**8*d*
*3*exp(9*I*c)*exp(9*I*d*x) - 253440*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) - 1
18272*I*a**8*d**3*exp(5*I*c)*exp(5*I*d*x))/(4730880*d**4), Ne(d**4, 0)), (x
```

```
*(a**8*exp(11*I*c)/8 + 3*a**8*exp(9*I*c)/8 + 3*a**8*exp(7*I*c)/8 + a**8*exp
(5*I*c)/8), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(112) = 224$.
time = 1.80, size = 2863, normalized size = 21.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/4844421120*(82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) +
7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 298581
74165820*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 821099789560
05*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^
8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(
16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^8*e^(10*I*d*x -
 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978956005*a^8*e^(8*I*d*x - 6*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 29858174165820*a^8*e^(6*I*d*x - 8*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c
) + 1) + 281519927849160*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 8202
7951005*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82004266575*a^8*e^(28*
I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^(26*I*d*x
+ 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^(24*I*d*x + 10*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(22*I*d*x + 8*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 164172541683150*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 246258812524725*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I
*c) - 1) + 246258812524725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 298495
53033300*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325
*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^
(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 281438642885400*a^8*e^(14*I
*d*x)*log(I*e^(I*d*x + I*c) - 1) + 82004266575*a^8*e^(-14*I*c)*log(I*e^(I*d
*x + I*c) - 1) - 82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*
c) + 1) - 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
29858174165820*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109
978956005*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1642199579
12010*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24632993686801
```

```
5*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 246329936868015*a^
8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 164219957912010*a^8*e^
(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109978956005*a^8*e^(8*I*
d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 29858174165820*a^8*e^(6*I*d*x -
8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 7464543541455*a^8*e^(4*I*d*x - 10*I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 281519927849160*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x +
 I*c) + 1) - 82027951005*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8200
4266575*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11480597320
50*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7462388258325*a^
8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^
(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(20*I
*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(18*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(12*I*d*x - 2*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 7462388258325*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 1148059732050*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 281438642885400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 82004266
575*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23684430*a^8*e^(28*I*d*x
+ 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 331582020*a^8*e^(26*I*d*x + 12*I*c)
*log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(24*I*d*x + 10*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 23708114430*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 47416228860*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 71
124343290*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7112434329
0*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 47416228860*a^8*e^
(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23708114430*a^8*e^(8*I*d*x
 - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(6*I*d*x - 8*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) - 331582020*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e
^(-I*c)) - 81284963760*a^8*e^(14*I*d*x)*log(I*e...
```

Mupad [B]

time = 3.80, size = 65, normalized size = 0.48

$$-\frac{a^8\left(\frac{{\mathrm{e}}^{c\,5{}\mathrm{i}+d\,x\,5{}\mathrm{i}}\,1{}\mathrm{i}}{40}+\frac{{\mathrm{e}}^{c\,7{}\mathrm{i}+d\,x\,7{}\mathrm{i}}\,3{}\mathrm{i}}{56}+\frac{{\mathrm{e}}^{c\,9{}\mathrm{i}+d\,x\,9{}\mathrm{i}}\,1{}\mathrm{i}}{24}+\frac{{\mathrm{e}}^{c\,11{}\mathrm{i}+d\,x\,11{}\mathrm{i}}\,1{}\mathrm{i}}{88}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*((exp(c*5i + d*x*5i)*1i)/40 + (exp(c*7i + d*x*7i)*3i)/56 + (exp(c*9i
+ d*x*9i)*1i)/24 + (exp(c*11i + d*x*11i)*1i)/88))/d
```

3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=211

$$-\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}-\frac{5ia\cos^{11}(c+dx)(a+ia}{143d}$$

```
[Out] -20/3003*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d-20/1287*I*a^2*cos(d*x+c)
^9*(a+I*a*tan(d*x+c))^6/d-5/143*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^7/d-1/
13*I*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8/d-8/9009*I*a^2*cos(d*x+c)^3*(a^2+I*
a^2*tan(d*x+c))^3/d-8/3003*I*cos(d*x+c)^5*(a^2+I*a^2*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.20, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3578, 3569}

$$-\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}-\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d}-\frac{8ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{9009d}-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}-\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-20*I)/3003)*a^3*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d - (((20*I)/1
287)*a^2*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^6)/d - (((5*I)/143)*a*Cos[c
+ d*x]^11*(a + I*a*Tan[c + d*x])^7)/d - ((I/13)*Cos[c + d*x]^13*(a + I*a*Ta
n[c + d*x])^8)/d - (((8*I)/9009)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*
x])^3)/d - (((8*I)/3003)*Cos[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x])^4)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} + \frac{1}{13}(5a)\int \cos^{11}(c+dx$$
$$= -\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} - \frac{i\cos^{13}(c+dx)(a+i}{13d}$$
$$= -\frac{20ia^2\cos^{9}(c+dx)(a+ia\tan(c+dx))^6}{1287d} - \frac{5ia\cos^{11}(c+dx)(a}{14}$$
$$= -\frac{20ia^3\cos^{7}(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^{9}(c+dx)(}{12}$$
$$= -\frac{20ia^3\cos^{7}(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^{9}(c+dx)(}{12}$$
$$= -\frac{8ia^5\cos^{3}(c+dx)(a+ia\tan(c+dx))^3}{9009d} - \frac{20ia^3\cos^{7}(c+dx)(a}{300}$$

Mathematica [**A**]
time = 0.99, size = 111, normalized size = 0.53

$$\frac{a^8(11440\cos(c+dx)+6552\cos(3(c+dx))+1848\cos(5(c+dx))-1430i\sin(c+dx)-2457i\sin(3(c+dx))-1155i\sin(5(c+dx)))(-i\cos(8(c+2dx))+\sin(8(c+2dx)))}{144144d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 1848*Cos[5*(c + d*x)] -
(1430*I)*Sin[c + d*x] - (2457*I)*Sin[3*(c + d*x)] - (1155*I)*Sin[5*(c + d*x
)])*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(144144*d*(Cos[d*x] + I
*Sin[d*x])^8)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(187) = 374.
time = 0.29, size = 617, normalized size = 2.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/13*sin(d*x+c)^6*cos(d
*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x+c)^7-16/
3003*cos(d*x+c)^7)-28*a^8*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)
^3*cos(d*x+c)^8-5/429*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5
*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/13*sin(d*x+c)^4*co
s(d*x+c)^9-4/143*sin(d*x+c)^2*cos(d*x+c)^9-8/1287*cos(d*x+c)^9)+70*a^8*(-1/
13*sin(d*x+c)^3*cos(d*x+c)^10-3/143*cos(d*x+c)^10*sin(d*x+c)+1/429*(128/35+
```

```
cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*
x+c))-56*I*a^8*(-1/13*sin(d*x+c)^2*cos(d*x+c)^11-2/143*cos(d*x+c)^11)-28*a^
8*(-1/13*sin(d*x+c)*cos(d*x+c)^12+1/143*(256/63+cos(d*x+c)^10+10/9*cos(d*x+
c)^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
-8/13*I*a^8*cos(d*x+c)^13+1/13*a^8*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)
^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(175) = 350$.
time = 0.30, size = 405, normalized size = 1.92

$$-\frac{5544i\,a^8\cos\left(dx+c\right)^{13}+24i\left(231\cos\left(dx+c\right)^{13}-819\cos\left(dx+c\right)^{11}+1001\cos\left(dx+c\right)^{9}-429\cos\left(dx+c\right)^{7}\right)a^8+392i\left(99\cos\left(dx+c\right)^{13}-234\cos\left(dx+c\right)^{11}+143\cos\left(dx+c\right)^{9}\right)a^8+3528i\left(11\cos\left(dx+c\right)^{13}-13\cos\left(dx+c\right)^{11}\right)a^8-42\left(1155\sin\left(dx+c\right)^{13}-5460\sin\left(dx+c\right)^{11}+10010\sin\left(dx+c\right)^{9}-8580\sin\left(dx+c\right)^{7}+3003\sin\left(dx+c\right)^{5}\right)a^8-28\left(693\sin\left(dx+c\right)^{13}-4095\sin\left(dx+c\right)^{11}+10010\sin\left(dx+c\right)^{9}-12870\sin\left(dx+c\right)^{7}+9009\sin\left(dx+c\right)^{5}-3003\sin\left(dx+c\right)^{3}\right)a^8-84\left(231\sin\left(dx+c\right)^{13}-819\sin\left(dx+c\right)^{11}+1001\sin\left(dx+c\right)^{9}-429\sin\left(dx+c\right)^{7}\right)a^8-3\left(231\sin\left(dx+c\right)^{13}-1638\sin\left(dx+c\right)^{11}+5005\sin\left(dx+c\right)^{9}-8580\sin\left(dx+c\right)^{7}+9009\sin\left(dx+c\right)^{5}-6006\sin\left(dx+c\right)^{3}+3003\sin\left(dx+c\right)\right)a^8-7\left(99\sin\left(dx+c\right)^{13}-234\sin\left(dx+c\right)^{11}+143\sin\left(dx+c\right)^{9}\right)a^8}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9009*(5544*I*a^8*cos(d*x + c)^13 + 24*I*(231*cos(d*x + c)^13 - 819*cos(d
*x + c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*a^8 + 392*I*(99*cos(
d*x + c)^13 - 234*cos(d*x + c)^11 + 143*cos(d*x + c)^9)*a^8 + 3528*I*(11*co
s(d*x + c)^13 - 13*cos(d*x + c)^11)*a^8 - 42*(1155*sin(d*x + c)^13 - 5460*s
in(d*x + c)^11 + 10010*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 3003*sin(d*x
+ c)^5)*a^8 - 28*(693*sin(d*x + c)^13 - 4095*sin(d*x + c)^11 + 10010*sin(d*
x + c)^9 - 12870*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 3003*sin(d*x + c)^3
)*a^8 - 84*(231*sin(d*x + c)^13 - 819*sin(d*x + c)^11 + 1001*sin(d*x + c)^9
 - 429*sin(d*x + c)^7)*a^8 - 3*(231*sin(d*x + c)^13 - 1638*sin(d*x + c)^11
+ 5005*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 6006*si
n(d*x + c)^3 + 3003*sin(d*x + c))*a^8 - 7*(99*sin(d*x + c)^13 - 234*sin(d*x
 + c)^11 + 143*sin(d*x + c)^9)*a^8)/d
```

Fricas [**A**]
time = 0.43, size = 90, normalized size = 0.43

$$\frac{-693i\,a^8e^{(13i\,dx+13i\,c)}-4095i\,a^8e^{(11i\,dx+11i\,c)}-10010i\,a^8e^{(9i\,dx+9i\,c)}-12870i\,a^8e^{(7i\,dx+7i\,c)}-9009i\,a^8e^{(5i\,dx+5i\,c)}-3003i\,a^8e^{(3i\,dx+3i\,c)}}{288288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(-693*I*a^8*e^(13*I*d*x + 13*I*c) - 4095*I*a^8*e^(11*I*d*x + 11*I*
c) - 10010*I*a^8*e^(9*I*d*x + 9*I*c) - 12870*I*a^8*e^(7*I*d*x + 7*I*c) - 90
09*I*a^8*e^(5*I*d*x + 5*I*c) - 3003*I*a^8*e^(3*I*d*x + 3*I*c))/d
```

Sympy [**A**]
time = 0.79, size = 240, normalized size = 1.14

$$\begin{cases} \frac{-17439916032ia^8d^5e^{13ic}e^{13idx}-103054049280ia^8d^5e^{11ic}e^{11idx}-251909898240ia^8d^5e^{9ic}e^{9idx}-323884154880ia^8d^5e^{7ic}e^{7idx}-226718908416ia^8d^5e^{5ic}e^{5idx}-75572969472ia^8d^5e^{3ic}e^{3idx}}{7255005069312d^6} & \text{for}\ d^6 \neq 0 \\ x\left(\frac{a^8e^{13ic}}{32}+\frac{5a^8e^{11ic}}{32}+\frac{5a^8e^{9ic}}{16}+\frac{5a^8e^{7ic}}{16}+\frac{5a^8e^{5ic}}{32}+\frac{a^8e^{3ic}}{32}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**13*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-17439916032*I*a**8*d**5*exp(13*I*c)*exp(13*I*d*x) - 1030540492
80*I*a**8*d**5*exp(11*I*c)*exp(11*I*d*x) - 251909898240*I*a**8*d**5*exp(9*I
*c)*exp(9*I*d*x) - 323884154880*I*a**8*d**5*exp(7*I*c)*exp(7*I*d*x) - 22671
8908416*I*a**8*d**5*exp(5*I*c)*exp(5*I*d*x) - 75572969472*I*a**8*d**5*exp(3
*I*c)*exp(3*I*d*x))/(7255005069312*d**6), Ne(d**6, 0)), (x*(a**8*exp(13*I*c
)/32 + 5*a**8*exp(11*I*c)/32 + 5*a**8*exp(9*I*c)/16 + 5*a**8*exp(7*I*c)/16
+ 5*a**8*exp(5*I*c)/32 + a**8*exp(3*I*c)/32), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2891 vs. $2(175) = 350$.
time = 1.92, size = 2891, normalized size = 13.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/151145938944*(1945052766657*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 27230738733198*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) +
1) + 176999801765787*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 707999207063148*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1946
997819423657*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 38939956
38847314*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099345827
0971*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5840993458270971
*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3893995638847314*a^8
*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1946997819423657*a^8*e^(
8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 707999207063148*a^8*e^(6*I*d*
x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 176999801765787*a^8*e^(4*I*d*x - 10
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 27230738733198*a^8*e^(2*I*d*x - 12*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 6675421095166824*a^8*e^(14*I*d*x)*log(I*e^(I*d
*x + I*c) + 1) + 1945052766657*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 1944080407269*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 27217
125701766*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1769113170
61479*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 70764526824591
6*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1946024487676269*a^
8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892048975352538*a^8*e^
(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838073463028807*a^8*e^(16*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838073463028807*a^8*e^(12*I*d*
x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892048975352538*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1946024487676269*a^8*e^(8*I*d*x - 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 707645268245916*a^8*e^(6*I*d*x - 8*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 176911317061479*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(
```

```
I*d*x + I*c) - 1) + 27217125701766*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 6672083957747208*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1)
+ 1944080407269*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1945052766657*
a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 27230738733198*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 176999801765787*a^8*e^(
24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 707999207063148*a^8*e^(22*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1946997819423657*a^8*e^(20*I*d
*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3893995638847314*a^8*e^(18*I*d*x
+ 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5840993458270971*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5840993458270971*a^8*e^(12*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 3893995638847314*a^8*e^(10*I*d*x - 4*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 1946997819423657*a^8*e^(8*I*d*x - 6*I*c)*log(
-I*e^(I*d*x + I*c) + 1) - 707999207063148*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 176999801765787*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 27230738733198*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x +
I*c) + 1) - 6675421095166824*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) -
 1945052766657*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1944080407269*
a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 27217125701766*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 176911317061479*a^8*e^(
24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 707645268245916*a^8*e^(22*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1946024487676269*a^8*e^(20*I*d
*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3892048975352538*a^8*e^(18*I*d*x
+ 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5838073463028807*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5838073463028807*a^8*e^(12*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 3892048975352538*a^8*e^(10*I*d*x - 4*I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 1946024487676269*a^8*e^(8*I*d*x - 6*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 707645268245916*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 176911317061479*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d
*x + I*c) - 1) - 27217125701766*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 6672083957747208*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) -
 1944080407269*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 972359388*a^8*
e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13613031432*a^8*e^(26*I
*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 88484704308*a^8*e^(24*I*d*x +
10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 353938817232*a^8*e^(22*I*d*x + 8*I*c)
*log(I*e^(I*d*x) + e^(-I*c)) - 973331747388*a^8*e^(20*I*d*x + 6*I*c)*log(I*
e^(I*d*x) + e^(-I*c)) - 1946663494776*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 2919995242164*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) - 2919995242164*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 1946663494776*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
973331747388*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 35393881
7232*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 88484704308*a^8*
e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)...
```

Mupad [B]

time = 4.08, size = 93, normalized size = 0.44

$$-\frac{a^8\left(\frac{\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,1\mathrm{i}}{96}+\frac{\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,1\mathrm{i}}{32}+\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,5\mathrm{i}}{112}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,5\mathrm{i}}{144}+\frac{\mathrm{e}^{c\,11\mathrm{i}+d\,x\,11\mathrm{i}}\,5\mathrm{i}}{352}+\frac{\mathrm{e}^{c\,13\mathrm{i}+d\,x\,13\mathrm{i}}\,1\mathrm{i}}{416}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*((exp(c*3i + d*x*3i)*1i)/96 + (exp(c*5i + d*x*5i)*1i)/32 + (exp(c*7i
+ d*x*7i)*5i)/112 + (exp(c*9i + d*x*9i)*5i)/144 + (exp(c*11i + d*x*11i)*5i)
/352 + (exp(c*13i + d*x*13i)*1i)/416))/d
```

3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=212

$$\frac{7a^8\sin(c+dx)}{1287d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{a^8\sin^7(c+dx)}{1287d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))}{195d}$$

```
[Out] 7/1287*a^8*sin(d*x+c)/d-7/1287*a^8*sin(d*x+c)^3/d+7/2145*a^8*sin(d*x+c)^5/d
-1/1287*a^8*sin(d*x+c)^7/d-2/195*I*a^3*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^5/d
-2/15*I*a*cos(d*x+c)^15*(a+I*a*tan(d*x+c))^7/d-2/715*I*a^2*cos(d*x+c)^11*(a
^2+I*a^2*tan(d*x+c))^3/d-2/1287*I*cos(d*x+c)^9*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A]
time = 0.16, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 2713}

$$-\frac{a^8\sin^7(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin(c+dx)}{1287d}-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2ia^2\cos^{11}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{715d}-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (7*a^8*Sin[c + d*x])/(1287*d) - (7*a^8*Sin[c + d*x]^3)/(1287*d) + (7*a^8*Si
n[c + d*x]^5)/(2145*d) - (a^8*Sin[c + d*x]^7)/(1287*d) - (((2*I)/195)*a^3*C
os[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/15)*a*Cos[c + d*x]^15*
(a + I*a*Tan[c + d*x])^7)/d - (((2*I)/715)*a^2*Cos[c + d*x]^11*(a^2 + I*a^2
*Tan[c + d*x])^3)/d - (((2*I)/1287)*Cos[c + d*x]^9*(a^8 + I*a^8*Tan[c + d*x
]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}+\frac{1}{15}a^2\int \cos^{13}(c+dx \\
&= -\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2ia\cos^{15}(c+dx)(a}{15} \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a}{19} \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a}{19} \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a}{19} \\
&= \frac{7a^8\sin(c+dx)}{1287d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{a^8\sin^7}{12}
\end{aligned}
$$

Mathematica [A]
time = 1.81, size = 133, normalized size = 0.63

$$\frac{a^8(28600\cos(c+dx)+19656\cos(3(c+dx))+9240\cos(5(c+dx))+3432\cos(7(c+dx))-3575i\sin(c+dx)-7371i\sin(3(c+dx))-5775i\sin(5(c+dx))-3003i\sin(7(c+dx)))(-i\cos(8(c+2dx))+\sin(8(c+2dx)))}{411840d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] + 9240*Cos[5*(c + d*x)] +
 3432*Cos[7*(c + d*x)] - (3575*I)*Sin[c + d*x] - (7371*I)*Sin[3*(c + d*x)]
- (5775*I)*Sin[5*(c + d*x)] - (3003*I)*Sin[7*(c + d*x)])*((-I)*Cos[8*(c + 2
*d*x)] + Sin[8*(c + 2*d*x)]))/(411840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(188) = 376.
time = 0.30, size = 667, normalized size = 3.15

method	result
risch	$-\frac{ia^8e^{15i(dx+c)}}{1920d}-\frac{7ia^8e^{13i(dx+c)}}{1664d}-\frac{21ia^8e^{11i(dx+c)}}{1408d}-\frac{35ia^8e^{9i(dx+c)}}{1152d}-\frac{5ia^8e^{7i(dx+c)}}{128d}-\frac{21ia^8e^{5i(dx+c)}}{640d}-7$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/15*sin(d*x+c)^7*cos(d*x+c)^8-7/195*sin(d*x+c)^5*cos(d*x+c)^8-7
/429*sin(d*x+c)^3*cos(d*x+c)^8-7/1287*sin(d*x+c)*cos(d*x+c)^8+1/1287*(16/5+
```

```
cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/15*
sin(d*x+c)^6*cos(d*x+c)^9-2/65*sin(d*x+c)^4*cos(d*x+c)^9-8/715*sin(d*x+c)^2
*cos(d*x+c)^9-16/6435*cos(d*x+c)^9)-28*a^8*(-1/15*sin(d*x+c)^5*cos(d*x+c)^1
0-1/39*sin(d*x+c)^3*cos(d*x+c)^10-1/143*cos(d*x+c)^10*sin(d*x+c)+1/1287*(12
8/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*s
in(d*x+c))+56*I*a^8*(-1/15*sin(d*x+c)^4*cos(d*x+c)^11-4/195*sin(d*x+c)^2*co
s(d*x+c)^11-8/2145*cos(d*x+c)^11)+70*a^8*(-1/15*sin(d*x+c)^3*cos(d*x+c)^12-
1/65*sin(d*x+c)*cos(d*x+c)^12+1/715*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8
+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-56*
I*a^8*(-1/15*sin(d*x+c)^2*cos(d*x+c)^13-2/195*cos(d*x+c)^13)-28*a^8*(-1/15*
sin(d*x+c)*cos(d*x+c)^14+1/195*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)^10+
40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos(d*x
+c)^2)*sin(d*x+c))-8/15*I*a^8*cos(d*x+c)^15+1/15*a^8*(2048/429+cos(d*x+c)^1
4+14/13*cos(d*x+c)^12+168/143*cos(d*x+c)^10+560/429*cos(d*x+c)^8+640/429*co
s(d*x+c)^6+256/143*cos(d*x+c)^4+1024/429*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(180) = 360$.
time = 0.30, size = 453, normalized size = 2.14

$$-\frac{3432i\,a^8\cos(dx+c)^{15} + 8i\left(429\cos(dx+c)^{15} - 1485\cos(dx+c)^{13} + 1755\cos(dx+c)^{11} - 715\cos(dx+c)^9\right)a^8 + 168i\left(143\cos(dx+c)^{15} - 330\cos(dx+c)^{13} + 195\cos(dx+c)^{11}\right)a^8 + 1848i\left(13\cos(dx+c)^{15} - 15\cos(dx+c)^{13}\right)a^8 + 4\left(3003\sin(dx+c)^{15} - 13860\sin(dx+c)^{13} + 24570\sin(dx+c)^{11} - 20020\sin(dx+c)^9 + 6435\sin(dx+c)^7\right)a^8 + 10\left(3003\sin(dx+c)^{15} - 17325\sin(dx+c)^{13} + 40950\sin(dx+c)^{11} - 50050\sin(dx+c)^9 + 32175\sin(dx+c)^7 - 9009\sin(dx+c)^5\right)a^8 + 4\left(3003\sin(dx+c)^{15} - 20790\sin(dx+c)^{13} + 61425\sin(dx+c)^{11} - 100100\sin(dx+c)^9 + 96525\sin(dx+c)^7 - 54054\sin(dx+c)^5 + 15015\sin(dx+c)^3\right)a^8 + \left(429\sin(dx+c)^{15} - 1485\sin(dx+c)^{13} + 1755\sin(dx+c)^{11} - 715\sin(dx+c)^9\right)a^8 + \left(429\sin(dx+c)^{15} - 3465\sin(dx+c)^{13} + 12285\sin(dx+c)^{11} - 25025\sin(dx+c)^9 + 32175\sin(dx+c)^7 - 27027\sin(dx+c)^5 + 15015\sin(dx+c)^3 - 6435\sin(dx+c)\right)a^8}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/6435*(3432*I*a^8*cos(d*x + c)^15 + 8*I*(429*cos(d*x + c)^15 - 1485*cos(d
*x + c)^13 + 1755*cos(d*x + c)^11 - 715*cos(d*x + c)^9)*a^8 + 168*I*(143*co
s(d*x + c)^15 - 330*cos(d*x + c)^13 + 195*cos(d*x + c)^11)*a^8 + 1848*I*(13
*cos(d*x + c)^15 - 15*cos(d*x + c)^13)*a^8 + 4*(3003*sin(d*x + c)^15 - 1386
0*sin(d*x + c)^13 + 24570*sin(d*x + c)^11 - 20020*sin(d*x + c)^9 + 6435*sin
(d*x + c)^7)*a^8 + 10*(3003*sin(d*x + c)^15 - 17325*sin(d*x + c)^13 + 40950
*sin(d*x + c)^11 - 50050*sin(d*x + c)^9 + 32175*sin(d*x + c)^7 - 9009*sin(d
*x + c)^5)*a^8 + 4*(3003*sin(d*x + c)^15 - 20790*sin(d*x + c)^13 + 61425*si
n(d*x + c)^11 - 100100*sin(d*x + c)^9 + 96525*sin(d*x + c)^7 - 54054*sin(d*
x + c)^5 + 15015*sin(d*x + c)^3)*a^8 + (429*sin(d*x + c)^15 - 1485*sin(d*x
+ c)^13 + 1755*sin(d*x + c)^11 - 715*sin(d*x + c)^9)*a^8 + (429*sin(d*x + c
)^15 - 3465*sin(d*x + c)^13 + 12285*sin(d*x + c)^11 - 25025*sin(d*x + c)^9
+ 32175*sin(d*x + c)^7 - 27027*sin(d*x + c)^5 + 15015*sin(d*x + c)^3 - 6435
*sin(d*x + c))*a^8)/d
```

Fricas [**A**]
time = 0.43, size = 118, normalized size = 0.56

$$\frac{-429i\,a^8e^{(15i\,dx+15i\,c)} - 3465i\,a^8e^{(13i\,dx+13i\,c)} - 12285i\,a^8e^{(11i\,dx+11i\,c)} - 25025i\,a^8e^{(9i\,dx+9i\,c)} - 32175i\,a^8e^{(7i\,dx+7i\,c)} - 27027i\,a^8e^{(5i\,dx+5i\,c)} - 15015i\,a^8e^{(3i\,dx+3i\,c)} - 6435i\,a^8e^{(i\,dx+i\,c)}}{823680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(-429*I*a^8*e^(15*I*d*x + 15*I*c) - 3465*I*a^8*e^(13*I*d*x + 13*I*
c) - 12285*I*a^8*e^(11*I*d*x + 11*I*c) - 25025*I*a^8*e^(9*I*d*x + 9*I*c) -
32175*I*a^8*e^(7*I*d*x + 7*I*c) - 27027*I*a^8*e^(5*I*d*x + 5*I*c) - 15015*I
*a^8*e^(3*I*d*x + 3*I*c) - 6435*I*a^8*e^(I*d*x + I*c))/d
```

Sympy [A]
time = 0.94, size = 313, normalized size = 1.48

$$\begin{cases} \frac{-1086774885079842816 0ia^8d^7e^{15ic}e^{15idx} - 8777797148721807360 0ia^8d^7e^{13ic}e^{13idx} - 311212808000136806400ia^8d^7e^{11ic}e^{11idx} - 633952016296574976000ia^8d^7e^{9ic}e^{9idx} - 815081163809882112000ia^8d^7e^{7ic}e^{7idx} - 684668177600300974080ia^8d^7e^{5ic}e^{5idx} - 380371209777944985600ia^8d^7e^{3ic}e^{3idx} - 163016232761976422400ia^8d^7e^{ic}e^{idx}}{20866077793532982067200d^8} & \text{for}\: d^8 \neq 0 \\ x\left(\frac{a^8e^{15ic}}{128} + \frac{7a^8e^{13ic}}{128} + \frac{21a^8e^{11ic}}{128} + \frac{35a^8e^{9ic}}{128} + \frac{35a^8e^{7ic}}{128} + \frac{21a^8e^{5ic}}{128} + \frac{7a^8e^{3ic}}{128} + \frac{a^8e^{ic}}{128}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**15*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-10867748850798428160*I*a**8*d**7*exp(15*I*c)*exp(15*I*d*x) - 8
7777971487218073600*I*a**8*d**7*exp(13*I*c)*exp(13*I*d*x) - 311212808000136
806400*I*a**8*d**7*exp(11*I*c)*exp(11*I*d*x) - 633952016296574976000*I*a**8
*d**7*exp(9*I*c)*exp(9*I*d*x) - 815081163809882112000*I*a**8*d**7*exp(7*I*c
)*exp(7*I*d*x) - 684668177600300974080*I*a**8*d**7*exp(5*I*c)*exp(5*I*d*x)
- 380371209777944985600*I*a**8*d**7*exp(3*I*c)*exp(3*I*d*x) - 1630162327619
76422400*I*a**8*d**7*exp(I*c)*exp(I*d*x))/(20866077793532982067200*d**8), N
e(d**8, 0)), (x*(a**8*exp(15*I*c)/128 + 7*a**8*exp(13*I*c)/128 + 21*a**8*ex
p(11*I*c)/128 + 35*a**8*exp(9*I*c)/128 + 35*a**8*exp(7*I*c)/128 + 21*a**8*e
xp(5*I*c)/128 + 7*a**8*exp(3*I*c)/128 + a**8*exp(I*c)/128), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(180) = 360$.
time = 1.81, size = 2919, normalized size = 13.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/863691079680*(5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) +
1) + 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 568
7783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1137556
6892529840*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706335033
8794760*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706335033879
4760*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 113755668925298
40*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446264920*a^
8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2068284889550880*a^8*e^(
6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^8*e^(4*I*d*
```

x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19500971815765440*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 5682101344920*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378254079140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5679790198717635*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11359580397435270*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11359580397435270*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5679790198717635*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378254079140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516344563519785*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 79437625156890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 19473566395603320*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 5674116082635*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5687783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5687783446264920*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2068284889550880*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222387720*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549418828880*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 19500971815765440*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 5682101344920*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2065378254079140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5679790198717635*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397435270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 17039370596152905*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 17039370596152905*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397435270*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5679790198717635*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2065378254079140*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516344563519785*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 79437625156890*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19473566395603320*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 5674116082635*a^8*e^(-14*I*c)*log(-I*e^(I*d*x

```
+ I*c) - 1) - 7985262285*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 111793671990*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
726658867935*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 290663
5471740*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 799324754728
5*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8
*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(1
6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(12*I*d
*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7993247547285*a^8*e^(8*I*d*x - 6*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 2906635471740*a^8*e^(6*I*d*x - 8*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 726658867935*a^8*e^(4*I*...
```

Mupad [B]
time = 5.26, size = 222, normalized size = 1.05

$$\frac{2\,a^8\,\left(2\,\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\,\left(-\frac{44779\,\sin(c+d\,x)^2}{32}+\frac{\sin(c+d\,x)\,32175\mathrm{i}}{128}-\frac{26075\,\sin(2\,c+2\,d\,x)^2}{16}-\frac{\sin(2\,c+2\,d\,x)\,3575\mathrm{i}}{8}+\frac{114583\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64}-\frac{57925\,\sin(3\,c+3\,d\,x)^2}{32}+\frac{\sin(3\,c+3\,d\,x)\,84227\mathrm{i}}{128}+\frac{116585\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2}{64}+\frac{119315\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)^2}{64}+\frac{122285\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)^2}{64}-\sin\left(4\,c+4\,d\,x\right)\,754\mathrm{i}+\frac{\sin(5\,c+5\,d\,x)\,111527\mathrm{i}}{128}-\frac{\sin(6\,c+6\,d\,x)\,7187\mathrm{i}}{8}+\frac{\sin(7\,c+7\,d\,x)\,121427\mathrm{i}}{128}-952\right)}{6435\,d\,\left(-\sin\left(\frac{15\,c}{4}+\frac{15\,d\,x}{4}\right)^2\,2\mathrm{i}+\sin\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^15*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (2*a^8*(2*sin(c/4 + (d*x)/4)^2 - 1)*((sin(c + d*x)*32175i)/128 - (sin(2*c +
 2*d*x)*3575i)/8 + (sin(3*c + 3*d*x)*84227i)/128 - sin(4*c + 4*d*x)*754i +
(sin(5*c + 5*d*x)*111527i)/128 - (sin(6*c + 6*d*x)*7187i)/8 + (sin(7*c + 7*
d*x)*121427i)/128 - (26075*sin(2*c + 2*d*x)^2)/16 + (114583*sin(c/2 + (d*x)
/2)^2)/64 - (57925*sin(3*c + 3*d*x)^2)/32 + (116585*sin((3*c)/2 + (3*d*x)/2
)^2)/64 + (119315*sin((5*c)/2 + (5*d*x)/2)^2)/64 + (122285*sin((7*c)/2 + (7
*d*x)/2)^2)/64 - (44779*sin(c + d*x)^2)/32 - 952))/(6435*d*(sin((15*c)/2 +
(15*d*x)/2) - sin((15*c)/4 + (15*d*x)/4)^2*2i + 1i))
```

3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{8i(a-ia\tan(c+dx))^5}{5a^6d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}$$

```
[Out] 8/5*I*(a-I*a*tan(d*x+c))^5/a^6/d-2*I*(a-I*a*tan(d*x+c))^6/a^7/d+6/7*I*(a-I*
a*tan(d*x+c))^7/a^8/d-1/8*I*(a-I*a*tan(d*x+c))^8/a^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{8i(a-ia\tan(c+dx))^5}{5a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((8*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^6*d) - ((2*I)*(a - I*a*Tan[c + d*x]
)^6)/(a^7*d) + (((6*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^8*d) - ((I/8)*(a - I
*a*Tan[c + d*x])^8)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^3\,dx,x,ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a-x)^4-12a^2(a-x)^5+6a(a-x)^6-(a-x)^7\right)dx,x,ia\tan(c\right.}{a^9d}\\ &= \frac{8i(a-ia\tan(c+dx))^5}{5a^6d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{i}{}\end{aligned}$$

Mathematica [A]
time = 0.53, size = 71, normalized size = 0.66

$$\frac{\sec(c)\sec^8(c+dx)(-35i\cos(c)-35\sin(c)+56\sin(c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx))}{280ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^8*((-35*I)*Cos[c] - 35*Sin[c] + 56*Sin[c + 2*d*x] + 28
*Sin[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(280*a*d)
```

Maple [A]
time = 0.30, size = 93, normalized size = 0.87

method	result
risch	$\frac{32i\left(56\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{35da\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{i\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^6(dx+c)\right)}{2}-\frac{3i\left(\tan^5(dx+c)\right)}{5}-\frac{3\left(\tan^4(dx+c)\right)}{4}-i\left(\tan^3(dx+c)\right)-\right.}{da}$
default	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{i\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^6(dx+c)\right)}{2}-\frac{3i\left(\tan^5(dx+c)\right)}{5}-\frac{3\left(\tan^4(dx+c)\right)}{4}-i\left(\tan^3(dx+c)\right)-\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a*(-I*tan(d*x+c)-1/8*tan(d*x+c)^8-1/7*I*tan(d*x+c)^7-1/2*tan(d*x+c)^6-3
/5*I*tan(d*x+c)^5-3/4*tan(d*x+c)^4-I*tan(d*x+c)^3-1/2*tan(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.81

$$-\frac{35i\tan(dx+c)^8-40\tan(dx+c)^7+140i\tan(dx+c)^6-168\tan(dx+c)^5+210i\tan(dx+c)^4-280\tan(dx+c)^3+140i\tan(dx+c)^2-280\tan(dx+c)}{280\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(d*
x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Fricas [A]
time = 0.38, size = 146, normalized size = 1.36

$$-\frac{32\left(-56i\,e^{(6i\,dx+6i\,c)}-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-i\right)}{35\left(ade^{(16i\,dx+16i\,c)}+8\,ade^{(14i\,dx+14i\,c)}+28\,ade^{(12i\,dx+12i\,c)}+56\,ade^{(10i\,dx+10i\,c)}+70\,ade^{(8i\,dx+8i\,c)}+56\,ade^{(6i\,dx+6i\,c)}+28\,ade^{(4i\,dx+4i\,c)}+8\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -32/35*(-56*I*e^(6*I*d*x + 6*I*c) - 28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d
*x + 2*I*c) - I)/(a*d*e^(16*I*d*x + 16*I*c) + 8*a*d*e^(14*I*d*x + 14*I*c) +
 28*a*d*e^(12*I*d*x + 12*I*c) + 56*a*d*e^(10*I*d*x + 10*I*c) + 70*a*d*e^(8*
I*d*x + 8*I*c) + 56*a*d*e^(6*I*d*x + 6*I*c) + 28*a*d*e^(4*I*d*x + 4*I*c) +
8*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^{10}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**10/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.52, size = 87, normalized size = 0.81

$$-\frac{35i\tan(dx+c)^8-40\tan(dx+c)^7+140i\tan(dx+c)^6-168\tan(dx+c)^5+210i\tan(dx+c)^4-280\tan(dx+c)^3+140i\tan(dx+c)^2-280\tan(dx+c)}{280\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(d*
x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Mupad [B]
time = 3.53, size = 92, normalized size = 0.86

$$\frac{\cos(c+d\,x)^8\,35\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^7+64\,\sin(c+d\,x)\,\cos(c+d\,x)^5+48\,\sin(c+d\,x)\,\cos(c+d\,x)^3+40\,\sin(c+d\,x)\,\cos(c+d\,x)-35\mathrm{i}}{280\,a\,d\cos(c+d\,x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (40*cos(c + d*x)*sin(c + d*x) + 48*cos(c + d*x)^3*sin(c + d*x) + 64*cos(c +
 d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + cos(c + d*x)^8*35i
 - 35i)/(280*a*d*cos(c + d*x)^8)
```

3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{i(a-ia\tan(c+dx))^4}{a^5d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^6}{6a^7d}$$

```
[Out] I*(a-I*a*tan(d*x+c))^4/a^5/d-4/5*I*(a-I*a*tan(d*x+c))^5/a^6/d+1/6*I*(a-I*a*
tan(d*x+c))^6/a^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^6}{6a^7d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*(a - I*a*Tan[c + d*x])^4)/(a^5*d) - (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)
/(a^6*d) + ((I/6)*(a - I*a*Tan[c + d*x])^6)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{a^5d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^6}{6a^7d}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 60, normalized size = 0.75

$$\frac{\sec(c)\sec^6(c+dx)(-10i\cos(c)-10\sin(c)+15\sin(c+2dx)+6\sin(3c+4dx)+\sin(5c+6dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^6*((-10*I)*Cos[c] - 10*Sin[c] + 15*Sin[c + 2*d*x] + 6*
Sin[3*c + 4*d*x] + Sin[5*c + 6*d*x]))/(60*a*d)
```

Maple [A]
time = 0.25, size = 72, normalized size = 0.90

method	result	size
risch	$\frac{16i\left(15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15da\left(e^{2i(dx+c)}+1\right)^6}$	47
derivativedivides	$-\frac{i\left(\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{\left(\tan^4(dx+c)\right)}{2}+\frac{i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{2i\left(\tan^3(dx+c)\right)}{3}+i\tan(dx+c)\right)}{da}$	72
default	$-\frac{i\left(\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{\left(\tan^4(dx+c)\right)}{2}+\frac{i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{2i\left(\tan^3(dx+c)\right)}{3}+i\tan(dx+c)\right)}{da}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a*(1/6*tan(d*x+c)^6+1/2*tan(d*x+c)^4+1/5*I*tan(d*x+c)^5+1/2*tan(d*x+c)
^2+2/3*I*tan(d*x+c)^3+I*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.84

$$\frac{-5i\,\tan(dx+c)^6+6\,\tan(dx+c)^5-15i\,\tan(dx+c)^4+20\,\tan(dx+c)^3-15i\,\tan(dx+c)^2+30\,\tan(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(-5*I*tan(d*x + c)^6 + 6*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 + 20*tan
(d*x + c)^3 - 15*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a*d)
```

Fricas [A]
time = 0.37, size = 109, normalized size = 1.36

$$-\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(12
*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*c)
 + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I*d
*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^8(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**8/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.57, size = 67, normalized size = 0.84

$$-\frac{5i\tan(dx+c)^6-6\tan(dx+c)^5+15i\tan(dx+c)^4-20\tan(dx+c)^3+15i\tan(dx+c)^2-30\tan(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

Mupad [B]
time = 3.32, size = 114, normalized size = 1.42

$$\frac{\sin(c+d\,x)\,\left(30\cos(c+d\,x)^5-\cos(c+d\,x)^4\,\sin(c+d\,x)\,15\mathrm{i}+20\cos(c+d\,x)^3\sin(c+d\,x)^2-\cos(c+d\,x)^2\sin(c+d\,x)^3\,15\mathrm{i}+6\,\cos(c+d\,x)\,\sin(c+d\,x)^4-\sin(c+d\,x)^5\,5\mathrm{i}\right)}{30\,a\,d\cos(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)*
15i + 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c + d*x)^3
*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a*d*cos(c + d*x)^6)
```

3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)} dx$

Optimal. Leaf size=55

$$\frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

```
[Out] 2/3*I*(a-I*a*tan(d*x+c))^3/a^4/d-1/4*I*(a-I*a*tan(d*x+c))^4/a^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$\frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^3)/(a^4*d) - ((I/4)*(a - I*a*Tan[c + d*x]
)^4)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}\end{aligned}$$

Mathematica [**A**]
time = 0.24, size = 49, normalized size = 0.89

$$\frac{\sec(c)\sec^4(c+dx)(-3i\cos(c)-3\sin(c)+4\sin(c+2dx)+\sin(3c+4dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^4*((-3*I)*Cos[c] - 3*Sin[c] + 4*Sin[c + 2*d*x] + Sin[3
*c + 4*d*x]))/(12*a*d)
```

Maple [**A**]
time = 0.25, size = 51, normalized size = 0.93

method	result	size
risch	$\frac{4i\left(4\,\mathrm{e}^{2i(dx+c)}+1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	36
derivativedivides	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}\right)}{da}$	51
default	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}\right)}{da}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a*(-I*tan(d*x+c)-1/4*tan(d*x+c)^4-1/3*I*tan(d*x+c)^3-1/2*tan(d*x+c)^2)
```

Maxima [**A**]
time = 0.28, size = 47, normalized size = 0.85

$$\frac{-3i\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^3-6i\,\tan\left(dx+c\right)^2+12\,\tan\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(-3*I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 + 12*tan(
d*x + c))/(a*d)
```

Fricas [**A**]
time = 0.39, size = 72, normalized size = 1.31

$$-\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*I
*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^6(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**6/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.53, size = 47, normalized size = 0.85

$$-\frac{3i\,\tan(dx+c)^4-4\,\tan(dx+c)^3+6i\,\tan(dx+c)^2-12\,\tan(dx+c)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

Mupad [B]
time = 3.34, size = 77, normalized size = 1.40

$$\frac{\sin(c+d\,x)\,\left(12\cos(c+d\,x)^3-\cos(c+d\,x)^2\,\sin(c+d\,x)\,6\mathrm{i}+4\,\cos(c+d\,x)\,\sin(c+d\,x)^2-\sin(c+d\,x)^3\,3\mathrm{i}\right)}{12\,a\,d\cos(c+d\,x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 - cos(c + d*x)^2*sin(c + d*x)*
6i + 12*cos(c + d*x)^3 - sin(c + d*x)^3*3i))/(12*a*d*cos(c + d*x)^4)
```

3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}$$

```
[Out] tan(d*x+c)/a/d-1/2*I*tan(d*x+c)^2/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3568}

$$\frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 35, normalized size = 1.03

$$\frac{\sec(c+dx)(-i\sec(c+dx)+2\sec(c)\sin(dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

[Out] (Sec[c + d*x]*((-I)*Sec[c + d*x] + 2*Sec[c]*Sin[d*x]))/(2*a*d)

Maple [A]
time = 0.72, size = 30, normalized size = 0.88

method	result	size
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)^{2}}$	23
derivativedivides	$-\frac{i\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{da}$	30
default	$-\frac{i\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{da}$	30

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -I/d/a*(1/2*tan(d*x+c)^2+I*tan(d*x+c))

Maxima [A]
time = 0.29, size = 27, normalized size = 0.79

$$-\frac{i\tan\left(dx+c\right)^{2}-2\tan\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)

Fricas [A]
time = 0.36, size = 33, normalized size = 0.97

$$\frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^{4}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**4/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.50, size = 27, normalized size = 0.79

$$-\frac{i\tan\left(dx+c\right)^{2}-2\tan\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Mupad [B]
time = 3.33, size = 25, normalized size = 0.74

$$-\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] -(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[Out] x/a+I*ln(cos(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 31}

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,ia\tan(c+dx)\right)}{ad}\\ &= \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 31, normalized size = 1.35

$$\frac{2\text{ArcTan}(\tan(dx))+i\log\left(\cos^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*ArcTan[Tan[d*x]] + I*Log[Cos[c + d*x]^2])/(2*a*d)
```

Maple [A]
time = 0.36, size = 23, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
default	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
risch	$\frac{2x}{a}+\frac{2c}{da}+\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{da}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/a/d*ln(a+I*a*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.87

$$-\frac{i\log\left(i\,a\tan\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*tan(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.36, size = 26, normalized size = 1.13

$$\frac{2\,dx+i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^2\left(c+dx\right)}{\tan\left(c+dx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**2/(tan(c + d*x) - I), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.
time = 0.52, size = 57, normalized size = 2.48

$$-\frac{-\frac{i \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}+\frac{2i \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a}-\frac{i \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
- I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

Mupad [B]
time = 3.35, size = 19, normalized size = 0.83

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] -(log(tan(c + d*x) - 1i)*1i)/(a*d)
```

3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[Out] 1/2*x/a+1/2*I/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3560, 8}

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\tan(c+dx)}\,dx &= \frac{i}{2d(a+ia\tan(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 45, normalized size = 1.36

$$\frac{1-2idx+(-i+2dx)\tan(c+dx)}{4ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] (1 - (2*I)*d*x + (-I + 2*d*x)*Tan[c + d*x])/(4*a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.09, size = 48, normalized size = 1.45

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}$	26
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
norman	$\frac{\frac{x}{2a}+\frac{i}{2da}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{\tan(dx+c)}{2da}}{1+\tan^2(dx+c)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/4*I*ln(tan(d*x+c)-I)+1/2/(tan(d*x+c)-I)+1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 32, normalized size = 0.97

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.08, size = 60, normalized size = 1.82

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*
((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.
time = 0.47, size = 60, normalized size = 1.82

$$-\frac{\frac{i\log(\tan(dx+c)-i)}{a} - \frac{i\log(-i\tan(dx+c)+1)}{a} + \frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 3.36, size = 29, normalized size = 0.88

$$\frac{x}{2\,a} + \frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] x/(2*a) + 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3x}{8a}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia}{8d(a+ia\tan(c+dx))^2}+\frac{i}{4d(a+ia\tan(c+dx))}$$

```
[Out] 3/8*x/a-1/8*I/d/(a-I*a*tan(d*x+c))+1/8*I*a/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(
a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3568, 46, 212}

$$\frac{ia}{8d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{i}{4d(a+ia\tan(c+dx))}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) - (I/8)/(d*(a - I*a*Tan[c + d*x])) + ((I/8)*a)/(d*(a + I*a*Tan[
c + d*x])^2) + (I/4)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2} + \frac{1}{4a^2(a+x)^3} + \frac{1}{4a^3(a+x)^2} + \frac{3}{8a^3(a^2-x^2)}\right) dx, x, ia\tan(c+\right.}{d}$$

$$= -\frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia}{8d(a+ia\tan(c+dx))^2} + \frac{i}{4d(a+ia\tan(c+dx))} -$$

$$= \frac{3x}{8a} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia}{8d(a+ia\tan(c+dx))^2} + \frac{i}{4d(a+ia\tan(c+dx}$$

Mathematica [A]
time = 0.33, size = 78, normalized size = 0.95

$$-\frac{-7+12idx+2\cos(2(c+dx))+3i\sec(c+dx)\sin(3(c+dx))+6i\tan(c+dx)-12dx\tan(c+dx)}{32ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/32*(-7 + (12*I)*d*x + 2*Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c +
 d*x)] + (6*I)*Tan[c + d*x] - 12*d*x*Tan[c + d*x])/(a*d*(-I + Tan[c + d*x])
)
```

Maple [A]
time = 0.42, size = 75, normalized size = 0.91

method	result	size
risch	$\frac{3x}{8a}+\frac{ie^{-4i(dx+c)}}{32da}+\frac{i\cos(2dx+2c)}{8da}+\frac{\sin(2dx+2c)}{4da}$	61
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-3/16*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2+1/4/(tan(d*x+c)-I)+3
/16*I*ln(tan(d*x+c)+I)+1/8/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.35, size = 54, normalized size = 0.66

$$\frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A]
time = 0.16, size = 151, normalized size = 1.84

$$\begin{cases}\frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } a^3d^3e^{6ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise}\end{cases}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*exp(
4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(8*a)
```

Giac [A]
time = 0.54, size = 99, normalized size = 1.21

$$-\frac{\frac{6i\,\log\left(i\,\tan\left(dx+c\right)+1\right)}{a}-\frac{6i\,\log\left(i\,\tan\left(dx+c\right)-1\right)}{a}+\frac{2\,\left(3\,\tan\left(dx+c\right)+5i\right)}{a\left(-i\,\tan\left(dx+c\right)+1\right)}+\frac{-9i\,\tan\left(dx+c\right)^2-26\,\tan\left(dx+c\right)+21i}{a\left(\tan\left(dx+c\right)-i\right)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(d*x + c) + 1)/a - 6*I*log(I*tan(d*x + c) - 1)/a + 2*(3
*tan(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*
tan(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

Mupad [B]
time = 3.45, size = 60, normalized size = 0.73

$$\frac{3\,x}{8\,a}-\frac{\frac{3\,{\tan\left(c+d\,x\right)}^2}{8}-\frac{\tan\left(c+d\,x\right)\,3{}\mathrm{i}}{8}+\frac{1}{4}}{a\,d\,{\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (3*x)/(8*a) - ((3*tan(c + d*x)^2)/8 - (tan(c + d*x)*3i)/8 + 1/4)/(a*d*(tan(
c + d*x)*1i + 1)^2*(tan(c + d*x) + 1i))
```

3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{5x}{16a}-\frac{ia}{32d(a-ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia^2}{24d(a+ia\tan(c+dx))^3}+\frac{3ia}{32d(a+ia\tan(c+dx))}$$

```
[Out] 5/16*x/a-1/32*I*a/d/(a-I*a*tan(d*x+c))^2-1/8*I/d/(a-I*a*tan(d*x+c))+1/24*I*
a^2/d/(a+I*a*tan(d*x+c))^3+3/32*I*a/d/(a+I*a*tan(d*x+c))^2+3/16*I/d/(a+I*a*
tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$\frac{ia^2}{24d(a+ia\tan(c+dx))^3}-\frac{ia}{32d(a-ia\tan(c+dx))^2}+\frac{3ia}{32d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{3i}{16d(a+ia\tan(c+dx))}+\frac{5x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*x)/(16*a) - ((I/32)*a)/(d*(a - I*a*Tan[c + d*x])^2) - (I/8)/(d*(a - I*a*
Tan[c + d*x])) + ((I/24)*a^2)/(d*(a + I*a*Tan[c + d*x])^3) + (((3*I)/32)*a)
/(d*(a + I*a*Tan[c + d*x])^2) + ((3*I)/16)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\mathrm{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \frac{1}{8a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{3}{16a^5(a+x)^2} + \frac{}{16a}\right.\right.}{d}$$

$$= -\frac{ia}{32d(a-ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia^2}{24d(a+ia\tan(c+dx))}$$

$$= \frac{5x}{16a} - \frac{ia}{32d(a-ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia^2}{24d(a+ia\tan(c+}$$

Mathematica [A]

time = 0.29, size = 109, normalized size = 0.81

$$-\frac{\sec(c+dx)(60i(i+2dx)\cos(c+dx)+15\cos(3(c+dx))+\cos(5(c+dx))+60i\sin(c+dx)-120dx\sin(c+dx)+45i\sin(3(c+dx))+5i\sin(5(c+dx)))}{384ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/384*(Sec[c + d*x]*((60*I)*(I + 2*d*x)*Cos[c + d*x] + 15*Cos[3*(c + d*x)]
 + Cos[5*(c + d*x)] + (60*I)*Sin[c + d*x] - 120*d*x*Sin[c + d*x] + (45*I)*S
in[3*(c + d*x)] + (5*I)*Sin[5*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Maple [A]

time = 0.73, size = 102, normalized size = 0.76

method	result
risch	$\frac{5x}{16a}+\frac{i e^{-6i(dx+c)}}{192da}+\frac{i\cos(4dx+4c)}{32da}+\frac{3\sin(4dx+4c)}{64da}+\frac{5i\cos(2dx+2c)}{64da}+\frac{15\sin(2dx+2c)}{64da}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{}{8\tan}}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{}{8\tan}}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-5/32*I*ln(tan(d*x+c)-I)-3/32*I/(tan(d*x+c)-I)^2-1/24/(tan(d*x+c)-I)
^3+3/16/(tan(d*x+c)-I)+1/32*I/(tan(d*x+c)+I)^2+5/32*I*ln(tan(d*x+c)+I)+1/8/
(tan(d*x+c)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.57

$$\frac{\left(120\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(10i\,dx+10i\,c)}-30i\,e^{(8i\,dx+8i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8*
I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)
*e^(-6*I*d*x - 6*I*c)/(a*d)
```

Sympy [A]
time = 0.22, size = 219, normalized size = 1.63

$$\begin{cases}\frac{\left(-50331648ia^{4}d^{4}e^{16ic}e^{4idx}-503316480ia^{4}d^{4}e^{14ic}e^{2idx}+1006632960ia^{4}d^{4}e^{10ic}e^{-2idx}+251658240ia^{4}d^{4}e^{8ic}e^{-4idx}+33554432ia^{4}d^{4}e^{6ic}e^{-6idx}\right)e^{-12ic}}{6442450944a^{5}d^{5}} & \text{for}\ a^{5}d^{5}e^{12ic}\neq 0\\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-6ic}}{32a}-\frac{5}{16a}\right) & \text{otherwise}\end{cases}+\frac{5x}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(a**
5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) +
10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), True)) +
5*x/(16*a)
```

Giac [A]
time = 0.51, size = 116, normalized size = 0.87

$$-\frac{-\frac{30i\,\log\left(\tan\left(dx+c\right)+i\right)}{a}+\frac{30i\,\log\left(\tan\left(dx+c\right)-i\right)}{a}+\frac{3\left(-15i\,\tan\left(dx+c\right)^{2}+38\,\tan\left(dx+c\right)+25i\right)}{a\left(-i\,\tan\left(dx+c\right)+1\right)^{2}}-\frac{55i\,\tan\left(dx+c\right)^{3}+201\,\tan\left(dx+c\right)^{2}-255i\,\tan\left(dx+c\right)-117}{a\left(\tan\left(dx+c\right)-i\right)^{3}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
```

```
(55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.72, size = 123, normalized size = 0.92

$$\frac{5\,x}{16\,a}+\frac{\frac{25\,\tan\left(c+d\,x\right)}{48\,a}+\frac{1{}\mathrm{i}}{6\,a}+\frac{{\tan\left(c+d\,x\right)}^2\,25{}\mathrm{i}}{48\,a}+\frac{5\,{\tan\left(c+d\,x\right)}^3}{16\,a}+\frac{{\tan\left(c+d\,x\right)}^4\,5{}\mathrm{i}}{16\,a}}{d\,\left({\tan\left(c+d\,x\right)}^5\,1{}\mathrm{i}+{\tan\left(c+d\,x\right)}^4+{\tan\left(c+d\,x\right)}^3\,2{}\mathrm{i}+2\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,1{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (5*x)/(16*a) + ((25*tan(c + d*x))/(48*a) + 1i/(6*a) + (tan(c + d*x)^2*25i)/
(48*a) + (5*tan(c + d*x)^3)/(16*a) + (tan(c + d*x)^4*5i)/(16*a))/(d*(tan(c
+ d*x)*1i + 2*tan(c + d*x)^2 + tan(c + d*x)^3*2i + tan(c + d*x)^4 + tan(c +
 d*x)^5*1i + 1))
```

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+c
)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3582, 3853, 3855}

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\sec^5(c+dx)}{5ad} + \frac{\int \sec^5(c+dx)\,dx}{a}$$
$$= -\frac{i\sec^5(c+dx)}{5ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{3\int \sec^3(c+dx)\,dx}{4a}$$
$$= -\frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{3\int s$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+$$

Mathematica [A]
time = 0.43, size = 60, normalized size = 0.71

$$\frac{240\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(-64i+70\sin(2(c+dx))+15\sin(4(c+dx)))}{320ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (240*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(-64*I + 70*Sin
[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(320*a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(75) = 150.
time = 0.30, size = 206, normalized size = 2.45

method	result
risch	$-\frac{i\left(15\,e^{9i(dx+c)}+70\,e^{7i(dx+c)}+128\,e^{5i(dx+c)}-70\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{20da\left(e^{2i(dx+c)}+1\right)^5}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{da}$
default	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/10*I/(tan(1/2*d*x+1/2*c)-1)^5+(7/16+5/16*I)/(tan(1/2*d*x+1/2*c)-1)
^2+(5/16+3/16*I)/(tan(1/2*d*x+1/2*c)-1)+(1/4+3/8*I)/(tan(1/2*d*x+1/2*c)-1)^
3+(1/8+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^4-3/16*ln(tan(1/2*d*x+1/2*c)-1)-1/10*I
/(tan(1/2*d*x+1/2*c)+1)^5+(5/16-3/16*I)/(tan(1/2*d*x+1/2*c)+1)+(1/4-3/8*I)/
```

```
(tan(1/2*d*x+1/2*c)+1)^3+(-1/8+1/4*I)/(tan(1/2*d*x+1/2*c)+1)^4+(-7/16+5/16*
I)/(tan(1/2*d*x+1/2*c)+1)^2+3/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(74) = 148$.
time = 0.30, size = 288, normalized size = 3.43

$$-\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200*I*
a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.
time = 0.40, size = 266, normalized size = 3.17

$$\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(9i\,dx+9i\,c)}-140i\,e^{(7i\,dx+7i\,c)}-256i\,e^{(5i\,dx+5i\,c)}+140i\,e^{(3i\,dx+3i\,c)}+30i\,e^{(i\,dx+i\,c)}}{40\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^7(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**7/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.58, size = 138, normalized size = 1.64

$$\frac{\frac{15 \log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{15 \log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(25 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+40i \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-10 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+80i \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+10 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-25 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1)
/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(1/
2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^3
- 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

Mupad [B]
time = 7.01, size = 193, normalized size = 2.30

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2\,a}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4\,a}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}4\mathrm{i}}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}2\mathrm{i}}{a}+\frac{2\mathrm{i}}{5\,a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan(
c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)^8
*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/2))
/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
 (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)} dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+c
)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3582, 3853, 3855}

$$-\frac{i\sec^3(c+dx)}{3ad} + \frac{\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\sec^3(c+dx)}{3ad} + \frac{\int \sec^3(c+dx)\,dx}{a}$$
$$= -\frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \sec(c+dx)\,dx}{2a}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

Mathematica [A]
time = 0.28, size = 50, normalized size = 0.83

$$\frac{12\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-4i+3\sin(2(c+dx)))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-4*I + 3*Sin[2*
(c + d*x)]))/(12*a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(53) = 106$.
time = 0.30, size = 138, normalized size = 2.30

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{da}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/6*I/(tan(1/2*d*x+1/2*c)-1)^3+(1/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^2+
(1/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)-1/4*ln(tan(1/2*d*x+1/2*c)-1)-1/6*I/(tan(
1/2*d*x+1/2*c)+1)^3+(1/4-1/4*I)/(tan(1/2*d*x+1/2*c)+1)+(-1/4+1/4*I)/(tan(1/
2*d*x+1/2*c)+1)^2+1/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.

time = 0.29, size = 186, normalized size = 3.10

$$\frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6i\,a-\frac{18i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(52) = 104.
time = 0.35, size = 174, normalized size = 2.90

$$\frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(5i\,dx+5i\,c)}-16i\,e^{(3i\,dx+3i\,c)}+6i\,e^{(i\,dx+i\,c)}}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sec^5(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**5/(tan(c + d*x) - I), x)/a
```

Giac [A]
time = 0.55, size = 99, normalized size = 1.65

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
+ 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x
+ 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B]
time = 5.38, size = 116, normalized size = 1.93

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,2{}\mathrm{i}}{a}+\frac{2{}\mathrm{i}}{3\,a}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
(d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

```
[Out] arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3582, 3855}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\sec(c+dx)}{ad}+\frac{\int \sec(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 34, normalized size = 1.10

$$\frac{2\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
[Out] (2*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x])/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.
time = 0.53, size = 70, normalized size = 2.26

method	result	size
derivativedivides	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	70
default	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	70
risch	$-\frac{2i e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/a*(1/2*I/(tan(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2*I/(tan
(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.
time = 0.30, size = 83, normalized size = 2.68

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.
time = 0.37, size = 80, normalized size = 2.58

$$\frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
+ 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\sec^3(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**3/(tan(c + d*x) - I), x)/a
```

Giac [**A**]
time = 0.56, size = 58, normalized size = 1.87

$$\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/((
tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [**B**]
time = 3.44, size = 43, normalized size = 1.39

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\mathrm{i}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

```
[Out] I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3569\}$

$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] (I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.89

$$\frac{\sec(c+dx)}{ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] Sec[c + d*x]/(a*d*(-I + Tan[c + d*x]))
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.82

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{da}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a/(-I+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 29, normalized size = 1.04

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A]
time = 0.43, size = 17, normalized size = 0.61

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A]
time = 0.35, size = 34, normalized size = 1.21

$$\begin{cases} \frac{\sec\left(c+dx\right)}{ad\tan\left(c+dx\right)-iad} & \text{for}\ d\neq 0\\ \frac{x\sec\left(c\right)}{ia\tan\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((sec(c + d*x)/(a*d*tan(c + d*x) - I*a*d), Ne(d, 0)), (x*sec(c)/(I
*a*tan(c) + a), True))
```

Giac [A]
time = 0.54, size = 21, normalized size = 0.75

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

Mupad [B]
time = 3.35, size = 25, normalized size = 0.89

$$\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

```
[Out] 2/3*sin(d*x+c)/a/d+1/3*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3583, 2717}

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] (2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}+\frac{2\int \cos(c+dx)\,dx}{3a}\\ &= \frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 50, normalized size = 1.06

$$-\frac{\sec(c+dx)(-3+\cos(2(c+dx))+2i\sin(2(c+dx)))}{6ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/6*(Sec[c + d*x]*(-3 + Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(a*d*(
-I + Tan[c + d*x]))
```

Maple [A]
time = 0.35, size = 75, normalized size = 1.60

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{12da}+\frac{i\cos(dx+c)}{4da}+\frac{3\sin(dx+c)}{4da}$
derivativedivides	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{da}$
default	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{da}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{2\tan(dx+c)}{3da}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}(dx+c)\right)}{3ad}-\frac{2i\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{2i\left(\tan^{2}(dx+c)\right)}{3da}-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan(dx+c)}{3da}-}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(1+\tan^{2}(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/3/(-I+tan(1/2*d*x+1/2*c))^3+1/2*I/(-I+tan(1/2*d*x+1/2*c))^2+3/4/(
-I+tan(1/2*d*x+1/2*c))+1/4/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.34, size = 41, normalized size = 0.87

$$\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(36) = 72$.
time = 0.15, size = 126, normalized size = 2.68

$$\begin{cases} \frac{(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx})e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic} \neq 0 \\ \frac{x(e^{4ic}+2e^{2ic}+1)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)*ex
p(-3*I*c)/(4*a), True))
```

Giac [A]
time = 0.49, size = 67, normalized size = 1.43

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

Mupad [B]
time = 3.55, size = 78, normalized size = 1.66

$$\frac{\left(-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] ((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 + 1
i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

```
[Out] 4/5*sin(d*x+c)/a/d-4/15*sin(d*x+c)^3/a/d+1/5*I*cos(d*x+c)^3/d/(a+I*a*tan(d*
x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 2713}

$$-\frac{4\sin^3(c+dx)}{15ad}+\frac{4\sin(c+dx)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (4*Sin[c + d*x])/(5*a*d) - (4*Sin[c + d*x]^3)/(15*a*d) + ((I/5)*Cos[c + d*x
]^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))} + \frac{4\int \cos^3(c+dx)\,dx}{5a}$$
$$= \frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))} - \frac{4\text{Subst}\left(\int (1-x^2)\ dx, x, -\sin(c+dx)\right)}{5ad}$$
$$= \frac{4\sin(c+dx)}{5ad} - \frac{4\sin^3(c+dx)}{15ad} + \frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

Mathematica [A]
time = 0.21, size = 72, normalized size = 1.07

$$-\frac{\sec(c+dx)(-45+20\cos(2(c+dx))+\cos(4(c+dx))+40i\sin(2(c+dx))+4i\sin(4(c+dx)))}{120ad(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]*(-45 + 20*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + (40*I)
*Sin[2*(c + d*x)] + (4*I)*Sin[4*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(59) = 118.
time = 0.60, size = 141, normalized size = 2.10

method	result
risch	$\frac{i e^{-5i(dx+c)}}{80da} + \frac{i\cos(dx+c)}{8da} + \frac{5\sin(dx+c)}{8da} + \frac{i\cos(3dx+3c)}{16da} + \frac{5\sin(3dx+3c)}{48da}$
derivativedivides	$\frac{-\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{3i}{2(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}+\frac{2}{5(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{5}{3(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{11}{8(-i+\tan(\frac{dx}{2}+\frac{c}{2}))}-\frac{i}{4(\tan(\frac{dx}{2}}}{da}$
default	$\frac{-\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{3i}{2(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}+\frac{2}{5(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{5}{3(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{11}{8(-i+\tan(\frac{dx}{2}+\frac{c}{2}))}-\frac{i}{4(\tan(\frac{dx}{2}}}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/2*I/(-I+tan(1/2*d*x+1/2*c))^4+3/4*I/(-I+tan(1/2*d*x+1/2*c))^2+1/5
/(-I+tan(1/2*d*x+1/2*c))^5-5/6/(-I+tan(1/2*d*x+1/2*c))^3+11/16/(-I+tan(1/2*
d*x+1/2*c))-1/8*I/(tan(1/2*d*x+1/2*c)+I)^2-1/12/(tan(1/2*d*x+1/2*c)+I)^3+5/
16/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.35, size = 63, normalized size = 0.94

$$\frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(53) = 106$.
time = 0.24, size = 196, normalized size = 2.93

$$\begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for}\ a^5d^5e^{9ic}\neq 0 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)), (
x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5*I*c)
/(16*a), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.
time = 0.52, size = 119, normalized size = 1.78

$$\frac{\frac{5\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^3}+\frac{165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-650\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+400i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+113}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B]
time = 4.79, size = 134, normalized size = 2.00

$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,15{}\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,25{}\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,21{}\mathrm{i}+9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3{}\mathrm{i}\right)\,2{}\mathrm{i}}{15\,a\,d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}^3\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) + 1i)
^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{6\sin(c+dx)}{7ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin^5(c+dx)}{35ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

```
[Out] 6/7*sin(d*x+c)/a/d-4/7*sin(d*x+c)^3/a/d+6/35*sin(d*x+c)^5/a/d+1/7*I*cos(d*x
+c)^5/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3583, 2713}

$$\frac{6\sin^5(c+dx)}{35ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin(c+dx)}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (6*Sin[c + d*x])/(7*a*d) - (4*Sin[c + d*x]^3)/(7*a*d) + (6*Sin[c + d*x]^5)/
(35*a*d) + ((I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}+\frac{6\int\cos^5(c+dx)\,dx}{7a} \\
&= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}-\frac{6\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7ad} \\
&= \frac{6\sin(c+dx)}{7ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin^5(c+dx)}{35ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 94, normalized size = 1.11

$$
-\frac{\sec(c+dx)(-350+175\cos(2(c+dx))+14\cos(4(c+dx))+\cos(6(c+dx))+350i\sin(2(c+dx))+56i\sin(4(c+dx))+6i\sin(6(c+dx)))}{1120ad(-i+\tan(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/1120*(Sec[c + d*x]*(-350 + 175*Cos[2*(c + d*x)] + 14*Cos[4*(c + d*x)] +
Cos[6*(c + d*x)] + (350*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)] + (6*
I)*Sin[6*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(75) = 150.
time = 0.30, size = 207, normalized size = 2.44

method	result
risch	$\frac{i e^{-7i(dx+c)}}{448da}+\frac{5i\cos(dx+c)}{64da}+\frac{35\sin(dx+c)}{64da}+\frac{i\cos(5dx+5c)}{64da}+\frac{7\sin(5dx+5c)}{320da}+\frac{3i\cos(3dx+3c)}{64da}+\frac{7\sin(3dx+3c)}{64da}$
derivativedivides	$-\frac{2}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{15i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{11i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{21}{10\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-$
default	$-\frac{2}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{15i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{11i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{21}{10\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/7/(-I+tan(1/2*d*x+1/2*c))^7+1/2*I/(-I+tan(1/2*d*x+1/2*c))^6+15/16
*I/(-I+tan(1/2*d*x+1/2*c))^2-11/8*I/(-I+tan(1/2*d*x+1/2*c))^4+21/20/(-I+tan
(1/2*d*x+1/2*c))^5-11/8/(-I+tan(1/2*d*x+1/2*c))^3+21/32/(-I+tan(1/2*d*x+1/2
*c))+1/8*I/(tan(1/2*d*x+1/2*c)+I)^4-1/4*I/(tan(1/2*d*x+1/2*c)+I)^2+1/20/(ta
n(1/2*d*x+1/2*c)+I)^5-1/4/(tan(1/2*d*x+1/2*c)+I)^3+11/32/(tan(1/2*d*x+1/2*c
)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 85, normalized size = 1.00

$$\frac{\left(-7i\,e^{(12i\,dx+12i\,c)}-70i\,e^{(10i\,dx+10i\,c)}-525i\,e^{(8i\,dx+8i\,c)}+700i\,e^{(6i\,dx+6i\,c)}+175i\,e^{(4i\,dx+4i\,c)}+42i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{2240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2240*(-7*I*e^(12*I*d*x + 12*I*c) - 70*I*e^(10*I*d*x + 10*I*c) - 525*I*e^(
8*I*d*x + 8*I*c) + 700*I*e^(6*I*d*x + 6*I*c) + 175*I*e^(4*I*d*x + 4*I*c) +
42*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(68) = 136$.
time = 0.33, size = 264, normalized size = 3.11

$$\begin{cases} \frac{\left(-150323855360ia^6d^6e^{21ic}e^{5idx}-1503238553600ia^6d^6e^{19ic}e^{3idx}-11274289152000ia^6d^6e^{17ic}e^{idx}+15032385536000ia^6d^6e^{15ic}e^{-idx}+3758096384000ia^6d^6e^{13ic}e^{-3idx}+901943132160ia^6d^6e^{11ic}e^{-5idx}+107374182400ia^6d^6e^{9ic}e^{-7idx}\right)e^{-16ic}}{48103633715200a^7d^7} & \text{for } a^7d^7e^{16ic}\neq 0 \\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-7ic}}{64a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-150323855360*I*a**6*d**6*exp(21*I*c)*exp(5*I*d*x) - 1503238553
600*I*a**6*d**6*exp(19*I*c)*exp(3*I*d*x) - 11274289152000*I*a**6*d**6*exp(1
7*I*c)*exp(I*d*x) + 15032385536000*I*a**6*d**6*exp(15*I*c)*exp(-I*d*x) + 37
58096384000*I*a**6*d**6*exp(13*I*c)*exp(-3*I*d*x) + 901943132160*I*a**6*d**
6*exp(11*I*c)*exp(-5*I*d*x) + 107374182400*I*a**6*d**6*exp(9*I*c)*exp(-7*I*
d*x))*exp(-16*I*c)/(48103633715200*a**7*d**7), Ne(a**7*d**7*exp(16*I*c), 0)
), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp
(4*I*c) + 6*exp(2*I*c) + 1)*exp(-7*I*c)/(64*a), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(73) = 146$.
time = 0.50, size = 171, normalized size = 2.01

$$\frac{\frac{7\left(55\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+180i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-250\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-160i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+43\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^5}+\frac{735\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3360i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-7315\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8820i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6321\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2492i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-461}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/560*(7*(55*tan(1/2*d*x + 1/2*c)^4 + 180*I*tan(1/2*d*x + 1/2*c)^3 - 250*ta
n(1/2*d*x + 1/2*c)^2 - 160*I*tan(1/2*d*x + 1/2*c) + 43)/(a*(tan(1/2*d*x + 1
/2*c) + I)^5) + (735*tan(1/2*d*x + 1/2*c)^6 - 3360*I*tan(1/2*d*x + 1/2*c)^5
 - 7315*tan(1/2*d*x + 1/2*c)^4 + 8820*I*tan(1/2*d*x + 1/2*c)^3 + 6321*tan(1
/2*d*x + 1/2*c)^2 - 2492*I*tan(1/2*d*x + 1/2*c) - 461)/(a*(tan(1/2*d*x + 1/
2*c) - I)^7))/d
```

Mupad [B]
time = 7.10, size = 188, normalized size = 2.21

$$\frac{\left(-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}105\mathrm{i}-126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}182\mathrm{i}+26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}130\mathrm{i}-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}55\mathrm{i}+25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+5\mathrm{i}\right)\,2\mathrm{i}}{35\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^{5}\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] ((25*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*55i - 15*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*130i + 26*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*182i - 126*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*105i - 35*tan(c/
2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10*35i - 35*tan(c/2 + (d*x)/2)^11 + 5i)
*2i)/(35*a*d*(tan(c/2 + (d*x)/2) + 1i)^5*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{4i(a-ia\tan(c+dx))^5}{5a^7d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{i(a-ia\tan(c+dx))^7}{7a^9d}$$

```
[Out] 4/5*I*(a-I*a*tan(d*x+c))^5/a^7/d-2/3*I*(a-I*a*tan(d*x+c))^6/a^8/d+1/7*I*(a-
I*a*tan(d*x+c))^7/a^9/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^7}{7a^9d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{4i(a-ia\tan(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^7*d) - (((2*I)/3)*(a - I*a*Tan[c +
d*x])^6)/(a^8*d) + ((I/7)*(a - I*a*Tan[c + d*x])^7)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx&=-\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^9d}\\&=-\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^4-4a(a-x)^5+(a-x)^6\right)\,dx,x,ia\tan(c+dx)\right)}{a^9d}\\&=\frac{4i(a-ia\tan(c+dx))^5}{5a^7d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{i(a-ia\tan(c+dx))^7}{7a^9d}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 90, normalized size = 1.10

$$\frac{\sec(c)\sec^7(c+dx)(-35i\cos(dx)-35i\cos(2c+dx)+35\sin(dx)-35\sin(2c+dx)+42\sin(2c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx))}{210a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*((-35*I)*Cos[d*x] - (35*I)*Cos[2*c + d*x] + 35*Sin[d
*x] - 35*Sin[2*c + d*x] + 42*Sin[2*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*Sin
[6*c + 7*d*x]))/(210*a^2*d)
```

Maple [A]
time = 0.29, size = 78, normalized size = 0.95

method	result	size
risch	$\frac{128i\left(21\,{\rm e}^{4i(dx+c)}+7\,{\rm e}^{2i(dx+c)}+1\right)}{105d\,a^2\left({\rm e}^{2i(dx+c)}+1\right)^7}$	47
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{i\left(\tan^6(dx+c)\right)}{3}-\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)+\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)}{d\,a^2}$	78
default	$\frac{\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{i\left(\tan^6(dx+c)\right)}{3}-\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)+\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)}{d\,a^2}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/7*tan(d*x+c)^7-1/3*I*tan(d*x+c)^6-1/5*tan(d*x+c)^5-I*
tan(d*x+c)^4+1/3*tan(d*x+c)^3-I*tan(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.94

$$-\frac{15\,\tan\left(dx+c\right)^7+35i\,\tan\left(dx+c\right)^6+21\,\tan\left(dx+c\right)^5+105i\,\tan\left(dx+c\right)^4-35\,\tan\left(dx+c\right)^3+105i\,\tan\left(dx+c\right)^2-105\,\tan\left(dx+c\right)}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I
*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x +
c))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(64) = 128.
time = 0.35, size = 138, normalized size = 1.68

$$-\frac{128\left(-21i\,e^{(4i\,dx+4i\,c)}-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{105\left(a^2de^{(14i\,dx+14i\,c)}+7\,a^2de^{(12i\,dx+12i\,c)}+21\,a^2de^{(10i\,dx+10i\,c)}+35\,a^2de^{(8i\,dx+8i\,c)}+35\,a^2de^{(6i\,dx+6i\,c)}+21\,a^2de^{(4i\,dx+4i\,c)}+7\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -128/105*(-21*I*e^(4*I*d*x + 4*I*c) - 7*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e
^(14*I*d*x + 14*I*c) + 7*a^2*d*e^(12*I*d*x + 12*I*c) + 21*a^2*d*e^(10*I*d*x
 + 10*I*c) + 35*a^2*d*e^(8*I*d*x + 8*I*c) + 35*a^2*d*e^(6*I*d*x + 6*I*c) +
21*a^2*d*e^(4*I*d*x + 4*I*c) + 7*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^{10}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**10/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Giac [A]
time = 0.57, size = 77, normalized size = 0.94

$$-\frac{15\tan(dx+c)^7+35i\tan(dx+c)^6+21\tan(dx+c)^5+105i\tan(dx+c)^4-35\tan(dx+c)^3+105i\tan(dx+c)^2-105\tan(dx+c)}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I
*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x +
c))/(a^2*d)
```

Mupad [B]
time = 3.46, size = 93, normalized size = 1.13

$$\frac{\cos(c+d\,x)^7\,35\mathrm{i}+64\sin(c+d\,x)\cos(c+d\,x)^6+32\sin(c+d\,x)\cos(c+d\,x)^4+24\sin(c+d\,x)\cos(c+d\,x)^2-\cos(c+d\,x)\,35\mathrm{i}-15\sin(c+d\,x)}{105\,a^2\,d\cos(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (24*cos(c + d*x)^2*sin(c + d*x) - 15*sin(c + d*x) - cos(c + d*x)*35i + 32*c
os(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + cos(c + d*x)^
7*35i)/(105*a^2*d*cos(c + d*x)^7)
```

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[Out] 1/2*I*(a-I*a*tan(d*x+c))^4/a^6/d-1/5*I*(a-I*a*tan(d*x+c))^5/a^7/d
```

Rubi [A]

time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2)*(a - I*a*Tan[c + d*x])^4)/(a^6*d) - ((I/5)*(a - I*a*Tan[c + d*x])^5)
/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\
&= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\
&= \frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 77, normalized size = 1.40

$$\frac{\sec(c)\sec^5(c+dx)(-5i\cos(dx)-5i\cos(2c+dx)+5\sin(dx)-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx))}{20a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*((-5*I)*Cos[d*x] - (5*I)*Cos[2*c + d*x] + 5*Sin[d*x]
 - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*a^2*d)
```

Maple [A]
time = 0.26, size = 47, normalized size = 0.85

method	result	size
risch	$\frac{8i\left(5\,{\mathrm e}^{2i(dx+c)}+1\right)}{5d\,a^{2}\left({\mathrm e}^{2i(dx+c)}+1\right)^{5}}$	36
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^{5}(dx+c)\right)}{5}-\frac{i\left(\tan^{4}(dx+c)\right)}{2}-i\left(\tan^{2}(dx+c)\right)}{d\,a^{2}}$	47
default	$\frac{\tan(dx+c)-\frac{\left(\tan^{5}(dx+c)\right)}{5}-\frac{i\left(\tan^{4}(dx+c)\right)}{2}-i\left(\tan^{2}(dx+c)\right)}{d\,a^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-I*tan(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.85

$$-\frac{2\,\tan\left(dx+c\right)^{5}+5i\,\tan\left(dx+c\right)^{4}+10i\,\tan\left(dx+c\right)^{2}-10\,\tan\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.
time = 0.37, size = 97, normalized size = 1.76

$$-\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^{2}de^{(10i\,dx+10i\,c)}+5\,a^{2}de^{(8i\,dx+8i\,c)}+10\,a^{2}de^{(6i\,dx+6i\,c)}+10\,a^{2}de^{(4i\,dx+4i\,c)}+5\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*
e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^{8}(c+dx)}{\tan^{2}(c+dx)-2i\tan(c+dx)-1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**8/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [A]
time = 0.57, size = 47, normalized size = 0.85

$$-\frac{2\tan(dx+c)^{5}+5i\tan(dx+c)^{4}+10i\tan(dx+c)^{2}-10\tan(dx+c)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Mupad [B]
time = 3.36, size = 77, normalized size = 1.40

$$-\frac{\sin(c+d\,x)\,\left(-10\cos(c+d\,x)^{4}+\cos(c+d\,x)^{3}\,\sin(c+d\,x)\,10\mathrm{i}+\cos(c+d\,x)\,\sin(c+d\,x)^{3}\,5\mathrm{i}+2\sin(c+d\,x)^{4}\right)}{10\,a^{2}\,d\cos(c+d\,x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] -(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i + cos(c + d*x)^3*sin(c + d*x
)*10i - 10*cos(c + d*x)^4 + 2*sin(c + d*x)^4))/(10*a^2*d*cos(c + d*x)^5)
```

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

```
[Out] 1/3*I*(a-I*a*tan(d*x+c))^3/a^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{i(a-ia\tan(c+dx))^3}{3a^5d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 0.29, size = 68, normalized size = 2.52

$$\frac{\sec(c)\sec^3(c+dx)(-3i\cos(dx)-3i\cos(2c+dx)+3\sin(dx)-3\sin(2c+dx)+2\sin(2c+3dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^3*((-3*I)*Cos[d*x] - (3*I)*Cos[2*c + d*x] + 3*Sin[d*x]
 - 3*Sin[2*c + d*x] + 2*Sin[2*c + 3*d*x]))/(6*a^2*d)
```

Maple [A]
time = 0.25, size = 20, normalized size = 0.74

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
risch	$\frac{8i}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d/a^2*(tan(d*x+c)+I)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 1.30

$$-\frac{\tan\left(dx+c\right)^3+3i\,\tan\left(dx+c\right)^2-3\,\tan\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.
time = 0.36, size = 54, normalized size = 2.00

$$\frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^6(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sec(c + d*x)**6/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [A]
time = 0.58, size = 35, normalized size = 1.30

$$-\frac{\tan(dx+c)^3+3i\,\tan(dx+c)^2-3\,\tan(dx+c)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Mupad [B]
time = 3.34, size = 33, normalized size = 1.22

$$-\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}-3\right)}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2),x)
[Out] -(tan(c + d*x)*(tan(c + d*x)*3i + tan(c + d*x)^2 - 3))/(3*a^2*d)
```

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{2x}{a^2}+\frac{2i\log(\cos(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}$$

```
[Out] 2*x/a^2+2*I*ln(cos(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\cos(c+dx))}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + ((2*I)*Log[Cos[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\text{Subst}\left(\int \frac{a-x}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{2x}{a^2}+\frac{2i\log(\cos(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 71, normalized size = 1.87

$$\frac{4\text{ArcTan}(\tan(dx)) + i\sec(c)\sec(c+dx)\left(\cos(dx)\log\left(\cos^2(c+dx)\right) + \cos(2c+dx)\log\left(\cos^2(c+dx)\right) + 2i\sin(dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (4*ArcTan[Tan[d*x]] + I*Sec[c]*Sec[c + d*x]*(Cos[d*x]*Log[Cos[c + d*x]^2] +
 Cos[2*c + d*x]*Log[Cos[c + d*x]^2] + (2*I)*Sin[d*x]))/(2*a^2*d)
```

Maple [A]
time = 0.27, size = 30, normalized size = 0.79

method	result	size
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
default	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(e^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-tan(d*x+c)-2*I*ln(tan(d*x+c)-I))
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.84

$$\frac{-\frac{2i\,\log(i\,\tan(dx+c)+1)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (-2*I*log(I*tan(d*x + c) + 1)/a^2 - tan(d*x + c)/a^2)/d
```

Fricas [A]
time = 0.38, size = 70, normalized size = 1.84

$$\frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^(
2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^4(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**4/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(36) = 72.
time = 0.59, size = 100, normalized size = 2.63

$$\frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2}+\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

Mupad [B]
time = 3.35, size = 28, normalized size = 0.74

$$-\frac{\tan(c+d\,x)+\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] -(log(tan(c + d*x) - 1i)*2i + tan(c + d*x))/(a^2*d)
```

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=26

$$\frac{i}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i}{d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
[Out] I/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.23

$$\frac{i\sec^2(c+dx)}{2d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2)*Sec[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^2)
```

Maple [A]
time = 0.61, size = 24, normalized size = 0.92

method	result	size
risch	$\frac{i e^{-2i(dx+c)}}{2a^2 d}$	19
derivativedivides	$\frac{i}{ad(a+ia\tan(dx+c))}$	24
default	$\frac{i}{ad(a+ia\tan(dx+c))}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I/a/d/(a+I*a*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.81

$$\frac{i}{(i\,a\tan(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] I/((I*a*tan(d*x + c) + a)*a*d)
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.65

$$\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.
time = 0.55, size = 65, normalized size = 2.50

$$\begin{cases} -\frac{i\sec^2(c+dx)}{2a^2d\tan^2(c+dx)-4ia^2d\tan(c+dx)-2a^2d} & \text{for } d \neq 0 \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*sec(c + d*x)**2/(2*a**2*d*tan(c + d*x)**2 - 4*I*a**2*d*tan(c
+ d*x) - 2*a**2*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**2, True))
```

Giac [A]
time = 0.55, size = 30, normalized size = 1.15

$$-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

Mupad [B]
time = 3.34, size = 22, normalized size = 0.85

$$\frac{1\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] 1i/(a^2*d*(tan(c + d*x)*1i + 1))
```

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 1/4*x/a^2+1/4*I/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {3560, 8}

$$\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) + (I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (I/4)/(d*(a^2 + I*a^2*Tan[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx&=\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{2a}\\&=\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\int 1\,dx}{4a^2}\\&=\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 68, normalized size = 1.11

$$-\frac{\sec^2(c+dx)(4i+(i+4dx)\cos(2(c+dx))+(1+4idx)\sin(2(c+dx)))}{16a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] -1/16*(Sec[c + d*x]^2*(4*I + (I + 4*d*x)*Cos[2*(c + d*x)] + (1 + (4*I)*d*x)
*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 62, normalized size = 1.02

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
norman	$\frac{\frac{x}{4a}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{x\left(\tan^4(dx+c)\right)}{4a}+\frac{i}{2da}+\frac{3\tan(dx+c)}{4da}+\frac{\tan^3(dx+c)}{4da}}{a(1+\tan^2(dx+c))^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/8*I*ln(tan(d*x+c)-I)-1/4*I/(tan(d*x+c)-I)^2+1/4/(tan(d*x+c)-I)+
1/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 43, normalized size = 0.70

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.13, size = 117, normalized size = 1.92

$$\begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{(e^{4ic}+2e^{2ic}+1)e^{-4ic}}{4a^2} - \frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(
(exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True))
+ x/(4*a**2)
```

Giac [A]
time = 0.47, size = 72, normalized size = 1.18

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2} - \frac{2i\log(i\tan(dx+c)-1)}{a^2} + \frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

Mupad [B]
time = 3.40, size = 39, normalized size = 0.64

$$\frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)}{4} - \frac{1}{2}\mathrm{i}}{a^2\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - (tan(c + d*x)/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}-\frac{i}{16d\,(a^2-ia^2\tan(c+dx))}+\frac{3i}{16d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 1/4*x/a^2+1/12*I*a/d/(a+I*a*tan(d*x+c))^3+1/8*I/d/(a+I*a*tan(d*x+c))^2-1/16
*I/d/(a^2-I*a^2*tan(d*x+c))+3/16*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{i}{16d\,(a^2-ia^2\tan(c+dx))}+\frac{3i}{16d\,(a^2+ia^2\tan(c+dx))}+\frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/12)*a)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(d*(a + I*a*Tan
[c + d*x])^2) - (I/16)/(d*(a^2 - I*a^2*Tan[c + d*x])) + ((3*I)/16)/(d*(a^2
+ I*a^2*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\operatorname{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^2} + \frac{1}{4a^2(a+x)^4} + \frac{1}{4a^3(a+x)^3} + \frac{3}{16a^4(a+x)^2} + \frac{1}{4a^4(a^2-x^2)}\right)\,dx\right.}{d}$$

$$= \frac{ia}{12d(a+ia\tan(c+dx))^3} + \frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\left(a^2-ia^2\tan(c+d\right.}$$

$$= \frac{x}{4a^2} + \frac{ia}{12d(a+ia\tan(c+dx))^3} + \frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\left(a^2-ia^2\tan\right.}$$

Mathematica [A]
time = 0.34, size = 95, normalized size = 0.83

$$\frac{i\sec^2(c+dx)(-9+(-3+12idx)\cos(2(c+dx))+\cos(4(c+dx))+3i\sin(2(c+dx))-12dx\sin(2(c+dx))+2i\sin(4(c+dx)))}{48a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/48)*Sec[c + d*x]^2*(-9 + (-3 + (12*I)*d*x)*Cos[2*(c + d*x)] + Cos[4*(c
+ d*x)] + (3*I)*Sin[2*(c + d*x)] - 12*d*x*Sin[2*(c + d*x)] + (2*I)*Sin[4*(c
 + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.82, size = 88, normalized size = 0.77

method	result	size
risch	$\frac{x}{4a^2} + \frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d} + \frac{i\mathrm{e}^{-6i(dx+c)}}{96a^2d} + \frac{5i\cos(2dx+2c)}{32a^2d} + \frac{7\sin(2dx+2c)}{32a^2d}$	79
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8} - \frac{i}{8(\tan(dx+c)-i)^2} - \frac{1}{12(\tan(dx+c)-i)^3} + \frac{3}{16(\tan(dx+c)-i)} + \frac{i\ln(\tan(dx+c)+i)}{8} + \frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8} - \frac{i}{8(\tan(dx+c)-i)^2} - \frac{1}{12(\tan(dx+c)-i)^3} + \frac{3}{16(\tan(dx+c)-i)} + \frac{i\ln(\tan(dx+c)+i)}{8} + \frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/8*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2-1/12/(tan(d*x+c)-I)
^3+3/16/(tan(d*x+c)-I)+1/8*I*ln(tan(d*x+c)+I)+1/16/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.57

$$\frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [A]
time = 0.21, size = 189, normalized size = 1.66

$$\begin{cases}\frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0\\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*
c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x/(4*a
**2)
```

Giac [A]
time = 0.57, size = 103, normalized size = 0.90

$$-\frac{-\frac{6i\,\log(\tan(dx+c)+i)}{a^2}+\frac{6i\,\log(\tan(dx+c)-i)}{a^2}+\frac{3\,(2i\,\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\,\tan(dx+c)^3-42\,\tan(dx+c)^2+57i\,\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.47, size = 71, normalized size = 0.62

$$\frac{x}{4\,a^2}-\frac{\frac{\tan(c+d\,x)^3\,1\mathrm{i}}{4}+\frac{\tan(c+d\,x)^2}{2}-\frac{\tan(c+d\,x)\,1\mathrm{i}}{12}+\frac{1}{3}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - (tan(c + d*x)^2/2 - (tan(c + d*x)*1i)/12 + (tan(c + d*x)^3*1i)/
4 + 1/3)/(a^2*d*(tan(c + d*x)*1i + 1)^3*(tan(c + d*x) + 1i))
```

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$\frac{15x}{64a^2}-\frac{i}{64d(a-ia\tan(c+dx))^2}+\frac{ia^2}{32d(a+ia\tan(c+dx))^4}+\frac{ia}{16d(a+ia\tan(c+dx))^3}+\frac{3i}{32d(a+ia\tan(c+$$

```
[Out] 15/64*x/a^2-1/64*I/d/(a-I*a*tan(d*x+c))^2+1/32*I*a^2/d/(a+I*a*tan(d*x+c))^4
+1/16*I*a/d/(a+I*a*tan(d*x+c))^3+3/32*I/d/(a+I*a*tan(d*x+c))^2-5/64*I/d/(a^
2-I*a^2*tan(d*x+c))+5/32*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$\frac{ia^2}{32d(a+ia\tan(c+dx))^4}-\frac{5i}{64d\,(a^2-ia^2\tan(c+dx))}+\frac{5i}{32d\,(a^2+ia^2\tan(c+dx))}+\frac{15x}{64a^2}+\frac{ia}{16d(a+ia\tan(c+dx))^3}-\frac{i}{64d(a-ia\tan(c+dx))^2}+\frac{3i}{32d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (15*x)/(64*a^2) - (I/64)/(d*(a - I*a*Tan[c + d*x])^2) + ((I/32)*a^2)/(d*(a
+ I*a*Tan[c + d*x])^4) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/3
2)/(d*(a + I*a*Tan[c + d*x])^2) - ((5*I)/64)/(d*(a^2 - I*a^2*Tan[c + d*x]))
 + ((5*I)/32)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3} + \frac{5}{64a^6(a-x)^2} + \frac{1}{8a^3(a+x)^5} + \frac{3}{16a^4(a+x)^4} + \frac{3}{16a^5(a+x)^3} + \right.\right.}{d}$$

$$= -\frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{ia^2}{32d(a+ia\tan(c+dx))^4} + \frac{ia}{16d(a+ia\tan(c+d}$$

$$= \frac{15x}{64a^2} - \frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{ia^2}{32d(a+ia\tan(c+dx))^4} + \frac{ia}{16d(a+ia\tan}$$

Mathematica [A]
time = 0.37, size = 120, normalized size = 0.73

$$\frac{i\sec^2(c+dx)(-80+30i(i+4dx)\cos(2(c+dx))+16\cos(4(c+dx))+\cos(6(c+dx))+30i\sin(2(c+dx))-120dx\sin(2(c+dx))+32i\sin(4(c+dx))+3i\sin(6(c+dx)))}{512a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/512)*Sec[c + d*x]^2*(-80 + (30*I)*(I + 4*d*x)*Cos[2*(c + d*x)] + 16*Cos
[4*(c + d*x)] + Cos[6*(c + d*x)] + (30*I)*Sin[2*(c + d*x)] - 120*d*x*Sin[2*
(c + d*x)] + (32*I)*Sin[4*(c + d*x)] + (3*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I
+ Tan[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 116, normalized size = 0.70

method	result
risch	$\frac{15x}{64a^2} + \frac{ie^{-6i(dx+c)}}{64a^2d} + \frac{ie^{-8i(dx+c)}}{512a^2d} + \frac{7i\cos(4dx+4c)}{128a^2d} + \frac{\sin(4dx+4c)}{16a^2d} + \frac{7i\cos(2dx+2c)}{64a^2d} + \frac{13\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128} + \frac{i}{32(\tan(dx+c)-i)^4} - \frac{3i}{32(\tan(dx+c)-i)^2} - \frac{1}{16(\tan(dx+c)-i)^3} + \frac{5}{32(\tan(dx+c)-i)} + \frac{i}{64(\tan(dx+c)+i)^2} + \frac{15i\ln}{}}{d\,a^2}$
default	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128} + \frac{i}{32(\tan(dx+c)-i)^4} - \frac{3i}{32(\tan(dx+c)-i)^2} - \frac{1}{16(\tan(dx+c)-i)^3} + \frac{5}{32(\tan(dx+c)-i)} + \frac{i}{64(\tan(dx+c)+i)^2} + \frac{15i\ln}{}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-15/128*I*ln(tan(d*x+c)-I)+1/32*I/(tan(d*x+c)-I)^4-3/32*I/(tan(d*x
+c)-I)^2-1/16/(tan(d*x+c)-I)^3+5/32/(tan(d*x+c)-I)+1/64*I/(tan(d*x+c)+I)^2+
15/128*I*ln(tan(d*x+c)+I)+5/64/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 87, normalized size = 0.53

$$\frac{\left(120\, dxe^{(8i\, dx+8i\, c)}-2i\, e^{(12i\, dx+12i\, c)}-24i\, e^{(10i\, dx+10i\, c)}+80i\, e^{(6i\, dx+6i\, c)}+30i\, e^{(4i\, dx+4i\, c)}+8i\, e^{(2i\, dx+2i\, c)}+i\right)e^{(-8i\, dx-8i\, c)}}{512\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*d*x*e^(8*I*d*x + 8*I*c) - 2*I*e^(12*I*d*x + 12*I*c) - 24*I*e^(10
*I*d*x + 10*I*c) + 80*I*e^(6*I*d*x + 6*I*c) + 30*I*e^(4*I*d*x + 4*I*c) + 8*
I*e^(2*I*d*x + 2*I*c) + I)*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

Sympy [A]
time = 0.27, size = 258, normalized size = 1.56

$$\begin{cases} \frac{\left(-17179869184ia^{10}d^5e^{24ic}e^{4idx}-206158430208ia^{10}d^5e^{22ic}e^{2idx}+687194767360ia^{10}d^5e^{18ic}e^{-2idx}+257698037760ia^{10}d^5e^{16ic}e^{-4idx}+68719476736ia^{10}d^5e^{14ic}e^{-6idx}+8589934592ia^{10}d^5e^{12ic}e^{-8idx}\right)e^{-20ic}}{4398046511104a^{12}d^6} & \text{for}\: a^{12}d^6e^{20ic} \neq 0 \\ x\left(\frac{\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-8ic}}{64a^2}-\frac{15}{64a^2}\right) & \text{otherwise} \end{cases} + \frac{15x}{64a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-17179869184*I*a**10*d**5*exp(24*I*c)*exp(4*I*d*x) - 2061584302
08*I*a**10*d**5*exp(22*I*c)*exp(2*I*d*x) + 687194767360*I*a**10*d**5*exp(18
*I*c)*exp(-2*I*d*x) + 257698037760*I*a**10*d**5*exp(16*I*c)*exp(-4*I*d*x) +
 68719476736*I*a**10*d**5*exp(14*I*c)*exp(-6*I*d*x) + 8589934592*I*a**10*d*
*5*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(4398046511104*a**12*d**6), Ne(a
**12*d**6*exp(20*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c
) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-8*I*c)/(64*a**2)
 - 15/(64*a**2)), True)) + 15*x/(64*a**2)
```

Giac [A]
time = 0.59, size = 127, normalized size = 0.77

$$-\frac{\frac{60i\, \log(i\, \tan(dx+c)+1)}{a^2}-\frac{60i\, \log(i\, \tan(dx+c)-1)}{a^2}+\frac{2\left(45i\, \tan(dx+c)^2-110\, \tan(dx+c)-69i\right)}{a^2(\tan(dx+c)+i)^2}+\frac{-125i\, \tan(dx+c)^4-580\, \tan(dx+c)^3+1038i\, \tan(dx+c)^2+868\, \tan(dx+c)-301i}{a^2(\tan(dx+c)-i)^4}}{512\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/512*(60*I*log(I*tan(d*x + c) + 1)/a^2 - 60*I*log(I*tan(d*x + c) - 1)/a^2
 + 2*(45*I*tan(d*x + c)^2 - 110*tan(d*x + c) - 69*I)/(a^2*(tan(d*x + c) + I
)^2) + (-125*I*tan(d*x + c)^4 - 580*tan(d*x + c)^3 + 1038*I*tan(d*x + c)^2
+ 868*tan(d*x + c) - 301*I)/(a^2*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 4.18, size = 149, normalized size = 0.90

$$\frac{15\,x}{64\,a^2}+\frac{\frac{1}{4\,a^2}-\frac{\tan(c+d\,x)\,17\mathrm{i}}{64\,a^2}+\frac{25\tan(c+d\,x)^2}{32\,a^2}+\frac{\tan(c+d\,x)^3\,5\mathrm{i}}{32\,a^2}+\frac{15\tan(c+d\,x)^4}{32\,a^2}+\frac{\tan(c+d\,x)^5\,15\mathrm{i}}{64\,a^2}}{d\,\left(\tan\left(c+d\,x\right)^6\,1\mathrm{i}+2\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (15*x)/(64*a^2) + (1/(4*a^2) - (tan(c + d*x)*17i)/(64*a^2) + (25*tan(c + d*
x)^2)/(32*a^2) + (tan(c + d*x)^3*5i)/(32*a^2) + (15*tan(c + d*x)^4)/(32*a^2
) + (tan(c + d*x)^5*15i)/(64*a^2))/(d*(2*tan(c + d*x) - tan(c + d*x)^2*1i +
 4*tan(c + d*x)^3 + tan(c + d*x)^4*1i + 2*tan(c + d*x)^5 + tan(c + d*x)^6*1
i - 1i))
```

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d}+\frac{7\sec(c+dx)\tan(c+dx)}{16a^2d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d}+\frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}-$$

```
[Out] 7/16*arctanh(sin(d*x+c))/a^2/d+7/16*sec(d*x+c)*tan(d*x+c)/a^2/d+7/24*sec(d*
x+c)^3*tan(d*x+c)/a^2/d+7/30*sec(d*x+c)^5*tan(d*x+c)/a^2/d-2/5*I*sec(d*x+c)
^7/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$\frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d}-\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{7\tan(c+dx)\sec^5(c+dx)}{30a^2d}+\frac{7\tan(c+dx)\sec^3(c+dx)}{24a^2d}+\frac{7\tan(c+dx)\sec(c+dx)}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(16*a^2*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(16*a^
2*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(24*a^2*d) + (7*Sec[c + d*x]^5*Tan[c
 + d*x])/(30*a^2*d) - (((2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{7\int \sec^7(c+dx)\,dx}{5a^2}$$

$$= \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{7\int \sec^5(c+dx)\,dx}{6a^2}$$

$$= \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+ \cdots}$$

$$= \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c \cdots}{30a^2d}$$

$$= \frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx \cdots}{24a^2d}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 294 vs. $2(124) = 248$.
time = 2.30, size = 294, normalized size = 2.37

$$-\frac{\sec^6(c+dx)\left(3072i\cos(c+dx)+5\left(210\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+21\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+315\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+126\cos(4(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-210\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-21\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\sin(c+dx)-238\sin(3(c+dx))-42\sin(5(c+dx))\right)\right)}{7680a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/7680*(Sec[c + d*x]^6*((3072*I)*Cos[c + d*x] + 5*(210*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 21*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 315*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 126*Cos[4*(c + d*x)]*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) - 210*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 21*Cos[6*(c + d*x)]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x] - 238*Sin[3*(c + d*
x)] - 42*Sin[5*(c + d*x)])))/(a^2*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(112) = 224$.
time = 0.27, size = 238, normalized size = 1.92

method	result
risch	$-\frac{i\left(105\,e^{11i(dx+c)}+595\,e^{9i(dx+c)}+1386\,e^{7i(dx+c)}+1686\,e^{5i(dx+c)}-595\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{120d\,a^2\left(e^{2i(dx+c)}+1\right)^6}+\frac{7\ln\left(e^{i(dx+c)}+i\right)}{16a^2d}-$

derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}-7}{}$
default	$\frac{\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}-7}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((-1/4+1/2*I)/(tan(1/2*d*x+1/2*c)-1)^4+(9/32+5/8*I)/(tan(1/2*d*x+1/
2*c)-1)^2+(9/32+3/8*I)/(tan(1/2*d*x+1/2*c)-1)+(-1/12+3/4*I)/(tan(1/2*d*x+1/
2*c)-1)^3+(-1/4+1/5*I)/(tan(1/2*d*x+1/2*c)-1)^5-1/12/(tan(1/2*d*x+1/2*c)-1)
^6-7/32*ln(tan(1/2*d*x+1/2*c)-1)+(1/4+1/2*I)/(tan(1/2*d*x+1/2*c)+1)^4+(9/32
-3/8*I)/(tan(1/2*d*x+1/2*c)+1)-(1/12+3/4*I)/(tan(1/2*d*x+1/2*c)+1)^3-(1/4+1
/5*I)/(tan(1/2*d*x+1/2*c)+1)^5+(-9/32+5/8*I)/(tan(1/2*d*x+1/2*c)+1)^2+1/12/
(tan(1/2*d*x+1/2*c)+1)^6+7/32*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(110) = 220$.
time = 0.31, size = 421, normalized size = 3.40

$$\frac{\frac{2\left(\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96i\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{445\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{960i\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{330\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{960i\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{330\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{480i\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{445\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{480i\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{135\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-96i\right)}{a^{2}-\frac{6a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{2}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}-\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(2*(135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 960*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*I*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 330*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 480*I*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + 480*I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 135*sin(d*x +
 c)^11/(cos(d*x + c) + 1)^11 - 96*I)/(a^2 - 6*a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^2*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*
a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 - 105*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(110) = 220$.
time = 0.43, size = 326, normalized size = 2.63

$$\frac{105\left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(11i\,dx+11i\,c)}-1190i\,e^{(9i\,dx+9i\,c)}-2772i\,e^{(7i\,dx+7i\,c)}-3372i\,e^{(5i\,dx+5i\,c)}+1190i\,e^{(3i\,dx+3i\,c)}+210i\,e^{(i\,dx+i\,c)}}{240\left(a^{2}de^{(12i\,dx+12i\,c)}+6\,a^{2}de^{(10i\,dx+10i\,c)}+15\,a^{2}de^{(8i\,dx+8i\,c)}+20\,a^{2}de^{(6i\,dx+6i\,c)}+15\,a^{2}de^{(4i\,dx+4i\,c)}+6\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(105*(e^(12*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x
 + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x
+ 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(12*I*d*x + 12*I*c) + 6*e^(
10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e
^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) -
210*I*e^(11*I*d*x + 11*I*c) - 1190*I*e^(9*I*d*x + 9*I*c) - 2772*I*e^(7*I*d*
x + 7*I*c) - 3372*I*e^(5*I*d*x + 5*I*c) + 1190*I*e^(3*I*d*x + 3*I*c) + 210*
I*e^(I*d*x + I*c))/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d*x + 10*
I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c) + 15*a^2
*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^9(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**9/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [A]
time = 0.59, size = 203, normalized size = 1.64

$$\frac{\frac{105\log(\tan(\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^2}-\frac{105\log(\tan(\frac{1}{2}dx+\frac{1}{2}c)-1)}{a^2}+\frac{2\left(135\tan(\frac{1}{2}dx+\frac{1}{2}c)^{11}+480i\tan(\frac{1}{2}dx+\frac{1}{2}c)^{10}-445\tan(\frac{1}{2}dx+\frac{1}{2}c)^{9}-480i\tan(\frac{1}{2}dx+\frac{1}{2}c)^{8}-330\tan(\frac{1}{2}dx+\frac{1}{2}c)^{7}+960i\tan(\frac{1}{2}dx+\frac{1}{2}c)^{6}-330\tan(\frac{1}{2}dx+\frac{1}{2}c)^{5}-960i\tan(\frac{1}{2}dx+\frac{1}{2}c)^{4}-445\tan(\frac{1}{2}dx+\frac{1}{2}c)^{3}+96i\tan(\frac{1}{2}dx+\frac{1}{2}c)^{2}+135\tan(\frac{1}{2}dx+\frac{1}{2}c)-96i\right)}{\left(\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-1\right)^6a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^2 + 2*(135*tan(1/2*d*x + 1/2*c)^11 + 480*I*tan(1/2*d*x + 1/2*c)^10
- 445*tan(1/2*d*x + 1/2*c)^9 - 480*I*tan(1/2*d*x + 1/2*c)^8 - 330*tan(1/2*d
*x + 1/2*c)^7 + 960*I*tan(1/2*d*x + 1/2*c)^6 - 330*tan(1/2*d*x + 1/2*c)^5 -
 960*I*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 + 96*I*tan(1/2*d
*x + 1/2*c)^2 + 135*tan(1/2*d*x + 1/2*c) - 96*I)/((tan(1/2*d*x + 1/2*c)^2 -
 1)^6*a^2))/d
```

Mupad [B]
time = 6.31, size = 191, normalized size = 1.54

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}-\frac{-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}8\mathrm{i}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}8\mathrm{i}+\frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}4\mathrm{i}}{5}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{4}{5}\mathrm{i}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2),x)

[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(8*a^2*d) - ((89*tan(c/2 + (d*x)/2)^3)/24 - (tan(c/2 + (d*x)/2)^2*4i)/5 - (9*tan(c/2 + (d*x)/2))/8 + tan(c/2 + (d*x)/2)^4*8i + (11*tan(c/2 + (d*x)/2)^5)/4 - tan(c/2 + (d*x)/2)^6*8i + (11*tan(c/2 + (d*x)/2)^7)/4 + tan(c/2 + (d*x)/2)^8*4i + (89*tan(c/2 + (d*x)/2)^9)/24 - tan(c/2 + (d*x)/2)^10*4i - (9*tan(c/2 + (d*x)/2)^11)/8 + 4i/5)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^6)

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 5/8*arctanh(sin(d*x+c))/a^2/d+5/8*sec(d*x+c)*tan(d*x+c)/a^2/d+5/12*sec(d*x+
c)^3*tan(d*x+c)/a^2/d-2/3*I*sec(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{5\tan(c+dx)\sec^3(c+dx)}{12a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*
d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^2*d) - (((2*I)/3)*Sec[c + d*x]^5
)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int \sec^5(c+dx)\,dx}{3a^2}\\ &= \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int \sec^3(c+dx)\,dx}{4a^2}\\ &= \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx}{3d\left(a^2+ia^2\tan(c\right.}\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}+\frac{5\sec^3(c+dx)\tan(c+}{12a^2d}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(100) = 200.
time = 1.19, size = 215, normalized size = 2.15

$$-\frac{\sec^4(c+dx)\left(128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+15\cos(4(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\sin(c+dx)-30\sin(3(c+dx))\right)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30
*Sin[3*(c + d*x)]))/(a^2*d)
```

Maple [**A**]
time = 0.26, size = 170, normalized size = 1.70

method	result
risch	$-\frac{i\left(15\,e^{7i(dx+c)}+55\,e^{5i(dx+c)}+73\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{12d\,a^2\left(e^{2i(dx+c)}+1\right)^4}-\frac{5\ln\left(e^{i(dx+c)}-i\right)}{8a^2d}+\frac{5\ln\left(e^{i(dx+c)}+i\right)}{8a^2d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{}{(\mathrm{ta}}}{a^2d}$
default	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{}{(\mathrm{ta}}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((3/16+1/2*I)/(tan(1/2*d*x+1/2*c)-1)+(1/16+1/2*I)/(tan(1/2*d*x+1/2*
c)-1)^2+(-1/4+1/3*I)/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^4-
5/16*ln(tan(1/2*d*x+1/2*c)-1)+(3/16-1/2*I)/(tan(1/2*d*x+1/2*c)+1)+(-1/16+1/
2*I)/(tan(1/2*d*x+1/2*c)+1)^2-(1/4+1/3*I)/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan
(1/2*d*x+1/2*c)+1)^4+5/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(88) = 176.
time = 0.30, size = 295, normalized size = 2.95

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}-\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(88) = 176.
time = 0.38, size = 230, normalized size = 2.30

$$\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(7i\,dx+7i\,c)}-110i\,e^{(5i\,dx+5i\,c)}-146i\,e^{(3i\,dx+3i\,c)}+30i\,e^{(i\,dx+i\,c)}}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^7(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**2,x)

[Out] -Integral(sec(c + d*x)**7/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2

Giac [A]
time = 0.84, size = 151, normalized size = 1.51

$$\frac{\frac{15 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{15 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}+\frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*tan (1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c) ^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan(1/2 *d*x + 1/2*c)^2 - 1)^4*a^2))/d

Mupad [B]
time = 5.92, size = 136, normalized size = 1.36

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{3}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{4}{3}\mathrm{i}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2),x)

[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)^4 *4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2 + (d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a^2/d+3/2*sec(d*x+c)*tan(d*x+c)/a^2/d-2*I*sec(d*x+c
)^3/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}+\frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) - ((2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}+\frac{3\int\sec^3(c+dx)\,dx}{a^2}\\
&= \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}+\frac{3\int\sec(c+dx)\,dx}{2a^2}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.52, size = 146, normalized size = 1.97

$$
-\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin(c+dx)\right)}{4a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.24, size = 102, normalized size = 1.38

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^2d}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^2d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^2d}$
default	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((-1/4+I)/(tan(1/2*d*x+1/2*c)-1)-1/4/(tan(1/2*d*x+1/2*c)-1)^2-3/4*l
n(tan(1/2*d*x+1/2*c)-1)-(1/4+I)/(tan(1/2*d*x+1/2*c)+1)+1/4/(tan(1/2*d*x+1/2
*c)+1)^2+3/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(66) = 132.
time = 0.32, size = 167, normalized size = 2.26

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.
time = 0.37, size = 134, normalized size = 1.81

$$\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(3i\,dx+3i\,c)}-10i\,e^{(i\,dx+i\,c)}}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec^5(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**5/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [A]
time = 0.60, size = 95, normalized size = 1.28

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [B]
time = 3.93, size = 104, normalized size = 1.41

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4{}\mathrm{i}}{a^2}+\frac{4{}\mathrm{i}}{a^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c/
2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (d*
x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] -arctanh(sin(d*x+c))/a^2/d+2*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3581, 3855}

$$-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan
[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx&=\frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}-\frac{\int \sec(c+dx)\,dx}{a^2}\\&=-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. $2(48) = 96$.
time = 0.22, size = 184, normalized size = 3.83

$$-\frac{\sec^2(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(2+i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{3}{2}(c+dx)\right)+i\sin\left(\frac{3}{2}(c+dx)\right)\right)}{a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

Maple [A]
time = 0.26, size = 54, normalized size = 1.12

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2d}$	54
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2d}$	54
risch	$\frac{2i\mathrm{e}^{-i(dx+c)}}{a^2d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)+1)+2/(-I+ta
n(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.
time = 0.52, size = 117, normalized size = 2.44

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx+c\right)+\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Fricas [A]
time = 0.39, size = 64, normalized size = 1.33

$$-\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^3(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**3/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Giac [A]
time = 0.62, size = 57, normalized size = 1.19

$$-\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 - 4
/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

Mupad [B]
time = 3.50, size = 44, normalized size = 0.92

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] 4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^2
*d)
```

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 1/3*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^2+1/3*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d
*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3583, 3569}

$$\frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(d
*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a}\\ &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 38, normalized size = 0.58

$$\frac{\sec(c+dx)(-2i+\tan(c+dx))}{3a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-2*I + Tan[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.36, size = 57, normalized size = 0.88

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{2a^2d}+\frac{i e^{-3i(dx+c)}}{6a^2d}$	38
derivativedivides	$\frac{-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2d}$	57
default	$\frac{-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-2/3/(-I+tan(1/2*d*x+1/2*c))^3+I/(-I+tan(1/2*d*x+1/2*c))^2+1/(-I+t
an(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.30, size = 45, normalized size = 0.69

$$\frac{i\cos\left(3\,dx+3\,c\right)+3i\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+3\sin\left(dx+c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.46

$$\frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.
time = 0.56, size = 112, normalized size = 1.72

$$\begin{cases} \frac{\tan(c+dx)\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} - \frac{2i\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} & \text{for } d \neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((tan(c + d*x)*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*I*a**2*d
*tan(c + d*x) - 3*a**2*d) - 2*I*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*
I*a**2*d*tan(c + d*x) - 3*a**2*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**2
, True))
```

Giac [A]
time = 0.63, size = 47, normalized size = 0.72

$$\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\right)}{3a^2d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

Mupad [B]
time = 3.51, size = 79, normalized size = 1.22

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1{}\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c/
2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{3\sin(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 3/5*sin(d*x+c)/a^2/d-1/5*sin(d*x+c)^3/a^2/d+2/5*I*cos(d*x+c)^3/d/(a^2+I*a^2
*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {3581, 2713}

$$-\frac{\sin^3(c+dx)}{5a^2d}+\frac{3\sin(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*Sin[c + d*x])/(5*a^2*d) - Sin[c + d*x]^3/(5*a^2*d) + (((2*I)/5)*Cos[c +
d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2i\cos^3(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{3\int \cos^3(c+dx)\,dx}{5a^2}$$
$$= \frac{2i\cos^3(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))} - \frac{3\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{5a^2d}$$
$$= \frac{3\sin(c+dx)}{5a^2d} - \frac{\sin^3(c+dx)}{5a^2d} + \frac{2i\cos^3(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [A]
time = 0.38, size = 68, normalized size = 0.96

$$\frac{\sec(c+dx)(-12i+4i\cos(2(c+dx))-3\sec(c+dx)\sin(3(c+dx))+5\tan(c+dx))}{20a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-12*I + (4*I)*Cos[2*(c + d*x)] - 3*Sec[c + d*x]*Sin[3*(c + d
*x)] + 5*Tan[c + d*x]))/(20*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.47, size = 108, normalized size = 1.52

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^2d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{40a^2d}+\frac{i\cos(dx+c)}{4a^2d}+\frac{\sin(dx+c)}{2a^2d}$
derivativedivides	$\frac{\frac{2}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8i}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4(-i+\tan(\frac{dx}{2}}}{a^2d}$
default	$\frac{\frac{2}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8i}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4(-i+\tan(\frac{dx}{2}}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/8/(tan(1/2*d*x+1/2*c)+I)-I/(-I+tan(1/2*d*x+1/2*c))^4+5/4*I/(-I+t
an(1/2*d*x+1/2*c))^2+2/5/(-I+tan(1/2*d*x+1/2*c))^5-3/2/(-I+tan(1/2*d*x+1/2*
c))^3+7/8/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 52, normalized size = 0.73

$$\frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.
time = 0.22, size = 163, normalized size = 2.30

$$\begin{cases} \frac{\left(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx}\right)e^{-9ic}}{20480a^8d^4} & \text{for}\ a^8d^4e^{9ic}\neq 0 \\ \frac{x\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*d**
4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-5
*I*c)/(8*a**2), True))
```

Giac [A]
time = 0.68, size = 93, normalized size = 1.31

$$\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B]
time = 3.89, size = 90, normalized size = 1.27

$$-\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{5\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*
10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*(t
an(c/2 + (d*x)/2) + 1i))
```

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{5\sin(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{\sin^5(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 5/7*sin(d*x+c)/a^2/d-10/21*sin(d*x+c)^3/a^2/d+1/7*sin(d*x+c)^5/a^2/d+2/7*I*
cos(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3581, 2713}

$$\frac{\sin^5(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{5\sin(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*Sin[c + d*x])/(7*a^2*d) - (10*Sin[c + d*x]^3)/(21*a^2*d) + Sin[c + d*x]^
5/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}+\frac{5\int\cos^5(c+dx)\,dx}{7a^2}\\
&= \frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}-\frac{5\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7a^2d}\\
&= \frac{5\sin(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{\sin^5(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 95, normalized size = 1.07

$$
\frac{i\sec^2(c+dx)(-140\cos(c+dx)+42\cos(3(c+dx))+2\cos(5(c+dx))-70i\sin(c+dx)+63i\sin(3(c+dx))+5i\sin(5(c+dx)))}{336a^2d(-i+\tan(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/336)*Sec[c + d*x]^2*(-140*Cos[c + d*x] + 42*Cos[3*(c + d*x)] + 2*Cos[5*
(c + d*x)] - (70*I)*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c
 + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(79) = 158.
time = 0.26, size = 174, normalized size = 1.96

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{32a^2d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{224a^2d}+\frac{5i\cos(dx+c)}{32a^2d}+\frac{15\sin(dx+c)}{32a^2d}+\frac{3i\cos(3dx+3c)}{32a^2d}+\frac{11\sin(3dx+3c)}{96a^2d}$
derivativedivides	$-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\cdots$ over a^2d
default	$-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\cdots$ over a^2d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-1/16*I/(tan(1/2*d*x+1/2*c)+I)^2-1/24/(tan(1/2*d*x+1/2*c)+I)^3+3/1
6/(tan(1/2*d*x+1/2*c)+I)+I/(-I+tan(1/2*d*x+1/2*c))^6-5/2*I/(-I+tan(1/2*d*x+
1/2*c))^4+23/16*I/(-I+tan(1/2*d*x+1/2*c))^2-2/7/(-I+tan(1/2*d*x+1/2*c))^7+2
/(-I+tan(1/2*d*x+1/2*c))^5-55/24/(-I+tan(1/2*d*x+1/2*c))^3+13/16/(-I+tan(1/
2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.83

$$\frac{\left(-7 i e^{(10 i dx+10 i c)}-105 i e^{(8 i dx+8 i c)}+210 i e^{(6 i dx+6 i c)}+70 i e^{(4 i dx+4 i c)}+21 i e^{(2 i dx+2 i c)}+3 i\right) e^{(-7 i dx-7 i c)}}{672 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(76) = 152.
time = 0.32, size = 231, normalized size = 2.60

$$\begin{cases} \frac{\left(-176160768 i a^{10} d^5 e^{19 i c} e^{3 i d x}-2642411520 i a^{10} d^5 e^{17 i c} e^{i d x}+5284823040 i a^{10} d^5 e^{15 i c} e^{-i d x}+1761607680 i a^{10} d^5 e^{13 i c} e^{-3 i d x}+528482304 i a^{10} d^5 e^{11 i c} e^{-5 i d x}+75497472 i a^{10} d^5 e^{9 i c} e^{-7 i d x}\right) e^{-16 i c}}{16911433728 a^{12} d^6} & \text{for}\: a^{12} d^6 e^{16 i c} \neq 0 \\ \frac{x \left(e^{10 i c}+5 e^{8 i c}+10 e^{6 i c}+10 e^{4 i c}+5 e^{2 i c}+1\right) e^{-7 i c}}{32 a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(16*I*
c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5
*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Giac [A]
time = 0.64, size = 145, normalized size = 1.63

$$\frac{\frac{7 \left(9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+15 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-8\right)}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+i\right)^3}+\frac{273 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6-1155 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-2450 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+2870 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+2037 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-791 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-152}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)^7}}{168 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B]
time = 6.87, size = 161, normalized size = 1.81

$$\frac{\left(-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 28\mathrm{i}+76\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 24\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-6\mathrm{i}\right)\,2\mathrm{i}}{21\,a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^3
 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2 + (
d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/
2)*1i + 1)^7)
```

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$\frac{7\sin(c+dx)}{9a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{\sin^7(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 7/9*sin(d*x+c)/a^2/d-7/9*sin(d*x+c)^3/a^2/d+7/15*sin(d*x+c)^5/a^2/d-1/9*sin
(d*x+c)^7/a^2/d+2/9*I*cos(d*x+c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 2713}

$$-\frac{\sin^7(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*Sin[c + d*x])/(9*a^2*d) - (7*Sin[c + d*x]^3)/(9*a^2*d) + (7*Sin[c + d*x]
^5)/(15*a^2*d) - Sin[c + d*x]^7/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(
a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))} + \frac{7\int \cos^7(c+dx)\,dx}{9a^2}$$
$$= \frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))} - \frac{7\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin(c+ \ldots}{9a^2d}$$
$$= \frac{7\sin(c+dx)}{9a^2d} - \frac{7\sin^3(c+dx)}{9a^2d} + \frac{7\sin^5(c+dx)}{15a^2d} - \frac{\sin^7(c+dx)}{9a^2d} + \frac{2i\cos \ldots}{9d\,(a^2+ia \ldots}$$

Mathematica [A]
time = 0.53, size = 117, normalized size = 1.09

$$\frac{i\sec^2(c+dx)(-1050\cos(c+dx)+378\cos(3(c+dx))+30\cos(5(c+dx))+2\cos(7(c+dx))-525i\sin(c+dx)+567i\sin(3(c+dx))+75i\sin(5(c+dx))+7i\sin(7(c+dx)))}{2880a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2880)*Sec[c + d*x]^2*(-1050*Cos[c + d*x] + 378*Cos[3*(c + d*x)] + 30*Co
s[5*(c + d*x)] + 2*Cos[7*(c + d*x)] - (525*I)*Sin[c + d*x] + (567*I)*Sin[3*
(c + d*x)] + (75*I)*Sin[5*(c + d*x)] + (7*I)*Sin[7*(c + d*x)]))/(a^2*d*(-I
+ Tan[c + d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(95) = 190.
time = 0.27, size = 240, normalized size = 2.24

method	result
risch	$\frac{i e^{-7i(dx+c)}}{128a^2d} + \frac{i e^{-9i(dx+c)}}{1152a^2d} + \frac{7i\cos(dx+c)}{64a^2d} + \frac{7\sin(dx+c)}{16a^2d} + \frac{i\cos(5dx+5c)}{32a^2d} + \frac{11\sin(5dx+5c)}{320a^2d} + \frac{7i\cos(3dx+3c \ldots}{96a^2d}$
derivativedivides	$\frac{\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4} - \frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2} + \frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5} - \frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3} + \frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)} - \frac{2i}{\left(-i+\tan\left(\frac{dx}{2} \ldots\right.\right.}}{}$
default	$\frac{\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4} - \frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2} + \frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5} - \frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3} + \frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)} - \frac{2i}{\left(-i+\tan\left(\frac{dx}{2} \ldots\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/16*I/(tan(1/2*d*x+1/2*c)+I)^4-9/64*I/(tan(1/2*d*x+1/2*c)+I)^2+1/
40/(tan(1/2*d*x+1/2*c)+I)^5-13/96/(tan(1/2*d*x+1/2*c)+I)^3+29/128/(tan(1/2*
d*x+1/2*c)+I)-I/(-I+tan(1/2*d*x+1/2*c))^8+51/32*I/(-I+tan(1/2*d*x+1/2*c))^2
+49/12*I/(-I+tan(1/2*d*x+1/2*c))^6-35/8*I/(-I+tan(1/2*d*x+1/2*c))^4+2/9/(-I
+tan(1/2*d*x+1/2*c))^9-5/2/(-I+tan(1/2*d*x+1/2*c))^7+49/10/(-I+tan(1/2*d*x+
1/2*c))^5-49/16/(-I+tan(1/2*d*x+1/2*c))^3+99/128/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.36, size = 96, normalized size = 0.90

$$\frac{\left(-9i\,e^{(14i\,dx+14i\,c)}-105i\,e^{(12i\,dx+12i\,c)}-945i\,e^{(10i\,dx+10i\,c)}+1575i\,e^{(8i\,dx+8i\,c)}+525i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-9i\,dx-9i\,c)}}{5760\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-9*I*e^(14*I*d*x + 14*I*c) - 105*I*e^(12*I*d*x + 12*I*c) - 945*I*e^
(10*I*d*x + 10*I*c) + 1575*I*e^(8*I*d*x + 8*I*c) + 525*I*e^(6*I*d*x + 6*I*c
) + 189*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9*I*d*x
 - 9*I*c)/(a^2*d)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(94) = 188.
time = 0.41, size = 299, normalized size = 2.79

$$\begin{cases} \frac{\left(-227994731135631360ia^{14}d^7e^{30ic}e^{5idx}-2659938529915699200ia^{14}d^7e^{28ic}e^{3idx}-23939446769241292800ia^{14}d^7e^{26ic}e^{idx}+39899077948735488000ia^{14}d^7e^{24ic}e^{-idx}+13299692649578496000ia^{14}d^7e^{22ic}e^{-3idx}+4787889353848258560ia^{14}d^7e^{20ic}e^{-5idx}+1139973655678156800ia^{14}d^7e^{18ic}e^{-7idx}+126663739519795200ia^{14}d^7e^{16ic}e^{-9idx}\right)e^{-25ic}}{145916627926804070400a^{16}d^8} & \text{for}\ a^{16}d^8e^{25ic}\neq 0 \\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-9ic}}{128a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-227994731135631360*I*a**14*d**7*exp(30*I*c)*exp(5*I*d*x) - 265
9938529915699200*I*a**14*d**7*exp(28*I*c)*exp(3*I*d*x) - 239394467692412928
00*I*a**14*d**7*exp(26*I*c)*exp(I*d*x) + 39899077948735488000*I*a**14*d**7*
exp(24*I*c)*exp(-I*d*x) + 13299692649578496000*I*a**14*d**7*exp(22*I*c)*exp
(-3*I*d*x) + 4787889353848258560*I*a**14*d**7*exp(20*I*c)*exp(-5*I*d*x) + 1
139973655678156800*I*a**14*d**7*exp(18*I*c)*exp(-7*I*d*x) + 126663739519795
200*I*a**14*d**7*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(14591662792680407
0400*a**16*d**8), Ne(a**16*d**8*exp(25*I*c), 0)), (x*(exp(14*I*c) + 7*exp(1
2*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*c) + 7
*exp(2*I*c) + 1)*exp(-9*I*c)/(128*a**2), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(93) = 186$.
time = 0.78, size = 197, normalized size = 1.84

$$\frac{\frac{3\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+1470i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-2060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1330i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+353\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{5}}+\frac{4455\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-26460i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-78120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+137340i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+157374\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-118356i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-57744\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+16596i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2339}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{9}}}{2880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2880*(3*(435*tan(1/2*d*x + 1/2*c)^4 + 1470*I*tan(1/2*d*x + 1/2*c)^3 - 206
0*tan(1/2*d*x + 1/2*c)^2 - 1330*I*tan(1/2*d*x + 1/2*c) + 353)/(a^2*(tan(1/2
*d*x + 1/2*c) + I)^5) + (4455*tan(1/2*d*x + 1/2*c)^8 - 26460*I*tan(1/2*d*x
+ 1/2*c)^7 - 78120*tan(1/2*d*x + 1/2*c)^6 + 137340*I*tan(1/2*d*x + 1/2*c)^5
 + 157374*tan(1/2*d*x + 1/2*c)^4 - 118356*I*tan(1/2*d*x + 1/2*c)^3 - 57744*
tan(1/2*d*x + 1/2*c)^2 + 16596*I*tan(1/2*d*x + 1/2*c) + 2339)/(a^2*(tan(1/2
*d*x + 1/2*c) - I)^9))/d
```

Mupad [B]
time = 5.65, size = 216, normalized size = 2.02

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{191\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{1289\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}+\frac{649\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{41\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}+\frac{41\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}-\frac{7\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{64}+\frac{7\sin\left(\frac{13c}{2}+\frac{13dx}{2}\right)}{64}+\frac{\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)525i}{32}-\frac{\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)205i}{32}+\frac{\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)1i}{2}-\frac{\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)1i}{2}+\frac{\cos\left(\frac{11c}{2}+\frac{11dx}{2}\right)1i}{32}-\frac{\cos\left(\frac{13c}{2}+\frac{13dx}{2}\right)1i}{32}\right)2i}{45\,a^{2}\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^{9}\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*525i)/32 - (cos((5*c)/2 + (5
*d*x)/2)*205i)/32 + (cos((7*c)/2 + (7*d*x)/2)*1i)/2 - (cos((9*c)/2 + (9*d*x
)/2)*1i)/2 + (cos((11*c)/2 + (11*d*x)/2)*1i)/32 - (cos((13*c)/2 + (13*d*x)/
2)*1i)/32 + (191*sin(c/2 + (d*x)/2))/16 - (1289*sin((3*c)/2 + (3*d*x)/2))/6
4 + (649*sin((5*c)/2 + (5*d*x)/2))/64 - (41*sin((7*c)/2 + (7*d*x)/2))/32 +
(41*sin((9*c)/2 + (9*d*x)/2))/32 - (7*sin((11*c)/2 + (11*d*x)/2))/64 + (7*s
in((13*c)/2 + (13*d*x)/2))/64)*2i)/(45*a^2*d*(cos(c/2 + (d*x)/2) + sin(c/2
+ (d*x)/2)*1i)^9*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^5)
```

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$\frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}$$

```
[Out] 8/7*I*(a-I*a*tan(d*x+c))^7/a^10/d-3/2*I*(a-I*a*tan(d*x+c))^8/a^11/d+2/3*I*(
a-I*a*tan(d*x+c))^9/a^12/d-1/10*I*(a-I*a*tan(d*x+c))^10/a^13/d
```

Rubi [A]

time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((8*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^10*d) - (((3*I)/2)*(a - I*a*Tan[c +
 d*x])^8)/(a^11*d) + (((2*I)/3)*(a - I*a*Tan[c + d*x])^9)/(a^12*d) - ((I/10
)*(a - I*a*Tan[c + d*x])^10)/(a^13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^6(a+x)^3\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a-x)^6-12a^2(a-x)^7+6a(a-x)^8-(a-x)^9\right)\,dx, x, ia\tan\right.}{a^{13}d}\\ &= \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\end{aligned}$$

Mathematica [A]
time = 1.07, size = 117, normalized size = 1.07

$$\frac{\sec(c)\sec^{10}(c+dx)(-126i\cos(c)-105i\cos(c+2dx)-105i\cos(3c+2dx)-126\sin(c)+105\sin(c+2dx)-105\sin(3c+2dx)+120\sin(3c+4dx)+45\sin(5c+6dx)+10\sin(7c+8dx)+\sin(9c+10dx))}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
[Out] (Sec[c]*Sec[c + d*x]^10*((-126*I)*Cos[c] - (105*I)*Cos[c + 2*d*x] - (105*I)
*Cos[3*c + 2*d*x] - 126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*x]
+ 120*Sin[3*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin[9*
c + 10*d*x]))/(840*a^3*d)
```

Maple [A]
time = 0.30, size = 91, normalized size = 0.83

method	result
risch	$\frac{128i\left(120\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{10}(dx+c)\right)}{10}-\frac{i\left(\tan^9(dx+c)\right)}{3}-\frac{8i\left(\tan^7(dx+c)\right)}{7}+\tan^6(dx+c)-\frac{6i\left(\tan^5(dx+c)\right)}{5}+2\left(\tan^4(dx+c)\right)+\frac{3(t}{}\right.}{d\,a^3}$
default	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{10}(dx+c)\right)}{10}-\frac{i\left(\tan^9(dx+c)\right)}{3}-\frac{8i\left(\tan^7(dx+c)\right)}{7}+\tan^6(dx+c)-\frac{6i\left(\tan^5(dx+c)\right)}{5}+2\left(\tan^4(dx+c)\right)+\frac{3(t}{}\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -I/d/a^3*(I*tan(d*x+c)-1/10*tan(d*x+c)^10-1/3*I*tan(d*x+c)^9-8/7*I*tan(d*x+
c)^7+tan(d*x+c)^6-6/5*I*tan(d*x+c)^5+2*tan(d*x+c)^4+3/2*tan(d*x+c)^2)
```

Maxima [A]
time = 0.30, size = 87, normalized size = 0.80

$$-\frac{-21i\,\tan\left(dx+c\right)^{10}+70\,\tan\left(dx+c\right)^{9}+240\,\tan\left(dx+c\right)^{7}+210i\,\tan\left(dx+c\right)^{6}+252\,\tan\left(dx+c\right)^{5}+420i\,\tan\left(dx+c\right)^{4}+315i\,\tan\left(dx+c\right)^{2}-210\,\tan\left(dx+c\right)}{210\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 21
0*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*tan(
d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.
time = 0.37, size = 194, normalized size = 1.78

$$-\frac{128\left(-120i\,e^{(6i\,dx+6i\,c)}-45i\,e^{(4i\,dx+4i\,c)}-10i\,e^{(2i\,dx+2i\,c)}-i\right)}{105\left(a^3de^{(20i\,dx+20i\,c)}+10\,a^3de^{(18i\,dx+18i\,c)}+45\,a^3de^{(16i\,dx+16i\,c)}+120\,a^3de^{(14i\,dx+14i\,c)}+210\,a^3de^{(12i\,dx+12i\,c)}+252\,a^3de^{(10i\,dx+10i\,c)}+210\,a^3de^{(8i\,dx+8i\,c)}+120\,a^3de^{(6i\,dx+6i\,c)}+45\,a^3de^{(4i\,dx+4i\,c)}+10\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -128/105*(-120*I*e^(6*I*d*x + 6*I*c) - 45*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(2
*I*d*x + 2*I*c) - I)/(a^3*d*e^(20*I*d*x + 20*I*c) + 10*a^3*d*e^(18*I*d*x +
18*I*c) + 45*a^3*d*e^(16*I*d*x + 16*I*c) + 120*a^3*d*e^(14*I*d*x + 14*I*c)
+ 210*a^3*d*e^(12*I*d*x + 12*I*c) + 252*a^3*d*e^(10*I*d*x + 10*I*c) + 210*a
^3*d*e^(8*I*d*x + 8*I*c) + 120*a^3*d*e^(6*I*d*x + 6*I*c) + 45*a^3*d*e^(4*I*
d*x + 4*I*c) + 10*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^{14}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**14/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Giac [A]
time = 0.72, size = 87, normalized size = 0.80

$$-\frac{-21i\tan(dx+c)^{10}+70\tan(dx+c)^{9}+240\tan(dx+c)^{7}+210i\tan(dx+c)^{6}+252\tan(dx+c)^{5}+420i\tan(dx+c)^{4}+315i\tan(dx+c)^{2}-210\tan(dx+c)}{210\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 21
0*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*tan(
d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

Mupad [B]
time = 3.60, size = 119, normalized size = 1.09

$$\frac{\cos(c+d\,x)^{10}\,84{}\mathrm{i}+128\sin(c+d\,x)\cos(c+d\,x)^{9}+64\sin(c+d\,x)\cos(c+d\,x)^{7}+48\sin(c+d\,x)\cos(c+d\,x)^{5}+40\sin(c+d\,x)\cos(c+d\,x)^{3}-\cos(c+d\,x)^{2}\,105{}\mathrm{i}-70\sin(c+d\,x)\cos(c+d\,x)+21{}\mathrm{i}}{210\,a^{3}\,d\cos(c+d\,x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (40*cos(c + d*x)^3*sin(c + d*x) - 70*cos(c + d*x)*sin(c + d*x) + 48*cos(c +
 d*x)^5*sin(c + d*x) + 64*cos(c + d*x)^7*sin(c + d*x) + 128*cos(c + d*x)^9*
sin(c + d*x) - cos(c + d*x)^2*105i + cos(c + d*x)^10*84i + 21i)/(210*a^3*d*
cos(c + d*x)^10)
```

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{2i(a-ia\tan(c+dx))^6}{3a^9d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}$$

```
[Out] 2/3*I*(a-I*a*tan(d*x+c))^6/a^9/d-4/7*I*(a-I*a*tan(d*x+c))^7/a^10/d+1/8*I*(a
-I*a*tan(d*x+c))^8/a^11/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{2i(a-ia\tan(c+dx))^6}{3a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^6)/(a^9*d) - (((4*I)/7)*(a - I*a*Tan[c +
d*x])^7)/(a^10*d) + ((I/8)*(a - I*a*Tan[c + d*x])^8)/(a^11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^5(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^5-4a(a-x)^6+(a-x)^7\right)\,dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{2i(a-ia\tan(c+dx))^6}{3a^9d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}\end{aligned}$$

Mathematica [A]
time = 0.71, size = 106, normalized size = 1.29

$$\frac{\sec(c)\sec^8(c+dx)(-35i\cos(c)-28i\cos(c+2dx)-28i\cos(3c+2dx)-35\sin(c)+28\sin(c+2dx)-28\sin(3c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx))}{168a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^8*((-35*I)*Cos[c] - (28*I)*Cos[c + 2*d*x] - (28*I)*Cos
[3*c + 2*d*x] - 35*Sin[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 28*Si
n[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(168*a^3*d)
```

Maple [A]
time = 0.30, size = 93, normalized size = 1.13

method	result
risch	$\frac{32i\left(28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{21d\,a^3\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{i\left(\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{3i\left(\tan^7(dx+c)\right)}{7}-\frac{5\left(\tan^4(dx+c)\right)}{4}+i\left(\tan^5(dx+c)\right)-\frac{3\left(\tan^2(dx+c)\right)}{2}+\frac{i\left(\tan^3(dx+c)\right)}{3}-i\tan(\right.}{d\,a^3}$
default	$\frac{i\left(\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{3i\left(\tan^7(dx+c)\right)}{7}-\frac{5\left(\tan^4(dx+c)\right)}{4}+i\left(\tan^5(dx+c)\right)-\frac{3\left(\tan^2(dx+c)\right)}{2}+\frac{i\left(\tan^3(dx+c)\right)}{3}-i\tan(\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a^3*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+3/7*I*tan(d*x+c)^7-5/4*tan(d*x+c
)^4+I*tan(d*x+c)^5-3/2*tan(d*x+c)^2+1/3*I*tan(d*x+c)^3-I*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 87, normalized size = 1.06

$$-\frac{-21i\tan(dx+c)^8+72\tan(dx+c)^7+28i\tan(dx+c)^6+168\tan(dx+c)^5+210i\tan(dx+c)^4+56\tan(dx+c)^3+252i\tan(dx+c)^2-168\tan(dx+c)}{168\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.
time = 0.37, size = 153, normalized size = 1.87

$$-\frac{32\left(-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\left(a^3de^{(16i\,dx+16i\,c)}+8\,a^3de^{(14i\,dx+14i\,c)}+28\,a^3de^{(12i\,dx+12i\,c)}+56\,a^3de^{(10i\,dx+10i\,c)}+70\,a^3de^{(8i\,dx+8i\,c)}+56\,a^3de^{(6i\,dx+6i\,c)}+28\,a^3de^{(4i\,dx+4i\,c)}+8\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/21*(-28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(
16*I*d*x + 16*I*c) + 8*a^3*d*e^(14*I*d*x + 14*I*c) + 28*a^3*d*e^(12*I*d*x +
 12*I*c) + 56*a^3*d*e^(10*I*d*x + 10*I*c) + 70*a^3*d*e^(8*I*d*x + 8*I*c) +
56*a^3*d*e^(6*I*d*x + 6*I*c) + 28*a^3*d*e^(4*I*d*x + 4*I*c) + 8*a^3*d*e^(2*
I*d*x + 2*I*c) + a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^{12}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**12/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Giac [**A**]
time = 0.81, size = 87, normalized size = 1.06

$$-\frac{-21i\tan(dx+c)^8+72\tan(dx+c)^7+28i\tan(dx+c)^6+168\tan(dx+c)^5+210i\tan(dx+c)^4+56\tan(dx+c)^3+252i\tan(dx+c)^2-168\tan(dx+c)}{168\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Mupad [**B**]
time = 3.49, size = 103, normalized size = 1.26

$$\frac{\cos(c+d\,x)^8\,91\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^7+64\,\sin(c+d\,x)\,\cos(c+d\,x)^5+48\,\sin(c+d\,x)\,\cos(c+d\,x)^3-\cos(c+d\,x)^2\,112\mathrm{i}-72\,\sin(c+d\,x)\,\cos(c+d\,x)+21\mathrm{i}}{168\,a^3\,d\cos(c+d\,x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (48*cos(c + d*x)^3*sin(c + d*x) - 72*cos(c + d*x)*sin(c + d*x) + 64*cos(c +
 d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) - cos(c + d*x)^2*112
i + cos(c + d*x)^8*91i + 21i)/(168*a^3*d*cos(c + d*x)^8)
```

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=55

$$\frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

```
[Out] 2/5*I*(a-I*a*tan(d*x+c))^5/a^8/d-1/6*I*(a-I*a*tan(d*x+c))^6/a^9/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^8*d) - ((I/6)*(a - I*a*Tan[c + d*x]
)^6)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^4-(a-x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}\end{aligned}$$

Mathematica [A]
time = 0.66, size = 97, normalized size = 1.76

$$\frac{\sec(c)\sec^6(c+dx)(-20i\cos(c)-15i\cos(c+2dx)-15i\cos(3c+2dx)-20\sin(c)+15\sin(c+2dx)-15\sin(3c+2dx)+12\sin(3c+4dx)+2\sin(5c+6dx))}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^6*((-20*I)*Cos[c] - (15*I)*Cos[c + 2*d*x] - (15*I)*Cos
[3*c + 2*d*x] - 20*Sin[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 12*Si
n[3*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(60*a^3*d)
```

Maple [A]
time = 0.29, size = 72, normalized size = 1.31

method	result	size
risch	$\frac{32i\left(6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$	36
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{3i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{2}-\frac{2i\left(\tan^3(dx+c)\right)}{3}+\frac{3\left(\tan^2(dx+c)\right)}{2}\right)}{d\,a^3}$	72
default	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{3i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{2}-\frac{2i\left(\tan^3(dx+c)\right)}{3}+\frac{3\left(\tan^2(dx+c)\right)}{2}\right)}{d\,a^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a^3*(I*tan(d*x+c)-1/6*tan(d*x+c)^6-3/5*I*tan(d*x+c)^5+1/2*tan(d*x+c)^4
-2/3*I*tan(d*x+c)^3+3/2*tan(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 67, normalized size = 1.22

$$\frac{5i\,\tan\left(dx+c\right)^6-18\,\tan\left(dx+c\right)^5-15i\,\tan\left(dx+c\right)^4-20\,\tan\left(dx+c\right)^3-45i\,\tan\left(dx+c\right)^2+30\,\tan\left(dx+c\right)}{30\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*tan(d*x + c)^6 - 18*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 - 45*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.
time = 0.38, size = 112, normalized size = 2.04

$$-\frac{32\left(-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/15*(-6*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^3*
d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*d*
x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c) + a
^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{\sec^{10}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**10/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Giac [A]
time = 1.01, size = 67, normalized size = 1.22

$$-\frac{-5i\tan(dx+c)^6+18\tan(dx+c)^5+15i\tan(dx+c)^4+20\tan(dx+c)^3+45i\tan(dx+c)^2-30\tan(dx+c)}{30\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(-5*I*tan(d*x + c)^6 + 18*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 20*t
an(d*x + c)^3 + 45*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a^3*d)
```

Mupad [B]
time = 3.34, size = 114, normalized size = 2.07

$$-\frac{\sin(c+d\,x)\,\left(-30\cos(c+d\,x)^5+\cos(c+d\,x)^4\sin(c+d\,x)\,45\mathrm{i}+20\cos(c+d\,x)^3\sin(c+d\,x)^2+\cos(c+d\,x)^2\sin(c+d\,x)^3\,15\mathrm{i}+18\,\cos(c+d\,x)\,\sin(c+d\,x)^4-\sin(c+d\,x)^5\,5\mathrm{i}\right)}{30\,a^3\,d\cos(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*x
)*45i - 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a^3*d*cos(c + d*x)^6)
```

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

```
[Out] 1/4*I*(a-I*a*tan(d*x+c))^4/a^7/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^3\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{4a^7d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.
time = 0.52, size = 84, normalized size = 3.11

$$\frac{\sec(c)\sec^4(c+dx)(-3i\cos(c)-2i\cos(c+2dx)-2i\cos(3c+2dx)-3\sin(c)+2\sin(c+2dx)-2\sin(3c+2dx)+\sin(3c+4dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^4*((-3*I)*Cos[c] - (2*I)*Cos[c + 2*d*x] - (2*I)*Cos[3*
c + 2*d*x] - 3*Sin[c] + 2*Sin[c + 2*d*x] - 2*Sin[3*c + 2*d*x] + Sin[3*c + 4
*d*x]))/(4*a^3*d)
```

Maple [A]
time = 0.28, size = 21, normalized size = 0.78

method	result	size
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$	21
default	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$	21
risch	$\frac{4i}{d\,a^3\left(e^{2i(dx+c)}+1\right)^4}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/d/a^3*(tan(d*x+c)+I)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.
time = 0.29, size = 47, normalized size = 1.74

$$-\frac{-i\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^3+6i\,\tan\left(dx+c\right)^2-4\,\tan\left(dx+c\right)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.
time = 0.36, size = 69, normalized size = 2.56

$$\frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^8(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**8/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.
time = 0.75, size = 47, normalized size = 1.74

$$-\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Mupad [B]
time = 3.34, size = 77, normalized size = 2.85

$$-\frac{\sin(c+dx)\left(-4\cos(c+dx)^3+\cos(c+dx)^2\sin(c+dx)\,6\mathrm{i}+4\cos(c+dx)\sin(c+dx)^2-\sin(c+dx)^3\,1\mathrm{i}\right)}{4a^3d\cos(c+dx)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 + cos(c + d*x)^2*sin(c + d*x)
*6i - 4*cos(c + d*x)^3 - sin(c + d*x)^3*1i))/(4*a^3*d*cos(c + d*x)^4)
```

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=58

$$\frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

```
[Out] 4*x/a^3+4*I*ln(cos(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/
d
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\cos(c+dx))}{a^3d}+\frac{4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Cos[c + d*x]])/(a^3*d) - (3*Tan[c + d*x])/(a^3*d) +
((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= \frac{4x}{a^3} + \frac{4i\log(\cos(c+dx))}{a^3 d} - \frac{3\tan(c+dx)}{a^3 d} + \frac{i\tan^2(c+dx)}{2a^3 d}$$

Mathematica [A]
time = 0.53, size = 113, normalized size = 1.95

$$\frac{\sec(c)\sec^2(c+dx)(2dx\cos(3c+2dx)+2\cos(c+2dx)(dx+i\log(\cos(c+dx)))+\cos(c)(i+4dx+4i\log(\cos(c+dx)))+2i\cos(3c+2dx)\log(\cos(c+dx))+3\sin(c)-3\sin(c+2dx))}{2a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^2*(2*d*x*Cos[3*c + 2*d*x] + 2*Cos[c + 2*d*x]*(d*x + I*
Log[Cos[c + d*x]]) + Cos[c]*(I + 4*d*x + (4*I)*Log[Cos[c + d*x]]) + (2*I)*C
os[3*c + 2*d*x]*Log[Cos[c + d*x]] + 3*Sin[c] - 3*Sin[c + 2*d*x]))/(2*a^3*d)
```

Maple [A]
time = 0.30, size = 41, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$	41
default	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$	41
risch	$\frac{8x}{a^3}+\frac{8c}{a^3 d}-\frac{2i\left(2\,e^{2i(dx+c)}+3\right)}{d\,a^3\left(e^{2i(dx+c)}+1\right)^2}+\frac{4i\ln\left(e^{2i(dx+c)}+1\right)}{a^3 d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-3*tan(d*x+c)+1/2*I*tan(d*x+c)^2-4*I*ln(tan(d*x+c)-I))
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.78

$$\frac{\frac{i\,\tan(dx+c)^2-6\,\tan(dx+c)}{a^3}-\frac{8i\,\log(i\,\tan(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((I*tan(d*x + c)^2 - 6*tan(d*x + c))/a^3 - 8*I*log(I*tan(d*x + c) + 1)/
a^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(52) = 104$.
time = 0.37, size = 113, normalized size = 1.95

$$\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3i\right)}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^6(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**6/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(52) = 104$.
time = 0.87, size = 128, normalized size = 2.21

$$\frac{2\left(\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{4i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^3}+\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}+\frac{-3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) - I
)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c)^
```

```
4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x +
 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B]
time = 3.37, size = 41, normalized size = 0.71

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}+6\tan(c+d\,x)-\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3),x)
[Out] -(log(tan(c + d*x) - 1i)*8i + 6*tan(c + d*x) - tan(c + d*x)^2*1i)/(2*a^3*d)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$

Optimal. Leaf size=50

$$-\frac{x}{a^3}-\frac{i\log(\cos(c+dx))}{a^3d}+\frac{2i}{d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] -x/a^3-I*ln(cos(d*x+c))/a^3/d+2*I/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{2i}{d\,(a^3+ia^3\tan(c+dx))}-\frac{i\log(\cos(c+dx))}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (I*Log[Cos[c + d*x]])/(a^3*d) + (2*I)/(d*(a^3 + I*a^3*Tan[c + d*
x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx&=-\frac{i\text{Subst}\left(\int\frac{a-x}{(a+x)^2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{i\text{Subst}\left(\int\left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{x}{a^3}-\frac{i\log(\cos(c+dx))}{a^3d}+\frac{2i}{d\,(a^3+ia^3\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 88, normalized size = 1.76

$$\frac{\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))(-1-idx+\log(\cos(c+dx))+(i+dx+i\log(\cos(c+dx)))\tan(c+dx))}{a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(-1 - I*d*x + Log[C
os[c + d*x]] + (I + d*x + I*Log[Cos[c + d*x]])*Tan[c + d*x]))/(a^3*d*(-I +
Tan[c + d*x])^3)
```

Maple [A]
time = 0.30, size = 35, normalized size = 0.70

method	result	size
derivativedivides	$\frac{i\ln(\tan(dx+c)-i)+\frac{2}{\tan(dx+c)-i}}{d\,a^3}$	35
default	$\frac{i\ln(\tan(dx+c)-i)+\frac{2}{\tan(dx+c)-i}}{d\,a^3}$	35
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{a^3d}-\frac{2x}{a^3}-\frac{2c}{a^3d}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(I*ln(tan(d*x+c)-I)+2/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.29, size = 66, normalized size = 1.32

$$-\frac{\frac{4(-i\tan(dx+c)-1)}{2i\,a^3\tan(dx+c)^2+4\,a^3\tan(dx+c)-2i\,a^3}-\frac{i\log(i\tan(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*(-I*tan(d*x + c) - 1)/(2*I*a^3*tan(d*x + c)^2 + 4*a^3*tan(d*x + c) - 2*
I*a^3) - I*log(I*tan(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.37, size = 55, normalized size = 1.10

$$-\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{\sec^4(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**4/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 0.97, size = 100, normalized size = 2.00

$$-\frac{\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}+\frac{3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 + 10
*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

Mupad [B]
time = 3.41, size = 42, normalized size = 0.84

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a^3\,d}+\frac{2\mathrm{i}}{a^3\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(a^3*d) + 2i/(a^3*d*(tan(c + d*x)*1i + 1))
```

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

```
[Out] 1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx,x,ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{2ad(a+ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 42, normalized size = 1.56

$$-\frac{i\sec^2(c+dx)(-3i+\tan(c+dx))}{8a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-1/8*I)*Sec[c + d*x]^2*(-3*I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^
3)
```

Maple [A]
time = 0.19, size = 24, normalized size = 0.89

method	result	size
derivativedivides	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
default	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
risch	$\frac{i e^{-2i(dx+c)}}{4a^3d}+\frac{i e^{-4i(dx+c)}}{8a^3d}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.78

$$\frac{i}{2\left(i\,a\tan\left(dx+c\right)+a\right)^2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*I/((I*a*tan(d*x + c) + a)^2*a*d)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 1.11

$$\frac{\left(2i\,e^{\left(2i\,dx+2i\,c\right)}+i\right)e^{\left(-4i\,dx-4i\,c\right)}}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(19) = 38.
time = 0.84, size = 153, normalized size = 5.67

$$\begin{cases} -\frac{i\tan\left(c+dx\right)\sec^2\left(c+dx\right)}{8a^3d\tan^3\left(c+dx\right)-24ia^3d\tan^2\left(c+dx\right)-24a^3d\tan\left(c+dx\right)+8ia^3d}-\frac{3\sec^2\left(c+dx\right)}{8a^3d\tan^3\left(c+dx\right)-24ia^3d\tan^2\left(c+dx\right)-24a^3d\tan\left(c+dx\right)+8ia^3d} & \text{for}\: d\neq 0 \\ \frac{x\sec^2\left(c\right)}{\left(ia\tan\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)*sec(c + d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I
*a**3*d*tan(c + d*x)**2 - 24*a**3*d*tan(c + d*x) + 8*I*a**3*d) - 3*sec(c +
d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I*a**3*d*tan(c + d*x)**2 - 24*a**3*d
*tan(c + d*x) + 8*I*a**3*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**3, T
rue))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.
time = 0.91, size = 57, normalized size = 2.11

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

Mupad [B]
time = 3.35, size = 20, normalized size = 0.74

$$-\frac{1\mathrm{i}}{2\,a^{3}\,d\,\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -1i/(2*a^3*d*(tan(c + d*x) - 1i)^2)
```

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 1/8*x/a^3+1/6*I/d/(a+I*a*tan(d*x+c))^3+1/8*I/a/d/(a+I*a*tan(d*x+c))^2+1/8*I
/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{3560, 8\}$

$$\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{x}{8a^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) + (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(a*d*(a + I*a*Tan[c
+ d*x])^2) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{2a}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d(a^3+ia^3\tan(c+dx}\\
&= \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d(a^3+ia^3\tan(}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 93, normalized size = 1.06

$$\frac{i\sec^3(c+dx)(27i\cos(c+dx)+2(i+6dx)\cos(3(c+dx))-9\sin(c+dx)+2\sin(3(c+dx))+12idx\sin(3(c+dx)))}{96a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] ((I/96)*Sec[c + d*x]^3*((27*I)*Cos[c + d*x] + 2*(I + 6*d*x)*Cos[3*(c + d*x)
] - 9*Sin[c + d*x] + 2*Sin[3*(c + d*x)] + (12*I)*d*x*Sin[3*(c + d*x)]))/(a^
3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.85

method	result
risch	$\frac{x}{8a^3}+\frac{3i\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{48a^3d}$
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$
norman	$\frac{\frac{x}{8a}+\frac{\tan^5(dx+c)}{8da}+\frac{3x\left(\tan^2(dx+c)\right)}{8a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{8a}+\frac{5i}{12da}+\frac{7\tan(dx+c)}{8da}+\frac{\tan^3(dx+c)}{3da}-\frac{i\left(\tan^2(dx+c)\right)}{4da}}{a^2\left(1+\tan^2(dx+c)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/16*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2-1/6/(tan(d*x+c)-I)
^3+1/8/(tan(d*x+c)-I)+1/16*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.61

$$\frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.16, size = 155, normalized size = 1.76

$$\begin{cases} \frac{\left(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), True))
+ x/(8*a**3)
```

Giac [A]
time = 0.67, size = 80, normalized size = 0.91

$$-\frac{\frac{6i\log(\tan(dx+c)-i)}{a^3}-\frac{6i\log(i\tan(dx+c)-1)}{a^3}+\frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.49, size = 50, normalized size = 0.57

$$\frac{x}{8\,a^3}-\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,1{}\mathrm{i}}{8}+\frac{3\,\mathrm{tan}\left(c+d\,x\right)}{8}-\frac{5}{12}{}\mathrm{i}}{a^3\,d\,{\left(1+\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] x/(8*a^3) - ((3*tan(c + d*x))/8 + (tan(c + d*x)^2*1i)/8 - 5i/12)/(a^3*d*(ta
n(c + d*x)*1i + 1)^3)
```

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$\frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}-\frac{i}{32d\left(a^3-ia^3\tan(\right.}$$

```
[Out] 5/32*x/a^3+1/16*I*a/d/(a+I*a*tan(d*x+c))^4+1/12*I/d/(a+I*a*tan(d*x+c))^3+3/
32*I/a/d/(a+I*a*tan(d*x+c))^2-1/32*I/d/(a^3-I*a^3*tan(d*x+c))+1/8*I/d/(a^3+
I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{i}{32d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(d*(a + I
*a*Tan[c + d*x])^3) + ((3*I)/32)/(a*d*(a + I*a*Tan[c + d*x])^2) - (I/32)/(d
*(a^3 - I*a^3*Tan[c + d*x])) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2} + \frac{1}{4a^2(a+x)^5} + \frac{1}{4a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{1}{8a^5(a+x)^2} + \frac{}{32}\right.\right.}{d}$$

$$= \frac{ia}{16d(a+ia\tan(c+dx))^4} + \frac{i}{12d(a+ia\tan(c+dx))^3} + \frac{3i}{32ad(a+ia\tan(c+d}$$

$$= \frac{5x}{32a^3} + \frac{ia}{16d(a+ia\tan(c+dx))^4} + \frac{i}{12d(a+ia\tan(c+dx))^3} + \frac{3i}{32ad(a+ia\,\text{ta}}$$

Mathematica [A]
time = 0.34, size = 115, normalized size = 0.82

$$\frac{\sec^3(c+dx)(-180\cos(c+dx)+20i(i+6dx)\cos(3(c+dx))+9\cos(5(c+dx))-60i\sin(c+dx)+20i\sin(3(c+dx))-120dx\sin(3(c+dx))+15i\sin(5(c+dx)))}{768a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-180*Cos[c + d*x] + (20*I)*(I + 6*d*x)*Cos[3*(c + d*x)] +
9*Cos[5*(c + d*x)] - (60*I)*Sin[c + d*x] + (20*I)*Sin[3*(c + d*x)] - 120*d*
x*Sin[3*(c + d*x)] + (15*I)*Sin[5*(c + d*x)]))/(768*a^3*d*(-I + Tan[c + d*x
])^3)
```

Maple [A]
time = 0.29, size = 102, normalized size = 0.72

method	result
risch	$\frac{5x}{32a^3}+\frac{5i e^{-4i(dx+c)}}{64a^3d}+\frac{5i e^{-6i(dx+c)}}{192a^3d}+\frac{i e^{-8i(dx+c)}}{256a^3d}+\frac{9i\cos(2dx+2c)}{64a^3d}+\frac{11\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan(}}{d\,a^3}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan(}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-5/64*I*ln(tan(d*x+c)-I)+1/16*I/(tan(d*x+c)-I)^4-3/32*I/(tan(d*x+c
)-I)^2-1/12/(tan(d*x+c)-I)^3+1/8/(tan(d*x+c)-I)+5/64*I*ln(tan(d*x+c)+I)+1/3
2/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.54

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A]
time = 0.28, size = 224, normalized size = 1.59

$$\begin{cases}\frac{\left(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4e^{12ic}e^{-8idx}\right)e^{-20ic}}{6442450944a^{15}d^5} & \text{for}\ a^{15}d^5e^{20ic}\neq 0\\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) & \text{otherwise}\end{cases}+\frac{5x}{32a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) - 5/(32*
a**3)), True)) + 5*x/(32*a**3)
```

Giac [A]
time = 0.81, size = 119, normalized size = 0.84

$$-\frac{-\frac{60i\,\log(-i\,\tan(dx+c)+1)}{a^3}+\frac{60i\,\log(-i\,\tan(dx+c)-1)}{a^3}-\frac{12\,(5\,\tan(dx+c)+7i)}{a^3(i\,\tan(dx+c)-1)}+\frac{-125i\,\tan(dx+c)^4-596\,\tan(dx+c)^3+1110i\,\tan(dx+c)^2+996\,\tan(dx+c)-405i}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(-I*tan(d*x + c) + 1)/a^3 + 60*I*log(-I*tan(d*x + c) - 1)/
a^3 - 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*
x + c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) -
405*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.68, size = 124, normalized size = 0.88

$$\frac{5\,x}{32\,a^3}+\frac{\frac{1}{3\,a^3}+\frac{35\,\tan(c+d\,x)^2}{96\,a^3}-\frac{5\,\tan(c+d\,x)^4}{32\,a^3}+\frac{\tan(c+d\,x)\,5\mathrm{i}}{32\,a^3}+\frac{\tan(c+d\,x)^3\,15\mathrm{i}}{32\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,3\mathrm{i}+2\,\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,2\mathrm{i}+3\,\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (5*x)/(32*a^3) + ((tan(c + d*x)*5i)/(32*a^3) + 1/(3*a^3) + (35*tan(c + d*x)
^2)/(96*a^3) + (tan(c + d*x)^3*15i)/(32*a^3) - (5*tan(c + d*x)^4)/(32*a^3))
/(d*(3*tan(c + d*x) + tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4
*3i - tan(c + d*x)^5 - 1i))
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{21x}{128a^3}-\frac{i}{128ad(a-ia\tan(c+dx))^2}+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}+\frac{i}{16d(a+ia\tan(}$$

```
[Out] 21/128*x/a^3-1/128*I/a/d/(a-I*a*tan(d*x+c))^2+1/40*I*a^2/d/(a+I*a*tan(d*x+c
))^5+3/64*I*a/d/(a+I*a*tan(d*x+c))^4+1/16*I/d/(a+I*a*tan(d*x+c))^3+5/64*I/a
/d/(a+I*a*tan(d*x+c))^2-3/64*I/d/(a^3-I*a^3*tan(d*x+c))+15/128*I/d/(a^3+I*a
^3*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{3i}{64d(a^3-ia^3\tan(c+dx))}+\frac{15i}{128d(a^3+ia^3\tan(c+dx))}+\frac{21x}{128a^3}+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}+\frac{i}{16d(a+ia\tan(c+dx))^3}-\frac{i}{128ad(a-ia\tan(c+dx))^2}+\frac{5i}{64ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (21*x)/(128*a^3) - (I/128)/(a*d*(a - I*a*Tan[c + d*x])^2) + ((I/40)*a^2)/(d
*(a + I*a*Tan[c + d*x])^5) + (((3*I)/64)*a)/(d*(a + I*a*Tan[c + d*x])^4) +
(I/16)/(d*(a + I*a*Tan[c + d*x])^3) + ((5*I)/64)/(a*d*(a + I*a*Tan[c + d*x]
)^2) - ((3*I)/64)/(d*(a^3 - I*a^3*Tan[c + d*x])) + ((15*I)/128)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^5)\,\mathrm{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \right.\right.}{d}$$
$$= -\frac{i}{128ad(a-ia\tan(c+dx))^2} + \frac{ia^2}{40d(a+ia\tan(c+dx))^5} + \frac{3ia}{64d(a+ia\tan(c+}$$
$$= \frac{21x}{128a^3} - \frac{i}{128ad(a-ia\tan(c+dx))^2} + \frac{ia^2}{40d(a+ia\tan(c+dx))^5} + \frac{}{64d(a+ia}$$

Mathematica [A]
time = 0.59, size = 137, normalized size = 0.70

$$\frac{\sec^3(c+dx)(-1050\cos(c+dx) + 140i(i+6dx)\cos(3(c+dx)) + 105\cos(5(c+dx)) + 6\cos(7(c+dx)) - 350i\sin(c+dx) + 140i\sin(3(c+dx)) - 840dx\sin(3(c+dx)) + 175i\sin(5(c+dx)) + 14i\sin(7(c+dx)))}{5120a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-1050*Cos[c + d*x] + (140*I)*(I + 6*d*x)*Cos[3*(c + d*x)]
+ 105*Cos[5*(c + d*x)] + 6*Cos[7*(c + d*x)] - (350*I)*Sin[c + d*x] + (140*I
)*Sin[3*(c + d*x)] - 840*d*x*Sin[3*(c + d*x)] + (175*I)*Sin[5*(c + d*x)] +
(14*I)*Sin[7*(c + d*x)]))/(5120*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.27, size = 129, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{21i\ln(\tan(dx+c)-i)}{256} + \frac{3i}{64(\tan(dx+c)-i)^4} - \frac{5i}{64(\tan(dx+c)-i)^2} + \frac{1}{40(\tan(dx+c)-i)^5} - \frac{1}{16(\tan(dx+c)-i)^3} + \frac{15}{128(\tan(dx+c)-i)} + \frac{}{128(}}{d\,a^3}$
default	$\frac{-\frac{21i\ln(\tan(dx+c)-i)}{256} + \frac{3i}{64(\tan(dx+c)-i)^4} - \frac{5i}{64(\tan(dx+c)-i)^2} + \frac{1}{40(\tan(dx+c)-i)^5} - \frac{1}{16(\tan(dx+c)-i)^3} + \frac{15}{128(\tan(dx+c)-i)} + \frac{}{128(}}{d\,a^3}$
risch	$\frac{21x}{128a^3} + \frac{7i\mathrm{e}^{-6i(dx+c)}}{256a^3d} + \frac{7i\mathrm{e}^{-8i(dx+c)}}{1024a^3d} + \frac{i\mathrm{e}^{-10i(dx+c)}}{1280a^3d} + \frac{17i\cos(4dx+4c)}{256a^3d} + \frac{9\sin(4dx+4c)}{128a^3d} + \frac{7i\cos(2dx+2c)}{64a^3d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-21/256*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-5/64*I/(tan(d*x
+c)-I)^2+1/40/(tan(d*x+c)-I)^5-1/16/(tan(d*x+c)-I)^3+15/128/(tan(d*x+c)-I)+
1/128*I/(tan(d*x+c)+I)^2+21/256*I*ln(tan(d*x+c)+I)+3/64/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.50

$$\frac{\left(840\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(14i\,dx+14i\,c)}-140i\,e^{(12i\,dx+12i\,c)}+700i\,e^{(8i\,dx+8i\,c)}+350i\,e^{(6i\,dx+6i\,c)}+140i\,e^{(4i\,dx+4i\,c)}+35i\,e^{(2i\,dx+2i\,c)}+4i\right)e^{(-10i\,dx-10i\,c)}}{5120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5120*(840*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(14*I*d*x + 14*I*c) - 140*I*
e^(12*I*d*x + 12*I*c) + 700*I*e^(8*I*d*x + 8*I*c) + 350*I*e^(6*I*d*x + 6*I*
c) + 140*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 4*I)*e^(-10*I*d
*x - 10*I*c)/(a^3*d)
```

Sympy [A]
time = 0.31, size = 292, normalized size = 1.50

$$\begin{cases} \frac{\left(-11258999068426240ia^{18}d^6e^{34ic}e^{4idx}-157625986957967360ia^{18}d^6e^{32ic}e^{2idx}+788129934789836800ia^{18}d^6e^{28ic}e^{-2idx}+394064967394918400ia^{18}d^6e^{26ic}e^{-4idx}+157625986957967360ia^{18}d^6e^{24ic}e^{-6idx}+39406496739491840ia^{18}d^6e^{22ic}e^{-8idx}+4503599627370496ia^{18}d^6e^{20ic}e^{-10idx}\right)e^{-30ic}}{5764607523034234880a^{21}d^7} & \text{for}\ a^{21}d^7e^{30ic}\neq 0 \\ x\left(\frac{\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-10ic}}{128a^3}-\frac{21}{128a^3}\right) & \text{otherwise} \end{cases}+\frac{21x}{128a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-11258999068426240*I*a**18*d**6*exp(34*I*c)*exp(4*I*d*x) - 1576
25986957967360*I*a**18*d**6*exp(32*I*c)*exp(2*I*d*x) + 788129934789836800*I
*a**18*d**6*exp(28*I*c)*exp(-2*I*d*x) + 394064967394918400*I*a**18*d**6*exp
(26*I*c)*exp(-4*I*d*x) + 157625986957967360*I*a**18*d**6*exp(24*I*c)*exp(-6
*I*d*x) + 39406496739491840*I*a**18*d**6*exp(22*I*c)*exp(-8*I*d*x) + 450359
9627370496*I*a**18*d**6*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(576460752
3034234880*a**21*d**7), Ne(a**21*d**7*exp(30*I*c), 0)), (x*((exp(14*I*c) +
7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I
*c) + 7*exp(2*I*c) + 1)*exp(-10*I*c)/(128*a**3) - 21/(128*a**3)), True)) +
21*x/(128*a**3)
```

Giac [A]
time = 0.74, size = 136, normalized size = 0.70

$$-\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^3}+\frac{420i\log(\tan(dx+c)-i)}{a^3}+\frac{10\left(-63i\tan(dx+c)^2+150\tan(dx+c)+91i\right)}{a^3(i\tan(dx+c)-1)^2}-\frac{959i\tan(dx+c)^5+5395\tan(dx+c)^4-12390i\tan(dx+c)^3-14710\tan(dx+c)^2+9275i\tan(dx+c)+2647}{a^3(\tan(dx+c)-i)^5}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*(-420*I*log(tan(d*x + c) + I)/a^3 + 420*I*log(tan(d*x + c) - I)/a^3
 + 10*(-63*I*tan(d*x + c)^2 + 150*tan(d*x + c) + 91*I)/(a^3*(I*tan(d*x + c)
 - 1)^2) - (959*I*tan(d*x + c)^5 + 5395*tan(d*x + c)^4 - 12390*I*tan(d*x +
c)^3 - 14710*tan(d*x + c)^2 + 9275*I*tan(d*x + c) + 2647)/(a^3*(tan(d*x + c
) - I)^5))/d
```

Mupad [B]
time = 5.04, size = 173, normalized size = 0.89

$$\frac{21\,x}{128\,a^3}+\frac{\frac{7\tan(c+d\,x)}{640\,a^3}+\frac{11\mathrm{i}}{40\,a^3}+\frac{\tan(c+d\,x)^2\,469\mathrm{i}}{640\,a^3}-\frac{21\tan(c+d\,x)^3}{32\,a^3}+\frac{\tan(c+d\,x)^4\,7\mathrm{i}}{32\,a^3}-\frac{63\tan(c+d\,x)^5}{128\,a^3}-\frac{\tan(c+d\,x)^6\,21\mathrm{i}}{128\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^7\,1\mathrm{i}-3\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,1\mathrm{i}-5\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,5\mathrm{i}-\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (21*x)/(128*a^3) + ((7*tan(c + d*x))/(640*a^3) + 11i/(40*a^3) + (tan(c + d*
x)^2*469i)/(640*a^3) - (21*tan(c + d*x)^3)/(32*a^3) + (tan(c + d*x)^4*7i)/(
32*a^3) - (63*tan(c + d*x)^5)/(128*a^3) - (tan(c + d*x)^6*21i)/(128*a^3))/(
d*(tan(c + d*x)*3i - tan(c + d*x)^2 + tan(c + d*x)^3*5i - 5*tan(c + d*x)^4
+ tan(c + d*x)^5*1i - 3*tan(c + d*x)^6 - tan(c + d*x)^7*1i + 1))
```

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] 7/8*arctanh(sin(d*x+c))/a^3/d-7/15*I*sec(d*x+c)^5/a^3/d+7/8*sec(d*x+c)*tan(
d*x+c)/a^3/d+7/12*sec(d*x+c)^3*tan(d*x+c)/a^3/d-2/3*I*sec(d*x+c)^7/a/d/(a+I
*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3581, 3582, 3853, 3855}

$$-\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{7\tan(c+dx)\sec^3(c+dx)}{12a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((7*I)/15)*Sec[c + d*x]^5)/(a^3*d) +
 (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/
(12*a^3*d) - (((2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+\frac{7\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a^2} \\
&= -\frac{7i\sec^5(c+dx)}{15a^3d}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+\frac{7\int \sec^5(c+dx)\,dx}{3a^3} \\
&= -\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+ \\
&= -\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\sec(c+dx)\tan(c+dx)}{8a^3d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d}- \\
&= \frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\sec(c+dx)\tan(c+dx)}{8a^3d}+
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 113, normalized size = 0.95

$$\frac{\sec^8(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))\left(448+1680i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^5(c+dx)+640\cos(2(c+dx))-150i\sin(2(c+dx))+105i\sin(4(c+dx))\right)}{960a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^8*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(448 + (1680*I)*Arc
Tanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c + d*x)] -
(150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(960*a^3*d*(-I + Tan[
c + d*x])^3)
```

Maple [A]
time = 0.30, size = 206, normalized size = 1.73

method	result

risch	$-\frac{i\left(105\,e^{9i(dx+c)}+490\,e^{7i(dx+c)}+896\,e^{5i(dx+c)}+790\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{60d\,a^{3}\left(e^{2i(dx+c)}+1\right)^{5}}+\frac{7\ln\left(e^{i(dx+c)}+i\right)}{8a^{3}d}-\frac{7\ln\left(e^{i(dx+c)}-i\right.}{8a^{3}d}$
derivativedivides	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{a^{3}}$
default	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-1/10*I/(tan(1/2*d*x+1/2*c)-1)^5+(1/16+13/16*I)/(tan(1/2*d*x+1/2*c
)-1)-(3/8+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^4+(-5/16+11/16*I)/(tan(1/2*d*x+1/2*
c)-1)^2+(-3/4+7/24*I)/(tan(1/2*d*x+1/2*c)-1)^3-7/16*ln(tan(1/2*d*x+1/2*c)-1
)+1/10*I/(tan(1/2*d*x+1/2*c)+1)^5+(5/16+11/16*I)/(tan(1/2*d*x+1/2*c)+1)^2+(
3/8-1/4*I)/(tan(1/2*d*x+1/2*c)+1)^4+(1/16-13/16*I)/(tan(1/2*d*x+1/2*c)+1)-(
3/4+7/24*I)/(tan(1/2*d*x+1/2*c)+1)^3+7/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(103) = 206$.
time = 0.31, size = 341, normalized size = 2.87

$$\frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{390i\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{400\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{960\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{390i\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{360\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{15i\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-136\right)}{-120i\,a^{3}+\frac{600i\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1200i\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1200i\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{600i\,a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{120i\,a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}-\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(103) = 206$.
time = 0.37, size = 278, normalized size = 2.34

$$\frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(9i\,dx+9i\,c)}-980i\,e^{(7i\,dx+7i\,c)}-1792i\,e^{(5i\,dx+5i\,c)}-1580i\,e^{(3i\,dx+3i\,c)}+210i\,e^{(i\,dx+i\,c)}}{120\left(a^{3}de^{(10i\,dx+10i\,c)}+5\,a^{3}de^{(8i\,dx+8i\,c)}+10\,a^{3}de^{(6i\,dx+6i\,c)}+10\,a^{3}de^{(4i\,dx+4i\,c)}+5\,a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^{9}(c+dx)}{\tan^{3}(c+dx)-3i\tan^{2}(c+dx)-3\tan(c+dx)+i}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**9/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [A]
time = 0.74, size = 164, normalized size = 1.38

$$\frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{3}}-\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^{3}}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-960i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-320i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+136i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{5}a^{3}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 - 3
90*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2*d*
x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^2 -
15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))/d
```

Mupad [B]
time = 6.08, size = 150, normalized size = 1.26

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^{3}\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\,6\mathrm{i}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\,16\mathrm{i}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,20\mathrm{i}}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,16\mathrm{i}}{3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{34}{15}\mathrm{i}}{a^{3}\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (t
an(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4*2
0i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2 +
(d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x)/2)
^2 - 1)^5)
```

3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] 5/2*arctanh(sin(d*x+c))/a^3/d-5/3*I*sec(d*x+c)^3/a^3/d+5/2*sec(d*x+c)*tan(d
*x+c)/a^3/d-2*I*sec(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3581, 3582, 3853, 3855}

$$-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{5\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (((5*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((2*I)*Sec[c + d*x]^5)/(a*d*(a +
I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2}\\ &= -\frac{5i\sec^3(c+dx)}{3a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int \sec^3(c+dx)\,dx}{a^3}\\ &= -\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+ 5\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2}{ad(a}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 63, normalized size = 0.68

$$\frac{60\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec^3(c+dx)(20+24\cos(2(c+dx))-9i\sin(2(c+dx)))}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
[Out] (60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x]^3*(20 + 24*Cos[2
*(c + d*x)] - (9*I)*Sin[2*(c + d*x)]))/(12*a^3*d)
```

Maple [A]
time = 0.28, size = 138, normalized size = 1.48

method	result
risch	$-\frac{i\left(15\,e^{5i(dx+c)}+40\,e^{3i(dx+c)}+33\,e^{i(dx+c)}\right)}{3d\,a^3\left(e^{2i(dx+c)}+1\right)^3}+\frac{5\ln\left(e^{i(dx+c)}+i\right)}{2a^3d}-\frac{5\ln\left(e^{i(dx+c)}-i\right)}{2a^3d}$
derivativedivides	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}{a^3d}$

default	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+}{a^{3}d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 2/d/a^3*(-1/6*I/(tan(1/2*d*x+1/2*c)-1)^3-(3/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^2+(-3/4+7/4*I)/(tan(1/2*d*x+1/2*c)-1)-5/4*ln(tan(1/2*d*x+1/2*c)-1)+1/6*I/(tan(1/2*d*x+1/2*c)+1)^3+(3/4-1/4*I)/(tan(1/2*d*x+1/2*c)+1)^2-(3/4+7/4*I)/(tan(1/2*d*x+1/2*c)+1)+5/4*ln(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(81) = 162.
time = 0.30, size = 215, normalized size = 2.31

$$\frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6i\,a^3-\frac{18i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(81) = 162.
time = 0.38, size = 182, normalized size = 1.96

$$\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(5i\,dx+5i\,c)}-80i\,e^{(3i\,dx+3i\,c)}-66i\,e^{(i\,dx+i\,c)}}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{\sec^7(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**7/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [A]
time = 0.72, size = 112, normalized size = 1.20

$$\frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}-\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-22i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*ta
n(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/2*c
)^2 - 1)^3*a^3))/d
```

Mupad [B]
time = 5.44, size = 135, normalized size = 1.45

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}}{a^3}+\frac{22\mathrm{i}}{3\,a^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (t
an(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3) +
 (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)
/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3i\sec(c+dx)}{a^3d}+\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+3*I*sec(d*x+c)/a^3/d+2*I*sec(d*x+c)^3/a/d/(a+I
*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3582, 3855}

$$\frac{3i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((3*I)*Sec[c + d*x])/(a^3*d) + ((2*I)*
Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2}$$
$$= \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \sec(c+dx)\,dx}{a^3}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Mathematica [A]
time = 0.38, size = 108, normalized size = 1.66

$$\frac{\sec^3(c+dx)(i\cos(dx)-\sin(dx))^3\left(6\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)(\cos(3c)+i\sin(3c))+(\cos(2c-dx)+i\sin(2c-dx))(-5i+\tan(c+dx))\right)}{a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
[Out] (Sec[c + d*x]^3*(I*Cos[d*x] - Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d
*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-5*I
 + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.28, size = 86, normalized size = 1.32

method	result	size
derivativedivides	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	86
default	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	86
risch	$\frac{4i\,e^{-i(dx+c)}}{a^3d}+\frac{2i\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}+1\right)}+\frac{3\ln\left(e^{i(dx+c)}-i\right)}{a^3d}-\frac{3\ln\left(e^{i(dx+c)}+i\right)}{a^3d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 2/d/a^3*(-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan(1/2*d*x+1/2*c)-1)+1/2*I/(
tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+1)+4/(-I+tan(1/2*d*x+1/2*c)
))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(59) = 118.
time = 0.52, size = 319, normalized size = 4.91

$$\frac{6 \left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)+6\left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+3\left(i\,\cos\left(3\,dx+3\,c\right)+i\,\cos\left(dx+c\right)-\sin\left(3\,dx+3\,c\right)-\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)+3\left(-i\,\cos\left(3\,dx+3\,c\right)-i\,\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)+12\,\cos\left(2\,dx+2\,c\right)+12i\,\sin\left(2\,dx+2\,c\right)+8}{-2\left(i\,a^3\cos\left(3\,dx+3\,c\right)+i\,a^3\cos\left(dx+c\right)-a^3\sin\left(3\,dx+3\,c\right)-a^3\sin\left(dx+c\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*
arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x + c
) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c
) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - sin(d*
x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(-I*c
os(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x + 2*c) +
12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*cos(d*x +
c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

Fricas [A]
time = 0.41, size = 112, normalized size = 1.72

$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}-4i}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sec^5(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**5/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Giac [A]
time = 0.73, size = 110, normalized size = 1.69

$$-\frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}-\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^3
 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*x
+ 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/d
```

Mupad [B]
time = 3.77, size = 105, normalized size = 1.62

$$-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] - (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=32

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

```
[Out] 1/3*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3569\}$

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 1.00

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Maple [A]
time = 0.31, size = 57, normalized size = 1.78

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{3a^3d}$	19
derivativedivides	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
default	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 2/d/a^3*(-4/3/(-I+tan(1/2*d*x+1/2*c))^3+1/(-I+tan(1/2*d*x+1/2*c))+2*I/(-I+t
an(1/2*d*x+1/2*c))^2)
```

Maxima [A]
time = 0.30, size = 29, normalized size = 0.91

$$\frac{i \cos(3\,dx+3\,c)+\sin(3\,dx+3\,c)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A]
time = 0.36, size = 17, normalized size = 0.53

$$\frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.
time = 0.90, size = 80, normalized size = 2.50

$$\begin{cases} -\frac{\sec^3(c+dx)}{3a^3d\tan^3(c+dx)-9ia^3d\tan^2(c+dx)-9a^3d\tan(c+dx)+3ia^3d} & \text{for } d\neq 0 \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-sec(c + d*x)**3/(3*a**3*d*tan(c + d*x)**3 - 9*I*a**3*d*tan(c +
d*x)**2 - 9*a**3*d*tan(c + d*x) + 3*I*a**3*d), Ne(d, 0)), (x*sec(c)**3/(I*a
*tan(c) + a)**3, True))
```

Giac [A]
time = 0.66, size = 36, normalized size = 1.12

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B]
time = 3.44, size = 68, normalized size = 2.12

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 1/5*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^3+2/15*I*sec(d*x+c)/a/d/(a+I*a*tan(d*
x+c))^2+2/15*I*sec(d*x+c)/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3583, 3569}

$$\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/15)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^2) + (((2*I)/15)*Sec[c + d*x])/(d*(a^3 + I*a
^3*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{5a}$$
$$= \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{2\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{15a^2}$$
$$= \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{2i\sec(c+dx)}{15d\,(a^3+ia^3\tan(c+$$

Mathematica [A]
time = 0.23, size = 54, normalized size = 0.55

$$-\frac{\sec^3(c+dx)(5+9\cos(2(c+dx))+6i\sin(2(c+dx)))}{30a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/30*(Sec[c + d*x]^3*(5 + 9*Cos[2*(c + d*x)] + (6*I)*Sin[2*(c + d*x)]))/(a
^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.15, size = 90, normalized size = 0.92

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-3i(dx+c)}}{6a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}$	56
derivativedivides	$\frac{-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	90
default	$\frac{-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-2*I/(-I+tan(1/2*d*x+1/2*c))^4+2*I/(-I+tan(1/2*d*x+1/2*c))^2+4/5/(
-I+tan(1/2*d*x+1/2*c))^5-8/3/(-I+tan(1/2*d*x+1/2*c))^3+1/(-I+tan(1/2*d*x+1/
2*c)))
```

Maxima [A]
time = 0.31, size = 69, normalized size = 0.70

$$\frac{3i\cos(5\,dx+5\,c)+10i\cos(3\,dx+3\,c)+15i\cos(dx+c)+3\sin(5\,dx+5\,c)+10\sin(3\,dx+3\,c)+15\sin(dx+c)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Fricas [A]
time = 0.38, size = 41, normalized size = 0.42

$$\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.
time = 0.91, size = 219, normalized size = 2.23

$$\begin{cases} \frac{2\tan^2(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d}-\frac{6i\tan(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d}-\frac{7\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d} & \text{for } d\neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((2*tan(c + d*x)**2*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I
*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 6*I*tan(c
 + d*x)*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)*
*2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 7*sec(c + d*x)/(15*a**3*d*tan(
c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a
**3*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**3, True))
```

Giac [A]
time = 0.59, size = 73, normalized size = 0.74

$$\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

Mupad [B]
time = 3.68, size = 133, normalized size = 1.36

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i - 10
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=101

$$\frac{12\sin(c+dx)}{35a^3d}-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] 12/35*sin(d*x+c)/a^3/d-4/35*sin(d*x+c)^3/a^3/d+1/7*I*cos(d*x+c)/d/(a+I*a*ta
n(d*x+c))^3+8/35*I*cos(d*x+c)^3/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3583, 3581, 2713}

$$-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{12\sin(c+dx)}{35a^3d}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (12*Sin[c + d*x])/(35*a^3*d) - (4*Sin[c + d*x]^3)/(35*a^3*d) + ((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((8*I)/35)*Cos[c + d*x]^3)/(d*(a^3
 + I*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
```

```
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{4\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{7a}\\ &= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}+\frac{12\int \cos^3(c+dx)\,dx}{35a^3}\\ &= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}-\frac{12\text{Subst}\left(\int (1-x^2)\,d\right.}{35a^3}\\ &= \frac{12\sin(c+dx)}{35a^3d}-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+}{35d\,(a^3+ia^3\tan}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 76, normalized size = 0.75

$$-\frac{\sec^3(c+dx)(35+84\cos(2(c+dx))-15\cos(4(c+dx))+56i\sin(2(c+dx))-20i\sin(4(c+dx)))}{280a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/280*(Sec[c + d*x]^3*(35 + 84*Cos[2*(c + d*x)] - 15*Cos[4*(c + d*x)] + (5
6*I)*Sin[2*(c + d*x)] - (20*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x]
)^3)
```

Maple [A]
time = 0.31, size = 141, normalized size = 1.40

method	result
risch	$\frac{i e^{-3i(dx+c)}}{8a^3d}+\frac{i e^{-5i(dx+c)}}{20a^3d}+\frac{i e^{-7i(dx+c)}}{112a^3d}+\frac{3i\cos(dx+c)}{16a^3d}+\frac{5\sin(dx+c)}{16a^3d}$
derivativedivides	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^3d}$
default	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(1/16/(tan(1/2*d*x+1/2*c)+I)+2*I/(-I+tan(1/2*d*x+1/2*c))^6-9/2*I/(-
I+tan(1/2*d*x+1/2*c))^4+17/8*I/(-I+tan(1/2*d*x+1/2*c))^2-4/7/(-I+tan(1/2*d*
x+1/2*c))^7+19/5/(-I+tan(1/2*d*x+1/2*c))^5-15/4/(-I+tan(1/2*d*x+1/2*c))^3+1
5/16/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 63, normalized size = 0.62

$$\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(87) = 174$.
time = 0.31, size = 197, normalized size = 1.95

$$\begin{cases} \frac{\left(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} & \text{for } a^{15}d^5e^{16ic}\neq 0 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) +
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(16*I
*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*
exp(-7*I*c)/(16*a**3), True))
```

Giac [A]
time = 0.77, size = 119, normalized size = 1.18

$$\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-243}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B]
time = 5.90, size = 134, normalized size = 1.33

$$-\frac{\left(35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,105\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,105\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,77\mathrm{i}+43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-13\mathrm{i}\right)\,2\mathrm{i}}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/
2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x)/2
) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$\frac{10\sin(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{2\sin^5(c+dx)}{21a^3d}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 10/21*sin(d*x+c)/a^3/d-20/63*sin(d*x+c)^3/a^3/d+2/21*sin(d*x+c)^5/a^3/d+1/9
*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3+4/21*I*cos(d*x+c)^5/d/(a^3+I*a^3*tan
(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$\frac{2\sin^5(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{10\sin(c+dx)}{21a^3d}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (10*Sin[c + d*x])/(21*a^3*d) - (20*Sin[c + d*x]^3)/(63*a^3*d) + (2*Sin[c +
d*x]^5)/(21*a^3*d) + ((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) +
(((4*I)/21)*Cos[c + d*x]^5)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{2\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{3a}\\
&= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{10\int \cos^5(c+dx)\,dx}{21a^3}\\
&= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{10\text{Subst}\left(\int (1-2x^2+\cdots\right.}{2\cdots}\\
&= \frac{10\sin(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{2\sin^5(c+dx)}{21a^3d}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}-\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.33, size = 98, normalized size = 0.81

$$
\frac{\sec^3(c+dx)(-210-567\cos(2(c+dx))+162\cos(4(c+dx))+7\cos(6(c+dx))-378i\sin(2(c+dx))+216i\sin(4(c+dx))+14i\sin(6(c+dx)))}{2016a^3d(-i+\tan(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-210 - 567*Cos[2*(c + d*x)] + 162*Cos[4*(c + d*x)] + 7*Cos
[6*(c + d*x)] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14*I
)*Sin[6*(c + d*x)]))/(2016*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.29, size = 207, normalized size = 1.71

method	result
risch	$\frac{3i\mathrm{e}^{-5i(dx+c)}}{64a^3d}+\frac{3i\mathrm{e}^{-7i(dx+c)}}{224a^3d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{576a^3d}+\frac{9i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{19i\cos(3dx+3c)}{192a^3d}+\frac{7\sin(3dx+3c)}{64a^3d}\cdots$
derivativedivides	$\frac{-\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{9i}{2\left(-i+\tan\left(\cdots\right.\right.}\cdots}{\cdots}$
default	$\frac{-\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{9i}{2\left(-i+\tan\left(\cdots\right.\right.}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-1/32*I/(tan(1/2*d*x+1/2*c)+I)^2-1/48/(tan(1/2*d*x+1/2*c)+I)^3+7/6
4/(tan(1/2*d*x+1/2*c)+I)+23/3*I/(-I+tan(1/2*d*x+1/2*c))^6-2*I/(-I+tan(1/2*d
*x+1/2*c))^8+9/4*I/(-I+tan(1/2*d*x+1/2*c))^2-59/8*I/(-I+tan(1/2*d*x+1/2*c))
^4+4/9/(-I+tan(1/2*d*x+1/2*c))^9-34/7/(-I+tan(1/2*d*x+1/2*c))^7+35/4/(-I+ta
n(1/2*d*x+1/2*c))^5-19/4/(-I+tan(1/2*d*x+1/2*c))^3+57/64/(-I+tan(1/2*d*x+1/
2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.70

$$\frac{\left(-21i\,e^{(12i\,dx+12i\,c)}-378i\,e^{(10i\,dx+10i\,c)}+945i\,e^{(8i\,dx+8i\,c)}+420i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+4i\,c)}+54i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{4032\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4032*(-21*I*e^(12*I*d*x + 12*I*c) - 378*I*e^(10*I*d*x + 10*I*c) + 945*I*e
^(8*I*d*x + 8*I*c) + 420*I*e^(6*I*d*x + 6*I*c) + 189*I*e^(4*I*d*x + 4*I*c)
+ 54*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(105) = 210.
time = 0.38, size = 265, normalized size = 2.19

$$\begin{cases}\frac{\left(-811748818944ia^{18}d^6e^{28ic}e^{3idx}-14611478740992ia^{18}d^6e^{26ic}e^{idx}+36528696852480ia^{18}d^6e^{24ic}e^{-idx}+16234976378880ia^{18}d^6e^{22ic}e^{-3idx}+7305739370496ia^{18}d^6e^{20ic}e^{-5idx}+2087354105856ia^{18}d^6e^{18ic}e^{-7idx}+270582939648ia^{18}d^6e^{16ic}e^{-9idx}\right)e^{-25ic}}{155855773237248a^{21}d^7} & \text{for } a^{21}d^7e^{25ic}\neq 0\\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-9ic}}{64a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-811748818944*I*a**18*d**6*exp(28*I*c)*exp(3*I*d*x) - 146114787
40992*I*a**18*d**6*exp(26*I*c)*exp(I*d*x) + 36528696852480*I*a**18*d**6*exp
(24*I*c)*exp(-I*d*x) + 16234976378880*I*a**18*d**6*exp(22*I*c)*exp(-3*I*d*x
) + 7305739370496*I*a**18*d**6*exp(20*I*c)*exp(-5*I*d*x) + 2087354105856*I*
a**18*d**6*exp(18*I*c)*exp(-7*I*d*x) + 270582939648*I*a**18*d**6*exp(16*I*c
```

```
)*exp(-9*I*d*x))*exp(-25*I*c)/(155855773237248*a**21*d**7), Ne(a**21*d**7*e
xp(25*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6
*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-9*I*c)/(64*a**3), True))
```

Giac [A]
time = 0.75, size = 171, normalized size = 1.41

$$\frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+36i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-19\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-19656i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+95760i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+107730\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-79464i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-38484\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+10944i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1615}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{9}}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 + 36*I*tan(1/2*d*x + 1/2*c) - 19)/(a^
3*(tan(1/2*d*x + 1/2*c) + I)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 - 19656*I*ta
n(1/2*d*x + 1/2*c)^7 - 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*I*tan(1/2*d*x +
 1/2*c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 - 79464*I*tan(1/2*d*x + 1/2*c)^3
- 38484*tan(1/2*d*x + 1/2*c)^2 + 10944*I*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*
(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B]
time = 6.61, size = 188, normalized size = 1.55

$$\frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}189\mathrm{i}-273\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}63\mathrm{i}-378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}294\mathrm{i}-306\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}450\mathrm{i}+235\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}39\mathrm{i}+51\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-19\mathrm{i}\right)2\mathrm{i}}{63\,a^{3}\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^{3}\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((51*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*39i + 235*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*450i - 306*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)
/2)^6*294i - 378*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*63i - 273*tan(
c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*189i + 63*tan(c/2 + (d*x)/2)^11 -
19i)*2i)/(63*a^3*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/2)*1i + 1)^
9)
```

3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{56\sin(c+dx)}{99a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(\ldots}{99d\,(a^3+ia^3\ldots}$$

```
[Out] 56/99*sin(d*x+c)/a^3/d-56/99*sin(d*x+c)^3/a^3/d+56/165*sin(d*x+c)^5/a^3/d-8
/99*sin(d*x+c)^7/a^3/d+1/11*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^3+16/99*I*c
os(d*x+c)^7/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin(c+dx)}{99a^3d}+\frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (56*Sin[c + d*x])/(99*a^3*d) - (56*Sin[c + d*x]^3)/(99*a^3*d) + (56*Sin[c +
 d*x]^5)/(165*a^3*d) - (8*Sin[c + d*x]^7)/(99*a^3*d) + ((I/11)*Cos[c + d*x]
^5)/(d*(a + I*a*Tan[c + d*x])^3) + (((16*I)/99)*Cos[c + d*x]^7)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{8\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{11a}\\ &= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{56\int \cos^7(c+dx)\,dx}{99a^3}\\ &= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{56\text{Subst}\left(\int (1-3x^2 \right.}{}\\ &= \frac{56\sin(c+dx)}{99a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{i}{11d(a}\end{aligned}$$

Mathematica [A]
time = 0.76, size = 120, normalized size = 0.86

$$\frac{\sec^3(c+dx)(-5775-16632\cos(2(c+dx))+5940\cos(4(c+dx))+440\cos(6(c+dx))+27\cos(8(c+dx))-11088i\sin(2(c+dx))+7920i\sin(4(c+dx))+880i\sin(6(c+dx))+72i\sin(8(c+dx)))}{63360a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-5775 - 16632*Cos[2*(c + d*x)] + 5940*Cos[4*(c + d*x)] + 4
40*Cos[6*(c + d*x)] + 27*Cos[8*(c + d*x)] - (11088*I)*Sin[2*(c + d*x)] + (7
920*I)*Sin[4*(c + d*x)] + (880*I)*Sin[6*(c + d*x)] + (72*I)*Sin[8*(c + d*x)
]))/(63360*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(123) = 246.
time = 0.30, size = 273, normalized size = 1.96

method	result
risch	$\frac{i e^{-7i(dx+c)}}{64a^3d}+\frac{i e^{-9i(dx+c)}}{288a^3d}+\frac{i e^{-11i(dx+c)}}{2816a^3d}+\frac{7i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{11i\cos(5dx+5c)}{256a^3d}+\frac{57\sin(5dx+5c)}{1280a^3d}$
derivativedivides	$\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$
default	$\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(217/12*I/(-I+tan(1/2*d*x+1/2*c))^6+1/32*I/(tan(1/2*d*x+1/2*c)+I)^4
+1/80/(tan(1/2*d*x+1/2*c)+I)^5-7/96/(tan(1/2*d*x+1/2*c)+I)^3+37/256/(tan(1/
2*d*x+1/2*c)+I)-23/2*I/(-I+tan(1/2*d*x+1/2*c))^8+2*I/(-I+tan(1/2*d*x+1/2*c)
)^10+303/128*I/(-I+tan(1/2*d*x+1/2*c))^2-169/16*I/(-I+tan(1/2*d*x+1/2*c))^4
-5/64*I/(tan(1/2*d*x+1/2*c)+I)^2-4/11/(-I+tan(1/2*d*x+1/2*c))^11+53/9/(-I+t
an(1/2*d*x+1/2*c))^9-33/2/(-I+tan(1/2*d*x+1/2*c))^7+623/40/(-I+tan(1/2*d*x+
1/2*c))^5-365/64/(-I+tan(1/2*d*x+1/2*c))^3+219/256/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 107, normalized size = 0.77

$$\frac{\left(-99i\,e^{(16i\,dx+16i\,c)}-1320i\,e^{(14i\,dx+14i\,c)}-13860i\,e^{(12i\,dx+12i\,c)}+27720i\,e^{(10i\,dx+10i\,c)}+11550i\,e^{(8i\,dx+8i\,c)}+5544i\,e^{(6i\,dx+6i\,c)}+1980i\,e^{(4i\,dx+4i\,c)}+440i\,e^{(2i\,dx+2i\,c)}+45i\right)e^{(-11i\,dx-11i\,c)}}{126720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/126720*(-99*I*e^(16*I*d*x + 16*I*c) - 1320*I*e^(14*I*d*x + 14*I*c) - 1386
0*I*e^(12*I*d*x + 12*I*c) + 27720*I*e^(10*I*d*x + 10*I*c) + 11550*I*e^(8*I*
d*x + 8*I*c) + 5544*I*e^(6*I*d*x + 6*I*c) + 1980*I*e^(4*I*d*x + 4*I*c) + 44
0*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-11*I*d*x - 11*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(122) = 244.
time = 0.53, size = 333, normalized size = 2.40

$$\begin{cases} \frac{\left(-626985510622986240ia^{24}d^{8}e^{41ic}e^{5idx}-8359806808306483200ia^{24}d^{8}e^{39ic}e^{3idx}-87777971487218073600ia^{24}d^{8}e^{37ic}e^{idx}+175555942974436147200ia^{24}d^{8}e^{35ic}e^{-idx}+73148309572681728000ia^{24}d^{8}e^{33ic}e^{-3idx}+35111188594887229440ia^{24}d^{8}e^{31ic}e^{-5idx}+12539710212459724800ia^{24}d^{8}e^{29ic}e^{-7idx}+2786602269435494400ia^{24}d^{8}e^{27ic}e^{-9idx}+284993413919539200ia^{24}d^{8}e^{25ic}e^{-11idx}\right)e^{-36ic}}{802541453597422387200a^{27}d^{9}} & \text{for}\: a^{27}d^{9}e^{36ic} \neq 0 \\ \frac{x\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-11ic}}{256a^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-626985510622986240*I*a**24*d**8*exp(41*I*c)*exp(5*I*d*x) - 835
9806808306483200*I*a**24*d**8*exp(39*I*c)*exp(3*I*d*x) - 877779714872180736
00*I*a**24*d**8*exp(37*I*c)*exp(I*d*x) + 175555942974436147200*I*a**24*d**8
*exp(35*I*c)*exp(-I*d*x) + 73148309572681728000*I*a**24*d**8*exp(33*I*c)*ex
p(-3*I*d*x) + 35111188594887229440*I*a**24*d**8*exp(31*I*c)*exp(-5*I*d*x) +
 12539710212459724800*I*a**24*d**8*exp(29*I*c)*exp(-7*I*d*x) + 278660226943
5494400*I*a**24*d**8*exp(27*I*c)*exp(-9*I*d*x) + 284993413919539200*I*a**24
*d**8*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(802541453597422387200*a**27
*d**9), Ne(a**27*d**9*exp(36*I*c), 0)), (x*(exp(16*I*c) + 8*exp(14*I*c) + 2
8*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) + 28*exp(4*I
*c) + 8*exp(2*I*c) + 1)*exp(-11*I*c)/(256*a**3), True))
```

Giac [A]
time = 0.81, size = 223, normalized size = 1.60

$$\frac{\frac{33\left(555\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+1920i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-2710\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1760i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+463\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{5}}+\frac{108405\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-784080i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-2901195\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+6652800i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+10407474\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-11435424i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-8949270\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+4899840i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+1816265\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-411664i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-47279}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{11}}}{63360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/63360*(33*(555*tan(1/2*d*x + 1/2*c)^4 + 1920*I*tan(1/2*d*x + 1/2*c)^3 - 2
710*tan(1/2*d*x + 1/2*c)^2 - 1760*I*tan(1/2*d*x + 1/2*c) + 463)/(a^3*(tan(1
/2*d*x + 1/2*c) + I)^5) + (108405*tan(1/2*d*x + 1/2*c)^10 - 784080*I*tan(1/
2*d*x + 1/2*c)^9 - 2901195*tan(1/2*d*x + 1/2*c)^8 + 6652800*I*tan(1/2*d*x +
 1/2*c)^7 + 10407474*tan(1/2*d*x + 1/2*c)^6 - 11435424*I*tan(1/2*d*x + 1/2*
c)^5 - 8949270*tan(1/2*d*x + 1/2*c)^4 + 4899840*I*tan(1/2*d*x + 1/2*c)^3 +
1816265*tan(1/2*d*x + 1/2*c)^2 - 411664*I*tan(1/2*d*x + 1/2*c) - 47279)/(a^
3*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B]
time = 5.52, size = 136, normalized size = 0.98

$$\frac{\left(\frac{\cos(7c+7dx)}{64}+\frac{\cos(9c+9dx)}{288}+\frac{\cos(11c+11dx)}{2816}-\frac{\sin(7c+7dx)\,1\mathrm{i}}{64}-\frac{\sin(9c+9dx)\,1\mathrm{i}}{288}-\frac{\sin(11c+11dx)\,1\mathrm{i}}{2816}+\frac{\sqrt{224}\,\cos\left(5c+5dx+\mathrm{atanh}\left(\frac{57}{55}\right)1\mathrm{i}\right)1\mathrm{i}}{1280}+\frac{\sqrt{560}\,\cos\left(3c+3dx+\mathrm{atanh}\left(\frac{39}{31}\right)1\mathrm{i}\right)1\mathrm{i}}{384}+\frac{\sqrt{2}\,\cos\left(c+dx+\mathrm{atanh}(3)\,1\mathrm{i}\right)7\mathrm{i}}{32}\right)1\mathrm{i}}{a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((cos(7*c + 7*d*x)/64 + cos(9*c + 9*d*x)/288 + cos(11*c + 11*d*x)/2816 - (s
in(7*c + 7*d*x)*1i)/64 - (sin(9*c + 9*d*x)*1i)/288 - (sin(11*c + 11*d*x)*1i
)/2816 + (224^(1/2)*cos(5*c + atanh(57/55)*1i + 5*d*x)*1i)/1280 + (560^(1/2
)*cos(3*c + atanh(39/31)*1i + 3*d*x)*1i)/384 + (2^(1/2)*cos(c + atanh(3)*1i
 + d*x)*7i)/32)*1i)/(a^3*d)
```

3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=82

$$\frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}$$

```
[Out] 4/7*I*(a-I*a*tan(d*x+c))^7/a^11/d-1/2*I*(a-I*a*tan(d*x+c))^8/a^12/d+1/9*I*(
a-I*a*tan(d*x+c))^9/a^13/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((4*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^11*d) - ((I/2)*(a - I*a*Tan[c + d*x
])^8)/(a^12*d) + ((I/9)*(a - I*a*Tan[c + d*x])^9)/(a^13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^6(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^6-4a(a-x)^7+(a-x)^8\right)\,dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}\end{aligned}$$

Mathematica [A]
time = 0.87, size = 136, normalized size = 1.66

$$\frac{\sec(c)\sec^9(c+dx)(-63i\cos(dx)-63i\cos(2c+dx)-42i\cos(2c+3dx)-42i\cos(4c+3dx)+63\sin(dx)-63\sin(2c+dx)+42\sin(2c+3dx)-42\sin(4c+3dx)+36\sin(4c+5dx)+9\sin(6c+7dx)+\sin(8c+9dx))}{252a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
[Out] (Sec[c]*Sec[c + d*x]^9*((-63*I)*Cos[d*x] - (63*I)*Cos[2*c + d*x] - (42*I)*C
os[2*c + 3*d*x] - (42*I)*Cos[4*c + 3*d*x] + 63*Sin[d*x] - 63*Sin[2*c + d*x]
 + 42*Sin[2*c + 3*d*x] - 42*Sin[4*c + 3*d*x] + 36*Sin[4*c + 5*d*x] + 9*Sin[
6*c + 7*d*x] + Sin[8*c + 9*d*x]))/(252*a^4*d)
```

Maple [A]
time = 0.30, size = 99, normalized size = 1.21

method	result
risch	$\frac{128i\left(36\,e^{4i(dx+c)}+9\,e^{2i(dx+c)}+1\right)}{63d\,a^4\left(e^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{\tan(dx+c)+\frac{\left(\tan^9(dx+c)\right)}{9}+\frac{i\left(\tan^8(dx+c)\right)}{2}-\frac{4\left(\tan^7(dx+c)\right)}{7}+\frac{2i\left(\tan^6(dx+c)\right)}{3}-2\left(\tan^5(dx+c)\right)-i\left(\tan^4(dx+c)\right)-\frac{4\left(\tan^3(dx+\right.}{3}}{d\,a^4}$
default	$\frac{\tan(dx+c)+\frac{\left(\tan^9(dx+c)\right)}{9}+\frac{i\left(\tan^8(dx+c)\right)}{2}-\frac{4\left(\tan^7(dx+c)\right)}{7}+\frac{2i\left(\tan^6(dx+c)\right)}{3}-2\left(\tan^5(dx+c)\right)-i\left(\tan^4(dx+c)\right)-\frac{4\left(\tan^3(dx+\right.}{3}}{d\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d/a^4*(tan(d*x+c)+1/9*tan(d*x+c)^9+1/2*I*tan(d*x+c)^8-4/7*tan(d*x+c)^7+2/
3*I*tan(d*x+c)^6-2*tan(d*x+c)^5-I*tan(d*x+c)^4-4/3*tan(d*x+c)^3-2*I*tan(d*x
+c)^2)
```

Maxima [A]
time = 0.30, size = 97, normalized size = 1.18

$$\frac{14\tan\left(dx+c\right)^9+63i\tan\left(dx+c\right)^8-72\tan\left(dx+c\right)^7+84i\tan\left(dx+c\right)^6-252\tan\left(dx+c\right)^5-126i\tan\left(dx+c\right)^4-168\tan\left(dx+c\right)^3-252i\tan\left(dx+c\right)^2+126\tan\left(dx+c\right)}{126\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*t
an(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c
)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.
time = 0.39, size = 168, normalized size = 2.05

$$-\frac{128\left(-36i\,e^{(4i\,dx+4i\,c)}-9i\,e^{(2i\,dx+2i\,c)}-i\right)}{63\left(a^4de^{(18i\,dx+18i\,c)}+9\,a^4de^{(16i\,dx+16i\,c)}+36\,a^4de^{(14i\,dx+14i\,c)}+84\,a^4de^{(12i\,dx+12i\,c)}+126\,a^4de^{(10i\,dx+10i\,c)}+126\,a^4de^{(8i\,dx+8i\,c)}+84\,a^4de^{(6i\,dx+6i\,c)}+36\,a^4de^{(4i\,dx+4i\,c)}+9\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] -128/63*(-36*I*e^(4*I*d*x + 4*I*c) - 9*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^(18*I*d*x + 18*I*c) + 9*a^4*d*e^(16*I*d*x + 16*I*c) + 36*a^4*d*e^(14*I*d*x + 14*I*c) + 84*a^4*d*e^(12*I*d*x + 12*I*c) + 126*a^4*d*e^(10*I*d*x + 10*I*c) + 126*a^4*d*e^(8*I*d*x + 8*I*c) + 84*a^4*d*e^(6*I*d*x + 6*I*c) + 36*a^4*d *e^(4*I*d*x + 4*I*c) + 9*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{14}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)**14/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4

Giac [A]
time = 0.70, size = 97, normalized size = 1.18

$$\frac{14\tan(dx+c)^9+63i\tan(dx+c)^8-72\tan(dx+c)^7+84i\tan(dx+c)^6-252\tan(dx+c)^5-126i\tan(dx+c)^4-168\tan(dx+c)^3-252i\tan(dx+c)^2+126\tan(dx+c)}{126\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*tan(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)

Mupad [B]
time = 3.52, size = 120, normalized size = 1.46

$$\frac{\cos(c+dx)^9\,105\mathrm{i}+128\sin(c+dx)\cos(c+dx)^8+64\sin(c+dx)\cos(c+dx)^6+48\sin(c+dx)\cos(c+dx)^4-\cos(c+dx)^3\,168\mathrm{i}-128\sin(c+dx)\cos(c+dx)^2+\cos(c+dx)\,63\mathrm{i}+14\sin(c+dx)}{126\,a^4\,d\cos(c+dx)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^4),x)

[Out] (cos(c + d*x)*63i + 14*sin(c + d*x) - 128*cos(c + d*x)^2*sin(c + d*x) + 48*cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + 128*cos(c + d*x)^8*sin(c + d*x) - cos(c + d*x)^3*168i + cos(c + d*x)^9*105i)/(126*a^4*d *cos(c + d*x)^9)

3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=55

$$\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[Out] 1/3*I*(a-I*a*tan(d*x+c))^6/a^10/d-1/7*I*(a-I*a*tan(d*x+c))^7/a^11/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^6)/(a^10*d) - ((I/7)*(a - I*a*Tan[c + d*x])^7
)/(a^11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^5(a+x)\,dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^5-(a-x)^6\right)\,dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. $2(55) = 110$.
time = 0.56, size = 127, normalized size = 2.31

$$\frac{\sec(c)\sec^7(c+dx)(-35i\cos(dx)-35i\cos(2c+dx)-21i\cos(2c+3dx)-21i\cos(4c+3dx)+35\sin(dx)-35\sin(2c+dx)+21\sin(2c+3dx)-21\sin(4c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx))}{84a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*((-35*I)*Cos[d*x] - (35*I)*Cos[2*c + d*x] - (21*I)*C
os[2*c + 3*d*x] - (21*I)*Cos[4*c + 3*d*x] + 35*Sin[d*x] - 35*Sin[2*c + d*x]
 + 21*Sin[2*c + 3*d*x] - 21*Sin[4*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*Sin[
6*c + 7*d*x]))/(84*a^4*d)
```

Maple [A]
time = 0.28, size = 67, normalized size = 1.22

method	result	size
risch	$\frac{64i\left(7\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$	36
derivativedivides	$\frac{\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{2i\left(\tan^6(dx+c)\right)}{3}-\left(\tan^5(dx+c)\right)-\frac{5\left(\tan^3(dx+c)\right)}{3}-2i\left(\tan^2(dx+c)\right)}{d\,a^4}$	67
default	$\frac{\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{2i\left(\tan^6(dx+c)\right)}{3}-\left(\tan^5(dx+c)\right)-\frac{5\left(\tan^3(dx+c)\right)}{3}-2i\left(\tan^2(dx+c)\right)}{d\,a^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+1/7*tan(d*x+c)^7+2/3*I*tan(d*x+c)^6-tan(d*x+c)^5-5/3*ta
n(d*x+c)^3-2*I*tan(d*x+c)^2)
```

Maxima [A]
time = 0.30, size = 67, normalized size = 1.22

$$\frac{3\,\tan\left(dx+c\right)^7+14i\,\tan\left(dx+c\right)^6-21\,\tan\left(dx+c\right)^5-35\,\tan\left(dx+c\right)^3-42i\,\tan\left(dx+c\right)^2+21\,\tan\left(dx+c\right)}{21\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(43) = 86$.
time = 0.37, size = 127, normalized size = 2.31

$$-\frac{64\left(-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\left(a^4de^{(14i\,dx+14i\,c)}+7\,a^4de^{(12i\,dx+12i\,c)}+21\,a^4de^{(10i\,dx+10i\,c)}+35\,a^4de^{(8i\,dx+8i\,c)}+35\,a^4de^{(6i\,dx+6i\,c)}+21\,a^4de^{(4i\,dx+4i\,c)}+7\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -64/21*(-7*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^(14*I*d*x + 14*I*c) + 7*a^4*
d*e^(12*I*d*x + 12*I*c) + 21*a^4*d*e^(10*I*d*x + 10*I*c) + 35*a^4*d*e^(8*I*
d*x + 8*I*c) + 35*a^4*d*e^(6*I*d*x + 6*I*c) + 21*a^4*d*e^(4*I*d*x + 4*I*c)
+ 7*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{12}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**12/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.90, size = 67, normalized size = 1.22

$$\frac{3\tan(dx+c)^7+14i\tan(dx+c)^6-21\tan(dx+c)^5-35\tan(dx+c)^3-42i\tan(dx+c)^2+21\tan(dx+c)}{21a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Mupad [B]
time = 3.34, size = 113, normalized size = 2.05

$$\frac{\sin(c+dx)\left(21\cos(c+dx)^6-\cos(c+dx)^5\sin(c+dx)\,42\mathrm{i}-35\cos(c+dx)^4\sin(c+dx)^2-21\cos(c+dx)^2\sin(c+dx)^4+\cos(c+dx)\sin(c+dx)^5\,14\mathrm{i}+3\sin(c+dx)^6\right)}{21a^4d\cos(c+dx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*14i - cos(c + d*x)^5*sin(c + d*x
)*42i + 21*cos(c + d*x)^6 + 3*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin(c + d*
x)^4 - 35*cos(c + d*x)^4*sin(c + d*x)^2))/(21*a^4*d*cos(c + d*x)^7)
```

3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

[Out] 1/5*I*(a-I*a*tan(d*x+c))^5/a^9/d

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]

[Out] ((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^4\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{i(a-ia\tan(c+dx))^5}{5a^9d}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 116 vs. 2(27) = 54.
time = 0.49, size = 116, normalized size = 4.30

$$\frac{\sec(c)\sec^5(c+dx)(-10i\cos(dx)-10i\cos(2c+dx)-5i\cos(2c+3dx)-5i\cos(4c+3dx)+10\sin(dx)-10\sin(2c+dx)+5\sin(2c+3dx)-5\sin(4c+3dx)+2\sin(4c+5dx))}{10a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*((-10*I)*Cos[d*x] - (10*I)*Cos[2*c + d*x] - (5*I)*Co
s[2*c + 3*d*x] - (5*I)*Cos[4*c + 3*d*x] + 10*Sin[d*x] - 10*Sin[2*c + d*x] +
 5*Sin[2*c + 3*d*x] - 5*Sin[4*c + 3*d*x] + 2*Sin[4*c + 5*d*x]))/(10*a^4*d)
```

Maple [A]
time = 0.27, size = 20, normalized size = 0.74

method	result	size
derivativedivides	$\frac{(\tan(dx+c)+i)^5}{5d\,a^4}$	20
default	$\frac{(\tan(dx+c)+i)^5}{5d\,a^4}$	20
risch	$\frac{32i}{5d\,a^4\left(e^{2i(dx+c)}+1\right)^5}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d/a^4*(tan(d*x+c)+I)^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.
time = 0.30, size = 55, normalized size = 2.04

$$\frac{\tan\left(dx+c\right)^5+5i\,\tan\left(dx+c\right)^4-10\,\tan\left(dx+c\right)^3-10i\,\tan\left(dx+c\right)^2+5\,\tan\left(dx+c\right)}{5\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*x
 + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(21) = 42.
time = 0.37, size = 84, normalized size = 3.11

$$\frac{32i}{5\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 32/5*I/(a^4*d*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*
d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x +
 2*I*c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{10}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**10/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.
time = 0.88, size = 55, normalized size = 2.04

$$\frac{\tan(dx+c)^5 + 5i\tan(dx+c)^4 - 10\tan(dx+c)^3 - 10i\tan(dx+c)^2 + 5\tan(dx+c)}{5a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*x
 + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Mupad [B]
time = 3.39, size = 93, normalized size = 3.44

$$\frac{\sin(c+dx)\,\left(5\cos(c+dx)^4 - \cos(c+dx)^3\sin(c+dx)\,10\mathrm{i} - 10\cos(c+dx)^2\sin(c+dx)^2 + \cos(c+dx)\,\sin(c+dx)^3\,5\mathrm{i} + \sin(c+dx)^4\right)}{5a^4d\cos(c+dx)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i - cos(c + d*x)^3*sin(c + d*x)
*10i + 5*cos(c + d*x)^4 + sin(c + d*x)^4 - 10*cos(c + d*x)^2*sin(c + d*x)^2
))/(5*a^4*d*cos(c + d*x)^5)
```

3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=90

$$\frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}$$

```
[Out] 8*x/a^4+8*I*ln(cos(d*x+c))/a^4/d-4*tan(d*x+c)/a^4/d-I*(a-I*a*tan(d*x+c))^2/
a^6/d-1/3*I*(a-I*a*tan(d*x+c))^3/a^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{4\tan(c+dx)}{a^4d}+\frac{8i\log(\cos(c+dx))}{a^4d}+\frac{8x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*x)/a^4 + ((8*I)*Log[Cos[c + d*x]])/(a^4*d) - (4*Tan[c + d*x])/(a^4*d) -
(I*(a - I*a*Tan[c + d*x])^2)/(a^6*d) - ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^
7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$

$$= -\frac{i\text{Subst}\left(\int \left(-4a^2-2a(a-x)-(a-x)^2+\frac{8a^3}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^7d}$$

$$= \frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-i}{}$$

Mathematica [A]
time = 0.84, size = 168, normalized size = 1.87

$$\frac{\sec(c)\sec^3(c+dx)(6dx\cos(2c+3dx)+6dx\cos(4c+3dx)+6\cos(dx)(i+3dx+3i\log(\cos(c+dx)))+6\cos(2c+dx)(i+3dx+3i\log(\cos(c+dx)))+6i\cos(2c+3dx)\log(\cos(c+dx))+6i\cos(4c+3dx)\log(\cos(c+dx))-21\sin(dx)+12\sin(2c+dx)-11\sin(2c+3dx))}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^3*(6*d*x*Cos[2*c + 3*d*x] + 6*d*x*Cos[4*c + 3*d*x] + 6
*Cos[d*x]*(I + 3*d*x + (3*I)*Log[Cos[c + d*x]]) + 6*Cos[2*c + d*x]*(I + 3*d
*x + (3*I)*Log[Cos[c + d*x]]) + (6*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]] +
(6*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]] - 21*Sin[d*x] + 12*Sin[2*c + d*x]
- 11*Sin[2*c + 3*d*x]))/(6*a^4*d)
```

Maple [A]
time = 0.29, size = 51, normalized size = 0.57

method	result	size
derivativedivides	$\frac{-7\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)-8i\ln(\tan(dx+c)-i)}{d\,a^4}$	51
default	$\frac{-7\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)-8i\ln(\tan(dx+c)-i)}{d\,a^4}$	51
risch	$\frac{16x}{a^4}+\frac{16c}{a^4d}-\frac{4i\left(6\,e^{4i(dx+c)}+15\,e^{2i(dx+c)}+11\right)}{3d\,a^4\left(e^{2i(dx+c)}+1\right)^3}+\frac{8i\ln\left(e^{2i(dx+c)}+1\right)}{a^4d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-7*tan(d*x+c)+1/3*tan(d*x+c)^3+2*I*tan(d*x+c)^2-8*I*ln(tan(d*x+c)-
I))
```

Maxima [A]
time = 0.30, size = 53, normalized size = 0.59

$$\frac{\frac{\tan(dx+c)^3+6i\,\tan(dx+c)^2-21\,\tan(dx+c)}{a^4}-\frac{24i\,\log(i\,\tan(dx+c)+1)}{a^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 21*tan(d*x + c))/a^4 - 24*I*log
(I*tan(d*x + c) + 1)/a^4)/d
```

Fricas [A]
time = 0.39, size = 156, normalized size = 1.73

$$\frac{4\left(12\,dxe^{(6i\,dx+6i\,c)}+12\,dx+6\,(6\,dx-i)e^{(4i\,dx+4i\,c)}+3\,(12\,dx-5i)e^{(2i\,dx+2i\,c)}-6\left(-i\,e^{(6i\,dx+6i\,c)}-3i\,e^{(4i\,dx+4i\,c)}-3i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-11i\right)}{3\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 4/3*(12*d*x*e^(6*I*d*x + 6*I*c) + 12*d*x + 6*(6*d*x - I)*e^(4*I*d*x + 4*I*c
) + 3*(12*d*x - 5*I)*e^(2*I*d*x + 2*I*c) - 6*(-I*e^(6*I*d*x + 6*I*c) - 3*I*
e^(4*I*d*x + 4*I*c) - 3*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2*I*c)
+ 1) - 11*I)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a
^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^8(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**8/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.83, size = 154, normalized size = 1.71

$$-\frac{2\left(-\frac{12i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}+\frac{24i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^4}-\frac{12i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}+\frac{22i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-78i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+46\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+78i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-22i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^4}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(-12*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 24*I*log(tan(1/2*d*x + 1/2*
c) - I)/a^4 - 12*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (22*I*tan(1/2*d*x +
1/2*c)^6 - 21*tan(1/2*d*x + 1/2*c)^5 - 78*I*tan(1/2*d*x + 1/2*c)^4 + 46*tan
(1/2*d*x + 1/2*c)^3 + 78*I*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c)
 - 22*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

Mupad [B]
time = 3.38, size = 60, normalized size = 0.67

$$-\frac{\frac{7\tan(c+d\,x)}{a^4}-\frac{\tan(c+d\,x)^3}{3\,a^4}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,8\mathrm{i}}{a^4}-\frac{\tan(c+d\,x)^2\,2\mathrm{i}}{a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] -((log(tan(c + d*x) - 1i)*8i)/a^4 + (7*tan(c + d*x))/a^4 - (tan(c + d*x)^2*
2i)/a^4 - tan(c + d*x)^3/(3*a^4))/d
```

3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=63

$$-\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] -4*x/a^4-4*I*ln(cos(d*x+c))/a^4/d+tan(d*x+c)/a^4/d+4*I/d/(a^4+I*a^4*tan(d*x
+c))
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{4i\log(\cos(c+dx))}{a^4d}-\frac{4x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 - ((4*I)*Log[Cos[c + d*x]])/(a^4*d) + Tan[c + d*x]/(a^4*d) + (4*
I)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}$$
$$= -\frac{i\text{Subst}\left(\int \left(1+\frac{4a^2}{(a+x)^2}-\frac{4a}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}$$
$$= -\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\,(a^4+ia^4\tan(c+dx))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 214 vs. $2(63) = 126$.
time = 0.79, size = 214, normalized size = 3.40

$$\frac{\sec(c)\sec(c+dx)(-\cos(c+dx)+i\sin(c+dx))(-i\cos(3c+2dx)+2dx\cos(3c+2dx)+2\cos(c+2dx)(dx+i\log(\cos(c+dx)))+\cos(c)(-3i+4dx+4i\log(\cos(c+dx)))+2i\cos(3c+2dx)\log(\cos(c+dx))+\sin(c)-2\sin(c+2dx)+2idx\sin(c+2dx)-2\log(\cos(c+dx))\sin(c+2dx)-\sin(3c+2dx)+2idx\sin(3c+2dx)-2\log(\cos(c+dx))\sin(3c+2dx))}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]*(-Cos[c + d*x] + I*Sin[c + d*x])*((-I)*Cos[3*c + 2*d*x
] + 2*d*x*Cos[3*c + 2*d*x] + 2*Cos[c + 2*d*x]*(d*x + I*Log[Cos[c + d*x]]) +
 Cos[c]*(-3*I + 4*d*x + (4*I)*Log[Cos[c + d*x]]) + (2*I)*Cos[3*c + 2*d*x]*L
og[Cos[c + d*x]] + Sin[c] - 2*Sin[c + 2*d*x] + (2*I)*d*x*Sin[c + 2*d*x] - 2
*Log[Cos[c + d*x]]*Sin[c + 2*d*x] - Sin[3*c + 2*d*x] + (2*I)*d*x*Sin[3*c +
2*d*x] - 2*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x]))/(2*a^4*d)
```

Maple [A]
time = 0.29, size = 41, normalized size = 0.65

method	result	size
derivativedivides	$\frac{\tan(dx+c)+4i\ln(\tan(dx+c)-i)+\frac{4}{\tan(dx+c)-i}}{d\,a^4}$	41
default	$\frac{\tan(dx+c)+4i\ln(\tan(dx+c)-i)+\frac{4}{\tan(dx+c)-i}}{d\,a^4}$	41
risch	$\frac{2i\mathrm{e}^{-2i(dx+c)}}{a^4d}-\frac{8x}{a^4}-\frac{8c}{a^4d}+\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^4d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+4*I*ln(tan(d*x+c)-I)+4/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.30, size = 95, normalized size = 1.51

$$\frac{\frac{4\left(\tan(dx+c)^2-2i\,\tan(dx+c)-1\right)}{a^4\tan(dx+c)^3-3i\,a^4\tan(dx+c)^2-3\,a^4\tan(dx+c)+i\,a^4}+\frac{4i\,\log(i\,\tan(dx+c)+1)}{a^4}+\frac{\tan(dx+c)}{a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (4*(tan(d*x + c)^2 - 2*I*tan(d*x + c) - 1)/(a^4*tan(d*x + c)^3 - 3*I*a^4*ta
n(d*x + c)^2 - 3*a^4*tan(d*x + c) + I*a^4) + 4*I*log(I*tan(d*x + c) + 1)/a^
4 + tan(d*x + c)/a^4)/d
```

Fricas [A]
time = 0.40, size = 102, normalized size = 1.62

$$-\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+2\,(2\,dx-i)e^{(2i\,dx+2i\,c)}+2\left(i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2*(4*d*x*e^(4*I*d*x + 4*I*c) + 2*(2*d*x - I)*e^(2*I*d*x + 2*I*c) + 2*(I*e^
(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - I
)/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(57) = 114.
time = 0.77, size = 146, normalized size = 2.32

$$\frac{2\left(-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}+\frac{4i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^4}-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}+\frac{2i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{2\left(3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2*(-2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 4*I*log(tan(1/2*d*x + 1/2*c) -
I)/a^4 - 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (2*I*tan(1/2*d*x + 1/2*c)^
2 - tan(1/2*d*x + 1/2*c) - 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - 2*(3*I
```

```
*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 3*I)/(a^4*(tan(1/2*d*x +
 1/2*c) - I)^2))/d
```

Mupad [B]
time = 3.36, size = 55, normalized size = 0.87

$$\frac{\ln(\tan(c+dx)-\mathrm{i})\ 4\mathrm{i}}{a^4\,d}+\frac{\tan(c+dx)}{a^4\,d}+\frac{4\mathrm{i}}{a^4\,d\,(1+\tan(c+dx)\ 1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*4i)/(a^4*d) + tan(c + d*x)/(a^4*d) + 4i/(a^4*d*(tan
(c + d*x)*1i + 1))
```

3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=29

$$\frac{\tan(c+dx)}{d\,(a^2+ia^2\tan(c+dx))^2}$$

```
[Out] tan(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 34}

$$\frac{\tan(c+dx)}{d\,(a^2+ia^2\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{\tan(c+dx)}{d\,(a^2+ia^2\tan(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 1.10

$$\frac{i\sec^4(c+dx)}{4d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/4)*Sec[c + d*x]^4)/(d*(a + I*a*Tan[c + d*x])^4)
```

Maple [A]
time = 0.31, size = 36, normalized size = 1.24

method	result	size
risch	$\frac{i e^{-4i(dx+c)}}{4a^4d}$	19
derivativedivides	$\frac{-\frac{1}{\tan(dx+c)-i}-\frac{i}{(\tan(dx+c)-i)^2}}{d\,a^4}$	36
default	$\frac{-\frac{1}{\tan(dx+c)-i}-\frac{i}{(\tan(dx+c)-i)^2}}{d\,a^4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/(tan(d*x+c)-I)-I/(tan(d*x+c)-I)^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.
time = 0.29, size = 66, normalized size = 2.28

$$-\frac{\tan(dx+c)^2-i\tan(dx+c)}{\left(a^4\tan(dx+c)^3-3i\,a^4\tan(dx+c)^2-3\,a^4\tan(dx+c)+i\,a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -(tan(d*x + c)^2 - I*tan(d*x + c))/((a^4*tan(d*x + c)^3 - 3*I*a^4*tan(d*x +
 c)^2 - 3*a^4*tan(d*x + c) + I*a^4)*d)
```

Fricas [A]
time = 0.38, size = 17, normalized size = 0.59

$$\frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(24) = 48$.
time = 1.37, size = 95, normalized size = 3.28

$$\begin{cases} \frac{i\sec^4(c+dx)}{4a^4d\tan^4(c+dx)-16ia^4d\tan^3(c+dx)-24a^4d\tan^2(c+dx)+16ia^4d\tan(c+dx)+4a^4d} & \text{for } d\neq 0 \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((I*sec(c + d*x)**4/(4*a**4*d*tan(c + d*x)**4 - 16*I*a**4*d*tan(c
+ d*x)**3 - 24*a**4*d*tan(c + d*x)**2 + 16*I*a**4*d*tan(c + d*x) + 4*a**4*d
), Ne(d, 0)), (x*sec(c)**4/(I*a*tan(c) + a)**4, True))
```

Giac [A]
time = 0.77, size = 44, normalized size = 1.52

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

Mupad [B]
time = 3.39, size = 25, normalized size = 0.86

$$-\frac{\tan(c+d\,x)}{a^4\,d\,(\tan\left(c+d\,x\right)-\mathrm{i})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] -tan(c + d*x)/(a^4*d*(tan(c + d*x) - 1i)^2)
```

3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=27

$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

```
[Out] 1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{3ad(a+ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

time = 0.23, size = 56, normalized size = 2.07

$$\frac{i\sec^4(c+dx)(3+4\cos(2(c+dx))+2i\sin(2(c+dx)))}{24a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/24)*Sec[c + d*x]^4*(3 + 4*Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(
a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.22, size = 24, normalized size = 0.89

method	result	size
derivativedivides	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
default	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.78

$$\frac{i}{3\left(i\,a\tan\left(dx+c\right)+a\right)^3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*I/((I*a*tan(d*x + c) + a)^3*a*d)
```

Fricas [A]
time = 0.41, size = 41, normalized size = 1.52

$$\frac{\left(3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{24\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x -
6*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(19) = 38$.
time = 1.55, size = 272, normalized size = 10.07

$$\begin{cases} -\frac{i\tan^2(c+dx)\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} - \frac{4\tan(c+dx)\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} + \frac{7i\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} & \text{for } d \neq 0 \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x)**4 -
96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4*d*tan(
c + d*x) + 24*a**4*d) - 4*tan(c + d*x)*sec(c + d*x)**2/(24*a**4*d*tan(c + d
*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a*
*4*d*tan(c + d*x) + 24*a**4*d) + 7*I*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x
)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4
*d*tan(c + d*x) + 24*a**4*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**4,
True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(21) = 42$.
time = 0.73, size = 85, normalized size = 3.15

$$-\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3a^4d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 6*I*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*
x + 1/2*c)^3 + 6*I*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c))/(a^4*d*
(tan(1/2*d*x + 1/2*c) - I)^6)
```

Mupad [B]
time = 3.40, size = 19, normalized size = 0.70

$$-\frac{1}{3\,a^4\,d\,{\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] -1/(3*a^4*d*(tan(c + d*x) - 1i)^3)
```

3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{16d\left(a^4+ia^4\tan(c\right.}$$

```
[Out] 1/16*x/a^4+1/8*I/d/(a+I*a*tan(d*x+c))^4+1/12*I/a/d/(a+I*a*tan(d*x+c))^3+1/1
6*I/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3560, 8}

$$\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{x}{16a^4}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] x/(16*a^4) + (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(a*d*(a + I*a*Tan[
c + d*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) + (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{2a}$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\tan(c+}$$
$$= \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{i}{16d\,(a^2+ia^2\tan(c+}$$
$$= \frac{x}{16a^4} + \frac{i}{8d(a+ia\tan(c+dx))^4} + \frac{i}{12ad(a+ia\tan(c+dx))^3} + \frac{}{16d\,(a^2+ia^2}$$

Mathematica [A]
time = 0.27, size = 98, normalized size = 0.84

$$\frac{\sec^4(c+dx)(36i+64i\cos(2(c+dx))+3(i+8dx)\cos(4(c+dx))-32\sin(2(c+dx))+3\sin(4(c+dx))+24idx\sin(4(c+dx)))}{384a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] (Sec[c + d*x]^4*(36*I + (64*I)*Cos[2*(c + d*x)] + 3*(I + 8*d*x)*Cos[4*(c +
d*x)] - 32*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)] + (24*I)*d*x*Sin[4*(c + d*
x)]))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.16, size = 89, normalized size = 0.77

method	result
risch	$\frac{x}{16a^4}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{i}{16(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{16\tan(dx+c)-16i}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
default	$\frac{\frac{i}{8(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{32}-\frac{i}{16(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{16\tan(dx+c)-16i}+\frac{i\ln(\tan(dx+c)+i)}{32}}{d\,a^4}$
norman	$\frac{\frac{x}{16a}+\frac{11\left(\tan^5(dx+c)\right)}{48da}+\frac{\tan^7(dx+c)}{16da}+\frac{x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{4a}+\frac{x\left(\tan^8(dx+c)\right)}{16a}+\frac{i}{3da}+\frac{15\tan(dx+c}{16da}}{a^3(1+\tan^2(dx+c))^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/8*I/(tan(d*x+c)-I)^4-1/32*I*ln(tan(d*x+c)-I)-1/16*I/(tan(d*x+c)-
I)^2-1/12/(tan(d*x+c)-I)^3+1/16/(tan(d*x+c)-I)+1/32*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 65, normalized size = 0.56

$$\frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I*
d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A]
time = 0.23, size = 189, normalized size = 1.63

$$\begin{cases} \frac{(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx})e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases}+\frac{x}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d*
*3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x)
 + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**16*
d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), True))
+ x/(16*a**4)
```

Giac [A]
time = 0.52, size = 92, normalized size = 0.79

$$-\frac{-\frac{12i\,\log(-i\,\tan(dx+c)+1)}{a^4}+\frac{12i\,\log(-i\,\tan(dx+c)-1)}{a^4}+\frac{-25i\,\tan(dx+c)^4-124\,\tan(dx+c)^3+246i\,\tan(dx+c)^2+252\,\tan(dx+c)-153i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(-12*I*log(-I*tan(d*x + c) + 1)/a^4 + 12*I*log(-I*tan(d*x + c) - 1)/
a^4 + (-25*I*tan(d*x + c)^4 - 124*tan(d*x + c)^3 + 246*I*tan(d*x + c)^2 + 2
52*tan(d*x + c) - 153*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 3.50, size = 60, normalized size = 0.52

$$\frac{x}{16\,a^4}-\frac{-\frac{\tan(c+d\,x)^3}{16}+\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4}+\frac{19\tan(c+d\,x)}{48}-\frac{1}{3}\mathrm{i}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] x/(16*a^4) - ((19*tan(c + d*x))/48 + (tan(c + d*x)^2*1i)/4 - tan(c + d*x)^3
/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4} dx$

Optimal. Leaf size=169

$$\frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c\right.}$$

```
[Out] 3/32*x/a^4+1/20*I*a/d/(a+I*a*tan(d*x+c))^5+1/16*I/d/(a+I*a*tan(d*x+c))^4+1/
16*I/a/d/(a+I*a*tan(d*x+c))^3+1/16*I/d/(a^2+I*a^2*tan(d*x+c))^2-1/64*I/d/(a
^4-I*a^4*tan(d*x+c))+5/64*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]

time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{3x}{32a^4}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*x)/(32*a^4) + ((I/20)*a)/(d*(a + I*a*Tan[c + d*x])^5) + (I/16)/(d*(a + I
*a*Tan[c + d*x])^4) + (I/16)/(a*d*(a + I*a*Tan[c + d*x])^3) + (I/16)/(d*(a^
2 + I*a^2*Tan[c + d*x])^2) - (I/64)/(d*(a^4 - I*a^4*Tan[c + d*x])) + ((5*I)
/64)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(ia^3)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^2}+\frac{1}{4a^2(a+x)^6}+\frac{1}{4a^3(a+x)^5}+\frac{3}{16a^4(a+x)^4}+\frac{1}{8a^5(a+x)^3}+\right.\right.}{d}$$

$$= \frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+}$$

$$= \frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia}$$

Mathematica [A]
time = 0.38, size = 120, normalized size = 0.71

$$\frac{\sec^4(c+dx)(100i+200i\cos(2(c+dx))+15(i+8dx)\cos(4(c+dx))-8i\cos(6(c+dx))-100\sin(2(c+dx))+15\sin(4(c+dx))+120idx\sin(4(c+dx))+12\sin(6(c+dx)))}{1280a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(100*I + (200*I)*Cos[2*(c + d*x)] + 15*(I + 8*d*x)*Cos[4*(c
 + d*x)] - (8*I)*Cos[6*(c + d*x)] - 100*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*
x)] + (120*I)*d*x*Sin[4*(c + d*x)] + 12*Sin[6*(c + d*x)]))/(1280*a^4*d*(-I
+ Tan[c + d*x])^4)
```

Maple [A]
time = 0.29, size = 115, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{i}{16(\tan(dx+c)-i)^2}+\frac{1}{20(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{64(\tan(dx+c)-i)}+\frac{3i\ln}{}}{d\,a^4}$
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{i}{16(\tan(dx+c)-i)^2}+\frac{1}{20(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{64(\tan(dx+c)-i)}+\frac{3i\ln}{}}{d\,a^4}$
risch	$\frac{3x}{32a^4}+\frac{5i\mathrm{e}^{-4i(dx+c)}}{64a^4d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{128a^4d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{256a^4d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{640a^4d}+\frac{7i\cos(2dx+2c)}{64a^4d}+\frac{\sin(2dx+2c)}{8a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-3/64*I*ln(tan(d*x+c)-I)+1/16*I/(tan(d*x+c)-I)^4-1/16*I/(tan(d*x+c
)-I)^2+1/20/(tan(d*x+c)-I)^5-1/16/(tan(d*x+c)-I)^3+5/64/(tan(d*x+c)-I)+3/64
*I*ln(tan(d*x+c)+I)+1/64/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.42, size = 87, normalized size = 0.51

$$\frac{\left(120\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(12i\,dx+12i\,c)}+150i\,e^{(8i\,dx+8i\,c)}+100i\,e^{(6i\,dx+6i\,c)}+50i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-10i\,dx-10i\,c)}}{1280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(120*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(12*I*d*x + 12*I*c) + 150*I*
e^(8*I*d*x + 8*I*c) + 100*I*e^(6*I*d*x + 6*I*c) + 50*I*e^(4*I*d*x + 4*I*c)
+ 15*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

Sympy [A]
time = 0.33, size = 258, normalized size = 1.53

$$\begin{cases} \frac{\left(-171798691840ia^{20}d^5e^{32ic}e^{2idx}+2576980377600ia^{20}d^5e^{28ic}e^{-2idx}+1717986918400ia^{20}d^5e^{26ic}e^{-4idx}+858993459200ia^{20}d^5e^{24ic}e^{-6idx}+257698037760ia^{20}d^5e^{22ic}e^{-8idx}+34359738368ia^{20}d^5e^{20ic}e^{-10idx}\right)e^{-30ic}}{21990232555520a^{24}d^6} & \text{for}\ a^{24}d^6e^{30ic}\neq 0 \\ x\left(\frac{\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-10ic}}{64a^4}-\frac{3}{32a^4}\right) & \text{otherwise} \end{cases}+\frac{3x}{32a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-171798691840*I*a**20*d**5*exp(32*I*c)*exp(2*I*d*x) + 257698037
7600*I*a**20*d**5*exp(28*I*c)*exp(-2*I*d*x) + 1717986918400*I*a**20*d**5*ex
p(26*I*c)*exp(-4*I*d*x) + 858993459200*I*a**20*d**5*exp(24*I*c)*exp(-6*I*d*
x) + 257698037760*I*a**20*d**5*exp(22*I*c)*exp(-8*I*d*x) + 34359738368*I*a*
*20*d**5*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(21990232555520*a**24*d**
6), Ne(a**24*d**6*exp(30*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*e
xp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-10*I*c)/
(64*a**4) - 3/(32*a**4)), True)) + 3*x/(32*a**4)
```

Giac [A]
time = 0.85, size = 123, normalized size = 0.73

$$-\frac{-\frac{60i\,\log(\tan(dx+c)+i)}{a^4}+\frac{60i\,\log(\tan(dx+c)-i)}{a^4}+\frac{20\,(3i\,\tan(dx+c)-4)}{a^4(\tan(dx+c)+i)}+\frac{-137i\,\tan(dx+c)^5-785\,\tan(dx+c)^4+1850i\,\tan(dx+c)^3+2290\,\tan(dx+c)^2-1565i\,\tan(dx+c)-541}{a^4(\tan(dx+c)-i)^5}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1280*(-60*I*log(tan(d*x + c) + I)/a^4 + 60*I*log(tan(d*x + c) - I)/a^4 +
 20*(3*I*tan(d*x + c) - 4)/(a^4*(tan(d*x + c) + I)) + (-137*I*tan(d*x + c)^
5 - 785*tan(d*x + c)^4 + 1850*I*tan(d*x + c)^3 + 2290*tan(d*x + c)^2 - 1565
*I*tan(d*x + c) - 541)/(a^4*(tan(d*x + c) - I)^5))/d
```

Mupad [B]
time = 4.13, size = 90, normalized size = 0.53

$$\frac{3\,x}{32\,a^4}-\frac{-\frac{3\,{\mathrm{tan}\left(c+d\,x\right)}^5}{32}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^4\,3{}\mathrm{i}}{8}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^3}{2}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,1{}\mathrm{i}}{8}+\frac{47\,\mathrm{tan}\left(c+d\,x\right)}{160}-\frac{3}{10}{}\mathrm{i}}{a^4\,d\,{\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}^5\,\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (3*x)/(32*a^4) - ((47*tan(c + d*x))/160 - (tan(c + d*x)^2*1i)/8 + tan(c + d
*x)^3/2 + (tan(c + d*x)^4*3i)/8 - (3*tan(c + d*x)^5)/32 - 3i/10)/(a^4*d*(ta
n(c + d*x) - 1i)^5*(tan(c + d*x) + 1i))
```

3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=224

$$\frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6}+\frac{3ia}{80d(a+ia\tan(c+dx))^5}+\frac{3i}{64d(a+ia\tan(c+dx))^4}+\frac{5i}{96ad(a+ia\tan(c+$$

```
[Out] 7/64*x/a^4+1/48*I*a^2/d/(a+I*a*tan(d*x+c))^6+3/80*I*a/d/(a+I*a*tan(d*x+c))^
5+3/64*I/d/(a+I*a*tan(d*x+c))^4+5/96*I/a/d/(a+I*a*tan(d*x+c))^3-1/256*I/d/(
a^2-I*a^2*tan(d*x+c))^2+15/256*I/d/(a^2+I*a^2*tan(d*x+c))^2-7/256*I/d/(a^4-
I*a^4*tan(d*x+c))+21/256*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{7i}{256d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{21i}{256d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6}-\frac{i}{256d\left(a^2-ia^2\tan(c+dx)\right)^2}+\frac{15i}{256d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3ia}{80d(a+ia\tan(c+dx))^5}+\frac{3i}{64d(a+ia\tan(c+dx))^4}+\frac{5i}{96ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (7*x)/(64*a^4) + ((I/48)*a^2)/(d*(a + I*a*Tan[c + d*x])^6) + (((3*I)/80)*a)
/(d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/64)/(d*(a + I*a*Tan[c + d*x])^4) + (
(5*I)/96)/(a*d*(a + I*a*Tan[c + d*x])^3) - (I/256)/(d*(a^2 - I*a^2*Tan[c +
d*x])^2) + ((15*I)/256)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) - ((7*I)/256)/(d*(
a^4 - I*a^4*Tan[c + d*x])) + ((21*I)/256)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^7}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{128a^7(a-x)^3} + \frac{7}{256a^8(a-x)^2} + \frac{1}{8a^3(a+x)^7} + \frac{3}{16a^4(a+x)^6} + \frac{3}{16a^5(a+x)^5}\right.\right.}{}$$

$$= \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3i}{64d(a+ia\tan(c+d}$$

$$= \frac{7x}{64a^4} + \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3}{64d(a+ia\,\text{ta}}$$

Mathematica [A]
time = 0.80, size = 142, normalized size = 0.63

$$\frac{\sec^4(c+dx)(525i + 1120i\cos(2(c+dx)) + 105(i+8dx)\cos(4(c+dx)) - 96i\cos(6(c+dx)) - 5i\cos(8(c+dx)) - 560\sin(2(c+dx)) + 105\sin(4(c+dx)) + 840idx\sin(4(c+dx)) + 144\sin(6(c+dx)) + 10\sin(8(c+dx)))}{7680a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(525*I + (1120*I)*Cos[2*(c + d*x)] + 105*(I + 8*d*x)*Cos[4*
(c + d*x)] - (96*I)*Cos[6*(c + d*x)] - (5*I)*Cos[8*(c + d*x)] - 560*Sin[2*(
c + d*x)] + 105*Sin[4*(c + d*x)] + (840*I)*d*x*Sin[4*(c + d*x)] + 144*Sin[6
*(c + d*x)] + 10*Sin[8*(c + d*x)]))/(7680*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.30, size = 143, normalized size = 0.64

method	result
derivativedivides	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128} + \frac{3i}{64(\tan(dx+c)-i)^4} - \frac{i}{48(\tan(dx+c)-i)^6} - \frac{15i}{256(\tan(dx+c)-i)^2} + \frac{3}{80(\tan(dx+c)-i)^5} - \frac{5}{96(\tan(dx+c)-i)^3} + \frac{25}{}}{d\,a^4}$
default	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128} + \frac{3i}{64(\tan(dx+c)-i)^4} - \frac{i}{48(\tan(dx+c)-i)^6} - \frac{15i}{256(\tan(dx+c)-i)^2} + \frac{3}{80(\tan(dx+c)-i)^5} - \frac{5}{96(\tan(dx+c)-i)^3} + \frac{25}{}}{d\,a^4}$
risch	$\frac{7x}{64a^4} + \frac{7i\mathrm{e}^{-6i(dx+c)}}{192a^4d} + \frac{7i\mathrm{e}^{-8i(dx+c)}}{512a^4d} + \frac{i\mathrm{e}^{-10i(dx+c)}}{320a^4d} + \frac{i\mathrm{e}^{-12i(dx+c)}}{3072a^4d} + \frac{69i\cos(4dx+4c)}{1024a^4d} + \frac{71\sin(4dx+4c)}{1024a^4d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-7/128*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-1/48*I/(tan(d*x+
c)-I)^6-15/256*I/(tan(d*x+c)-I)^2+3/80/(tan(d*x+c)-I)^5-5/96/(tan(d*x+c)-I)
^3+21/256/(tan(d*x+c)-I)+1/256*I/(tan(d*x+c)+I)^2+7/128*I*ln(tan(d*x+c)+I)+
7/256/(tan(d*x+c)+I))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.39, size = 109, normalized size = 0.49

$$\frac{\left(1680\,dxe^{(12i\,dx+12i\,c)}-15i\,e^{(16i\,dx+16i\,c)}-240i\,e^{(14i\,dx+14i\,c)}+1680i\,e^{(10i\,dx+10i\,c)}+1050i\,e^{(8i\,dx+8i\,c)}+560i\,e^{(6i\,dx+6i\,c)}+210i\,e^{(4i\,dx+4i\,c)}+48i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{15360\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15360*(1680*d*x*e^(12*I*d*x + 12*I*c) - 15*I*e^(16*I*d*x + 16*I*c) - 240*
I*e^(14*I*d*x + 14*I*c) + 1680*I*e^(10*I*d*x + 10*I*c) + 1050*I*e^(8*I*d*x
+ 8*I*c) + 560*I*e^(6*I*d*x + 6*I*c) + 210*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(
2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

Sympy [**A**]
time = 0.42, size = 326, normalized size = 1.46

$$\begin{cases} \frac{\left(-202661983231672320ia^{28}d^7e^{46ic}e^{4idx}-3242591731706757120ia^{28}d^7e^{44ic}e^{2idx}+22698142121947299840ia^{28}d^7e^{40ic}e^{-2idx}+14186338826217062400ia^{28}d^7e^{38ic}e^{-4idx}+7566047373982433280ia^{28}d^7e^{36ic}e^{-6idx}+2837267765243412480ia^{28}d^7e^{34ic}e^{-8idx}+648518346341351424ia^{28}d^7e^{32ic}e^{-10idx}+67553994410557440ia^{28}d^7e^{30ic}e^{-12idx}\right)e^{-42ic}}{207525870829232455680a^{32}d^8} & \text{for}\ a^{32}d^8e^{42ic}\neq 0 \\ x\left(\frac{\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-12ic}}{256a^4}-\frac{7}{64a^4}\right) & \text{otherwise} \end{cases}+\frac{7x}{64a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-202661983231672320*I*a**28*d**7*exp(46*I*c)*exp(4*I*d*x) - 324
2591731706757120*I*a**28*d**7*exp(44*I*c)*exp(2*I*d*x) + 226981421219472998
40*I*a**28*d**7*exp(40*I*c)*exp(-2*I*d*x) + 14186338826217062400*I*a**28*d*
*7*exp(38*I*c)*exp(-4*I*d*x) + 7566047373982433280*I*a**28*d**7*exp(36*I*c)
*exp(-6*I*d*x) + 2837267765243412480*I*a**28*d**7*exp(34*I*c)*exp(-8*I*d*x)
 + 648518346341351424*I*a**28*d**7*exp(32*I*c)*exp(-10*I*d*x) + 67553994410
557440*I*a**28*d**7*exp(30*I*c)*exp(-12*I*d*x))*exp(-42*I*c)/(2075258708292
32455680*a**32*d**8), Ne(a**32*d**8*exp(42*I*c), 0)), (x*((exp(16*I*c) + 8*
exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*
```

```
c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-12*I*c)/(256*a**4) - 7/(64*a**4
)), True)) + 7*x/(64*a**4)
```

Giac [A]
time = 0.85, size = 147, normalized size = 0.66

$$-\frac{-\frac{420i\log\left(-i\tan\left(dx+c\right)+1\right)}{a^4}+\frac{420i\log\left(-i\tan\left(dx+c\right)-1\right)}{a^4}+\frac{30\left(21i\tan\left(dx+c\right)^2-49\tan\left(dx+c\right)-29i\right)}{a^4\left(\tan\left(dx+c\right)+i\right)^2}+\frac{-1029i\tan\left(dx+c\right)^6-6804\tan\left(dx+c\right)^5+19035i\tan\left(dx+c\right)^4+29080\tan\left(dx+c\right)^3-25995i\tan\left(dx+c\right)^2-13332\tan\left(dx+c\right)+3317i}{a^4\left(\tan\left(dx+c\right)-i\right)^6}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7680*(-420*I*log(-I*tan(d*x + c) + 1)/a^4 + 420*I*log(-I*tan(d*x + c) -
1)/a^4 + 30*(21*I*tan(d*x + c)^2 - 49*tan(d*x + c) - 29*I)/(a^4*(tan(d*x +
c) + I)^2) + (-1029*I*tan(d*x + c)^6 - 6804*tan(d*x + c)^5 + 19035*I*tan(d*
x + c)^4 + 29080*tan(d*x + c)^3 - 25995*I*tan(d*x + c)^2 - 13332*tan(d*x +
c) + 3317*I)/(a^4*(tan(d*x + c) - I)^6))/d
```

Mupad [B]
time = 4.94, size = 197, normalized size = 0.88

$$\frac{7\,x}{64\,a^4}+\frac{\frac{\mathrm{tan}\left(c+d\,x\right)\,169{}\mathrm{i}}{960\,a^4}+\frac{4}{15\,a^4}+\frac{119\,\mathrm{tan}\left(c+d\,x\right)^2}{240\,a^4}+\frac{\mathrm{tan}\left(c+d\,x\right)^3\,889{}\mathrm{i}}{960\,a^4}-\frac{7\,\mathrm{tan}\left(c+d\,x\right)^4}{24\,a^4}+\frac{\mathrm{tan}\left(c+d\,x\right)^5\,91{}\mathrm{i}}{192\,a^4}-\frac{7\,\mathrm{tan}\left(c+d\,x\right)^6}{16\,a^4}-\frac{\mathrm{tan}\left(c+d\,x\right)^7\,7{}\mathrm{i}}{64\,a^4}}{d\,\left(-{\mathrm{tan}\left(c+d\,x\right)}^8\,1{}\mathrm{i}-4\,{\mathrm{tan}\left(c+d\,x\right)}^7+{\mathrm{tan}\left(c+d\,x\right)}^6\,4{}\mathrm{i}-4\,{\mathrm{tan}\left(c+d\,x\right)}^5+{\mathrm{tan}\left(c+d\,x\right)}^4\,10{}\mathrm{i}+4\,{\mathrm{tan}\left(c+d\,x\right)}^3+{\mathrm{tan}\left(c+d\,x\right)}^2\,4{}\mathrm{i}+4\,\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (7*x)/(64*a^4) + ((tan(c + d*x)*169i)/(960*a^4) + 4/(15*a^4) + (119*tan(c +
 d*x)^2)/(240*a^4) + (tan(c + d*x)^3*889i)/(960*a^4) - (7*tan(c + d*x)^4)/(
24*a^4) + (tan(c + d*x)^5*91i)/(192*a^4) - (7*tan(c + d*x)^6)/(16*a^4) - (t
an(c + d*x)^7*7i)/(64*a^4))/(d*(4*tan(c + d*x) + tan(c + d*x)^2*4i + 4*tan(
c + d*x)^3 + tan(c + d*x)^4*10i - 4*tan(c + d*x)^5 + tan(c + d*x)^6*4i - 4*
tan(c + d*x)^7 - tan(c + d*x)^8*1i - 1i))
```

3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=133

$$\frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d}+\frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-$$

```
[Out] 35/8*arctanh(sin(d*x+c))/a^4/d+35/8*sec(d*x+c)*tan(d*x+c)/a^4/d+35/12*sec(d
*x+c)^3*tan(d*x+c)/a^4/d-2*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^3-14/3*I*s
ec(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$\frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d}-\frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{35\tan(c+dx)\sec^3(c+dx)}{12a^4d}+\frac{35\tan(c+dx)\sec(c+dx)}{8a^4d}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(8*a^4*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(8*a^
4*d) + (35*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^4*d) - ((2*I)*Sec[c + d*x]^7)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (((14*I)/3)*Sec[c + d*x]^5)/(d*(a^4 + I*a
^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}+\frac{7\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\ &= -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{35\int \sec^5(c+dx)\,dx}{3a^4}\\ &= \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)}\\ &= \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\\ &= \frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d}+\frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}\end{aligned}$$

Mathematica [A]
time = 1.31, size = 237, normalized size = 1.78

$$-\frac{\sec^4(c+dx)\left(896i\cos(c+dx)+3\left(128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+35\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+140\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-35\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+42\sin(c+dx)+58\sin(3(c+dx))\right)\right)}{192a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((896*I)*Cos[c + d*x] + 3*((128*I)*Cos[3*(c + d*x)]
+ 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 105*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])))/(a^
4*d)
```

Maple [A]
time = 0.30, size = 170, normalized size = 1.28

method	result
risch	$-\frac{i\left(105\,e^{7i(dx+c)}+385\,e^{5i(dx+c)}+511\,e^{3i(dx+c)}+279\,e^{i(dx+c)}\right)}{12d\,a^4\left(e^{2i(dx+c)}+1\right)^4}+\frac{35\ln\left(e^{i(dx+c)}+i\right)}{8a^4d}-\frac{35\ln\left(e^{i(dx+c)}-i\right)}{8a^4d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}{a^4d}$

default	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+}{a^4 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*((1/4-2/3*I)/(tan(1/2*d*x+1/2*c)-1)^3-(25/16+I)/(tan(1/2*d*x+1/2*c)
-1)^2+(-27/16+3*I)/(tan(1/2*d*x+1/2*c)-1)+1/8/(tan(1/2*d*x+1/2*c)-1)^4-35/1
6*ln(tan(1/2*d*x+1/2*c)-1)+(25/16-I)/(tan(1/2*d*x+1/2*c)+1)^2+(1/4+2/3*I)/(
tan(1/2*d*x+1/2*c)+1)^3-(27/16+3*I)/(tan(1/2*d*x+1/2*c)+1)-1/8/(tan(1/2*d*x
+1/2*c)+1)^4+35/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(117) = 234$.
time = 0.31, size = 295, normalized size = 2.22

$$-\frac{\frac{2\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{544i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{105\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{480i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{96i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+160i\right)}{a^4-\frac{4a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 544*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 480*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 96*I*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 160*I)/(a^4 - 4*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^4*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 105*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A]
time = 0.37, size = 230, normalized size = 1.73

$$\frac{105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(7i\,dx+7i\,c)}-770i\,e^{(5i\,dx+5i\,c)}-1022i\,e^{(3i\,dx+3i\,c)}-558i\,e^{(i\,dx+i\,c)}}{24\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(6i\,dx+6i\,c)}+6\,a^4de^{(4i\,dx+4i\,c)}+4\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 210*I*e^(7*I*d*x + 7*I*c) - 770*I*e^
(5*I*d*x + 5*I*c) - 1022*I*e^(3*I*d*x + 3*I*c) - 558*I*e^(I*d*x + I*c))/(a^
```

```
4*d*e^(8*I*d*x + 8*I*c) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x
+ 4*I*c) + 4*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^9(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**9/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.84, size = 151, normalized size = 1.14

$$\frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^4}-\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4}-\frac{2\left(81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-544i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+160i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 105*log(tan(1/2*d*x + 1/2*c)
- 1)/a^4 - 2*(81*tan(1/2*d*x + 1/2*c)^7 - 96*I*tan(1/2*d*x + 1/2*c)^6 - 105
*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan(1/2*d*x + 1/2*c)^4 - 105*tan(1/2*d*x +
1/2*c)^3 - 544*I*tan(1/2*d*x + 1/2*c)^2 + 81*tan(1/2*d*x + 1/2*c) + 160*I)/
((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/d
```

Mupad [B]
time = 6.87, size = 197, normalized size = 1.48

$$\frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^4\,d}+\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4\,a^4}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4\,a^4}-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4\,a^4}-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,136\mathrm{i}}{3\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,40\mathrm{i}}{a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,8\mathrm{i}}{a^4}-\frac{40\mathrm{i}}{3\,a^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (35*atanh(tan(c/2 + (d*x)/2)))/(4*a^4*d) + ((tan(c/2 + (d*x)/2)^2*136i)/(3*
a^4) + (35*tan(c/2 + (d*x)/2)^3)/(4*a^4) - (tan(c/2 + (d*x)/2)^4*40i)/a^4 +
 (35*tan(c/2 + (d*x)/2)^5)/(4*a^4) + (tan(c/2 + (d*x)/2)^6*8i)/a^4 - (27*ta
n(c/2 + (d*x)/2)^7)/(4*a^4) - 40i/(3*a^4) - (27*tan(c/2 + (d*x)/2))/(4*a^4)
)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2
)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=107

$$-\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{15\sec(c+dx)\tan(c+dx)}{2a^4d}+\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}+\frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] -15/2*arctanh(sin(d*x+c))/a^4/d-15/2*sec(d*x+c)*tan(d*x+c)/a^4/d+2*I*sec(d*
x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+10*I*sec(d*x+c)^3/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$-\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}-\frac{15\tan(c+dx)\sec(c+dx)}{2a^4d}+\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (15*Sec[c + d*x]*Tan[c + d*x])/(2*a
^4*d) + ((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) + ((10*I)*Sec
[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{5\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2} \\
&= \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{15\int \sec^3(c+dx)\,dx}{a^4} \\
&= -\frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx}{d\left(a^4+ia^4\tan(c+\right.} \\
&= -\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 988 vs. 2(107) = 214.
time = 6.23, size = 988, normalized size = 9.23

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(C
os[d*x] + I*Sin[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) - (15*Cos[4*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d
*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) + (Cos[d*x]*Sec[c + d*x]^4*((8*I)*Co
s[3*c] - 8*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4
) + (Sec[c]*Sec[c + d*x]^4*((4*I)*Cos[4*c] - 4*Sin[4*c])*(Cos[d*x] + I*Sin[
d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4) + (((15*I)/2)*Log[Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d
*(a + I*a*Tan[c + d*x])^4) - (((15*I)/2)*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 +
 (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*T
an[c + d*x])^4) + (Sec[c + d*x]^4*(8*Cos[3*c] + (8*I)*Sin[3*c])*(Cos[d*x] +
 I*Sin[d*x])^4*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(Co
s[4*c]/4 + (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(-1/4
*Cos[4*c] - (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (4*Sec[c + d*x]^4*(C
os[d*x] + I*Sin[d*x])^4*(Cos[4*c - (d*x)/2]/2 - Cos[4*c + (d*x)/2]/2 + (I/2
)*Sin[4*c - (d*x)/2] - (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos[c/2] + Sin[c/2])*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^4) + (4*Se
c[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-1/2*Cos[4*c - (d*x)/2] + Cos[4*c +
 (d*x)/2]/2 - (I/2)*Sin[4*c - (d*x)/2] + (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos
```

```
[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c
+ d*x])^4)
```

Maple [A]
time = 0.28, size = 118, normalized size = 1.10

method	result
risch	$\frac{8 i \,{\mathrm e}^{-i (d x +c)}}{a^{4} d}+\frac{i \left(7 \,{\mathrm e}^{3 i (d x +c)}+9 \,{\mathrm e}^{i (d x +c)}\right)}{d \,a^{4} \left({\mathrm e}^{2 i (d x +c)}+1\right)^{2}}+\frac{15 \ln \left({\mathrm e}^{i (d x +c)}-i\right)}{2 a^{4} d}-\frac{15 \ln \left({\mathrm e}^{i (d x +c)}+i\right)}{2 a^{4} d}$
derivativedivides	$\frac{\frac{2 \left(\frac{1}{4}-2 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{15 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2 \left(\frac{1}{4}+2 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1}-\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{15 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{2}+}{a^{4} d}$
default	$\frac{\frac{2 \left(\frac{1}{4}-2 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{15 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2 \left(\frac{1}{4}+2 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1}-\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{15 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{2}+}{a^{4} d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*((1/4-2*I)/(tan(1/2*d*x+1/2*c)-1)+1/4/(tan(1/2*d*x+1/2*c)-1)^2+15/4
*ln(tan(1/2*d*x+1/2*c)-1)+(1/4+2*I)/(tan(1/2*d*x+1/2*c)+1)-1/4/(tan(1/2*d*x
+1/2*c)+1)^2-15/4*ln(tan(1/2*d*x+1/2*c)+1)+8/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(95) = 190.
time = 0.54, size = 457, normalized size = 4.27

$$\frac{30 (\cos(5dx+5c)+2\cos(3dx+3c)+\cos(dx+c)+i\sin(5dx+5c)+2i\sin(3dx+3c)+i\sin(dx+c))\arctan(\cos(dx+c),\sin(dx+c)+1)+30(\cos(5dx+5c)+2\cos(3dx+3c)+\cos(dx+c)+i\sin(5dx+5c)+2i\sin(3dx+3c)+i\sin(dx+c))\arctan(\cos(dx+c),-\sin(dx+c)+1)+15(i\cos(5dx+5c)+2i\cos(3dx+3c)+i\cos(dx+c)-\sin(5dx+5c)-2\sin(3dx+3c)-\sin(dx+c))\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1)+15(-i\cos(5dx+5c)-2i\cos(3dx+3c)-i\cos(dx+c)+\sin(5dx+5c)+2\sin(3dx+3c)+\sin(dx+c))\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1)+60\cos(4dx+4c)+100\cos(2dx+2c)+60i\sin(4dx+4c)+100i\sin(2dx+2c)+32}{-4(ia^4\cos(5dx+5c)+2ia^4\cos(3dx+3c)+ia^4\cos(dx+c)-a^4\sin(5dx+5c)-2a^4\sin(3dx+3c)-a^4\sin(dx+c))d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(5*d*x + 5
*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x
+ c) + 1) + 30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*si
n(5*d*x + 5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c
), -sin(d*x + c) + 1) + 15*(I*cos(5*d*x + 5*c) + 2*I*cos(3*d*x + 3*c) + I*c
os(d*x + c) - sin(5*d*x + 5*c) - 2*sin(3*d*x + 3*c) - sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 15*(-I*cos(5*d*x + 5*c
) - 2*I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(5*d*x + 5*c) + 2*sin(3*d*x
+ 3*c) + sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
 + 1) + 60*cos(4*d*x + 4*c) + 100*cos(2*d*x + 2*c) + 60*I*sin(4*d*x + 4*c)
+ 100*I*sin(2*d*x + 2*c) + 32)/((-4*I*a^4*cos(5*d*x + 5*c) - 8*I*a^4*cos(3*
d*x + 3*c) - 4*I*a^4*cos(d*x + c) + 4*a^4*sin(5*d*x + 5*c) + 8*a^4*sin(3*d*
x + 3*c) + 4*a^4*sin(d*x + c))*d)
```

Fricas [A]
time = 0.39, size = 160, normalized size = 1.50

$$-\frac{15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(4i\,dx+4i\,c)}-50i\,e^{(2i\,dx+2i\,c)}-16i}{2\left(a^4de^{(5i\,dx+5i\,c)}+2\,a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*lo
g(e^(I*d*x + I*c) + I) - 15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) +
e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 30*I*e^(4*I*d*x + 4*I*c) - 50*I
*e^(2*I*d*x + 2*I*c) - 16*I)/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*
x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^7(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**7/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.78, size = 113, normalized size = 1.06

$$-\frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}-\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{32}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/2*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^4 - 2*(tan(1/2*d*x + 1/2*c)^3 - 8*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d
*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - 32/(a^4*(tan(1/2*
d*x + 1/2*c) - I)))/d
```

Mupad [B]
time = 5.52, size = 162, normalized size = 1.51

$$-\frac{15\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}+\frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 39\mathrm{i}}{a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 17\mathrm{i}}{a^4}+\frac{24\mathrm{i}}{a^4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 2\mathrm{i}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4),x)

[Out] ((9*tan(c/2 + (d*x)/2)^3)/a^4 - (tan(c/2 + (d*x)/2)^2*39i)/a^4 + (tan(c/2 + (d*x)/2)^4*17i)/a^4 + 24i/a^4 - (7*tan(c/2 + (d*x)/2))/a^4)/(d*(tan(c/2 + (d*x)/2)*1i - 2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*2i + tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*1i + 1)) - (15*atanh(tan(c/2 + (d*x)/2))) /(a^4*d)

3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/a^4/d+2/3*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3-2*I*s
ec(d*x+c)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3581, 3855}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}+\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Ta
n[c + d*x])^3) - ((2*I)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2} \\ &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{\int \sec(c+dx)\,dx}{a^4} \\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^4 d} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 247 vs. $2(82) = 164$.
time = 0.36, size = 247, normalized size = 3.01

$$\frac{\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\cos(3dx)\sin(c)+6\cos(dx)\sin(3c)-3i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4c)+3i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4c)+\cos(3c)(-6i\cos(dx)-6\sin(dx))-6i\sin(3c)\sin(dx)+2i\sin(c)\sin(3dx)+2\cos(c)(i\cos(3dx)+\sin(3dx))\right)}{3a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Sin[3*c] - (3*I)*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]]*Sin[4*c] + (3*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*Sin[4*c] + Cos[3*c]*((-6*I)*Cos[d*x] - 6*Sin[d*x]) - (6*I)*Sin[3*c]*Sin[d*
x] + (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c]*(I*Cos[3*d*x] + Sin[3*d*x])))/(3*a^
4*d*(-I + Tan[c + d*x])^4)
```

Maple [**A**]
time = 0.31, size = 71, normalized size = 0.87

method	result	size
derivativedivides	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^4d}$	71
default	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^4d}$	71
risch	$-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^4d}+\frac{2i\mathrm{e}^{-3i(dx+c)}}{3a^4d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^4d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^4d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(-1/2*ln(tan(1/2*d*x+1/2*c)-1)+1/2*ln(tan(1/2*d*x+1/2*c)+1)+4*I/(-I
+tan(1/2*d*x+1/2*c))^2-8/3/(-I+tan(1/2*d*x+1/2*c))^3)
```

Maxima [A]
time = 0.53, size = 141, normalized size = 1.72

$$\frac{-6i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-6i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+4i\cos\left(3\,dx+3\,c\right)-12i\cos\left(dx+c\right)+3\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\sin\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\sin\left(dx+c\right)+1\right)+4\sin\left(3\,dx+3\,c\right)-12\sin\left(dx+c\right)}{6\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(-6*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*I*arctan2(cos(d*x + c
), -sin(d*x + c) + 1) + 4*I*cos(3*d*x + 3*c) - 12*I*cos(d*x + c) + 3*log(co
s(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 3*log(cos(d*x + c)^2
+ sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*sin(3*d*x + 3*c) - 12*sin(d*x +
c))/(a^4*d)
```

Fricas [A]
time = 0.43, size = 76, normalized size = 0.93

$$\frac{\left(3\,e^{\left(3i\,dx+3i\,c\right)}\log\left(e^{\left(i\,dx+i\,c\right)}+i\right)-3\,e^{\left(3i\,dx+3i\,c\right)}\log\left(e^{\left(i\,dx+i\,c\right)}-i\right)-6i\,e^{\left(2i\,dx+2i\,c\right)}+2i\right)e^{\left(-3i\,dx-3i\,c\right)}}{3\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^(3*I*d*x + 3*I*c)*log(e^(I*d*x + I*c) + I) - 3*e^(3*I*d*x + 3*I*c)
*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-3*I*d*x - 3*
I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}\left(c+dx\right)}{\tan^{4}\left(c+dx\right)-4i\tan^{3}\left(c+dx\right)-6\tan^{2}\left(c+dx\right)+4i\tan\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.73, size = 71, normalized size = 0.87

$$\frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{4}}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{4}}+\frac{8\,\left(3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{4}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^4 + 8*(3*I*tan(1/2*d*x + 1/2*c) + 1)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^3))/
d
```

Mupad [B]
time = 3.68, size = 88, normalized size = 1.07

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}+\frac{8\mathrm{i}}{3\,a^4}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (8i/(3*a^4) - (8*tan(c/2 + (d*x)/2)
)/a^4)/(d*(tan(c/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)
/2)^3*1i + 1))
```

3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=68

$$\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}+\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}$$

```
[Out] 1/5*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+1/15*I*sec(d*x+c)^3/a/d/(a+I*a*ta
n(d*x+c))^3
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3583, 3569}

$$\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]^
3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4} + \frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{5a}$$
$$= \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4} + \frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}$$

Mathematica [A]
time = 0.12, size = 40, normalized size = 0.59

$$-\frac{\sec^3(c+dx)(-4i+\tan(c+dx))}{15a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/15*(Sec[c + d*x]^3*(-4*I + Tan[c + d*x]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.31, size = 90, normalized size = 1.32

method	result	size
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{6a^4d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{10a^4d}$	38
derivativedivides	$\frac{-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90
default	$\frac{-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(-4*I/(-I+tan(1/2*d*x+1/2*c))^4+3*I/(-I+tan(1/2*d*x+1/2*c))^2+1/(-I
+tan(1/2*d*x+1/2*c))-14/3/(-I+tan(1/2*d*x+1/2*c))^3+8/5/(-I+tan(1/2*d*x+1/2
*c))^5)
```

Maxima [A]
time = 0.30, size = 53, normalized size = 0.78

$$\frac{3i\cos(5\,dx+5\,c)+5i\cos(3\,dx+3\,c)+3\sin(5\,dx+5\,c)+5\sin(3\,dx+3\,c)}{30\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/30*(3*I*cos(5*d*x + 5*c) + 5*I*cos(3*d*x + 3*c) + 3*sin(5*d*x + 5*c) + 5*
sin(3*d*x + 3*c))/(a^4*d)
```

Fricas [A]
time = 0.40, size = 30, normalized size = 0.44

$$\frac{\left(5i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{30\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(5*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(54) = 108$.
time = 1.38, size = 182, normalized size = 2.68

$$\begin{cases} -\frac{\tan(c+dx)\sec^3(c+dx)}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\tan^2(c+dx)+60ia^4d\tan(c+dx)+15a^4d}+\frac{4i\sec^3(c+dx)}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\tan^2(c+dx)+60ia^4d\tan(c+dx)+15a^4d} & \text{for } d\neq 0 \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-tan(c + d*x)*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*
a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d*
x) + 15*a**4*d) + 4*I*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*a**
4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d*x)
+ 15*a**4*d), Ne(d, 0)), (x*sec(c)**3/(I*a*tan(c) + a)**4, True))
```

Giac [A]
time = 0.73, size = 73, normalized size = 1.07

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{15\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 15*I*tan(1/2*d*x + 1/2*c)^3 - 25*tan(1/2*
d*x + 1/2*c)^2 + 5*I*tan(1/2*d*x + 1/2*c) + 4)/(a^4*d*(tan(1/2*d*x + 1/2*c)
 - I)^5)
```

Mupad [B]
time = 3.66, size = 133, normalized size = 1.96

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,15{}\mathrm{i}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,25{}\mathrm{i}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4{}\mathrm{i}\right)}{15\,a^4\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,1{}\mathrm{i}+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,10{}\mathrm{i}-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (2*(15*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*25i - 5*tan(c/2 + (d*x)/
2) + tan(c/2 + (d*x)/2)^4*15i + 4i))/(15*a^4*d*(tan(c/2 + (d*x)/2)*5i - 10*
tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx\right.}$$

```
[Out] 1/7*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^4+3/35*I*sec(d*x+c)/a/d/(a+I*a*tan(d*
x+c))^3+2/35*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2+2/35*I*sec(d*x+c)/d/(a
^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3583, 3569}

$$\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((3*I)/35)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/35)*Sec[c + d*x])/(d*(a^2 + I*a
^2*Tan[c + d*x])^2) + (((2*I)/35)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{7a}$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{6\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{35a^2}$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{35d\,(a^2+ia^2\tan(c+$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{35d\,(a^2+ia^2\tan(c+$$

Mathematica [A]
time = 0.28, size = 73, normalized size = 0.55

$$\frac{i\sec^4(c+dx)(28\cos(c+dx)+20\cos(3(c+dx))+7i\sin(c+dx)+15i\sin(3(c+dx)))}{140a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/140)*Sec[c + d*x]^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] + (7*I)*Sin[
c + d*x] + (15*I)*Sin[3*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.15, size = 123, normalized size = 0.93

method	result
risch	$\frac{i e^{-i(dx+c)}}{8a^4d}+\frac{i e^{-3i(dx+c)}}{8a^4d}+\frac{3i e^{-5i(dx+c)}}{40a^4d}+\frac{i e^{-7i(dx+c)}}{56a^4d}$
derivativedivides	$\frac{\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^4d}$
default	$\frac{\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(3*I/(-I+tan(1/2*d*x+1/2*c))^2+1/(-I+tan(1/2*d*x+1/2*c))-8/7/(-I+ta
n(1/2*d*x+1/2*c))^7-8*I/(-I+tan(1/2*d*x+1/2*c))^4-6/(-I+tan(1/2*d*x+1/2*c))
^3+36/5/(-I+tan(1/2*d*x+1/2*c))^5+4*I/(-I+tan(1/2*d*x+1/2*c))^6)
```

Maxima [A]
time = 0.31, size = 91, normalized size = 0.69

$$\frac{5i\cos(7dx+7c)+21i\cos(5dx+5c)+35i\cos(3dx+3c)+35i\cos(dx+c)+5\sin(7dx+7c)+21\sin(5dx+5c)+35\sin(3dx+3c)+35\sin(dx+c)}{280a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(5*I*cos(7*d*x + 7*c) + 21*I*cos(5*d*x + 5*c) + 35*I*cos(3*d*x + 3*c)
 + 35*I*cos(d*x + c) + 5*sin(7*d*x + 7*c) + 21*sin(5*d*x + 5*c) + 35*sin(3*
d*x + 3*c) + 35*sin(d*x + c))/(a^4*d)
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.39

$$\frac{\left(35i\,e^{(6i\,dx+6i\,c)}+35i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*I*e^(6*I*d*x + 6*I*c) + 35*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*
x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(112) = 224$.
time = 1.40, size = 354, normalized size = 2.68

$$\begin{cases} \frac{2\tan^3(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} - \frac{8i\tan^2(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} - \frac{13\tan(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} + \frac{12i\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} & \text{for } d \neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3*sec(c + d*x)/(35*a**4*d*tan(c + d*x)**4 - 140*
I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*a**4*d*tan(c
+ d*x) + 35*a**4*d) - 8*I*tan(c + d*x)**2*sec(c + d*x)/(35*a**4*d*tan(c + d
*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*
a**4*d*tan(c + d*x) + 35*a**4*d) - 13*tan(c + d*x)*sec(c + d*x)/(35*a**4*d*
tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2
 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d) + 12*I*sec(c + d*x)/(35*a**4*d*ta
n(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 +
 140*I*a**4*d*tan(c + d*x) + 35*a**4*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) +
 a)**4, True))
```

Giac [A]
time = 0.66, size = 99, normalized size = 0.75

$$\frac{2\left(35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+210i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+147\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-49i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\right)}{35\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/35*(35*tan(1/2*d*x + 1/2*c)^6 - 105*I*tan(1/2*d*x + 1/2*c)^5 - 210*tan(1/
2*d*x + 1/2*c)^4 + 210*I*tan(1/2*d*x + 1/2*c)^3 + 147*tan(1/2*d*x + 1/2*c)^
2 - 49*I*tan(1/2*d*x + 1/2*c) - 12)/(a^4*d*(tan(1/2*d*x + 1/2*c) - I)^7)
```

Mupad [B]
time = 3.74, size = 64, normalized size = 0.48

$$\frac{\frac{e^{-c\,1i-d\,x\,1i}\,1i}{8}+\frac{e^{-c\,3i-d\,x\,3i}\,1i}{8}+\frac{e^{-c\,5i-d\,x\,5i}\,3i}{40}+\frac{e^{-c\,7i-d\,x\,7i}\,1i}{56}}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] ((exp(- c*1i - d*x*1i)*1i)/8 + (exp(- c*3i - d*x*3i)*1i)/8 + (exp(- c*5i -
d*x*5i)*3i)/40 + (exp(- c*7i - d*x*7i)*1i)/56)/(a^4*d)
```

3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=134

$$\frac{4\sin(c+dx)}{21a^4d}-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+$$

```
[Out] 4/21*sin(d*x+c)/a^4/d-4/63*sin(d*x+c)^3/a^4/d+1/9*I*cos(d*x+c)/d/(a+I*a*tan
(d*x+c))^4+5/63*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3+8/63*I*cos(d*x+c)^3/d
/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3583, 3581, 2713}

$$-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{4\sin(c+dx)}{21a^4d}+\frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (4*Sin[c + d*x])/(21*a^4*d) - (4*Sin[c + d*x]^3)/(63*a^4*d) + ((I/9)*Cos[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((5*I)/63)*Cos[c + d*x])/(a*d*(a +
I*a*Tan[c + d*x])^3) + (((8*I)/63)*Cos[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c +
d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{9a}\\ &= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{20\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{63a^2}\\ &= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+\right.}\\ &= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+\right.}\\ &= \frac{4\sin(c+dx)}{21a^4d}-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+d}{63ad(a+ia\tan(c}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 95, normalized size = 0.71

$$-\frac{i\sec^4(c+dx)(-168\cos(c+dx)-180\cos(3(c+dx))+28\cos(5(c+dx))-42i\sin(c+dx)-135i\sin(3(c+dx))+35i\sin(5(c+dx)))}{1008a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/1008*I)*Sec[c + d*x]^4*(-168*Cos[c + d*x] - 180*Cos[3*(c + d*x)] + 28*
Cos[5*(c + d*x)] - (42*I)*Sin[c + d*x] - (135*I)*Sin[3*(c + d*x)] + (35*I)*
Sin[5*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.31, size = 174, normalized size = 1.30

method	result
risch	$\frac{5i\mathrm{e}^{-3i(dx+c)}}{48a^4d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{16a^4d}+\frac{5i\mathrm{e}^{-7i(dx+c)}}{224a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^4d}+\frac{i\cos(dx+c)}{8a^4d}+\frac{3\sin(dx+c)}{16a^4d}$
derivativedivides	$\frac{\frac{2}{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32i}+\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{d}{}\right.\right.}}{a^4d}$
default	$\frac{\frac{2}{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32i}+\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{d}{}\right.\right.}}{a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(1/32/(tan(1/2*d*x+1/2*c)+I)+43/3*I/(-I+tan(1/2*d*x+1/2*c))^6-4*I/(
-I+tan(1/2*d*x+1/2*c))^8-49/4*I/(-I+tan(1/2*d*x+1/2*c))^4+49/16*I/(-I+tan(1
/2*d*x+1/2*c))^2+8/9/(-I+tan(1/2*d*x+1/2*c))^9-66/7/(-I+tan(1/2*d*x+1/2*c))
^7+31/2/(-I+tan(1/2*d*x+1/2*c))^5-173/24/(-I+tan(1/2*d*x+1/2*c))^3+31/32/(-
I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.39, size = 74, normalized size = 0.55

$$\frac{\left(-63i\,e^{(10i\,dx+10i\,c)}+315i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+126i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{2016\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-63*I*e^(10*I*d*x + 10*I*c) + 315*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(
6*I*d*x + 6*I*c) + 126*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 7
*I)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

Sympy [A]
time = 0.35, size = 231, normalized size = 1.72

$$\begin{cases} \frac{\left(-1585446912ia^{20}d^5e^{26ic}e^{idx}+7927234560ia^{20}d^5e^{24ic}e^{-idx}+5284823040ia^{20}d^5e^{22ic}e^{-3idx}+3170893824ia^{20}d^5e^{20ic}e^{-5idx}+1132462080ia^{20}d^5e^{18ic}e^{-7idx}+176160768ia^{20}d^5e^{16ic}e^{-9idx}\right)e^{-25ic}}{50734301184a^{24}d^6} & \text{for } a^{24}d^6e^{25ic}\neq 0 \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-9ic}}{32a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-1585446912*I*a**20*d**5*exp(26*I*c)*exp(I*d*x) + 7927234560*I*
a**20*d**5*exp(24*I*c)*exp(-I*d*x) + 5284823040*I*a**20*d**5*exp(22*I*c)*ex
```

```
p(-3*I*d*x) + 3170893824*I*a**20*d**5*exp(20*I*c)*exp(-5*I*d*x) + 113246208
0*I*a**20*d**5*exp(18*I*c)*exp(-7*I*d*x) + 176160768*I*a**20*d**5*exp(16*I*
c)*exp(-9*I*d*x))*exp(-25*I*c)/(50734301184*a**24*d**6), Ne(a**24*d**6*exp(
25*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*c
) + 5*exp(2*I*c) + 1)*exp(-9*I*c)/(32*a**4), True))
```

Giac [A]
time = 0.82, size = 145, normalized size = 1.08

$$\frac{\frac{63}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{1953\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-9450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-25998\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+42210i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+46368\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33054i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15858\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4374i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+703}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^9}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1008*(63/(a^4*(tan(1/2*d*x + 1/2*c) + I)) + (1953*tan(1/2*d*x + 1/2*c)^8
- 9450*I*tan(1/2*d*x + 1/2*c)^7 - 25998*tan(1/2*d*x + 1/2*c)^6 + 42210*I*ta
n(1/2*d*x + 1/2*c)^5 + 46368*tan(1/2*d*x + 1/2*c)^4 - 33054*I*tan(1/2*d*x +
 1/2*c)^3 - 15858*tan(1/2*d*x + 1/2*c)^2 + 4374*I*tan(1/2*d*x + 1/2*c) + 70
3)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B]
time = 7.41, size = 161, normalized size = 1.20

$$\frac{\left(63\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,252{}\mathrm{i}-588\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,672{}\mathrm{i}+378\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,168{}\mathrm{i}+372\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,288{}\mathrm{i}-97\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+20{}\mathrm{i}\right)\,2{}\mathrm{i}}{63\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] ((372*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*288i - 97*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*168i + 378*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x
)/2)^6*672i - 588*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*252i + 63*tan
(c/2 + (d*x)/2)^9 + 20i)*2i)/(63*a^4*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 +
 (d*x)/2)*1i + 1)^9)
```

3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=156

$$\frac{10\sin(c+dx)}{33a^4d}-\frac{20\sin^3(c+dx)}{99a^4d}+\frac{2\sin^5(c+dx)}{33a^4d}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+33$$

```
[Out] 10/33*sin(d*x+c)/a^4/d-20/99*sin(d*x+c)^3/a^4/d+2/33*sin(d*x+c)^5/a^4/d+1/1
1*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+7/99*I*cos(d*x+c)^3/a/d/(a+I*a*tan(
d*x+c))^3+4/33*I*cos(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$\frac{2\sin^5(c+dx)}{33a^4d}-\frac{20\sin^3(c+dx)}{99a^4d}+\frac{10\sin(c+dx)}{33a^4d}+\frac{4i\cos^5(c+dx)}{33d\,(a^4+ia^4\tan(c+dx))}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (10*Sin[c + d*x])/(33*a^4*d) - (20*Sin[c + d*x]^3)/(99*a^4*d) + (2*Sin[c +
d*x]^5)/(33*a^4*d) + ((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) +
 (((7*I)/99)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*C
os[c + d*x]^5)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{11a}\\ &= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{14\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,d}{33a^2}\\ &= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+\right.}\\ &= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+\right.}\\ &= \frac{10\sin(c+dx)}{33a^4d}-\frac{20\sin^3(c+dx)}{99a^4d}+\frac{2\sin^5(c+dx)}{33a^4d}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 117, normalized size = 0.75

$$-\frac{i\sec^4(c+dx)(-924\cos(c+dx)-1188\cos(3(c+dx))+308\cos(5(c+dx))+12\cos(7(c+dx))-231i\sin(c+dx)-891i\sin(3(c+dx))+385i\sin(5(c+dx))+21i\sin(7(c+dx)))}{6336a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/6336*I)*Sec[c + d*x]^4*(-924*Cos[c + d*x] - 1188*Cos[3*(c + d*x)] + 30
8*Cos[5*(c + d*x)] + 12*Cos[7*(c + d*x)] - (231*I)*Sin[c + d*x] - (891*I)*S
in[3*(c + d*x)] + (385*I)*Sin[5*(c + d*x)] + (21*I)*Sin[7*(c + d*x)]))/(a^4
*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.30, size = 240, normalized size = 1.54

method	result
risch	$\frac{7i\mathrm{e}^{-5i(dx+c)}}{128a^4d}+\frac{3i\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{7i\mathrm{e}^{-9i(dx+c)}}{1152a^4d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{1408a^4d}+\frac{7i\cos(dx+c)}{64a^4d}+\frac{7\sin(dx+c)}{32a^4d}+\frac{17i\cos(3dx+3c)}{192a^4d}$
derivativedivides	$\frac{-\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{44i}{\left(-i+\tan\left(\right.\right.}}{}$

default	$-\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{4}{(-i+\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(-1/64*I/(tan(1/2*d*x+1/2*c)+I)^2-1/96/(tan(1/2*d*x+1/2*c)+I)^3+1/1
6/(tan(1/2*d*x+1/2*c)+I)+4*I/(-I+tan(1/2*d*x+1/2*c))^10-67/4*I/(-I+tan(1/2*
d*x+1/2*c))^4-22*I/(-I+tan(1/2*d*x+1/2*c))^8+385/12*I/(-I+tan(1/2*d*x+1/2*c
))^6+201/64*I/(-I+tan(1/2*d*x+1/2*c))^2-8/11/(-I+tan(1/2*d*x+1/2*c))^11+104
/9/(-I+tan(1/2*d*x+1/2*c))^9-61/2/(-I+tan(1/2*d*x+1/2*c))^7+105/4/(-I+tan(1
/2*d*x+1/2*c))^5-267/32/(-I+tan(1/2*d*x+1/2*c))^3+15/16/(-I+tan(1/2*d*x+1/2
*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 96, normalized size = 0.62

$$\frac{\left(-33i\,e^{(14i\,dx+14i\,c)}-693i\,e^{(12i\,dx+12i\,c)}+2079i\,e^{(10i\,dx+10i\,c)}+1155i\,e^{(8i\,dx+8i\,c)}+693i\,e^{(6i\,dx+6i\,c)}+297i\,e^{(4i\,dx+4i\,c)}+77i\,e^{(2i\,dx+2i\,c)}+9i\right)e^{(-11i\,dx-11i\,c)}}{12672\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12672*(-33*I*e^(14*I*d*x + 14*I*c) - 693*I*e^(12*I*d*x + 12*I*c) + 2079*I
*e^(10*I*d*x + 10*I*c) + 1155*I*e^(8*I*d*x + 8*I*c) + 693*I*e^(6*I*d*x + 6*
I*c) + 297*I*e^(4*I*d*x + 4*I*c) + 77*I*e^(2*I*d*x + 2*I*c) + 9*I)*e^(-11*I
*d*x - 11*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(136) = 272.
time = 0.46, size = 299, normalized size = 1.92

$$\begin{cases} \frac{\left(-167196136166129664ia^{28}d^7e^{39ic}e^{3idx}-351111885948872294 4ia^{28}d^7e^{37ic}e^{idx}+10533356578466168832ia^{28}d^7e^{35ic}e^{-idx}+5851864765814538240ia^{28}d^7e^{33ic}e^{-3idx}+3511118859488722944ia^{28}d^7e^{31ic}e^{-5idx}+1504765225495166976ia^{28}d^7e^{29ic}e^{-7idx}+390124317720969216ia^{28}d^7e^{27ic}e^{-9idx}+45598946227126272ia^{28}d^7e^{25ic}e^{-11idx}\right)e^{-36ic}}{6420331628779379097 6a^{32}d^8} & \text{for}\ a^{32}d^8e^{36ic}\neq 0 \\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-11ic}}{128a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-167196136166129664*I*a**28*d**7*exp(39*I*c)*exp(3*I*d*x) - 351
1118859488722944*I*a**28*d**7*exp(37*I*c)*exp(I*d*x) + 10533356578466168832
*I*a**28*d**7*exp(35*I*c)*exp(-I*d*x) + 5851864765814538240*I*a**28*d**7*ex
p(33*I*c)*exp(-3*I*d*x) + 3511118859488722944*I*a**28*d**7*exp(31*I*c)*exp(
-5*I*d*x) + 1504765225495166976*I*a**28*d**7*exp(29*I*c)*exp(-7*I*d*x) + 39
0124317720969216*I*a**28*d**7*exp(27*I*c)*exp(-9*I*d*x) + 45598946227126272
*I*a**28*d**7*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(642033162877937909
76*a**32*d**8), Ne(a**32*d**8*exp(36*I*c), 0)), (x*(exp(14*I*c) + 7*exp(12*I
*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*c) + 7*ex
p(2*I*c) + 1)*exp(-11*I*c)/(128*a**4), True))
```

Giac [A]
time = 0.86, size = 197, normalized size = 1.26

$$\frac{\frac{33\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+21i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-11\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-39501i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-141075\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+313236i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+479556\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-516054i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-397914\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+214500i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+79024\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-17765i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2155}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{11}}}{3168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3168*(33*(12*tan(1/2*d*x + 1/2*c)^2 + 21*I*tan(1/2*d*x + 1/2*c) - 11)/(a^
4*(tan(1/2*d*x + 1/2*c) + I)^3) + (5940*tan(1/2*d*x + 1/2*c)^10 - 39501*I*t
an(1/2*d*x + 1/2*c)^9 - 141075*tan(1/2*d*x + 1/2*c)^8 + 313236*I*tan(1/2*d*
x + 1/2*c)^7 + 479556*tan(1/2*d*x + 1/2*c)^6 - 516054*I*tan(1/2*d*x + 1/2*c
)^5 - 397914*tan(1/2*d*x + 1/2*c)^4 + 214500*I*tan(1/2*d*x + 1/2*c)^3 + 790
24*tan(1/2*d*x + 1/2*c)^2 - 17765*I*tan(1/2*d*x + 1/2*c) - 2155)/(a^4*(tan(
1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B]
time = 5.61, size = 216, normalized size = 1.38

$$-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{269\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{1307\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}+\frac{1307\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{1099\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}+\frac{203\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}-\frac{21\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{64}+\frac{21\,\sin\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{64}+\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,231{}\mathrm{i}}{16}-\frac{\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,231{}\mathrm{i}}{16}+\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\,33{}\mathrm{i}-\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\,5{}\mathrm{i}+\frac{\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,3{}\mathrm{i}}{16}-\frac{\cos\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)\,3{}\mathrm{i}}{16}\right)\,2{}\mathrm{i}}{99\,a^4\,d\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^{11}\,{\left(\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*231i)/16 - (cos((5*c)/2 + (
5*d*x)/2)*231i)/16 + cos((7*c)/2 + (7*d*x)/2)*33i - cos((9*c)/2 + (9*d*x)/2
)*5i + (cos((11*c)/2 + (11*d*x)/2)*3i)/16 - (cos((13*c)/2 + (13*d*x)/2)*3i)
/16 + (269*sin(c/2 + (d*x)/2))/16 - (1307*sin((3*c)/2 + (3*d*x)/2))/64 + (1
307*sin((5*c)/2 + (5*d*x)/2))/64 - (1099*sin((7*c)/2 + (7*d*x)/2))/32 + (20
3*sin((9*c)/2 + (9*d*x)/2))/32 - (21*sin((11*c)/2 + (11*d*x)/2))/64 + (21*s
in((13*c)/2 + (13*d*x)/2))/64)*2i)/(99*a^4*d*(cos(c/2 + (d*x)/2) + sin(c/2
+ (d*x)/2)*1i)^11*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^3)
```

3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$\frac{56\sin(c+dx)}{143a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{8\sin^7(c+dx)}{143a^4d}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9i\cos^5}{143ad(a+i}$$

```
[Out] 56/143*sin(d*x+c)/a^4/d-56/143*sin(d*x+c)^3/a^4/d+168/715*sin(d*x+c)^5/a^4/
d-8/143*sin(d*x+c)^7/a^4/d+1/13*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^4+9/143
*I*cos(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+16/143*I*cos(d*x+c)^7/d/(a^4+I*a^4
*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$-\frac{8\sin^7(c+dx)}{143a^4d}+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}+\frac{56\sin(c+dx)}{143a^4d}+\frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (56*Sin[c + d*x])/(143*a^4*d) - (56*Sin[c + d*x]^3)/(143*a^4*d) + (168*Sin[
c + d*x]^5)/(715*a^4*d) - (8*Sin[c + d*x]^7)/(143*a^4*d) + ((I/13)*Cos[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (((9*I)/143)*Cos[c + d*x]^5)/(a*d*(a
 + I*a*Tan[c + d*x])^3) + (((16*I)/143)*Cos[c + d*x]^7)/(d*(a^4 + I*a^4*Tan
[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{13a}\\ &= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3}+\frac{72\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}}{143a^2}\\ &= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+d}{143d\left(a^4+ia^4\tan(c\right.}\\ &= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}+\frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+d}{143d\left(a^4+ia^4\tan(c\right.}\\ &= \frac{56\sin(c+dx)}{143a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{8\sin^7(c+dx)}{143a^4d}+\frac{}{13d(a}\end{aligned}$$

Mathematica [A]
time = 0.97, size = 139, normalized size = 0.80

$$-\frac{i\sec^4(c+dx)(-24024\cos(c+dx)-34320\cos(3(c+dx))+11440\cos(5(c+dx))+780\cos(7(c+dx))+44\cos(9(c+dx))-6006i\sin(c+dx)-25740i\sin(3(c+dx))+14300i\sin(5(c+dx))+1365i\sin(7(c+dx))+99i\sin(9(c+dx)))}{183040a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/183040*I)*Sec[c + d*x]^4*(-24024*Cos[c + d*x] - 34320*Cos[3*(c + d*x)]
 + 11440*Cos[5*(c + d*x)] + 780*Cos[7*(c + d*x)] + 44*Cos[9*(c + d*x)] - (6
006*I)*Sin[c + d*x] - (25740*I)*Sin[3*(c + d*x)] + (14300*I)*Sin[5*(c + d*x
)] + (1365*I)*Sin[7*(c + d*x)] + (99*I)*Sin[9*(c + d*x)]))/(a^4*d*(-I + Tan
[c + d*x])^4)
```

Maple [A]
time = 0.32, size = 306, normalized size = 1.76

method	result
risch	$\frac{3i\,e^{-7i(dx+c)}}{128a^4d}+\frac{i\,e^{-9i(dx+c)}}{128a^4d}+\frac{9i\,e^{-11i(dx+c)}}{5632a^4d}+\frac{i\,e^{-13i(dx+c)}}{6656a^4d}+\frac{3i\cos(dx+c)}{32a^4d}+\frac{15\sin(dx+c)}{64a^4d}+\frac{25i\cos(5dx+5c)}{512a^4d}$

derivativedivides	$\frac{-\frac{11i}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{465i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\cdots}{}$
default	$\frac{-\frac{11i}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{465i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(-11/256*I/(tan(1/2*d*x+1/2*c)+I)^2+465/8*I/(-I+tan(1/2*d*x+1/2*c))
^6+1/160/(tan(1/2*d*x+1/2*c)+I)^5-5/128/(tan(1/2*d*x+1/2*c)+I)^3+23/256/(ta
n(1/2*d*x+1/2*c)+I)-4*I/(-I+tan(1/2*d*x+1/2*c))^12-1375/64*I/(-I+tan(1/2*d*
x+1/2*c))^4+825/256*I/(-I+tan(1/2*d*x+1/2*c))^2+31*I/(-I+tan(1/2*d*x+1/2*c)
)^10+1/64*I/(tan(1/2*d*x+1/2*c)+I)^4-135/2*I/(-I+tan(1/2*d*x+1/2*c))^8+8/13
/(-I+tan(1/2*d*x+1/2*c))^13-150/11/(-I+tan(1/2*d*x+1/2*c))^11+52/(-I+tan(1/
2*d*x+1/2*c))^9-279/4/(-I+tan(1/2*d*x+1/2*c))^7+6291/160/(-I+tan(1/2*d*x+1/
2*c))^5-1207/128/(-I+tan(1/2*d*x+1/2*c))^3+233/256/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 118, normalized size = 0.68

$$\frac{\left(-143i\,e^{(18i\,dx+18i\,c)}-2145i\,e^{(16i\,dx+16i\,c)}-25740i\,e^{(14i\,dx+14i\,c)}+60060i\,e^{(12i\,dx+12i\,c)}+30030i\,e^{(10i\,dx+10i\,c)}+18018i\,e^{(8i\,dx+8i\,c)}+8580i\,e^{(6i\,dx+6i\,c)}+2860i\,e^{(4i\,dx+4i\,c)}+585i\,e^{(2i\,dx+2i\,c)}+55i\right)e^{(-13i\,dx-13i\,c)}}{366080\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/366080*(-143*I*e^(18*I*d*x + 18*I*c) - 2145*I*e^(16*I*d*x + 16*I*c) - 257
40*I*e^(14*I*d*x + 14*I*c) + 60060*I*e^(12*I*d*x + 12*I*c) + 30030*I*e^(10*
I*d*x + 10*I*c) + 18018*I*e^(8*I*d*x + 8*I*c) + 8580*I*e^(6*I*d*x + 6*I*c)
+ 2860*I*e^(4*I*d*x + 4*I*c) + 585*I*e^(2*I*d*x + 2*I*c) + 55*I)*e^(-13*I*d
*x - 13*I*c)/(a^4*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(153) = 306.
time = 0.55, size = 367, normalized size = 2.11

$$\begin{cases} \frac{\left(-1688246017625898163896320ia^{36}d^{9}e^{54ic}e^{5idx}-2532369026438847245844480 0ia^{36}d^{9}e^{52ic}e^{3idx}-30388428317266166950133760 0ia^{36}d^{9}e^{50ic}e^{idx}+70906332740287722883645440 0ia^{36}d^{9}e^{48ic}e^{-idx}+35453166370143861441822720 0ia^{36}d^{9}e^{46ic}e^{-3idx}+2127189982208631686509363 20ia^{36}d^{9}e^{44ic}e^{-5idx}+10129476105755388983377920 0ia^{36}d^{9}e^{42ic}e^{-7idx}+33764920352517963277926400ia^{36}d^{9}e^{40ic}e^{-9idx}+6906460981196856125030400ia^{36}d^{9}e^{38ic}e^{-11idx}+649325391394576216883200ia^{36}d^{9}e^{36ic}e^{-13idx}\right)e^{-49ic}}{4321909805122299299574579200a^{40}d^{10}} & \text{for}\ a^{40}d^{10}e^{49ic}\neq 0 \\ \frac{x\left(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1\right)e^{-13ic}}{512a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-16882460176258981638963 20*I*a**36*d**9*exp(54*I*c)*exp(5*I*d*x
) - 253236902643884724584448 00*I*a**36*d**9*exp(52*I*c)*exp(3*I*d*x) - 3038
842831726616695013376 00*I*a**36*d**9*exp(50*I*c)*exp(I*d*x) + 7090633274028
77228836454400*I*a**36*d**9*exp(48*I*c)*exp(-I*d*x) + 354531663701438614418
227200*I*a**36*d**9*exp(46*I*c)*exp(-3*I*d*x) + 212718998220863168650936320
*I*a**36*d**9*exp(44*I*c)*exp(-5*I*d*x) + 101294761057553889833779200*I*a**
36*d**9*exp(42*I*c)*exp(-7*I*d*x) + 33764920352517963277926400*I*a**36*d**9
*exp(40*I*c)*exp(-9*I*d*x) + 6906460981196856125030400*I*a**36*d**9*exp(38*
I*c)*exp(-11*I*d*x) + 649325391394576216883200*I*a**36*d**9*exp(36*I*c)*exp
(-13*I*d*x))*exp(-49*I*c)/(43219098051222992995745792 00*a**40*d**10), Ne(a*
*40*d**10*exp(49*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c) + 36*exp(14*I*c
) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84*exp(6*I*c) + 36*
exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-13*I*c)/(512*a**4), True))
```

Giac [A]
time = 0.86, size = 249, normalized size = 1.43

$$\frac{\frac{143\left(115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+405i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-575\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-375i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+98\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{166595\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-1409265i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-6232655\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+17535375i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+34610004\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-49771722i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-53349582\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+42730974i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+25431835\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10954229i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3278067\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+614627i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+60094}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{13}}}{91520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/91520*(143*(115*tan(1/2*d*x + 1/2*c)^4 + 405*I*tan(1/2*d*x + 1/2*c)^3 - 5
75*tan(1/2*d*x + 1/2*c)^2 - 375*I*tan(1/2*d*x + 1/2*c) + 98)/(a^4*(tan(1/2*
d*x + 1/2*c) + I)^5) + (166595*tan(1/2*d*x + 1/2*c)^12 - 1409265*I*tan(1/2*
d*x + 1/2*c)^11 - 6232655*tan(1/2*d*x + 1/2*c)^10 + 17535375*I*tan(1/2*d*x
+ 1/2*c)^9 + 34610004*tan(1/2*d*x + 1/2*c)^8 - 49771722*I*tan(1/2*d*x + 1/2
*c)^7 - 53349582*tan(1/2*d*x + 1/2*c)^6 + 42730974*I*tan(1/2*d*x + 1/2*c)^5
 + 25431835*tan(1/2*d*x + 1/2*c)^4 - 10954229*I*tan(1/2*d*x + 1/2*c)^3 - 32
78067*tan(1/2*d*x + 1/2*c)^2 + 614627*I*tan(1/2*d*x + 1/2*c) + 60094)/(a^4*
(tan(1/2*d*x + 1/2*c) - I)^13))/d
```

Mupad [B]
time = 6.97, size = 262, normalized size = 1.51

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15049\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}-\frac{4513\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{32}+\frac{4513\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{15461\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{64}+\frac{3941\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{64}-\frac{183\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{32}+\frac{183\sin\left(\frac{13c}{2}+\frac{13dx}{2}\right)}{32}-\frac{99\sin\left(\frac{15c}{2}+\frac{15dx}{2}\right)}{256}+\frac{99\sin\left(\frac{17c}{2}+\frac{17dx}{2}\right)}{256}+\frac{\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)3003i}{32}-\frac{\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)3003i}{32}+\frac{\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)7293i}{32}-\frac{\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)1533i}{32}+\frac{\cos\left(\frac{11c}{2}+\frac{11dx}{2}\right)103i}{32}-\frac{\cos\left(\frac{13c}{2}+\frac{13dx}{2}\right)103i}{32}+\frac{\cos\left(\frac{15c}{2}+\frac{15dx}{2}\right)11i}{64}-\frac{\cos\left(\frac{17c}{2}+\frac{17dx}{2}\right)11i}{64}\right)2i}{715\,a^4\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^{13}\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*3003i)/32 - (cos((5*c)/2 + (
5*d*x)/2)*3003i)/32 + (cos((7*c)/2 + (7*d*x)/2)*7293i)/32 - (cos((9*c)/2 +
```

```
(9*d*x)/2)*1533i)/32 + (cos((11*c)/2 + (11*d*x)/2)*103i)/32 - (cos((13*c)/2
 + (13*d*x)/2)*103i)/32 + (cos((15*c)/2 + (15*d*x)/2)*11i)/64 - (cos((17*c)
/2 + (17*d*x)/2)*11i)/64 + (15049*sin(c/2 + (d*x)/2))/128 - (4513*sin((3*c)
/2 + (3*d*x)/2))/32 + (4513*sin((5*c)/2 + (5*d*x)/2))/32 - (15461*sin((7*c)
/2 + (7*d*x)/2))/64 + (3941*sin((9*c)/2 + (9*d*x)/2))/64 - (183*sin((11*c)/
2 + (11*d*x)/2))/32 + (183*sin((13*c)/2 + (13*d*x)/2))/32 - (99*sin((15*c)/
2 + (15*d*x)/2))/256 + (99*sin((17*c)/2 + (17*d*x)/2))/256)*2i)/(715*a^4*d*
(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^13*(cos(c/2 + (d*x)/2)*1i + si
n(c/2 + (d*x)/2))^5)
```

3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=134

$$-\frac{192x}{a^8}-\frac{192i\log(\cos(c+dx))}{a^8d}+\frac{129\tan(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}+\frac{\tan^5}{}$$

```
[Out] -192*x/a^8-192*I*ln(cos(d*x+c))/a^8/d+129*tan(d*x+c)/a^8/d-36*I*tan(d*x+c)^
2/a^8/d-10*tan(d*x+c)^3/a^8/d+2*I*tan(d*x+c)^4/a^8/d+1/5*tan(d*x+c)^5/a^8/d
+64*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$,
Rules used = {3568, 45}

$$\frac{\tan^5(c+dx)}{5a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}+\frac{129\tan(c+dx)}{a^8d}+\frac{64i}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{192i\log(\cos(c+dx))}{a^8d}-\frac{192x}{a^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-192*x)/a^8 - ((192*I)*Log[Cos[c + d*x]])/(a^8*d) + (129*Tan[c + d*x])/(a^
8*d) - ((36*I)*Tan[c + d*x]^2)/(a^8*d) - (10*Tan[c + d*x]^3)/(a^8*d) + ((2*
I)*Tan[c + d*x]^4)/(a^8*d) + Tan[c + d*x]^5/(5*a^8*d) + (64*I)/(d*(a^8 + I*
a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^6}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}$$
$$= -\frac{i\text{Subst}\left(\int \left(129a^4-72a^3x+30a^2x^2-8ax^3+x^4+\frac{64a^6}{(a+x)^2}-\frac{192a^5}{a+x}\right)dx, x, ic$$
$$= -\frac{192x}{a^8}-\frac{192i\log(\cos(c+dx))}{a^8d}+\frac{129\tan(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}-10t$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 599 vs. 2(134) = 268.
time = 3.06, size = 599, normalized size = 4.47

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^13*(-Cos[7*(c + d*x)] - I*Sin[7*(c + d*x)])*((-220*I)*
Cos[3*c + 2*d*x] + 900*d*x*Cos[3*c + 2*d*x] + (238*I)*Cos[3*c + 4*d*x] + 36
0*d*x*Cos[3*c + 4*d*x] - (110*I)*Cos[5*c + 4*d*x] + 360*d*x*Cos[5*c + 4*d*x
] + (77*I)*Cos[5*c + 6*d*x] + 60*d*x*Cos[5*c + 6*d*x] - (10*I)*Cos[7*c + 6*
d*x] + 60*d*x*Cos[7*c + 6*d*x] + 10*Cos[c]*(-7*I + 120*d*x + (120*I)*Log[Co
s[c + d*x]]) + 5*Cos[c + 2*d*x]*(43*I + 180*d*x + (180*I)*Log[Cos[c + d*x]]
) + (900*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]] + (360*I)*Cos[3*c + 4*d*x]*L
og[Cos[c + d*x]] + (360*I)*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[
5*c + 6*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[7*c + 6*d*x]*Log[Cos[c + d*x]]
+ 870*Sin[c] - 985*Sin[c + 2*d*x] + (300*I)*d*x*Sin[c + 2*d*x] - 300*Log[Co
s[c + d*x]]*Sin[c + 2*d*x] + 320*Sin[3*c + 2*d*x] + (300*I)*d*x*Sin[3*c + 2
*d*x] - 300*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x] - 512*Sin[3*c + 4*d*x] + (24
0*I)*d*x*Sin[3*c + 4*d*x] - 240*Log[Cos[c + d*x]]*Sin[3*c + 4*d*x] + 10*Sin
[5*c + 4*d*x] + (240*I)*d*x*Sin[5*c + 4*d*x] - 240*Log[Cos[c + d*x]]*Sin[5*
c + 4*d*x] - 97*Sin[5*c + 6*d*x] + (60*I)*d*x*Sin[5*c + 6*d*x] - 60*Log[Cos
[c + d*x]]*Sin[5*c + 6*d*x] - 10*Sin[7*c + 6*d*x] + (60*I)*d*x*Sin[7*c + 6*
d*x] - 60*Log[Cos[c + d*x]]*Sin[7*c + 6*d*x]))/(20*a^8*d*(-I + Tan[c + d*x]
)^8)
```

Maple [A]
time = 0.37, size = 85, normalized size = 0.63

method	result

derivativedivides	$\frac{129\tan(dx+c)+\frac{\left(\tan^5(dx+c)\right)}{5}+2i\left(\tan^4(dx+c)\right)-10\left(\tan^3(dx+c)\right)-36i\left(\tan^2(dx+c)\right)+192i\ln(\tan(dx+c)-i)+\frac{64}{\tan(dx+c)-}}{d\,a^8}$
default	$\frac{129\tan(dx+c)+\frac{\left(\tan^5(dx+c)\right)}{5}+2i\left(\tan^4(dx+c)\right)-10\left(\tan^3(dx+c)\right)-36i\left(\tan^2(dx+c)\right)+192i\ln(\tan(dx+c)-i)+\frac{64}{\tan(dx+c)-}}{d\,a^8}$
risch	$\frac{32i\mathrm{e}^{-2i(dx+c)}}{a^8d}-\frac{384x}{a^8}-\frac{384c}{a^8d}+\frac{16i\left(50\,\mathrm{e}^{8i(dx+c)}+220\,\mathrm{e}^{6i(dx+c)}+370\,\mathrm{e}^{4i(dx+c)}+285\,\mathrm{e}^{2i(dx+c)}+87\right)}{5d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}-\frac{192i\ln\left(\mathrm{e}^{2i(c}\right.}{a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(129*tan(d*x+c)+1/5*tan(d*x+c)^5+2*I*tan(d*x+c)^4-10*tan(d*x+c)^3-3
6*I*tan(d*x+c)^2+192*I*ln(tan(d*x+c)-I)+64/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.31, size = 229, normalized size = 1.71

$$\frac{\frac{320\left(\tan(dx+c)^6-6i\tan(dx+c)^5-15\tan(dx+c)^4+20i\tan(dx+c)^3+15\tan(dx+c)^2-6i\tan(dx+c)-1\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\frac{\tan(dx+c)^5+10i\tan(dx+c)^4-50\tan(dx+c)^3-180i\tan(dx+c)^2+645\tan(dx+c)}{a^8}+\frac{960i\log(i\tan(dx+c)+1)}{a^8}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(320*(tan(d*x + c)^6 - 6*I*tan(d*x + c)^5 - 15*tan(d*x + c)^4 + 20*I*ta
n(d*x + c)^3 + 15*tan(d*x + c)^2 - 6*I*tan(d*x + c) - 1)/(a^8*tan(d*x + c)^
7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^
4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) +
I*a^8) + (tan(d*x + c)^5 + 10*I*tan(d*x + c)^4 - 50*tan(d*x + c)^3 - 180*I*
tan(d*x + c)^2 + 645*tan(d*x + c))/a^8 + 960*I*log(I*tan(d*x + c) + 1)/a^8)
/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(122) = 244.
time = 0.48, size = 273, normalized size = 2.04

$$-\frac{16\left(120\,dxe^{(12i\,dx+12i\,c)}+60\,(10\,dx-i)e^{(10i\,dx+10i\,c)}+30\,(40\,dx-9i)e^{(8i\,dx+8i\,c)}+10\,(120\,dx-47i)e^{(6i\,dx+6i\,c)}+5\,(120\,dx-77i)e^{(4i\,dx+4i\,c)}+(120\,dx-137i)e^{(2i\,dx+2i\,c)}+60\left(i\,e^{(12i\,dx+12i\,c)}+5i\,e^{(10i\,dx+10i\,c)}+10i\,e^{(8i\,dx+8i\,c)}+10i\,e^{(6i\,dx+6i\,c)}+5i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-10i\right)}{5\left(a^8de^{(12i\,dx+12i\,c)}+5\,a^8de^{(10i\,dx+10i\,c)}+10\,a^8de^{(8i\,dx+8i\,c)}+10\,a^8de^{(6i\,dx+6i\,c)}+5\,a^8de^{(4i\,dx+4i\,c)}+a^8de^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -16/5*(120*d*x*e^(12*I*d*x + 12*I*c) + 60*(10*d*x - I)*e^(10*I*d*x + 10*I*c
) + 30*(40*d*x - 9*I)*e^(8*I*d*x + 8*I*c) + 10*(120*d*x - 47*I)*e^(6*I*d*x
+ 6*I*c) + 5*(120*d*x - 77*I)*e^(4*I*d*x + 4*I*c) + (120*d*x - 137*I)*e^(2*
I*d*x + 2*I*c) + 60*(I*e^(12*I*d*x + 12*I*c) + 5*I*e^(10*I*d*x + 10*I*c) +
10*I*e^(8*I*d*x + 8*I*c) + 10*I*e^(6*I*d*x + 6*I*c) + 5*I*e^(4*I*d*x + 4*I*
c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 10*I)/(a^8*d*e^(
```

```
12*I*d*x + 12*I*c) + 5*a^8*d*e^(10*I*d*x + 10*I*c) + 10*a^8*d*e^(8*I*d*x +
8*I*c) + 10*a^8*d*e^(6*I*d*x + 6*I*c) + 5*a^8*d*e^(4*I*d*x + 4*I*c) + a^8*d
*e^(2*I*d*x + 2*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{14}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**14/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(122) = 244.
time = 1.16, size = 250, normalized size = 1.87

$$-\frac{2\left(\frac{480i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}-\frac{960i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}+\frac{480i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}+\frac{160\left(9i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9i\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^2}+\frac{-1096i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+645\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+5840i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-2780\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-12120i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+4286\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12120i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2780\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5840i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+645\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1096i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5a^8}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(480*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 960*I*log(tan(1/2*d*x + 1/2
*c) - I)/a^8 + 480*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + 160*(9*I*tan(1/2*d
*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) - 9*I)/(a^8*(tan(1/2*d*x + 1/2*c) -
 I)^2) + (-1096*I*tan(1/2*d*x + 1/2*c)^10 + 645*tan(1/2*d*x + 1/2*c)^9 + 58
40*I*tan(1/2*d*x + 1/2*c)^8 - 2780*tan(1/2*d*x + 1/2*c)^7 - 12120*I*tan(1/2
*d*x + 1/2*c)^6 + 4286*tan(1/2*d*x + 1/2*c)^5 + 12120*I*tan(1/2*d*x + 1/2*c
)^4 - 2780*tan(1/2*d*x + 1/2*c)^3 - 5840*I*tan(1/2*d*x + 1/2*c)^2 + 645*tan
(1/2*d*x + 1/2*c) + 1096*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^8))/d
```

Mupad [**B**]
time = 3.45, size = 105, normalized size = 0.78

$$\frac{\frac{129\tan(c+dx)}{a^8}-\frac{10\tan(c+dx)^3}{a^8}+\frac{\tan(c+dx)^5}{5a^8}+\frac{\ln(\tan(c+dx)-\mathrm{i})\,192\mathrm{i}}{a^8}+\frac{64\mathrm{i}}{a^8\,(1+\tan(c+dx)\,1\mathrm{i})}-\frac{\tan(c+dx)^2\,36\mathrm{i}}{a^8}+\frac{\tan(c+dx)^4\,2\mathrm{i}}{a^8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((log(tan(c + d*x) - 1i)*192i)/a^8 + (129*tan(c + d*x))/a^8 + 64i/(a^8*(tan
(c + d*x)*1i + 1)) - (tan(c + d*x)^2*36i)/a^8 - (10*tan(c + d*x)^3)/a^8 + (
tan(c + d*x)^4*2i)/a^8 + tan(c + d*x)^5/(5*a^8))/d
```

3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=126

$$\frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+dx)}{3a^8d}+\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}-\frac{}{d\,(a^8}$$

```
[Out] 80*x/a^8+80*I*ln(cos(d*x+c))/a^8/d-31*tan(d*x+c)/a^8/d+4*I*tan(d*x+c)^2/a^8
/d+1/3*tan(d*x+c)^3/a^8/d+16*I/d/(a^4+I*a^4*tan(d*x+c))^2-80*I/d/(a^8+I*a^8
*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{3568, 45\}$

$$\frac{\tan^3(c+dx)}{3a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}-\frac{31\tan(c+dx)}{a^8d}-\frac{80i}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{80i\log(\cos(c+dx))}{a^8d}+\frac{80x}{a^8}+\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (80*x)/a^8 + ((80*I)*Log[Cos[c + d*x]])/(a^8*d) - (31*Tan[c + d*x])/(a^8*d)
 + ((4*I)*Tan[c + d*x]^2)/(a^8*d) + Tan[c + d*x]^3/(3*a^8*d) + (16*I)/(d*(a
^4 + I*a^4*Tan[c + d*x])^2) - (80*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\mathrm{Subst}\left(\int \frac{(a-x)^5}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}$$

$$= -\frac{i\mathrm{Subst}\left(\int \left(-31a^2+8ax-x^2+\frac{32a^5}{(a+x)^3}-\frac{80a^4}{(a+x)^2}+\frac{80a^3}{a+x}\right)\,dx, x, ia\tan(c+d\right.}{a^{11}d}$$

$$= \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+}{3a^8d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 537 vs. 2(126) = 252.
time = 1.66, size = 537, normalized size = 4.26

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^11*(Cos[6*(c + d*x)] + I*Sin[6*(c + d*x)])*((66*I)*Cos
[2*c + 3*d*x] + 180*d*x*Cos[2*c + 3*d*x] - (75*I)*Cos[4*c + 3*d*x] + 180*d*
x*Cos[4*c + 3*d*x] + (50*I)*Cos[4*c + 5*d*x] + 60*d*x*Cos[4*c + 5*d*x] + (3
*I)*Cos[6*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] + 3*Cos[2*c + d*x]*(-71*I +
80*d*x + (80*I)*Log[Cos[c + d*x]]) + Cos[d*x]*(-119*I + 240*d*x + (240*I)*L
og[Cos[c + d*x]]) + (180*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]] + (180*I)*Co
s[4*c + 3*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]
] + (60*I)*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]] - 101*Sin[d*x] + (120*I)*d*x*
Sin[d*x] - 120*Log[Cos[c + d*x]]*Sin[d*x] + 87*Sin[2*c + d*x] + (120*I)*d*x
*Sin[2*c + d*x] - 120*Log[Cos[c + d*x]]*Sin[2*c + d*x] - 96*Sin[2*c + 3*d*x
] + (180*I)*d*x*Sin[2*c + 3*d*x] - 180*Log[Cos[c + d*x]]*Sin[2*c + 3*d*x] +
 45*Sin[4*c + 3*d*x] + (180*I)*d*x*Sin[4*c + 3*d*x] - 180*Log[Cos[c + d*x]]
*Sin[4*c + 3*d*x] - 44*Sin[4*c + 5*d*x] + (60*I)*d*x*Sin[4*c + 5*d*x] - 60*
Log[Cos[c + d*x]]*Sin[4*c + 5*d*x] + 3*Sin[6*c + 5*d*x] + (60*I)*d*x*Sin[6*
c + 5*d*x] - 60*Log[Cos[c + d*x]]*Sin[6*c + 5*d*x]))/(12*a^8*d*(-I + Tan[c
+ d*x])^8)
```

Maple [A]
time = 0.34, size = 78, normalized size = 0.62

method	result
derivativedivides	$\frac{-31\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}+4i\left(\tan^2(dx+c)\right)-80i\ln(\tan(dx+c)-i)-\frac{16i}{(\tan(dx+c)-i)^2}-\frac{80}{\tan(dx+c)-i}}{d\,a^8}$

default	$\frac{-31\tan(dx+c)+\frac{\left(\tan^3(dx+c)\right)}{3}+4i\left(\tan^2(dx+c)\right)-80i\ln(\tan(dx+c)-i)-\frac{16i}{(\tan(dx+c)-i)^2}-\frac{80}{\tan(dx+c)-i}}{d\,a^8}$	7
risch	$-\frac{32i\mathrm{e}^{-2i(dx+c)}}{a^8d}+\frac{4i\mathrm{e}^{-4i(dx+c)}}{a^8d}+\frac{160x}{a^8}+\frac{160c}{a^8d}-\frac{4i\left(36\,\mathrm{e}^{4i(dx+c)}+81\,\mathrm{e}^{2i(dx+c)}+47\right)}{3d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{80i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8d}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-31*tan(d*x+c)+1/3*tan(d*x+c)^3+4*I*tan(d*x+c)^2-80*I*ln(tan(d*x+c
)-I)-16*I/(tan(d*x+c)-I)^2-80/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.32, size = 212, normalized size = 1.68

$$-\frac{\frac{48\left(5\tan(dx+c)^6-29i\tan(dx+c)^5-70\tan(dx+c)^4+90i\tan(dx+c)^3+65\tan(dx+c)^2-25i\tan(dx+c)-4\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}-\frac{\tan(dx+c)^3+12i\tan(dx+c)^2-93\tan(dx+c)}{a^8}+\frac{240i\log(i\tan(dx+c)+1)}{a^8}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(48*(5*tan(d*x + c)^6 - 29*I*tan(d*x + c)^5 - 70*tan(d*x + c)^4 + 90*I
*tan(d*x + c)^3 + 65*tan(d*x + c)^2 - 25*I*tan(d*x + c) - 4)/(a^8*tan(d*x +
 c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
 c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) - (tan(d*x + c)^3 + 12*I*tan(d*x + c)^2 - 93*tan(d*x + c))/a^8 +
 240*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

Fricas [A]
time = 0.40, size = 199, normalized size = 1.58

$$\frac{4\left(120\,dxe^{(10i\,dx+10i\,c)}+60\,(6\,dx-i)e^{(8i\,dx+8i\,c)}+30\,(12\,dx-5i)e^{(6i\,dx+6i\,c)}+10\,(12\,dx-11i)e^{(4i\,dx+4i\,c)}-60\left(-i\,e^{(10i\,dx+10i\,c)}-3i\,e^{(8i\,dx+8i\,c)}-3i\,e^{(6i\,dx+6i\,c)}-i\,e^{(4i\,dx+4i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-15i\,e^{(2i\,dx+2i\,c)}+3i\right)}{3\left(a^8de^{(10i\,dx+10i\,c)}+3\,a^8de^{(8i\,dx+8i\,c)}+3\,a^8de^{(6i\,dx+6i\,c)}+a^8de^{(4i\,dx+4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 4/3*(120*d*x*e^(10*I*d*x + 10*I*c) + 60*(6*d*x - I)*e^(8*I*d*x + 8*I*c) + 3
0*(12*d*x - 5*I)*e^(6*I*d*x + 6*I*c) + 10*(12*d*x - 11*I)*e^(4*I*d*x + 4*I*
c) - 60*(-I*e^(10*I*d*x + 10*I*c) - 3*I*e^(8*I*d*x + 8*I*c) - 3*I*e^(6*I*d*
x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 15*I*e^(
2*I*d*x + 2*I*c) + 3*I)/(a^8*d*e^(10*I*d*x + 10*I*c) + 3*a^8*d*e^(8*I*d*x +
 8*I*c) + 3*a^8*d*e^(6*I*d*x + 6*I*c) + a^8*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{12}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**12/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Giac [A]
time = 1.84, size = 224, normalized size = 1.78

$$-\frac{2\left(-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}+\frac{240i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}+\frac{220i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-93\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-684i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+190\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+684i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-93\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-220i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^8}+\frac{4\left(-125i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-536\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+846i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+536\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-125i\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^4}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/3*(-120*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 + 240*I*log(tan(1/2*d*x + 1/
2*c) - I)/a^8 - 120*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + (220*I*tan(1/2*d*
x + 1/2*c)^6 - 93*tan(1/2*d*x + 1/2*c)^5 - 684*I*tan(1/2*d*x + 1/2*c)^4 + 1
90*tan(1/2*d*x + 1/2*c)^3 + 684*I*tan(1/2*d*x + 1/2*c)^2 - 93*tan(1/2*d*x +
 1/2*c) - 220*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^8) + 4*(-125*I*tan(1/2*d
*x + 1/2*c)^4 - 536*tan(1/2*d*x + 1/2*c)^3 + 846*I*tan(1/2*d*x + 1/2*c)^2 +
 536*tan(1/2*d*x + 1/2*c) - 125*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^4))/d
```

Mupad [B]
time = 3.44, size = 114, normalized size = 0.90

$$\frac{\tan(c+d\,x)^3}{3\,a^8\,d}-\frac{31\tan(c+d\,x)}{a^8\,d}+\frac{\tan(c+d\,x)^2\,4\mathrm{i}}{a^8\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,80\mathrm{i}}{a^8\,d}-\frac{\frac{64}{a^8}+\frac{\tan(c+d\,x)\,80\mathrm{i}}{a^8}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (tan(c + d*x)^2*4i)/(a^8*d) - (31*tan(c + d*x))/(a^8*d) - (log(tan(c + d*x)
 - 1i)*80i)/(a^8*d) + tan(c + d*x)^3/(3*a^8*d) - ((tan(c + d*x)*80i)/a^8 +
64/a^8)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=116

$$-\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8d}+\frac{\tan(c+dx)}{a^8d}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{2\cdots}{d\left(a^8+ia^8\,\mathrm{t}\cdots\right.}$$

```
[Out] -8*x/a^8-8*I*ln(cos(d*x+c))/a^8/d+tan(d*x+c)/a^8/d+16/3*I/a^5/d/(a+I*a*tan(
d*x+c))^3-16*I/d/(a^4+I*a^4*tan(d*x+c))^2+24*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]

time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\frac{\tan(c+dx)}{a^8d}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{8i\log(\cos(c+dx))}{a^8d}-\frac{8x}{a^8}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-8*x)/a^8 - ((8*I)*Log[Cos[c + d*x]])/(a^8*d) + Tan[c + d*x]/(a^8*d) + ((1
6*I)/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - (16*I)/(d*(a^4 + I*a^4*Tan[c + d
*x])^2) + (24*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^4}{(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{a^9 d}$$

$$= -\frac{i\text{Subst}\left(\int \left(1+\frac{16a^4}{(a+x)^4}-\frac{32a^3}{(a+x)^3}+\frac{24a^2}{(a+x)^2}-\frac{8a}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^9 d}$$

$$= -\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8 d}+\frac{\tan(c+dx)}{a^8 d}+\frac{16i}{3a^5 d(a+ia\tan(c+dx))^3}-\frac{}{d(}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 397 vs. 2(116) = 232.
time = 1.12, size = 397, normalized size = 3.42

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^9*(-Cos[5*(c + d*x)] - I*Sin[5*(c + d*x)])*((-12*I)*Co
s[c] - (10*I)*Cos[3*c + 2*d*x] + 12*d*x*Cos[3*c + 2*d*x] + (2*I)*Cos[3*c +
4*d*x] + 12*d*x*Cos[3*c + 4*d*x] - I*Cos[5*c + 4*d*x] + 12*d*x*Cos[5*c + 4*
d*x] + Cos[c + 2*d*x]*(-7*I + 12*d*x + (12*I)*Log[Cos[c + d*x]]) + (12*I)*C
os[3*c + 2*d*x]*Log[Cos[c + d*x]] + (12*I)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x
]] + (12*I)*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]] + 11*Sin[c + 2*d*x] + (12*I)
*d*x*Sin[c + 2*d*x] - 12*Log[Cos[c + d*x]]*Sin[c + 2*d*x] + 14*Sin[3*c + 2*
d*x] + (12*I)*d*x*Sin[3*c + 2*d*x] - 12*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x]
- 4*Sin[3*c + 4*d*x] + (12*I)*d*x*Sin[3*c + 4*d*x] - 12*Log[Cos[c + d*x]]*S
in[3*c + 4*d*x] - Sin[5*c + 4*d*x] + (12*I)*d*x*Sin[5*c + 4*d*x] - 12*Log[C
os[c + d*x]]*Sin[5*c + 4*d*x]))/(6*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.33, size = 68, normalized size = 0.59

method	result	siz
derivativedivides	$\frac{\tan(dx+c)+8i\ln(\tan(dx+c)-i)-\frac{16}{3(\tan(dx+c)-i)^3}+\frac{24}{\tan(dx+c)-i}+\frac{16i}{(\tan(dx+c)-i)^2}}{d\,a^8}$	68
default	$\frac{\tan(dx+c)+8i\ln(\tan(dx+c)-i)-\frac{16}{3(\tan(dx+c)-i)^3}+\frac{24}{\tan(dx+c)-i}+\frac{16i}{(\tan(dx+c)-i)^2}}{d\,a^8}$	68
risch	$\frac{6i\mathrm{e}^{-2i(dx+c)}}{a^8 d}-\frac{2i\mathrm{e}^{-4i(dx+c)}}{a^8 d}+\frac{2i\mathrm{e}^{-6i(dx+c)}}{3a^8 d}-\frac{16x}{a^8}-\frac{16c}{a^8 d}+\frac{2i}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8 d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(tan(d*x+c)+8*I*ln(tan(d*x+c)-I)-16/3/(tan(d*x+c)-I)^3+24/(tan(d*x+
c)-I)+16*I/(tan(d*x+c)-I)^2)
```

Maxima [A]
time = 0.31, size = 191, normalized size = 1.65

$$\frac{\frac{8\left(9\tan(dx+c)^{6}-48i\tan(dx+c)^{5}-107\tan(dx+c)^{4}+128i\tan(dx+c)^{3}+87\tan(dx+c)^{2}-32i\tan(dx+c)-5\right)}{a^{8}\tan(dx+c)^{7}-7i\,a^{8}\tan(dx+c)^{6}-21\,a^{8}\tan(dx+c)^{5}+35i\,a^{8}\tan(dx+c)^{4}+35\,a^{8}\tan(dx+c)^{3}-21i\,a^{8}\tan(dx+c)^{2}-7\,a^{8}\tan(dx+c)+i\,a^{8}}+\frac{24i\log(i\tan(dx+c)+1)}{a^{8}}+\frac{3\tan(dx+c)}{a^{8}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/3*(8*(9*tan(d*x + c)^6 - 48*I*tan(d*x + c)^5 - 107*tan(d*x + c)^4 + 128*I
*tan(d*x + c)^3 + 87*tan(d*x + c)^2 - 32*I*tan(d*x + c) - 5)/(a^8*tan(d*x +
 c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
 c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) + 24*I*log(I*tan(d*x + c) + 1)/a^8 + 3*tan(d*x + c)/a^8)/d
```

Fricas [A]
time = 0.40, size = 124, normalized size = 1.07

$$-\frac{2\left(24\,dxe^{(8i\,dx+8i\,c)}+12\,(2\,dx-i)e^{(6i\,dx+6i\,c)}+12\left(i\,e^{(8i\,dx+8i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(a^{8}de^{(8i\,dx+8i\,c)}+a^{8}de^{(6i\,dx+6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -2/3*(24*d*x*e^(8*I*d*x + 8*I*c) + 12*(2*d*x - I)*e^(6*I*d*x + 6*I*c) + 12*
(I*e^(8*I*d*x + 8*I*c) + I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) + 1
) - 6*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) - I)/(a^8*d*e^(8*I*d*
x + 8*I*c) + a^8*d*e^(6*I*d*x + 6*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{10}(c+dx)}{\tan^{8}(c+dx)-8i\tan^{7}(c+dx)-28\tan^{6}(c+dx)+56i\tan^{5}(c+dx)+70\tan^{4}(c+dx)-56i\tan^{3}(c+dx)-28\tan^{2}(c+dx)+8i\tan(c+dx)+1}\,dx}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**10/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Giac [A]
time = 1.67, size = 199, normalized size = 1.72

$$-\frac{2\left(\frac{60i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}+\frac{60i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}-\frac{15\left(4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^8}+\frac{2\left(147i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+942\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2445i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3460\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2445i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+942\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-147i\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^6}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(60*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 120*I*log(tan(1/2*d*x + 1/2
*c) - I)/a^8 + 60*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 - 15*(4*I*tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) - 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^8
) + 2*(147*I*tan(1/2*d*x + 1/2*c)^6 + 942*tan(1/2*d*x + 1/2*c)^5 - 2445*I*t
an(1/2*d*x + 1/2*c)^4 - 3460*tan(1/2*d*x + 1/2*c)^3 + 2445*I*tan(1/2*d*x +
1/2*c)^2 + 942*tan(1/2*d*x + 1/2*c) - 147*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I
)^6))/d
```

Mupad [B]
time = 3.50, size = 104, normalized size = 0.90

$$\frac{\tan(c+d\,x)}{a^8\,d}-\frac{\frac{32\tan(c+d\,x)}{a^8}-\frac{40\mathrm{i}}{3\,a^8}+\frac{\tan(c+d\,x)^2\,24\mathrm{i}}{a^8}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}}{a^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*8i)/(a^8*d) - ((32*tan(c + d*x))/a^8 - 40i/(3*a^8)
+ (tan(c + d*x)^2*24i)/a^8)/(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c
+ d*x)^3*1i + 1)) + tan(c + d*x)/(a^8*d)
```

3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=43

$$\frac{i(a-ia\tan(c+dx))^4}{8d\,(a^3+ia^3\tan(c+dx))^4}$$

```
[Out] 1/8*I*(a-I*a*tan(d*x+c))^4/d/(a^3+I*a^3*tan(d*x+c))^4
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 37}

$$\frac{i(a-ia\tan(c+dx))^4}{8d\,(a^3+ia^3\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/8)*(a - I*a*Tan[c + d*x])^4)/(d*(a^3 + I*a^3*Tan[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{8d\,(a^3+ia^3\tan(c+dx))^4}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 32, normalized size = 0.74

$$\frac{i\sec^8(c+dx)}{8d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/8)*Sec[c + d*x]^8)/(d*(a + I*a*Tan[c + d*x])^8)
```

Maple [A]
time = 0.34, size = 63, normalized size = 1.47

method	result	size
risch	$\frac{i e^{-8i(dx+c)}}{8a^8d}$	19
derivativedivides	$\frac{\frac{4}{(\tan(dx+c)-i)^3}-\frac{1}{\tan(dx+c)-i}+\frac{2i}{(\tan(dx+c)-i)^4}-\frac{3i}{(\tan(dx+c)-i)^2}}{d\,a^8}$	63
default	$\frac{\frac{4}{(\tan(dx+c)-i)^3}-\frac{1}{\tan(dx+c)-i}+\frac{2i}{(\tan(dx+c)-i)^4}-\frac{3i}{(\tan(dx+c)-i)^2}}{d\,a^8}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(4/(tan(d*x+c)-I)^3-1/(tan(d*x+c)-I)+2*I/(tan(d*x+c)-I)^4-3*I/(tan(
d*x+c)-I)^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(35) = 70.
time = 0.31, size = 158, normalized size = 3.67

$$-\frac{\tan(dx+c)^6-3i\tan(dx+c)^5-4\tan(dx+c)^4+4i\tan(dx+c)^3+3\tan(dx+c)^2-i\tan(dx+c)}{(a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(tan(d*x + c)^6 - 3*I*tan(d*x + c)^5 - 4*tan(d*x + c)^4 + 4*I*tan(d*x + c)
^3 + 3*tan(d*x + c)^2 - I*tan(d*x + c))/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(
d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d
*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.40

$$\frac{i\,e^{(-8i\,dx-8i\,c)}}{8\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/8*I*e^(-8*I*d*x - 8*I*c)/(a^8*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.
time = 10.73, size = 160, normalized size = 3.72

$$\begin{cases} \frac{i \sec^{8}(c+dx)}{8a^{8}d\tan^{8}(c+dx)-64ia^{8}d\tan^{7}(c+dx)-224a^{8}d\tan^{6}(c+dx)+448ia^{8}d\tan^{5}(c+dx)+560a^{8}d\tan^{4}(c+dx)-448ia^{8}d\tan^{3}(c+dx)-224a^{8}d\tan^{2}(c+dx)+64ia^{8}d\tan(c+dx)+8a^{8}d} & \text{for } d \neq 0 \\ \frac{x\sec^{8}(c)}{(ia\tan(c)+a)^{8}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((I*sec(c + d*x)**8/(8*a**8*d*tan(c + d*x)**8 - 64*I*a**8*d*tan(c
+ d*x)**7 - 224*a**8*d*tan(c + d*x)**6 + 448*I*a**8*d*tan(c + d*x)**5 + 560
*a**8*d*tan(c + d*x)**4 - 448*I*a**8*d*tan(c + d*x)**3 - 224*a**8*d*tan(c +
 d*x)**2 + 64*I*a**8*d*tan(c + d*x) + 8*a**8*d), Ne(d, 0)), (x*sec(c)**8/(I
*a*tan(c) + a)**8, True))
```

Giac [A]
time = 1.62, size = 70, normalized size = 1.63

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^7 - 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^8)
```

Mupad [B]
time = 3.51, size = 73, normalized size = 1.70

$$-\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^8\,d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,6\mathrm{i}-4\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] -(tan(c + d*x)*(tan(c + d*x)^2*1i - 1i))/(a^8*d*(4*tan(c + d*x)^3 - tan(c +
 d*x)^2*6i - 4*tan(c + d*x) + tan(c + d*x)^4*1i + 1i))
```

3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=81

$$\frac{4i}{5a^3d(a+ia\tan(c+dx))^5}+\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

```
[Out] 4/5*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/3*I/a^5/d/(a+I*a*tan(d*x+c))^3-I/d/(a^2+
I*a^2*tan(d*x+c))^4
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{3568, 45\}$

$$\frac{i}{3a^5d(a+ia\tan(c+dx))^3}+\frac{4i}{5a^3d(a+ia\tan(c+dx))^5}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((4*I)/5)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (I/3)/(a^5*d*(a + I*a*Tan[c +
d*x])^3) - I/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6} - \frac{4a}{(a+x)^5} + \frac{1}{(a+x)^4}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= \frac{4i}{5a^3 d(a+ia\tan(c+dx))^5} + \frac{i}{3a^5 d(a+ia\tan(c+dx))^3} - \frac{i}{d\left(a^2+ia^2\tan(c+\right.}$$

Mathematica [A]
time = 0.31, size = 56, normalized size = 0.69

$$\frac{i\sec^8(c+dx)(15+16\cos(2(c+dx))+4i\sin(2(c+dx)))}{240a^8 d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/240)*Sec[c + d*x]^8*(15 + 16*Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)])
)/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.33, size = 49, normalized size = 0.60

method	result	size
derivativedivides	$\frac{\frac{4}{5(\tan(dx+c)-i)^5}-\frac{1}{3(\tan(dx+c)-i)^3}-\frac{i}{(\tan(dx+c)-i)^4}}{d\,a^8}$	49
default	$\frac{\frac{4}{5(\tan(dx+c)-i)^5}-\frac{1}{3(\tan(dx+c)-i)^3}-\frac{i}{(\tan(dx+c)-i)^4}}{d\,a^8}$	49
risch	$\frac{i e^{-6i(dx+c)}}{24a^8 d}+\frac{i e^{-8i(dx+c)}}{16a^8 d}+\frac{i e^{-10i(dx+c)}}{40a^8 d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(4/5/(tan(d*x+c)-I)^5-1/3/(tan(d*x+c)-I)^3-I/(tan(d*x+c)-I)^4)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.
time = 0.30, size = 141, normalized size = 1.74

$$-\frac{5\,\tan(dx+c)^4 - 5i\,\tan(dx+c)^3 + 3\,\tan(dx+c)^2 - i\,\tan(dx+c) + 2}{15\left(a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+c)^3 - 21i\,a^8\tan(dx+c)^2 - 7\,a^8\tan(dx+c) + i\,a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*tan(d*x + c)^4 - 5*I*tan(d*x + c)^3 + 3*tan(d*x + c)^2 - I*tan(d*x
 + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x +
 c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x
+ c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Fricas [A]
time = 0.36, size = 41, normalized size = 0.51

$$\frac{\left(10i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+6i\right)e^{(-10i\,dx-10i\,c)}}{240\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(10*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(-10*I*
d*x - 10*I*c)/(a^8*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(65) = 130.
time = 10.85, size = 466, normalized size = 5.75

$$\begin{cases} -\frac{i\tan^2(c+dx)\sec^6(c+dx)}{240a^8d\tan^8(c+dx)-1920ia^8d\tan^7(c+dx)-6720a^8d\tan^6(c+dx)+13440ia^8d\tan^5(c+dx)+16800a^8d\tan^4(c+dx)-13440ia^8d\tan^3(c+dx)-6720a^8d\tan^2(c+dx)+1920ia^8d\tan(c+dx)+240a^8d}-\frac{8\tan(c+dx)\sec^6(c+dx)}{240a^8d\tan^8(c+dx)-1920ia^8d\tan^7(c+dx)-6720a^8d\tan^6(c+dx)+13440ia^8d\tan^5(c+dx)+16800a^8d\tan^4(c+dx)-13440ia^8d\tan^3(c+dx)-6720a^8d\tan^2(c+dx)+1920ia^8d\tan(c+dx)+240a^8d}+\frac{31i\sec^6(c+dx)}{240a^8d\tan^8(c+dx)-1920ia^8d\tan^7(c+dx)-6720a^8d\tan^6(c+dx)+13440ia^8d\tan^5(c+dx)+16800a^8d\tan^4(c+dx)-13440ia^8d\tan^3(c+dx)-6720a^8d\tan^2(c+dx)+1920ia^8d\tan(c+dx)+240a^8d} & \text{for}\ d\neq 0 \\ \frac{x\sec^6(c)}{(ia\tan(c)+a)^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 -
 1920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8
*d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c +
d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a*
*8*d) - 8*tan(c + d*x)*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I
*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(
c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3
 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d) +
 31*I*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I*a**8*d*tan(c + d
*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(c + d*x)**5 + 168
00*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3 - 6720*a**8*d*ta
n(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d), Ne(d, 0)), (x*sec
(c)**6/(I*a*tan(c) + a)**8, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(65) = 130.
time = 1.46, size = 137, normalized size = 1.69

$$-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+170i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+282\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-170i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{15a^8d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^9 - 30*I*tan(1/2*d*x + 1/2*c)^8 - 140*tan(1/
2*d*x + 1/2*c)^7 + 170*I*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^
5 - 170*I*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/2*c)^3 + 30*I*tan(1/
2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) -
I)^10)
```

Mupad [B]
time = 3.50, size = 85, normalized size = 1.05

$$\frac{-\tan(c+d\,x)^2\,5\mathrm{i}+5\tan(c+d\,x)+2\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+5\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,10\mathrm{i}-10\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (5*tan(c + d*x) - tan(c + d*x)^2*5i + 2i)/(15*a^8*d*(tan(c + d*x)*5i - 10*t
an(c + d*x)^2 - tan(c + d*x)^3*10i + 5*tan(c + d*x)^4 + tan(c + d*x)^5*1i +
 1))
```

3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=55

$$\frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] 1/3*I/a^2/d/(a+I*a*tan(d*x+c))^6-1/5*I/a^3/d/(a+I*a*tan(d*x+c))^5
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {3568, 45}

$$\frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) - (I/5)/(a^3*d*(a + I*a*Tan[c + d*x]
)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= \frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 78, normalized size = 1.42

$$\frac{i\sec^8(c+dx)(45+64\cos(2(c+dx))+20\cos(4(c+dx))+16i\sin(2(c+dx))+10i\sin(4(c+dx)))}{960a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/960)*Sec[c + d*x]^8*(45 + 64*Cos[2*(c + d*x)] + 20*Cos[4*(c + d*x)] + (
16*I)*Sin[2*(c + d*x)] + (10*I)*Sin[4*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x
])^8)
```

Maple [A]
time = 0.34, size = 36, normalized size = 0.65

method	result	size
derivativedivides	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{d\,a^8}$	36
default	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{d\,a^8}$	36
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{192a^8d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-1/3*I/(tan(d*x+c)-I)^6-1/5/(tan(d*x+c)-I)^5)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(43) = 86.
time = 0.29, size = 121, normalized size = 2.20

$$-\frac{3\,\tan(dx+c)^2-i\,\tan(dx+c)+2}{15\,(a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(3*tan(d*x + c)^2 - I*tan(d*x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^
8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8
*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 1.15

$$\frac{\left(15i\,e^{(8i\,dx+8i\,c)}+40i\,e^{(6i\,dx+6i\,c)}+45i\,e^{(4i\,dx+4i\,c)}+24i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{960\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(15*I*e^(8*I*d*x + 8*I*c) + 40*I*e^(6*I*d*x + 6*I*c) + 45*I*e^(4*I*d*
x + 4*I*c) + 24*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^8*d)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(42) = 84$.
time = 10.90, size = 774, normalized size = 14.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((I*tan(c + d*x)**4*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 -
7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8
*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c +
d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a
**8*d) + 8*tan(c + d*x)**3*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 76
80*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d
*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*
x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**
8*d) - 30*I*tan(c + d*x)**2*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7
680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*
d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d
*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a*
*8*d) - 72*tan(c + d*x)*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7680*
I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d*ta
n(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*x)*
*3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**8*d
) + 129*I*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7680*I*a**8*d*tan(c
 + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d*tan(c + d*x)**5
+ 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*x)**3 - 26880*a**
8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**8*d), Ne(d, 0)),
(x*sec(c)**4/(I*a*tan(c) + a)**8, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(43) = 86$.
time = 1.37, size = 163, normalized size = 2.96

$$-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-60i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-235\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+822\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-904i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-822\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+235\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-60i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{15a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^11 - 60*I*tan(1/2*d*x + 1/2*c)^10 - 235*tan(
1/2*d*x + 1/2*c)^9 + 480*I*tan(1/2*d*x + 1/2*c)^8 + 822*tan(1/2*d*x + 1/2*c
)^7 - 904*I*tan(1/2*d*x + 1/2*c)^6 - 822*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan
(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 - 60*I*tan(1/2*d*x + 1/2*c
)^2 - 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^12)
```

Mupad [B]
time = 3.50, size = 85, normalized size = 1.55

$$-\frac{-2+\tan(c+d\,x)\;3\mathrm{i}}{15\,a^8\,d\,\left({\tan\left(c+d\,x\right)}^6\,1\mathrm{i}+6\,{\tan\left(c+d\,x\right)}^5-{\tan\left(c+d\,x\right)}^4\,15\mathrm{i}-20\,{\tan\left(c+d\,x\right)}^3+{\tan\left(c+d\,x\right)}^2\,15\mathrm{i}+6\,\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] -(tan(c + d*x)*3i - 2)/(15*a^8*d*(6*tan(c + d*x) + tan(c + d*x)^2*15i - 20*
tan(c + d*x)^3 - tan(c + d*x)^4*15i + 6*tan(c + d*x)^5 + tan(c + d*x)^6*1i
- 1i))
```

3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=27

$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

```
[Out] 1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{7ad(a+ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. 2(27) = 54.
time = 0.33, size = 100, normalized size = 3.70

$$\frac{i\sec^8(c+dx)(35+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+dx))+14i\sin(2(c+dx))+14i\sin(4(c+dx))+6i\sin(6(c+dx)))}{896a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/896)*Sec[c + d*x]^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8
*Cos[6*(c + d*x)] + (14*I)*Sin[2*(c + d*x)] + (14*I)*Sin[4*(c + d*x)] + (6*
I)*Sin[6*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.21, size = 24, normalized size = 0.89

method	result
derivativedivides	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
default	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
risch	$\frac{i e^{-2i(dx+c)}}{128a^8d}+\frac{3i e^{-4i(dx+c)}}{128a^8d}+\frac{5i e^{-6i(dx+c)}}{128a^8d}+\frac{5i e^{-8i(dx+c)}}{128a^8d}+\frac{3i e^{-10i(dx+c)}}{128a^8d}+\frac{i e^{-12i(dx+c)}}{128a^8d}+\frac{i e^{-14i(dx+c)}}{896a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Maxima [A]
time = 0.30, size = 21, normalized size = 0.78

$$\frac{i}{7\,(i\,a\tan\left(dx+c\right)+a)^7ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/7*I/((I*a*tan(d*x + c) + a)^7*a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.
time = 0.38, size = 85, normalized size = 3.15

$$\frac{\left(7i\,e^{(12i\,dx+12i\,c)}+21i\,e^{(10i\,dx+10i\,c)}+35i\,e^{(8i\,dx+8i\,c)}+35i\,e^{(6i\,dx+6i\,c)}+21i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-14i\,dx-14i\,c)}}{896\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(7*I*e^(12*I*d*x + 12*I*c) + 21*I*e^(10*I*d*x + 10*I*c) + 35*I*e^(8*I
*d*x + 8*I*c) + 35*I*e^(6*I*d*x + 6*I*c) + 21*I*e^(4*I*d*x + 4*I*c) + 7*I*e
^(2*I*d*x + 2*I*c) + I)*e^(-14*I*d*x - 14*I*c)/(a^8*d)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(19) = 38$.
time = 11.01, size = 1081, normalized size = 40.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**6*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**
8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
 d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*
a**8*d) - 8*tan(c + d*x)**5*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7
168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*
d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d
*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a*
*8*d) + 29*I*tan(c + d*x)**4*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8
*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a
**8*d) + 64*tan(c + d*x)**3*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7
168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*
d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d
*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a*
*8*d) - 99*I*tan(c + d*x)**2*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8
*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a
**8*d) - 120*tan(c + d*x)*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 716
8*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*d*
tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d*x
)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a**8
*d) + 127*I*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7168*I*a**8*d*tan
(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*d*tan(c + d*x)**
5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d*x)**3 - 25088*a
**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a**8*d), Ne(d, 0))
, (x*sec(c)**2/(I*a*tan(c) + a)**8, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(21) = 42$.
time = 1.24, size = 189, normalized size = 7.00

$$-\frac{2\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-42i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-182\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+490i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+1001\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-1484i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-1716\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+1484i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+1001\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-490i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-182\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+42i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{7a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/7*(7*tan(1/2*d*x + 1/2*c)^13 - 42*I*tan(1/2*d*x + 1/2*c)^12 - 182*tan(1/
2*d*x + 1/2*c)^11 + 490*I*tan(1/2*d*x + 1/2*c)^10 + 1001*tan(1/2*d*x + 1/2*
c)^9 - 1484*I*tan(1/2*d*x + 1/2*c)^8 - 1716*tan(1/2*d*x + 1/2*c)^7 + 1484*I
*tan(1/2*d*x + 1/2*c)^6 + 1001*tan(1/2*d*x + 1/2*c)^5 - 490*I*tan(1/2*d*x +
 1/2*c)^4 - 182*tan(1/2*d*x + 1/2*c)^3 + 42*I*tan(1/2*d*x + 1/2*c)^2 + 7*ta
n(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^14)
```

Mupad [B]
time = 3.56, size = 19, normalized size = 0.70

$$-\frac{1}{7\,a^8\,d\,{\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] -1/(7*a^8*d*(tan(c + d*x) - 1i)^7)
```

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=229

$$\frac{x}{256a^8}+\frac{i}{16d(a+ia\tan(c+dx))^8}+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{i}{80a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] 1/256*x/a^8+1/16*I/d/(a+I*a*tan(d*x+c))^8+1/28*I/a/d/(a+I*a*tan(d*x+c))^7+1
/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/80*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/128*I/
d/(a^2+I*a^2*tan(d*x+c))^4+1/192*I/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+1/256*I/d
/(a^4+I*a^4*tan(d*x+c))^2+1/256*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3560, 8}

$$\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{x}{256a^8}+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{80a^3d(a+ia\tan(c+dx))^5}+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-8),x]
```

```
[Out] x/(256*a^8) + (I/16)/(d*(a + I*a*Tan[c + d*x])^8) + (I/28)/(a*d*(a + I*a*Ta
n[c + d*x])^7) + (I/48)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/80)/(a^3*d*(a
 + I*a*Tan[c + d*x])^5) + (I/128)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (I/192
)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (I/256)/(d*(a^4 + I*a^4*Tan[c + d*
x])^2) + (I/256)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^7}\,dx}{2a}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^6}\,dx}{4a^2}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+$$

$$= \frac{x}{256a^8} + \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+i$$

Mathematica [A]
time = 0.78, size = 148, normalized size = 0.65

$$\frac{\sec^8(c+dx)(14700i+25088i\cos(2(c+dx))+15680i\cos(4(c+dx))+7680i\cos(6(c+dx))+105i\cos(8(c+dx))+1680dx\cos(8(c+dx))-6272\sin(2(c+dx))-7840\sin(4(c+dx))-5760\sin(6(c+dx))+105\sin(8(c+dx))+1680idx\sin(8(c+dx)))}{430080a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-8),x]
```

```
[Out] (Sec[c + d*x]^8*(14700*I + (25088*I)*Cos[2*(c + d*x)] + (15680*I)*Cos[4*(c
+ d*x)] + (7680*I)*Cos[6*(c + d*x)] + (105*I)*Cos[8*(c + d*x)] + 1680*d*x*C
os[8*(c + d*x)] - 6272*Sin[2*(c + d*x)] - 7840*Sin[4*(c + d*x)] - 5760*Sin[
6*(c + d*x)] + 105*Sin[8*(c + d*x)] + (1680*I)*d*x*Sin[8*(c + d*x)]))/(4300
80*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.32, size = 143, normalized size = 0.62

method	result

derivativedivides	$\frac{\frac{i}{16(\tan(dx+c)-i)^8}+\frac{i}{128(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{512}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{256(\tan(dx+c)-i)^2}-\frac{1}{28(\tan(dx+c)-i)^7}+\frac{}{80(\text{t}}}{d\,a^8}$
default	$\frac{\frac{i}{16(\tan(dx+c)-i)^8}+\frac{i}{128(\tan(dx+c)-i)^4}-\frac{i\ln(\tan(dx+c)-i)}{512}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{256(\tan(dx+c)-i)^2}-\frac{1}{28(\tan(dx+c)-i)^7}+\frac{}{80(\text{t}}}{d\,a^8}$
risch	$\frac{x}{256a^8}+\frac{i\mathrm{e}^{-2i(dx+c)}}{64a^8d}+\frac{7i\mathrm{e}^{-4i(dx+c)}}{256a^8d}+\frac{7i\mathrm{e}^{-6i(dx+c)}}{192a^8d}+\frac{35i\mathrm{e}^{-8i(dx+c)}}{1024a^8d}+\frac{7i\mathrm{e}^{-10i(dx+c)}}{320a^8d}+\frac{7i\mathrm{e}^{-12i(dx+c)}}{768a^8d}+\frac{i\mathrm{e}}{}$
norman	$\frac{\frac{35x\left(\tan^8(dx+c)\right)}{128a}+\frac{961\left(\tan^7(dx+c)\right)}{8960da}+\frac{x}{256a}+\frac{x\left(\tan^2(dx+c)\right)}{32a}+\frac{3371\left(\tan^5(dx+c)\right)}{1280da}+\frac{255\tan(dx+c)}{256da}-\frac{1117\left(\tan^3(dx+c)\right)}{256da}+\frac{7x(\text{ta}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(1/16*I/(tan(d*x+c)-I)^8+1/128*I/(tan(d*x+c)-I)^4-1/512*I*ln(tan(d*
x+c)-I)-1/48*I/(tan(d*x+c)-I)^6-1/256*I/(tan(d*x+c)-I)^2-1/28/(tan(d*x+c)-I
)^7+1/80/(tan(d*x+c)-I)^5-1/192/(tan(d*x+c)-I)^3+1/256/(tan(d*x+c)-I)+1/512
*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]

time = 0.42, size = 109, normalized size = 0.48

$$\frac{\left(1680\,dxe^{(16i\,dx+16i\,c)}+6720i\,e^{(14i\,dx+14i\,c)}+11760i\,e^{(12i\,dx+12i\,c)}+15680i\,e^{(10i\,dx+10i\,c)}+14700i\,e^{(8i\,dx+8i\,c)}+9408i\,e^{(6i\,dx+6i\,c)}+3920i\,e^{(4i\,dx+4i\,c)}+960i\,e^{(2i\,dx+2i\,c)}+105i\right)e^{(-16i\,dx-16i\,c)}}{430080\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(1680*d*x*e^(16*I*d*x + 16*I*c) + 6720*I*e^(14*I*d*x + 14*I*c) + 1
1760*I*e^(12*I*d*x + 12*I*c) + 15680*I*e^(10*I*d*x + 10*I*c) + 14700*I*e^(8
*I*d*x + 8*I*c) + 9408*I*e^(6*I*d*x + 6*I*c) + 3920*I*e^(4*I*d*x + 4*I*c) +
 960*I*e^(2*I*d*x + 2*I*c) + 105*I)*e^(-16*I*d*x - 16*I*c)/(a^8*d)
```

Sympy [A]

time = 0.45, size = 325, normalized size = 1.42

$$\begin{cases} \frac{\left(226981421219472998840ia^{56}d^7e^{70ic}e^{-2idx}+397217487134077774720ia^{56}d^7e^{68ic}e^{-4idx}+529623316178770329 60ia^{56}d^7e^{66ic}e^{-6idx}+496521858917597184 00ia^{56}d^7e^{64ic}e^{-8idx}+317773989707262197 76ia^{56}d^7e^{62ic}e^{-10idx}+132405829044692582 40ia^{56}d^7e^{60ic}e^{-12idx}+324259173170675712 0ia^{56}d^7e^{58ic}e^{-14idx}+354658470655426560ia^{56}d^7e^{56ic}e^{-16idx}\right)e^{-72ic}}{14526810958046271897 60a^{64}d^8} & \text{for } a^{64}d^8e^{72ic}\neq 0 \\ x\left(\frac{\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-16ic}}{256a^8}-\frac{1}{256a^8}\right) & \text{otherwise} \end{cases}+\frac{x}{256a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((22698142121947299840*I*a**56*d**7*exp(70*I*c)*exp(-2*I*d*x) + 3
97217487134077774720*I*a**56*d**7*exp(68*I*c)*exp(-4*I*d*x) + 52962331617877
032960*I*a**56*d**7*exp(66*I*c)*exp(-6*I*d*x) + 49652185891759718400*I*a**5
6*d**7*exp(64*I*c)*exp(-8*I*d*x) + 31777398970726219776*I*a**56*d**7*exp(62
*I*c)*exp(-10*I*d*x) + 13240582904469258240*I*a**56*d**7*exp(60*I*c)*exp(-1
2*I*d*x) + 3242591731706757120*I*a**56*d**7*exp(58*I*c)*exp(-14*I*d*x) + 35
4658470655426560*I*a**56*d**7*exp(56*I*c)*exp(-16*I*d*x))*exp(-72*I*c)/(145
2681095804627189760*a**64*d**8), Ne(a**64*d**8*exp(72*I*c), 0)), (x*((exp(1
6*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) +
56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-16*I*c)/(256*a**8) -
 1/(256*a**8)), True)) + x/(256*a**8)
```

Giac [A]
time = 0.88, size = 132, normalized size = 0.58

$$-\frac{-\frac{840i\log(-i\tan(dx+c)+1)}{a^8}+\frac{840i\log(-i\tan(dx+c)-1)}{a^8}+\frac{-2283i\tan(dx+c)^8-19944\tan(dx+c)^7+77364i\tan(dx+c)^6+175448\tan(dx+c)^5-258370i\tan(dx+c)^4-261464\tan(dx+c)^3+192052i\tan(dx+c)^2+114152\tan(dx+c)-67819i}{a^8(\tan(dx+c)-i)^8}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/430080*(-840*I*log(-I*tan(d*x + c) + 1)/a^8 + 840*I*log(-I*tan(d*x + c)
- 1)/a^8 + (-2283*I*tan(d*x + c)^8 - 19944*tan(d*x + c)^7 + 77364*I*tan(d*x
 + c)^6 + 175448*tan(d*x + c)^5 - 258370*I*tan(d*x + c)^4 - 261464*tan(d*x
+ c)^3 + 192052*I*tan(d*x + c)^2 + 114152*tan(d*x + c) - 67819*I)/(a^8*(tan
(d*x + c) - I)^8))/d
```

Mupad [B]
time = 4.96, size = 198, normalized size = 0.86

$$\frac{x}{256\,a^8}-\frac{\frac{\tan(c+d\,x)\,5993{}\mathrm{i}}{26880\,a^8}+\frac{16}{105\,a^8}-\frac{143\tan(c+d\,x)^2}{480\,a^8}-\frac{\tan(c+d\,x)^3\,1193{}\mathrm{i}}{3840\,a^8}+\frac{11\tan(c+d\,x)^4}{48\,a^8}+\frac{\tan(c+d\,x)^5\,85{}\mathrm{i}}{768\,a^8}-\frac{\tan(c+d\,x)^6}{32\,a^8}-\frac{\tan(c+d\,x)^7\,1{}\mathrm{i}}{256\,a^8}}{d\,\left(\tan(c+d\,x)^8\,1{}\mathrm{i}+8\tan(c+d\,x)^7-\tan(c+d\,x)^6\,28{}\mathrm{i}-56\tan(c+d\,x)^5+\tan(c+d\,x)^4\,70{}\mathrm{i}+56\tan(c+d\,x)^3-\tan(c+d\,x)^2\,28{}\mathrm{i}-8\tan(c+d\,x)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] x/(256*a^8) - ((tan(c + d*x)*5993i)/(26880*a^8) + 16/(105*a^8) - (143*tan(c
 + d*x)^2)/(480*a^8) - (tan(c + d*x)^3*1193i)/(3840*a^8) + (11*tan(c + d*x)
^4)/(48*a^8) + (tan(c + d*x)^5*85i)/(768*a^8) - tan(c + d*x)^6/(32*a^8) - (
tan(c + d*x)^7*1i)/(256*a^8))/(d*(56*tan(c + d*x)^3 - tan(c + d*x)^2*28i -
8*tan(c + d*x) + tan(c + d*x)^4*70i - 56*tan(c + d*x)^5 - tan(c + d*x)^6*28
i + 8*tan(c + d*x)^7 + tan(c + d*x)^8*1i + 1i))
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=278

$$\frac{5x}{512a^8}+\frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan$$

```
[Out] 5/512*x/a^8+1/36*I*a/d/(a+I*a*tan(d*x+c))^9+1/32*I/d/(a+I*a*tan(d*x+c))^8+3
/112*I/a/d/(a+I*a*tan(d*x+c))^7+1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/64*I/a^
3/d/(a+I*a*tan(d*x+c))^5+7/768*I/a^5/d/(a+I*a*tan(d*x+c))^3+3/256*I/d/(a^2+
I*a^2*tan(d*x+c))^4+1/128*I/d/(a^4+I*a^4*tan(d*x+c))^2-1/1024*I/d/(a^8-I*a^
8*tan(d*x+c))+9/1024*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{i}{1024d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{9i}{1024d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{5x}{512a^8}+\frac{7i}{768a^5d(a+ia\tan(c+dx))^3}+\frac{i}{128d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{64a^3d(a+ia\tan(c+dx))^5}+\frac{3i}{256d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (5*x)/(512*a^8) + ((I/36)*a)/(d*(a + I*a*Tan[c + d*x])^9) + (I/32)/(d*(a +
I*a*Tan[c + d*x])^8) + ((3*I)/112)/(a*d*(a + I*a*Tan[c + d*x])^7) + (I/48)/
(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/64)/(a^3*d*(a + I*a*Tan[c + d*x])^5)
+ ((7*I)/768)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/256)/(d*(a^2 + I*a^
2*Tan[c + d*x])^4) + (I/128)/(d*(a^4 + I*a^4*Tan[c + d*x])^2) - (I/1024)/(d
*(a^8 - I*a^8*Tan[c + d*x])) + ((9*I)/1024)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2} + \frac{1}{4a^2(a+x)^{10}} + \frac{1}{4a^3(a+x)^9} + \frac{3}{16a^4(a+x)^8} + \frac{1}{8a^5(a+x)^7} + \cdots\right.\right.}{}$$

$$= \frac{ia}{36d(a+ia\tan(c+dx))^9} + \frac{i}{32d(a+ia\tan(c+dx))^8} + \frac{3i}{112ad(a+ia\tan(c+\cdots}$$

$$= \frac{5x}{512a^8} + \frac{ia}{36d(a+ia\tan(c+dx))^9} + \frac{i}{32d(a+ia\tan(c+dx))^8} + \frac{\cdots}{112ad(a+ia\cdots}$$

Mathematica [A]

time = 1.28, size = 170, normalized size = 0.61

$$\frac{\sec^8(c+dx)(15876i + 28224i\cos(2(c+dx)) + 20160i\cos(4(c+dx)) + 12960i\cos(6(c+dx)) + 315i\cos(8(c+dx)) + 5040dx\cos(8(c+dx)) - 224i\cos(10(c+dx)) - 7056\sin(2(c+dx)) - 10080\sin(4(c+dx)) - 9720\sin(6(c+dx)) + 315\sin(8(c+dx)) + 5040idx\sin(8(c+dx)) + 280\sin(10(c+dx)))}{516096a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*(15876*I + (28224*I)*Cos[2*(c + d*x)] + (20160*I)*Cos[4*(c
+ d*x)] + (12960*I)*Cos[6*(c + d*x)] + (315*I)*Cos[8*(c + d*x)] + 5040*d*x*
Cos[8*(c + d*x)] - (224*I)*Cos[10*(c + d*x)] - 7056*Sin[2*(c + d*x)] - 1008
0*Sin[4*(c + d*x)] - 9720*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] + (5040*I
)*d*x*Sin[8*(c + d*x)] + 280*Sin[10*(c + d*x)]))/(516096*a^8*d*(-I + Tan[c
+ d*x])^8)
```

Maple [A]

time = 0.29, size = 169, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024} + \frac{3i}{256(\tan(dx+c)-i)^4} + \frac{i}{32(\tan(dx+c)-i)^8} - \frac{i}{48(\tan(dx+c)-i)^6} - \frac{i}{128(\tan(dx+c)-i)^2} + \frac{1}{36(\tan(dx+c)-i)^9} - \frac{\cdots}{112\cdots}}{d\,a^8}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024} + \frac{3i}{256(\tan(dx+c)-i)^4} + \frac{i}{32(\tan(dx+c)-i)^8} - \frac{i}{48(\tan(dx+c)-i)^6} - \frac{i}{128(\tan(dx+c)-i)^2} + \frac{1}{36(\tan(dx+c)-i)^9} - \frac{\cdots}{112\cdots}}{d\,a^8}$
risch	$\frac{5x}{512a^8} + \frac{15ie^{-4i(dx+c)}}{512a^8d} + \frac{35ie^{-6i(dx+c)}}{1024a^8d} + \frac{63ie^{-8i(dx+c)}}{2048a^8d} + \frac{21ie^{-10i(dx+c)}}{1024a^8d} + \frac{5ie^{-12i(dx+c)}}{512a^8d} + \frac{45ie^{-14i(dx+c)}}{14336a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-5/1024*I*ln(tan(d*x+c)-I)+3/256*I/(tan(d*x+c)-I)^4+1/32*I/(tan(d*
x+c)-I)^8-1/48*I/(tan(d*x+c)-I)^6-1/128*I/(tan(d*x+c)-I)^2+1/36/(tan(d*x+c)
-I)^9-3/112/(tan(d*x+c)-I)^7+1/64/(tan(d*x+c)-I)^5-7/768/(tan(d*x+c)-I)^3+9
/1024/(tan(d*x+c)-I)+5/1024*I*ln(tan(d*x+c)+I)+1/1024/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 131, normalized size = 0.47

$$\frac{\left(5040\,dxe^{(18i\,dx+18i\,c)}-252i\,e^{(20i\,dx+20i\,c)}+11340i\,e^{(16i\,dx+16i\,c)}+15120i\,e^{(14i\,dx+14i\,c)}+17640i\,e^{(12i\,dx+12i\,c)}+15876i\,e^{(10i\,dx+10i\,c)}+10584i\,e^{(8i\,dx+8i\,c)}+5040i\,e^{(6i\,dx+6i\,c)}+1620i\,e^{(4i\,dx+4i\,c)}+315i\,e^{(2i\,dx+2i\,c)}+28i\right)e^{(-18i\,dx-18i\,c)}}{516096\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*d*x*e^(18*I*d*x + 18*I*c) - 252*I*e^(20*I*d*x + 20*I*c) + 11
340*I*e^(16*I*d*x + 16*I*c) + 15120*I*e^(14*I*d*x + 14*I*c) + 17640*I*e^(12
*I*d*x + 12*I*c) + 15876*I*e^(10*I*d*x + 10*I*c) + 10584*I*e^(8*I*d*x + 8*I
*c) + 5040*I*e^(6*I*d*x + 6*I*c) + 1620*I*e^(4*I*d*x + 4*I*c) + 315*I*e^(2*
I*d*x + 2*I*c) + 28*I)*e^(-18*I*d*x - 18*I*c)/(a^8*d)
```

Sympy [A]
time = 0.49, size = 394, normalized size = 1.42

$$\begin{cases} \frac{\left(-2495687119199326634196634435584ia^{72}d^{9}e^{92ic}e^{2idx}+112305920363969698538848549601280ia^{72}d^{9}e^{88ic}e^{-2idx}+149741227151959598051798066135040ia^{72}d^{9}e^{86ic}e^{-4idx}+174698098343952864393764410490880ia^{72}d^{9}e^{84ic}e^{-6idx}+157228288509555757795438796944 1792ia^{72}d^{9}e^{82ic}e^{-8idx}+104818859006371718636258646294528ia^{72}d^{9}e^{80ic}e^{-10idx}+49913742383986532683932688711680ia^{72}d^{9}e^{78ic}e^{-12idx}+16043702909138528362692649943040ia^{72}d^{9}e^{76ic}e^{-14idx}+3119608898999158292745793044480ia^{72}d^{9}e^{74ic}e^{-16idx}+277298568799925181577403826176ia^{72}d^{9}e^{72ic}e^{-18idx}\right)e^{-90ic}}{5111167220120220946834707324076032a^{80}d^{10}} & \text{for}\; a^{80}d^{10}e^{90ic}\neq 0 \\ x\left(\frac{\left(e^{20ic}+10e^{18ic}+45e^{16ic}+120e^{14ic}+210e^{12ic}+252e^{10ic}+210e^{8ic}+120e^{6ic}+45e^{4ic}+10e^{2ic}+1\right)e^{-18ic}}{1024a^{8}}-\frac{5}{512a^{8}}\right) & \text{otherwise} \end{cases}+\frac{5x}{512a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-2495687119199326634196634435584*I*a**72*d**9*exp(92*I*c)*exp(2
*I*d*x) + 112305920363969698538848549601280*I*a**72*d**9*exp(88*I*c)*exp(-2
*I*d*x) + 149741227151959598051798066135040*I*a**72*d**9*exp(86*I*c)*exp(-4
*I*d*x) + 174698098343952864393764410490880*I*a**72*d**9*exp(84*I*c)*exp(-6
*I*d*x) + 157228288509555757795438796944 1792*I*a**72*d**9*exp(82*I*c)*exp(-8
*I*d*x) + 104818859006371718636258646294528*I*a**72*d**9*exp(80*I*c)*exp(-1
0*I*d*x) + 49913742383986532683932688711680*I*a**72*d**9*exp(78*I*c)*exp(-1
```

```
2*I*d*x) + 160437029091385283626926499430 40*I*a**72*d**9*exp(76*I*c)*exp(-1
4*I*d*x) + 311960889899915829274579304448 0*I*a**72*d**9*exp(74*I*c)*exp(-16
*I*d*x) + 277298568799925181577403826176*I*a**72*d**9*exp(72*I*c)*exp(-18*I
*d*x))*exp(-90*I*c)/(511116722012022094683470732407 6032*a**80*d**10), Ne(a*
*80*d**10*exp(90*I*c), 0)), (x*((exp(20*I*c) + 10*exp(18*I*c) + 45*exp(16*I
*c) + 120*exp(14*I*c) + 210*exp(12*I*c) + 252*exp(10*I*c) + 210*exp(8*I*c)
+ 120*exp(6*I*c) + 45*exp(4*I*c) + 10*exp(2*I*c) + 1)*exp(-18*I*c)/(1024*a*
*8) - 5/(512*a**8)), True)) + 5*x/(512*a**8)
```

Giac [A]
time = 1.28, size = 163, normalized size = 0.59

$$-\frac{-\frac{2520i\log(\tan(dx+c)+i)}{a^8}+\frac{2520i\log(\tan(dx+c)-i)}{a^8}+\frac{504(5i\tan(dx+c)-6)}{a^8(\tan(dx+c)+i)}+\frac{-7129i\tan(dx+c)^9-68697\tan(dx+c)^8+296964i\tan(dx+c)^7+758772\tan(dx+c)^6-1271214i\tan(dx+c)^5-1465758\tan(dx+c)^4+1191540i\tan(dx+c)^3+693828\tan(dx+c)^2-295425i\tan(dx+c)-89553}{a^8(\tan(dx+c)-i)^9}}{516096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/516096*(-2520*I*log(tan(d*x + c) + I)/a^8 + 2520*I*log(tan(d*x + c) - I)
/a^8 + 504*(5*I*tan(d*x + c) - 6)/(a^8*(tan(d*x + c) + I)) + (-7129*I*tan(d
*x + c)^9 - 68697*tan(d*x + c)^8 + 296964*I*tan(d*x + c)^7 + 758772*tan(d*x
 + c)^6 - 1271214*I*tan(d*x + c)^5 - 1465758*tan(d*x + c)^4 + 1191540*I*tan
(d*x + c)^3 + 693828*tan(d*x + c)^2 - 295425*I*tan(d*x + c) - 89553)/(a^8*(
tan(d*x + c) - I)^9))/d
```

Mupad [B]
time = 5.29, size = 235, normalized size = 0.85

$$\frac{5x}{512a^8}+\frac{\frac{163\tan(c+dx)^2}{448a^8}-\frac{10}{63a^8}-\frac{\tan(c+dx)\,9019i}{32256a^8}+\frac{\tan(c+dx)^3\,393i}{1792a^8}+\frac{11\tan(c+dx)^4}{64a^8}+\frac{\tan(c+dx)^5\,1i}{2a^8}-\frac{95\tan(c+dx)^6}{192a^8}-\frac{\tan(c+dx)^7\,205i}{768a^8}+\frac{5\tan(c+dx)^8}{64a^8}+\frac{\tan(c+dx)^9\,5i}{512a^8}}{d\left(\tan(c+dx)^{10}\,1i+8\tan(c+dx)^9-\tan(c+dx)^8\,27i-48\tan(c+dx)^7+\tan(c+dx)^6\,42i+\tan(c+dx)^4\,42i+48\tan(c+dx)^3-\tan(c+dx)^2\,27i-8\tan(c+dx)+1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (5*x)/(512*a^8) + ((163*tan(c + d*x)^2)/(448*a^8) - 10/(63*a^8) - (tan(c +
d*x)*9019i)/(32256*a^8) + (tan(c + d*x)^3*393i)/(1792*a^8) + (11*tan(c + d*
x)^4)/(64*a^8) + (tan(c + d*x)^5*1i)/(2*a^8) - (95*tan(c + d*x)^6)/(192*a^8
) - (tan(c + d*x)^7*205i)/(768*a^8) + (5*tan(c + d*x)^8)/(64*a^8) + (tan(c
+ d*x)^9*5i)/(512*a^8))/(d*(48*tan(c + d*x)^3 - tan(c + d*x)^2*27i - 8*tan(
c + d*x) + tan(c + d*x)^4*42i + tan(c + d*x)^6*42i - 48*tan(c + d*x)^7 - ta
n(c + d*x)^8*27i + 8*tan(c + d*x)^9 + tan(c + d*x)^10*1i + 1i))
```

3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=333

$$\frac{33x}{2048a^8}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8}+\frac{5i}{224ad(a+ia\tan}$$

```
[Out] 33/2048*x/a^8+1/80*I*a^2/d/(a+I*a*tan(d*x+c))^10+1/48*I*a/d/(a+I*a*tan(d*x+
c))^9+3/128*I/d/(a+I*a*tan(d*x+c))^8+5/224*I/a/d/(a+I*a*tan(d*x+c))^7+5/256
*I/a^2/d/(a+I*a*tan(d*x+c))^6+21/1280*I/a^3/d/(a+I*a*tan(d*x+c))^5+3/256*I/
a^5/d/(a+I*a*tan(d*x+c))^3+7/512*I/d/(a^2+I*a^2*tan(d*x+c))^4-1/4096*I/d/(a
^4-I*a^4*tan(d*x+c))^2+45/4096*I/d/(a^4+I*a^4*tan(d*x+c))^2-11/4096*I/d/(a^
8-I*a^8*tan(d*x+c))+55/4096*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$-\frac{11i}{4096d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{55i}{4096d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{33x}{2048a^8}+\frac{3i}{256a^5d(a+ia\tan(c+dx))^3}-\frac{i}{4096d\left(a^4-ia^4\tan(c+dx)\right)^2}+\frac{45i}{4096d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{21i}{1280a^3d(a+ia\tan(c+dx))^5}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}+\frac{7i}{512d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{5i}{256a^2d(a+ia\tan(c+dx))^6}+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8}+\frac{5i}{224ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (33*x)/(2048*a^8) + ((I/80)*a^2)/(d*(a + I*a*Tan[c + d*x])^10) + ((I/48)*a)
/(d*(a + I*a*Tan[c + d*x])^9) + ((3*I)/128)/(d*(a + I*a*Tan[c + d*x])^8) +
((5*I)/224)/(a*d*(a + I*a*Tan[c + d*x])^7) + ((5*I)/256)/(a^2*d*(a + I*a*Ta
n[c + d*x])^6) + ((21*I)/1280)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/25
6)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((7*I)/512)/(d*(a^2 + I*a^2*Tan[c + d
*x])^4) - (I/4096)/(d*(a^4 - I*a^4*Tan[c + d*x])^2) + ((45*I)/4096)/(d*(a^4
 + I*a^4*Tan[c + d*x])^2) - ((11*I)/4096)/(d*(a^8 - I*a^8*Tan[c + d*x])) +
((55*I)/4096)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+\right.\right.}{}$$

$$= \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{3i}{128d(a+ia\tan(c+}$$

$$= \frac{33x}{2048a^8} + \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{}{128d(a+i}$$

Mathematica [A]
time = 1.80, size = 192, normalized size = 0.58

$$\frac{\sec^8(c+dx)(97020i + 177408i\cos(2(c+dx)) + 138600i\cos(4(c+dx)) + 105600i\cos(6(c+dx)) + 3465i\cos(8(c+dx)) + 55440dx\cos(8(c+dx)) - 4480i\cos(10(c+dx)) - 168i\cos(12(c+dx)) - 44352\sin(2(c+dx)) - 69300\sin(4(c+dx)) - 79200\sin(6(c+dx)) + 3465\sin(8(c+dx)) + 55440idx\sin(8(c+dx)) + 5600\sin(10(c+dx)) + 252\sin(12(c+dx)))}{3440640a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*(97020*I + (177408*I)*Cos[2*(c + d*x)] + (138600*I)*Cos[4*(
c + d*x)] + (105600*I)*Cos[6*(c + d*x)] + (3465*I)*Cos[8*(c + d*x)] + 55440
*d*x*Cos[8*(c + d*x)] - (4480*I)*Cos[10*(c + d*x)] - (168*I)*Cos[12*(c + d*
x)] - 44352*Sin[2*(c + d*x)] - 69300*Sin[4*(c + d*x)] - 79200*Sin[6*(c + d*
x)] + 3465*Sin[8*(c + d*x)] + (55440*I)*d*x*Sin[8*(c + d*x)] + 5600*Sin[10*
(c + d*x)] + 252*Sin[12*(c + d*x)]))/(3440640*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.30, size = 197, normalized size = 0.59

method	result
derivativedivides	$-\frac{33i\ln(\tan(dx+c)-i)}{4096} + \frac{7i}{512(\tan(dx+c)-i)^4} + \frac{3i}{128(\tan(dx+c)-i)^8} - \frac{i}{80(\tan(dx+c)-i)^{10}} - \frac{5i}{256(\tan(dx+c)-i)^6} - \frac{45i}{4096(\tan(dx+c)-i)^2}$
default	$-\frac{33i\ln(\tan(dx+c)-i)}{4096} + \frac{7i}{512(\tan(dx+c)-i)^4} + \frac{3i}{128(\tan(dx+c)-i)^8} - \frac{i}{80(\tan(dx+c)-i)^{10}} - \frac{5i}{256(\tan(dx+c)-i)^6} - \frac{45i}{4096(\tan(dx+c)-i)^2}$

risch	$\frac{33x}{2048a^8} + \frac{33ie^{-6i(dx+c)}}{1024a^8d} + \frac{231ie^{-8i(dx+c)}}{8192a^8d} + \frac{99ie^{-10i(dx+c)}}{5120a^8d} + \frac{165ie^{-12i(dx+c)}}{16384a^8d} + \frac{55ie^{-14i(dx+c)}}{14336a^8d} + \frac{33ie^{-16i}}{32768}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-33/4096*I*ln(tan(d*x+c)-I)+7/512*I/(tan(d*x+c)-I)^4+3/128*I/(tan(
d*x+c)-I)^8-1/80*I/(tan(d*x+c)-I)^10-5/256*I/(tan(d*x+c)-I)^6-45/4096*I/(ta
n(d*x+c)-I)^2+1/48/(tan(d*x+c)-I)^9-5/224/(tan(d*x+c)-I)^7+21/1280/(tan(d*x
+c)-I)^5-3/256/(tan(d*x+c)-I)^3+55/4096/(tan(d*x+c)-I)+1/4096*I/(tan(d*x+c)
+I)^2+33/4096*I*ln(tan(d*x+c)+I)+11/4096/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.38, size = 153, normalized size = 0.46

$$\frac{\left(55440\,dxe^{(20i\,dx+20i\,c)} - 210i\,e^{(24i\,dx+24i\,c)} - 5040i\,e^{(22i\,dx+22i\,c)} + 92400i\,e^{(18i\,dx+18i\,c)} + 103950i\,e^{(16i\,dx+16i\,c)} + 110880i\,e^{(14i\,dx+14i\,c)} + 97020i\,e^{(12i\,dx+12i\,c)} + 66528i\,e^{(10i\,dx+10i\,c)} + 34650i\,e^{(8i\,dx+8i\,c)} + 13200i\,e^{(6i\,dx+6i\,c)} + 3465i\,e^{(4i\,dx+4i\,c)} + 560i\,e^{(2i\,dx+2i\,c)} + 42i\right)e^{(-20i\,dx-20i\,c)}}{3440640\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3440640*(55440*d*x*e^(20*I*d*x + 20*I*c) - 210*I*e^(24*I*d*x + 24*I*c) -
5040*I*e^(22*I*d*x + 22*I*c) + 92400*I*e^(18*I*d*x + 18*I*c) + 103950*I*e^(
16*I*d*x + 16*I*c) + 110880*I*e^(14*I*d*x + 14*I*c) + 97020*I*e^(12*I*d*x +
 12*I*c) + 66528*I*e^(10*I*d*x + 10*I*c) + 34650*I*e^(8*I*d*x + 8*I*c) + 13
200*I*e^(6*I*d*x + 6*I*c) + 3465*I*e^(4*I*d*x + 4*I*c) + 560*I*e^(2*I*d*x +
 2*I*c) + 42*I)*e^(-20*I*d*x - 20*I*c)/(a^8*d)
```

Sympy [A]
time = 0.63, size = 462, normalized size = 1.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-11433487528543532372369386809707411904921600*I*a**88*d**11*exp
(114*I*c)*exp(4*I*d*x) - 274403700685044776936865283432977885718118400*I*a*
*88*d**11*exp(112*I*c)*exp(2*I*d*x) + 5030734512559154243842530196271261238
165504000*I*a**88*d**11*exp(108*I*c)*exp(-2*I*d*x) + 5659576326629048524322
846470805168892936192000*I*a**88*d**11*exp(106*I*c)*exp(-4*I*d*x) + 6036881
415070985092611036235525513485798604800*I*a**88*d**11*exp(104*I*c)*exp(-6*I
*d*x) + 5282271238187111956034656706084824300073779200*I*a**88*d**11*exp(10
2*I*c)*exp(-8*I*d*x) + 3622128849042591055566621741315308091479162880*I*a**
88*d**11*exp(100*I*c)*exp(-10*I*d*x) + 188652544220968284144094882360172296
4312064000*I*a**88*d**11*exp(98*I*c)*exp(-12*I*d*x) + 718676358937022034834
647170895894462595072000*I*a**88*d**11*exp(96*I*c)*exp(-14*I*d*x) + 1886525
44220968284144094882360172296431206400*I*a**88*d**11*exp(94*I*c)*exp(-16*I*
d*x) + 30489300076116086326318364825886431746457600*I*a**88*d**11*exp(92*I*
c)*exp(-18*I*d*x) + 2286697505708706474473877361941482380984320*I*a**88*d**
11*exp(90*I*c)*exp(-20*I*d*x))*exp(-110*I*c)/(18732625966765723438890003349
0246236650235494400*a**96*d**12), Ne(a**96*d**12*exp(110*I*c), 0)), (x*((ex
p(24*I*c) + 12*exp(22*I*c) + 66*exp(20*I*c) + 220*exp(18*I*c) + 495*exp(16*
I*c) + 792*exp(14*I*c) + 924*exp(12*I*c) + 792*exp(10*I*c) + 495*exp(8*I*c)
 + 220*exp(6*I*c) + 66*exp(4*I*c) + 12*exp(2*I*c) + 1)*exp(-20*I*c)/(4096*a
**8) - 33/(2048*a**8)), True)) + 33*x/(2048*a**8)
```

Giac [A]
time = 1.33, size = 188, normalized size = 0.56

$$-\frac{-\frac{27720i\log\left(-i\tan\left(dx+c\right)+1\right)}{a^8}+\frac{27720i\log\left(-i\tan\left(dx+c\right)-1\right)}{a^8}+\frac{420\left(99i\tan\left(dx+c\right)^2-220\tan\left(dx+c\right)-123i\right)}{a^8\left(\tan\left(dx+c\right)+i\right)^2}-\frac{81191i\tan\left(dx+c\right)^{10}+858110\tan\left(dx+c\right)^9-4107195i\tan\left(dx+c\right)^8-11748840\tan\left(dx+c\right)^7+22318590i\tan\left(dx+c\right)^6+29583540\tan\left(dx+c\right)^5-27983550i\tan\left(dx+c\right)^4-19002600\tan\left(dx+c\right)^3+9206235i\tan\left(dx+c\right)^2+3108990\tan\left(dx+c\right)-648327i}{a^8\left(\tan\left(dx+c\right)-i\right)^{10}}}{3440640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3440640*(-27720*I*log(-I*tan(d*x + c) + 1)/a^8 + 27720*I*log(-I*tan(d*x
+ c) - 1)/a^8 + 420*(99*I*tan(d*x + c)^2 - 220*tan(d*x + c) - 123*I)/(a^8*(
tan(d*x + c) + I)^2) - (81191*I*tan(d*x + c)^10 + 858110*tan(d*x + c)^9 - 4
107195*I*tan(d*x + c)^8 - 11748840*tan(d*x + c)^7 + 22318590*I*tan(d*x + c)
^6 + 29583540*tan(d*x + c)^5 - 27983550*I*tan(d*x + c)^4 - 19002600*tan(d*x
 + c)^3 + 9206235*I*tan(d*x + c)^2 + 3108990*tan(d*x + c) - 648327*I)/(a^8*
(tan(d*x + c) - I)^10))/d
```

Mupad [B]
time = 5.70, size = 294, normalized size = 0.88

$$\frac{33\,x}{2048\,a^8}-\frac{\frac{\mathrm{tan}\left(c+d\,x\right)\,66953{}\mathrm{i}}{215040\,a^8}+\frac{17}{105\,a^8}-\frac{9097\,{\mathrm{tan}\left(c+d\,x\right)}^2}{26880\,a^8}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^3\,4279{}\mathrm{i}}{43008\,a^8}-\frac{99\,{\mathrm{tan}\left(c+d\,x\right)}^4}{112\,a^8}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^5\,42537{}\mathrm{i}}{35840\,a^8}+\frac{341\,{\mathrm{tan}\left(c+d\,x\right)}^6}{640\,a^8}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^7\,1969{}\mathrm{i}}{5120\,a^8}+\frac{11\,{\mathrm{tan}\left(c+d\,x\right)}^8}{16\,a^8}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^9\,869{}\mathrm{i}}{2048\,a^8}-\frac{33\,{\mathrm{tan}\left(c+d\,x\right)}^{10}}{256\,a^8}-\frac{{\mathrm{tan}\left(c+d\,x\right)}^{11}\,33{}\mathrm{i}}{2048\,a^8}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^{12}\,1{}\mathrm{i}+8\,{\mathrm{tan}\left(c+d\,x\right)}^{11}-{\mathrm{tan}\left(c+d\,x\right)}^{10}\,26{}\mathrm{i}-40\,{\mathrm{tan}\left(c+d\,x\right)}^9+{\mathrm{tan}\left(c+d\,x\right)}^8\,15{}\mathrm{i}-48\,{\mathrm{tan}\left(c+d\,x\right)}^7+{\mathrm{tan}\left(c+d\,x\right)}^6\,84{}\mathrm{i}+48\,{\mathrm{tan}\left(c+d\,x\right)}^5+{\mathrm{tan}\left(c+d\,x\right)}^4\,15{}\mathrm{i}+40\,{\mathrm{tan}\left(c+d\,x\right)}^3-{\mathrm{tan}\left(c+d\,x\right)}^2\,26{}\mathrm{i}-8\,\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (33*x)/(2048*a^8) - ((tan(c + d*x)*66953i)/(215040*a^8) + 17/(105*a^8) - (9
097*tan(c + d*x)^2)/(26880*a^8) + (tan(c + d*x)^3*4279i)/(43008*a^8) - (99*
tan(c + d*x)^4)/(112*a^8) - (tan(c + d*x)^5*42537i)/(35840*a^8) + (341*tan(
c + d*x)^6)/(640*a^8) - (tan(c + d*x)^7*1969i)/(5120*a^8) + (11*tan(c + d*x
)^8)/(16*a^8) + (tan(c + d*x)^9*869i)/(2048*a^8) - (33*tan(c + d*x)^10)/(25
6*a^8) - (tan(c + d*x)^11*33i)/(2048*a^8))/(d*(40*tan(c + d*x)^3 - tan(c +
d*x)^2*26i - 8*tan(c + d*x) + tan(c + d*x)^4*15i + 48*tan(c + d*x)^5 + tan(
c + d*x)^6*84i - 48*tan(c + d*x)^7 + tan(c + d*x)^8*15i - 40*tan(c + d*x)^9
 - tan(c + d*x)^10*26i + 8*tan(c + d*x)^11 + tan(c + d*x)^12*1i + 1i))
```

3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=205

$$\frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d}+\frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d}+\frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d}+\frac{2i\sec^{11}(c+dx}{3ad(a+ia\tan(c+}$$

```
[Out] 1155/8*arctanh(sin(d*x+c))/a^8/d+1155/8*sec(d*x+c)*tan(d*x+c)/a^8/d+385/4*s
ec(d*x+c)^3*tan(d*x+c)/a^8/d+2/3*I*sec(d*x+c)^11/a/d/(a+I*a*tan(d*x+c))^7-2
2/3*I*sec(d*x+c)^9/a^3/d/(a+I*a*tan(d*x+c))^5-66*I*sec(d*x+c)^7/a^2/d/(a^2+
I*a^2*tan(d*x+c))^3-154*I*sec(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$\frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d}-\frac{154i\sec^5(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{385\tan(c+dx)\sec^3(c+dx)}{4a^8d}+\frac{1155\tan(c+dx)\sec(c+dx)}{8a^8d}-\frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5}-\frac{66i\sec^7(c+dx)}{a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (1155*ArcTanh[Sin[c + d*x]])/(8*a^8*d) + (1155*Sec[c + d*x]*Tan[c + d*x])/(
8*a^8*d) + (385*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^8*d) + (((2*I)/3)*Sec[c +
 d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((22*I)/3)*Sec[c + d*x]^9)/(a^3
*d*(a + I*a*Tan[c + d*x])^5) - ((66*I)*Sec[c + d*x]^7)/(a^2*d*(a^2 + I*a^2*
Tan[c + d*x])^3) - ((154*I)*Sec[c + d*x]^5)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{11\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{3a^2}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^{9}(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} + \frac{33\int \frac{\sec^{9}(c+dx)}{(a+ia\tan(c+dx))^4}}{a^4}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^{9}(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^{7}(c+dx}{a^5d(a+ia\tan(c+}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^{9}(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^{7}(c+dx}{a^5d(a+ia\tan(c+}$$

$$= \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^{9}(c+}{3a^3d(a+ia\tan(}$$

$$= \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(}{3ad(a+ia\,\mathrm{ta}}$$

$$= \frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d} + \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)}{4a^8c}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1704 vs. $2(205) = 410$.
time = 6.48, size = 1704, normalized size = 8.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-1155*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8
*(Cos[d*x] + I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (1155*Cos[8*c]
*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*
Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((
(32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*
a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*x]^8*((-160*I)*Cos[7*c] + 160*Sin[
7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((1155*I)/
8)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((1155*I)/8)*Log[Co
```

```
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] +
I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*(((-23
6*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-160*Cos[7*c] - (160*I)*Sin[7*c])*(Cos
[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x
]^8*((32*Cos[5*c])/3 + ((32*I)/3)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[3
*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/16 + (I/16)
*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^4*(a + I*a*Tan[c + d*x])^8) - ((1/96 + I/96)*Sec[c + d*x]^8*((-407*
I)*Cos[(15*c)/2] + 343*Cos[(17*c)/2] + 407*Sin[(15*c)/2] + (343*I)*Sin[(17*
c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)
/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-1
/16*Cos[8*c] - (I/16)*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2])^4*(a + I*a*Tan[c + d*x])^8) + ((1/96 + I/96)*S
ec[c + d*x]^8*(407*Cos[(15*c)/2] - (343*I)*Cos[(17*c)/2] + (407*I)*Sin[(15*
c)/2] + 343*Sin[(17*c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] + Sin[c/
2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) +
 (236*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]/2 - Cos[
8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x)/2]))/(3
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a
*Tan[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c -
(d*x)/2]/2 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*
c + (d*x)/2]))/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (
d*x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[
d*x])^8*(-1/2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (
d*x)/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
(d*x)/2] - Sin[c/2 + (d*x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (236*Sec[c + d
*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(-1/2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/
2]/2 - (I/2)*Sin[8*c - (d*x)/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2]
 + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x
])^8)
```

Maple [A]
time = 0.37, size = 219, normalized size = 1.07

method	result
risch	$-\frac{160 i \,{\mathrm e}^{-i (d x +c)}}{a^{8} d}+\frac{32 i \,{\mathrm e}^{-3 i (d x +c)}}{3 a^{8} d}-\frac{i \left(1545 \,{\mathrm e}^{7 i (d x +c)}+5153 \,{\mathrm e}^{5 i (d x +c)}+5855 \,{\mathrm e}^{3 i (d x +c)}+2295 \,{\mathrm e}^{i (d x +c)}\right)}{12 d \,a^{8} \left({\mathrm e}^{2 i (d x +c)}+1\right)^{4}}+\frac{1155 \ln \left({\mathrm e}^{i (\cdots}\right.}{8 a^{8} \cdots}$
derivativedivides	$\frac{\frac{2 \left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2 \left(-\frac{121}{16}-2 i\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2 \left(-\frac{123}{16}+38 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+\frac{1}{4 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2 \left(\frac{121}{16}-2 i\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}} \cdots}{\cdots}$
default	$\frac{\frac{2 \left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2 \left(-\frac{121}{16}-2 i\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2 \left(-\frac{123}{16}+38 i\right)}{\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1}+\frac{1}{4 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2 \left(\frac{121}{16}-2 i\right)}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}} \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*((1/4-4/3*I)/(tan(1/2*d*x+1/2*c)-1)^3-(121/16+2*I)/(tan(1/2*d*x+1/2
*c)-1)^2+(-123/16+38*I)/(tan(1/2*d*x+1/2*c)-1)+1/8/(tan(1/2*d*x+1/2*c)-1)^4
-1155/16*ln(tan(1/2*d*x+1/2*c)-1)+(121/16-2*I)/(tan(1/2*d*x+1/2*c)+1)^2+(1/
4+4/3*I)/(tan(1/2*d*x+1/2*c)+1)^3-(123/16+38*I)/(tan(1/2*d*x+1/2*c)+1)-1/8/
(tan(1/2*d*x+1/2*c)+1)^4+1155/16*ln(tan(1/2*d*x+1/2*c)+1)+64*I/(-I+tan(1/2*
d*x+1/2*c))^2-128/3/(-I+tan(1/2*d*x+1/2*c))^3-128/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(179) = 358$.
time = 0.64, size = 786, normalized size = 3.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(6930*(cos(11*d*x + 11*c) + 4*cos(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*co
s(5*d*x + 5*c) + cos(3*d*x + 3*c) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x +
9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*ar
ctan2(cos(d*x + c), sin(d*x + c) + 1) + 6930*(cos(11*d*x + 11*c) + 4*cos(9*
d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*cos(5*d*x + 5*c) + cos(3*d*x + 3*c) + I
*sin(11*d*x + 11*c) + 4*I*sin(9*d*x + 9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin
(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1
) + 3465*(I*cos(11*d*x + 11*c) + 4*I*cos(9*d*x + 9*c) + 6*I*cos(7*d*x + 7*c
) + 4*I*cos(5*d*x + 5*c) + I*cos(3*d*x + 3*c) - sin(11*d*x + 11*c) - 4*sin(
9*d*x + 9*c) - 6*sin(7*d*x + 7*c) - 4*sin(5*d*x + 5*c) - sin(3*d*x + 3*c))*
log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3465*(-I*cos(11
*d*x + 11*c) - 4*I*cos(9*d*x + 9*c) - 6*I*cos(7*d*x + 7*c) - 4*I*cos(5*d*x
+ 5*c) - I*cos(3*d*x + 3*c) + sin(11*d*x + 11*c) + 4*sin(9*d*x + 9*c) + 6*s
in(7*d*x + 7*c) + 4*sin(5*d*x + 5*c) + sin(3*d*x + 3*c))*log(cos(d*x + c)^2
 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 13860*cos(10*d*x + 10*c) + 50820*
cos(8*d*x + 8*c) + 67452*cos(6*d*x + 6*c) + 36828*cos(4*d*x + 4*c) + 5632*c
os(2*d*x + 2*c) + 13860*I*sin(10*d*x + 10*c) + 50820*I*sin(8*d*x + 8*c) + 6
7452*I*sin(6*d*x + 6*c) + 36828*I*sin(4*d*x + 4*c) + 5632*I*sin(2*d*x + 2*c
) - 512)/((-48*I*a^8*cos(11*d*x + 11*c) - 192*I*a^8*cos(9*d*x + 9*c) - 288*
I*a^8*cos(7*d*x + 7*c) - 192*I*a^8*cos(5*d*x + 5*c) - 48*I*a^8*cos(3*d*x +
3*c) + 48*a^8*sin(11*d*x + 11*c) + 192*a^8*sin(9*d*x + 9*c) + 288*a^8*sin(7
*d*x + 7*c) + 192*a^8*sin(5*d*x + 5*c) + 48*a^8*sin(3*d*x + 3*c))*d)
```

Fricas [A]
time = 0.43, size = 267, normalized size = 1.30

$$\frac{3465\left(e^{(11i\,dx+11i\,c)}+4\,e^{(9i\,dx+9i\,c)}+6\,e^{(7i\,dx+7i\,c)}+4\,e^{(5i\,dx+5i\,c)}+e^{(3i\,dx+3i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3465\left(e^{(11i\,dx+11i\,c)}+4\,e^{(9i\,dx+9i\,c)}+6\,e^{(7i\,dx+7i\,c)}+4\,e^{(5i\,dx+5i\,c)}+e^{(3i\,dx+3i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6930i\,e^{(10i\,dx+10i\,c)}-25410i\,e^{(8i\,dx+8i\,c)}-33726i\,e^{(6i\,dx+6i\,c)}-18414i\,e^{(4i\,dx+4i\,c)}-2816i\,e^{(2i\,dx+2i\,c)}+256i}{24\left(a^8de^{(11i\,dx+11i\,c)}+4\,a^8de^{(9i\,dx+9i\,c)}+6\,a^8de^{(7i\,dx+7i\,c)}+4\,a^8de^{(5i\,dx+5i\,c)}+a^8de^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) +
 I) - 3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) -
 I) - 6930*I*e^(10*I*d*x + 10*I*c) - 25410*I*e^(8*I*d*x + 8*I*c) - 33726*I*
e^(6*I*d*x + 6*I*c) - 18414*I*e^(4*I*d*x + 4*I*c) - 2816*I*e^(2*I*d*x + 2*I
*c) + 256*I)/(a^8*d*e^(11*I*d*x + 11*I*c) + 4*a^8*d*e^(9*I*d*x + 9*I*c) + 6
*a^8*d*e^(7*I*d*x + 7*I*c) + 4*a^8*d*e^(5*I*d*x + 5*I*c) + a^8*d*e^(3*I*d*x
 + 3*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{13}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**13/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Giac [A]
time = 2.18, size = 195, normalized size = 0.95

$$\frac{\frac{3465\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}-\frac{3465\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}-\frac{1024\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-7\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3}-\frac{2\left(369\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-1728i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-393\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+5568i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-393\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5696i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+369\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1856i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^8}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 3465*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 1024*(6*tan(1/2*d*x + 1/2*c)^2 - 15*I*tan(1/2*d*x + 1/2*c) - 7
)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^3) - 2*(369*tan(1/2*d*x + 1/2*c)^7 - 1728
*I*tan(1/2*d*x + 1/2*c)^6 - 393*tan(1/2*d*x + 1/2*c)^5 + 5568*I*tan(1/2*d*x
 + 1/2*c)^4 - 393*tan(1/2*d*x + 1/2*c)^3 - 5696*I*tan(1/2*d*x + 1/2*c)^2 +
369*tan(1/2*d*x + 1/2*c) + 1856*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^8))/d
```

Mupad [B]
time = 7.57, size = 344, normalized size = 1.68

$$\frac{\frac{33847\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{6a^8}-\frac{12041\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3a^8}-\frac{3585\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{a^8}+\frac{3505\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4a^8}+\frac{4293\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 27565\mathrm{i}}{12a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 4575\mathrm{i}}{a^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 25993\mathrm{i}}{6a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 5639\mathrm{i}}{3a^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} 1147\mathrm{i}}{4a^8}-\frac{1360\mathrm{i}}{3a^8}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 7\mathrm{i}+13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 18\mathrm{i}-22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 22\mathrm{i}+18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 13\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)3\mathrm{i}+1\right)}+\frac{1155\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*27565i)/(12*a^8) - (12041*tan(c/2 + (d*x)/2)^3)/(3*a
^8) - (tan(c/2 + (d*x)/2)^4*4575i)/a^8 + (33847*tan(c/2 + (d*x)/2)^5)/(6*a^
8) + (tan(c/2 + (d*x)/2)^6*25993i)/(6*a^8) - (3585*tan(c/2 + (d*x)/2)^7)/a^
8 - (tan(c/2 + (d*x)/2)^8*5639i)/(3*a^8) + (3505*tan(c/2 + (d*x)/2)^9)/(4*a
^8) + (tan(c/2 + (d*x)/2)^10*1147i)/(4*a^8) - 1360i/(3*a^8) + (4293*tan(c/2
 + (d*x)/2))/(4*a^8))/(d*(tan(c/2 + (d*x)/2)*3i - 7*tan(c/2 + (d*x)/2)^2 -
tan(c/2 + (d*x)/2)^3*13i + 18*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*2
2i - 22*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*18i + 13*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^9*7i - 3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x
)/2)^11*1i + 1)) + (1155*atanh(tan(c/2 + (d*x)/2)))/(4*a^8*d)
```

3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=183

$$-\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d}-\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d}+\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] -63/2*arctanh(sin(d*x+c))/a^8/d-63/2*sec(d*x+c)*tan(d*x+c)/a^8/d+2/5*I*sec(
d*x+c)^9/a/d/(a+I*a*tan(d*x+c))^7-6/5*I*sec(d*x+c)^7/a^3/d/(a+I*a*tan(d*x+c
))^5+42/5*I*sec(d*x+c)^5/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+42*I*sec(d*x+c)^3/d
/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3581, 3853, 3855}

$$-\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{63\tan(c+dx)\sec(c+dx)}{2a^8d}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{42i\sec^5(c+dx)}{5a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-63*ArcTanh[Sin[c + d*x]])/(2*a^8*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(2*a
^8*d) + (((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((6*I)
/5)*Sec[c + d*x]^7)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((42*I)/5)*Sec[c +
d*x]^5)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + ((42*I)*Sec[c + d*x]^3)/(d*(
a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{9\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{5a^2}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{21\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}}{5a^4}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx}{5a^5d(a+ia\tan(c+}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx}{5a^5d(a+ia\tan(c+}$$

$$= -\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+}{5a^3d(a+ia\tan(}$$

$$= -\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d} - \frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx}{5ad(a+ia\tan(c+}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1244 vs. 2(183) = 366.
time = 6.31, size = 1244, normalized size = 6.80

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(C
os[d*x] + I*Sin[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) - (63*Cos[8*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d
*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) + (Cos[5*d*x]*Sec[c + d*x]^8*(((8*I)
/5)*Cos[3*c] - (8*Sin[3*c])/5)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c
 + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((-8*I)*Cos[5*c] + 8*Sin[5*c])*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*
x]^8*((48*I)*Cos[7*c] - 48*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*((8*I)*Cos[8*c] - 8*Sin[8*c])*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((63*I)/2)*Log[Cos
[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I
```

```
*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((63*I)/2)*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I*Sin[d*x])^
8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(48*Cos[7*c] + (48*I)*Sin
[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (
Sec[c + d*x]^8*(-8*Cos[5*c] - (8*I)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin
[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*((8*Cos[3*c])/5 + (
(8*I)/5)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[5*d*x])/(d*(a + I*a*Tan[c
+ d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/4 + (I/4)*Sin[8*c])*(Cos[d*x] + I*Si
n[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c +
d*x])^8) + (Sec[c + d*x]^8*(-1/4*Cos[8*c] - (I/4)*Sin[8*c])*(Cos[d*x] + I*S
in[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c +
 d*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]
/2 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x
)/2]))/(d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(
a + I*a*Tan[c + d*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(-1/
2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (d*x)/2] + (I
/2)*Sin[8*c + (d*x)/2]))/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^8)
```

Maple [A]
time = 0.34, size = 184, normalized size = 1.01

method	result
risch	$\frac{48i\,\mathrm{e}^{-i(dx+c)}}{a^8 d}-\frac{8i\,\mathrm{e}^{-3i(dx+c)}}{a^8 d}+\frac{8i\,\mathrm{e}^{-5i(dx+c)}}{5a^8 d}+\frac{i\left(15\,\mathrm{e}^{3i(dx+c)}+17\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{63\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^8 d}+\frac{63\ln\left(\mathrm{e}^{i(dx+c}\right.}{2a^8 d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-}{a^8 d}$
default	$\frac{\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-}{a^8 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*((1/4-4*I)/(tan(1/2*d*x+1/2*c)-1)+1/4/(tan(1/2*d*x+1/2*c)-1)^2+63/4
*ln(tan(1/2*d*x+1/2*c)-1)+(1/4+4*I)/(tan(1/2*d*x+1/2*c)+1)-1/4/(tan(1/2*d*x
+1/2*c)+1)^2-63/4*ln(tan(1/2*d*x+1/2*c)+1)-16*I/(-I+tan(1/2*d*x+1/2*c))^2-6
4*I/(-I+tan(1/2*d*x+1/2*c))^4+128/5/(-I+tan(1/2*d*x+1/2*c))^5-32/(-I+tan(1/
2*d*x+1/2*c))^3+32/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(157) = 314.
time = 0.57, size = 531, normalized size = 2.90

$$\frac{630\left(\cos\left(9dx+9c\right)+2\cos\left(7dx+7c\right)+\cos\left(5dx+5c\right)+i\sin\left(9dx+9c\right)+2i\sin\left(7dx+7c\right)+i\sin\left(5dx+5c\right)\right)\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)+630\left(\cos\left(9dx+9c\right)+2\cos\left(7dx+7c\right)+\cos\left(5dx+5c\right)+i\sin\left(9dx+9c\right)+2i\sin\left(7dx+7c\right)+i\sin\left(5dx+5c\right)\right)\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+315\left(i\cos\left(9dx+9c\right)+2i\cos\left(7dx+7c\right)+i\cos\left(5dx+5c\right)-\sin\left(9dx+9c\right)-2\sin\left(7dx+7c\right)-\sin\left(5dx+5c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)+315\left(-i\cos\left(9dx+9c\right)-2i\cos\left(7dx+7c\right)-i\cos\left(5dx+5c\right)+\sin\left(9dx+9c\right)+2\sin\left(7dx+7c\right)+\sin\left(5dx+5c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)+1260\cos\left(8dx+8c\right)+2100\cos\left(6dx+6c\right)+672\cos\left(4dx+4c\right)-96\cos\left(2dx+2c\right)+1260i\sin\left(8dx+8c\right)+2100i\sin\left(6dx+6c\right)+672i\sin\left(4dx+4c\right)-96i\sin\left(2dx+2c\right)+32}{-20\left(i a^8\cos\left(9dx+9c\right)+2i a^8\cos\left(7dx+7c\right)+i a^8\cos\left(5dx+5c\right)-a^8\sin\left(9dx+9c\right)-2a^8\sin\left(7dx+7c\right)-a^8\sin\left(5dx+5c\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] (630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x + 5*c) + I*sin(9*d*
x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*arctan2(cos(d*x + c),
 sin(d*x + c) + 1) + 630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x
 + 5*c) + I*sin(9*d*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*a
rctan2(cos(d*x + c), -sin(d*x + c) + 1) + 315*(I*cos(9*d*x + 9*c) + 2*I*cos
(7*d*x + 7*c) + I*cos(5*d*x + 5*c) - sin(9*d*x + 9*c) - 2*sin(7*d*x + 7*c)
- sin(5*d*x + 5*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
1) + 315*(-I*cos(9*d*x + 9*c) - 2*I*cos(7*d*x + 7*c) - I*cos(5*d*x + 5*c) +
 sin(9*d*x + 9*c) + 2*sin(7*d*x + 7*c) + sin(5*d*x + 5*c))*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 1260*cos(8*d*x + 8*c) + 2100*co
s(6*d*x + 6*c) + 672*cos(4*d*x + 4*c) - 96*cos(2*d*x + 2*c) + 1260*I*sin(8*
d*x + 8*c) + 2100*I*sin(6*d*x + 6*c) + 672*I*sin(4*d*x + 4*c) - 96*I*sin(2*
d*x + 2*c) + 32)/((-20*I*a^8*cos(9*d*x + 9*c) - 40*I*a^8*cos(7*d*x + 7*c) -
 20*I*a^8*cos(5*d*x + 5*c) + 20*a^8*sin(9*d*x + 9*c) + 40*a^8*sin(7*d*x + 7
*c) + 20*a^8*sin(5*d*x + 5*c))*d)
```

Fricas [**A**]
time = 0.41, size = 182, normalized size = 0.99

$$-\frac{315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-630i\,e^{(8i\,dx+8i\,c)}-1050i\,e^{(6i\,dx+6i\,c)}-336i\,e^{(4i\,dx+4i\,c)}+48i\,e^{(2i\,dx+2i\,c)}-16i}{10\left(a^8de^{(9i\,dx+9i\,c)}+2\,a^8de^{(7i\,dx+7i\,c)}+a^8de^{(5i\,dx+5i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/10*(315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*I*c) + e^(5*I*d*x + 5*I*
c))*log(e^(I*d*x + I*c) + I) - 315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*
I*c) + e^(5*I*d*x + 5*I*c))*log(e^(I*d*x + I*c) - I) - 630*I*e^(8*I*d*x + 8
*I*c) - 1050*I*e^(6*I*d*x + 6*I*c) - 336*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(2*
I*d*x + 2*I*c) - 16*I)/(a^8*d*e^(9*I*d*x + 9*I*c) + 2*a^8*d*e^(7*I*d*x + 7*
I*c) + a^8*d*e^(5*I*d*x + 5*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^{11}\left(c+dx\right)}{\tan^8\left(c+dx\right)-8i\tan^7\left(c+dx\right)-28\tan^6\left(c+dx\right)+56i\tan^5\left(c+dx\right)+70\tan^4\left(c+dx\right)-56i\tan^3\left(c+dx\right)-28\tan^2\left(c+dx\right)+8i\tan\left(c+dx\right)+1}\,dx}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**11/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Giac [A]
time = 2.36, size = 165, normalized size = 0.90

$$-\frac{\frac{315 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^8}-\frac{315 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}{a^8}-\frac{10 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-16 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+16 i\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-1\right)^2 a^8}-\frac{64 \left(10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-45 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-85 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+55 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+13\right)}{a^8 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)^5}}{10 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/10*(315*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 315*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^8 - 10*(tan(1/2*d*x + 1/2*c)^3 - 16*I*tan(1/2*d*x + 1/2*c)^2 + tan(
1/2*d*x + 1/2*c) + 16*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^8) - 64*(10*tan(
1/2*d*x + 1/2*c)^4 - 45*I*tan(1/2*d*x + 1/2*c)^3 - 85*tan(1/2*d*x + 1/2*c)^
2 + 55*I*tan(1/2*d*x + 1/2*c) + 13)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B]
time = 7.44, size = 284, normalized size = 1.55

$$-\frac{63\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^8\,d}+\frac{\frac{1223\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^8}-\frac{1109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^8}+\frac{309\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{a^8}-\frac{431\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4407{}\mathrm{i}}{5\,a^8}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,7351{}\mathrm{i}}{5\,a^8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,761{}\mathrm{i}}{a^8}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,65{}\mathrm{i}}{a^8}+\frac{496{}\mathrm{i}}{5\,a^8}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,1{}\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,12{}\mathrm{i}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,26{}\mathrm{i}+26\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,20{}\mathrm{i}-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((1223*tan(c/2 + (d*x)/2)^3)/a^8 - (tan(c/2 + (d*x)/2)^2*4407i)/(5*a^8) + (
tan(c/2 + (d*x)/2)^4*7351i)/(5*a^8) - (1109*tan(c/2 + (d*x)/2)^5)/a^8 - (ta
n(c/2 + (d*x)/2)^6*761i)/a^8 + (309*tan(c/2 + (d*x)/2)^7)/a^8 + (tan(c/2 +
(d*x)/2)^8*65i)/a^8 + 496i/(5*a^8) - (431*tan(c/2 + (d*x)/2))/a^8)/(d*(tan(
c/2 + (d*x)/2)*5i - 12*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*20i + 26
*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*26i - 20*tan(c/2 + (d*x)/2)^6
- tan(c/2 + (d*x)/2)^7*12i + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*
1i + 1)) - (63*atanh(tan(c/2 + (d*x)/2)))/(a^8*d)
```

3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=156

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^8d}+\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^3(c+dx)}{3a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}-$$

```
[Out] arctanh(sin(d*x+c))/a^8/d+2/7*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^7-2/5*I
*sec(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5+2/3*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^
2*tan(d*x+c))^3-2*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]

time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3581, 3855}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^8d}-\frac{2i\sec(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^3(c+dx)}{3a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^8*d) + (((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Ta
n[c + d*x])^7) - (((2*I)/5)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan[c + d*x])^5
) + (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) - ((2*I
)*Sec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{a^2}$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{a^4}$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^8d} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}$$

Mathematica [A]
time = 1.13, size = 304, normalized size = 1.95

$$\frac{\sec^8(c+dx)\left(70i\cos\left(\frac{1}{2}(c+dx)\right)-42i\cos\left(\frac{3}{2}(c+dx)\right)-210i\cos\left(\frac{5}{2}(c+dx)\right)+30i\cos\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+105\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-70\sin\left(\frac{1}{2}(c+dx)\right)-42\sin\left(\frac{3}{2}(c+dx)\right)+210\sin\left(\frac{5}{2}(c+dx)\right)+30\sin\left(\frac{7}{2}(c+dx)\right)-105i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)+105i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)\right)\left(\cos\left(\frac{9}{2}(c+dx)\right)+i\sin\left(\frac{9}{2}(c+dx)\right)\right)}{105a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*((70*I)*Cos[(c + d*x)/2] - (42*I)*Cos[(3*(c + d*x))/2] - (2
10*I)*Cos[(5*(c + d*x))/2] + (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c +
d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2
]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(
3*(c + d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] - (105
*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + (105*I)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(9*(c
+ d*x))/2] + I*Sin[(9*(c + d*x))/2]))/(105*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.35, size = 137, normalized size = 0.88

method	result
risch	$-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^8d}+\frac{2i\mathrm{e}^{-3i(dx+c)}}{3a^8d}-\frac{2i\mathrm{e}^{-5i(dx+c)}}{5a^8d}+\frac{2i\mathrm{e}^{-7i(dx+c)}}{7a^8d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^8d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^8d}$
derivativedivides	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{25}{7\left(-i+\tan\right.}}{a^8d}$
default	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{25}{7\left(-i+\tan\right.}}{a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-1/2*ln(tan(1/2*d*x+1/2*c)-1)+1/2*ln(tan(1/2*d*x+1/2*c)+1)+64*I/(-
I+tan(1/2*d*x+1/2*c))^6+8*I/(-I+tan(1/2*d*x+1/2*c))^2-64*I/(-I+tan(1/2*d*x+
1/2*c))^4-128/7/(-I+tan(1/2*d*x+1/2*c))^7+448/5/(-I+tan(1/2*d*x+1/2*c))^5-8
0/3/(-I+tan(1/2*d*x+1/2*c))^3)
```

Maxima [A]
time = 0.56, size = 185, normalized size = 1.19

$$\frac{-210i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-210i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+60i\cos\left(7\,dx+7\,c\right)-84i\cos\left(5\,dx+5\,c\right)+140i\cos\left(3\,dx+3\,c\right)-420i\cos\left(dx+c\right)+105\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-105\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)+60\sin\left(7\,dx+7\,c\right)-84\sin\left(5\,dx+5\,c\right)+140\sin\left(3\,dx+3\,c\right)-420\sin\left(dx+c\right)}{210\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/210*(-210*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 210*I*arctan2(cos(d
*x + c), -sin(d*x + c) + 1) + 60*I*cos(7*d*x + 7*c) - 84*I*cos(5*d*x + 5*c)
 + 140*I*cos(3*d*x + 3*c) - 420*I*cos(d*x + c) + 105*log(cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*sin(d*x + c) + 1) - 105*log(cos(d*x + c)^2 + sin(d*x + c)
^2 - 2*sin(d*x + c) + 1) + 60*sin(7*d*x + 7*c) - 84*sin(5*d*x + 5*c) + 140*
sin(3*d*x + 3*c) - 420*sin(d*x + c))/(a^8*d)
```

Fricas [A]
time = 0.38, size = 98, normalized size = 0.63

$$\frac{\left(105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}-42i\,e^{(2i\,dx+2i\,c)}+30i\right)e^{(-7i\,dx-7i\,c)}}{105\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*e^(7*I*d*x + 7*I*c)*log(e^(I*d*x + I*c) + I) - 105*e^(7*I*d*x +
7*I*c)*log(e^(I*d*x + I*c) - I) - 210*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d
*x + 4*I*c) - 42*I*e^(2*I*d*x + 2*I*c) + 30*I)*e^(-7*I*d*x - 7*I*c)/(a^8*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^9\left(c+dx\right)}{\tan^8\left(c+dx\right)-8i\tan^7\left(c+dx\right)-28\tan^6\left(c+dx\right)+56i\tan^5\left(c+dx\right)+70\tan^4\left(c+dx\right)-56i\tan^3\left(c+dx\right)-28\tan^2\left(c+dx\right)+8i\tan\left(c+dx\right)+1}\,dx}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**8,x)
```

[Out] Integral(sec(c + d*x)**9/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8

Giac [A]
time = 1.59, size = 123, normalized size = 0.79

$$\frac{\frac{105 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^8} - \frac{105 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}{a^8} - \frac{16 \left(-105 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-175 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+490 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+294 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-133 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-19\right)}{a^8 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)^7}}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")

[Out] 1/105*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 105*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 - 16*(-105*I*tan(1/2*d*x + 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^4 + 490*I*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 - 133*I*tan(1/2*d*x + 1/2*c) - 19)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^7))/d

Mupad [B]
time = 6.94, size = 207, normalized size = 1.33

$$\frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^8 d} + \frac{\frac{16 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^8} - \frac{224 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3 a^8} + \frac{304 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15 a^8} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 224\mathrm{i}}{5 a^8} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 80\mathrm{i}}{3 a^8} - \frac{304\mathrm{i}}{105 a^8}}{d \left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 1\mathrm{i} - 7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 21\mathrm{i} + 35 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 35\mathrm{i} - 21 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right) 7\mathrm{i} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8),x)

[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^8*d) + ((tan(c/2 + (d*x)/2)^2*224i)/(5*a^8) - (224*tan(c/2 + (d*x)/2)^3)/(3*a^8) - (tan(c/2 + (d*x)/2)^4*80i)/(3*a^8) + (16*tan(c/2 + (d*x)/2)^5)/a^8 - 304i/(105*a^8) + (304*tan(c/2 + (d*x)/2))/(15*a^8))/(d*(tan(c/2 + (d*x)/2)*7i - 21*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*35i + 35*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*21i - 7*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*1i + 1))

3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=68

$$\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}+\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}$$

```
[Out] 1/9*I*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^8+1/63*I*sec(d*x+c)^7/a/d/(a+I*a*ta
n(d*x+c))^7
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 3569}

$$\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]^
7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8} + \frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{9a}$$
$$= \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8} + \frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}$$

Mathematica [A]
time = 0.15, size = 40, normalized size = 0.59

$$-\frac{\sec^7(c+dx)(-8i+\tan(c+dx))}{63a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -1/63*(Sec[c + d*x]^7*(-8*I + Tan[c + d*x]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(60) = 120.
time = 0.34, size = 156, normalized size = 2.29

method	result
risch	$\frac{i e^{-7i(dx+c)}}{14a^8d} + \frac{i e^{-9i(dx+c)}}{18a^8d}$
derivativedivides	$\frac{-\frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\cdots}{a^8d}$
default	$\frac{-\frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\cdots}{a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-86/3/(-I+tan(1/2*d*x+1/2*c))^3+128/9/(-I+tan(1/2*d*x+1/2*c))^9+49
6/3*I/(-I+tan(1/2*d*x+1/2*c))^6-76*I/(-I+tan(1/2*d*x+1/2*c))^4-928/7/(-I+ta
n(1/2*d*x+1/2*c))^7-64*I/(-I+tan(1/2*d*x+1/2*c))^8+1/(-I+tan(1/2*d*x+1/2*c)
)+7*I/(-I+tan(1/2*d*x+1/2*c))^2+136/(-I+tan(1/2*d*x+1/2*c))^5)
```

Maxima [A]
time = 0.32, size = 53, normalized size = 0.78

$$\frac{7i\cos(9\,dx+9\,c)+9i\cos(7\,dx+7\,c)+7\sin(9\,dx+9\,c)+9\sin(7\,dx+7\,c)}{126\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/126*(7*I*cos(9*d*x + 9*c) + 9*I*cos(7*d*x + 7*c) + 7*sin(9*d*x + 9*c) + 9
*sin(7*d*x + 7*c))/(a^8*d)
```

Fricas [A]
time = 0.38, size = 30, normalized size = 0.44

$$\frac{\left(9i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{126\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/126*(9*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^8*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(54) = 108.
time = 11.31, size = 311, normalized size = 4.57

$$\begin{cases} -\frac{\tan(c+dx)\sec^7(c+dx)}{63a^8d\tan^8(c+dx)-504ia^8d\tan^7(c+dx)-1764a^8d\tan^6(c+dx)+3528ia^8d\tan^5(c+dx)+4410a^8d\tan^4(c+dx)-3528ia^8d\tan^3(c+dx)-1764a^8d\tan^2(c+dx)+504ia^8d\tan(c+dx)+63a^8d}+\frac{8i\sec^7(c+dx)}{63a^8d\tan^8(c+dx)-504ia^8d\tan^7(c+dx)-1764a^8d\tan^6(c+dx)+3528ia^8d\tan^5(c+dx)+4410a^8d\tan^4(c+dx)-3528ia^8d\tan^3(c+dx)-1764a^8d\tan^2(c+dx)+504ia^8d\tan(c+dx)+63a^8d} & \text{for}\ d\neq 0 \\ \frac{x\sec^7(c)}{(ia\tan(c)+a)^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-tan(c + d*x)*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I
*a**8*d*tan(c + d*x)**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c
 + d*x)**5 + 4410*a**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 -
1764*a**8*d*tan(c + d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d) + 8*I*
sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I*a**8*d*tan(c + d*x)**7 -
 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c + d*x)**5 + 4410*a**8*d*
tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 - 1764*a**8*d*tan(c + d*x)*
*2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d), Ne(d, 0)), (x*sec(c)**7/(I*a*t
an(c) + a)**8, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.
time = 1.41, size = 125, normalized size = 1.84

$$\frac{2\left(63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-63i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-483\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+315i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+693\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-189i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-225\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8\right)}{63\,a^8d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/63*(63*tan(1/2*d*x + 1/2*c)^8 - 63*I*tan(1/2*d*x + 1/2*c)^7 - 483*tan(1/2
*d*x + 1/2*c)^6 + 315*I*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4
```

```
 - 189*I*tan(1/2*d*x + 1/2*c)^3 - 225*tan(1/2*d*x + 1/2*c)^2 + 9*I*tan(1/2*
d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^9)
```

Mupad [B]
time = 3.74, size = 37, normalized size = 0.54

$$\frac{2\left(\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{-c\,9\mathrm{i}-d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,a^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8),x)
[Out] (2*((exp(- c*7i - d*x*7i)*9i)/4 + (exp(- c*9i - d*x*9i)*7i)/4))/(63*a^8*d)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=138

$$\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] 1/11*I*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^8+1/33*I*sec(d*x+c)^5/a/d/(a+I*a*t
an(d*x+c))^7+2/231*I*sec(d*x+c)^5/a^2/d/(a+I*a*tan(d*x+c))^6+2/1155*I*sec(d
*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 3569}

$$\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/33)*Sec[c + d*x]
^5)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((2*I)/231)*Sec[c + d*x]^5)/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((2*I)/1155)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan
[c + d*x])^5)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{3\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{11a}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{33a^2}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx}{231a^2d(a+ia\tan(c}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx}{231a^2d(a+ia\tan(c}$$

Mathematica [A]
time = 0.37, size = 73, normalized size = 0.53

$$\frac{i\sec^8(c+dx)(440\cos(c+dx)+168\cos(3(c+dx))+55i\sin(c+dx)+63i\sin(3(c+dx)))}{4620a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/4620)*Sec[c + d*x]^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] + (55*I)*
Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.34, size = 189, normalized size = 1.37

method	result
risch	$\frac{ie^{-5i(dx+c)}}{40a^8d}+\frac{3ie^{-7i(dx+c)}}{56a^8d}+\frac{ie^{-9i(dx+c)}}{24a^8d}+\frac{ie^{-11i(dx+c)}}{88a^8d}$
derivativedivides	$\frac{-\frac{60}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{584i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{256}{11\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{}{(-i+\tan}}{a^8d}$
default	$\frac{-\frac{60}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{584i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{256}{11\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{}{(-i+\tan}}{a^8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-30/(-I+tan(1/2*d*x+1/2*c))^3+292*I/(-I+tan(1/2*d*x+1/2*c))^6-2376
/7/(-I+tan(1/2*d*x+1/2*c))^7-128/11/(-I+tan(1/2*d*x+1/2*c))^11+512/3/(-I+ta
n(1/2*d*x+1/2*c))^9+64*I/(-I+tan(1/2*d*x+1/2*c))^10+7*I/(-I+tan(1/2*d*x+1/2
*c))^2-88*I/(-I+tan(1/2*d*x+1/2*c))^4+932/5/(-I+tan(1/2*d*x+1/2*c))^5-288*I
/(-I+tan(1/2*d*x+1/2*c))^8+1/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [**A**]
time = 0.32, size = 97, normalized size = 0.70

$$\frac{105i\cos(11\,dx+11\,c)+385i\cos(9\,dx+9\,c)+495i\cos(7\,dx+7\,c)+231i\cos(5\,dx+5\,c)+105\sin(11\,dx+11\,c)+385\sin(9\,dx+9\,c)+495\sin(7\,dx+7\,c)+231\sin(5\,dx+5\,c)}{9240\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*I*cos(11*d*x + 11*c) + 385*I*cos(9*d*x + 9*c) + 495*I*cos(7*d*x
 + 7*c) + 231*I*cos(5*d*x + 5*c) + 105*sin(11*d*x + 11*c) + 385*sin(9*d*x +
 9*c) + 495*sin(7*d*x + 7*c) + 231*sin(5*d*x + 5*c))/(a^8*d)
```

Fricas [**A**]
time = 0.36, size = 52, normalized size = 0.38

$$\frac{\left(231i\,e^{(6i\,dx+6i\,c)}+495i\,e^{(4i\,dx+4i\,c)}+385i\,e^{(2i\,dx+2i\,c)}+105i\right)e^{(-11i\,dx-11i\,c)}}{9240\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(231*I*e^(6*I*d*x + 6*I*c) + 495*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(2*
I*d*x + 2*I*c) + 105*I)*e^(-11*I*d*x - 11*I*c)/(a^8*d)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(119) = 238.
time = 11.00, size = 620, normalized size = 4.49

$$\begin{cases} \frac{2\tan^3(c+dx)\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)-32340a^8d\tan^2(c+dx)+9240ia^8d\tan(c+dx)+1155a^8d} - \frac{16i\tan^2(c+dx)\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)-32340a^8d\tan^2(c+dx)+9240ia^8d\tan(c+dx)+1155a^8d} - \frac{61\tan(c+dx)\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)-32340a^8d\tan^2(c+dx)+9240ia^8d\tan(c+dx)+1155a^8d} + \frac{152i\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)-32340a^8d\tan^2(c+dx)+9240ia^8d\tan(c+dx)+1155a^8d} & \text{for}\: d \neq 0 \\ \frac{x\sec^5(c)}{(ia\tan(c)+a)^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8 -
 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a**
8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c +
 d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1155
*a**8*d) - 16*I*tan(c + d*x)**2*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**
8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*
a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(
c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1
155*a**8*d) - 61*tan(c + d*x)*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
- 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a*
*8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c
+ d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 115
5*a**8*d) + 152*I*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8 - 9240*I*a**
```

```
8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a**8*d*tan(c +
 d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c + d*x)**3 -
32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1155*a**8*d), N
e(d, 0)), (x*sec(c)**5/(I*a*tan(c) + a)**8, True))
```

Giac [A]
time = 1.43, size = 151, normalized size = 1.09

$$\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-3465i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-13860\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+23100i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+37422\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-32802i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-27060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+11220i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+4895\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-517i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-152\right)}{1155a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 - 3465*I*tan(1/2*d*x + 1/2*c)^9 - 1386
0*tan(1/2*d*x + 1/2*c)^8 + 23100*I*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d
*x + 1/2*c)^6 - 32802*I*tan(1/2*d*x + 1/2*c)^5 - 27060*tan(1/2*d*x + 1/2*c)
^4 + 11220*I*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 - 517*I*t
an(1/2*d*x + 1/2*c) - 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^11)
```

Mupad [B]
time = 3.91, size = 64, normalized size = 0.46

$$\frac{\frac{e^{-c\,5i-d\,x\,5i}\,1i}{40}+\frac{e^{-c\,7i-d\,x\,7i}\,3i}{56}+\frac{e^{-c\,9i-d\,x\,9i}\,1i}{24}+\frac{e^{-c\,11i-d\,x\,11i}\,1i}{88}}{a^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((exp(- c*5i - d*x*5i)*1i)/40 + (exp(- c*7i - d*x*7i)*3i)/56 + (exp(- c*9i
 - d*x*9i)*1i)/24 + (exp(- c*11i - d*x*11i)*1i)/88)/(a^8*d)
```

3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=213

$$\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}+\frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7}+\frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}+\frac{20i\sec^3(c+ \ldots}{3003a^3d(a+ia\tan \ldots}$$

[Out] 1/13*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+5/143*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^7+20/1287*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+20/3003*I*sec(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+8/3003*I*sec(d*x+c)^3/d/(a^2+I*a^2*tan(d*x+c))^4+8/9009*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^3

Rubi [A]
time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 3569}

$$\frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5}+\frac{8i\sec^3(c+dx)}{9009a^2d\,(a^2+ia^2\tan(c+dx))^3}+\frac{8i\sec^3(c+dx)}{3003d\,(a^2+ia^2\tan(c+dx))^4}+\frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}+\frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (((5*I)/143)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((20*I)/1287)*Sec[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (((20*I)/3003)*Sec[c + d*x]^3)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((8*I)/3003)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (((8*I)/9009)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{13a}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^6}}{143a^2}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$

Mathematica [A]
time = 0.36, size = 95, normalized size = 0.45

$$\frac{i\sec^8(c+dx)(11440\cos(c+dx)+6552\cos(3(c+dx))+1848\cos(5(c+dx))+1430i\sin(c+dx)+2457i\sin(3(c+dx))+1155i\sin(5(c+dx)))}{144144a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/144144)*Sec[c + d*x]^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 18
48*Cos[5*(c + d*x)] + (1430*I)*Sin[c + d*x] + (2457*I)*Sin[3*(c + d*x)] + (
1155*I)*Sin[5*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.34, size = 222, normalized size = 1.04

method	result
risch	$\frac{i e^{-3i(dx+c)}}{96a^8d}+\frac{i e^{-5i(dx+c)}}{32a^8d}+\frac{5i e^{-7i(dx+c)}}{112a^8d}+\frac{5i e^{-9i(dx+c)}}{144a^8d}+\frac{5i e^{-11i(dx+c)}}{352a^8d}+\frac{i e^{-13i(dx+c)}}{416a^8d}$
derivativedivides	$\frac{\frac{2}{-i+\tan(\frac{dx}{2}+\frac{c}{2})}-\frac{200i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}-\frac{9056}{7(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^7}+\frac{480}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{1472i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^8}+\frac{14i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2})}}{}$
default	$\frac{\frac{2}{-i+\tan(\frac{dx}{2}+\frac{c}{2})}-\frac{200i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}-\frac{9056}{7(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^7}+\frac{480}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{1472i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^8}+\frac{14i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2})}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(1/(-I+tan(1/2*d*x+1/2*c))-100*I/(-I+tan(1/2*d*x+1/2*c))^4-4528/7/(
-I+tan(1/2*d*x+1/2*c))^7+240/(-I+tan(1/2*d*x+1/2*c))^5-736*I/(-I+tan(1/2*d*
x+1/2*c))^8+7*I/(-I+tan(1/2*d*x+1/2*c))^2-2272/11/(-I+tan(1/2*d*x+1/2*c))^1
1+432*I/(-I+tan(1/2*d*x+1/2*c))^10-64*I/(-I+tan(1/2*d*x+1/2*c))^12+5840/9/(
-I+tan(1/2*d*x+1/2*c))^9-94/3/(-I+tan(1/2*d*x+1/2*c))^3+128/13/(-I+tan(1/2*
d*x+1/2*c))^13+1336/3*I/(-I+tan(1/2*d*x+1/2*c))^6)
```

Maxima [A]
time = 0.33, size = 141, normalized size = 0.66

$$\frac{693i\cos(13\,dx+13\,c)+4095i\cos(11\,dx+11\,c)+10010i\cos(9\,dx+9\,c)+12870i\cos(7\,dx+7\,c)+9009i\cos(5\,dx+5\,c)+3003i\cos(3\,dx+3\,c)+693\sin(13\,dx+13\,c)+4095\sin(11\,dx+11\,c)+10010\sin(9\,dx+9\,c)+12870\sin(7\,dx+7\,c)+9009\sin(5\,dx+5\,c)+3003\sin(3\,dx+3\,c)}{288288\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/288288*(693*I*cos(13*d*x + 13*c) + 4095*I*cos(11*d*x + 11*c) + 10010*I*co
s(9*d*x + 9*c) + 12870*I*cos(7*d*x + 7*c) + 9009*I*cos(5*d*x + 5*c) + 3003*
I*cos(3*d*x + 3*c) + 693*sin(13*d*x + 13*c) + 4095*sin(11*d*x + 11*c) + 100
10*sin(9*d*x + 9*c) + 12870*sin(7*d*x + 7*c) + 9009*sin(5*d*x + 5*c) + 3003
*sin(3*d*x + 3*c))/(a^8*d)
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.35

$$\frac{\left(3003i\,e^{(10i\,dx+10i\,c)}+9009i\,e^{(8i\,dx+8i\,c)}+12870i\,e^{(6i\,dx+6i\,c)}+10010i\,e^{(4i\,dx+4i\,c)}+4095i\,e^{(2i\,dx+2i\,c)}+693i\right)e^{(-13i\,dx-13i\,c)}}{288288\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(3003*I*e^(10*I*d*x + 10*I*c) + 9009*I*e^(8*I*d*x + 8*I*c) + 12870
*I*e^(6*I*d*x + 6*I*c) + 10010*I*e^(4*I*d*x + 4*I*c) + 4095*I*e^(2*I*d*x +
2*I*c) + 693*I)*e^(-13*I*d*x - 13*I*c)/(a^8*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(189) = 378.
time = 10.91, size = 928, normalized size = 4.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-8*tan(c + d*x)**5*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8
- 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I
```

```
*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*t
an(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c + d*x
) + 9009*a**8*d) + 64*I*tan(c + d*x)**4*sec(c + d*x)**3/(9009*a**8*d*tan(c
+ d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6
+ 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*
I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*t
an(c + d*x) + 9009*a**8*d) + 236*tan(c + d*x)**3*sec(c + d*x)**3/(9009*a**8
*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c +
 d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4
- 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I
*a**8*d*tan(c + d*x) + 9009*a**8*d) - 544*I*tan(c + d*x)**2*sec(c + d*x)**3
/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**
8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c
 + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**
2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 911*tan(c + d*x)*sec(c + d
*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 2522
52*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d
*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c +
d*x)**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) + 1240*I*sec(c + d*x)*
*3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a
**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan
(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)
**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d), Ne(d, 0)), (x*sec(c)**3/(
I*a*tan(c) + a)**8, True))
```

Giac [A]
time = 1.28, size = 177, normalized size = 0.83

$$\frac{2\left(9009\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-45045i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-183183\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+435435i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+810810\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-1051050i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-1076790\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+785070i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+451165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-171457i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-51675\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+7111i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1240\right)}{9009a^8d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 - 45045*I*tan(1/2*d*x + 1/2*c)^11 - 18
3183*tan(1/2*d*x + 1/2*c)^10 + 435435*I*tan(1/2*d*x + 1/2*c)^9 + 810810*tan
(1/2*d*x + 1/2*c)^8 - 1051050*I*tan(1/2*d*x + 1/2*c)^7 - 1076790*tan(1/2*d*
x + 1/2*c)^6 + 785070*I*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c
)^4 - 171457*I*tan(1/2*d*x + 1/2*c)^3 - 51675*tan(1/2*d*x + 1/2*c)^2 + 7111
*I*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^13)
```

Mupad [B]
time = 4.22, size = 159, normalized size = 0.75

$$\frac{\frac{\cos(3c+3dx)^3\,5\mathrm{i}}{36}+\frac{5\sin(3c+3dx)\cos(3c+3dx)^2}{36}-\frac{\cos(3c+3dx)\,3\mathrm{i}}{32}+\frac{\cos(5c+5dx)\,1\mathrm{i}}{32}+\frac{\cos(7c+7dx)\,5\mathrm{i}}{112}+\frac{\cos(11c+11dx)\,5\mathrm{i}}{352}+\frac{\cos(13c+13dx)\,1\mathrm{i}}{416}-\frac{7\sin(3c+3dx)}{288}+\frac{\sin(5c+5dx)}{32}+\frac{5\sin(7c+7dx)}{112}+\frac{5\sin(11c+11dx)}{352}+\frac{\sin(13c+13dx)}{416}}{a^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((cos(5*c + 5*d*x)*1i)/32 - (cos(3*c + 3*d*x)*3i)/32 + (cos(7*c + 7*d*x)*5i
)/112 + (cos(11*c + 11*d*x)*5i)/352 + (cos(13*c + 13*d*x)*1i)/416 - (7*sin(
3*c + 3*d*x))/288 + sin(5*c + 5*d*x)/32 + (5*sin(7*c + 7*d*x))/112 + (5*sin
(11*c + 11*d*x))/352 + sin(13*c + 13*d*x)/416 + (cos(3*c + 3*d*x)^3*5i)/36
+ (5*cos(3*c + 3*d*x)^2*sin(3*c + 3*d*x))/36)/(a^8*d)
```

3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=269

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}+\frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6}+\frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+$$

```
[Out] 1/15*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^8+7/195*I*sec(d*x+c)/a/d/(a+I*a*tan(
d*x+c))^7+14/715*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+14/1287*I*sec(d*x+
c)/a^3/d/(a+I*a*tan(d*x+c))^5+8/1287*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^
4+8/2145*I*sec(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+16/6435*I*sec(d*x+c)/d
/(a^4+I*a^4*tan(d*x+c))^2+16/6435*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]

time = 0.21, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3583, 3569}

$$\frac{16i\sec(c+dx)}{6435d(a^8+ia^8\tan(c+dx))}+\frac{16i\sec(c+dx)}{6435d(a^4+ia^4\tan(c+dx))^2}+\frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5}+\frac{8i\sec(c+dx)}{2145a^2d(a^2+ia^2\tan(c+dx))^3}+\frac{8i\sec(c+dx)}{1287d(a^2+ia^2\tan(c+dx))^4}+\frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6}+\frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((7*I)/195)*Sec[c + d
*x])/(a*d*(a + I*a*Tan[c + d*x])^7) + (((14*I)/715)*Sec[c + d*x])/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((14*I)/1287)*Sec[c + d*x])/(a^3*d*(a + I*a*Tan[
c + d*x])^5) + (((8*I)/1287)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
 + (((8*I)/2145)*Sec[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((16
*I)/6435)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x])^2) + (((16*I)/6435)*S
ec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
```

]

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{15a}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))}}{65a^2}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+}{715a^2d(a+ia\tan}$$

Mathematica [A]
time = 0.64, size = 117, normalized size = 0.43

$$\frac{i\sec^8(c+dx)(28600\cos(c+dx)+19656\cos(3(c+dx))+9240\cos(5(c+dx))+3432\cos(7(c+dx))+3575i\sin(c+dx)+7371i\sin(3(c+dx))+5775i\sin(5(c+dx))+3003i\sin(7(c+dx)))}{411840a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/411840)*Sec[c + d*x]^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] + 9
240*Cos[5*(c + d*x)] + 3432*Cos[7*(c + d*x)] + (3575*I)*Sin[c + d*x] + (737
1*I)*Sin[3*(c + d*x)] + (5775*I)*Sin[5*(c + d*x)] + (3003*I)*Sin[7*(c + d*x
)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.15, size = 255, normalized size = 0.95

method	result
risch	$\frac{ie^{-i(dx+c)}}{128a^8d}+\frac{7ie^{-3i(dx+c)}}{384a^8d}+\frac{21ie^{-5i(dx+c)}}{640a^8d}+\frac{5ie^{-7i(dx+c)}}{128a^8d}+\frac{35ie^{-9i(dx+c)}}{1152a^8d}+\frac{21ie^{-11i(dx+c)}}{1408a^8d}+\frac{7ie^{-13i(dx+c)}}{1664a^8d}$
derivativedivides	$-\frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}+\frac{2968}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{3752i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$
default	$-\frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{14}}+\frac{2968}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{3752i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-112*I/(-I+tan(1/2*d*x+1/2*c))^4+64*I/(-I+tan(1/2*d*x+1/2*c))^14+1
484/5/(-I+tan(1/2*d*x+1/2*c))^5+1/(-I+tan(1/2*d*x+1/2*c))+1876/3*I/(-I+tan(
1/2*d*x+1/2*c))^6-1792/3*I/(-I+tan(1/2*d*x+1/2*c))^12-1064/(-I+tan(1/2*d*x+
1/2*c))^7+7504/5*I/(-I+tan(1/2*d*x+1/2*c))^10+7*I/(-I+tan(1/2*d*x+1/2*c))^2
+14896/9/(-I+tan(1/2*d*x+1/2*c))^9-11872/11/(-I+tan(1/2*d*x+1/2*c))^11-1472
*I/(-I+tan(1/2*d*x+1/2*c))^8-128/15/(-I+tan(1/2*d*x+1/2*c))^15+3136/13/(-I+
tan(1/2*d*x+1/2*c))^13-98/3/(-I+tan(1/2*d*x+1/2*c))^3)
```

Maxima [A]
time = 0.32, size = 179, normalized size = 0.67

$$\frac{429i\cos(15dx+15c)+3465i\cos(13dx+13c)+12285i\cos(11dx+11c)+25025i\cos(9dx+9c)+32175i\cos(7dx+7c)+27027i\cos(5dx+5c)+15015i\cos(3dx+3c)+6435i\cos(dx+c)+429\sin(15dx+15c)+3465\sin(13dx+13c)+12285\sin(11dx+11c)+25025\sin(9dx+9c)+32175\sin(7dx+7c)+27027\sin(5dx+5c)+15015\sin(3dx+3c)+6435\sin(dx+c)}{823680a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*I*cos(15*d*x + 15*c) + 3465*I*cos(13*d*x + 13*c) + 12285*I*co
s(11*d*x + 11*c) + 25025*I*cos(9*d*x + 9*c) + 32175*I*cos(7*d*x + 7*c) + 27
027*I*cos(5*d*x + 5*c) + 15015*I*cos(3*d*x + 3*c) + 6435*I*cos(d*x + c) + 4
29*sin(15*d*x + 15*c) + 3465*sin(13*d*x + 13*c) + 12285*sin(11*d*x + 11*c)
+ 25025*sin(9*d*x + 9*c) + 32175*sin(7*d*x + 7*c) + 27027*sin(5*d*x + 5*c)
+ 15015*sin(3*d*x + 3*c) + 6435*sin(d*x + c))/(a^8*d)
```

Fricas [A]
time = 0.40, size = 96, normalized size = 0.36

$$\frac{\left(6435i\,e^{(14i\,dx+14i\,c)}+15015i\,e^{(12i\,dx+12i\,c)}+27027i\,e^{(10i\,dx+10i\,c)}+32175i\,e^{(8i\,dx+8i\,c)}+25025i\,e^{(6i\,dx+6i\,c)}+12285i\,e^{(4i\,dx+4i\,c)}+3465i\,e^{(2i\,dx+2i\,c)}+429i\right)e^{(-15i\,dx-15i\,c)}}{823680\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(6435*I*e^(14*I*d*x + 14*I*c) + 15015*I*e^(12*I*d*x + 12*I*c) + 27
027*I*e^(10*I*d*x + 10*I*c) + 32175*I*e^(8*I*d*x + 8*I*c) + 25025*I*e^(6*I*
```

```
d*x + 6*I*c) + 12285*I*e^(4*I*d*x + 4*I*c) + 3465*I*e^(2*I*d*x + 2*I*c) + 4
29*I)*e^(-15*I*d*x - 15*I*c)/(a^8*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1221 vs. 2(238) = 476.
time = 11.21, size = 1221, normalized size = 4.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
[Out] Piecewise((16*tan(c + d*x)**7*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 5
1480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*a*
*8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(
c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c + d*x) +
 6435*a**8*d) - 128*I*tan(c + d*x)**6*sec(c + d*x)/(6435*a**8*d*tan(c + d*x
)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360
360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**
8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c
+ d*x) + 6435*a**8*d) - 456*tan(c + d*x)**5*sec(c + d*x)/(6435*a**8*d*tan(c
 + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6
 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360
*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*
tan(c + d*x) + 6435*a**8*d) + 960*I*tan(c + d*x)**4*sec(c + d*x)/(6435*a**8
*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c +
 d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4
- 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I
*a**8*d*tan(c + d*x) + 6435*a**8*d) + 1350*tan(c + d*x)**3*sec(c + d*x)/(64
35*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*
tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d
*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 +
51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) - 1392*I*tan(c + d*x)**2*sec(c +
 d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 18018
0*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*
tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d
*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) - 1181*tan(c + d*x)*sec
(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 1
80180*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**
8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c
 + d*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 952*I*sec(c + d*x
)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a*
*8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(
c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)*
*2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d), Ne(d, 0)), (x*sec(c)/(I*a*
tan(c) + a)**8, True))
```

Giac [A]
time = 1.24, size = 203, normalized size = 0.75

$$\frac{2\left(6435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}-45045i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-210210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+630630i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1414413\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-2357355i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-3063060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+3063060i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+2407405\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-1444443i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-668850\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+222950i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+54915\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-7845i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-952\right)}{6435a^8d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/6435*(6435*tan(1/2*d*x + 1/2*c)^14 - 45045*I*tan(1/2*d*x + 1/2*c)^13 - 21
0210*tan(1/2*d*x + 1/2*c)^12 + 630630*I*tan(1/2*d*x + 1/2*c)^11 + 1414413*t
an(1/2*d*x + 1/2*c)^10 - 2357355*I*tan(1/2*d*x + 1/2*c)^9 - 3063060*tan(1/2
*d*x + 1/2*c)^8 + 3063060*I*tan(1/2*d*x + 1/2*c)^7 + 2407405*tan(1/2*d*x +
1/2*c)^6 - 1444443*I*tan(1/2*d*x + 1/2*c)^5 - 668850*tan(1/2*d*x + 1/2*c)^4
 + 222950*I*tan(1/2*d*x + 1/2*c)^3 + 54915*tan(1/2*d*x + 1/2*c)^2 - 7845*I*
tan(1/2*d*x + 1/2*c) - 952)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^15)
```

Mupad [B]
time = 5.36, size = 224, normalized size = 0.83

$$\frac{2\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\left(-\frac{\sin(c+d\,x)^2\,44779\mathrm{i}}{32}+\frac{32175\sin(c+d\,x)}{128}-\frac{\sin(2\,c+2\,d\,x)^2\,26075\mathrm{i}}{16}-\frac{3575\sin(2\,c+2\,d\,x)}{8}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,114583\mathrm{i}}{64}-\frac{\sin(3\,c+3\,d\,x)^2\,57925\mathrm{i}}{32}+\frac{84227\sin(3\,c+3\,d\,x)}{128}+\frac{\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,116585\mathrm{i}}{64}+\frac{\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)^2\,119315\mathrm{i}}{64}+\frac{\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)^2\,122285\mathrm{i}}{64}-754\sin\left(4\,c+4\,d\,x\right)+\frac{111527\sin(5\,c+5\,d\,x)}{128}-\frac{7187\sin(6\,c+6\,d\,x)}{8}+\frac{121427\sin(7\,c+7\,d\,x)}{128}-952\mathrm{i}\right)}{6435\,a^8\,d\left(-2\sin\left(\frac{15\,c}{4}+\frac{15\,d\,x}{4}\right)^2+\sin\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (2*(2*sin(c/4 + (d*x)/4)^2 - 1)*((32175*sin(c + d*x))/128 - (3575*sin(2*c +
 2*d*x))/8 + (84227*sin(3*c + 3*d*x))/128 - 754*sin(4*c + 4*d*x) + (111527*
sin(5*c + 5*d*x))/128 - (7187*sin(6*c + 6*d*x))/8 + (121427*sin(7*c + 7*d*x
))/128 - (sin(2*c + 2*d*x)^2*26075i)/16 + (sin(c/2 + (d*x)/2)^2*114583i)/64
 - (sin(3*c + 3*d*x)^2*57925i)/32 + (sin((3*c)/2 + (3*d*x)/2)^2*116585i)/64
 + (sin((5*c)/2 + (5*d*x)/2)^2*119315i)/64 + (sin((7*c)/2 + (7*d*x)/2)^2*12
2285i)/64 - (sin(c + d*x)^2*44779i)/32 - 952i))/(6435*a^8*d*(sin((15*c)/2 +
 (15*d*x)/2)*1i - 2*sin((15*c)/4 + (15*d*x)/4)^2 + 1))
```

3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=271

$$\frac{192\sin(c+dx)}{12155a^8d}-\frac{64\sin^3(c+dx)}{12155a^8d}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}+\frac{24i\cos(c-}{1105a^2d(a+ia\,\mathrm{ta}}$$

```
[Out] 192/12155*sin(d*x+c)/a^8/d-64/12155*sin(d*x+c)^3/a^8/d+1/17*I*cos(d*x+c)/d/
(a+I*a*tan(d*x+c))^8+3/85*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^7+24/1105*I*c
os(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+168/12155*I*cos(d*x+c)/a^3/d/(a+I*a*ta
n(d*x+c))^5+112/12155*I*cos(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^4+16/2431*I*cos
(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+128/12155*I*cos(d*x+c)^3/d/(a^8+I*a^
8*tan(d*x+c))
```

Rubi [A]

time = 0.25, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3583, 3581, 2713}

$$-\frac{64\sin^3(c+dx)}{12155a^8d}+\frac{192\sin(c+dx)}{12155a^8d}+\frac{128i\cos^3(c+dx)}{12155d\,(a^8+ia^8\tan(c+dx))}+\frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5}+\frac{16i\cos(c+dx)}{2431a^2d\,(a^2+ia^2\tan(c+dx))^3}+\frac{112i\cos(c+dx)}{12155d\,(a^2+ia^2\tan(c+dx))^4}+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (192*Sin[c + d*x])/(12155*a^8*d) - (64*Sin[c + d*x]^3)/(12155*a^8*d) + ((I/
17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((3*I)/85)*Cos[c + d*x])/
(a*d*(a + I*a*Tan[c + d*x])^7) + (((24*I)/1105)*Cos[c + d*x])/(a^2*d*(a + I
*a*Tan[c + d*x])^6) + (((168*I)/12155)*Cos[c + d*x])/(a^3*d*(a + I*a*Tan[c
+ d*x])^5) + (((112*I)/12155)*Cos[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4
) + (((16*I)/2431)*Cos[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((
128*I)/12155)*Cos[c + d*x]^3)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
```

```
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{9\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{17a} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^6}\,d}{85a^2} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx}{1105a^2d(a+ia\tan(c} \\
&= \frac{192\sin(c+dx)}{12155a^8d} - \frac{64\sin^3(c+dx)}{12155a^8d} + \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(}{85ad(a+ia\,\mathrm{t}}
\end{aligned}$$

Mathematica [A]
time = 1.20, size = 139, normalized size = 0.51

$$-\frac{i\sec^8(c+dx)(-194480\cos(c+dx)-148512\cos(3(c+dx))-89760\cos(5(c+dx))-58344\cos(7(c+dx))+5720\cos(9(c+dx))-24310i\sin(c+dx)-55692i\sin(3(c+dx))-56100i\sin(5(c+dx))-51051i\sin(7(c+dx))+6435i\sin(9(c+dx)))}{3111680a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3111680*I)*Sec[c + d*x]^8*(-194480*Cos[c + d*x] - 148512*Cos[3*(c + d*
x)] - 89760*Cos[5*(c + d*x)] - 58344*Cos[7*(c + d*x)] + 5720*Cos[9*(c + d*x
)] - (24310*I)*Sin[c + d*x] - (55692*I)*Sin[3*(c + d*x)] - (56100*I)*Sin[5*
(c + d*x)] - (51051*I)*Sin[7*(c + d*x)] + (6435*I)*Sin[9*(c + d*x)]))/(a^8*
d*(-I + Tan[c + d*x])^8)
```

Maple [A]
time = 0.31, size = 306, normalized size = 1.13

method	result
risch	$\frac{3ie^{-3i(dx+c)}}{128a^8d}+\frac{21ie^{-5i(dx+c)}}{640a^8d}+\frac{9ie^{-7i(dx+c)}}{256a^8d}+\frac{7ie^{-9i(dx+c)}}{256a^8d}+\frac{21ie^{-11i(dx+c)}}{1408a^8d}+\frac{9ie^{-13i(dx+c)}}{1664a^8d}+\frac{3ie^{-15i(d}}{2560a^8}$
derivativedivides	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{10241i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{5384i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}+\frac{}{5(-i+}$
default	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{10241i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{5384i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}+\frac{}{5(-i+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(1/512/(tan(1/2*d*x+1/2*c)+I)-64*I/(-I+tan(1/2*d*x+1/2*c))^16-7937/
64*I/(-I+tan(1/2*d*x+1/2*c))^4-10241/4*I/(-I+tan(1/2*d*x+1/2*c))^8-2692*I/(
-I+tan(1/2*d*x+1/2*c))^12+19109/5*I/(-I+tan(1/2*d*x+1/2*c))^10+1793/256*I/(
-I+tan(1/2*d*x+1/2*c))^2+784*I/(-I+tan(1/2*d*x+1/2*c))^14+13313/16*I/(-I+ta
n(1/2*d*x+1/2*c))^6+128/17/(-I+tan(1/2*d*x+1/2*c))^17-1376/5/(-I+tan(1/2*d*
x+1/2*c))^15+21400/13/(-I+tan(1/2*d*x+1/2*c))^13-38954/11/(-I+tan(1/2*d*x+1
/2*c))^11+6847/2/(-I+tan(1/2*d*x+1/2*c))^9-12799/8/(-I+tan(1/2*d*x+1/2*c))^
7+57083/160/(-I+tan(1/2*d*x+1/2*c))^5-4351/128/(-I+tan(1/2*d*x+1/2*c))^3+51
1/512/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 118, normalized size = 0.44

$$\frac{\left(-12155i\,e^{(18i\,dx+18i\,c)}+109395i\,e^{(16i\,dx+16i\,c)}+145860i\,e^{(14i\,dx+14i\,c)}+204204i\,e^{(12i\,dx+12i\,c)}+218790i\,e^{(10i\,dx+10i\,c)}+170170i\,e^{(8i\,dx+8i\,c)}+92820i\,e^{(6i\,dx+6i\,c)}+33660i\,e^{(4i\,dx+4i\,c)}+7293i\,e^{(2i\,dx+2i\,c)}+715i\right)e^{(-17i\,dx-17i\,c)}}{6223360\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6223360*(-12155*I*e^(18*I*d*x + 18*I*c) + 109395*I*e^(16*I*d*x + 16*I*c)
+ 145860*I*e^(14*I*d*x + 14*I*c) + 204204*I*e^(12*I*d*x + 12*I*c) + 218790*
I*e^(10*I*d*x + 10*I*c) + 170170*I*e^(8*I*d*x + 8*I*c) + 92820*I*e^(6*I*d*x
 + 6*I*c) + 33660*I*e^(4*I*d*x + 4*I*c) + 7293*I*e^(2*I*d*x + 2*I*c) + 715*
I)*e^(-17*I*d*x - 17*I*c)/(a^8*d)
```

Sympy [A]
time = 0.60, size = 367, normalized size = 1.35

$$\begin{cases} \frac{\left(-14350091149820134393118720 0 i a^{72} d^{9} e^{82 i c} e^{i d x} + 129150820348381209538068480 0 i a^{72} d^{9} e^{80 i c} e^{- i d x} + 1722010937978416127174246400 i a^{72} d^{9} e^{78 i c} e^{- 3 i d x} + 2410815313169782578043944960 i a^{72} d^{9} e^{76 i c} e^{- 5 i d x} + 2583016406967624190761369600 i a^{72} d^{9} e^{74 i c} e^{- 7 i d x} + 2009012760974818815036620800 i a^{72} d^{9} e^{72 i c} e^{- 9 i d x} + 1095825142349901171838156800 i a^{72} d^{9} e^{70 i c} e^{- 11 i d x} + 397387139533480644732518400 i a^{72} d^{9} e^{68 i c} e^{- 13 i d x} + 86100546898920806358712320 i a^{72} d^{9} e^{66 i c} e^{- 15 i d x} + 8441230088129490819481600 i a^{72} d^{9} e^{64 i c} e^{- 17 i d x}\right) e^{- 81 i c}}{7347246668707908809276784640 0 a^{80} d^{10}} & \text{for}\: a^{80} d^{10} e^{81 i c} \neq 0 \\ \frac{x \left(e^{18 i c} + 9 e^{16 i c} + 36 e^{14 i c} + 84 e^{12 i c} + 126 e^{10 i c} + 126 e^{8 i c} + 84 e^{6 i c} + 36 e^{4 i c} + 9 e^{2 i c} + 1\right) e^{- 17 i c}}{512 a^{8}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-143500911498201343931187200*I*a**72*d**9*exp(82*I*c)*exp(I*d*x
) + 1291508203483812095380684800*I*a**72*d**9*exp(80*I*c)*exp(-I*d*x) + 172
2010937978416127174246400*I*a**72*d**9*exp(78*I*c)*exp(-3*I*d*x) + 24108153
13169782578043944960*I*a**72*d**9*exp(76*I*c)*exp(-5*I*d*x) + 2583016406967
624190761369600*I*a**72*d**9*exp(74*I*c)*exp(-7*I*d*x) + 200901276097481881
5036620800*I*a**72*d**9*exp(72*I*c)*exp(-9*I*d*x) + 109582514234990117183811
56800*I*a**72*d**9*exp(70*I*c)*exp(-11*I*d*x) + 397387139533480644732518400
*I*a**72*d**9*exp(68*I*c)*exp(-13*I*d*x) + 86100546898920806358712320*I*a**
72*d**9*exp(66*I*c)*exp(-15*I*d*x) + 8441230088129490819481600*I*a**72*d**9
*exp(64*I*c)*exp(-17*I*d*x))*exp(-81*I*c)/(734724666870790880927678464000*a*
*80*d**10), Ne(a**80*d**10*exp(81*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c
) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84
*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-17*I*c)/(512*a**8), Tr
ue))
```

Giac [A]
time = 1.25, size = 249, normalized size = 0.92

$$\frac{\frac{12155}{a^{8} \left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + i\right)} + \frac{6211205 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{16} - 55791450 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{15} - 303072770 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{14} + 1091397450 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{13} + 2909561798 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{12} - 5901218466 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{11} - 9405145178 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{10} + 11877161010 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{9} + 12017308160 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{8} - 9710430158 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 6263238566 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 3172666718 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} + 1247921210 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 365303990 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 77883902 \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 10498214 i \, \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 982907}{a^{8} \left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - i\right)^{17}}}{3111680 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) + I)) + (6211205*tan(1/2*d*x +
1/2*c)^16 - 55791450*I*tan(1/2*d*x + 1/2*c)^15 - 303072770*tan(1/2*d*x + 1/
2*c)^14 + 1091397450*I*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1
/2*c)^12 - 5901218466*I*tan(1/2*d*x + 1/2*c)^11 - 9405145178*tan(1/2*d*x +
1/2*c)^10 + 11877161010*I*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x
```

```
+ 1/2*c)^8 - 9710430158*I*tan(1/2*d*x + 1/2*c)^7 - 6263238566*tan(1/2*d*x +
 1/2*c)^6 + 3172666718*I*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x +
1/2*c)^4 - 365303990*I*tan(1/2*d*x + 1/2*c)^3 - 77883902*tan(1/2*d*x + 1/2*
c)^2 + 10498214*I*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c)
 - I)^17))/d
```

Mupad [B]
time = 6.66, size = 262, normalized size = 0.97

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{152329\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}-\frac{41121\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{32}+\frac{41121\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{96165\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{64}+\frac{96165\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{64}-\frac{55095\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{32}+\frac{55095\sin\left(\frac{13c}{2}+\frac{13dx}{2}\right)}{32}-\frac{491811\sin\left(\frac{15c}{2}+\frac{15dx}{2}\right)}{256}+\frac{6435\sin\left(\frac{17c}{2}+\frac{17dx}{2}\right)}{256}+\frac{\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)12155\mathrm{i}}{16}-\frac{\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)12155\mathrm{i}}{16}+\frac{\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)21437\mathrm{i}}{16}-\frac{\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)21437\mathrm{i}}{16}+\frac{\cos\left(\frac{11c}{2}+\frac{11dx}{2}\right)27047\mathrm{i}}{16}-\frac{\cos\left(\frac{13c}{2}+\frac{13dx}{2}\right)27047\mathrm{i}}{16}+\frac{\cos\left(\frac{15c}{2}+\frac{15dx}{2}\right)61387\mathrm{i}}{32}-\frac{\cos\left(\frac{17c}{2}+\frac{17dx}{2}\right)715\mathrm{i}}{32}\right)2\mathrm{i}}{12155\,a^8\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}\right)^{17}\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*12155i)/16 - (cos((5*c)/2 +
(5*d*x)/2)*12155i)/16 + (cos((7*c)/2 + (7*d*x)/2)*21437i)/16 - (cos((9*c)/2
 + (9*d*x)/2)*21437i)/16 + (cos((11*c)/2 + (11*d*x)/2)*27047i)/16 - (cos((1
3*c)/2 + (13*d*x)/2)*27047i)/16 + (cos((15*c)/2 + (15*d*x)/2)*61387i)/32 -
(cos((17*c)/2 + (17*d*x)/2)*715i)/32 + (152329*sin(c/2 + (d*x)/2))/128 - (4
1121*sin((3*c)/2 + (3*d*x)/2))/32 + (41121*sin((5*c)/2 + (5*d*x)/2))/32 - (
96165*sin((7*c)/2 + (7*d*x)/2))/64 + (96165*sin((9*c)/2 + (9*d*x)/2))/64 -
(55095*sin((11*c)/2 + (11*d*x)/2))/32 + (55095*sin((13*c)/2 + (13*d*x)/2))/
32 - (491811*sin((15*c)/2 + (15*d*x)/2))/256 + (6435*sin((17*c)/2 + (17*d*x
)/2))/256)*2i)/(12155*a^8*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^17
*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2)))
```

3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=301

$$\frac{160\sin(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{32\sin^5(c+dx)}{4199a^8d}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+$$

```
[Out] 160/4199*sin(d*x+c)/a^8/d-320/12597*sin(d*x+c)^3/a^8/d+32/4199*sin(d*x+c)^5
/a^8/d+1/19*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+11/323*I*cos(d*x+c)^3/a/d
/(a+I*a*tan(d*x+c))^7+22/969*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+66/4
199*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+48/4199*I*cos(d*x+c)^3/d/(a^2
+I*a^2*tan(d*x+c))^4+112/12597*I*cos(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^
3+64/4199*I*cos(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$\frac{32\sin^5(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{160\sin(c+dx)}{4199a^8d}+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}+\frac{112i\cos^3(c+dx)}{12597a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (160*Sin[c + d*x])/(4199*a^8*d) - (320*Sin[c + d*x]^3)/(12597*a^8*d) + (32*
Sin[c + d*x]^5)/(4199*a^8*d) + ((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^8) + (((11*I)/323)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (
((22*I)/969)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (((66*I)/41
99)*Cos[c + d*x]^3)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((48*I)/4199)*Cos[c
 + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (((112*I)/12597)*Cos[c + d*x]
^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((64*I)/4199)*Cos[c + d*x]^5)/(
d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
```

```
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{19a}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{110\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx}}{323a^2}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+}{969a^2d(a+ia\tan}$$

$$= \frac{160\sin(c+dx)}{4199a^8d} - \frac{320\sin^3(c+dx)}{12597a^8d} + \frac{32\sin^5(c+dx)}{4199a^8d} + \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+a}$$

Mathematica [A]
time = 1.60, size = 161, normalized size = 0.53

$$-\frac{i\sec^8(c+dx)(-739024\cos(c+dx)-604656\cos(3(c+dx))-426360\cos(5(c+dx))-369512\cos(7(c+dx))+65208\cos(9(c+dx))+1768\cos(11(c+dx))-92378i\sin(c+dx)-226746i\sin(3(c+dx))-266475i\sin(5(c+dx))-323323i\sin(7(c+dx))+73359i\sin(9(c+dx))+2431i\sin(11(c+dx)))}{12899328a^8d(-i+\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/12899328*I)*Sec[c + d*x]^8*(-739024*Cos[c + d*x] - 604656*Cos[3*(c + d
*x)] - 426360*Cos[5*(c + d*x)] - 369512*Cos[7*(c + d*x)] + 65208*Cos[9*(c +
 d*x)] + 1768*Cos[11*(c + d*x)] - (92378*I)*Sin[c + d*x] - (226746*I)*Sin[3
*(c + d*x)] - (266475*I)*Sin[5*(c + d*x)] - (323323*I)*Sin[7*(c + d*x)] + (
73359*I)*Sin[9*(c + d*x)] + (2431*I)*Sin[11*(c + d*x)]))/(a^8*d*(-I + Tan[c
 + d*x])^8)
```

Maple [A]
time = 0.30, size = 372, normalized size = 1.24

method	result
risch	$\frac{33i e^{-5i(dx+c)}}{1024a^8d}+\frac{33i e^{-7i(dx+c)}}{1024a^8d}+\frac{77i e^{-9i(dx+c)}}{3072a^8d}+\frac{15i e^{-11i(dx+c)}}{1024a^8d}+\frac{165i e^{-13i(dx+c)}}{26624a^8d}+\frac{11i e^{-15i(dx+c)}}{6144a^8d}+\frac{11ie}{34}$
derivativedivides	$\frac{\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{18}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{7181i}{512\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{}{\left(-i+\tan\right.}}{}$
default	$\frac{\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{18}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{7181i}{512\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{}{\left(-i+\tan\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(64*I/(-I+tan(1/2*d*x+1/2*c))^18-1/1536/(tan(1/2*d*x+1/2*c))+I)^3+3/
512/(tan(1/2*d*x+1/2*c)+I)+7181/1024*I/(-I+tan(1/2*d*x+1/2*c))^2-1/1024*I/(
tan(1/2*d*x+1/2*c)+I)^2+4428*I/(-I+tan(1/2*d*x+1/2*c))^14+204605/192*I/(-I+
tan(1/2*d*x+1/2*c))^6-32525/8*I/(-I+tan(1/2*d*x+1/2*c))^8-2177/16*I/(-I+tan
(1/2*d*x+1/2*c))^4+32417/4*I/(-I+tan(1/2*d*x+1/2*c))^10-25468/3*I/(-I+tan(1
/2*d*x+1/2*c))^12-992*I/(-I+tan(1/2*d*x+1/2*c))^16-128/19/(-I+tan(1/2*d*x+1
/2*c))^19+5248/17/(-I+tan(1/2*d*x+1/2*c))^17-7096/3/(-I+tan(1/2*d*x+1/2*c))
^15+87508/13/(-I+tan(1/2*d*x+1/2*c))^13-18011/2/(-I+tan(1/2*d*x+1/2*c))^11+
6215/(-I+tan(1/2*d*x+1/2*c))^9-72425/32/(-I+tan(1/2*d*x+1/2*c))^7+26871/64/
(-I+tan(1/2*d*x+1/2*c))^5-54229/1536/(-I+tan(1/2*d*x+1/2*c))^3+509/512/(-I+
tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 140, normalized size = 0.47

$$\frac{\left(-4199i\,e^{(22i\,dx+22i\,c)}-138567i\,e^{(20i\,dx+20i\,c)}+692835i\,e^{(18i\,dx+18i\,c)}+692835i\,e^{(16i\,dx+16i\,c)}+831402i\,e^{(14i\,dx+14i\,c)}+831402i\,e^{(12i\,dx+12i\,c)}+646646i\,e^{(10i\,dx+10i\,c)}+377910i\,e^{(8i\,dx+8i\,c)}+159885i\,e^{(6i\,dx+6i\,c)}+46189i\,e^{(4i\,dx+4i\,c)}+8151i\,e^{(2i\,dx+2i\,c)}+663i\right)e^{(-19i\,dx-19i\,c)}}{25798656\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/25798656*(-4199*I*e^(22*I*d*x + 22*I*c) - 138567*I*e^(20*I*d*x + 20*I*c)
+ 692835*I*e^(18*I*d*x + 18*I*c) + 692835*I*e^(16*I*d*x + 16*I*c) + 831402*
I*e^(14*I*d*x + 14*I*c) + 831402*I*e^(12*I*d*x + 12*I*c) + 646646*I*e^(10*I
*d*x + 10*I*c) + 377910*I*e^(8*I*d*x + 8*I*c) + 159885*I*e^(6*I*d*x + 6*I*c
) + 46189*I*e^(4*I*d*x + 4*I*c) + 8151*I*e^(2*I*d*x + 2*I*c) + 663*I)*e^(-1
9*I*d*x - 19*I*c)/(a^8*d)
```

Sympy [A]
time = 0.72, size = 435, normalized size = 1.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-627910689858846946911347157688181273395 2*I*a**88*d**11*exp(103
*I*c)*exp(3*I*d*x) - 2072105276534194924807445620370998202 20416*I*a**88*d**
11*exp(101*I*c)*exp(I*d*x) + 10360526382670974624037228101854991011 02080*I*
a**88*d**11*exp(99*I*c)*exp(-I*d*x) + 1036052638267097462403722810185499101
102080*I*a**88*d**11*exp(97*I*c)*exp(-3*I*d*x) + 1243263165920516954884467 3
72222598921322496*I*a**88*d**11*exp(95*I*c)*exp(-5*I*d*x) + 124326316592051
69548844673722225989213 22496*I*a**88*d**11*exp(93*I*c)*exp(-7*I*d*x) + 9669
824623826242982434746228397991610286 08*I*a**88*d**11*exp(91*I*c)*exp(-9*I*d
*x) + 5651196208729622522202124419193631460556 80*I*a**88*d**11*exp(89*I*c)*
exp(-11*I*d*x) + 2390890703693301836316283408120382541004 80*I*a**88*d**11*e
xp(87*I*c)*exp(-13*I*d*x) + 690701758844731641602481873456999400734 72*I*a**
88*d**11*exp(85*I*c)*exp(-15*I*d*x) + 1218885456784820544004379776688224 71
8848*I*a**88*d**11*exp(83*I*c)*exp(-17*I*d*x) + 991437931356074126702127091
086602010624*I*a**88*d**11*exp(81*I*c)*exp(-19*I*d*x))*exp(-100*I*c)/(38578
8327849275564182331693683618574374010 88*a**96*d**12), Ne(a**96*d**12*exp(10
0*I*c), 0)), (x*(exp(22*I*c) + 11*exp(20*I*c) + 55*exp(18*I*c) + 165*exp(16
*I*c) + 330*exp(14*I*c) + 462*exp(12*I*c) + 462*exp(10*I*c) + 330*exp(8*I*c
) + 165*exp(6*I*c) + 55*exp(4*I*c) + 11*exp(2*I*c) + 1)*exp(-19*I*c)/(2048*
a**8), True))
```

Giac [A]
time = 1.23, size = 301, normalized size = 1.00

$$\frac{\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+33i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-17\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}-140368371i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+3693272440i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+11467502592\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}-27403194676i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-51919375300\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+79183835016i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+98304418212\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-99750226290i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-82860874122\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+56110430792i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+30766700912\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-13462452660i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-4616712644\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+1197851960i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+226248618\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-27911475i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2143959}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{19}}}{6449664\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 + 33*I*tan(1/2*d*x + 1/2*c) - 17
)/(a^8*(tan(1/2*d*x + 1/2*c) + I)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 -
140368371*I*tan(1/2*d*x + 1/2*c)^17 - 879644311*tan(1/2*d*x + 1/2*c)^16 + 3
693272440*I*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 -
 27403194676*I*tan(1/2*d*x + 1/2*c)^13 - 51919375300*tan(1/2*d*x + 1/2*c)^1
2 + 79183835016*I*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c
)^10 - 99750226290*I*tan(1/2*d*x + 1/2*c)^9 - 82860874122*tan(1/2*d*x + 1/2
*c)^8 + 56110430792*I*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/
2*c)^6 - 13462452660*I*tan(1/2*d*x + 1/2*c)^5 - 4616712644*tan(1/2*d*x + 1/
2*c)^4 + 1197851960*I*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*
c)^2 - 27911475*I*tan(1/2*d*x + 1/2*c) - 2143959)/(a^8*(tan(1/2*d*x + 1/2*c
) - I)^19))/d
```

Mupad [B]
time = 9.52, size = 308, normalized size = 1.02

$$-\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{46189\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{46189\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}-\frac{20995\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{16}+\frac{20995\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{16}-\frac{221255\cos\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{128}+\frac{221255\cos\left(\frac{13c}{2}+\frac{13dx}{2}\right)}{128}-\frac{66861\cos\left(\frac{15c}{2}+\frac{15dx}{2}\right)}{32}+\frac{2093\cos\left(\frac{17c}{2}+\frac{17dx}{2}\right)}{32}-\frac{221\cos\left(\frac{19c}{2}+\frac{19dx}{2}\right)}{128}+\frac{221\cos\left(\frac{21c}{2}+\frac{21dx}{2}\right)}{128}+\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)309861i}{256}-\frac{\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)665911i}{512}+\frac{\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)665911i}{512}-\frac{\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)194821i}{128}+\frac{\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)194821i}{128}-\frac{\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)1825043i}{1024}+\frac{\sin\left(\frac{13c}{2}+\frac{13dx}{2}\right)1825043i}{1024}-\frac{\sin\left(\frac{15c}{2}+\frac{15dx}{2}\right)1074183i}{512}+\frac{\sin\left(\frac{17c}{2}+\frac{17dx}{2}\right)37895i}{512}-\frac{\sin\left(\frac{19c}{2}+\frac{19dx}{2}\right)2431i}{1024}+\frac{\sin\left(\frac{21c}{2}+\frac{21dx}{2}\right)2431i}{1024}\right)}{12597a^8d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^{19}\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*((46189*cos((5*c)/2 + (5*d*x)/2))/64 - (46189*cos((3
*c)/2 + (3*d*x)/2))/64 - (20995*cos((7*c)/2 + (7*d*x)/2))/16 + (20995*cos((
9*c)/2 + (9*d*x)/2))/16 - (221255*cos((11*c)/2 + (11*d*x)/2))/128 + (221255
*cos((13*c)/2 + (13*d*x)/2))/128 - (66861*cos((15*c)/2 + (15*d*x)/2))/32 +
(2093*cos((17*c)/2 + (17*d*x)/2))/32 - (221*cos((19*c)/2 + (19*d*x)/2))/128
 + (221*cos((21*c)/2 + (21*d*x)/2))/128 + (sin(c/2 + (d*x)/2)*309861i)/256
- (sin((3*c)/2 + (3*d*x)/2)*665911i)/512 + (sin((5*c)/2 + (5*d*x)/2)*665911
i)/512 - (sin((7*c)/2 + (7*d*x)/2)*194821i)/128 + (sin((9*c)/2 + (9*d*x)/2)
*194821i)/128 - (sin((11*c)/2 + (11*d*x)/2)*1825043i)/1024 + (sin((13*c)/2
+ (13*d*x)/2)*1825043i)/1024 - (sin((15*c)/2 + (15*d*x)/2)*1074183i)/512 +
(sin((17*c)/2 + (17*d*x)/2)*37895i)/512 - (sin((19*c)/2 + (19*d*x)/2)*2431i
)/1024 + (sin((21*c)/2 + (21*d*x)/2)*2431i)/1024))/(12597*a^8*d*(cos(c/2 +
(d*x)/2) + sin(c/2 + (d*x)/2)*1i)^19*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*
x)/2))^3)
```

3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=123

$$-\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{6ae^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2ae(e\sec(c+\cdots}{}$$

```
[Out] 2/7*I*a*(e*sec(d*x+c))^(7/2)/d+2/5*a*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/d-6/
5*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*a*e^3*sin(d*
x+c)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3853, 3856, 2719}

$$-\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{6ae^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{2ae\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-6*a*e^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + (6*a*e^3*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx &= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + a\int (e\sec(c+dx))^{7/2}\,dx\\ &= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}(\\ &= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2ae}{}\\ &= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2ae}{}\\ &= -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.16, size = 156, normalized size = 1.27

$$\frac{aee^{-idx}(e\sec(c+dx))^{5/2}(\cos(dx)-i\sin(dx))(\cos(c+3dx)+i\sin(c+3dx))\left(-36i-28i\cos(2(c+dx))+7ie^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+7\sec(c+dx)\sin(3(c+dx))+27\tan(c+dx)\right)}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 3*d*x] + I*Sin
[c + 3*d*x])*(-36*I - (28*I)*Cos[2*(c + d*x)] + ((7*I)*(1 + E^((2*I)*(c + d
*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 7*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d*x]))/(70*d*E^
(I*d*x))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(130) = 260.
time = 6.70, size = 365, normalized size = 2.97

method	result

default	$-\frac{2a(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(21i(\cos^4(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(21*I*cos(d*x+c)^4*sin(d*x+c)*
(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+
cos(d*x+c))/sin(d*x+c),I)-21*I*cos(d*x+c)^4*sin(d*x+c)*(1/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+
c),I)+21*I*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*cos(d*x+c
)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*E
llipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+21*cos(d*x+c)^4-14*cos(d*x+c)^3-5*
I*sin(d*x+c)-7*cos(d*x+c))*(e/cos(d*x+c))^(7/2)/sin(d*x+c)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((I*a*tan(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 195, normalized size = 1.59

$$-\frac{2\left(\frac{\sqrt{2}\left(21i\,ae^{\left(7i\,dx+7i\,c+\frac{7}{2}\right)}+77i\,ae^{\left(5i\,dx+5i\,c+\frac{7}{2}\right)}+23i\,ae^{\left(3i\,dx+3i\,c+\frac{7}{2}\right)}+7i\,ae^{\left(i\,dx+i\,c+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+21\left(i\sqrt{2}\,ae^{\frac{7}{2}}+i\sqrt{2}\,ae^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)}+3i\sqrt{2}\,ae^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+3i\sqrt{2}\,ae^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(21*I*a*e^(7*I*d*x + 7*I*c + 7/2) + 77*I*a*e^(5*I*d*x + 5*I*
c + 7/2) + 23*I*a*e^(3*I*d*x + 3*I*c + 7/2) + 7*I*a*e^(I*d*x + I*c + 7/2))*
e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 21*(I*sqrt(2)*a*e^(
7/2) + I*sqrt(2)*a*e^(6*I*d*x + 6*I*c + 7/2) + 3*I*sqrt(2)*a*e^(4*I*d*x + 4
*I*c + 7/2) + 3*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c + 7/2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3
*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(7/2)*sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.186 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}+\frac{2ae(e\sec(c+dx))^{3/2}\sin(c+d}{3d}$$

```
[Out] 2/5*I*a*(e*sec(d*x+c))^(5/2)/d+2/3*a*e*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/
3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3853, 3856, 2720}

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}+\frac{2ae\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d) + (((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + (2*a*e*(e*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + a\int (e\sec(c+dx))^{5/2}\,dx$$
$$= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}(\ldots$$
$$= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\Big(\ldots$$
$$= \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2ia(e\,\mathrm{s}\ldots}{\ldots}$$

Mathematica [A]
time = 0.46, size = 57, normalized size = 0.61

$$\frac{a(e\sec(c+dx))^{5/2}\left(6i+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(e*Sec[c + d*x])^(5/2)*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/
2, 2] + 5*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A]
time = 0.46, size = 192, normalized size = 2.04

method	result
default	$\frac{2a(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^3(dx+c))+5i\sqrt{\frac{\ldots}{1+\mathrm{c}\ldots}}\right.}{15d\sin(dx+c)^4\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(5*I*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*
cos(d*x+c)^3+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

```
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^2+5*sin(d*x+c)*cos(d*
x+c)+3*I)*(e/cos(d*x+c))^(5/2)/sin(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((I*a*tan(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 143, normalized size = 1.52

$$-\frac{2\left(\frac{\sqrt{2}\left(-5i\,ae^{\frac{5}{2}}+5i\,ae^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}-12i\,ae^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+5\left(i\sqrt{2}\,ae^{\frac{5}{2}}+i\sqrt{2}\,ae^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2i\sqrt{2}\,ae^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-5*I*a*e^(5/2) + 5*I*a*e^(4*I*d*x + 4*I*c + 5/2) - 12*I*a*e
^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c)
+ 1) + 5*(I*sqrt(2)*a*e^(5/2) + I*sqrt(2)*a*e^(4*I*d*x + 4*I*c + 5/2) + 2*I
*sqrt(2)*a*e^(2*I*d*x + 2*I*c + 5/2))*weierstrassPInverse(-4, 0, e^(I*d*x +
 I*c)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(e\sec{(c+dx)})^{\frac{5}{2}}\right)\,dx+\int(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**(5/2), x) + Integral((e*sec(c + d*x))**(
5/2)*tan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(5/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}+\frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] 2/3*I*a*(e*sec(d*x+c))^(3/2)/d-2*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)+2*a*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3853, 3856, 2719}

$$-\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}+\frac{2ae\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*a*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + (((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + (2*a*e*Sqrt[e*Sec[c + d*x]
]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + a\int (e\sec(c+dx))^{3/2}\,dx$$
$$= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{d} - (ae^2$$
$$= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{(}{\sqrt{c}}$$
$$= -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.82, size = 102, normalized size = 1.13

$$\frac{2aee^{-2idx}\sqrt{e\sec(c+dx)}\,(\cos(c+3dx)+i\sin(c+3dx))\left(-2i+i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[e*Sec[c + d*x]]*(Cos[c + 3*d*x] + I*Sin[c + 3*d*x])*(-2*I + I*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))] + Tan[c + d*x]))/(3*d*E^((2*I)*d*x))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(105) = 210.
time = 0.52, size = 351, normalized size = 3.90

method	result
default	$-\frac{2a(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(3i(\cos^2(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)^2*(1/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*sin(d*x+c)-3*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+
c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*sin(d*x+c)+3*cos(d*x+c)^2-3*cos(d*x+c))*(
e/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 113, normalized size = 1.26

$$-\frac{2\left(\frac{\sqrt{2}\left(3i\,ae^{\left(3i\,dx+3i\,c+\frac{3}{2}\right)}+i\,ae^{\left(i\,dx+i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+3\left(i\sqrt{2}\,ae^{\frac{3}{2}}+i\sqrt{2}\,ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(3*I*a*e^(3*I*d*x + 3*I*c + 3/2) + I*a*e^(I*d*x + I*c + 3/2))
*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 3*(I*sqrt(2)*a*e^(
3/2) + I*sqrt(2)*a*e^(2*I*d*x + 2*I*c + 3/2))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(e\sec\left(c+dx\right))^{\frac{3}{2}}\right)dx+\int(e\sec\left(c+dx\right))^{\frac{3}{2}}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**(
3/2)*tan(c + d*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.188 $\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{2ia\sqrt{e\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}$$

```
[Out] 2*I*a*(e*sec(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))
^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {3567, 3856, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))\,dx = \frac{2ia\sqrt{e\sec(c+dx)}}{d} + a\int \sqrt{e\sec(c+dx)}\,dx$$
$$= \frac{2ia\sqrt{e\sec(c+dx)}}{d} + \left(a\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int -$$
$$= \frac{2ia\sqrt{e\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec}}{d}$$

Mathematica [A]
time = 0.22, size = 44, normalized size = 0.73

$$\frac{2a\left(i+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*(I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d*x]
])/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(79) = 158.
time = 0.92, size = 164, normalized size = 2.73

method	result
default	$\frac{2ia\sqrt{\frac{e}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{d\sin(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*a/d*(e/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(EllipticF(
I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1)/sin(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 55, normalized size = 0.92

$$-\frac{2\left(i\sqrt{2}\,ae^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)-\frac{i\sqrt{2}\,ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{1}{2}\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -2*(I*sqrt(2)*a*e^(1/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) - I*sqr
t(2)*a*e^(1/2*I*d*x + 1/2*I*c + 1/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt{e\sec\left(c+dx\right)}\right)dx+\int\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*t
an(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [B]
time = 3.78, size = 40, normalized size = 0.67

$$\frac{2\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+1\mathrm{i}\right)\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
[Out] (2*a*(cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2) + 1i)*(e/cos(c + d*x))
^(1/2))/d
```

3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] -2*I*a/d/(e*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3567, 3856, 2719}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sq
rt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + a\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx$$
$$= -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + \frac{a\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.34, size = 73, normalized size = 1.22

$$-\frac{4iae^{2i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (((-4*I)/3)*a*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(79) = 158.
time = 0.59, size = 910, normalized size = 15.17

method	result
risch	$-\frac{2ia\sqrt{2}}{d\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}} + \frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*(-1+cos(d*x+c))*(4*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*EllipticF(I*(-1+cos(d*x+
c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2-4*I*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*Elliptic
E(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2+8*I*(1/(1+cos(d*x
```

```
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2
)^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-8*I
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1
+cos(d*x+c))^2)^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*
cos(d*x+c)+4*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),
I)*sin(d*x+c)-4*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+
c),I)*sin(d*x+c)-4*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*sin(d*x+c)*cos(d*
x+c)^2-4*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*sin(d*x+c)*cos(d*x+c)-4*(-c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^3-I*cos(d*x+c)*ln(-(2*(-cos(d*
x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+co
s(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+I*ln(-2*(2*(-co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)*sin(d*x+c)+
4*cos(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2))/(-cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)/sin(d*x+c)^3/cos(d*x+c)/(e/cos(d*x+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 26, normalized size = 0.43

$$\frac{2i\sqrt{2}\,ae^{\left(-\frac{1}{2}\right)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*I*sqrt(2)*a*e^(-1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\sqrt{e\sec\left(c+dx\right)}}\right)dx+\int\frac{\tan\left(c+dx\right)}{\sqrt{e\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] I*a*(Integral(-I/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*se
c(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2), x)
```

3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

```
[Out] -2/3*I*a/d/(e*sec(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(1/2)+2
/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2
```

Rubi [A]

time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3854, 3856, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*a*Sin[c + d*x])/(3*
d*e*Sqrt[e*Sec[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+a\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx\\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}+\frac{a\int \sqrt{e\sec(c+dx)}\,dx}{3e^2}\\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}+\frac{\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right.}{3e^2}\\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\end{aligned}$$

Mathematica [A]
time = 0.40, size = 62, normalized size = 0.65

$$\frac{2a\left(-i\cos(c+dx)+\frac{F\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}}+\sin(c+dx)\right)}{3de\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*((-I)*Cos[c + d*x] + EllipticF[(c + d*x)/2, 2]/Sqrt[Cos[c + d*x]] + Si
n[c + d*x]))/(3*d*e*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.48, size = 170, normalized size = 1.77

method	result
default	$\frac{2a\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{Elliptic}}{3d\cos(dx+c)^2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}}$

risch	$-\frac{i e^{i(dx+c)} a \sqrt{2}}{3de\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} + \frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\right.}{3d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a/d*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)
-I*cos(d*x+c)^2+sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^2/(e/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 68, normalized size = 0.71

$$\frac{\left(-2i\sqrt{2}\,a\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*sqrt(2)*a*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(
-I*a*e^(2*I*d*x + 2*I*c) - I*a)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2
*I*c) + 1))*e^(-3/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/5*I*a/d/(e*sec(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(3/2)+6
/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3854, 3856, 2719}

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + (6*a*EllipticE[(c + d*x)/2, 2])
/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*a*Sin[c + d*x])/(5*
d*e*(e*Sec[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+a\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}+\frac{(3a)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}+\frac{(3a)\int \sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+}$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a\sin(c+d}{5de(e\sec(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.78, size = 99, normalized size = 1.03

$$-\frac{a\left(2+2\cos(2(c+dx))-2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3i\sin(2(c+dx))\right)(-i+\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/5*(a*(2 + 2*Cos[2*(c + d*x)] - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - (3*I)*Sin[2*(c + d*x)])*(-I
 + Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(107) = 214.
time = 0.46, size = 339, normalized size = 3.53

method	result

risch	$-\frac{i\left(e^{2i(dx+c)}+7\right)a\sqrt{2}}{10d\,e^{2}\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}}{}\right.}{}$
default	$-\frac{2a\left(3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+}{1+\cos(da}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a/d*(3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*El
lipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-3*I*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c
))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*
x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*sin(d*x+c)*cos(d*x+c)^3+cos
(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/(e/cos(d*x+c))^(5/2)/sin(d*x+c)/cos(
d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 100, normalized size = 1.04

$$\frac{\left(12i\sqrt{2}\,ae^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(-i\,ae^{(4i\,dx+4i\,c)}+4i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-i\,dx-i\,c-\frac{5}{2}\right)}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*I*sqrt(2)*a*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) + 4*I*a*e
```

```
^(2*I*d*x + 2*I*c) + 5*I*a)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c
) + 1))*e^(-I*d*x - I*c - 5/2)/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\right)dx+\int\frac{\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(5/2),x)
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(5/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2), x)
```

3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}+\frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}+\frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}+\frac{10a\,s}{21de^3\sqrt{}}$$

```
[Out] -2/7*I*a/d/(e*sec(d*x+c))^(7/2)+2/7*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(5/2)+1
0/21*a*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+10/21*a*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3567, 3854, 3856, 2720}

$$\frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}+\frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}+\frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + (10*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*d*e^4) + (2*a*Sin[c + d*x])/(
7*d*e*(e*Sec[c + d*x])^(5/2)) + (10*a*Sin[c + d*x])/(21*d*e^3*Sqrt[e*Sec[c
+ d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + a\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{(5a)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} + \frac{(5a)}{}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} + \frac{(5a\sqrt{}}{}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{10a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4} +$$

Mathematica [A]

time = 0.74, size = 121, normalized size = 0.97

$$\frac{a\sqrt{e\sec(c+dx)}\,(\cos(c+dx)+i\sin(c+dx))\left(-14i\cos(c+dx)+2i\cos(3(c+dx))+20\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c+dx))+5\sin(c+dx)+5\sin(3(c+dx))\right)}{42de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a*Sqrt[e*Sec[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*((-14*I)*Cos[c + d*
x] + (2*I)*Cos[3*(c + d*x)] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 5*Sin[c + d*x] + 5*Sin[3*(c + d*x)]))/
(42*d*e^4)
```

Maple [A]

time = 0.48, size = 187, normalized size = 1.50

method	result

default	$\frac{2a\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-3i\left(\cos^4(dx+c)\right)+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos}{1+\cos}}\right.}{21d\cos(dx+c)^4\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*a/d*(5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*cos(d*x+c)^4+5*I*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)+3*sin(d*x+c)*cos(d*x+c)^3+5*sin(d*x+c)*cos(d*x+c))/c
os(d*x+c)^4/(e/cos(d*x+c))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((I*a*tan(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 109, normalized size = 0.87

$$\frac{\left(-40i\sqrt{2}\,ae^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-3i\,ae^{(6i\,dx+6i\,c)}-19i\,ae^{(4i\,dx+4i\,c)}-9i\,ae^{(2i\,dx+2i\,c)}+7i\,a\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-2i\,dx-2i\,c-\frac{7}{2}\right)}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(-40*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^(I*d
*x + I*c)) + sqrt(2)*(-3*I*a*e^(6*I*d*x + 6*I*c) - 19*I*a*e^(4*I*d*x + 4*I*
c) - 9*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I
*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c - 7/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\right)dx+\int\frac{\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(7/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\tan(c + d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2), x)
```

3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2i(e\sec(c$$

```
[Out] 14/15*I*a^2*(e*sec(d*x+c))^(3/2)/d-14/5*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(e*sec(d*x+c))^(1/2)+14/5*a^2*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/5*I*
(e*sec(d*x+c))^(3/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3567, 3853, 3856, 2719}

$$-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-14*a^2*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((14*I)/15)*a^2*(e*Sec[c + d*x])^(3/2))/d + (14*a^2*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2
+ I*a^2*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
```

```
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx &= \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}(7a)\int (e\sec(\\ &= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c\right.}{5d}\\ &= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d}\\ &= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d}\\ &= -\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/}}{15d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.57, size = 267, normalized size = 1.93

$$\frac{(e\sec(c+dx))^{3/2}\left(-\frac{14i\sqrt{2}\left(3\sqrt{1+e^{2i(c+dx)}}-e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}}+\frac{1}{2}\csc(c)\sec^{\frac{5}{2}}(c+dx)(\cos(2c)-i\sin(2c))(36\cos(dx)+27\cos(2c+dx)+21\cos(2c+3dx)-20i\sin(dx)+20i\sin(2c+dx))\right)(a+ia\tan(c+dx))^2}{15d\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(3/2)*(((-14*I)*Sqrt[2]*(3*Sqrt[1 + E^((2*I)*(c + d*x))]
- E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
```

```
I)*(c + d*x))]))/((-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Csc[c]*Sec[c + d*x]^(5/2)*(Cos[
2*c] - I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*d*x] -
 (20*I)*Sin[d*x] + (20*I)*Sin[2*c + d*x]))/2)*(a + I*a*Tan[c + d*x])^2)/(15
*d*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(143) = 286$.
time = 0.49, size = 374, normalized size = 2.71

method	result
default	$\frac{2a^2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(21i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(21*I*cos(d*x+c)^3*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c)
)/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(
d*x+c)+21*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+
c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I
*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*cos(d*x+c)^3+10*I*cos(d*x+c)*s
in(d*x+c)+24*cos(d*x+c)^2-3)*(e/cos(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 166, normalized size = 1.20

$$-\frac{2\left(\frac{\sqrt{2}\left(21i\,a^2e^{\left(5i\,dx+5i\,c+\frac{3}{2}\right)}+16i\,a^2e^{\left(3i\,dx+3i\,c+\frac{3}{2}\right)}+7i\,a^2e^{\left(i\,dx+i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+21\left(i\sqrt{2}\,a^2e^{\frac{3}{2}}+i\sqrt{2}\,a^2e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+2i\sqrt{2}\,a^2e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(21*I*a^2*e^(5*I*d*x + 5*I*c + 3/2) + 16*I*a^2*e^(3*I*d*x +
3*I*c + 3/2) + 7*I*a^2*e^(I*d*x + I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(
e^(2*I*d*x + 2*I*c) + 1) + 21*(I*sqrt(2)*a^2*e^(3/2) + I*sqrt(2)*a^2*e^(4*I
*d*x + 4*I*c + 3/2) + 2*I*sqrt(2)*a^2*e^(2*I*d*x + 2*I*c + 3/2))*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(4*I*d*x +
4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-(e\sec(c+dx))^{\frac{3}{2}}\right)dx+\int(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\,dx+\int\left(-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(-(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**(
3/2)*tan(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**(3/2)*tan(c + d*
x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.194 $\int \sqrt{e \sec(c+dx)}\,(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2i\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}{3d}$$

```
[Out] 10/3*I*a^2*(e*sec(d*x+c))^(1/2)/d+10/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*s
ec(d*x+c))^(1/2)/d+2/3*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3579, 3567, 3856, 2720}

$$\frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((10*I)/3)*a^2*Sqrt[e*Sec[c + d*x]])/d + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (((2*I)/3)*Sqrt[e*Sec[c +
 d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2\,dx = \frac{2i\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}{3d} + \frac{1}{3}(5a)\int \sqrt{e\sec(}$$
$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+}{3d}$$
$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+}{3d}$$
$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [A]
time = 0.56, size = 67, normalized size = 0.63

$$\frac{2a^2(e\sec(c+dx))^{3/2}\left(6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\sin(c+dx)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
[Out] (2*a^2*(e*Sec[c + d*x])^(3/2)*((6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*El
lipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*d*e)
```

Maple [A]
time = 0.49, size = 201, normalized size = 1.90

method	result
default	$\frac{2a^2\sqrt{\frac{e}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^2(dx+c))+5i\sqrt{\frac{}{1}}\right.}{3d\cos(dx+c)\sin(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 2/3*a^2/d*(e/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(5*I*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c
```

```
),I)*cos(d*x+c)^2+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+6*I*cos(d*x+c)-s
in(d*x+c))*(1+cos(d*x+c))^2/cos(d*x+c)/sin(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 110, normalized size = 1.04

$$-\frac{2\left(\frac{\sqrt{2}\left(-5i\,a^2e^{\frac{1}{2}}-7i\,a^2e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+5\left(i\,\sqrt{2}\,a^2e^{\frac{1}{2}}+i\,\sqrt{2}\,a^2e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-5*I*a^2*e^(1/2) - 7*I*a^2*e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2
*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 5*(I*sqrt(2)*a^2*e^(1/2)
+ I*sqrt(2)*a^2*e^(2*I*d*x + 2*I*c + 1/2))*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\sqrt{e\sec(c+dx)}\right)dx+\int\sqrt{e\sec(c+dx)}\tan^2(c+dx)\,dx+\int\left(-2i\sqrt{e\sec(c+dx)}\tan(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(-sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*t
an(c + d*x)**2, x) + Integral(-2*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] 6*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*a^2*sin(d*x+c)*(
e*sec(d*x+c))^(1/2)/d/e-4*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3853, 3856, 2719}

$$-\frac{6a^2\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}+\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (6*a^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) - (6*a^2*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) - ((4*I)*(a^2 + I*a^2*
Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx &= -\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}-\frac{(3a^2)\int(e\sec(c+dx))^{3/2}\,dx}{e^2}\\ &= -\frac{6a^2\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}+(3a^2)\int\frac{}{\sqrt{e}}\\ &= -\frac{6a^2\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}+\frac{(3a^2)\int\sqrt{}}{\sqrt{\cos(c+dx}}\\ &= \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{6a^2\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}-\frac{4i(a^2}{d\sqrt{}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.01, size = 132, normalized size = 1.23

$$-\frac{2i\sqrt{2}\,a^2e^{2i(c+dx)}\left(-\sqrt{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{d\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\left(1+e^{2i(c+dx)}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^2*E^((2*I)*(c + d*x))*(-Sqrt[1 + E^((2*I)*(c + d*x))] + (
1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))]))/(d*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(123) = 246.
time = 0.51, size = 1099, normalized size = 10.27

method	result	size
default	Expression too large to display	1099

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1+cos(d*x+c))*(12*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(1+cos(d*x
+c))^2)^(3/2)+6*I*cos(d*x+c)^3*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)+12*I*cos(d*x+c)^2*sin(d*x+c)*EllipticF(I*(-1+
cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*cos(d*x+c)^5*(-cos(d*x+c)/
(1+cos(d*x+c))^2)^(3/2)-12*I*cos(d*x+c)^2*sin(d*x+c)*EllipticE(I*(-1+cos(d*
x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+4*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(
d*x+c)/(1+cos(d*x+c))^2)^(3/2)+I*ln(-2*(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1
/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(
d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)^2*sin(d*x+c)+6*cos(d*x+c)^4*(-cos(d*x+c)
/(1+cos(d*x+c))^2)^(3/2)+4*I*cos(d*x+c)*sin(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+
c))^2)^(3/2)+6*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(
1/2)*cos(d*x+c)*sin(d*x+c)-6*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)*EllipticE(I*
(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-4*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos
(d*x+c))^2)^(3/2)-I*cos(d*x+c)^2*ln(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2
)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*
x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-6*I*cos(d*x+c)^3*sin(d*x+c)*EllipticE(I*(-
1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+12*I*cos(d*x+c)^3*sin(d*x+
c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-8*cos(d*x+c)^2*(-cos(d*x+c)/(1+cos(
d*x+c))^2)^(3/2)+2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2))/(1+cos(d*x+c))^2/c
os(d*x+c)/sin(d*x+c)^3/(e/cos(d*x+c))^(1/2)/(-cos(d*x+c)/(1+cos(d*x+c))^2)^
(3/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 61, normalized size = 0.57

$$-\frac{2\left(-3i\sqrt{2}\,a^2\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)-\frac{i\sqrt{2}\,a^2e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-3*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I
*d*x + I*c))) - I*sqrt(2)*a^2*e^(3/2*I*d*x + 3/2*I*c)/sqrt(e^(2*I*d*x + 2*I
*c) + 1))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{\sqrt{e\sec(c+dx)}}\right)dx+\int\frac{\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int\left(-\frac{2i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] -a**2*(Integral(-1/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**2/sqrt
(e*sec(c + d*x)), x) + Integral(-2*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+a\tan(c+d\,x)\ 1\text{i})^2}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2), x)
```

3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*(a^2+
I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3577, 3856, 2720}

$$-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2
))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

`EqQ[n^2, 1/4]`

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}} - \frac{a^2\int \sqrt{e\sec(c+dx)}\,dx}{3e^2}$$

$$= -\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{3e^2}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4i(a^2+ia^2\tan(c+}{3d(e\sec(c+dx))^3}$$

Mathematica [A]
time = 0.43, size = 114, normalized size = 1.34

$$-\frac{2a^2\sec^2(c+dx)\left(2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c+dx))\right)(\cos(c+3dx)+i\sin(c+3dx))}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^2*((2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]))*(Cos[c + 3*d*x] + I*Sin[c +
 3*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.47, size = 173, normalized size = 2.04

method	result
default	$-\frac{2a^2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{Elli}\right.}{3d\cos(dx+c)^2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}}$
risch	$-\frac{2ie^{i(dx+c)}a^2\sqrt{2}}{3de\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)}\right.}{3d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx}}{e^{2i(dx+}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^2/d*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/(1+cos(d*x+c)))^(1/
```

```
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)+2*I*cos(d*x+c)^2-2*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^2/(e/cos(d*x+c))^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 74, normalized size = 0.87

$$-\frac{2\left(-i\sqrt{2}\,a^2\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\,\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/3*(-I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*
(I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x
 + 2*I*c) + 1))*e^(-3/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(3/2),x)
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2), x)
```

3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] 2/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/5*I*(a^2+I
*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3577, 3856, 2719}

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2)
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}+\frac{a^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$
$$= -\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}+\frac{a^2\int\sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.96, size = 114, normalized size = 1.34

$$-\frac{i\sqrt{2}\,a^2\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(3\sqrt{1+e^{2i(c+dx)}}+2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{15de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
[Out] ((-1/15*I)*Sqrt[2]*a^2*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(3/2
)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(3*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*e^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(99) = 198.
time = 0.45, size = 343, normalized size = 4.04

method	result
risch	$-\frac{i(e^{2i(dx+c)}+2)a^2\sqrt{2}}{5d\,e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}}{\cdots}\right.}{\cdots}$
default	$-\frac{2a^2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx}{1+\cos(d}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a^2/d*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*Ellip
ticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+2*I*sin(d*x+c)*cos(d*x+c)^3+I*(1/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(
-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+2*cos(d
*x+c)^4-cos(d*x+c)^2-cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)/(e/cos(d*x+c))^(5/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 86, normalized size = 1.01

$$\frac{\left(2i\sqrt{2}\,a^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(-i\,a^2e^{(3i\,dx+3i\,c)}-i\,a^2e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{5}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I
*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(3*I*d*x + 3*I*c) - I*a^2*e^(I*d*x + I*c)
)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(5/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2), x)
```

3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4}+\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*a^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x
+c))^(7/2)
```

Rubi [A]

time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2720}

$$\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4}+\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*d*e^4) + (2*a^2*Sin[c + d*x])/(7*d*e^3*Sqrt[e*Sec[c + d*x]]) - (((4*I)/7)
*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}} + \frac{(3a^2)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$
$$= \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}} + \frac{a^2\int\sqrt{e\sec(c+dx)}\,dx}{7e^4}$$
$$= \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}} + \frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec\right.}{}$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4} + \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}$$

Mathematica [A]
time = 0.81, size = 133, normalized size = 1.15

$$\frac{a^2\sqrt{e\sec(c+dx)}\left(-2i-2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))-i\sin(2(c+dx)))-\sin(2(c+dx))\right)(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{7de^4(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-2*I - (2*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) - Si
n[2*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(7*d*e^4*(Cos[
d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.90, size = 188, normalized size = 1.62

method	result

default	$\frac{2a^2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-2i\left(\cos^4(dx+c)\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(}{1+\cos}}\right.}{7d\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\cos(dx+c)^4}$
risch	$-\frac{ie^{i(dx+c)}\left(e^{2i(dx+c)}+3\right)a^2\sqrt{2}}{14d\,e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c}\right.\right.}{7d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+1\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a^2/d*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos
(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-2*I*cos(d*x+c)^4+I*(1/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*
x+c))/sin(d*x+c),I)+2*sin(d*x+c)*cos(d*x+c)^3+sin(d*x+c)*cos(d*x+c))/(e/cos
(d*x+c))^(7/2)/cos(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((I*a*tan(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 0.76

$$\frac{\left(-4i\sqrt{2}\,a^2\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-4i\,a^2e^{(2i\,dx+2i\,c)}-3i\,a^2\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{7}{2}\right)}}{14\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/14*(-4*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2
)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c) - 3*I*a^2)*e^(1
/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\right)dx+\int\frac{\tan^2\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\left(-\frac{2i\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(7/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
7/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2), x)
```

3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(3/2)+2/3*a^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/cos(
d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x
+c))^(9/2)
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2719}

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(3*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Sec[c + d*x])^(3/2)) - (((4*I)/
9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx &= -\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{(5a^2)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}\\ &= \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{3e^4}\\ &= \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int\sqrt{\cos(c+dx)}\ldots}{3e^4\sqrt{\cos(c+dx)}\sqrt{e\sec\ldots}}\\ &= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan\ldots}{9d(e\sec(c\ldots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.76, size = 133, normalized size = 1.15

$$\frac{ia^2\left(9-4e^{2i(c+dx)}-e^{4i(c+dx)}-\frac{8e^{2i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)}{18\sqrt{2}\,de^4\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((I/18)*a^2*(9 - 4*E^((2*I)*(c + d*x)) - E^((4*I)*(c + d*x)) - (8*E^((2*I)*
(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 +
 E^((2*I)*(c + d*x))]))/(Sqrt[2]*d*e^4*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(126) = 252.
time = 0.53, size = 351, normalized size = 3.03

method	result
risch	$-\frac{i(e^{4i(dx+c)}+4\,e^{2i(dx+c)}+15)a^2\sqrt{2}}{36d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx\cdots\right.}}}{\cdots}\right.}{\cdots}$
default	$\frac{2a^2\left(-2i(\cos^5(dx+c))\sin(dx+c)+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-3i\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*a^2/d*(-2*I*cos(d*x+c)^5*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d
*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-2*cos(
d*x+c)^6+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c)
)/sin(d*x+c),I)+cos(d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))/cos(d*x+c)^5/sin(
d*x+c)/(e/cos(d*x+c))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((I*a*tan(d*x + c) + a)^2/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 122, normalized size = 1.05

$$\frac{\left(24i\sqrt{2}\,a^2e^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}+5i\,a^2e^{(2i\,dx+2i\,c)}+9i\,a^2\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-i\,dx-i\,c-\frac{9}{2}\right)}}{36\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/36*(24*I*sqrt(2)*a^2*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I
*a^2*e^(4*I*d*x + 4*I*c) + 5*I*a^2*e^(2*I*d*x + 2*I*c) + 9*I*a^2)*e^(1/2*I*
d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c - 9/2)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-9/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2), x)
```

3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=147

$$\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] 2/11*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(5/2)+10/33*a^2*sin(d*x+c)/d/e^5/(
e*sec(d*x+c))^(1/2)+10/33*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1
/2)/d/e^6-4/11*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(11/2)
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2720}

$$\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(33*d*e^6) + (2*a^2*Sin[c + d*x])/(11*d*e^3*(e*Sec[c + d*x])^(5/2)) + (10*a
^2*Sin[c + d*x])/(33*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)/11)*(a^2 + I*a^2
*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx &= -\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}+\frac{(7a^2)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\ &= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}+\frac{(5a^2)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11e^4}\\ &= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\\ &= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\\ &= \frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5}}\end{aligned}$$

Mathematica [A]
time = 1.21, size = 155, normalized size = 1.05

$$\frac{a^2\sqrt{e\sec(c+dx)}\left(-28i-24i\cos(2(c+dx))+4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))-i\sin(2(c+dx)))-6\sin(2(c+dx))+7\sin(4(c+dx))\right)(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{132de^6(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-28*I - (24*I)*Cos[2*(c + d*x)] + (4*I)*Cos[4*(c
 + d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)
] - I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)])*(Cos[2*(
c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(132*d*e^6*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.58, size = 205, normalized size = 1.39

method	result
default	$-\frac{2a^2\left(6i\left(\cos^6(dx+c)\right)-6\sin(dx+c)\left(\cos^5(dx+c)\right)-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\right.}{33d\cos(dx+c)^6\left(\frac{}{co}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/33*a^2/d*(6*I*cos(d*x+c)^6-6*sin(d*x+c)*cos(d*x+c)^5-5*I*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(
d*x+c))/sin(d*x+c),I)-5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*sin(d*x+c)*cos(d*x+c)
^3-5*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((I*a*tan(d*x + c) + a)^2/sec(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 133, normalized size = 0.90

$$\frac{\left(-80i\sqrt{2}\,a^2e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\,\left(-3i\,a^2e^{(8i\,dx+8i\,c)}-18i\,a^2e^{(6i\,dx+6i\,c)}-56i\,a^2e^{(4i\,dx+4i\,c)}-30i\,a^2e^{(2i\,dx+2i\,c)}+11i\,a^2\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-2i\,dx-2i\,c-\frac{11}{2}\right)}}{264\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/264*(-80*I*sqrt(2)*a^2*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)) + sqrt(2)*(-3*I*a^2*e^(8*I*d*x + 8*I*c) - 18*I*a^2*e^(6*I*d*x
 + 6*I*c) - 56*I*a^2*e^(4*I*d*x + 4*I*c) - 30*I*a^2*e^(2*I*d*x + 2*I*c) + 1
1*I*a^2)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x
 - 2*I*c - 11/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(11/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2*e^(-11/2)/sec(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2), x)
```

3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=202

$$-\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^3e(e\sec(c$$

```
[Out] 10/21*I*a^3*(e*sec(d*x+c))^(7/2)/d+2/3*a^3*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c
)/d-2*a^3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2*a^3*e^3*
sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/11*I*a*(e*sec(d*x+c))^(7/2)*(a+I*a*tan(
d*x+c))^2/d+10/33*I*(e*sec(d*x+c))^(7/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3567, 3853, 3856, 2719}

$$-\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^3e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e\sin(c+dx)(e\sec(c+dx))^{5/2}}{3d}+\frac{10i(a^3+ia^3\tan(c+dx))(e\sec(c+dx))^{7/2}}{33d}+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*e^4*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((10*I)/21)*a^3*(e*Sec[c + d*x])^(7/2))/d + (2*a^3*e^3*Sqrt[e*Se
c[c + d*x]]*Sin[c + d*x])/d + (2*a^3*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d) + (((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d +
 (((10*I)/33)*(e*Sec[c + d*x])^(7/2)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx &= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d}+\frac{1}{11}(15a)\int (e\,\mathrm{se}\\
&= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d}+\frac{10i(e\sec(c+d}{}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c}{11d}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{d}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{d}\\
&= -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.92, size = 442, normalized size = 2.19

$$
\frac{2i\sqrt{2}\,e^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3}{3d\left(-1+e^{2ic}\right)\sec^{\frac{13}{2}}(c+dx)(\cos(dx)+i\sin(dx))^3}+\frac{\cos^6(c+dx)(e\sec(c+dx))^{7/2}\left(\sec^5(c+dx)\left(-\frac{2}{11}i\cos(3c)-\frac{2}{11}\sin(3c)\right)+\cos(dx)\csc(c)(2\cos(3c)-2i\sin(3c))+\sec(c)\sec^3(c+dx)(12\cos(c)+7i\sin(c))\left(\frac{2}{21}i\cos(3c)+\frac{2}{21}\sin(3c)\right)+\sec(c)\sec^2(c+dx)\left(\frac{2}{3}\cos(3c)-\frac{2}{3}i\sin(3c)\right)\sin(dx)+\sec(c)\sec^4(c+dx)\left(-\frac{2}{3}\cos(3c)+\frac{2}{3}i\sin(3c)\right)\sin(dx)+\sec(c+dx)\left(\frac{2}{3}\cos(3c)-\frac{2}{3}i\sin(3c)\right)\tan(c)\right)(a+ia\tan(c+dx))^3}{d(\cos(dx)+i\sin(dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/3)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(e*
Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3)/(d*E^(I*(2*c + d*x))*(-1 + E^
((2*I)*c))*Sec[c + d*x]^(13/2)*(Cos[d*x] + I*Sin[d*x])^3) + (Cos[c + d*x]^6
*(e*Sec[c + d*x])^(7/2)*(Sec[c + d*x]^5*(((-2*I)/11)*Cos[3*c] - (2*Sin[3*c]
)/11) + Cos[d*x]*Csc[c]*(2*Cos[3*c] - (2*I)*Sin[3*c]) + Sec[c]*Sec[c + d*x]
^3*(12*Cos[c] + (7*I)*Sin[c])*(((2*I)/21)*Cos[3*c] + (2*Sin[3*c])/21) + Sec
[c]*Sec[c + d*x]^2*((2*Cos[3*c])/3 - ((2*I)/3)*Sin[3*c])*Sin[d*x] + Sec[c]*
Sec[c + d*x]^4*((-2*Cos[3*c])/3 + ((2*I)/3)*Sin[3*c])*Sin[d*x] + Sec[c + d*
x]*((2*Cos[3*c])/3 - ((2*I)/3)*Sin[3*c])*Tan[c])*(a + I*a*Tan[c + d*x])^3)/
(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.61, size = 402, normalized size = 1.99

method	result
default	$\frac{2a^3(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(231i\sqrt{\frac{1}{1+\cos(dx+c)}}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ (\cos^6(dx+c))\ \mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(231*I*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^6*EllipticE(I*(-1+cos(d*x+
c))/sin(d*x+c),I)*sin(d*x+c)-231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^6*s
in(d*x+c)+231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)^5*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-231*I*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^5*sin(d*x+c)-231*cos(d*x+c)^6+154*cos(d*x
+c)^5+132*I*sin(d*x+c)*cos(d*x+c)^2+154*cos(d*x+c)^3-21*I*sin(d*x+c)-77*cos
(d*x+c))*(e/cos(d*x+c))^(7/2)/sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((I*a*tan(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 301, normalized size = 1.49

$$-\frac{2\left(\frac{\sqrt{2}\left(231i\,a^{3}e^{\left(11i\,dx+11i\,c+\frac{7}{2}\right)}+1309i\,a^{3}e^{\left(9i\,dx+9i\,c+\frac{7}{2}\right)}+946i\,a^{3}e^{\left(7i\,dx+7i\,c+\frac{7}{2}\right)}+870i\,a^{3}e^{\left(5i\,dx+5i\,c+\frac{7}{2}\right)}+407i\,a^{3}e^{\left(3i\,dx+3i\,c+\frac{7}{2}\right)}+77i\,a^{3}e^{\left(i\,dx+i\,c+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+231\left(i\sqrt{2}\,a^{3}e^{\frac{7}{2}}+i\sqrt{2}\,a^{3}e^{\left(10i\,dx+10i\,c+\frac{7}{2}\right)}+5i\sqrt{2}\,a^{3}e^{\left(8i\,dx+8i\,c+\frac{7}{2}\right)}+10i\sqrt{2}\,a^{3}e^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)}+10i\sqrt{2}\,a^{3}e^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+5i\sqrt{2}\,a^{3}e^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{231\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/231*(sqrt(2)*(231*I*a^3*e^(11*I*d*x + 11*I*c + 7/2) + 1309*I*a^3*e^(9*I*
d*x + 9*I*c + 7/2) + 946*I*a^3*e^(7*I*d*x + 7*I*c + 7/2) + 870*I*a^3*e^(5*I
*d*x + 5*I*c + 7/2) + 407*I*a^3*e^(3*I*d*x + 3*I*c + 7/2) + 77*I*a^3*e^(I*d
*x + I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 23
1*(I*sqrt(2)*a^3*e^(7/2) + I*sqrt(2)*a^3*e^(10*I*d*x + 10*I*c + 7/2) + 5*I*
sqrt(2)*a^3*e^(8*I*d*x + 8*I*c + 7/2) + 10*I*sqrt(2)*a^3*e^(6*I*d*x + 6*I*c
 + 7/2) + 10*I*sqrt(2)*a^3*e^(4*I*d*x + 4*I*c + 7/2) + 5*I*sqrt(2)*a^3*e^(2
*I*d*x + 2*I*c + 7/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c))))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*
e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) +
d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(7/2)*sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=175

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d}+\frac{26ia^3(e\sec(c+dx))^{5/2}}{35d}+\frac{26a^3e(e\sec(c+dx))^{3/2}\sin(\ldots}{21d}$$

```
[Out] 26/35*I*a^3*(e*sec(d*x+c))^(5/2)/d+26/21*a^3*e*(e*sec(d*x+c))^(3/2)*sin(d*x
+c)/d+26/21*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d+2/9*
I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^2/d+26/63*I*(e*sec(d*x+c))^(5/2
)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3567, 3853, 3856, 2720}

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d}+\frac{26ia^3(e\sec(c+dx))^{5/2}}{35d}+\frac{26a^3e\sin(c+dx)(e\sec(c+dx))^{3/2}}{21d}+\frac{26i(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{5/2}}{63d}+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (26*a^3*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(21*d) + (((26*I)/35)*a^3*(e*Sec[c + d*x])^(5/2))/d + (26*a^3*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*
(a + I*a*Tan[c + d*x])^2)/d + (((26*I)/63)*(e*Sec[c + d*x])^(5/2)*(a^3 + I*
a^3*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{1}{9}(13a)\int (e\sec$$

$$= \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e\sec(c+d$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c}{9d}$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d}$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d}$$

$$= \frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26}{}$$

Mathematica [A]
time = 1.22, size = 89, normalized size = 0.51

$$\frac{a^3\sec^2(c+dx)(e\sec(c+dx))^{5/2}\left(728i+1008i\cos(2(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-150\sin(2(c+dx))+195\sin(4(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(728*I + (1008*I)*Cos[2*(c + d*x
)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2] - 150*Sin[2*(c + d*x
)] + 195*Sin[4*(c + d*x)]))/(1260*d)
```

Maple [A]
time = 0.55, size = 229, normalized size = 1.31

method	result
default	$\frac{2a^3(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(195i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^5(dx+c))+195i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(195*I*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c
),I)*cos(d*x+c)^5+195*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^4+195*sin(d*x+
c)*cos(d*x+c)^3+252*I*cos(d*x+c)^2-135*sin(d*x+c)*cos(d*x+c)-35*I)*(e/cos(d
*x+c))^(5/2)/cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((I*a*tan(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 245, normalized size = 1.40

$$-\frac{2\left(\frac{\sqrt{2}\left(-195i\,a^3e^{\frac{5}{2}}+195i\,a^3e^{\left(8i\,dx+8i\,c+\frac{5}{2}\right)}-1158i\,a^3e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}-1456i\,a^3e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}-858i\,a^3e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+195\left(i\sqrt{2}\,a^3e^{\frac{5}{2}}+i\sqrt{2}\,a^3e^{\left(8i\,dx+8i\,c+\frac{5}{2}\right)}+4i\sqrt{2}\,a^3e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}+6i\sqrt{2}\,a^3e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+4i\sqrt{2}\,a^3e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(-195*I*a^3*e^(5/2) + 195*I*a^3*e^(8*I*d*x + 8*I*c + 5/2) -
 1158*I*a^3*e^(6*I*d*x + 6*I*c + 5/2) - 1456*I*a^3*e^(4*I*d*x + 4*I*c + 5/2
) - 858*I*a^3*e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*
I*d*x + 2*I*c) + 1) + 195*(I*sqrt(2)*a^3*e^(5/2) + I*sqrt(2)*a^3*e^(8*I*d*x
 + 8*I*c + 5/2) + 4*I*sqrt(2)*a^3*e^(6*I*d*x + 6*I*c + 5/2) + 6*I*sqrt(2)*a
^3*e^(4*I*d*x + 4*I*c + 5/2) + 4*I*sqrt(2)*a^3*e^(2*I*d*x + 2*I*c + 5/2))*w
```

```
eierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^
(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(5/2)*sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.203 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2ia(e\sec(c$$

```
[Out] 22/15*I*a^3*(e*sec(d*x+c))^(3/2)/d-22/5*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(e*sec(d*x+c))^(1/2)+22/5*a^3*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/7*I*
a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2/d+22/35*I*(e*sec(d*x+c))^(3/2)*
(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3567, 3853, 3856, 2719}

$$-\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{22i(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{35d}+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*a^3*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/15)*a^3*(e*Sec[c + d*x])^(3/2))/d + (22*a^3*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a
+ I*a*Tan[c + d*x])^2)/d + (((22*I)/35)*(e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3
*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx &= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d}+\frac{1}{7}(11a)\int (e\sec \\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d}+\frac{22i(e\sec(c+d \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c}{7d} \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d} \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5d} \\
&= -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/}}{15d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.31, size = 129, normalized size = 0.74

$$\frac{a^3(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))\left(-116i-308i\cos(2(c+dx))+77ie^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+77\sec(c+dx)\sin(3(c+dx))+17\tan(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])*(-116*I - (308*I)*Cos[2*(c
 + d*x)] + ((77*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 77*Sec[c + d*x]*Sin[3
*(c + d*x)] + 17*Tan[c + d*x]))/(210*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(174) = 348.
time = 0.54, size = 392, normalized size = 2.24

method	result
default	$-\frac{2a^3(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(231i(\cos^4(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(231*I*sin(d*x+c)*cos(d*x+c
)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*
(-1+cos(d*x+c))/sin(d*x+c),I)-231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin
(d*x+c),I)+231*I*sin(d*x+c)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-231*I*si
n(d*x+c)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-140*I*sin(d*x+c)*cos(d*x+c)
^2+231*cos(d*x+c)^4-294*cos(d*x+c)^3+15*I*sin(d*x+c)+63*cos(d*x+c))*(e/cos(
d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 211, normalized size = 1.21

$$-\frac{2\left(\frac{\sqrt{2}\left(231i\,a^3e^{\left(7i\,dx+7i\,c+\frac{3}{2}\right)}+287i\,a^3e^{\left(5i\,dx+5i\,c+\frac{3}{2}\right)}+253i\,a^3e^{\left(3i\,dx+3i\,c+\frac{3}{2}\right)}+77i\,a^3e^{\left(i\,dx+i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+231\left(i\sqrt{2}\,a^3e^{\frac{3}{2}}+i\sqrt{2}\,a^3e^{\left(6i\,dx+6i\,c+\frac{3}{2}\right)}+3i\sqrt{2}\,a^3e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+3i\sqrt{2}\,a^3e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{105\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(sqrt(2)*(231*I*a^3*e^(7*I*d*x + 7*I*c + 3/2) + 287*I*a^3*e^(5*I*d*x
 + 5*I*c + 3/2) + 253*I*a^3*e^(3*I*d*x + 3*I*c + 3/2) + 77*I*a^3*e^(I*d*x +
 I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 231*(I
*sqrt(2)*a^3*e^(3/2) + I*sqrt(2)*a^3*e^(6*I*d*x + 6*I*c + 3/2) + 3*I*sqrt(2
)*a^3*e^(4*I*d*x + 4*I*c + 3/2) + 3*I*sqrt(2)*a^3*e^(2*I*d*x + 2*I*c + 3/2)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e
^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i(e\sec(c+dx))^{\frac{3}{2}}\,dx+\int\left(-3(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)dx+\int(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)\,dx+\int\left(-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(e*sec(c + d*x))**(3/2), x) + Integral(-3*(e*sec(c + d*
x))**(3/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)
**3, x) + Integral(-3*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.204 $\int \sqrt{e \sec(c+dx)}\,(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=139

$$\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c$$

```
[Out] 6*I*a^3*(e*sec(d*x+c))^(1/2)/d+6*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x
+c))^(1/2)/d+2/5*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2/d+6/5*I*(e*s
ec(d*x+c))^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3579, 3567, 3856, 2720}

$$\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6i(a^3+ia^3\tan(c+dx))\sqrt{e\sec(c+dx)}}{5d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((6*I)*a^3*Sqrt[e*Sec[c + d*x]])/d + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d + (((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(
a + I*a*Tan[c + d*x])^2)/d + (((6*I)/5)*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*T
an[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
```

```
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3\,dx = \frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2}{5d}+\frac{1}{5}(9a)\int \sqrt{e\sec(}$$

$$= \frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2}{5d}+\frac{6i\sqrt{e\sec(c+dx}}{}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+d}{5d}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+d}{5d}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}$$

Mathematica [A]
time = 1.08, size = 79, normalized size = 0.57

$$\frac{a^3\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(18i+20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-5\sin(2(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(18*I + (20*I)*Cos[2*(c + d*x)] +
30*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*d
)
```

Maple [A]
time = 0.57, size = 213, normalized size = 1.53

method	result

default	$\frac{2a^3(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^3(dx+c))+15i\sqrt{\ldots}\right.}{5d\sin(dx\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(15*I*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I
)*cos(d*x+c)^3+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^2+20*I*cos(d*x+c)^
2-5*sin(d*x+c)*cos(d*x+c)-I)*(e/cos(d*x+c))^(1/2)/sin(d*x+c)^4/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 155, normalized size = 1.12

$$-\frac{2\left(\frac{\sqrt{2}\left(-15ia^3e^{\frac{1}{2}}-25ia^3e^{\left(4idx+4ic+\frac{1}{2}\right)}-36ia^3e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+15\left(i\sqrt{2}\,a^3e^{\frac{1}{2}}+i\sqrt{2}\,a^3e^{\left(4idx+4ic+\frac{1}{2}\right)}+2i\sqrt{2}\,a^3e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)\text{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)}{5\left(de^{(4idx+4ic)}+2\,de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(2)*(-15*I*a^3*e^(1/2) - 25*I*a^3*e^(4*I*d*x + 4*I*c + 1/2) - 36*
I*a^3*e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x +
2*I*c) + 1) + 15*(I*sqrt(2)*a^3*e^(1/2) + I*sqrt(2)*a^3*e^(4*I*d*x + 4*I*c
+ 1/2) + 2*I*sqrt(2)*a^3*e^(2*I*d*x + 2*I*c + 1/2))*weierstrassPInverse(-4,
 0, e^(I*d*x + I*c)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int i\sqrt{e\sec(c+dx)}\,dx+\int\left(-3\sqrt{e\sec(c+dx)}\,\tan(c+dx)\right)dx+\int\sqrt{e\sec(c+dx)}\,\tan^3(c+dx)\,dx+\int\left(-3i\sqrt{e\sec(c+dx)}\,\tan^2(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(e*sec(c + d*x)), x) + Integral(-3*sqrt(e*sec(c + d
*x))*tan(c + d*x), x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**3, x) +
 Integral(-3*I*sqrt(e*sec(c + d*x))*tan(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{26ia^3}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^3\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}-\frac{2ia^3\tan^2(c+dx)}{3d\sqrt{e\sec(c+dx)}}$$

```
[Out] -26/3*I*a^3/d/(e*sec(d*x+c))^(1/2)+14*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)-6*a^3*tan(d*x+c)/d/(e*sec(d*x+c))^(1/2)-2/3*I*a^3*tan(d*x
+c)^2/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3577, 3853, 3856, 2719}

$$-\frac{14a^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x
]]) - (14*a^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) + (((2*I)/3)*a*(a +
I*a*Tan[c + d*x])^2)/(d*Sqrt[e*Sec[c + d*x]]) - (((28*I)/3)*(a^3 + I*a^3*Ta
n[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}+\frac{1}{3}(7a)\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx\\ &= \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}}-\frac{(7a^3)\int(e\sec(c+dx}{e^2}\\ &= -\frac{14a^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i(a^3+ia}{3d\sqrt{e\,se}}\\ &= -\frac{14a^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i(a^3+ia}{3d\sqrt{e\,se}}\\ &= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{14a^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{de}+\frac{2ia(}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.57, size = 101, normalized size = 0.81

$$\frac{2a^3\sqrt{e\sec(c+dx)}\,(\cos(c)+i\sin(c))(-i\cos(dx)+\sin(dx))\left(-8+7\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-i\tan(c+dx)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (2*a^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*((-I)*Cos[d*x] + Sin[d*x])*
(-8 + 7*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] - I*Tan[c + d*x]))/(3*d*e)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1563 vs. 2(134) = 268.
time = 0.56, size = 1564, normalized size = 12.61

method	result	size
default	Expression too large to display	1564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^3/d*(-1+cos(d*x+c))^2*(-I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*
x+c)+21*I*sin(d*x+c)*cos(d*x+c)^5*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)
*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-126*I*sin(d*x+c)*cos(d*x+c)^3*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15*I*sin(d*x+c)*cos(d*x+c)^2*(-cos(
d*x+c)/(1+cos(d*x+c))^2)^(3/2)+126*I*sin(d*x+c)*cos(d*x+c)^3*EllipticF(I*(-
1+cos(d*x+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*I*sin(d*x+c)*cos(d*x+c)
*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*sin(d*x
+c)*cos(d*x+c)^3*ln(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-c
os(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+
c)^2)-84*I*sin(d*x+c)*cos(d*x+c)^4*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I
)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+84*I*sin(d*x+c)*cos(d*x+c)^4*EllipticF(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x
+c)/(1+cos(d*x+c))^2)^(3/2)+84*I*sin(d*x+c)*cos(d*x+c)^2*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-12*I*sin(d*x+c)*cos(d*x+c)^5*(
-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-12*cos(d*x+c)^6*(-cos(d*x+c)/(1+cos(d*x
+c))^2)^(3/2)+21*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/
2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(
-1+cos(d*x+c))/sin(d*x+c),I)-15*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)-37*I*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+18
*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-3*I*sin(d*x+c)*cos(d*x+c
)^3*ln(-2*(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2
-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)+24*co
s(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-36*I*sin(d*x+c)*cos(d*x+c)^
```

```
4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-21*I*sin(d*x+c)*cos(d*x+c)^5*Ellipti
cE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-84*I*sin(d*x+c)*cos
(d*x+c)^2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-6*
cos(d*x+c)^2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-9*cos(d*x+c)*(-cos(d*x+c)
/(1+cos(d*x+c))^2)^(3/2))/(1+cos(d*x+c))/(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/
2)/sin(d*x+c)^5/cos(d*x+c)^2/(e/cos(d*x+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 120, normalized size = 0.97

$$-\frac{2\left(\frac{\sqrt{2}\left(-9i\,a^3e^{(3i\,dx+3i\,c)}-7i\,a^3e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+21\left(-i\sqrt{2}\,a^3e^{(2i\,dx+2i\,c)}-i\sqrt{2}\,a^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{3\left(de^{\frac{1}{2}}+de^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-9*I*a^3*e^(3*I*d*x + 3*I*c) - 7*I*a^3*e^(I*d*x + I*c))*e^(1
/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 21*(-I*sqrt(2)*a^3*e^(2
*I*d*x + 2*I*c) - I*sqrt(2)*a^3)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))))/(d*e^(1/2) + d*e^(2*I*d*x + 2*I*c + 1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{\sqrt{e\sec\left(c+dx\right)}}\,dx+\int\left(-\frac{3\tan\left(c+dx\right)}{\sqrt{e\sec\left(c+dx\right)}}\right)dx+\int\frac{\tan^3\left(c+dx\right)}{\sqrt{e\sec\left(c+dx\right)}}\,dx+\int\left(-\frac{3i\tan^2\left(c+dx\right)}{\sqrt{e\sec\left(c+dx\right)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] -I*a**3*(Integral(I/sqrt(e*sec(c + d*x)), x) + Integral(-3*tan(c + d*x)/sqr
t(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**3/sqrt(e*sec(c + d*x)), x) +
 Integral(-3*I*tan(c + d*x)**2/sqrt(e*sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2), x)
```

3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -10/3*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^2-10/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c))^(
3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3567, 3856, 2720}

$$-\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-10*I)/3)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^3*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) - (((4*I)/3)*a*(
a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
```

```
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{(5a^2)\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))\,dx}{3e^2}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{(5a^3)\int \sqrt{e\sec(c+dx)}}{3e^2}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(5a^3\sqrt{\cos(c+dx)}\,\sqrt{e}\right.}{}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}$$

Mathematica [A]
time = 0.55, size = 123, normalized size = 1.11

$$-\frac{2a^3\sec^2(c+dx)\left(7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c+dx))+3\sin(c+dx)\right)(\cos(c+4dx)+i\sin(c+4dx))}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^2*((7*I)*Cos[c + d*x] + 5*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 3*Sin[c + d*x])*(Cos[c +
 4*d*x] + I*Sin[c + 4*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[
d*x])^3)
```

Maple [A]
time = 0.53, size = 175, normalized size = 1.58

method	result

default	$\frac{2a^3\left(-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{El}\right.}{3d\cos(dx+c)^2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^3/d*(-5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*
x+c),I)-4*I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-3*I)/cos(d*x+c)^2/(e/cos(d
*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 74, normalized size = 0.67

$$-\frac{2\left(-5i\sqrt{2}\,a^3\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\,(2i\,a^3e^{(2i\,dx+2i\,c)}+5i\,a^3)e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{3}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(-5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2
)*(2*I*a^3*e^(2*I*d*x + 2*I*c) + 5*I*a^3)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2
*I*d*x + 2*I*c) + 1))*e^(-3/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{3\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{3i\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(3/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(3
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2), x)
```

3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=111

$$\frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] 6/5*I*a^3/d/e^2/(e*sec(d*x+c))^(1/2)-6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1
/2)/(e*sec(d*x+c))^(1/2)-4/5*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c))^(5/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3567, 3856, 2719}

$$\frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*a^3)/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (6*a^3*EllipticE[(c + d*x)/2
, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) - (((4*I)/5)*a*(a +
 I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
```

```
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{(3a^2)\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{(3a^3)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{(3a^3)\int \sqrt{\cos(c+dx)}\ldots}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(\ldots}}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(\ldots}{5d(e\sec(c+\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.28, size = 108, normalized size = 0.97

$$-\frac{4ia^3e^{2i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^3*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/
(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(120) = 240.
time = 0.55, size = 1086, normalized size = 9.78

method	result
risch	$-\frac{2i\left(e^{2i(dx+c)}-3\right)a^{3}\sqrt{2}}{5d\,e^{2}\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^3/d*(1+cos(d*x+c))*(-1+cos(d*x+c))^2*(-20*I*sin(d*x+c)*cos(d*x+c)*(
-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+24*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)
/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*ln(-2*(2*(-cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)*sin(d*x+c)+16*I*si
n(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-5*I*cos(d*x+c)*l
n(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+1
2*I*cos(d*x+c)^2*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1
+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)-20*I*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(1+cos(d*x+c))^2)
^(1/2)+12*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*
sin(d*x+c)+16*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-24*I*sin(d*
x+c)*cos(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)+16*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+16*I*sin(d*x+c)*co
s(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-12*I*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*sin(d*x+c)-12*I*cos(d*x+c)^2*sin(d
*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)-28*(-c
os(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^3-16*(-cos(d*x+c)/(1+cos(d*x+c
))^2)^(1/2)*cos(d*x+c)^2+12*cos(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)
)/cos(d*x+c)^3/sin(d*x+c)^5/(e/cos(d*x+c))^(5/2)/(-cos(d*x+c)/(1+cos(d*x+c)
)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 86, normalized size = 0.77

$$-\frac{2\left(3i\sqrt{2}\,a^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(i\,a^3e^{(3i\,dx+3i\,c)}+i\,a^3e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{5}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2/5*(3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(
I*d*x + I*c))) + sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + I*a^3*e^(I*d*x + I*c)
)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{3\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{3i\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(5/2),x)
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(5/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(5
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2), x)
```

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-2/7*I*(a+I
*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)-4/21*I*(a^3+I*a^3*tan(d*x+c))/d/e^2
/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3578, 3577, 3856, 2720}

$$-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)
) - (((4*I)/21)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
```

```
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}+\frac{a\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}-\frac{a^3\int\sqrt{e\sec(c+dx)}\,d}{21e^4}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}-\frac{\left(a^3\sqrt{\cos(c+dx)}\,\sqrt{e}\right.}{}\\ &= -\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2i(a+ia\tan(c+dx)}{7d(e\sec(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.82, size = 133, normalized size = 1.07

$$-\frac{a^3\sqrt{e\sec(c+dx)}\left(5i+5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))-i\sin(2(c+dx))-\sin(2(c+dx)))\right)(\cos(2c+5dx)+i\sin(2c+5dx))}{21de^4(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] -1/21*(a^3*Sqrt[e*Sec[c + d*x]]*(5*I + (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])
 - Sin[2*(c + d*x)])*(Cos[2*c + 5*d*x] + I*Sin[2*c + 5*d*x]))/(d*e^4*(Cos[d
*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.50, size = 199, normalized size = 1.60

method	result

default	$-\frac{2a^3\left(12i(\cos^4(dx+c))+i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\cdots}\right.}{21d\cos(dx+c)^4\left(\frac{e}{\cos(d\cdots}\right.}$
risch	$-\frac{ie^{i(dx+c)}\left(3\,e^{2i(dx+c)}+2\right)a^3\sqrt{2}}{21d\,e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(e^{i(d\cdots}\right.\right.}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*a^3/d*(12*I*cos(d*x+c)^4+I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)-12*sin(d*x+c)*cos(d*x+c)^3-7*I*cos(d*x+c)^2+sin(d*x+
c)*cos(d*x+c))/cos(d*x+c)^4/(e/cos(d*x+c))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 0.71

$$\frac{\left(2i\sqrt{2}\,a^3\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-3i\,a^3e^{(4i\,dx+4i\,c)}-5i\,a^3e^{(2i\,dx+2i\,c)}-2i\,a^3\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{7}{2}\right)}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)
*(-3*I*a^3*e^(4*I*d*x + 4*I*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c) - 2*I*a^3)*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^3\left(\int\frac{i}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\left(-\frac{3\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\right)dx+\int\frac{\tan^3\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx+\int\left(-\frac{3i\tan^2\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(7/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(7
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(7/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2), x)
```

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=124

$$\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

```
[Out] 2/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*
a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)-4/15*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/
(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3578, 3577, 3856, 2719}

$$\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^3*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2))
 - (((4*I)/15)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{a\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int \frac{1}{\sqrt{e\sec(c+dx)}}\,d}{15e^4}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int \sqrt{\cos(c+}}{15e^4\sqrt{\cos(c+dx)}\,\sqrt{}}\\ &= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3}{15de^2(e\sec}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.49, size = 118, normalized size = 0.95

$$-\frac{a^3e^{-2i(c+dx)}\left(11+16e^{2i(c+dx)}+5e^{4i(c+dx)}+4\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(-i+\tan(c+dx))^3}{90de^2(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
[Out] -1/90*(a^3*(11 + 16*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 4*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*(-I + Tan[c + d*x])^3)/(d*e^2*E^((2*I)*(c + d*x))*(e*Sec[c + d*x])^(5/2
))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(132) = 264.
time = 0.56, size = 370, normalized size = 2.98

method	result
risch	$-\frac{i(5\,\mathrm{e}^{4i(dx+c)}+11\,\mathrm{e}^{2i(dx+c)}+12)a^3\sqrt{2}}{90d\,e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}} + \frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i(\mathrm{e}}}{}\right.}{}$
default	$\frac{2a^3\left(-20i(\cos^5(dx+c))\sin(dx+c)+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-3i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)

[Out] 2/45*a^3/d*(-20*I*sin(d*x+c)*cos(d*x+c)^5+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-20*cos(d*x+c)^6+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*I*sin(d*x+c)*cos(d*x+c)^3+19*cos(d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^5/(e/cos(d*x+c))^(9/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")

[Out] e^(-9/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 100, normalized size = 0.81

$$\frac{\left(12i\sqrt{2}\,a^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(-5i\,a^3e^{(5i\,dx+5i\,c)}-16i\,a^3e^{(3i\,dx+3i\,c)}-11i\,a^3e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{9}{2}\right)}}{90\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")

[Out] 1/90*(12*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^3*e^(5*I*d*x + 5*I*c) - 16*I*a^3*e^(3*I*d

```
*x + 3*I*c) - 11*I*a^3*e^(I*d*x + I*c))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I
*d*x + 2*I*c) + 1))*e^(-9/2)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-9/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^3}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2), x)
```

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=155

$$\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}+\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20}{7}$$

```
[Out] 10/77*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)+10/77*a^3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-2/11*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(
d*x+c))^(11/2)-20/77*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2)
```

Rubi [A]
time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3578, 3577, 3854, 3856, 2720}

$$\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}+\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) + (10*a^3*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((2*I
)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (((20*I)/77)*
(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}+\frac{(5a)\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}+\frac{(15a^3)\int\frac{1}{(e\sec(c+dx))^{3/2}}}{77e^4}\\
&= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}\\
&= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}\\
&= \frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}+\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 1.15, size = 148, normalized size = 0.95

$$
\frac{a^3\sqrt{e\sec(c+dx)}\left(-46i\cos(c+dx)-22i\cos(3(c+dx))-15\sin(c+dx)+20\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(3(c+dx))-i\sin(3(c+dx)))-15\sin(3(c+dx))\right)(\cos(3(c+2dx))+i\sin(3(c+2dx)))}{154de^6(\cos(dx)+i\sin(dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-46*I)*Cos[c + d*x] - (22*I)*Cos[3*(c + d*x)] -
 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(
c + d*x)] - I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x)]
+ I*Sin[3*(c + 2*d*x)]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.98, size = 216, normalized size = 1.39

method	result
default	$\frac{2a^3\left(-28i\left(\cos^6(dx+c)\right)+28\sin(dx+c)\left(\cos^5(dx+c)\right)+11i\left(\cos^4(dx+c)\right)+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\mathrm{Elliptic}}{77d\cos(}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(7\,\mathrm{e}^{4i(dx+c)}+24\,\mathrm{e}^{2i(dx+c)}+37\right)a^3\sqrt{2}}{308d\,e^5\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{Elliptic}}{77d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77*a^3/d*(-28*I*cos(d*x+c)^6+28*sin(d*x+c)*cos(d*x+c)^5+11*I*cos(d*x+c)^4
+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*sin(d*x+c)*cos(d*x+c)^3+5*I*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)+5*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))
^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 102, normalized size = 0.66

$$\frac{\left(-40i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-7i\,a^3e^{(6i\,dx+6i\,c)}-31i\,a^3e^{(4i\,dx+4i\,c)}-61i\,a^3e^{(2i\,dx+2i\,c)}-37i\,a^3\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{11}{2}\right)}}{308\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/308*(-40*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt
(2)*(-7*I*a^3*e^(6*I*d*x + 6*I*c) - 31*I*a^3*e^(4*I*d*x + 4*I*c) - 61*I*a^3
*e^(2*I*d*x + 2*I*c) - 37*I*a^3)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x +
2*I*c) + 1))*e^(-11/2)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(11/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-11/2)/sec(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2), x)
```

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=155

$$\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{28i(a^3+ia^3\tan}{117de^2(e\sec(c-}$$

```
[Out] 14/117*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+14/39*a^3*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6
/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/13*I*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(13/2)-28/117*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9/2)
```

Rubi [A]

time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3578, 3577, 3854, 3856, 2719}

$$\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*a^3*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((
2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((28*I)/1
17)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} + \frac{(7a)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}$$

$$= -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} + \frac{(35a^3)\int \frac{1}{(e\sec(c+dx))^{5/2}}}{117e^4}$$

$$= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx)}{117de^2(e\sec(c+dx))^{9/}}$$

$$= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx)}{117de^2(e\sec(c+dx))^{9/}}$$

$$= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\,\mathrm{t}}{13d(e\sec(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.50, size = 155, normalized size = 1.00

$$-\frac{a^3e^{-4i(c+dx)}\left(-117-34e^{2i(c+dx)}+124e^{4i(c+dx)}+50e^{6i(c+dx)}+9e^{8i(c+dx)}+112e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(-i+\tan(c+dx))^3}{936de^4(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] -1/936*(a^3*(-117 - 34*E^((2*I)*(c + d*x)) + 124*E^((4*I)*(c + d*x)) + 50*E
^((6*I)*(c + d*x)) + 9*E^((8*I)*(c + d*x)) + 112*E^((2*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*(-I + Tan[c + d*x])^3)/(d*e^4*E^((4*I)*(c + d*x))*(e*Sec[c + d*x])^(5/
2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(159) = 318.
time = 0.68, size = 380, normalized size = 2.45

method	result
risch	$-\frac{i\left(9\,e^{6i(dx+c)}+41\,e^{4i(dx+c)}+83\,e^{2i(dx+c)}+219\right)a^{3}\sqrt{2}}{936d\,e^{6}\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{7i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right.}}{}\right.}{}$
default	$-\frac{2a^{3}\left(36i\sin(dx+c)\left(\cos^{7}(dx+c)\right)+36\left(\cos^{8}(dx+c)\right)-13i\left(\cos^{5}(dx+c)\right)\sin(dx+c)+21i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/117*a^3/d*(36*I*sin(d*x+c)*cos(d*x+c)^7+36*cos(d*x+c)^8-13*I*sin(d*x+c)*
cos(d*x+c)^5+21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(
d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-31*cos(d*x+c)^6+21*I*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1
+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*cos(
d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^7/(e/cos(d*x+
c))^(13/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] e^(-13/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 136, normalized size = 0.88

$$\frac{\left(336i\,\sqrt{2}\,a^{3}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(-9i\,a^{3}e^{(8i\,dx+8i\,c)}-50i\,a^{3}e^{(6i\,dx+6i\,c)}-124i\,a^{3}e^{(4i\,dx+4i\,c)}+34i\,a^{3}e^{(2i\,dx+2i\,c)}+117i\,a^{3}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-i\,dx-i\,c-\frac{13}{2}\right)}}{936\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/936*(336*I*sqrt(2)*a^3*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^3*e^(8*I*d*x + 8*I*c) -
 50*I*a^3*e^(6*I*d*x + 6*I*c) - 124*I*a^3*e^(4*I*d*x + 4*I*c) + 34*I*a^3*e^
(2*I*d*x + 2*I*c) + 117*I*a^3)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*
I*c) + 1))*e^(-I*d*x - I*c - 13/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-13/2)/sec(d*x + c)^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2), x)
```

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11de^8}+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}-\frac{2i(a}{15d}$$

```
[Out] 6/55*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/11*a^3*sin(d*x+c)/d/e^7/(e
*sec(d*x+c))^(1/2)+2/11*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2
)/d/e^8-2/15*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-12/55*I*(a^3+I*
a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

Rubi [A]

time = 0.15, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3578, 3577, 3854, 3856, 2720}

$$\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11de^8}+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
11*d*e^8) + (6*a^3*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^3
*Sin[c + d*x])/(11*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((2*I)/15)*(a + I*a*Tan[c
 + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((12*I)/55)*(a^3 + I*a^3*Tan[c +
 d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} + \frac{(3a)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}$$

$$= -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} + \frac{(21a^3)\int \frac{1}{(e\sec(c+dx))^{7/2}}$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} -$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} -$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11de^8} + \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}$$

Mathematica [A]
time = 1.71, size = 170, normalized size = 0.91

$$\frac{a^3\sqrt{e\sec(c+dx)}\left(-332i\cos(c+dx)-154i\cos(3(c+dx))+22i\cos(5(c+dx))-114\sin(c+dx)+240\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(3(c+dx))-i\sin(3(c+dx)))-81\sin(3(c+dx))+33\sin(5(c+dx))\right)(\cos(3(c+2dx))+i\sin(3(c+2dx)))}{1320de^8(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-332*I)*Cos[c + d*x] - (154*I)*Cos[3*(c + d*x)]
 + (22*I)*Cos[5*(c + d*x)] - 114*Sin[c + d*x] + 240*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 81*Sin[3*(c
 + d*x)] + 33*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*x)])
)/(1320*d*e^8*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.70, size = 232, normalized size = 1.25

method	result
default	$\frac{2a^3\left(-44i\left(\cos^8(dx+c)\right)+44\sin(dx+c)\left(\cos^7(dx+c)\right)+15i\left(\cos^6(dx+c)\right)+7\sin(dx+c)\left(\cos^5(dx+c)\right)+15i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/165*a^3/d*(-44*I*cos(d*x+c)^8+44*sin(d*x+c)*cos(d*x+c)^7+15*I*cos(d*x+c)^
6+7*sin(d*x+c)*cos(d*x+c)^5+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*sin
(d*x+c)*cos(d*x+c)^3+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+15*sin(d*x+c)*cos(d*x+c
))/cos(d*x+c)^8/(e/cos(d*x+c))^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
```

```
[Out] e^(-15/2)*integrate((I*a*tan(d*x + c) + a)^3/sec(d*x + c)^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 147, normalized size = 0.79

$$\frac{\left(-480i\sqrt{2}\,a^3e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-11i\,a^3e^{(10i\,dx+10i\,c)}-73i\,a^3e^{(8i\,dx+8i\,c)}-218i\,a^3e^{(6i\,dx+6i\,c)}-446i\,a^3e^{(4i\,dx+4i\,c)}-235i\,a^3e^{(2i\,dx+2i\,c)}+55i\,a^3\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-2i\,dx-2i\,c-\frac{15}{2}\right)}}{2640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/2640*(-480*I*sqrt(2)*a^3*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(2)*(-11*I*a^3*e^(10*I*d*x + 10*I*c) - 73*I*a^3*e^(8*
I*d*x + 8*I*c) - 218*I*a^3*e^(6*I*d*x + 6*I*c) - 446*I*a^3*e^(4*I*d*x + 4*I
*c) - 235*I*a^3*e^(2*I*d*x + 2*I*c) + 55*I*a^3)*e^(1/2*I*d*x + 1/2*I*c)/sqr
t(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c - 15/2)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(15/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*e^(-15/2)/sec(d*x + c)^(15/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{15/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2), x)
```

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=215

$$-\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2ia(e\sec$$

```
[Out] 22/9*I*a^4*(e*sec(d*x+c))^(3/2)/d-22/3*a^4*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2
)/(e*sec(d*x+c))^(1/2)+22/3*a^4*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/9*I*a
*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3/d+10/21*I*(e*sec(d*x+c))^(3/2)*(
a^2+I*a^2*tan(d*x+c))^2/d+22/21*I*(e*sec(d*x+c))^(3/2)*(a^4+I*a^4*tan(d*x+c
))/d
```

Rubi [A]
time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3579, 3567, 3853, 3856, 2719}

$$-\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sin(c+dx)\sqrt{e\sec(c+dx)}}{3d}+\frac{22i(a^4+ia^4\tan(c+dx))(e\sec(c+dx))^{3/2}}{21d}+\frac{10i(a^2+ia^2\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{21d}+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-22*a^4*e^2*EllipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/9)*a^4*(e*Sec[c + d*x])^(3/2))/d + (22*a^4*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/(3*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a +
 I*a*Tan[c + d*x])^3)/d + (((10*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((22*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c
 + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
```

```
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx &= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d}+\frac{1}{3}(5a)\int (e\sec(c\\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d}+\frac{10i(e\sec(c+dx)\\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d}+\frac{10i(e\sec(c+dx)\\
&= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+}{9d}\\
&= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{3d}+\\
&= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{3d}+\\
&= -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.89, size = 429, normalized size = 2.00

$$
\frac{22i\sqrt{2}\,e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4}{9d\,(-1+e^{2ic})\sec^{\frac{11}{2}}(c+dx)(\cos(dx)+i\sin(dx))^4}+\frac{\cos^5(c+dx)(e\sec(c+dx))^{3/2}\left(\sec(c)\sec^3(c+dx)(36\cos(c)+7i\sin(c))\left(-\frac{2}{63}i\cos(4c)-\frac{2}{63}\sin(4c)\right)+\cos(dx)\csc(c)\left(\frac{22}{3}\cos(4c)-\frac{22}{3}i\sin(4c)\right)+\sec(c)\sec(c+dx)(24\cos(c)+13i\sin(c))\left(\frac{2}{9}i\cos(4c)+\frac{2}{9}\sin(4c)\right)+\sec(c)\sec^4(c+dx)\left(\frac{2}{9}\cos(4c)-\frac{2}{9}i\sin(4c)\right)\sin(dx)+\sec(c)\sec^2(c+dx)\left(-\frac{26}{9}\cos(4c)+\frac{26}{9}i\sin(4c)\right)\sin(dx)\right)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((22*I)/9)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1
+ E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(e
*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4)/(d*E^(I*(3*c + d*x))*(-1 + E
^((2*I)*c))*Sec[c + d*x]^(11/2)*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^
5*(e*Sec[c + d*x])^(3/2)*(Sec[c]*Sec[c + d*x]^3*(36*Cos[c] + (7*I)*Sin[c])*
(((-2*I)/63)*Cos[4*c] - (2*Sin[4*c])/63) + Cos[d*x]*Csc[c]*((22*Cos[4*c])/3
 - ((22*I)/3)*Sin[4*c]) + Sec[c]*Sec[c + d*x]*(24*Cos[c] + (13*I)*Sin[c])*(
((2*I)/9)*Cos[4*c] + (2*Sin[4*c])/9) + Sec[c]*Sec[c + d*x]^4*((2*Cos[4*c])/
9 - ((2*I)/9)*Sin[4*c])*Sin[d*x] + Sec[c]*Sec[c + d*x]^2*((-26*Cos[4*c])/9
+ ((26*I)/9)*Sin[4*c])*Sin[d*x])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I
*Sin[d*x])^4)
```

Maple [A]
time = 0.56, size = 401, normalized size = 1.87

method	result
default	$\frac{2a^4(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(231i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^5(dx+c))\,\text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(d\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/63*a^4/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(231*I*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*EllipticE(I*(-1+cos(d*x+c
))/sin(d*x+c),I)*sin(d*x+c)-231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^5*si
n(d*x+c)+231*I*cos(d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-231*I*cos(
d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+168*I*sin(d*x+c)*cos(d*x+c)^3
-231*cos(d*x+c)^5+322*cos(d*x+c)^4-36*I*cos(d*x+c)*sin(d*x+c)-98*cos(d*x+c)
^2+7)*(e/cos(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 256, normalized size = 1.19

$$-\frac{2\left(\frac{\sqrt{2}\left(231ia^4e^{\left(9idx+9ic+\frac{3}{2}\right)}+406ia^4e^{\left(7idx+7ic+\frac{3}{2}\right)}+540ia^4e^{\left(5idx+5ic+\frac{3}{2}\right)}+330ia^4e^{\left(3idx+3ic+\frac{3}{2}\right)}+77ia^4e^{\left(idx+ic+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+231\left(i\sqrt{2}a^4e^{\frac{3}{2}}+i\sqrt{2}a^4e^{\left(8idx+8ic+\frac{3}{2}\right)}+4i\sqrt{2}a^4e^{\left(6idx+6ic+\frac{3}{2}\right)}+6i\sqrt{2}a^4e^{\left(4idx+4ic+\frac{3}{2}\right)}+4i\sqrt{2}a^4e^{\left(2idx+2ic+\frac{3}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)\right)}{63\left(de^{(8idx+8ic)}+4de^{(6idx+6ic)}+6de^{(4idx+4ic)}+4de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/63*(sqrt(2)*(231*I*a^4*e^(9*I*d*x + 9*I*c + 3/2) + 406*I*a^4*e^(7*I*d*x
+ 7*I*c + 3/2) + 540*I*a^4*e^(5*I*d*x + 5*I*c + 3/2) + 330*I*a^4*e^(3*I*d*x
 + 3*I*c + 3/2) + 77*I*a^4*e^(I*d*x + I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/s
qrt(e^(2*I*d*x + 2*I*c) + 1) + 231*(I*sqrt(2)*a^4*e^(3/2) + I*sqrt(2)*a^4*e
^(8*I*d*x + 8*I*c + 3/2) + 4*I*sqrt(2)*a^4*e^(6*I*d*x + 6*I*c + 3/2) + 6*I*
sqrt(2)*a^4*e^(4*I*d*x + 4*I*c + 3/2) + 4*I*sqrt(2)*a^4*e^(2*I*d*x + 2*I*c
+ 3/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))
)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c
) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int(e\sec(c+dx))^{\frac{3}{2}}\,dx+\int\left(-6(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\right)dx+\int(e\sec(c+dx))^{\frac{3}{2}}\tan^4(c+dx)\,dx+\int 4i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\,dx+\int\left(-4i(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(3/2), x) + Integral(-6*(e*sec(c + d*x))**
(3/2)*tan(c + d*x)**2, x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)**
4, x) + Integral(4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x), x) + Integral(-4
*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

3.214 $\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=183

$$\frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan}{7d}$$

```
[Out] 78/7*I*a^4*(e*sec(d*x+c))^(1/2)/d+78/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*s
ec(d*x+c))^(1/2)/d+2/7*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3/d+26/3
5*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))^2/d+78/35*I*(e*sec(d*x+c))^
(1/2)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3579, 3567, 3856, 2720}

$$\frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{78i(a^4+ia^4\tan(c+dx))\sqrt{e\sec(c+dx)}}{35d}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7d}+\frac{26i(a^2+ia^2\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{35d}+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((78*I)/7)*a^4*Sqrt[e*Sec[c + d*x]])/d + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*d) + (((2*I)/7)*a*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (((26*I)/35)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x])^2)/d + (((78*I)/35)*Sqrt[e*Sec[c + d*x]]*(a^4 + I*
a^4*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^4\,dx &= \frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}{7d}+\frac{1}{7}(13a)\int \sqrt{e\sec\\
&= \frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}{7d}+\frac{26i\sqrt{e\sec(c+dx}\\
&= \frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}{7d}+\frac{26i\sqrt{e\sec(c+dx}\\
&= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+d}{7d}\\
&= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{2ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+d}{7d}\\
&= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}\end{aligned}$$

Mathematica [A]
time = 1.02, size = 101, normalized size = 0.55

$$\frac{a^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(728i+1008i\cos(2(c+dx))+280i\cos(4(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-150\sin(2(c+dx))-85\sin(4(c+dx))\right)}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(728*I + (1008*I)*Cos[2*(c + d*x)]
 + (280*I)*Cos[4*(c + d*x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2
, 2] - 150*Sin[2*(c + d*x)] - 85*Sin[4*(c + d*x)]))/(140*d)
```

Maple [A]
time = 0.58, size = 230, normalized size = 1.26

method	result
default	$-\frac{2a^4(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(-195i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^4(dx+c))-19\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*a^4/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(-195*I*EllipticF(I*(-1+cos(
d*x+c))/sin(d*x+c),I)*cos(d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)-195*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+
c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-280*I*cos(d
*x+c)^3+85*cos(d*x+c)^2*sin(d*x+c)+28*I*cos(d*x+c)-5*sin(d*x+c))*(e/cos(d*x
+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 200, normalized size = 1.09

$$-\frac{2\left(\frac{\sqrt{2}\left(-195i\,a^4e^{\frac{1}{2}}-365i\,a^4e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}-793i\,a^4e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}-663i\,a^4e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+195\left(i\sqrt{2}\,a^4e^{\frac{1}{2}}+i\sqrt{2}\,a^4e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}+3i\sqrt{2}\,a^4e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+3i\sqrt{2}\,a^4e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(-195*I*a^4*e^(1/2) - 365*I*a^4*e^(6*I*d*x + 6*I*c + 1/2) -
793*I*a^4*e^(4*I*d*x + 4*I*c + 1/2) - 663*I*a^4*e^(2*I*d*x + 2*I*c + 1/2))*
e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 195*(I*sqrt(2)*a^4*
e^(1/2) + I*sqrt(2)*a^4*e^(6*I*d*x + 6*I*c + 1/2) + 3*I*sqrt(2)*a^4*e^(4*I*
d*x + 4*I*c + 1/2) + 3*I*sqrt(2)*a^4*e^(2*I*d*x + 2*I*c + 1/2))*weierstrass
PInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\sqrt{e\sec(c+dx)}\,dx+\int\left(-6\sqrt{e\sec(c+dx)}\tan^2(c+dx)\right)dx+\int\sqrt{e\sec(c+dx)}\tan^4(c+dx)\,dx+\int 4i\sqrt{e\sec(c+dx)}\tan(c+dx)\,dx+\int\left(-4i\sqrt{e\sec(c+dx)}\tan^3(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-6*sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**4, x) + I
ntegral(4*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) + Integral(-4*I*sqrt(e*se
c(c + d*x))*tan(c + d*x)**3, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=178

$$\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2}-\frac{154a^4\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5de}-\frac{4ia(a+ia}{d\sqrt{e\,se}}$$

```
[Out] -154/15*I*a^4*(e*sec(d*x+c))^(3/2)/d/e^2+154/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^
(1/2)/(e*sec(d*x+c))^(1/2)-154/5*a^4*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e-4*
I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(1/2)-22/5*I*(e*sec(d*x+c))^(3/2)
*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

Rubi [A]
time = 0.13, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3577, 3579, 3567, 3853, 3856, 2719}

$$-\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2}-\frac{22i(a^4+ia^4\tan(c+dx))(e\sec(c+dx))^{3/2}}{5de^2}-\frac{154a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de}+\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (154*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((154*I)/15)*a^4*(e*Sec[c + d*x])^(3/2))/(d*e^2) - (154*a^4*Sqrt[
e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e) - ((4*I)*a*(a + I*a*Tan[c + d*x])^3)/
(d*Sqrt[e*Sec[c + d*x]]) - (((22*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*
Tan[c + d*x]))/(d*e^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
```

```
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}dx = -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{(11a^2)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2 d}{e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2} - (7$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}}{5d}$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\,t}{d\sqrt{e\sec}}$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\,t}{d\sqrt{e\sec}}$$

$$= \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - 154a^4\sqrt{e\sec(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.99, size = 123, normalized size = 0.69

$$-\frac{2ia^4e^{i(c+dx)}\left(-77-176e^{2i(c+dx)}-111e^{4i(c+dx)}+77\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{15de\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (((-2*I)/15)*a^4*E^(I*(c + d*x))*(-77 - 176*E^((2*I)*(c + d*x)) - 111*E^((4
*I)*(c + d*x)) + 77*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e*(1 + E^((2*I)*(
c + d*x)))^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(179) = 358.
time = 0.59, size = 1618, normalized size = 9.09

method	result	size
default	Expression too large to display	1618

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -2/15*a^4/d*(-1+cos(d*x+c))^3*(3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-120*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^6-360*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^5-380*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^4-180*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^3-30*I*cos(d*x+c)^4*ln(-2*(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+30*I*cos(d*x+c)^4*ln(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-60*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^2-20*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)-120*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^7+231*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+219*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+231*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-108*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-105*cos(d*x+c)^2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-129*cos(d*x+c)^6*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-1386*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+924*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*cos(d*x+c)*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-924*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+231*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2-231*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2-231*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+924*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-924*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1386*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/cos(d*x+c)^3/(e/cos(d*x+c))^(1/2)/(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 164, normalized size = 0.92

$$-\frac{2\left(\frac{\sqrt{2}\left(-111i\,a^4e^{(5i\,dx+5i\,c)}-176i\,a^4e^{(3i\,dx+3i\,c)}-77i\,a^4e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+231\left(-i\sqrt{2}\,a^4e^{(4i\,dx+4i\,c)}-2i\sqrt{2}\,a^4e^{(2i\,dx+2i\,c)}-i\sqrt{2}\,a^4\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{15\left(de^{\frac{1}{2}}+de^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-111*I*a^4*e^(5*I*d*x + 5*I*c) - 176*I*a^4*e^(3*I*d*x + 3*I
*c) - 77*I*a^4*e^(I*d*x + I*c))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2
*I*c) + 1) + 231*(-I*sqrt(2)*a^4*e^(4*I*d*x + 4*I*c) - 2*I*sqrt(2)*a^4*e^(2
*I*d*x + 2*I*c) - I*sqrt(2)*a^4)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))))/(d*e^(1/2) + d*e^(4*I*d*x + 4*I*c + 1/2) + 2*d*e
^(2*I*d*x + 2*I*c + 1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx+\int\frac{\tan^4(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int\frac{4i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a**4*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-6*tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**4/sqrt(e*sec(c + d*x)), x) +
 Integral(4*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x) + Integral(-4*I*tan(c +
 d*x)**3/sqrt(e*sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^4}{\sqrt{\frac{e}{\cos\left(c + d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2), x)
```

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{de^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-2$$

```
[Out] -10*I*a^4*(e*sec(d*x+c))^(1/2)/d/e^2-10*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*
sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(3/2)
-2*I*(e*sec(d*x+c))^(1/2)*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3577, 3579, 3567, 3856, 2720}

$$-\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{2i(a^4+ia^4\tan(c+dx))\sqrt{e\sec(c+dx)}}{de^2}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{de^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-10*I)*a^4*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^4*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - ((2*I)*Sqrt[e*Sec[c + d*x]]
*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
```

```
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-\frac{(3a^2)\int\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2\,dx}{e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}{de^2}-\frac{(5a\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{e\sec(c+dx)}\,(a^4}{de^2}\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{e\sec(c+dx)}\,(a^4}{de^2}\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+d}}{de^2}
\end{aligned}$$

Mathematica [A]
time = 0.96, size = 130, normalized size = 0.89

$$\frac{a^4\sec^3(c+dx)\left(21+19\cos(2(c+dx))-30i\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c+dx))-11i\sin(2(c+dx))\right)(-i\cos(c+5dx)+\sin(c+5dx))}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^3*(21 + 19*Cos[2*(c + d*x)] - (30*I)*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) - (11*I)*Sin[2*(c
+ d*x)])*((-I)*Cos[c + 5*d*x] + Sin[c + 5*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2
)*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A]
time = 0.55, size = 198, normalized size = 1.36

method	result
default	$\frac{2a^4\left(-15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^2(dx+c))-15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{3d\cos(dx+c)^3\left(\frac{e}{\cos(dx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^4/d*(-15*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-15*I*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)-8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(
d*x+c)+sin(d*x+c))/cos(d*x+c)^3/(e/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 122, normalized size = 0.84

$$-\frac{2\left(\frac{\sqrt{2}\,(4i\,a^4e^{(4i\,dx+4i\,c)}+21i\,a^4e^{(2i\,dx+2i\,c)}+15i\,a^4)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+15\left(-i\sqrt{2}\,a^4e^{(2i\,dx+2i\,c)}-i\sqrt{2}\,a^4\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\left(de^{\frac{3}{2}}+de^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(4*I*a^4*e^(4*I*d*x + 4*I*c) + 21*I*a^4*e^(2*I*d*x + 2*I*c) +
 15*I*a^4)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 15*(-I*s
```

```
qrt(2)*a^4*e^(2*I*d*x + 2*I*c) - I*sqrt(2)*a^4)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)))/(d*e^(3/2) + d*e^(2*I*d*x + 2*I*c + 3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(3/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2), x)
```

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{42a^4\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5de^3}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4 \cdots}{5de^2 \cdots}$$

```
[Out] -42/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*a^4*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e^3-4/5*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(
d*x+c))^(5/2)+28/5*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3853, 3856, 2719}

$$\frac{42a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de^3}+\frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}-\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-42*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*a^4*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e^3) - (((4*I)
/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) + (((28*I)/5)*(a
^4 + I*a^4*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx &= -\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}-\frac{(7a^2)\int\frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\ &= -\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}+\frac{(21a^4)\int(e\sec(c+\\ &= \frac{42a^4\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5de^3}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4+ia^4\mathrm{t}}{5de^2\sqrt{e\,\mathrm{sec}}}\\ &= \frac{42a^4\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5de^3}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4+ia^4\mathrm{t}}{5de^2\sqrt{e\,\mathrm{sec}}}\\ &= -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{42a^4\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5de^3}-\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.56, size = 110, normalized size = 0.71

$$-\frac{4ia^4e^{2i(c+dx)}\left(7+2e^{2i(c+dx)}-7\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^4*E^((2*I)*(c + d*x))*(7 + 2*E^((2*I)*(c + d*x)) - 7*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
]))/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3761 vs. $2(160) = 320$.
time = 0.68, size = 3762, normalized size = 24.12

method	result	size
default	Expression too large to display	3762

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^4/d*(-1+cos(d*x+c))*(20*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+160*(-
cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*cos(d*x+c)^7-288*cos(d*x+c)^5*(-cos(d*x+
c)/(1+cos(d*x+c))^2)^(3/2)-36*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)+224*cos(d*x+c)^3*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-28*cos(d*x+c)^2*
(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+144*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(
3/2)*sin(d*x+c)*cos(d*x+c)^5-32*I*sin(d*x+c)*cos(d*x+c)^8*(-cos(d*x+c)/(1+c
os(d*x+c))^2)^(3/2)-32*I*sin(d*x+c)*cos(d*x+c)^7*(-cos(d*x+c)/(1+cos(d*x+c)
)^2)^(3/2)+144*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)
^6-192*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^4-192*I
*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^3+20*I*cos(d*x+
c)^4*ln(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-
2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*
x+c)-20*I*cos(d*x+c)^4*ln(-2*(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*
x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/
sin(d*x+c)^2)*sin(d*x+c)+80*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*sin(d*x+
c)*cos(d*x+c)^2-40*I*cos(d*x+c)^3*ln(-(2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/
2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d
*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+40*I*cos(d*x+c)^3*ln(-2*(2*(-cos(d*x+c)/(
1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+cos(d*x+
c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+80*I*(-cos(d*x+c)/(1+
cos(d*x+c))^2)^(3/2)*sin(d*x+c)*cos(d*x+c)+20*I*cos(d*x+c)^2*ln(-(2*(-cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(1+c
os(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-20*I*ln(-2*(2*
(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+
c)/(1+cos(d*x+c))^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)^2*sin(d
*x+c)+76*cos(d*x+c)^6*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)-64*cos(d*x+c)*(-
cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)+21*I*sin(d*x+c)*cos(d*x+c)^7*(-cos(d*x+c
)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(d*x+c)*cos(d*x
+c)^7*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-420*I*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c
))^2)^(3/2)+420*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(
```

```
d*x+c)^5*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d
*x+c)/(1+cos(d*x+c))^2)^(3/2)+42*I*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(1+
cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-42*I*sin(d*x+c)*cos(d*x+c)^6
*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-315*I*Ellip
ticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^4*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)/(1+cos(d*x+c))^2)
^(3/2)+315*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c
)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c)
/(1+cos(d*x+c))^2)^(3/2)-63*I*sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(1+cos(d
*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+63*I*sin(d*x+c)*cos(d*x+c)^5*(-co
s(d*x+c)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+336*I*(-cos(d*x+
c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^
3-336*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(
d*x+c)*cos(d*x+c)^3-168*I*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(1+cos(d*x+c
))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+168*I*sin(d*x+c)*cos(d*x+c)^4*(-cos(d
*x+c)/(1+cos(d*x+c))^2)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+504*I*(-cos(d*x+c)/
(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2-5
04*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x
+c)*cos(d*x+c)^2+168*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin
(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-168*I*(-cos(d*x+c)/(1+cos(d*x+c))^2)^(3/2)
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1
+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.10, size = 86, normalized size = 0.55

$$-\frac{2\left(21i\sqrt{2}\,a^4\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(2i\,a^4e^{(3i\,dx+3i\,c)}+7i\,a^4e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{5}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(21*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c))) + sqrt(2)*(2*I*a^4*e^(3*I*d*x + 3*I*c) + 7*I*a^4*e^(I*d*x +
 I*c))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\frac{1}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx+\int\frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(5/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(5/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2), x)
```

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}+\frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

```
[Out] 10/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*a*(a
+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)+20/21*I*(a^4+I*a^4*tan(d*x+c))/d/
e^2/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3577, 3856, 2720}

$$\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}+\frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/
2)) + (((20*I)/21)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2
))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{(5a^2)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} + \frac{(5a^4)\int \sqrt{e\sec(c+\ldots}}{21e^4}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} + \frac{\left(5a^4\sqrt{\cos(c+dx)}\ldots\right.}{\ldots}$$

$$= \frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4ia(a+ia\tan(c+dx\ldots}{7d(e\sec(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.94, size = 133, normalized size = 1.06

$$\frac{2a^4\sqrt{e\sec(c+dx)}\left(2i+2i\cos(2(c+dx))+5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))-i\sin(2(c+dx)))+8\sin(2(c+dx))\right)(\cos(2(c+3dx))+i\sin(2(c+3dx)))}{21de^4(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*a^4*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 5*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) + 8
*Sin[2*(c + d*x)])*(Cos[2*(c + 3*d*x)] + I*Sin[2*(c + 3*d*x)]))/(21*d*e^4*(
Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A]
time = 0.53, size = 200, normalized size = 1.60

method	result
default	$\frac{2a^4\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-24i(\cos^4(dx+c))+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{c\ldots}{1+\ldots}}\right.}{21d\cos(dx+c)^4\left(\frac{e}{\cos(dx\ldots}\right.}$
risch	$-\frac{2ie^{i(dx+c)}(3e^{2i(dx+c)}-5)a^4\sqrt{2}}{21de^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i(e^{i(dx+c)}+i)}\sqrt{i(e^{i(dx+c)}-i)}\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i(e^{i(d\ldots}}\right.}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3(e^{2i(dx+c)}+1\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*a^4/d*(5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-24*I*cos(d*x+c)^4+5*I*
(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+
cos(d*x+c))/sin(d*x+c),I)+24*sin(d*x+c)*cos(d*x+c)^3+28*I*cos(d*x+c)^2-16*s
in(d*x+c)*cos(d*x+c))/cos(d*x+c)^4/(e/cos(d*x+c))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 88, normalized size = 0.70

$$-\frac{2\left(5i\sqrt{2}\,a^4\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\,(3i\,a^4e^{(4i\,dx+4i\,c)}-2i\,a^4e^{(2i\,dx+2i\,c)}-5i\,a^4)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{7}{2}\right)}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/21*(5*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2
)*(3*I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*I*d*x + 2*I*c) - 5*I*a^4)*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^4}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2), x)
```

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

```
[Out] -2/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*a*(a
+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)+4/15*I*(a^4+I*a^4*tan(d*x+c))/d/e
^2/(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3577, 3856, 2719}

$$-\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (-2*a^4*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/
2)) + (((4*I)/15)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2)
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx &= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} - \frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2} \\ &= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}} - \frac{a^4\int \frac{1}{\sqrt{e\sec(c+dx)}}}{15e^4} \\ &= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}} - \frac{a^4\int \sqrt{\cos(c+}}{15e^4\sqrt{\cos(c+dx)}\,\sqrt{}} \\ &= -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i(a^4+}{15de^2(e}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.79, size = 108, normalized size = 0.86

$$-\frac{ia^4e^{i(c+dx)}\left(2+7e^{2i(c+dx)}+5e^{4i(c+dx)}-2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{45de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((-1/45*I)*a^4*E^(I*(c + d*x))*(2 + 7*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c +
 d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(133) = 266.
time = 0.99, size = 370, normalized size = 2.96

method	result
risch	$-\frac{i\left(5\,e^{4i(dx+c)}+2\,e^{2i(dx+c)}-6\right)a^4\sqrt{2}}{45d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx}\right.}}{}\right.}{}$

default	$-\frac{2a^4\left(40i(\cos^5(dx+c))\sin(dx+c)-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)+3i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*a^4/d*(40*I*sin(d*x+c)*cos(d*x+c)^5-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos
(d*x+c)*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+40*c
os(d*x+c)^6-36*I*sin(d*x+c)*cos(d*x+c)^3-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x
+c),I)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-56*cos(d*x+c)^4+13*cos(d*x+
c)^2+3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^5/(e/cos(d*x+c))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 100, normalized size = 0.80

$$\frac{\left(-6i\sqrt{2}\,a^4\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)+\frac{\sqrt{2}\left(-5ia^4e^{(5idx+5ic)}-7ia^4e^{(3idx+3ic)}-2ia^4e^{(idx+ic)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}\right)e^{\left(-\frac{9}{2}\right)}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(-6*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c))) + sqrt(2)*(-5*I*a^4*e^(5*I*d*x + 5*I*c) - 7*I*a^4*e^(3*I*d*
x + 3*I*c) - 2*I*a^4*e^(I*d*x + I*c))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d
*x + 2*I*c) + 1))*e^(-9/2)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-9/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2), x)
```

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}+\frac{4}{7}$$

```
[Out] -2/77*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-2/77*a^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-4/11*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(11/2)+4/77*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2720}

$$-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}+\frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) - (2*a^4*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)
/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (((4*I)/77)*
(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx &= -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2} \\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} - \frac{(3a^4)\int\frac{1}{(e\sec(c+dx))^{3/2}}}{77e^4} \\
&= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx)}{77de^2(e\sec(c+dx))^{7/2}} \\
&= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx)}{77de^2(e\sec(c+dx))^{7/2}} \\
&= -\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 1.25, size = 148, normalized size = 0.95

$$
-\frac{a^4\sqrt{e\sec(c+dx)}\left(37i\cos(c+dx)+11i\cos(3(c+dx))-3\sin(c+dx)+4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(3(c+dx))-i\sin(3(c+dx)))-3\sin(3(c+dx))\right)(\cos(3c+7dx)+i\sin(3c+7dx))}{154de^6(\cos(dx)+i\sin(dx))^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] -1/154*(a^4*Sqrt[e*Sec[c + d*x]]*((37*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d
*x)] - 3*Sin[c + d*x] + 4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos
[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 3*Sin[3*(c + d*x)])*(Cos[3*c + 7*d*x]
 + I*Sin[3*c + 7*d*x]))/(d*e^6*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A]
time = 0.59, size = 215, normalized size = 1.38

method	result
default	$-\frac{2a^4\left(56i(\cos^6(dx+c))-56\sin(dx+c)(\cos^5(dx+c))-44i(\cos^4(dx+c))+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\right.}{77d\cos(}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}(7\,\mathrm{e}^{4i(dx+c)}+13\,\mathrm{e}^{2i(dx+c)}+4)a^4\sqrt{2}}{154d\,e^5\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\right.}{77d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*a^4/d*(56*I*cos(d*x+c)^6-56*sin(d*x+c)*cos(d*x+c)^5-44*I*cos(d*x+c)^4
+I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*El
lipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+16*sin(
d*x+c)*cos(d*x+c)^3+sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))^(11/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 102, normalized size = 0.65

$$\frac{\left(4i\sqrt{2}\,a^4\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-7i\,a^4e^{(6i\,dx+6i\,c)}-20i\,a^4e^{(4i\,dx+4i\,c)}-17i\,a^4e^{(2i\,dx+2i\,c)}-4i\,a^4\right)e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{11}{2}\right)}}{154\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/154*(4*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2
)*(-7*I*a^4*e^(6*I*d*x + 6*I*c) - 20*I*a^4*e^(4*I*d*x + 4*I*c) - 17*I*a^4*e
^(2*I*d*x + 2*I*c) - 4*I*a^4)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I
*c) + 1))*e^(-11/2)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-11/2)/sec(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2), x)
```

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=156

$$\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i(a^4+ia^4\tan$$

```
[Out] 2/117*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+2/39*a^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/13*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(13/2)-4/117*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9/2)
```

Rubi [A]
time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2719}

$$\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (2*a^4*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*a^4*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((4*
I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((4*I)/11
7)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx &= -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}+\frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}+\frac{(5a^4)\int \frac{1}{(e\sec(c+dx))^{5/2}}}{117e^4}\\
&= \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i(a^4+ia^4\tan(c+dx)}{117de^2(e\sec(c+dx))^{9}}\\
&= \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i(a^4+ia^4\tan(c+dx)}{117de^2(e\sec(c+dx))^{9}}\\
&= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia}{13d(e\sec}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.89, size = 450, normalized size = 2.88

$$-\frac{2i\sqrt{2}\,e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^4}{117d\,(-1+e^{2ic})\,(e\sec(c+dx))^{13/2}(\cos(dx)+i\sin(dx))^4}+\frac{\sec^3(c+dx)\left(\cos(3dx)\left(-\frac{59}{468}i\cos(c)-\frac{59\sin(c)}{468}\right)+\cos(5dx)\left(-\frac{37}{468}i\cos(c)+\frac{37\sin(c)}{468}\right)+\cos(dx)\csc(c)(24\cos(c)+31i\sin(c))\left(-\frac{1}{468}\cos(3c)+\frac{1}{468}i\sin(3c)\right)+\cos(7dx)\left(-\frac{1}{52}i\cos(3c)+\frac{1}{52}\sin(3c)\right)+\left(\frac{55}{468}\cos(3c)-\frac{55}{468}i\sin(3c)\right)\sin(dx)+\left(\frac{59\cos(c)}{468}-\frac{59}{468}i\sin(c)\right)\sin(3dx)+\left(\frac{37\cos(c)}{468}+\frac{37}{468}i\sin(c)\right)\sin(5dx)+\left(\frac{1}{52}\cos(3c)+\frac{1}{52}i\sin(3c)\right)\sin(7dx)\right)(a+ia\tan(c+dx))^4}{d(e\sec(c+dx))^{13/2}(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (((-2*I)/117)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[
1 + E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*
Sec[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^4)/(d*E^(I*(3*c + d*x))*(-1 + E^(
(2*I)*c))*(e*Sec[c + d*x])^(13/2)*(Cos[d*x] + I*Sin[d*x])^4) + (Sec[c + d*x
]^3*(Cos[3*d*x]*(((-59*I)/468)*Cos[c] - (59*Sin[c])/468) + Cos[5*d*x]*(((-3
```

7*I)/468)*Cos[c] + (37*Sin[c])/468) + Cos[d*x]*Csc[c]*(24*Cos[c] + (31*I)*Sin[c])*(-1/468*Cos[3*c] + (I/468)*Sin[3*c]) + Cos[7*d*x]*((-1/52*I)*Cos[3*c] + Sin[3*c]/52) + ((55*Cos[3*c])/468 - ((55*I)/468)*Sin[3*c])*Sin[d*x] + ((59*Cos[c])/468 - ((59*I)/468)*Sin[c])*Sin[3*d*x] + ((37*Cos[c])/468 + ((37*I)/468)*Sin[c])*Sin[5*d*x] + (Cos[3*c]/52 + (I/52)*Sin[3*c])*Sin[7*d*x])*(a + I*a*Tan[c + d*x])^4)/(d*(e*Sec[c + d*x])^(13/2)*(Cos[d*x] + I*Sin[d*x])^4)

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(160) = 320$.
time = 0.64, size = 380, normalized size = 2.44

method	result
risch	$-\frac{i(9\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+31\,e^{2i(dx+c)}+24)a^4\sqrt{2}}{468d\,e^6\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}}{\cdots}\right.}{\cdots}$
default	$-\frac{2a^4\left(72i\sin(dx+c)\left(\cos^7(dx+c)\right)+72\left(\cos^8(dx+c)\right)-52i\left(\cos^5(dx+c)\right)\sin(dx+c)-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \text{Elliptic}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)

[Out] -2/117*a^4/d*(72*I*sin(d*x+c)*cos(d*x+c)^7+72*cos(d*x+c)^8-52*I*sin(d*x+c)*cos(d*x+c)^5-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-88*cos(d*x+c)^6-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+17*cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^7/(e/cos(d*x+c))^(13/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")

[Out] e^(-13/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(13/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 114, normalized size = 0.73

$$\frac{\left(24i\,\sqrt{2}\,a^{4}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(-9i\,a^{4}e^{(7i\,dx+7i\,c)}-37i\,a^{4}e^{(5i\,dx+5i\,c)}-59i\,a^{4}e^{(3i\,dx+3i\,c)}-31i\,a^{4}e^{(i\,dx+i\,c)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{13}{2}\right)}}{468\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/468*(24*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e
^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^4*e^(7*I*d*x + 7*I*c) - 37*I*a^4*e^(5*I*
d*x + 5*I*c) - 59*I*a^4*e^(3*I*d*x + 3*I*c) - 31*I*a^4*e^(I*d*x + I*c))*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-13/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-13/2)/sec(d*x + c)^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2), x)
```

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^8}+\frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}}-\frac{4ia(}{15d}$$

```
[Out] 2/55*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/33*a^4*sin(d*x+c)/d/e^7/(e
*sec(d*x+c))^(1/2)+2/33*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2
)/d/e^8-4/15*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-4/55*I*(a^4+I
*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3577, 3854, 3856, 2720}

$$\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^8}+\frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
33*d*e^8) + (2*a^4*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^4
*Sin[c + d*x])/(33*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((4*I)/15)*a*(a + I*a*Tan
[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((4*I)/55)*(a^4 + I*a^4*Tan[c
+ d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} + \frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} + \frac{(7a^4)\int \frac{1}{(e\sec(c+dx))^{7/2}}}{55e^4}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} -$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} -$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}$$

Mathematica [A]

time = 2.07, size = 155, normalized size = 0.83

$$-\frac{ia^4\sqrt{e\sec(c+dx)}\left(64+112\cos(2(c+dx))+48\cos(4(c+dx))-54i\sin(2(c+dx))-37i\sin(4(c+dx))+40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(i\cos(4(c+dx))+\sin(4(c+dx)))\right)(\cos(4(c+2dx))+i\sin(4(c+2dx)))}{660de^8(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] ((-1/660*I)*a^4*Sqrt[e*Sec[c + d*x]]*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4*
(c + d*x)] - (54*I)*Sin[2*(c + d*x)] - (37*I)*Sin[4*(c + d*x)] + 40*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)
```

```
]))*(Cos[4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(d*e^8*(Cos[d*x] + I*Sin[d
*x])^4)
```

Maple [A]
time = 0.67, size = 232, normalized size = 1.24

method	result
default	$-\frac{2a^4\left(88i\left(\cos^8(dx+c)\right)-88\sin(dx+c)\left(\cos^7(dx+c)\right)-60i\left(\cos^6(dx+c)\right)+16\sin(dx+c)\left(\cos^5(dx+c)\right)-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos}{1+\cos}}\right.}{}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(11\,\mathrm{e}^{6i(dx+c)}+47\,\mathrm{e}^{4i(dx+c)}+81\,\mathrm{e}^{2i(dx+c)}+85\right)a^4\sqrt{2}}{1320d\,e^7\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx}}}{33d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/165*a^4/d*(88*I*cos(d*x+c)^8-88*sin(d*x+c)*cos(d*x+c)^7-60*I*cos(d*x+c)^
6+16*sin(d*x+c)*cos(d*x+c)^5-5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*(
1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+c
os(d*x+c))/sin(d*x+c),I)-3*sin(d*x+c)*cos(d*x+c)^3-5*sin(d*x+c)*cos(d*x+c))
/cos(d*x+c)^8/(e/cos(d*x+c))^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
```

```
[Out] e^(-15/2)*integrate((I*a*tan(d*x + c) + a)^4/sec(d*x + c)^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 116, normalized size = 0.62

$$\frac{\left(-80i\,\sqrt{2}\,a^4\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\,\left(-11i\,a^4e^{(8i\,dx+8i\,c)}-58i\,a^4e^{(6i\,dx+6i\,c)}-128i\,a^4e^{(4i\,dx+4i\,c)}-166i\,a^4e^{(2i\,dx+2i\,c)}-85i\,a^4\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{15}{2}\right)}}{1320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/1320*(-80*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqr
t(2)*(-11*I*a^4*e^(8*I*d*x + 8*I*c) - 58*I*a^4*e^(6*I*d*x + 6*I*c) - 128*I*
```

```
a^4*e^(4*I*d*x + 4*I*c) - 166*I*a^4*e^(2*I*d*x + 2*I*c) - 85*I*a^4)*e^(1/2*
I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-15/2)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(15/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4*e^(-15/2)/sec(d*x + c)^(15/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2), x)
```

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5ad}+\frac{2e^3(e\sec(c$$

```
[Out] -2/7*I*e^2*(e*sec(d*x+c))^(7/2)/a/d+2/5*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)
/a/d-6/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*e^5*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3582, 3853, 3856, 2719}

$$-\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{6e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{5ad}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-6*e^6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) - (((2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (6*e^5*Sqrt[e*Sec[
c + d*x]]*Sin[c + d*x])/(5*a*d) + (2*e^3*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx &= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{e^2\int(e\sec(c+dx))^{7/2}\,dx}{a}\\ &= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}+\frac{(3e^4)\int(e\sec(c+d}{5a}\\ &= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5ad}+\frac{2e^3(e\sec(c+dx))^{5/}}{5ad}\\ &= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5ad}+\frac{2e^3(e\sec(c+dx))^{5/}}{5ad}\\ &= -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sqrt{e\sec(c+a}}{5a}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.57, size = 128, normalized size = 0.94

$$\frac{e^4(e\sec(c+dx))^{3/2}\left(76+28\cos(2(c+dx))-7e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+7i\sec(c+dx)\sin(3(c+dx))-13i\tan(c+dx)\right)(-i+\tan(c+dx))}{70ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(3/2)*(76 + 28*Cos[2*(c + d*x)] - (7*(1 + E^((2*I)*(c
 + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(
(2*I)*(c + d*x)) + (7*I)*Sec[c + d*x]*Sin[3*(c + d*x)] - (13*I)*Tan[c + d*x
])*(-I + Tan[c + d*x]))/(70*a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(143) = 286.
time = 0.68, size = 375, normalized size = 2.76

method	result
default	$-\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(21i(\cos^4(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(21*I*cos(d*x+c)^4*sin(d*x+c)*
(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+
cos(d*x+c))/sin(d*x+c),I)-21*I*cos(d*x+c)^4*sin(d*x+c)*(1/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+
c),I)+21*I*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*cos(d*x+c
)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*E
llipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+21*cos(d*x+c)^4-14*cos(d*x+c)^3+5*
I*sin(d*x+c)-7*cos(d*x+c))*(e/cos(d*x+c))^(11/2)*cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 192, normalized size = 1.41

$$-\frac{2\left(\frac{\sqrt{2}\left(21ie^{\left(7idx+7ic+\frac{11}{2}\right)}+77ie^{\left(5idx+5ic+\frac{11}{2}\right)}+103ie^{\left(3idx+3ic+\frac{11}{2}\right)}+7ie^{\left(idx+ic+\frac{11}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+21\left(i\sqrt{2}e^{\frac{11}{2}}+i\sqrt{2}e^{\left(6idx+6ic+\frac{11}{2}\right)}+3i\sqrt{2}e^{\left(4idx+4ic+\frac{11}{2}\right)}+3i\sqrt{2}e^{\left(2idx+2ic+\frac{11}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)\right)}{35\left(ade^{(6idx+6ic)}+3ade^{(4idx+4ic)}+3ade^{(2idx+2ic)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(21*I*e^(7*I*d*x + 7*I*c + 11/2) + 77*I*e^(5*I*d*x + 5*I*c +
 11/2) + 103*I*e^(3*I*d*x + 3*I*c + 11/2) + 7*I*e^(I*d*x + I*c + 11/2))*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 21*(I*sqrt(2)*e^(11/2)
 + I*sqrt(2)*e^(6*I*d*x + 6*I*c + 11/2) + 3*I*sqrt(2)*e^(4*I*d*x + 4*I*c +
11/2) + 3*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 11/2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*
e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(11/2)*sec(d*x + c)^(11/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i), x)
```

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+d}{3ad}$$

```
[Out] -2/5*I*e^2*(e*sec(d*x+c))^(5/2)/a/d+2/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)
/a/d+2/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3582, 3853, 3856, 2720}

$$\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3ad}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*a*d) - (((2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (2*e^3*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{e^2\int (e\sec(c+dx))^{5/2}\,dx}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad} + \frac{e^4\int \sqrt{e\sec(c+dx}}{3a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad} + \frac{\left(e^4\sqrt{\cos(c+dx)}\right.}{}$$
$$= \frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{3ad} - \frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e}{}$$

Mathematica [A]
time = 0.64, size = 62, normalized size = 0.59

$$\frac{e^2(e\sec(c+dx))^{5/2}\left(-6i+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))\right)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(-6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*
x)/2, 2] + 5*Sin[2*(c + d*x)]))/(15*a*d)
```

Maple [A]
time = 0.65, size = 202, normalized size = 1.92

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^3(dx+c))+5i\sqrt{\frac{}{1+\mathrm{co}}}\right.}{15ad\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(5*I*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*
```

```
cos(d*x+c)^3+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^2+5*sin(d*x+c)*cos(d*
x+c)-3*I)*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 141, normalized size = 1.34

$$-\frac{2\left(\frac{\sqrt{2}\left(-5i\,e^{\frac{9}{2}}+5i\,e^{\left(4i\,dx+4i\,c+\frac{9}{2}\right)}+12i\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+5\left(i\sqrt{2}\,e^{\frac{9}{2}}+i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{9}{2}\right)}+2i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-5*I*e^(9/2) + 5*I*e^(4*I*d*x + 4*I*c + 9/2) + 12*I*e^(2*I*
d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) +
 5*(I*sqrt(2)*e^(9/2) + I*sqrt(2)*e^(4*I*d*x + 4*I*c + 9/2) + 2*I*sqrt(2)*e
^(2*I*d*x + 2*I*c + 9/2))*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d
*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(9/2)*sec(d*x + c)^(9/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i), x)
```

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}$$

```
[Out] -2/3*I*e^2*(e*sec(d*x+c))^(3/2)/a/d-2*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(
e*sec(d*x+c))^(1/2)+2*e^3*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3582, 3853, 3856, 2719}

$$-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) - (((2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (2*e^3*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/(a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{e^2\int(e\sec(c+dx))^{3/2}\,dx}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{ad}-\frac{e^4\int\frac{1}{\sqrt{e\sec(c+dx)}}}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{ad}-\frac{e^4\int\sqrt{\cos(c+}}{a\sqrt{\cos(c+dx)}\,\sqrt{e}}$$
$$= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx}}{ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.84, size = 102, normalized size = 1.01

$$\frac{2ie^3\sqrt{e\sec(c+dx)}\,(\cos(c)+i\sin(c))(\cos(dx)+i\sin(dx))\left(-4+\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+i\tan(c+dx)\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*e^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*(Cos[d*x] + I*Sin[d
*x])*(-4 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))] + I*Tan[c + d*x]))/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(116) = 232.
time = 0.64, size = 361, normalized size = 3.57

method	result

default	$-\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(3i(\cos^2(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*cos(d*x+c)^2*(1/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)*sin(d*x+c)-3*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+
c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+c)+3*cos(d*x+c)^2-3*cos(d*x+c))*(
e/cos(d*x+c))^(7/2)*cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 1.11

$$-\frac{2\left(\frac{\sqrt{2}\left(3ie^{\left(3idx+3ic+\frac{7}{2}\right)}+5ie^{\left(idx+ic+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+3\left(i\sqrt{2}\,e^{\frac{7}{2}}+i\sqrt{2}\,e^{\left(2idx+2ic+\frac{7}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)\right)}{3\left(ade^{(2idx+2ic)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(3*I*e^(3*I*d*x + 3*I*c + 7/2) + 5*I*e^(I*d*x + I*c + 7/2))*e
^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 3*(I*sqrt(2)*e^(7/2)
 + I*sqrt(2)*e^(2*I*d*x + 2*I*c + 7/2))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i), x)
```

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}$$

```
[Out] -2*I*e^2*(e*sec(d*x+c))^(1/2)/a/d+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(
d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3582, 3856, 2720}

$$\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{a}$$
$$= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,d}{a}$$
$$= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}$$

Mathematica [A]
time = 0.34, size = 49, normalized size = 0.70

$$\frac{2e^2\left(-i+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*e^2*(-I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d
*x]])/(a*d)
```

Maple [A]
time = 0.64, size = 174, normalized size = 2.49

method	result
default	$\frac{2i(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\mathrm{EllipticF}\left(\frac{i(-1+}{\sin}\right.\right.}{ad\sin(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/a/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(EllipticF(I*(-1+cos(d*x+c))/sin
(d*x+c),I)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)+EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-1)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2/sin(d*
x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 56, normalized size = 0.80

$$-\frac{2\left(i\sqrt{2}\,e^{\frac{5}{2}}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{i\sqrt{2}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{5}{2}\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(2)*e^(5/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + I*sqrt(
2)*e^(1/2*I*d*x + 1/2*I*c + 5/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}}{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i), x)
```

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] 2*I*e^2/a/d/(e*sec(d*x+c))^(1/2)+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3582, 3856, 2719}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a}$$
$$= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{e^2\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.38, size = 74, normalized size = 1.06

$$\frac{2iee^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
[Out] ((2*I)*e*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])/(a*d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(89) = 178.
time = 4.61, size = 347, normalized size = 4.96

method	result
default	$-\frac{2\cos(dx+c)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;\text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -2/a/d*cos(d*x+c)*(e/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2*(
I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin
(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(-1+c
os(d*x+c))/sin(d*x+c),I)+I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-
```

```
1+cos(d*x+c))/sin(d*x+c),I)-I*cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-cos(d*x+c)
)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 91, normalized size = 1.30

$$-\frac{2\left(-i\sqrt{2}\,e^{\left(i\,dx+i\,c+\frac{3}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)+\frac{\sqrt{2}\left(-i\,e^{\frac{3}{2}}-i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*(-I*sqrt(2)*e^(I*d*x + I*c + 3/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*e^(3/2) - I*e^(2*I*d*x + 2*I*c
+ 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*x -
I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\tan\left(c+dx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i), x)
```

3.228 $\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d+2/3*I*(e*sec(d*x+c
))^(1/2)/d/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3583, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a*
d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx &= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}+\frac{\int\sqrt{e\sec(c+dx)}\,dx}{3a}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,d}{3a}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 83, normalized size = 1.04

$$\frac{2(e\sec(c+dx))^{3/2}\left(\cos(c+dx)+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(-i\cos(c+dx)+\sin(c+dx))\right)}{3ade(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
[Out] (2*(e*Sec[c + d*x])^(3/2)*(Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*((-I)*Cos[c + d*x] + Sin[c + d*x])))/(3*a*d*e*(-I + Tan[c + d*x
]))
```

Maple [A]
time = 0.58, size = 164, normalized size = 2.05

method	result
default	$\frac{2\sqrt{\frac{e}{\cos(dx+c)}}\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx}{1+\cos(}}\right.}{3ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/3/a/d*(e/cos(d*x+c))^(1/2)*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(
d*x+c))/sin(d*x+c),I)+I*cos(d*x+c)^2+sin(d*x+c)*cos(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 1.10

$$\frac{\left(-2i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\text{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)+\frac{\sqrt{2}\left(i\,e^{\frac{1}{2}}+i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-2i\,dx-2i\,c\right)}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 1/2)*weierstrassPInverse(-4, 0, e^(I
*d*x + I*c)) + sqrt(2)*(I*e^(1/2) + I*e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d
*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt{e\sec\left(c+dx\right)}}{\tan\left(c+dx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i), x)
```

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=80

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))}$$

```
[Out] 6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/5*I/d/(e*sec(d*x
+c))^(1/2)/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3583, 3856, 2719}

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))}\,dx = \frac{2i}{5d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))} + \frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}}{5a}$$

$$= \frac{2i}{5d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))} + \frac{3\int \sqrt{\cos(c+}}{5a\sqrt{\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{2i}{5d\sqrt{e\sec(c+dx)}\,(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.94, size = 109, normalized size = 1.36

$$\frac{\left(4+4\cos(2(c+dx))-2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3i\sin(2(c+dx))\right)(i+\tan(c+dx))}{5ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((4 + 4*Cos[2*(c + d*x)] - 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c
 + d*x)])*(I + Tan[c + d*x]))/(5*a*d*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(94) = 188.
time = 3.33, size = 358, normalized size = 4.48

method	result
default	$\frac{2\left(3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a/d*(3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c)
)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+c)*cos(d*x+c)^3+3*I*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x
```

```
+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-cos(
d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(e
/cos(d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 98, normalized size = 1.22

$$\frac{\left(12i\sqrt{2}\,e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(7i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-3i\,dx-3i\,c-\frac{1}{2}\right)}}{10\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(12*I*sqrt(2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(7*I*e^(4*I*d*x + 4*I*c) + 8*I*e^
(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1
))*e^(-3*I*d*x - 3*I*c - 1/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)-i\sqrt{e\sec\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x) - I*sqrt(e*sec(c + d*x))),
 x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{e}{\cos (c+d\,x)}}\,\left(a+a\tan (c+d\,x)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=114

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2}+\frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(}$$

```
[Out] 10/21*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/a/d/e^2+2/7*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x
+c))
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3583, 3854, 3856, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2}+\frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*
a*d*e^2) + (10*Sin[c + d*x])/(21*a*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/7)/(d
*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))} + \frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}}{7a}$$

$$= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+$$

$$= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10}{21ade}$$

Mathematica [A]
time = 0.59, size = 125, normalized size = 1.10

$$-\frac{\sec^3(c+dx)\left(-14\cos(c+dx)+2\cos(3(c+dx))+20i\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))+5i\sin(c+dx)+5i\sin(3(c+dx))\right)}{42ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] -1/42*(Sec[c + d*x]^3*(-14*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + (20*I)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + (
5*I)*Sin[c + d*x] + (5*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-
I + Tan[c + d*x]))
```

Maple [A]
time = 0.77, size = 218, normalized size = 1.91

method	result
default	$\frac{2\cos(dx+c)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(3i(\cos^4(dx+c))+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a/d*cos(d*x+c)*(e/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2
*(3*I*cos(d*x+c)^4+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*(1/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)+3*sin(d*x+c)*cos(d*x+c)^3+5*sin(d*x+c)*cos(d*x+c))/e^3/sin(d
*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 106, normalized size = 0.93

$$\frac{\left(-40i\sqrt{2}\,e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-7i\,e^{(6i\,dx+6i\,c)}+9i\,e^{(4i\,dx+4i\,c)}+19i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-4i\,dx-4i\,c-\frac{3}{2}\right)}}{84\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/84*(-40*I*sqrt(2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x
 + I*c)) + sqrt(2)*(-7*I*e^(6*I*d*x + 6*I*c) + 9*I*e^(4*I*d*x + 4*I*c) + 19
*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I
*c) + 1))*e^(-4*I*d*x - 4*I*c - 3/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(
3/2)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=114

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}$$

```
[Out] 14/45*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/e^2/cos(d*
x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/9*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x
+c))
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3583, 3854, 3856, 2719}

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Sec[c + d*x])^(3/2)) + ((2*I)/9)/
(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}dx = \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))} + \frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}}{9a}$$

$$= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c}$$

$$= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c}$$

$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.16, size = 134, normalized size = 1.18

$$\frac{\left(106+104\cos(2(c+dx))-2\cos(4(c+dx))-56e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+70i\sin(2(c+dx))-7i\sin(4(c+dx))\right)(i+\tan(c+dx))}{180ade^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((106 + 104*Cos[2*(c + d*x)] - 2*Cos[4*(c + d*x)] - 56*E^((2*I)*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] + (70*I)*Sin[2*(c + d*x)] - (7*I)*Sin[4*(c + d*x)])*(I + Tan[c +
d*x]))/(180*a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(124) = 248.
time = 1.13, size = 376, normalized size = 3.30

method	result

default	$-\frac{2(\cos^2(dx+c))\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(-5i(\cos^5(dx+c))\sin(dx+c)+5(\cos^6(dx+c))-21i\sqrt{\frac{1}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/a/d*cos(d*x+c)^2*(e/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c)
)^2*(-5*I*cos(d*x+c)^5*sin(d*x+c)+5*cos(d*x+c)^6-21*I*sin(d*x+c)*cos(d*x+c)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)-21*I*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*I*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/
sin(d*x+c),I)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/e^5/sin(d*x+c)^
5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 1.05

$$\frac{\left(336i\sqrt{2}\,e^{(5i\,dx+5i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,(-9i\,e^{(8i\,dx+8i\,c)}+174i\,e^{(6i\,dx+6i\,c)}+212i\,e^{(4i\,dx+4i\,c)}+34i\,e^{(2i\,dx+2i\,c)}+5i)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-5i\,dx-5i\,c-\frac{5}{2}\right)}}{360\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/360*(336*I*sqrt(2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*e^(8*I*d*x + 8*I*c) + 174
*I*e^(6*I*d*x + 6*I*c) + 212*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(2*I*d*x + 2*I*
c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*
x - 5*I*c - 5/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(
5/2)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-5/2)/((I*a*tan(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=145

$$\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4}+\frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}+\frac{}{11d(e}$$

```
[Out] 18/77*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(5/2)+30/77*sin(d*x+c)/a/d/e^3/(e*sec
(d*x+c))^(1/2)+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d/e
^4+2/11*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3583, 3854, 3856, 2720}

$$\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4}+\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}+\frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a*d*e^4) + (18*Sin[c + d*x])/(77*a*d*e*(e*Sec[c + d*x])^(5/2)) + (30*Sin[c
+ d*x])/(77*a*d*e^3*Sqrt[e*Sec[c + d*x]]) + ((2*I)/11)/(d*(e*Sec[c + d*x])^
(7/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))} + \frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}}{11a}$$

$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c$$

$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{}{11d(e\,\mathrm{s}}$$

$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{}{11d(e\,\mathrm{s}}$$

$$= \frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{18}{77ade(}$$

Mathematica [A]
time = 0.85, size = 142, normalized size = 0.98

$$-\frac{(e\sec(c+dx))^{3/2}\left(-148\cos(c+dx)+34\cos(3(c+dx))+2\cos(5(c+dx))+240i\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))+78i\sin(c+dx)+87i\sin(3(c+dx))+9i\sin(5(c+dx))\right)}{616ade^5(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] -1/616*((e*Sec[c + d*x])^(3/2)*(-148*Cos[c + d*x] + 34*Cos[3*(c + d*x)] + 2
*Cos[5*(c + d*x)] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(C
os[c + d*x] + I*Sin[c + d*x]) + (78*I)*Sin[c + d*x] + (87*I)*Sin[3*(c + d*x
)] + (9*I)*Sin[5*(c + d*x)]))/(a*d*e^5*(-I + Tan[c + d*x]))
```

Maple [A]
time = 1.44, size = 236, normalized size = 1.63

method	result

default	$\frac{2(-1+\cos(dx+c))^2(1+\cos(dx+c))^2(\cos^3(dx+c))\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(7i(\cos^6(dx+c))+7\sin(dx+c)(\cos^5(dx+c))+15i\sqrt{\frac{1}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/a/d*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2*cos(d*x+c)^3*(e/cos(d*x+c))^(7/
2)*(7*I*cos(d*x+c)^6+7*sin(d*x+c)*cos(d*x+c)^5+15*I*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/
sin(d*x+c),I)+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*sin(d*x+c)*cos(d*x+c)^3+15*s
in(d*x+c)*cos(d*x+c))/e^7/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 128, normalized size = 0.88

$$\frac{\left(-480i\sqrt{2}e^{(6i\,dx+6i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-11i\,e^{(10i\,dx+10i\,c)}-121i\,e^{(8i\,dx+8i\,c)}+70i\,e^{(6i\,dx+6i\,c)}+226i\,e^{(4i\,dx+4i\,c)}+53i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-6i\,dx-6i\,c-\frac{7}{2}\right)}}{1232\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1232*(-480*I*sqrt(2)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)) + sqrt(2)*(-11*I*e^(10*I*d*x + 10*I*c) - 121*I*e^(8*I*d*x + 8*I
*c) + 70*I*e^(6*I*d*x + 6*I*c) + 226*I*e^(4*I*d*x + 4*I*c) + 53*I*e^(2*I*d*
x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^
(-6*I*d*x - 6*I*c - 7/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}\tan\left(c+dx\right)-i\left(e\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/((e*sec(c + d*x))**(7/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(
7/2)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((I*a*tan(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=183

$$-\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{15a^2d}+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx}{45a^2d}$$

```
[Out] 22/45*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+22/63*e^3*(e*sec(d*x+c))^(9
/2)*sin(d*x+c)/a^2/d-22/15*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x
+c))^(1/2)+22/15*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^2/d-4/7*I*e^2*(e*sec
(d*x+c))^(11/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2719}

$$-\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{15a^2d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{45a^2d}+\frac{22e^3\sin(c+dx)(e\sec(c+dx))^{9/2}}{63a^2d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (22*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (22*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(45*a^2*d) + (22*e^3*(e*Sec[c + d*x
])^(9/2)*Sin[c + d*x])/(63*a^2*d) - (((4*I)/7)*e^2*(e*Sec[c + d*x])^(11/2))
/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(11e^2\right)\int(e\sec(c+dx))^{11/2}\,dx}{7a^2}\\
&= \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(11e^4\right)\int(e\,\mathrm{s}\ldots}{}\\
&= \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d}+\frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d}-\frac{4i\ldots}{7d\ldots}\\
&= \frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{15a^2d}+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d}+\frac{22e^3\ldots}{}\\
&= \frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{15a^2d}+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d}+\frac{22e^3\ldots}{}\\
&= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{15a^2d}+\ldots
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.73, size = 302, normalized size = 1.65

$$\frac{(e\sec(c+dx))^{15/2}(\cos(dx)+i\sin(dx))^2\left(\frac{22i\sqrt{2}\,e^{3ic-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}\left(-1+e^{2ic}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{-1+e^{2ic}}+\frac{1}{56}\csc(c)\sec^{\frac{9}{2}}(c+dx)(\cos(2c)+i\sin(2c))(1260\cos(dx)+1050\cos(2c+dx)+1078\cos(2c+3dx)+77\cos(4c+3dx)+231\cos(4c+5dx)+720i\sin(dx)-720i\sin(2c+dx))\right)}{45d\sec^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(15/2)*(Cos[d*x] + I*Sin[d*x])^2*(((22*I)*Sqrt[2]*E^((3*I
)*c - I*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^(
```

```
(2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(-1 + E^
((2*I)*c)) + (Csc[c]*Sec[c + d*x]^(9/2)*(Cos[2*c] + I*Sin[2*c])*(1260*Cos[d
*x] + 1050*Cos[2*c + d*x] + 1078*Cos[2*c + 3*d*x] + 77*Cos[4*c + 3*d*x] + 2
31*Cos[4*c + 5*d*x] + (720*I)*Sin[d*x] - (720*I)*Sin[2*c + d*x]))/56))/(45*
d*Sec[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(185) = 370$.
time = 0.73, size = 384, normalized size = 2.10

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(231i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^5(dx+c))\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(231*I*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5*EllipticE(I*(-1+cos(d*x+
c))/sin(d*x+c),I)*sin(d*x+c)-231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^5*s
in(d*x+c)+231*I*cos(d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-231*I*cos
(d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipt
icF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-231*cos(d*x+c)^5+154*cos(d*x
+c)^4-90*I*cos(d*x+c)*sin(d*x+c)+112*cos(d*x+c)^2-35)*(e/cos(d*x+c))^(15/2)
*cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 242, normalized size = 1.32

$$-\frac{2\left(\frac{\sqrt{2}\left(231i\,e^{\left(9i\,dx+9i\,c+\frac{15}{2}\right)}+1078i\,e^{\left(7i\,dx+7i\,c+\frac{15}{2}\right)}+1980i\,e^{\left(5i\,dx+5i\,c+\frac{15}{2}\right)}+1770i\,e^{\left(3i\,dx+3i\,c+\frac{15}{2}\right)}+77i\,e^{\left(i\,dx+i\,c+\frac{15}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+231\left(i\sqrt{2}\,e^{\frac{15}{2}}+i\sqrt{2}\,e^{\left(8i\,dx+8i\,c+\frac{15}{2}\right)}+4i\sqrt{2}\,e^{\left(6i\,dx+6i\,c+\frac{15}{2}\right)}+6i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{15}{2}\right)}+4i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{15}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{315\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(231*I*e^(9*I*d*x + 9*I*c + 15/2) + 1078*I*e^(7*I*d*x + 7*I
*c + 15/2) + 1980*I*e^(5*I*d*x + 5*I*c + 15/2) + 1770*I*e^(3*I*d*x + 3*I*c
+ 15/2) + 77*I*e^(I*d*x + I*c + 15/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*
d*x + 2*I*c) + 1) + 231*(I*sqrt(2)*e^(15/2) + I*sqrt(2)*e^(8*I*d*x + 8*I*c
+ 15/2) + 4*I*sqrt(2)*e^(6*I*d*x + 6*I*c + 15/2) + 6*I*sqrt(2)*e^(4*I*d*x +
 4*I*c + 15/2) + 4*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 15/2))*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^(8*I*d*x + 8*I*
c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(15/2)*sec(d*x + c)^(15/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{15/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}+\frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d}+\frac{18e^3(e\sec(c+dx))^{7/}}{35a^2d}$$

```
[Out] 6/7*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+18/35*e^3*(e*sec(d*x+c))^(7/2
)*sin(d*x+c)/a^2/d+6/7*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)
/a^2/d-4/5*I*e^2*(e*sec(d*x+c))^(9/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2720}

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}+\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\sec(c+dx))^{7/2}}{35a^2d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*a^2*d) + (6*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*a^2*d) + (18*e^3*
(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(35*a^2*d) - (((4*I)/5)*e^2*(e*Sec[c +
 d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{(9e^2)\int (e\sec(c+dx))^{9/2}\,dx}{5a^2}$$

$$= \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{(9e^4)\int (e\,\mathrm{se}}{}$$

$$= \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie}{5d\,(}$$

$$= \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie}{5d\,(}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\,\mathrm{s}}{7a^2d}$$

Mathematica [A]
time = 0.65, size = 85, normalized size = 0.56

$$\frac{e^6\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-56i\cos(c+dx)+60\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-5\sin(c+dx)+15\sin(3(c+dx))\right)}{70a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]

[Out] (e^6*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-56*I)*Cos[c + d*x] + 60*Cos[c +
 d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[c + d*x] + 15*Sin[3*(c + d*x)
]))/(70*a^2*d)
```

Maple [A]
time = 0.69, size = 219, normalized size = 1.44

method	result

default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^4(dx+c))+15i\sqrt{\frac{}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(15*I*cos(d*x+c)^4*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c)
)/sin(d*x+c),I)+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3+15*cos(d*x+c)^2
*sin(d*x+c)-14*I*cos(d*x+c)-5*sin(d*x+c))*(e/cos(d*x+c))^(13/2)*cos(d*x+c)^
3/sin(d*x+c)^4
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 189, normalized size = 1.24

$$-\frac{2\left(\frac{\sqrt{2}\left(-15ie^{\frac{13}{2}}+15ie^{\left(6idx+6ic+\frac{13}{2}\right)}+51ie^{\left(4idx+4ic+\frac{13}{2}\right)}+61ie^{\left(2idx+2ic+\frac{13}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+15\left(i\sqrt{2}e^{\frac{13}{2}}+i\sqrt{2}e^{\left(6idx+6ic+\frac{13}{2}\right)}+3i\sqrt{2}e^{\left(4idx+4ic+\frac{13}{2}\right)}+3i\sqrt{2}e^{\left(2idx+2ic+\frac{13}{2}\right)}\right)\text{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)}{35\left(a^2de^{(6idx+6ic)}+3a^2de^{(4idx+4ic)}+3a^2de^{(2idx+2ic)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(-15*I*e^(13/2) + 15*I*e^(6*I*d*x + 6*I*c + 13/2) + 51*I*e^(
4*I*d*x + 4*I*c + 13/2) + 61*I*e^(2*I*d*x + 2*I*c + 13/2))*e^(1/2*I*d*x + 1
/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 15*(I*sqrt(2)*e^(13/2) + I*sqrt(2)*
e^(6*I*d*x + 6*I*c + 13/2) + 3*I*sqrt(2)*e^(4*I*d*x + 4*I*c + 13/2) + 3*I*s
qrt(2)*e^(2*I*d*x + 2*I*c + 13/2))*weierstrassPInverse(-4, 0, e^(I*d*x + I*
c)))/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(
2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(13/2)*sec(d*x + c)^(13/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d}+\frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d}$$

```
[Out] 14/15*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d-14/5*e^6*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+14/5*e^5*sin(d*x+c)*(e*sec(d*x+c))^(1/
2)/a^2/d-4/3*I*e^2*(e*sec(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2719}

$$-\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^2d}+\frac{14e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-14*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*d) + (14*e^3
*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^2*d) - (((4*I)/3)*e^2*(e*Sec[c
+ d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))} + \frac{(7e^2)\int(e\sec(c+dx))^{7/2}\,dx}{3a^2}$$

$$= \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))} + \frac{(7e^4)\int(e\,\mathrm{se}}{}$$

$$= \frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4i}{3d\,(}$$

$$= \frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4i}{3d\,(}$$

$$= -\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^2d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.17, size = 123, normalized size = 0.81

$$\frac{2ie^5e^{i(c+dx)}\left(-47-56e^{2i(c+dx)}-21e^{4i(c+dx)}+7\left(1+e^{2i(c+dx)}\right)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{15a^2d\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((2*I)/15)*e^5*E^(I*(c + d*x))*(-47 - 56*E^((2*I)*(c + d*x)) - 21*E^((4*I)
*(c + d*x)) + 7*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4,
 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*(1 + E^((2*I)*(c
+ d*x)))^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(158) = 316.
time = 0.70, size = 374, normalized size = 2.46

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(21i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(21*I*cos(d*x+c)^3*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c)
)/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(
d*x+c)+21*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+
c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I
*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*cos(d*x+c)^3-10*I*cos(d*x+c)*s
in(d*x+c)+24*cos(d*x+c)^2-3)*(e/cos(d*x+c))^(11/2)*cos(d*x+c)^3/sin(d*x+c)^
5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 158, normalized size = 1.04

$$-\frac{2\left(\frac{\sqrt{2}\left(21i\,e^{\left(5i\,dx+5i\,c+\frac{11}{2}\right)}+56i\,e^{\left(3i\,dx+3i\,c+\frac{11}{2}\right)}+47i\,e^{\left(i\,dx+i\,c+\frac{11}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+21\left(i\sqrt{2}\,e^{\frac{11}{2}}+i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{11}{2}\right)}+2i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{11}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(21*I*e^(5*I*d*x + 5*I*c + 11/2) + 56*I*e^(3*I*d*x + 3*I*c +
 11/2) + 47*I*e^(I*d*x + I*c + 11/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d
*x + 2*I*c) + 1) + 21*(I*sqrt(2)*e^(11/2) + I*sqrt(2)*e^(4*I*d*x + 4*I*c +
11/2) + 2*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 11/2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^
2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(11/2)*sec(d*x + c)^(11/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d}-\frac{4ie^2(e\sec(c+dx$$

```
[Out] 10/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/3*e^4*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d-4*I*e^2*(e*sec(d*x+c))^(5/2)/d/(a^2+I*a
^2*tan(d*x+c))
```

Rubi [A]

time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2720}

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{10e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (10*e^3*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((4*I)
*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))} + \frac{(5e^2)\int(e\sec(c+dx))^{5/2}\,dx}{a^2}$$

$$= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))} + \frac{(5e^4)\int\sqrt{e\,\mathrm{se}}}{3}$$

$$= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(5e^4\sqrt{\cos(c}\right.}{}$$

$$= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{10e^3(e\sec(c+dx))^{3/}}{3a^2d}$$

Mathematica [A]
time = 0.43, size = 67, normalized size = 0.56

$$\frac{2e^3(e\sec(c+dx))^{3/2}\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\sin(c+dx)\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
[Out] (2*e^3*(e*Sec[c + d*x])^(3/2)*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*a^2*d)
```

Maple [A]
time = 0.66, size = 201, normalized size = 1.69

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^2(dx+c))+5i\sqrt{\frac{}{1+}}\right.}{3a^2d\sin(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^2/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(5*I*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)
*cos(d*x+c)^2+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-6*I*cos(d*x+c)-sin(d
*x+c))*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^3/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 105, normalized size = 0.88

$$-\frac{2\left(\frac{\sqrt{2}\left(7ie^{\frac{9}{2}}+5ie^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+5\left(i\sqrt{2}\,e^{\frac{9}{2}}+i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(7*I*e^(9/2) + 5*I*e^(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x +
1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 5*(I*sqrt(2)*e^(9/2) + I*sqrt(2)*e
^(2*I*d*x + 2*I*c + 9/2))*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2
*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(9/2)*sec(d*x + c)^(9/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 6*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*e^3*sin(d*x+
c)*(e*sec(d*x+c))^(1/2)/a^2/d+4*I*e^2*(e*sec(d*x+c))^(3/2)/d/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A]

time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2719}

$$\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^2d}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^4*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (6*e^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((4*I)*e^2*(e*
Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(3e^2\right)\int (e\sec(c+dx))^{3/2}\,dx}{a^2}\\ &= -\frac{6e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^4\right)\int \frac{}{\sqrt{e\,\mathrm{se}}}}{a}\\ &= -\frac{6e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^4\right)\int \surd}{a^2\sqrt{\cos(c+}}\\ &= \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{4ie}{d\,(a}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.59, size = 80, normalized size = 0.70

$$\frac{2ie^3e^{-i(c+dx)}\left(-1+3\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((2*I)*e^3*(-1 + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*E^(I*(c + d*x))
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(131) = 262.
time = 0.70, size = 352, normalized size = 3.06

method	result

default	$\frac{2(1+\cos(dx+c))^2(\cos^3(dx+c))(-1+\cos(dx+c))^2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2/d*(1+cos(d*x+c))^2*cos(d*x+c)^3*(-1+cos(d*x+c))^2*(e/cos(d*x+c))^(7/2
)*(3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF
(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(
d*x+c),I)*cos(d*x+c)*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*
I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Ell
ipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+
c)^2+3*cos(d*x+c)-1)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 91, normalized size = 0.79

$$-\frac{2\left(-3i\sqrt{2}\,e^{\left(idx+ic+\frac{7}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)+\frac{\sqrt{2}\left(-2ie^{\frac{7}{2}}-3ie^{\left(2idx+2ic+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}\right)e^{(-idx-ic)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(-3*I*sqrt(2)*e^(I*d*x + I*c + 7/2)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-2*I*e^(7/2) - 3*I*e^(2*I*d*x +
2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*
d*x - I*c)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] -2/3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d+4/3*I*e^2*(
e*sec(d*x+c))^(1/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3581, 3856, 2720}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\,(a^2+ia^2\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+}}}{3a^2}$$
$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{4ie^2\sqrt{e\sec(c+da}}{3d\left(a^2+ia^2\tan(c+\right.}$$

Mathematica [A]
time = 0.47, size = 101, normalized size = 1.12

$$\frac{2(e\sec(c+dx))^{5/2}\left(-2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))\right)(\cos(c+dx)+i\sin(c+dx))}{3a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*(e*Sec[c + d*x])^(5/2)*((-2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]))*(Cos[c + d*x] + I*Sin[c
 + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.71, size = 201, normalized size = 2.23

method	result
default	$-\frac{2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{Ellipt}}{3a^2d\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^2/d*(I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)-2*I*cos(d*x+c)^2-2*sin(d*x+c)*cos(d*x+c))*(e/cos(d*x+c))^(5/2)*(-1+cos(
d*x+c))^2*(1+cos(d*x+c))^2*cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 0.98

$$-\frac{2\left(-i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-i\,e^{\frac{5}{2}}-i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(-I*sqrt(2)*e^(2*I*d*x + 2*I*c + 5/2)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)) + sqrt(2)*(-I*e^(5/2) - I*e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d
*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/5*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2/d/
(e*sec(d*x+c))^(1/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3581, 3856, 2719}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\,(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x
]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2}$$

$$= \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{e^2\int\sqrt{\cos(c+dx)}\,dx}{5a^2\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx}}$$

$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.64, size = 102, normalized size = 1.13

$$\frac{iee^{-3i(c+dx)}\left(1+e^{2i(c+dx)}+2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/5)*e*(1 + E^((2*I)*(c + d*x)) + 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[
e*Sec[c + d*x]])/(a^2*d*E^((3*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(104) = 208.
time = 0.69, size = 357, normalized size = 3.97

method	result
default	$\frac{2\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a^2/d*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c
)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-I*(1/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*Ellipt
```

```
icE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c
),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*Ell
ipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+2*I*sin(d*x+c)*cos(d*x+c)^3-2*cos(d*
x+c)^4+cos(d*x+c)^2+cos(d*x+c))*cos(d*x+c)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))
^2*(e/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 1.14

$$\frac{\left(2i\sqrt{2}\,e^{\left(3i\,dx+3i\,c+\frac{3}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)+\frac{\sqrt{2}\left(i\,e^{\frac{3}{2}}+2i\,e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+3i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-3i\,dx-3i\,c\right)}}{5\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(2*I*sqrt(2)*e^(3*I*d*x + 3*I*c + 3/2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(I*e^(3/2) + 2*I*e^(4*I*d*x +
 4*I*c + 3/2) + 3*I*e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt
(e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}{\tan^{2}\left(c+dx\right)-2i\tan\left(c+dx\right)-1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.240 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}+\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/7*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(e*sec(d*x+c))^(1/2)/a^2/d+4/7*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2720}

$$\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d) + (2*e*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)/7)*e^2)/
(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(3e^2)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2}$$

$$= \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\int\sqrt{e\,\mathrm{s}}}{}$$

$$= \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(\sqrt{\cos}\right.}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} +$$

Mathematica [A]
time = 0.52, size = 112, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(2i+2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-\sin(2(c+dx))\right)}{7a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/7*(Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]) - Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.98, size = 180, normalized size = 1.55

method	result
default	$\frac{2\sqrt{\frac{e}{\cos(dx+c)}}\left(2i(\cos^4(dx+c))+i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+2\sin(dx+c)(\mathrm{c}\right.}{7a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/a^2/d*(e/cos(d*x+c))^(1/2)*(2*I*cos(d*x+c)^4+I*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)+2*sin(d*x+c)*cos(d*x+c)^3+I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+sin(d*x+
c)*cos(d*x+c))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 100, normalized size = 0.86

$$\frac{\left(-4i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(i\,e^{\frac{1}{2}}+3i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+4i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-4i\,dx-4i\,c)}}{14\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/14*(-4*I*sqrt(2)*e^(4*I*d*x + 4*I*c + 1/2)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)) + sqrt(2)*(I*e^(1/2) + 3*I*e^(4*I*d*x + 4*I*c + 1/2) + 4*I*e^
(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
 1))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.241 $\int \frac{1}{\sqrt{e \sec(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/9*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)
^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2719}

$$\frac{4ie^2}{9d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{5/2}}+\frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*
x]]) + (2*e*Sin[c + d*x])/(9*a^2*d*(e*Sec[c + d*x])^(3/2)) + (((4*I)/9)*e^2
)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))} + \frac{(5e^2)\int \frac{1}{(e\sec(c+}}{9a^2}$$

$$= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c}$$

$$= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c}$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.58, size = 123, normalized size = 1.06

$$\frac{\left(-\frac{8e^{4i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2(2+8\cos(2(c+dx))+7i\sin(2(c+dx)))\right)(i\cos(2(c+dx))+\sin(2(c+dx)))}{18a^2d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-8*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(2 + 8*Cos[2*(c + d*x)] + (7*I)*Si
n[2*(c + d*x)]))*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(18*a^2*d*Sqrt[e*
Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(126) = 252.
time = 0.88, size = 368, normalized size = 3.17

method	result
default	$-\frac{2\left(-2i(\cos^5(dx+c))\sin(dx+c)+2(\cos^6(dx+c))+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\,\mathrm{s}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/a^2/d*(-2*I*cos(d*x+c)^5*sin(d*x+c)+2*cos(d*x+c)^6+3*I*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin
(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*c
os(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*El
lipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+
c),I)*sin(d*x+c)-cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))*(1+cos(d*x+c))^2
*(-1+cos(d*x+c))^2*(e/cos(d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 109, normalized size = 0.94

$$\frac{\left(24i\sqrt{2}\,e^{(5i\,dx+5i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(15i\,e^{(6i\,dx+6i\,c)}+19i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-5i\,dx-5i\,c-\frac{1}{2}\right)}}{36\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/36*(24*I*sqrt(2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(15*I*e^(6*I*d*x + 6*I*c) + 19*I*
e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c)/
sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*x - 5*I*c - 1/2)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}-\sqrt{e\sec{(c+dx)}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*sec(c + d*x)
)*tan(c + d*x) - sqrt(e*sec(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{e}{\cos{(c+d\,x)}}}\,{\left(a+a\tan{(c+d\,x)}\,1\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2}+\frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}}+\frac{}{11d(e}$$

```
[Out] 2/11*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(5/2)+10/33*sin(d*x+c)/a^2/d/e/(e*se
c(d*x+c))^(1/2)+10/33*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/
d/e^2+4/11*I*e^2/d/(e*sec(d*x+c))^(7/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2720}

$$\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2}+\frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*
a^2*d*e^2) + (2*e*Sin[c + d*x])/(11*a^2*d*(e*Sec[c + d*x])^(5/2)) + (10*Sin
[c + d*x])/(33*a^2*d*e*Sqrt[e*Sec[c + d*x]]) + (((4*I)/11)*e^2)/(d*(e*Sec[c
 + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2\right)\int \frac{}{(e\sec(\cdots}}{11a\cdots}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan\cdots\right.}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{}{11d(e\cdots}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{}{11d(e\cdots}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2\cdots}{11a^2d\cdots}$$

Mathematica [A]
time = 0.67, size = 134, normalized size = 0.89

$$-\frac{\sec^4(c+dx)\left(28i+24i\cos(2(c+dx))-4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-6\sin(2(c+dx))+7\sin(4(c+dx))\right)}{132a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -1/132*(Sec[c + d*x]^4*(28*I + (24*I)*Cos[2*(c + d*x)] - (4*I)*Cos[4*(c + d
*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)]))/(a^2*d*(e*S
ec[c + d*x])^(3/2)*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.90, size = 234, normalized size = 1.56

method	result
default	$\frac{2\cos(dx+c)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(6i(\cos^6(dx+c))+6\sin(dx+c)(\cos^5(dx+c))+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/33/a^2/d*cos(d*x+c)*(e/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))
^2*(6*I*cos(d*x+c)^6+6*sin(d*x+c)*cos(d*x+c)^5+5*I*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/s
in(d*x+c),I)+3*sin(d*x+c)*cos(d*x+c)^3+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*sin(
d*x+c)*cos(d*x+c))/e^3/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 117, normalized size = 0.78

$$\frac{\left(-80i\sqrt{2}\,e^{(6i\,dx+6i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-11i\,e^{(8i\,dx+8i\,c)}+30i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,dx+4i\,c)}+18i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-6i\,dx-6i\,c-\frac{3}{2}\right)}}{264\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/264*(-80*I*sqrt(2)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e^(I*d*
x + I*c)) + sqrt(2)*(-11*I*e^(8*I*d*x + 8*I*c) + 30*I*e^(6*I*d*x + 6*I*c) +
 56*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x +
1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-6*I*d*x - 6*I*c - 3/2)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x))
**(3/2)*tan(c + d*x) - (e*sec(c + d*x))**(3/2)), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}+\frac{}{13d(e\sec(c+d}$$

```
[Out] 2/13*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(7/2)+14/65*sin(d*x+c)/a^2/d/e/(e*se
c(d*x+c))^(3/2)+42/65*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^
(1/2)+4/13*I*e^2/d/(e*sec(d*x+c))^(9/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2719}

$$\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}+\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (42*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (2*e*Sin[c + d*x])/(13*a^2*d*(e*Sec[c + d*x])^(7/2)) + (14*Sin
[c + d*x])/(65*a^2*d*e*(e*Sec[c + d*x])^(3/2)) + (((4*I)/13)*e^2)/(d*(e*Sec
[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx &= \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{(9e^2)\int \frac{}{(e\sec(}}{13a}\\ &= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\,\mathrm{ta}\right.}\\ &= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}+\frac{}{13d(e}\\ &= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}+\frac{}{13d(e}\\ &= \frac{42E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx}{13a^2d(e\sec(c+a}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.46, size = 149, normalized size = 0.99

$$\frac{(\cos(2(c+dx))-i\sin(2(c+dx)))\left(88i+416i\cos(2(c+dx))-8i\cos(4(c+dx))-\frac{224ie^{4i(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-356\sin(2(c+dx))+18\sin(4(c+dx))\right)}{520a^2de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(88*I + (416*I)*Cos[2*(c + d*x)] -
 (8*I)*Cos[4*(c + d*x)] - ((224*I)*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/
2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - 356*Sin
[2*(c + d*x)] + 18*Sin[4*(c + d*x)]))/(520*a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(156) = 312.
time = 1.12, size = 386, normalized size = 2.57

method	result
default	$-\frac{2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(\cos^2(dx+c)\right)\left(-10i\sin(dx+c)\left(\cos^7(dx+c)\right)+10\left(\cos^8(dx+c)\right)-5\left(\cos^6(dx+c)\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/65/a^2/d*(e/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2*cos(d*x
+c)^2*(-10*I*cos(d*x+c)^7*sin(d*x+c)+10*cos(d*x+c)^8-5*cos(d*x+c)^6+21*I*(1
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x
+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(d*x+c)*cos(d*x+c)*El
lipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)+21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(
d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+
c))/e^5/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 131, normalized size = 0.87

$$\frac{\left(672i\sqrt{2}\,e^{(7i\,dx+7i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(-13i\,e^{(10i\,dx+10i\,c)}+373i\,e^{(8i\,dx+8i\,c)}+474i\,e^{(6i\,dx+6i\,c)}+118i\,e^{(4i\,dx+4i\,c)}+35i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-7i\,dx-7i\,c-\frac{5}{2}\right)}}{1040\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1040*(672*I*sqrt(2)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-13*I*e^(10*I*d*x + 10*I*c) +
 373*I*e^(8*I*d*x + 8*I*c) + 474*I*e^(6*I*d*x + 6*I*c) + 118*I*e^(4*I*d*x +
```

```
4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2
*I*d*x + 2*I*c) + 1))*e^(-7*I*d*x - 7*I*c - 5/2)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{5}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{5}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x))
**(5/2)*tan(c + d*x) - (e*sec(c + d*x))**(5/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(-5/2)/((I*a*tan(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=181

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4}+\frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}}+\frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}}+\cdots$$

```
[Out] 2/15*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(9/2)+6/35*sin(d*x+c)/a^2/d/e/(e*sec
(d*x+c))^(5/2)+2/7*sin(d*x+c)/a^2/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d/e^4+4/15*I*e^2/d/(e*sec(d*x+c)
)^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4}+\frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}+\frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}}+\frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d*e^4) + (2*e*Sin[c + d*x])/(15*a^2*d*(e*Sec[c + d*x])^(9/2)) + (6*Sin[c
+ d*x])/(35*a^2*d*e*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c + d*x])/(7*a^2*d*e^3
*Sqrt[e*Sec[c + d*x]]) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)*(a^2 +
 I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(11e^2)\int \frac{}{(e\,\mathrm{se}}}{1}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\,\mathrm{t}\right.}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{}{15d(e}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{}{7a^2de}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{}{7a^2de}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2e}{15a^2d(}$$

Mathematica [A]
time = 1.09, size = 151, normalized size = 0.83

$$-\frac{(e\sec(c+dx))^{5/2}\left(296i+228i\cos(2(c+dx))-72i\cos(4(c+dx))-4i\cos(6(c+dx))+480\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-17\sin(2(c+dx))+128\sin(4(c+dx))+11\sin(6(c+dx))\right)}{1680a^2de^6(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -1/1680*((e*Sec[c + d*x])^(5/2)*(296*I + (228*I)*Cos[2*(c + d*x)] - (72*I)*
Cos[4*(c + d*x)] - (4*I)*Cos[6*(c + d*x)] + 480*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 17*Sin[2*(c + d
*x)] + 128*Sin[4*(c + d*x)] + 11*Sin[6*(c + d*x)]))/(a^2*d*e^6*(-I + Tan[c
+ d*x])^2)
```

Maple [A]
time = 1.27, size = 252, normalized size = 1.39

method	result
default	$\frac{2(\cos^3(dx+c))\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(14i(\cos^8(dx+c))+14\sin(dx+c)(\cos^7(dx+c))+7\sin(dx+c)(\cos^5(dx-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/a^2/d*cos(d*x+c)^3*(e/cos(d*x+c))^(7/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+
c))^2*(14*I*cos(d*x+c)^8+14*sin(d*x+c)*cos(d*x+c)^7+7*sin(d*x+c)*cos(d*x+c)
^5+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*sin(d*x+c)*cos(d*x+c)^3+15*I
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1
+cos(d*x+c))/sin(d*x+c),I)+15*sin(d*x+c)*cos(d*x+c))/e^7/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 139, normalized size = 0.77

$$\frac{\left(-960i\sqrt{2}\,e^{(8i\,dx+8i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-15i\,e^{(12i\,dx+12i\,c)}-200i\,e^{(10i\,dx+10i\,c)}+245i\,e^{(8i\,dx+8i\,c)}+592i\,e^{(6i\,dx+6i\,c)}+211i\,e^{(4i\,dx+4i\,c)}+56i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-8i\,dx-8i\,c-\frac{7}{2}\right)}}{3360\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/3360*(-960*I*sqrt(2)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)) + sqrt(2)*(-15*I*e^(12*I*d*x + 12*I*c) - 200*I*e^(10*I*d*x + 10
*I*c) + 245*I*e^(8*I*d*x + 8*I*c) + 592*I*e^(6*I*d*x + 6*I*c) + 211*I*e^(4*
I*d*x + 4*I*c) + 56*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c)/sq
rt(e^(2*I*d*x + 2*I*c) + 1))*e^(-8*I*d*x - 8*I*c - 7/2)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((I*a*tan(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d}+\frac{22e^5(e\sec}{}$$

```
[Out] -22/21*I*e^4*(e*sec(d*x+c))^(7/2)/a^3/d+22/15*e^5*(e*sec(d*x+c))^(5/2)*sin(
d*x+c)/a^3/d-22/5*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2
)+22/5*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d-4/3*I*e^2*(e*sec(d*x+c))^(
11/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.15, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3581, 3582, 3853, 3856, 2719}

$$-\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^3d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^3d}-\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((22*I)/21)*e^4*(e*Sec[c + d*x])^(7/2))/(a^3*d) + (22*e^7*Sqr
t[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) + (22*e^5*(e*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(15*a^3*d) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a
+ I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2} + \frac{(11e^2)\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx}{3a^2}$$

$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2} + \frac{(11e^4)\int(e\sec(c+dx}{3a^3}$$

$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d} - \frac{4ie^2(e\sec(}{3ad(a+ia\,t}$$

$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c-}{}$$

$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c-}{}$$

$$= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\,\mathrm{se}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.87, size = 128, normalized size = 0.72

$$-\frac{e^6(e\sec(c+dx))^{3/2}\left(-556-868\cos(2(c+dx))+77e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+203i\sec(c+dx)\sin(3(c+dx))+143i\tan(c+dx)\right)(-i+\tan(c+dx))}{210a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/210*(e^6*(e*Sec[c + d*x])^(3/2)*(-556 - 868*Cos[2*(c + d*x)] + (77*(1 +
E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^((2*I)*(c + d*x)) + (203*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (143*
I)*Tan[c + d*x])*(-I + Tan[c + d*x]))/(a^3*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(179) = 358$.
time = 0.78, size = 392, normalized size = 2.20

method	result
default	$-\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(231i(\cos^4(dx+c))\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(231*I*sin(d*x+c)*cos(d*x+c
)^4*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*
(-1+cos(d*x+c))/sin(d*x+c),I)-231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin
(d*x+c),I)+231*I*sin(d*x+c)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-231*I*si
n(d*x+c)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+140*I*cos(d*x+c)^2*sin(d*x+
c)+231*cos(d*x+c)^4-294*cos(d*x+c)^3-15*I*sin(d*x+c)+63*cos(d*x+c))*(e/cos(
d*x+c))^(15/2)*cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 200, normalized size = 1.12

$$-\frac{2\left(\frac{\sqrt{2}\left(231ie^{\left(7idx+7ic+\frac{15}{2}\right)}+847ie^{\left(5idx+5ic+\frac{15}{2}\right)}+1133ie^{\left(3idx+3ic+\frac{15}{2}\right)}+637ie^{\left(idx+ic+\frac{15}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+231\left(i\sqrt{2}e^{\frac{15}{2}}+i\sqrt{2}e^{\left(6idx+6ic+\frac{15}{2}\right)}+3i\sqrt{2}e^{\left(4idx+4ic+\frac{15}{2}\right)}+3i\sqrt{2}e^{\left(2idx+2ic+\frac{15}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)\right)}{105\left(a^3de^{(6idx+6ic)}+3a^3de^{(4idx+4ic)}+3a^3de^{(2idx+2ic)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(sqrt(2)*(231*I*e^(7*I*d*x + 7*I*c + 15/2) + 847*I*e^(5*I*d*x + 5*I*
c + 15/2) + 1133*I*e^(3*I*d*x + 3*I*c + 15/2) + 637*I*e^(I*d*x + I*c + 15/2
))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 231*(I*sqrt(2)*e
^(15/2) + I*sqrt(2)*e^(6*I*d*x + 6*I*c + 15/2) + 3*I*sqrt(2)*e^(4*I*d*x + 4
*I*c + 15/2) + 3*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 15/2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^3*d*e^(6*I*d*x + 6*I*c)
 + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(15/2)*sec(d*x + c)^(15/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.246 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^3d}-\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}+\frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d}$$

```
[Out] -18/5*I*e^4*(e*sec(d*x+c))^(5/2)/a^3/d+6*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c
)/a^3/d+6*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d-4*I*e^
2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.13, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3581, 3582, 3853, 3856, 2720}

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^3d}+\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^3d}-\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
a^3*d) - (((18*I)/5)*e^4*(e*Sec[c + d*x])^(5/2))/(a^3*d) + (6*e^5*(e*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(a^3*d) - ((4*I)*e^2*(e*Sec[c + d*x])^(9/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(9e^2)\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(9e^4)\int(e\sec(c+dx))}{a^3}$$

$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c}{ad(a+ia\tan}$$

$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c}{ad(a+ia\tan}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}$$

Mathematica [A]
time = 0.75, size = 74, normalized size = 0.52

$$\frac{e^4(e\sec(c+dx))^{5/2}\left(-18i-20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)-5\sin(2(c+dx))\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(5/2)*(-18*I - (20*I)*Cos[2*(c + d*x)] + 30*Cos[c + d
*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*a^3*d)
```

Maple [A]
time = 0.76, size = 213, normalized size = 1.51

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^3(dx+c))+15i\sqrt{\frac{}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a^3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(15*I*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I
)*cos(d*x+c)^3+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^2-20*I*cos(d*x+c)^
2-5*sin(d*x+c)*cos(d*x+c)+I)*(e/cos(d*x+c))^(13/2)*cos(d*x+c)^4/sin(d*x+c)^
4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 147, normalized size = 1.04

$$-\frac{2\left(\frac{\sqrt{2}\left(25ie^{\frac{13}{2}}+15ie^{\left(4i\,dx+4i\,c+\frac{13}{2}\right)}+36ie^{\left(2i\,dx+2i\,c+\frac{13}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+15\left(i\sqrt{2}\,e^{\frac{13}{2}}+i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{13}{2}\right)}+2i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{13}{2}\right)}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{5\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(2)*(25*I*e^(13/2) + 15*I*e^(4*I*d*x + 4*I*c + 13/2) + 36*I*e^(2*
I*d*x + 2*I*c + 13/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1
) + 15*(I*sqrt(2)*e^(13/2) + I*sqrt(2)*e^(4*I*d*x + 4*I*c + 13/2) + 2*I*sqr
t(2)*e^(2*I*d*x + 2*I*c + 13/2))*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)
))/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(13/2)*sec(d*x + c)^(13/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}-\frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^3d}+\frac{4ie^2(e\sec(c$$

```
[Out] 14/3*I*e^4*(e*sec(d*x+c))^(3/2)/a^3/d+14*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1
/2)/(e*sec(d*x+c))^(1/2)-14*e^5*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d+4*I*e
^2*(e*sec(d*x+c))^(7/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3581, 3582, 3853, 3856, 2719}

$$\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^3d}+\frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (14*e^6*EllipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((14*I)/3)*e^4*(e*Sec[c + d*x])^(3/2))/(a^3*d) - (14*e^5*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(a^3*d) + ((4*I)*e^2*(e*Sec[c + d*x])^(7/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} - \frac{(7e^2)\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} - \frac{(7e^4)\int (e\sec(c+dx))^{3/}}{a^3}$$

$$= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+}{ad(a+ia\tan(c}$$

$$= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+}{ad(a+ia\tan(c}$$

$$= \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.14, size = 93, normalized size = 0.66

$$\frac{ie^4(e\sec(c+dx))^{3/2}\left(35+33\cos(2(c+dx))-7\left(1+e^{2i(c+dx)}\right)^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+9i\sin(2(c+dx))\right)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*e^4*(e*Sec[c + d*x])^(3/2)*(35 + 33*Cos[2*(c + d*x)] - 7*(1 + E^((2*
I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
 + (9*I)*Sin[2*(c + d*x)]))/(a^3*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(152) = 304$.
time = 0.81, size = 388, normalized size = 2.75

method	result
default	$-\frac{2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{11}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(\cos^4(dx+c)\right)\left(21i\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticE}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3/d*(e/cos(d*x+c))^(11/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2*cos(d*x
+c)^4*(21*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+
c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I
*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+21*I*(1/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*
x+c))/sin(d*x+c),I)-21*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/
sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-12
*I*cos(d*x+c)^2*sin(d*x+c)+12*cos(d*x+c)^3-21*cos(d*x+c)^2-I*sin(d*x+c)+9*c
os(d*x+c))/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 137, normalized size = 0.97

$$-\frac{2\left(\frac{\sqrt{2}\left(-12i\,e^{\frac{11}{2}}-21i\,e^{\left(4i\,dx+4i\,c+\frac{11}{2}\right)}-35i\,e^{\left(2i\,dx+2i\,c+\frac{11}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+21\left(-i\,\sqrt{2}\,e^{\left(3i\,dx+3i\,c+\frac{11}{2}\right)}-i\,\sqrt{2}\,e^{\left(i\,dx+i\,c+\frac{11}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{3\left(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-12*I*e^(11/2) - 21*I*e^(4*I*d*x + 4*I*c + 11/2) - 35*I*e^(2
*I*d*x + 2*I*c + 11/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
1) + 21*(-I*sqrt(2)*e^(3*I*d*x + 3*I*c + 11/2) - I*sqrt(2)*e^(I*d*x + I*c +
 11/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))
)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(11/2)*sec(d*x + c)^(11/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{11/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] 10/3*I*e^4*(e*sec(d*x+c))^(1/2)/a^3/d-10/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
(e*sec(d*x+c))^(1/2)/a^3/d+4/3*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*
x+c))^2
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3582, 3856, 2720}

$$\frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((10*I)/3)*e^4*Sqrt[e*Sec[c + d*x]])/(a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a^3*d) + (((4*I)/3)*e^2*
(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{(5e^2)\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx}{3a^2}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{(5e^4)\int \sqrt{e\sec(c+dx)}}{3a^3}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{\left(5e^4\sqrt{\cos(c+dx)}\,\sqrt{e}\right.}{}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d}$$

Mathematica [A]
time = 0.50, size = 125, normalized size = 1.08

$$\frac{2e^4\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))+3\sin(c+dx)\right)(-i\cos(2(c+dx))+\sin(2(c+dx)))}{3a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e^4*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-7*I)*Cos[c + d*x] + 5*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + 3*S
in[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*a^3*d*(-I + Tan
[c + d*x])^3)
```

Maple [A]
time = 0.78, size = 203, normalized size = 1.75

method	result

default	$\frac{2\left(-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-5i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{Ellipti}}{3a^3d\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^3/d*(-5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*
x+c),I)+4*I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)+3*I)*cos(d*x+c)^4*(e/cos(d
*x+c))^(9/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 88, normalized size = 0.76

$$-\frac{2\left(-5i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-2i\,e^{\frac{9}{2}}-5i\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(-5*I*sqrt(2)*e^(2*I*d*x + 2*I*c + 9/2)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)) + sqrt(2)*(-2*I*e^(9/2) - 5*I*e^(2*I*d*x + 2*I*c + 9/2))*e^(1
/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a^
3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(e^(9/2)*sec(d*x + c)^(9/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.249 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

```
[Out] -6/5*I*e^4/a^3/d/(e*sec(d*x+c))^(1/2)-6/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(
1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x
+c))^2
```

Rubi [A]

time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3582, 3856, 2719}

$$-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-6*I)/5)*e^4)/(a^3*d*Sqrt[e*Sec[c + d*x]]) - (6*e^4*EllipticE[(c + d*x)/
2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (((4*I)/5)*e^2*(
e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2} - \frac{(3e^2)\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx}{5a^2}$$

$$= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2} - \frac{(3e^4)\int \frac{1}{\sqrt{e\sec(c+dx}}}{5a^3}$$

$$= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2} - \frac{(3e^4)\int \sqrt{\cos(c+}}{5a^3\sqrt{\cos(c+dx)}\sqrt{e}}$$

$$= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} - \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec}{5ad(a+ia}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.71, size = 117, normalized size = 1.01

$$\frac{2ee^{-idx}\left(-2+\frac{6e^{2i(c+dx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)(e\sec(c+dx))^{5/2}(\cos(c+2dx)+i\sin(c+2dx))}{5a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e*(-2 + (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))])*(e*Sec[c + d*x])^(5/2)*(Cos[
c + 2*d*x] + I*Sin[c + 2*d*x]))/(5*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(125) = 250.
time = 0.80, size = 378, normalized size = 3.26

method	result
default	$\frac{2\left(-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a^3/d*(-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+4*I*cos(d*x+c)^3*sin(d*x+c)-3*I*(1
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)
-4*cos(d*x+c)^4-5*I*cos(d*x+c)*sin(d*x+c)+7*cos(d*x+c)^2-3*cos(d*x+c))*cos(
d*x+c)^3*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(e/cos(d*x+c))^(7/2)/sin(d*x+c)
^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 0.89

$$-\frac{2\left(3i\sqrt{2}\,e^{\left(3i\,dx+3i\,c+\frac{7}{2}\right)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)+\frac{\sqrt{2}\left(-i\,e^{\frac{7}{2}}+3i\,e^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+2i\,e^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-3i\,dx-3i\,c\right)}}{5\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(3*I*sqrt(2)*e^(3*I*d*x + 3*I*c + 7/2)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*e^(7/2) + 3*I*e^(4*I*d*x
 + 4*I*c + 7/2) + 2*I*e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sq
rt(e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] -2/21*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+4/7*I*e^2*
(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^2-2/21*I*e^2*(e*sec(d*x+c))^(1/
2)/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]

time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3583, 3856, 2720}

$$-\frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^3*d) + (((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x
])^2) - (((2*I)/21)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^3 + I*a^3*Tan[c + d*x])
)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx}{7a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{e^2\int \sqrt{e\sec(c+dx}}{21a^3}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx}}{7ad(a+ia\tan(c+}$$

Mathematica [A]
time = 0.69, size = 104, normalized size = 0.79

$$\frac{(e\sec(c+dx))^{5/2}\left(-5i-5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-\sin(2(c+dx))\right)}{21a^3d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((e*Sec[c + d*x])^(5/2)*(-5*I - (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Sin[
2*(c + d*x)]))/(21*a^3*d*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 0.75, size = 228, normalized size = 1.73

method	result

default	$\frac{2\left(12i(\cos^4(dx+c))-i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+12\sin(dx+c)(\cos^3(dx+c))-i\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a^3/d*(12*I*cos(d*x+c)^4-I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+12*sin
(d*x+c)*cos(d*x+c)^3-I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-7*I*cos(d*x+c)^2-sin(d*x+c
)*cos(d*x+c))*cos(d*x+c)^2*(e/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))^2*(1+cos(d*
x+c))^2/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 100, normalized size = 0.76

$$\frac{\left(2i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(3i\,e^{\frac{5}{2}}+2i\,e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+5i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-4i\,dx-4i\,c)}}{21\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/21*(2*I*sqrt(2)*e^(4*I*d*x + 4*I*c + 5/2)*weierstrassPInverse(-4, 0, e^(I
*d*x + I*c)) + sqrt(2)*(3*I*e^(5/2) + 2*I*e^(4*I*d*x + 4*I*c + 5/2) + 5*I*e
^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c)
+ 1))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 -
 3*tan(c + d*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2}+\frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+$$

```
[Out] 2/15*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2/a
/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/45*I*e^2/d/(e*sec(d*x+c))^(1
/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3583, 3856, 2719}

$$\frac{2ie^2}{45d\,(a^3+ia^3\tan(c+dx))\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]
)^2) + (((2*I)/45)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx}}{9a^2}$$

$$= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan}$$

$$= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan}$$

$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.93, size = 140, normalized size = 1.06

$$-\frac{e^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(8+8\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+3i\sin(2(c+dx))\right)}{45a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/45*(Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(8 + 8
*Cos[2*(c + d*x)] + 6*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x
)]))/(a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(140) = 280.
time = 0.81, size = 388, normalized size = 2.94

method	result
default	$\frac{2\left(20i(\cos^5(dx+c))\sin(dx+c)+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-3i\sqrt{\frac{}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/a^3/d*(20*I*cos(d*x+c)^5*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(
d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-20*co
s(d*x+c)^6-9*I*cos(d*x+c)^3*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*
x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+19*cos(d*x+c)^4-2*cos(d*x+c)^
2+3*cos(d*x+c))*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(e/cos(d*x+c))^(3/2)*cos
(d*x+c)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 115, normalized size = 0.87

$$\frac{\left(12i\sqrt{2}\,e^{\left(5i\,dx+5i\,c+\frac{3}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(5ie^{\frac{3}{2}}+12ie^{\left(6i\,dx+6i\,c+\frac{3}{2}\right)}+23ie^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+16ie^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-5i\,dx-5i\,c)}}{90\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/90*(12*I*sqrt(2)*e^(5*I*d*x + 5*I*c + 3/2)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(5*I*e^(3/2) + 12*I*e^(6*I*
d*x + 6*I*c + 3/2) + 23*I*e^(4*I*d*x + 4*I*c + 3/2) + 16*I*e^(2*I*d*x + 2*I
*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d
*x - 5*I*c)/(a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 -
 3*tan(c + d*x) + I), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.252 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d}+\frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}+\frac{}{77d(}$$

```
[Out] 10/77*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(1/2)+10/77*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+2/11*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*
x+c))^3+20/77*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3583, 3581, 3854, 3856, 2720}

$$\frac{20ie^2}{77d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a^3*d) + (10*e*Sin[c + d*x])/(77*a^3*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/11)*
Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (((20*I)/77)*e^2)/(d*(
e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\,(a^3+ia^3\tan(c+dx))} + \frac{(1\cdots}{\cdots}$$

$$= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie\cdots}{77d(e\sec(c+dx))^{3/2}\,(a\cdots}$$

$$= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie\cdots}{77d(e\sec(c+dx))^{3/2}\,(a\cdots}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} \cdots$$

Mathematica [A]

time = 0.54, size = 129, normalized size = 0.85

$$\frac{i\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(46i\cos(c+dx)+22i\cos(3(c+dx))-15\sin(c+dx)+20\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(3(c+dx))+i\sin(3(c+dx)))-15\sin(3(c+dx))\right)}{154a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/154)*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((46*I)*Cos[c + d*x] + (22*I)*
Cos[3*(c + d*x)] - 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)]))/(
a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 1.12, size = 208, normalized size = 1.37

method	result
default	$\frac{2\sqrt{\frac{e}{\cos(dx+c)}}\left(28i\left(\cos^6(dx+c)\right)+28\sin(dx+c)\left(\cos^5(dx+c)\right)-11i\left(\cos^4(dx+c)\right)+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/a^3/d*(e/cos(d*x+c))^(1/2)*(28*I*cos(d*x+c)^6+28*sin(d*x+c)*cos(d*x+c)
^5-11*I*cos(d*x+c)^4+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*sin(d*x+c)*
cos(d*x+c)^3+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*sin(d*x+c)*cos(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 0.74

$$\frac{\left(-40i\sqrt{2}\,e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)+\frac{\sqrt{2}\left(7i\,e^{\frac{1}{2}}+37i\,e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}+61i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+31i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-6i\,dx-6i\,c\right)}}{308\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/308*(-40*I*sqrt(2)*e^(6*I*d*x + 6*I*c + 1/2)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(2)*(7*I*e^(1/2) + 37*I*e^(6*I*d*x + 6*I*c + 1/2) + 6
```

```
1*I*e^(4*I*d*x + 4*I*c + 1/2) + 31*I*e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*
x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{\sqrt{e\sec(c+dx)}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*
tan(c + d*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.253 $\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{2i}{13d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}+$$

```
[Out] 14/117*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(3/2)+14/39*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d
*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/13*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d
*x+c))^3+28/117*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]

time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3583, 3581, 3854, 3856, 2719}

$$\frac{28ie^2}{117d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (14*e*Sin[c + d*x])/(117*a^3*d*(e*Sec[c + d*x])^(3/2)) + ((2*I)/13
)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((28*I)/117)*e^2)/(d
*(e*Sec[c + d*x])^(5/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}\,dx = \frac{2i}{13d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{7\int \frac{}{\sqrt{e\sec(c+}}}{}$$

$$= \frac{2i}{13d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{}{117d(e\sec(c+}$$

$$= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{2i}{13d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{2i}{13d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c}$$

$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.61, size = 145, normalized size = 0.95

$$\frac{\sqrt{e\sec(c+dx)}\,(i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+176\cos(2(c+dx))+114\cos(4(c+dx))-56e^{4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+126i\sin(2(c+dx))+105i\sin(4(c+dx))\right)}{468a^3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sqrt[e*Sec[c + d*x]]*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 + 176*Cos
[2*(c + d*x)] + 114*Cos[4*(c + d*x)] - 56*E^((4*I)*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (
126*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(468*a^3*d*e)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(156) = 312$.
time = 1.09, size = 395, normalized size = 2.60

method	result
default	$-\frac{2\left(-36i\sin(dx+c)\left(\cos^7(dx+c)\right)+36\left(\cos^8(dx+c)\right)+13i\left(\cos^5(dx+c)\right)\sin(dx+c)-31\left(\cos^6(dx+c)\right)+21i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(d}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/117/a^3/d*(-36*I*cos(d*x+c)^7*sin(d*x+c)+36*cos(d*x+c)^8+13*I*cos(d*x+c)
^5*sin(d*x+c)-31*cos(d*x+c)^6+21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*
x+c),I)-21*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I
)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*I*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-
1+cos(d*x+c))/sin(d*x+c),I)-21*I*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin
(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*cos
(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*(e/cos(d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 0.79

$$\frac{\left(336i\sqrt{2}\,e^{(7i\,dx+7i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\,\left(219i\,e^{(8i\,dx+8i\,c)}+302i\,e^{(6i\,dx+6i\,c)}+124i\,e^{(4i\,dx+4i\,c)}+50i\,e^{(2i\,dx+2i\,c)}+9i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-7i\,dx-7i\,c-\frac{1}{2}\right)}}{936\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/936*(336*I*sqrt(2)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(219*I*e^(8*I*d*x + 8*I*c) + 30
2*I*e^(6*I*d*x + 6*I*c) + 124*I*e^(4*I*d*x + 4*I*c) + 50*I*e^(2*I*d*x + 2*I
*c) + 9*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7*I*d
*x - 7*I*c - 1/2)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{\sqrt{e\sec\left(c+dx\right)}\tan^3\left(c+dx\right)-3i\sqrt{e\sec\left(c+dx\right)}\tan^2\left(c+dx\right)-3\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)+i\sqrt{e\sec\left(c+dx\right)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**3 - 3*I*sqrt(e*sec(c + d*x
))*tan(c + d*x)**2 - 3*sqrt(e*sec(c + d*x))*tan(c + d*x) + I*sqrt(e*sec(c +
 d*x))), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=186

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}+\frac{}{15d(e\,\mathrm{s}}$$

```
[Out] 6/55*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(5/2)+2/11*sin(d*x+c)/a^3/d/e/(e*sec
(d*x+c))^(1/2)+2/11*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d/
e^2+2/15*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3+12/55*I*e^2/d/(e*sec
(d*x+c))^(7/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3583, 3581, 3854, 3856, 2720}

$$\frac{12ie^2}{55d\,(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{7/2}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(11*a
^3*d*e^2) + (6*e*Sin[c + d*x])/(55*a^3*d*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c
 + d*x])/(11*a^3*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/15)/(d*(e*Sec[c + d*x])
^(3/2)*(a + I*a*Tan[c + d*x])^3) + (((12*I)/55)*e^2)/(d*(e*Sec[c + d*x])^(7
/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{3\int \frac{}{(e\sec(c+dx))}}{}$$

$$= \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{}{55d(e\sec(c+}$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(}$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{}{15d(e}$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{}{15d(e}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2} + \frac{6e}{55a^3d(}$$

Mathematica [A]
time = 0.79, size = 151, normalized size = 0.81

$$\frac{\sec^5(c+dx)\left(-332\cos(c+dx)-154\cos(3(c+dx))+22\cos(5(c+dx))-114i\sin(c+dx)+240i\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(3(c+dx))+i\sin(3(c+dx)))-81i\sin(3(c+dx))+33i\sin(5(c+dx))\right)}{1320a^3d(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sec[c + d*x]^5*(-332*Cos[c + d*x] - 154*Cos[3*(c + d*x)] + 22*Cos[5*(c + d
*x)] - (114*I)*Sin[c + d*x] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (81*I)*Sin[3*(c + d*x)] +
 (33*I)*Sin[5*(c + d*x)]))/(1320*a^3*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c +
 d*x])^3)
```

Maple [A]
time = 1.06, size = 261, normalized size = 1.40

method	result
default	$\frac{2\cos(dx+c)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(44i(\cos^8(dx+c))+44\sin(dx+c)(\cos^7(dx+c))-15i(\cos^6(dx+c))+7\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/165/a^3/d*cos(d*x+c)*(e/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c)
)^2*(44*I*cos(d*x+c)^8+44*sin(d*x+c)*cos(d*x+c)^7-15*I*cos(d*x+c)^6+7*sin(d
*x+c)*cos(d*x+c)^5+15*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+15*I*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c
))/sin(d*x+c),I)+9*sin(d*x+c)*cos(d*x+c)^3+15*sin(d*x+c)*cos(d*x+c))/e^3/si
n(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 128, normalized size = 0.69

$$\frac{\left(-480i\sqrt{2}\,e^{(8i\,dx+8i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(-55i\,e^{(10i\,dx+10i\,c)}+235i\,e^{(8i\,dx+8i\,c)}+446i\,e^{(6i\,dx+6i\,c)}+218i\,e^{(4i\,dx+4i\,c)}+73i\,e^{(2i\,dx+2i\,c)}+11i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-8i\,dx-8i\,c-\frac{3}{2}\right)}}{2640\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/2640*(-480*I*sqrt(2)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)) + sqrt(2)*(-55*I*e^(10*I*d*x + 10*I*c) + 235*I*e^(8*I*d*x + 8*I
*c) + 446*I*e^(6*I*d*x + 6*I*c) + 218*I*e^(4*I*d*x + 4*I*c) + 73*I*e^(2*I*d
*x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*
e^(-8*I*d*x - 8*I*c - 3/2)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int\frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\tan^{3}{(c+dx)}-3i(e\sec{(c+dx)})^{\frac{3}{2}}\tan^{2}{(c+dx)}-3(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}+i(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**3 - 3*I*(e*sec(c + d*x)
)**(3/2)*tan(c + d*x)**2 - 3*(e*sec(c + d*x))**(3/2)*tan(c + d*x) + I*(e*se
c(c + d*x))**(3/2)), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$\frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d}-\frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d}$$

```
[Out] -154/15*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+154/5*e^8*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/
d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-154/5*e^7*sin(d*x+c)*(e*sec(d*x+c))
^(1/2)/a^4/d+4*I*e^2*(e*sec(d*x+c))^(11/2)/a/d/(a+I*a*tan(d*x+c))^3+44/3*I*
e^4*(e*sec(d*x+c))^(7/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2719}

$$\frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{154e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{154e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^4d}+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (154*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (154*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) - (154*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^4*d) + ((4*I)*e^2*(e*Sec[c +
d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + (((44*I)/3)*e^4*(e*Sec[c + d
*x])^(7/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} - \frac{(11e^2)\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2} \\
&= \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))} - \frac{(77e^4)\int(e\sec(c+dx}{3a^4} \\
&= -\frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\,s}{3d\,(a^4+i} \\
&= -\frac{154e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \\
&= -\frac{154e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \\
&= \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} - \frac{154e^7\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} -
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.59, size = 124, normalized size = 0.65

$$-\frac{ie^5(e\sec(c+dx))^{5/2}\left(-1133\cos(c+dx)+77e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3(117\cos(3(c+dx))+33i\sin(c+dx)+37i\sin(3(c+dx)))\right)}{30a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/30*I)*e^5*(e*Sec[c + d*x])^(5/2)*(-1133*Cos[c + d*x] + (77*(1 + E^((2*
I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)/E^(I*(c + d*x)) - 3*(117*Cos[3*(c + d*x)] + (33*I)*Sin[c + d*x] + (37*I)*
Sin[3*(c + d*x)])))/(a^4*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(194) = 388$.
time = 0.88, size = 401, normalized size = 2.09

method	result
default	$-\frac{2\left(\cos^5(dx+c)\right)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{15}{2}}(-1+\cos(dx+c))^2(1+\cos(dx+c))^2\left(-231i\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/a^4/d*cos(d*x+c)^5*(e/cos(d*x+c))^(15/2)*(-1+cos(d*x+c))^2*(1+cos(d*x
+c))^2*(-231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^3*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+231*I*(1/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x
+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-231*I*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF(I*(-1
+cos(d*x+c))/sin(d*x+c),I)+231*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*
x+c),I)-120*I*cos(d*x+c)^3*sin(d*x+c)+120*cos(d*x+c)^4-231*cos(d*x+c)^3-20*
I*cos(d*x+c)*sin(d*x+c)+114*cos(d*x+c)^2-3)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 179, normalized size = 0.93

$$-\frac{2\left(\frac{\sqrt{2}\left(-120ie^{\frac{15}{2}}-231ie^{\left(6idx+6ic+\frac{15}{2}\right)}-616ie^{\left(4idx+4ic+\frac{15}{2}\right)}-517ie^{\left(2idx+2ic+\frac{15}{2}\right)}\right)e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+231\left(-i\sqrt{2}\,e^{\left(5idx+5ic+\frac{15}{2}\right)}-2i\sqrt{2}\,e^{\left(3idx+3ic+\frac{15}{2}\right)}-i\sqrt{2}\,e^{\left(idx+ic+\frac{15}{2}\right)}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(idx+ic)}\right)\right)\right)}{15\left(a^4de^{(5idx+5ic)}+2a^4de^{(3idx+3ic)}+a^4de^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-120*I*e^(15/2) - 231*I*e^(6*I*d*x + 6*I*c + 15/2) - 616*I*
e^(4*I*d*x + 4*I*c + 15/2) - 517*I*e^(2*I*d*x + 2*I*c + 15/2))*e^(1/2*I*d*x
 + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 231*(-I*sqrt(2)*e^(5*I*d*x + 5*
```

```
I*c + 15/2) - 2*I*sqrt(2)*e^(3*I*d*x + 3*I*c + 15/2) - I*sqrt(2)*e^(I*d*x +
 I*c + 15/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
I*c))))/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^
(I*d*x + I*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate(e^(15/2)*sec(d*x + c)^(15/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d}-\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}+\frac{4ie^2(e\sec(c+}{3ad(a+ia\tan(c}$$

```
[Out] -10*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^4/d-10*e^6*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/3*I*e^2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*
tan(d*x+c))^3+12*I*e^4*(e*sec(d*x+c))^(5/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2720}

$$-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d}-\frac{10e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^4d}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(a^4*d) - (10*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(a^4*d) + (((4*I)/3
)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + ((12*I)*e^4*
(e*Sec[c + d*x])^(5/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} - \frac{(3e^2)\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2}$$
$$= \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))} - \frac{(15e^4)\int(e\sec(c+dx}{a^4}$$
$$= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\,\mathrm{se}}{d\,(a^4+ia}$$
$$= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\,\mathrm{se}}{d\,(a^4+ia}$$
$$= -\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d} - \frac{10e^5(e\sec(c+dx))}{a^4d}$$

Mathematica [A]
time = 0.64, size = 134, normalized size = 0.85

$$\frac{ie^6\sec^5(c+dx)\sqrt{e\sec(c+dx)}\left(21+19\cos(2(c+dx))+30i\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))+11i\sin(2(c+dx))\right)(\cos(3(c+dx))+i\sin(3(c+dx)))}{3a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*e^6*Sec[c + d*x]^5*Sqrt[e*Sec[c + d*x]]*(21 + 19*Cos[2*(c + d*x)] +
(30*I)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c
 + d*x]) + (11*I)*Sin[2*(c + d*x)])*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])
)/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.88, size = 226, normalized size = 1.44

method	result

default	$\frac{2\left(-15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^2(dx+c))-15i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^4/d*(-15*I*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-15*I*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)+8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)+12*I*cos(
d*x+c)+sin(d*x+c))*cos(d*x+c)^5*(e/cos(d*x+c))^(13/2)*(-1+cos(d*x+c))^2*(1+
cos(d*x+c))^2/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 134, normalized size = 0.85

$$-\frac{2\left(\frac{\sqrt{2}\left(-4ie^{\frac{13}{2}}-15ie^{\left(4i\,dx+4i\,c+\frac{13}{2}\right)}-21ie^{\left(2i\,dx+2i\,c+\frac{13}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+15\left(-i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{13}{2}\right)}-i\sqrt{2}\,e^{\left(2i\,dx+2i\,c+\frac{13}{2}\right)}\right)\operatorname{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\left(a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-4*I*e^(13/2) - 15*I*e^(4*I*d*x + 4*I*c + 13/2) - 21*I*e^(2*
I*d*x + 2*I*c + 13/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1
) + 15*(-I*sqrt(2)*e^(4*I*d*x + 4*I*c + 13/2) - I*sqrt(2)*e^(2*I*d*x + 2*I*
c + 13/2))*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^4*d*e^(4*I*d*x +
 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(e^(13/2)*sec(d*x + c)^(13/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{13/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$-\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{28ie^4(}{5d\,(a^4-}$$

```
[Out] -42/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*e^5*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/a^4/d+4/5*I*e^2*(e*sec(d*x+c))^(7/2)/a/d/(a+
I*a*tan(d*x+c))^3-28/5*I*e^4*(e*sec(d*x+c))^(3/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3853, 3856, 2719}

$$-\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{42e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\,(a^4+ia^4\tan(c+dx))}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-42*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) + (((4*I)
/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (((28*I)/5
)*e^4*(e*Sec[c + d*x])^(3/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{(7e^2)\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(21e^4)\int (e\sec(c+d}{5a^4}$$

$$= \frac{42e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(}{5d\left(a^4+ia^4\mathrm{t}\right.}$$

$$= \frac{42e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(}{5d\left(a^4+ia^4\mathrm{t}\right.}$$

$$= -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{42e^5\sqrt{e\sec(c+dx)}\,\sin(c+dx)}{5a^4d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.69, size = 106, normalized size = 0.65

$$-\frac{2ie^5e^{-3i(c+dx)}\left(-2-7e^{2i(c+dx)}+21e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((-2*I)/5)*e^5*(-2 - 7*E^((2*I)*(c + d*x)) + 21*E^((2*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])*Sqrt[e*Sec[c + d*x]])/(a^4*d*E^((3*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(167) = 334.
time = 0.84, size = 379, normalized size = 2.33

method	result
default	$-\frac{2\left(21i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-21i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx}{1+\cos(d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/a^4/d*(21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-21*I*(1/(1
+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-8*I*cos(d*x+c)^3*sin(d*x+c)+21*I
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1
+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)+8*cos(d*x+c)^4+20*I*cos(d*x+c)*sin(d*x+c)-24*cos(d*x+c)^2+21*cos(d*x+c
)-5)*(e/cos(d*x+c))^(11/2)*(-1+cos(d*x+c))^2*cos(d*x+c)^5*(1+cos(d*x+c))^2/
sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 0.63

$$-\frac{2\left(21i\sqrt{2}\,e^{\left(3i\,dx+3i\,c+\frac{11}{2}\right)}\operatorname{weierstrassZeta}\left(-4,0,\operatorname{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\frac{\sqrt{2}\left(-2i\,e^{\frac{11}{2}}+21i\,e^{\left(4i\,dx+4i\,c+\frac{11}{2}\right)}+14i\,e^{\left(2i\,dx+2i\,c+\frac{11}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-3i\,dx-3i\,c)}}{5\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/5*(21*I*sqrt(2)*e^(3*I*d*x + 3*I*c + 11/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-2*I*e^(11/2) + 21*I*e^(4
*I*d*x + 4*I*c + 11/2) + 14*I*e^(2*I*d*x + 2*I*c + 11/2))*e^(1/2*I*d*x + 1/
2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate(e^(11/2)*sec(d*x + c)^(11/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 10/21*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/7*I*e^2*
(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^3-20/21*I*e^4*(e*sec(d*x+c))^(1
/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3581, 3856, 2720}

$$-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))}+\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^4*d) + (((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((20*I)/21)*e^4*Sqrt[e*Sec[c + d*x]])/(d*(a^4 + I*a^4*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{(5e^2)\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx}{7a^2}\\ &= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))} + \frac{(5e^4)\int \sqrt{e\sec(c+ }{21a^4}\\ &= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))} + \frac{\left(5e^4\sqrt{\cos(c+dx)}\right.}{}\\ &= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d} + \frac{4ie^2(e\sec(c+dx))^5}{7ad(a+ia\tan(c+d}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 137, normalized size = 1.04

$$\frac{2e^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(5\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))-2i(1+\cos(2(c+dx))+4i\sin(2(c+dx)))\right)(\cos(2(c+dx))+i\sin(2(c+dx)))}{21a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*e^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(5*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - (2*I)*(1 + Cos[2*
(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)
]))/(21*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.79, size = 228, normalized size = 1.73

method	result
default	$\frac{2\left(24i(\cos^4(dx+c))+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+24\sin(dx+c)(\cos^3(dx+c))+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a^4/d*(24*I*cos(d*x+c)^4+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+24*s
```

```
in(d*x+c)*cos(d*x+c)^3+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-28*I*cos(d*x+c)^2-16*s
in(d*x+c)*cos(d*x+c))*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(-1+cos(d*x+c))^2*(
1+cos(d*x+c))^2/sin(d*x+c)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 100, normalized size = 0.76

$$-\frac{2\left(5i\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{9}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)+\frac{\sqrt{2}\left(-3i\,e^{\frac{9}{2}}+5i\,e^{\left(4i\,dx+4i\,c+\frac{9}{2}\right)}+2i\,e^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-4i\,dx-4i\,c\right)}}{21\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/21*(5*I*sqrt(2)*e^(4*I*d*x + 4*I*c + 9/2)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)) + sqrt(2)*(-3*I*e^(9/2) + 5*I*e^(4*I*d*x + 4*I*c + 9/2) + 2*I
*e^(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c
) + 1))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate(e^(9/2)*sec(d*x + c)^(9/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] -2/15*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2*
(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^3-4/15*I*e^4/d/(e*sec(d*x+c))^(
1/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3581, 3856, 2719}

$$-\frac{4ie^4}{15d\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((4*I)/15)*e^4)/(d*Sqrt[e*Sec[c + d*x]]*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx}{3a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))} - \frac{e^4 \ldots}{\ldots}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))} - \frac{\ldots}{15\ldots}$$

$$= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{\ldots}{15d\sqrt{\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.92, size = 149, normalized size = 1.13

$$\frac{e^3e^{-idx}\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-7-7\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+3i\sin(2(c+dx))\right)(-i\cos(c+2dx)+\sin(c+2dx))}{45a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-7 - 7*Cos[2*(c + d*x)] + 6*E^((2
*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)])*((-I)*Cos[c + 2*d*x] +
Sin[c + 2*d*x]))/(45*a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(140) = 280.
time = 0.83, size = 390, normalized size = 2.95

method	result
default	$-\frac{2\left(-40i(\cos^5(dx+c))\sin(dx+c)+40(\cos^6(dx+c))+3i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/a^4/d*(-40*I*cos(d*x+c)^5*sin(d*x+c)+40*cos(d*x+c)^6+3*I*(1/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/
sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c
)*sin(d*x+c)+36*I*cos(d*x+c)^3*sin(d*x+c)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin
(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-56*cos(d*x+c)^4+13*cos(d*x
+c)^2+3*cos(d*x+c))*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(e/cos(d*x+c))^(7/2)
*cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 115, normalized size = 0.87

$$\frac{\left(-6i\sqrt{2}\,e^{\left(5i\,dx+5i\,c+\frac{7}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)+\frac{\sqrt{2}\left(5i\,e^{\frac{7}{2}}-6i\,e^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)}-4i\,e^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+7i\,e^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-5i\,dx-5i\,c\right)}}{45\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/45*(-6*I*sqrt(2)*e^(5*I*d*x + 5*I*c + 7/2)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(5*I*e^(7/2) - 6*I*e^(6*I*d
*x + 6*I*c + 7/2) - 4*I*e^(4*I*d*x + 4*I*c + 7/2) + 7*I*e^(2*I*d*x + 2*I*c
+ 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5*I*d*x
- 5*I*c)/(a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^4d}-\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-$$

```
[Out] -2/77*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)-2/77*e^2*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/11*I*e^2*(e*sec(d*x+c))^(1/2)/a/d/(a
+I*a*tan(d*x+c))^3-4/77*I*e^4/d/(e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]

time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2720}

$$-\frac{4ie^4}{77d\,(a^4+ia^4\tan(c+dx))\,(e\sec(c+dx))^{3/2}}-\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}}-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^4d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*a^4*d) - (2*e^3*Sin[c + d*x])/(77*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)
/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (((4*I)/77)
*e^4)/(d*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{e^2\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))} -$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{}{77d(e\sec(c+dx))^{3/}}$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{}{77d(e\sec(c+dx))^{3/}}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx}}$$

Mathematica [A]

time = 0.70, size = 144, normalized size = 0.88

$$\frac{\sec^2(c+dx)(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\left(37i\cos(c+dx)+11i\cos(3(c+dx))+3\sin(c+dx)-4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)(\cos(3(c+dx))+i\sin(3(c+dx)))+3\sin(3(c+dx))\right)}{154a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(Cos[c + d*x] + I*Sin[c + d*x])*((37
*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] + 3*Sin[c + d*x] - 4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) +
 3*Sin[3*(c + d*x)]))/(154*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]

time = 0.78, size = 244, normalized size = 1.50

method	result
default	$\frac{2\left(56i(\cos^6(dx+c))+56\sin(dx+c)(\cos^5(dx+c))-44i(\cos^4(dx+c))-i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1}{\text{si}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 2/77/a^4/d*(56*I*cos(d*x+c)^6+56*sin(d*x+c)*cos(d*x+c)^5-44*I*cos(d*x+c)^4-I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-16*sin(d*x+c)*cos(d*x+c)^3-I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-sin(d*x+c)*cos(d*x+c))*cos(d*x+c)^2*(e/cos(d*x+c))^(5/2)*(-1+cos(d*x+c))^2*(1+cos(d*x+c))^2/sin(d*x+c)^4

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 112, normalized size = 0.69

$$\frac{\left(4i\sqrt{2}\,e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}\text{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)+\frac{\sqrt{2}\left(7ie^{\frac{5}{2}}+4ie^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}+17ie^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+20ie^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-6i\,dx-6i\,c\right)}}{154\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/154*(4*I*sqrt(2)*e^(6*I*d*x + 6*I*c + 5/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(7*I*e^(5/2) + 4*I*e^(6*I*d*x + 6*I*c + 5/2) + 17*I*e^(4*I*d*x + 4*I*c + 5/2) + 20*I*e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-6*I*d*x - 6*I*c)/(a^4*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6
*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}}+\frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3}$$

```
[Out] 2/117*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(3/2)+2/39*e^2*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/13*I*e^2/a/d/(e*sec(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^3+4/117*I*e^4/d/(e*sec(d*x+c))^(5/2)/(a^4+I*a^4*tan(d*x+c)
)
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2719}

$$\frac{4ie^4}{117d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*e^3*Sin[c + d*x])/(117*a^4*d*(e*Sec[c + d*x])^(3/2)) + (((4*
I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((4*I)/1
17)*e^4)/(d*(e*Sec[c + d*x])^(5/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+}}{13a^2}$$

$$= \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\,(a^4+ia^4}$$

$$= \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{}{117}$$

$$= \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^3} + \frac{}{117}$$

$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{39a^4d\sqrt{\cos(c+dx)}\,\sqrt{e\sec(c+dx)}} + \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{}{13ad\sqrt{e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.69, size = 142, normalized size = 0.87

$$\frac{ie^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(28+40\cos(2(c+dx))+\frac{24e^{4i(c+dx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+22i\sin(2(c+dx))\right)}{234a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/234)*Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(28
+ 40*Cos[2*(c + d*x)] + (24*E^((4*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + (22*I)*Sin[2*
(c + d*x)]))/(a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(167) = 334$.
time = 1.03, size = 398, normalized size = 2.44

method	result
default	$\frac{2\left(72 i \sin(dx+c)(\cos^7(dx+c)) - 72(\cos^8(dx+c)) - 52 i (\cos^5(dx+c)) \sin(dx+c) + 3 i \sqrt{\frac{1}{1+\cos(dx+c)}}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/117/a^4/d*(72*I*sin(d*x+c)*cos(d*x+c)^7-72*cos(d*x+c)^8-52*I*sin(d*x+c)*c
os(d*x+c)^5+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+88*cos(d*x+c)^6+3*I*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/si
n(d*x+c),I)*sin(d*x+c)-3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-17*cos(d*x+
c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(e/cos
(d*x+c))^(3/2)*cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 127, normalized size = 0.78

$$\frac{\left(24 i \sqrt{2}\, e^{\left(7 i dx + 7 i c + \frac{3}{2}\right)} \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, e^{(i dx + i c)}\right)\right) + \frac{\sqrt{2}\left(9 i e^{\frac{3}{2}} + 24 i e^{\left(8 i dx + 8 i c + \frac{3}{2}\right)} + 55 i e^{\left(6 i dx + 6 i c + \frac{3}{2}\right)} + 59 i e^{\left(4 i dx + 4 i c + \frac{3}{2}\right)} + 37 i e^{\left(2 i dx + 2 i c + \frac{3}{2}\right)}\right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c\right)}}{\sqrt{e^{(2 i dx + 2 i c)} + 1}}\right) e^{(-7 i dx - 7 i c)}}{468\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/468*(24*I*sqrt(2)*e^(7*I*d*x + 7*I*c + 3/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(9*I*e^(3/2) + 24*I*e^(8*I
```

```
*d*x + 8*I*c + 3/2) + 55*I*e^(6*I*d*x + 6*I*c + 3/2) + 59*I*e^(4*I*d*x + 4*
I*c + 3/2) + 37*I*e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e
^(2*I*d*x + 2*I*c) + 1))*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6
*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.262 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^4d}+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}+\frac{14\cdots}{165ad\cdots}$$

```
[Out] 2/33*e*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)+2/33*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/a^4/d+2/15*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+
c))^4+14/165*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^3+4/33*I*e^2/d/(
e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3583, 3581, 3854, 3856, 2720}

$$\frac{4ie^2}{33d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^4d}+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*a
^4*d) + (2*e*Sin[c + d*x])/(33*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/15)*Sq
rt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (((14*I)/165)*Sqrt[e*Sec
[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*e^2)/(d*(e*Sec[c +
 d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx}{15a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{7\int \frac{\sqrt{e\sec(c+dx}}{(a+ia\tan(c+dx))}}{33a^2}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{}{33d(e\sec(c+dx))}$$

$$= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+d}$$

$$= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+d}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\sec(c+dx)}}{33a^4d} + \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} +$$

Mathematica [A]

time = 0.68, size = 137, normalized size = 0.72

$$\frac{\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(4(c+dx))+i\sin(4(c+dx)))+i(64+112\cos(2(c+dx))+48\cos(4(c+dx))+54i\sin(2(c+dx))+37i\sin(4(c+dx)))\right)}{660a^4d(-i+\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) + I*(64 + 112*Cos[2*(c +
 d*x)] + 48*Cos[4*(c + d*x)] + (54*I)*Sin[2*(c + d*x)] + (37*I)*Sin[4*(c +
d*x)])))/(660*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A]
time = 0.76, size = 224, normalized size = 1.17

method	result
default	$\frac{2\sqrt{\frac{e}{\cos(dx+c)}}\left(88i(\cos^8(dx+c))+88\sin(dx+c)(\cos^7(dx+c))-60i(\cos^6(dx+c))-16\sin(dx+c)(\cos^5(dx+c))+5i\sqrt{\frac{1}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/165/a^4/d*(e/cos(d*x+c))^(1/2)*(88*I*cos(d*x+c)^8+88*sin(d*x+c)*cos(d*x+c
)^7-60*I*cos(d*x+c)^6-16*sin(d*x+c)*cos(d*x+c)^5+5*I*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)+5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*sin(d*x+c)*cos(d*x+c)^3+5*si
n(d*x+c)*cos(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 124, normalized size = 0.65

$$\frac{\left(-80i\sqrt{2}\,e^{\left(8i\,dx+8i\,c+\frac{1}{2}\right)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\frac{\sqrt{2}\left(11i\,e^{\frac{1}{2}}+85i\,e^{\left(8i\,dx+8i\,c+\frac{1}{2}\right)}+166i\,e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}+128i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+58i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-8i\,dx-8i\,c)}}{1320\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/1320*(-80*I*sqrt(2)*e^(8*I*d*x + 8*I*c + 1/2)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(11*I*e^(1/2) + 85*I*e^(8*I*d*x + 8*I*c + 1/2) +
 166*I*e^(6*I*d*x + 6*I*c + 1/2) + 128*I*e^(4*I*d*x + 4*I*c + 1/2) + 58*I*e
^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c)
+ 1))*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*ta
n(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{{(a+a\tan(c+d\,x)\ 1\mathrm{i})}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=69

$$\frac{6i2^{5/6}a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] 6/5*I*2^(5/6)*a*hypergeom([-5/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f
*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A]
time = 0.13, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x]
)/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{5/3}\int(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+f}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{(a+iax)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx\right.}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(2^{5/6}a^2(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+\right.}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}$$

$$= \frac{6i2^{5/6}a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/}}{5f(1+i\tan(e+fx))^{5/6}}$$

Mathematica [A]
time = 1.25, size = 104, normalized size = 1.51

$$\frac{3ade^{-2ifx}(d\sec(e+fx))^{2/3}(\cos(e+3fx)+i\sin(e+3fx))\left(-3i+i\left(1+e^{2i(e+fx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)+2\tan(e+fx)\right)}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
[Out] (3*a*d*(d*Sec[e + f*x])^(2/3)*(Cos[e + 3*f*x] + I*Sin[e + 3*f*x])*(-3*I + I
*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I
)*(e + f*x))] + 2*Tan[e + f*x]))/(10*E^((2*I)*f*x)*f)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+ia\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/10*(3*2^(2/3)*(5*I*a*d*e^(3*I*f*x + 3*I*e) + I*a*d*e^(I*f*x + I*e))*(d/(
e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 10*(f*e^(2*I*f*x
+ 2*I*e) + f)*integral(1/2*I*2^(2/3)*a*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(d\sec\left(e+fx\right))^{\frac{5}{3}}\right)\,dx+\int(d\sec\left(e+fx\right))^{\frac{5}{3}}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*sec(e + f*x))**(5/3), x) + Integral((d*sec(e + f*x))**(
5/3)*tan(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/3}\,\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i), x)
```

3.264 $\int \sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{6i\sqrt[6]{2}\,a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] 6*I*2^(1/6)*a*hypergeom([-1/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+
e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A]
time = 0.12, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{6i\sqrt[6]{2}\,a\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]*
(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}\,(a+ia\tan(e+fx))}{\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+iax}}{(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(\sqrt[6]{2}\,a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{5/6}}\,dx, x, \tan(e+\right.}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}$$

$$= \frac{6i\sqrt[6]{2}\,a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A]
time = 0.56, size = 92, normalized size = 1.37

$$\frac{3ade^{-ie}\left(-1+\sqrt[3]{1+e^{2i(e+fx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)\right)(\cos(fx)-i\sin(fx))(-i+\tan(e+fx))}{f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
[Out] (3*a*d*(-1 + (1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/
6, -E^((2*I)*(e + f*x))])*(Cos[f*x] - I*Sin[f*x])*(-I + Tan[e + f*x]))/(E^(
I*e)*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+ia\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (3*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e)
+ f*integral(-I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x
 - 2/3*I*e)/f, x))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt[3]{d\sec\left(e+fx\right)}\right)dx+\int\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] I*a*(Integral(-I*(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**(
1/3)*tan(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{1/3}\,\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i), x)
```

3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=67

$$-\frac{3i2^{5/6}a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3*I*2^(5/6)*a*hypergeom([-1/6, 1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f
*x+e))^(1/6)/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2]
*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{7/6}\sqrt[6]{a+iax}}\,d\right.}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}\,(a-iax)^{7/}}\right.}{\sqrt[6]{2}\,f\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i2^{5/6}a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.45, size = 98, normalized size = 1.46

$$-\frac{3i2^{2/3}ae^{2i(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)}{5\sqrt[3]{\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt[3]{1+e^{2i(e+fx)}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
[Out] (((-3*I)/5)*2^(2/3)*a*E^((2*I)*(e + f*x))*Hypergeometric2F1[2/3, 5/6, 11/6,
 -E^((2*I)*(e + f*x))])/(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1
/3)*(1 + E^((2*I)*(e + f*x)))^(1/3)*f)
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{a+ia\tan\left(fx+e\right)}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] -(3*2^(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e) - (d*f*e^(I*f*x + I*e) - d*f)*integral(-2*2^(2
/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a*e^(I*f*x + I*e) + I*a)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f
*e^(2*I*f*x + 2*I*e) + d*f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int \left(-\frac{i}{\sqrt[3]{d\sec\left(e+fx\right)}}\right) dx+\int \frac{\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] I*a*(Integral(-I/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)/(d*sec
(e + f*x))**(1/3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\tan(e + f\,x)\ 1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3), x)
```

3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=69

$$-\frac{3i\sqrt[6]{2}\,a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/5*I*2^(1/6)*a*hypergeom([-5/6, 5/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan
(f*x+e))^(5/6)/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A]
time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i\sqrt[6]{2}\,a(1+i\tan(e+fx))^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x]
)/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{\sqrt[6]{a+ia\tan(e+fx)}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{5/6}}\,d\right.}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}(a-iax)^{11/6}}\,dx,\right.}{2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\sqrt[6]{2}\,a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

Mathematica [A]
time = 0.45, size = 106, normalized size = 1.54

$$-\frac{3iae^{i(e+fx)}\left(1+e^{2i(e+fx)}+4\sqrt[3]{1+e^{2i(e+fx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)\right)}{5d\left(1+e^{2i(e+fx)}\right)f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-3*I)/5)*a*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)) + 4*(1 + E^((2*I)*(e
 + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))]))/(d
*(1 + E^((2*I)*(e + f*x)))*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{a+ia\tan(fx+e)}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/10*(10*d^2*f*integral(-2/5*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3
)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a*e^(2*I*f*x + 2*I*e)
 + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^2*f
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx+\int\frac{\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] I*a*(Integral(-I/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)/(d*sec
(e + f*x))**(5/3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\tan(e + f\,x)\ 1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3), x)
```

3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=71

$$\frac{12i2^{5/6}a^2\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] 12/5*I*2^(5/6)*a^2*hypergeom([-11/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*s
ec(f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A]
time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+f}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{(a+iax)^{11/6}}{\sqrt[6]{a-iax}}\,dx, x, \tan(e+f\right.}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(2\,2^{5/6}a^3(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}}{\sqrt[6]{a-iax}}\,dx, x, \tan(\right.}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}$$

$$= \frac{12i2^{5/6}a^2\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+f}{5f(1+i\tan(e+fx))^{5/6}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 267 vs. $2(71) = 142$.
time = 2.72, size = 267, normalized size = 3.76

$$\frac{(d\sec(e+fx))^{5/3}\left(-\frac{33i2^{2/3}\left(5\sqrt[3]{1+e^{2i(e+fx)}}-e^{2ifx}(-1+e^{2ie})\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)\right)}{(-1+e^{2ie})\sqrt[3]{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt[3]{1+e^{2i(e+fx)}}}+\frac{3}{4}\csc(e)\sec^{\frac{8}{3}}(e+fx)(\cos(2e)-i\sin(2e))(90\cos(fx)+75\cos(2e+fx)+55\cos(2e+3fx)-64i\sin(fx)+64i\sin(2e+fx))\right)(a+ia\tan(e+fx))^2}{80f\sec^{\frac{11}{3}}(e+fx)(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/3)*(((-33*I)*2^(2/3)*(5*(1 + E^((2*I)*(e + f*x)))^(1/3
) - E^((2*I)*f*x)*(-1 + E^((2*I)*e))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^(
(2*I)*(e + f*x))]))/((-1 + E^((2*I)*e))*(E^(I*(e + f*x))/(1 + E^((2*I)*(e +
 f*x))))^(1/3)*(1 + E^((2*I)*(e + f*x)))^(1/3)) + (3*Csc[e]*Sec[e + f*x]^(8
/3)*(Cos[2*e] - I*Sin[2*e])*(90*Cos[f*x] + 75*Cos[2*e + f*x] + 55*Cos[2*e +
 3*f*x] - (64*I)*Sin[f*x] + (64*I)*Sin[2*e + f*x]))/4)*(a + I*a*Tan[e + f*x
])^2)/(80*f*Sec[e + f*x]^(11/3)*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{\frac{5}{3}} (a+ia\tan(fx+e))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/80*(3*2^(2/3)*(55*I*a^2*d*e^(5*I*f*x + 5*I*e) + 26*I*a^2*d*e^(3*I*f*x +
3*I*e) + 11*I*a^2*d*e^(I*f*x + I*e))*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^
(2/3*I*f*x + 2/3*I*e) - 80*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
 + f)*integral(11/16*I*2^(2/3)*a^2*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^
(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*
e) + f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/3}\,{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.268 $\int \sqrt[3]{d \sec(e+fx)}\,(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=69

$$\frac{12i\sqrt[6]{2}\,a^2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] 12*I*2^(1/6)*a^2*hypergeom([-7/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f
*x+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A]
time = 0.13, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{12i\sqrt[6]{2}\,a^2\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/
2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))^2\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}\,(a+ia\tan(e+fx)}{\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{7/6}}{(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(2\sqrt[6]{2}\,a^3\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}}{(a-iax)^{5/6}}\,dx, x, \tan(e+\right.}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}$$

$$= \frac{12i\sqrt[6]{2}\,a^2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A]
time = 0.90, size = 128, normalized size = 1.86

$$-\frac{3a^2e^{-2ie}\sqrt[3]{d\sec(e+fx)}\,(\cos(2(e+fx))+i\sin(2(e+fx)))\left(-8i+7i\sqrt[3]{1+e^{2i(e+fx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)+\sec(e)\sec(e+fx)\sin(fx)+\tan(e)\right)}{4f(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a^2*(d*Sec[e + f*x])^(1/3)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-8*
I + (7*I)*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6,
-E^((2*I)*(e + f*x))] + Sec[e]*Sec[e + f*x]*Sin[f*x] + Tan[e]))/(4*E^((2*I)
*e)*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*2^(1/3)*(-9*I*a^2*e^(2*I*f*x + 2*I*e) - 7*I*a^2)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e) - 4*(f*e^(2*I*f*x + 2*I*e) + f)*i
ntegral(-7/4*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*
x - 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\sqrt[3]{d\sec\left(e+fx\right)}\right)dx+\int\sqrt[3]{d\sec\left(e+fx\right)}\tan^2\left(e+fx\right)dx+\int\left(-2i\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral(-(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**(
1/3)*tan(e + f*x)**2, x) + Integral(-2*I*(d*sec(e + f*x))**(1/3)*tan(e + f*
x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{1/3}\,{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=83

$$-\frac{6i2^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(a^2+ia^2\tan(e+fx))}{f\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}$$

```
[Out] -6*I*2^(5/6)*hypergeom([-5/6, -1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^2*
tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A]
time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$-\frac{6i2^{5/6}(a^2+ia^2\tan(e+fx))\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-6*I)*2^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/2]*
(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\tan\right.}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(2^{5/6}a^2\sqrt[6]{a-ia\tan(e+fx)}\,(a+ia\tan(e+fx))\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\right.}{f\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}$$

$$= -\frac{6i2^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}$$

Mathematica [A]
time = 1.17, size = 132, normalized size = 1.59

$$-\frac{3ia^2e^{2i(e+fx)}\left(-\sqrt[3]{1+e^{2i(e+fx)}}+\left(1+e^{2i(e+fx)}\right)\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)\right)}{\sqrt[3]{2}\sqrt[3]{\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(1+e^{2i(e+fx)}\right)^{4/3}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-3*I)*a^2*E^((2*I)*(e + f*x))*(-(1 + E^((2*I)*(e + f*x)))^(1/3) + (1 + E^
((2*I)*(e + f*x)))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))])
)/(2^(1/3)*((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*(1 + E^((2
*I)*(e + f*x)))^(4/3)*f)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+ia\tan\left(fx+e\right)\right)^{2}}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] -1/2*(3*2^(2/3)*(4*I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2*e^(I*f*x + I*e) + 5*I*
a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 2*(d*f*e
^(I*f*x + I*e) - d*f)*integral(-5*2^(2/3)*(I*a^2*e^(2*I*f*x + 2*I*e) + I*a^
2*e^(I*f*x + I*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x
 + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f*e^(2*I*f*x + 2*I*e) + d*f*e^(I
*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int\left(-\frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}}\right)dx+\int\frac{\tan^{2}\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx+\int\left(-\frac{2i\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] -a**2*(Integral(-1/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)**2/(
d*sec(e + f*x))**(1/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))**(
1/3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3), x)
```

3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{6i\sqrt[6]{2}\ {}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] -6/5*I*2^(1/6)*hypergeom([-5/6, -1/6],[1/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^
2*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A]
time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{6i\sqrt[6]{2}\left(a^2+ia^2\tan(e+fx)\right)\ {}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}\,(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-6*I)/5)*2^(1/6)*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*x])
/2]*(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{(a+ia\tan(e+fx))^{7/6}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+iax}}{(a-iax)^{11/6}}\,dx, x$$

$$= \frac{\left(\sqrt[6]{2}\,a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\right)\text{Subst}\left(\int \frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{11/6}}\,dx\right.}{f(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}$$

$$= -\frac{6i\sqrt[6]{2}\;{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A]
time = 0.57, size = 105, normalized size = 1.24

$$-\frac{12ia^2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}-\sqrt[3]{1+e^{2i(e+fx)}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)\right)}{5\left(1+e^{2i(e+fx)}\right)^2f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
[Out] (((-12*I)/5)*a^2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)) - (1 + E^((2*
I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))]
))/((1 + E^((2*I)*(e + f*x)))^2*f*(d*Sec[e + f*x])^(5/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/5*(5*d^2*f*integral(1/5*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)
*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a^2*e^(2*I*f*x + 2*I*e
) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^
2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-\frac{1}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx+\int\frac{\tan^2(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx+\int\left(-\frac{2i\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] -a**2*(Integral(-1/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)**2/(
d*sec(e + f*x))**(5/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))**(
5/3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3),x)
[Out] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3), x)
```

3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=83

$$\frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{5\sqrt[6]{2}\,f(a+ia\tan(e+fx))}$$

```
[Out] 3/10*I*hypergeom([5/6, 7/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(5/
3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]
time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,(d\sec(e+fx))^{5/3}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{5\sqrt[6]{2}\,f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e + f*
x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{a+ia\tan(e+fx)}}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}\,(a+iax)^{7/6}}\,dx, x, \tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(a(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}\sqrt[6]{a-iax}}\,dx, x\right.}{2\sqrt[6]{2}\,f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$= \frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{5\sqrt[6]{2}\,f(a+ia\tan(e+fx))}$$

Mathematica [**A**]
time = 0.45, size = 84, normalized size = 1.01

$$\frac{6de^{i(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}{a\sqrt[3]{1+e^{2i(e+fx)}}\,f(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (6*d*E^(I*(e + f*x))*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x))
]*(d*Sec[e + f*x])^(2/3))/(a*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*(-I + Tan[e
+ f*x]))
```

Maple [**F**]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (a*f*e^(I*f*x + I*e)*integral(-I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2
/3)*e^(2/3*I*f*x + 2/3*I*e)/(a*f), x) - 3*2^(2/3)*(-I*d*e^(2*I*f*x + 2*I*e)
 - I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-I*
f*x - I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(d\sec{(e+fx)})^{\frac{5}{3}}}{\tan{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i), x)
```

3.272 $\int \frac{\sqrt[3]{d \sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}$$

```
[Out] 3/2*I*hypergeom([1/6, 11/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/3
)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a+I*a*tan(f*x+e))
```

Rubi [A]

time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e +
 f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx &= \frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}\\ &= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}\\ &= \frac{\left(a\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}(a-iax)^{5/6}}\,dx,x,\tan(e\right.}{2\,2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}\,(a+ia\tan(e+fx))}\\ &= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 103, normalized size = 1.27

$$-\frac{3ie^{-2i(e+fx)}\left(-1-e^{2i(e+fx)}+4e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)\right)\sqrt[3]{d\sec(e+fx)}}{10af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((-3*I)/10)*(-1 - E^((2*I)*(e + f*x)) + 4*E^((2*I)*(e + f*x))*(1 + E^((2*I
)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])
*(d*Sec[e + f*x])^(1/3))/(a*E^((2*I)*(e + f*x))*f)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+ia\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/10*(10*a*f*e^(2*I*f*x + 2*I*e)*integral(-2/5*I*2^(1/3)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*f), x) - 3*2^(1/3)*(d/(e^(2*I
*f*x + 2*I*e) + 1))^(1/3)*(-I*e^(2*I*f*x + 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I
*e))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\sqrt[3]{d\sec{\left(e+fx\right)}}}{\tan{\left(e+fx\right)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i), x)
```

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}\,af\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3/4*I*hypergeom([-1/6, 13/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))^
(1/6)*2^(5/6)/a/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]

time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}\,af\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{}{\sqrt[6]{a-ia\,\mathrm{ta}}}}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\mathrm{Subst}\Big(\int \frac{}{(a}}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\mathrm{Subst}\Big(\int \frac{}{(\frac{1}{2}}}{4\sqrt[6]{2}\,f\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}\,af\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.97, size = 112, normalized size = 1.58

$$\frac{3\left(-8e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)+5(5+5\cos(2(e+fx))+4i\sin(2(e+fx)))\right)(i+\tan(e+fx))}{70af\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (3*(-8*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F
1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] + 5*(5 + 5*Cos[2*(e + f*x)] + (4*I)
*Sin[2*(e + f*x)]))*(I + Tan[e + f*x]))/(70*a*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+ia\tan\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/28*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(7*I*e^(5*I*f*x + 5*I*
e) + 9*I*e^(4*I*f*x + 4*I*e) + 6*I*e^(3*I*f*x + 3*I*e) + 10*I*e^(2*I*f*x +
2*I*e) - I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 28*(a*d*f*e^(4*I*
f*x + 4*I*e) - a*d*f*e^(3*I*f*x + 3*I*e))*integral(-8/7*2^(2/3)*(d/(e^(2*I*
f*x + 2*I*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^
(2/3*I*f*x + 2/3*I*e)/(a*d*f*e^(3*I*f*x + 3*I*e) - 2*a*d*f*e^(2*I*f*x + 2*I
*e) + a*d*f*e^(I*f*x + I*e)), x))/(a*d*f*e^(4*I*f*x + 4*I*e) - a*d*f*e^(3*I
*f*x + 3*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)-i\sqrt[3]{d\sec\left(e+fx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x) - I*(d*sec(e + f*x))**(
1/3)), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}\,\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)),x)
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)), x)
```

3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{17}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/20*I*hypergeom([-5/6, 17/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(5/6)*2^(1/6)/a/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A]
time = 0.15, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{17}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{}{(a-ia\tan(e}}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\Big(\int}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\Big(\int \frac{}{(\frac{1}{2}+\frac{i}{2}}}{4\ 2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{17}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

Mathematica [**A**]
time = 0.91, size = 119, normalized size = 1.68

$$-\frac{3\sec^2(e+fx)\left(-26+6\cos(2(e+fx))+\frac{128e^{2i(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^{2/3}}+16i\sin(2(e+fx))\right)}{220af(d\sec(e+fx))^{5/3}(-i+\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (-3*Sec[e + f*x]^2*(-26 + 6*Cos[2*(e + f*x)] + (128*E^((2*I)*(e + f*x))*Hyp
ergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])/(1 + E^((2*I)*(e + f*x
)))^(2/3) + (16*I)*Sin[2*(e + f*x)]))/(220*a*f*(d*Sec[e + f*x])^(5/3)*(-I +
 Tan[e + f*x]))
```

Maple [**F**]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(a+ia\tan(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/440*(440*a*d^2*f*e^(4*I*f*x + 4*I*e)*integral(-16/55*I*2^(1/3)*(d/(e^(2*I
*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*d^2*f), x) - 3*2^(1/3
)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(11*I*e^(6*I*f*x + 6*I*e) - 15*I*e^(4
*I*f*x + 4*I*e) - 31*I*e^(2*I*f*x + 2*I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*
e^(-4*I*f*x - 4*I*e)/(a*d^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{1}{(d\sec{(e+fx)})^{\frac{5}{3}}\tan{(e+fx)}-i(d\sec{(e+fx)})^{\frac{5}{3}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] -I*Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x) - I*(d*sec(e + f*x))**(
5/3)), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)),x)
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)), x)
```

3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{10\sqrt[6]{2}\,f\,(a^2+ia^2\tan(e+fx))}$$

```
[Out] 3/20*I*hypergeom([5/6, 13/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(5
/3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]
time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,(d\sec(e+fx))^{5/3}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{10\sqrt[6]{2}\,f\,(a^2+ia^2\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*S
ec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a^2 + I*a^2*Tan[
e + f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx &= \frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(a+ia\tan(e+fx))^{7/6}}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}\,(a+iax)^{13/6}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left((d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}\sqrt[6]{a-iax}}\,dx\right.}{4\sqrt[6]{2}\,f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}\\
&= \frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{10\sqrt[6]{2}\,f\left(a^2+ia^2\tan(e+fx)\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.82, size = 128, normalized size = 1.47

$$
-\frac{3e^{-i(4e+5fx)}\left(1+e^{2i(e+fx)}\right)\left(1+e^{2i(e+fx)}+2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-e^{2i(e+fx)}\right)\right)(d\sec(e+fx))^{5/3}(-i\cos(fx)+\sin(fx))}{28a^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-3*(1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)) + 2*E^((2*I)*(e + f*
x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((
2*I)*(e + f*x))])*(d*Sec[e + f*x])^(5/3)*((-I)*Cos[f*x] + Sin[f*x]))/(28*a^
2*E^(I*(4*e + 5*f*x))*f)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$
\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(a+ia\tan(fx+e))^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/14*(14*a^2*f*e^(3*I*f*x + 3*I*e)*integral(-1/7*I*2^(2/3)*d*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a^2*f), x) - 3*2^(2/3)*(-2*I
*d*e^(4*I*f*x + 4*I*e) - 3*I*d*e^(2*I*f*x + 2*I*e) - I*d)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}{2\,2^{5/6}f\,(a^2+ia^2\tan(e+fx))}$$

```
[Out] 3/4*I*hypergeom([1/6, 17/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/3
)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]

time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}f\,(a^2+ia^2\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a^2 + I*a^2*Tan[e
+ f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx &= \frac{\sqrt[3]{d\sec(e+fx)}\int\frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{11/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\mathrm{Subst}\left(\int\frac{1}{(a-iax)^{5/6}(a+iax)^{17/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\mathrm{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{17/6}(a-iax)^{5/6}}\,dx,x,\tan(\right.}{4\,2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}\,(a+ia\tan(e+fx))}\\
&= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}\,(1+i\tan(e+fx))^{5/6}}{2\,2^{5/6}f\,(a^2+ia^2\tan(e+fx))}
\end{aligned}$$

Mathematica [A]
time = 0.68, size = 121, normalized size = 1.39

$$\frac{3\sec^2(e+fx)\sqrt[3]{d\sec(e+fx)}\left(-2i-2i\cos(2(e+fx))+4ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)+\sin(2(e+fx))\right)}{22a^2f(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(1/3)*(-2*I - (2*I)*Cos[2*(e + f*x)] + (
4*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[
1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] + Sin[2*(e + f*x)]))/(22*a^2*f*(-I + T
an[e + f*x])^2)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/44*(44*a^2*f*e^(4*I*f*x + 4*I*e)*integral(-2/11*I*2^(1/3)*(d/(e^(2*I*f*x
+ 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*f), x) - 3*2^(1/3)*(d/(e
^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-3*I*e^(4*I*f*x + 4*I*e) - 4*I*e^(2*I*f*x +
 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{d\sec{(e+fx)}}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}\,a^2f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3/8*I*hypergeom([-1/6, 19/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))^
(1/6)*2^(5/6)/a^2/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}\,a^2f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+ia\tan(e+fx))^2}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{}{\sqrt[6]{a-ia\,\mathrm{t}}}}{\sqrt[3]{d\sec(e+fx)}}$$
$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{a+ia\tan(e+fx)}\right)\mathrm{Subst}\Big(\int}{f\sqrt[3]{d\sec(e+fx)}}$$
$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\,\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\mathrm{Subst}\Big(\int}{8\sqrt[6]{2}\,af\sqrt[3]{d\sec(e+fx)}}$$
$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx}}{4\sqrt[6]{2}\,a^2f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A]
time = 1.58, size = 141, normalized size = 1.99

$$\frac{(d\sec(e+fx))^{2/3}\left(16e^{3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2i(e+fx)}\right)-10(7\cos(e+fx)+5\cos(3(e+fx))+18i\cos^2(e+fx)\sin(e+fx))\right)(-3i\cos(2(e+fx))-3\sin(2(e+fx)))}{260a^2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((d*Sec[e + f*x])^(2/3)*(16*E^((3*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(
2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] - 10*(7*Cos[e
+ f*x] + 5*Cos[3*(e + f*x)] + (18*I)*Cos[e + f*x]^2*Sin[e + f*x]))*((-3*I)*
Cos[2*(e + f*x)] - 3*Sin[2*(e + f*x)]))/(260*a^2*d*f)
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] -1/104*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(13*I*e^(7*I*f*x + 7*
I*e) + 19*I*e^(6*I*f*x + 6*I*e) + 9*I*e^(5*I*f*x + 5*I*e) + 23*I*e^(4*I*f*x
 + 4*I*e) - 5*I*e^(3*I*f*x + 3*I*e) + 5*I*e^(2*I*f*x + 2*I*e) - I*e^(I*f*x
+ I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 104*(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^
2*d*f*e^(5*I*f*x + 5*I*e))*integral(-8/13*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2
/3*I*e)/(a^2*d*f*e^(3*I*f*x + 3*I*e) - 2*a^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*
d*f*e^(I*f*x + I*e)), x))/(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^2*d*f*e^(5*I*f*x
 + 5*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt[3]{d\sec{\left(e+fx \right)}}\tan^{2}{\left(e+fx \right)}-2i\sqrt[3]{d\sec{\left(e+fx \right)}}\tan{\left(e+fx \right)}-\sqrt[3]{d\sec{\left(e+fx \right)}}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x))
**(1/3)*tan(e + f*x) - (d*sec(e + f*x))**(1/3)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{23}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/40*I*hypergeom([-5/6, 23/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(5/6)*2^(1/6)/a^2/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A]
time = 0.15, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{23}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \frac{((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6})\int \frac{}{(a-ia\tan(}}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6})\,\mathrm{Subst}\Big(}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\mathrm{Subst}\left(\int \frac{}{(\frac{1}{2}+}\right.}{8\ 2^{5/6}af(d\sec(e+fx))^{5/}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{23}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx)}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.
time = 0.97, size = 143, normalized size = 2.01

$$\frac{3i\sec^4(e+fx)\left(-46-40\cos(2(e+fx))+6\cos(4(e+fx))+128e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{7}{6};-e^{2i(e+fx)}\right)-10i\sin(2(e+fx))+11i\sin(4(e+fx))\right)}{680a^2f(d\sec(e+fx))^{5/3}(-i+\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((3*I)/680)*Sec[e + f*x]^4*(-46 - 40*Cos[2*(e + f*x)] + 6*Cos[4*(e + f*x)]
 + 128*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F
1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] - (10*I)*Sin[2*(e + f*x)] + (11*I)*S
in[4*(e + f*x)]))/(a^2*f*(d*Sec[e + f*x])^(5/3)*(-I + Tan[e + f*x])^2)
```

Maple [**F**]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(a+ia\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1360*(1360*a^2*d^2*f*e^(6*I*f*x + 6*I*e)*integral(-16/85*I*2^(1/3)*(d/(e^
(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*d^2*f), x) - 3*
2^(1/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(17*I*e^(8*I*f*x + 8*I*e) - 50*
I*e^(6*I*f*x + 6*I*e) - 92*I*e^(4*I*f*x + 4*I*e) - 30*I*e^(2*I*f*x + 2*I*e)
 - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-6*I*f*x - 6*I*e)/(a^2*d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\tan^{2}\left(e+fx\right)-2i\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\tan\left(e+fx\right)-\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x))
**(5/3)*tan(e + f*x) - (d*sec(e + f*x))**(5/3)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d}-\frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d}+\frac{2i(a+ia\tan(c+dx}{15a^7d}$$

```
[Out] -16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d+24/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^
5/d-12/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^6/d+2/15*I*(a+I*a*tan(d*x+c))^(15/2
)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3568, 45\}$

$$\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}-\frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^4*d) + (((24*I)/11)*(a + I*a*
Tan[c + d*x])^(11/2))/(a^5*d) - (((12*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))
/(a^6*d) + (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{a^7 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{7/2} - 12a^2(a+x)^{9/2} + 6a(a+x)^{11/2} - (a\right.\right.}{a^7 d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4 d} + \frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5 d} - \frac{12}{}$$

Mathematica [A]
time = 0.95, size = 95, normalized size = 0.81

$$\frac{2\sec^7(c+dx)(510\cos(c+dx)+731\cos(3(c+dx))-3i(90\sin(c+dx)+233\sin(3(c+dx))))(-i\cos(4(c+dx))+\sin(4(c+dx)))\sqrt{a+ia\tan(c+dx)}}{6435d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^7*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] - (3*I)*(90*Sin[
c + d*x] + 233*Sin[3*(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]
)*Sqrt[a + I*a*Tan[c + d*x]])/(6435*d)
```

Maple [A]
time = 4.78, size = 141, normalized size = 1.21

method	result
default	$-\frac{2\left(1024i\left(\cos^7(dx+c)\right)-1024\sin(dx+c)\left(\cos^6(dx+c)\right)+128i\left(\cos^5(dx+c)\right)-640\sin(dx+c)\left(\cos^4(dx+c)\right)+56i\left(\cos^3(dx+c)\right)-504\left(\cos\right.\right.}{6435d\cos(dx+c)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/6435/d*(1024*I*cos(d*x+c)^7-1024*sin(d*x+c)*cos(d*x+c)^6+128*I*cos(d*x+c
)^5-640*sin(d*x+c)*cos(d*x+c)^4+56*I*cos(d*x+c)^3-504*cos(d*x+c)^2*sin(d*x+
c)+33*I*cos(d*x+c)-429*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)/cos(d*x+c)^7
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.65

$$\frac{2i\left(429\left(ia\tan(dx+c)+a\right)^{\frac{15}{2}} - 2970\left(ia\tan(dx+c)+a\right)^{\frac{13}{2}}a + 7020\left(ia\tan(dx+c)+a\right)^{\frac{11}{2}}a^2 - 5720\left(ia\tan(dx+c)+a\right)^{\frac{9}{2}}a^3\right)}{6435\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/6435*I*(429*(I*a*tan(d*x + c) + a)^(15/2) - 2970*(I*a*tan(d*x + c) + a)^(
13/2)*a + 7020*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 5720*(I*a*tan(d*x + c) +
 a)^(9/2)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.42, size = 154, normalized size = 1.32

$$-\frac{256\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(16i\,e^{(15i\,dx+15i\,c)}+120i\,e^{(13i\,dx+13i\,c)}+390i\,e^{(11i\,dx+11i\,c)}+715i\,e^{(9i\,dx+9i\,c)}\right)}{6435\,\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(15*I*d*x + 15*
I*c) + 120*I*e^(13*I*d*x + 13*I*c) + 390*I*e^(11*I*d*x + 11*I*c) + 715*I*e^
(9*I*d*x + 9*I*c))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 2
1*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*
I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^{8}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^8, x)
```

Mupad [B]
time = 12.12, size = 474, normalized size = 4.05

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{6435\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{6435\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{2145\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,40960\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,52736\mathrm{i}}{715\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11776\mathrm{i}}{195\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^8,x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4
0960i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(6435*d*(exp(c*2i + d*x*
2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*512i)/(2145*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(1287*d*(exp(c*
2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*4096i)/(6435*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1
i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*52736i)/(715*d*(exp(c*2i + d*x*2i) +
 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*11776i)/(195*d*(exp(c*2i + d*x*2i) + 1)^6) - ((a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*d*(exp(c*2i +
 d*x*2i) + 1)^7)
```

3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}$$

```
[Out] -8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d+8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d-
2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^3*d) + (((8*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^4*d) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^5
*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)\,dx, x, ia\tan\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{2i(a+\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 77, normalized size = 0.88

$$\frac{2\sec^5(c+dx)(44+107\cos(2(c+dx))-91i\sin(2(c+dx)))(-i\cos(3(c+dx))+\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{693d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*Sec[c + d*x]^5*(44 + 107*Cos[2*(c + d*x)] - (91*I)*Sin[2*(c + d*x)])*((-
I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(693*d)
```

Maple [A]
time = 0.97, size = 114, normalized size = 1.30

method	result
default	$\frac{2\left(-128i\left(\cos^5(dx+c)\right)+128\sin(dx+c)\left(\cos^4(dx+c)\right)-16i\left(\cos^3(dx+c)\right)+80\left(\cos^2(dx+c)\right)\sin(dx+c)-7i\cos(dx+c)+63\sin(dx+c)\right)\sqrt{}}{693d\cos(dx+c)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/693/d*(-128*I*cos(d*x+c)^5+128*sin(d*x+c)*cos(d*x+c)^4-16*I*cos(d*x+c)^3+
80*cos(d*x+c)^2*sin(d*x+c)-7*I*cos(d*x+c)+63*sin(d*x+c))*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.66

$$-\frac{2i\left(63\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-308\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a+396\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/693*I*(63*(I*a*tan(d*x + c) + a)^(11/2) - 308*(I*a*tan(d*x + c) + a)^(9/
2)*a + 396*(I*a*tan(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.44, size = 119, normalized size = 1.35

$$-\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(11i\,dx+11i\,c)}+44i\,e^{(9i\,dx+9i\,c)}+99i\,e^{(7i\,dx+7i\,c)}\right)}{693\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(11*I*d*x + 11*I*c
) + 44*I*e^(9*I*d*x + 9*I*c) + 99*I*e^(7*I*d*x + 7*I*c))/(d*e^(10*I*d*x + 1
0*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d
*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^6, x)
```

Mupad [B]
time = 7.01, size = 352, normalized size = 4.00

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{693\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,7232\mathrm{i}}{693\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1472\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^6,x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*7
232i)/(693*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(693*d*(exp(c*2i + d*x*2i)
+ 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))
^(1/2)*64i)/(231*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*d) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1472i)/(99*d
*(exp(c*2i + d*x*2i) + 1)^4) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(e
xp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*x*2i) + 1)^5)
```

3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

```
[Out] -4/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^2/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}-\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^2*d) + (((2*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 58, normalized size = 0.98

$$\frac{2\sqrt{a+ia\tan(c+dx)}\,(8(-i+\tan(c+dx))+\sec^2(c+dx)(-i+5\tan(c+dx)))}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + I*a*Tan[c + d*x]]*(8*(-I + Tan[c + d*x]) + Sec[c + d*x]^2*(-I +
 5*Tan[c + d*x])))/(35*d)
```

Maple [A]
time = 1.02, size = 87, normalized size = 1.47

method	result	size
default	$-\frac{2\left(8i\left(\cos^3(dx+c)\right)-8\left(\cos^2(dx+c)\right)\sin(dx+c)+i\cos(dx+c)-5\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{35d\cos(dx+c)^3}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/d*(8*I*cos(d*x+c)^3-8*cos(d*x+c)^2*sin(d*x+c)+I*cos(d*x+c)-5*sin(d*x+
c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.68

$$\frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 14*(I*a*tan(d*x + c) + a)^(5/2)*a)
/(a^3*d)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 1.42

$$-\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(7i\,dx+7i\,c)}+7i\,e^{(5i\,dx+5i\,c)}\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(7*I*d*x + 7*I*c) +
 7*I*e^(5*I*d*x + 5*I*c))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c)
+ 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [B]
time = 6.25, size = 230, normalized size = 3.90

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{35\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{35\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
28i)/(35*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i -
1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(35*d*(exp(c*2i + d*x*2i) + 1)
) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*32i)/(35*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int \sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 34, normalized size = 1.17

$$\frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d)
```

Maple [A]
time = 0.30, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3da}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3da}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/a
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.72

$$-\frac{2i\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/3*I*(I*a*tan(d*x + c) + a)^(3/2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.
time = 0.35, size = 46, normalized size = 1.59

$$-\frac{4i\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(3i\,dx+3i\,c)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.
time = 0.61, size = 55, normalized size = 1.90

$$-\frac{2i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/
2*d*x + 1/2*c)^2 - 1))^(3/2)/(a*d)
```

Mupad [B]
time = 0.58, size = 82, normalized size = 2.83

$$-\frac{\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,2\mathrm{i}}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] -((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)*((a*(cos(2*c + 2*d*x) + sin(
2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*2i)/(3*d*(cos(2*c + 2*d
*x) + 1))
```

3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{3i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}+\frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*2^(1
/2)+3/4*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)
/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-3*I)/4)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
(Sqrt[2]*d) + (((3*I)/4)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^2)/(d
*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia^2)\,\text{Subst}\Big(\int$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c}$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c}$$

$$= -\frac{3i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+}$$

Mathematica [A]
time = 0.55, size = 105, normalized size = 0.88

$$-\frac{ie^{-2i(c+dx)}\left(-2-e^{2i(c+dx)}+e^{4i(c+dx)}+3e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/8*I)*(-2 - E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) + 3*E^(I*(c + d*x
))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*E^((2*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(94) = 188$.
time = 3.66, size = 397, normalized size = 3.31

method	result
default	$\frac{\left(3i\sqrt{2}\,\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(3*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*si
n(d*x+c)+3*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+3
*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+3*2^(1/2)*arctan(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*sin(d*x+c)-8*I*cos(d*x+c)^4-4*I*cos(d*x+c)^3+8*sin(d*x+c)*cos(d*x+c)^3+
12*I*cos(d*x+c)^2-12*cos(d*x+c)^2*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/
cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)
```

Maxima [A]
time = 0.50, size = 122, normalized size = 1.02

$$\frac{i\left(3\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(3\left(i\,a\tan(dx+c)+a\right)a^{2}-4\,a^{3}\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}-2\sqrt{i\,a\tan\left(dx+c\right)+a}\,a}\right)}{16\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*I*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x + c)
 + a)*a^2 - 4*a^3)/((I*a*tan(d*x + c) + a)^(3/2) - 2*sqrt(I*a*tan(d*x + c)
+ a)*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(87) = 174$.
time = 0.39, size = 253, normalized size = 2.11

$$-\frac{\left(3\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^2}}\,e^{(i\,dx+i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^2}}\,e^{(i\,dx+i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}+2i\right)\right)e^{(-i\,dx-i\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*
(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d
^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(
I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x
- I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c)
 + I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-I*d*x - I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{35i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\,\mathrm{ta}}$$

```
[Out] -35/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*2
^(1/2)+35/64*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+35/96*I*a^2/d/(a+I*a*tan(d*x+c)
)^(3/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-7/16*I*a^
3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}}+\frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{64\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-35*I)/64)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((35*I)/96)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/4)
*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((7*I)/1
6)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((35*I)/
64)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{(7ia^4)\,\text{Subst}\Big(}{}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{}{16d(a-ia\tan}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(}$$

$$= -\frac{35i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d} + \frac{35ia^2}{96d(a+ia\tan(c+}$$

Mathematica [**A**]
time = 0.51, size = 133, normalized size = 0.69

$$-\frac{ie^{-4i(c+dx)}\left(-8-88e^{2i(c+dx)}-41e^{4i(c+dx)}+45e^{6i(c+dx)}+6e^{8i(c+dx)}+105e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/384*I)*(-8 - 88*E^((2*I)*(c + d*x)) - 41*E^((4*I)*(c + d*x)) + 45*E^((
6*I)*(c + d*x)) + 6*E^((8*I)*(c + d*x)) + 105*E^((3*I)*(c + d*x))*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/
(d*E^((4*I)*(c + d*x)))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(154) = 308.
time = 0.98, size = 741, normalized size = 3.84

method	result
default	$\frac{\left(-128i(\cos^7(dx+c))-560i(\cos^5(dx+c))+105\sin(dx+c)\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}(\cos^3(dx+c))\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072/d*(-128*I*cos(d*x+c)^7-560*I*cos(d*x+c)^5+105*sin(d*x+c)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(7/2)*cos(d*x+c)^3*2^(1/2)+315*I*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*cos(d*x+c)*2^(1/2)+315*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c
)^2*2^(1/2)+315*I*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(
d*x+c)^2*2^(1/2)+315*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*2^(1/2)+105*2
^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-768*I*cos(d*x+c)^8+768*sin(d*x+c)*cos(
d*x+c)^7+105*I*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x
+c)^3*2^(1/2)-896*sin(d*x+c)*cos(d*x+c)^6+1680*I*cos(d*x+c)^4+1120*sin(d*x+
c)*cos(d*x+c)^5-224*I*cos(d*x+c)^6-1680*sin(d*x+c)*cos(d*x+c)^4+105*I*2^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c))*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3
```

Maxima [A]
time = 0.51, size = 176, normalized size = 0.91

$$\frac{i\left(105\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(105\left(i\,a\tan(dx+c)+a\right)^{3}a^{2}-350\left(i\,a\tan(dx+c)+a\right)^{2}a^{3}+224\left(i\,a\tan(dx+c)+a\right)a^{4}+64\,a^{5}\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{7}{2}}-4\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}a+4\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a^{2}}\right)}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/768*I*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d*x
 + c) + a)^3*a^2 - 350*(I*a*tan(d*x + c) + a)^2*a^3 + 224*(I*a*tan(d*x + c)
 + a)*a^4 + 64*a^5)/((I*a*tan(d*x + c) + a)^(7/2) - 4*(I*a*tan(d*x + c) + a
)^(5/2)*a + 4*(I*a*tan(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

Fricas [A]
time = 0.37, size = 275, normalized size = 1.42

$$-\frac{105\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^{2}}}\,e^{\left(3i\,dx+3i\,c\right)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{\left(2i\,dx+2i\,c\right)}+i\,d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{-\frac{a}{d^{2}}}-ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)-105\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^{2}}}\,e^{\left(3i\,dx+3i\,c\right)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-i\,de^{\left(2i\,dx+2i\,c\right)}-i\,d\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\sqrt{-\frac{a}{d^{2}}}-ae^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}\right)-\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(-6i\,e^{\left(8i\,dx+8i\,c\right)}-45i\,e^{\left(6i\,dx+6i\,c\right)}+41i\,e^{\left(4i\,dx+4i\,c\right)}+88i\,e^{\left(2i\,dx+2i\,c\right)}+8i\right)\right)e^{\left(-3i\,dx-3i\,c\right)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/384*(105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sq
rt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 105*sqrt(1/2)*d*sqrt(-
a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I
*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*
c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-6*I*e^(
8*I*d*x + 8*I*c) - 45*I*e^(6*I*d*x + 6*I*c) + 41*I*e^(4*I*d*x + 4*I*c) + 88
*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cos^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=266

$$-\frac{231i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,d}+\frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] -231/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d
*2^(1/2)+231/512*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+231/640*I*a^3/d/(a+I*a*tan(
d*x+c))^(5/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(5/2)-11/
48*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-33/64*I*a^4/d/(a-I
*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+77/256*I*a^2/d/(a+I*a*tan(d*x+c))^(
3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}-\frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}}+\frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}}+\frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}-\frac{231i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-231*I)/512)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((231*I)/640)*a^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((
I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) - (((11
*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - ((
(33*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
((77*I)/256)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((231*I)/512)*a)/(d*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{(11ia^6)\,\text{Subst}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{48d(a-ia\tan}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{48d(a-ia\tan}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan}$$

$$= -\frac{231i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,d} + \frac{231ia^3}{640d(a+ia\tan(c}$$

Mathematica [A]
time = 0.70, size = 159, normalized size = 0.60

$$-\frac{ie^{-6i(c+dx)}\left(-48-464e^{2i(c+dx)}-3184e^{4i(c+dx)}-1433e^{6i(c+dx)}+1645e^{8i(c+dx)}+350e^{10i(c+dx)}+40e^{12i(c+dx)}+3465e^{5i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/15360*I)*(-48 - 464*E^((2*I)*(c + d*x)) - 3184*E^((4*I)*(c + d*x)) - 1
433*E^((6*I)*(c + d*x)) + 1645*E^((8*I)*(c + d*x)) + 350*E^((10*I)*(c + d*x
)) + 40*E^((12*I)*(c + d*x)) + 3465*E^((5*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((6*I
)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(214) = 428$.
time = 1.00, size = 1085, normalized size = 4.08

method	result	size
default	Expression too large to display	1085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/491520/d*(-101376*cos(d*x+c)^9*sin(d*x+c)-3465*2^(1/2)*arctan(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/
2)*sin(d*x+c)+81920*I*cos(d*x+c)^12+8192*I*cos(d*x+c)^11+11264*I*cos(d*x+c)
^10+16896*I*cos(d*x+c)^9+29568*I*cos(d*x+c)^8+73920*I*cos(d*x+c)^7-221760*I
*cos(d*x+c)^6-17325*I*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(11/2)-3465*I*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(11/2)-17325*I*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)-34650*I*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)-34650*I*cos(d*x+c)^2*sin(d*x+c)*
2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)-81920*sin(d*x+c)*cos(d*x
+c)^11+118272*sin(d*x+c)*cos(d*x+c)^8+90112*cos(d*x+c)^10*sin(d*x+c)-3465*c
os(d*x+c)^5*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-17325*cos(d*x+c)^4*sin(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-34650*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))-34650*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-
17325*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-3465*I*2^(1/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)-147840*sin(d*x+c)*cos(d*x
+c)^7+221760*sin(d*x+c)*cos(d*x+c)^6)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+
c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^5
```

Maxima [A]
time = 0.53, size = 230, normalized size = 0.86

$$\frac{i\left(3465\,\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5a^2-18480\,(i\,a\tan(dx+c)+a)^4a^3+30492\,(i\,a\tan(dx+c)+a)^3a^4-12672\,(i\,a\tan(dx+c)+a)^2a^5-2816\,(i\,a\tan(dx+c)+a)a^6-1536\,a^7\right)}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+12\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2-8\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^3}\right)}{30720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30720*I*(3465*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3465*(I*a*tan
(d*x + c) + a)^5*a^2 - 18480*(I*a*tan(d*x + c) + a)^4*a^3 + 30492*(I*a*tan(
d*x + c) + a)^3*a^4 - 12672*(I*a*tan(d*x + c) + a)^2*a^5 - 2816*(I*a*tan(d*
x + c) + a)*a^6 - 1536*a^7)/((I*a*tan(d*x + c) + a)^(11/2) - 6*(I*a*tan(d*x
 + c) + a)^(9/2)*a + 12*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 8*(I*a*tan(d*x +
 c) + a)^(5/2)*a^3))/(a*d)
```

Fricas [A]
time = 0.38, size = 297, normalized size = 1.12

$$-\frac{\left(3465 \sqrt{\frac{1}{2}}\, d \sqrt{-\frac{a}{d^{2}}}\, e^{\left(5 i\, d x+5 i\, c\right)} \log\left(-4 \left(\sqrt{2} \sqrt{\frac{1}{2}} \left(i\, d e^{\left(2 i\, d x+2 i\, c\right)}+i\, d\right) \sqrt{\frac{a}{e^{\left(2 i\, d x+2 i\, c\right)}+1}} \sqrt{-\frac{a}{d^{2}}}-a e^{\left(i\, d x+i\, c\right)}\right) e^{\left(-i\, d x-i\, c\right)}\right)-3465 \sqrt{\frac{1}{2}}\, d \sqrt{-\frac{a}{d^{2}}}\, e^{\left(5 i\, d x+5 i\, c\right)} \log\left(-4 \left(\sqrt{2} \sqrt{\frac{1}{2}} \left(-i\, d e^{\left(2 i\, d x+2 i\, c\right)}-i\, d\right) \sqrt{\frac{a}{e^{\left(2 i\, d x+2 i\, c\right)}+1}} \sqrt{-\frac{a}{d^{2}}}-a e^{\left(i\, d x+i\, c\right)}\right) e^{\left(-i\, d x-i\, c\right)}\right)-\sqrt{2} \sqrt{\frac{a}{e^{\left(2 i\, d x+2 i\, c\right)}+1}} \left(-40 i\, e^{\left(12 i\, d x+12 i\, c\right)}-350 i\, e^{\left(10 i\, d x+10 i\, c\right)}-1645 i\, e^{\left(8 i\, d x+8 i\, c\right)}+1433 i\, e^{\left(6 i\, d x+6 i\, c\right)}+3184 i\, e^{\left(4 i\, d x+4 i\, c\right)}+464 i\, e^{\left(2 i\, d x+2 i\, c\right)}+48 i\right)\right) e^{\left(-5 i\, d x-5 i\, c\right)}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15360*(3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)
*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3465*sqrt(1/2)*d*sq
rt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-40*
I*e^(12*I*d*x + 12*I*c) - 350*I*e^(10*I*d*x + 10*I*c) - 1645*I*e^(8*I*d*x +
 8*I*c) + 1433*I*e^(6*I*d*x + 6*I*c) + 3184*I*e^(4*I*d*x + 4*I*c) + 464*I*e
^(2*I*d*x + 2*I*c) + 48*I))*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i a \left(\tan \left(c + d x\right) - i\right)} \cos^{6}\left(c + d x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/13*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(1/2)+256/3003*I*a^4*sec(d*x+c)^
7/d/(a+I*a*tan(d*x+c))^(7/2)+64/429*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))
^(5/2)+24/143*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((256*I)/3003)*a^4*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((6
4*I)/429)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((24*I)/1
43)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/13)*a*Se
c[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{13}(12a)\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{143}($$
$$= \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{}{13}$$
$$= \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{}{1}$$

Mathematica [A]
time = 0.72, size = 95, normalized size = 0.65

$$\frac{2\sec^6(c+dx)(390\cos(c+dx)+445\cos(3(c+dx))+7i(26\sin(c+dx)+59\sin(3(c+dx))))(i\cos(4(c+dx))+\sin(4(c+dx)))\sqrt{a+ia\tan(c+dx)}}{3003d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] + (7*I)*(26*Sin[
c + d*x] + 59*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*Sq
rt[a + I*a*Tan[c + d*x]])/(3003*d)
```

Maple [A]
time = 1.14, size = 141, normalized size = 0.96

method	result
default	$\frac{2(1024i(\cos^7(dx+c))+1024\sin(dx+c)(\cos^6(dx+c))-128i(\cos^5(dx+c))+384\sin(dx+c)(\cos^4(dx+c))-40i(\cos^3(dx+c))+280(\cos^2(}{3003d\cos(dx+c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3003/d*(1024*I*cos(d*x+c)^7+1024*sin(d*x+c)*cos(d*x+c)^6-128*I*cos(d*x+c)
^5+384*sin(d*x+c)*cos(d*x+c)^4-40*I*cos(d*x+c)^3+280*cos(d*x+c)^2*sin(d*x+c
)-21*I*cos(d*x+c)+231*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^
(1/2)/cos(d*x+c)^6
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.43, size = 132, normalized size = 0.90

$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-429i\,e^{(6i\,dx+6i\,c)}-286i\,e^{(4i\,dx+4i\,c)}-104i\,e^{(2i\,dx+2i\,c)}-16i\right)}{3003\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-429*I*e^(6*I*d*x + 6*
I*c) - 286*I*e^(4*I*d*x + 4*I*c) - 104*I*e^(2*I*d*x + 2*I*c) - 16*I)/(d*e^(
12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) +
 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^{7}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^7, x)
```

Mupad [B]
time = 8.35, size = 289, normalized size = 1.97

$$\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,384\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}-\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^7,x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*128i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (exp(- c*1i -
d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^
(1/2)*128i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) + (exp(- c*1i - d*x*1i)*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*384i)/(1
1*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i
 + d*x*2i) + 1)^6)
```

3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/9*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+64/315*I*a^3*sec(d*x+c)^5/d
/(a+I*a*tan(d*x+c))^(5/2)+16/63*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/315)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((16*
I)/63)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/9)*a*
Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{9}(8a)\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{63}(32a$$
$$= \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}} + \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}} + \frac{}{9d}$$

Mathematica [A]
time = 0.50, size = 77, normalized size = 0.70

$$\frac{2\sec^4(c+dx)(36+71\cos(2(c+dx))+55i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*Sec[c + d*x]^4*(36 + 71*Cos[2*(c + d*x)] + (55*I)*Sin[2*(c + d*x)])*(I*C
os[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

Maple [A]
time = 0.84, size = 114, normalized size = 1.04

method	result
default	$\frac{2\left(128i\left(\cos^5(dx+c)\right)+128\sin(dx+c)\left(\cos^4(dx+c)\right)-16i\left(\cos^3(dx+c)\right)+48\left(\cos^2(dx+c)\right)\sin(dx+c)-5i\cos(dx+c)+35\sin(dx+c)\right)\sqrt{}}{315d\cos(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/315/d*(128*I*cos(d*x+c)^5+128*sin(d*x+c)*cos(d*x+c)^4-16*I*cos(d*x+c)^3+4
8*cos(d*x+c)^2*sin(d*x+c)-5*I*cos(d*x+c)+35*sin(d*x+c))*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A]
time = 0.36, size = 97, normalized size = 0.88

$$-\frac{32 \sqrt{2} \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-63i\,e^{(4i\,dx+4i\,c)}-36i\,e^{(2i\,dx+2i\,c)}-8i\right)}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-63*I*e^(4*I*d*x + 4*I*c
) - 36*I*e^(2*I*d*x + 2*I*c) - 8*I)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x
 + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^5, x)
```

Mupad [B]
time = 6.08, size = 102, normalized size = 0.93

$$\frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,63\mathrm{i}+8\mathrm{i}\right)}{315\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^5,x)
```

```
[Out] (32*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*36i + exp(c*4i + d*x*4i)*63i + 8
i))/(315*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/5*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/15*I*a^2*sec(d*x+c)^3/d/(
a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/
5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx&=\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{5}(4a)\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\&=\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 63, normalized size = 0.86

$$-\frac{2\sec(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-7i+3\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-7*I + 3*Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)
```

Maple [A]
time = 0.80, size = 87, normalized size = 1.19

method	result	size
default	$\frac{2\left(8i\left(\cos^{3}(dx+c)\right)+8\left(\cos^{2}(dx+c)\right)\sin(dx+c)-i\cos(dx+c)+3\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos(dx+c)^{2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-I*cos(d*x+c)+3*sin(d*x+c
))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(57) = 114.
time = 19.52, size = 222, normalized size = 3.04

$$\frac{8\left(5i\sqrt{2}\cos\left(2dx+2c\right)-5\sqrt{2}\sin\left(2dx+2c\right)+2i\sqrt{2}\right)\sqrt{a}}{15\left(\cos\left(2dx+2c\right)^{2}+\sin\left(2dx+2c\right)^{2}+2\cos\left(2dx+2c\right)+1\right)^{\frac{1}{4}}\left(\left(\cos\left(4dx+4c\right)+2\cos\left(2dx+2c\right)+i\sin\left(4dx+4c\right)+2i\sin\left(2dx+2c\right)+1\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)+1\right)\right)+\left(i\cos\left(4dx+4c\right)+2i\cos\left(2dx+2c\right)-\sin\left(4dx+4c\right)-2\sin\left(2dx+2c\right)+i\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)+1\right)\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(5*I*sqrt(2)*cos(2*d*x + 2*c) - 5*sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt(
2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c) + 2
*I*sin(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + (I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) - sin(4*d*x + 4*c) - 2*
sin(2*d*x + 2*c) + I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*d)
```

Fricas [A]
time = 0.39, size = 62, normalized size = 0.85

$$-\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(2i\,dx+2i\,c)}-2i\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(2*I*d*x + 2*I*c) -
 2*I)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [B]
time = 5.95, size = 88, normalized size = 1.21

$$\frac{8\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,5{}\mathrm{i}+2{}\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}}{15\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] (8*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*5i + 2i)*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(15*d*(exp(c*2i + d*x*2
i) + 1)^2)
```

3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*a*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3574\}$

$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.21, size = 39, normalized size = 1.26

$$\frac{2(i\cos(c+dx)+\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*(I*Cos[c + d*x] + Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A]
time = 0.58, size = 50, normalized size = 1.61

method	result	size
default	$\frac{2(i\cos(dx+c)+\sin(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{d}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A]
time = 0.37, size = 25, normalized size = 0.81

$$\frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [B]
time = 0.35, size = 61, normalized size = 1.97

$$\frac{2\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x),x)
[Out] (2*(cos(c + d*x)*1i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x
)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] 1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)/d*2^(1/2)-I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3571, 3570, 212}

$$\frac{i\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{(ia)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, -\right.}{d}$$

$$= \frac{i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,d} - \frac{i\cos(c+dx)\sqrt{a+i}}{d}$$

Mathematica [A]
time = 0.64, size = 87, normalized size = 1.05

$$-\frac{ie^{-i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/2*I)*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*
x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(68) = 136.
time = 0.76, size = 217, normalized size = 2.61

method	result
default	$-\frac{\left(i\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{}}{2}\right.\right.}{2d(i\sin(dx+c)+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*I*cos(d*x+c)^2-2*I*cos(d*x+c)-2*sin(d*x
```

```
+c)*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c
)+cos(d*x+c)-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(64) = 128$.
time = 0.61, size = 774, normalized size = 9.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(I*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a)
+ (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(64) = 128$.
time = 0.36, size = 184, normalized size = 2.22

$$\frac{\sqrt{2}\, d \sqrt{-\frac{a}{d^2}} \log\left(\frac{2 \left(\left(d e^{(2i\,dx+2i\,c)}+d\right) \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \sqrt{-\frac{a}{d^2}}+i\,a\right) e^{(-i\,dx-i\,c)}}{d}\right) - \sqrt{2}\, d \sqrt{-\frac{a}{d^2}} \log\left(-\frac{2 \left(\left(d e^{(2i\,dx+2i\,c)}+d\right) \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \sqrt{-\frac{a}{d^2}}-i\,a\right) e^{(-i\,dx-i\,c)}}{d}\right) - 2\sqrt{2} \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \left(i\, e^{(2i\,dx+2i\,c)}+i\right)}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*d*sqrt(-a/d^2)*log(2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - sqrt(2)*d*s
qrt(-a/d^2)*log(-2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) - 2*sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{5i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx}}{8d}$$

```
[Out] 5/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)/d*2^(1/2)+5/12*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*cos(d*x
+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}+\frac{5i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((5*I)/8)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan
[c + d*x]])])/(Sqrt[2]*d) + (((5*I)/12)*a*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/8)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/3
)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{1}{6}(5a)\int \frac{\cos(c+dx}{\sqrt{a+ia\tan(c-}}$$

$$= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}$$

$$= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}$$

$$= \frac{5i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} + \frac{5ia\cos(c+d}{12d\sqrt{a+ia\tan(c}}$$

Mathematica [A]
time = 0.60, size = 126, normalized size = 0.82

$$-\frac{ie^{-3i(c+dx)}\left(-3+11e^{2i(c+dx)}+16e^{4i(c+dx)}+2e^{6i(c+dx)}-15e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/48*I)*(-3 + 11*E^((2*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 2*E^((6*
I)*(c + d*x)) - 15*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(
c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(123) = 246$.
time = 0.86, size = 569, normalized size = 3.69

method	result
default	$\dfrac{\left(-15i \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sqrt{2}\,\sin(dx+c)+15\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/d*(-15*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*2^(1/
2)*sin(d*x+c)+15*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-30*I*ar
ctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*2^(1/2)*sin(d*x+c)+30*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)*sin(d*x+c)-15*I*2^(1/2)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+15*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x
+c)-64*I*cos(d*x+c)^6-16*I*cos(d*x+c)^5+64*sin(d*x+c)*cos(d*x+c)^5-40*I*cos
(d*x+c)^4-80*sin(d*x+c)*cos(d*x+c)^4+120*I*cos(d*x+c)^3+120*sin(d*x+c)*cos(
d*x+c)^3)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(
d*x+c)-1)/cos(d*x+c)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(115) = 230$.
time = 0.63, size = 935, normalized size = 6.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/192*(8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*(I*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a
) + 12*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 4*I*sqrt(2)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2
*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 4*sqrt(2))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*
log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(115) = 230$.
time = 0.39, size = 245, normalized size = 1.59

$$\frac{\left(15 \sqrt{\frac{1}{2}}\, d \sqrt{-\frac{a}{d^2}}\, e^{(2i\,dx+2i\,c)} \log\left(\frac{5 \left(\sqrt{2} \sqrt{\frac{1}{2}}\, (de^{(2i\,dx+2i\,c)}+d) \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \sqrt{-\frac{a}{d^2}} + i\,a\right) e^{(-i\,dx-i\,c)}}{4\,d}\right) - 15 \sqrt{\frac{1}{2}}\, d \sqrt{-\frac{a}{d^2}}\, e^{(2i\,dx+2i\,c)} \log\left(-\frac{5 \left(\sqrt{2} \sqrt{\frac{1}{2}}\, (de^{(2i\,dx+2i\,c)}+d) \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \sqrt{-\frac{a}{d^2}} - i\,a\right) e^{(-i\,dx-i\,c)}}{4\,d}\right) + \sqrt{2} \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \left(-2i\,e^{(6i\,dx+6i\,c)} - 16i\,e^{(4i\,dx+4i\,c)} - 11i\,e^{(2i\,dx+2i\,c)} + 3i\right)\right) e^{(-2i\,dx-2i\,c)}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(5/4*(sqrt(2)*sqrt
(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a
/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x +
 2*I*c)*log(-5/4*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sqrt(2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(6*I*d*x + 6*I*c) - 16*I*e^(4*I*d*x
 + 4*I*c) - 11*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=223

$$\frac{63i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,d}+\frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos}{}$$

```
[Out] 63/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a
^(1/2)/d*2^(1/2)+21/64*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+9/40*I*a*c
os(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-63/128*I*cos(d*x+c)*(a+I*a*tan(d*x+c
))^(1/2)/d-21/80*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-1/5*I*cos(d*x+c)
^5*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}+\frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{63i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((63*I)/128)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) + (((21*I)/64)*a*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (((9*I)/40)*a*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x
]]) - (((63*I)/128)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (((21*I)/8
0)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c + d*x]^5*Sqr
t[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{1}{10}(9a)\int \frac{\cos^3(c+}{\sqrt{a+ia\tan(}}$$

$$= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx}}{80d}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{21i\cos}{}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos}{}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos}{}$$

$$= \frac{63i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,d}+\frac{21ia\cos(c-}{64d\sqrt{a+ia\tan}}$$

Mathematica [A]
time = 0.71, size = 152, normalized size = 0.68

$$-\frac{ie^{-5i(c+dx)}\left(-10-95e^{2i(c+dx)}+203e^{4i(c+dx)}+344e^{6i(c+dx)}+64e^{8i(c+dx)}+8e^{10i(c+dx)}-315e^{4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{1280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/1280*I)*(-10 - 95*E^((2*I)*(c + d*x)) + 203*E^((4*I)*(c + d*x)) + 344*
E^((6*I)*(c + d*x)) + 64*E^((8*I)*(c + d*x)) + 8*E^((10*I)*(c + d*x)) - 315
*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((5*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(180) = 360.
time = 0.84, size = 913, normalized size = 4.09

method	result	size
default	Expression too large to display	913

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20480/d*(512*I*cos(d*x+c)^9-315*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+
c)*2^(1/2)+1344*I*cos(d*x+c)^7-1260*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x
+c)*2^(1/2)+315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x
+c)-1890*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-10080*I*cos(d*x
+c)^5-1260*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)+1260*I*cos(d*x+
c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*2^(1/2)-315*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*sin(d*x+c)+1890*I*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*sin(d*x+c)*2^(1/2)+1260*I*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*2^(1/2)-4096*cos(d*x+c)^9*sin(d*x+c)+315
*I*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*2^(1/2
)+4608*sin(d*x+c)*cos(d*x+c)^8+768*I*cos(d*x+c)^8-5376*sin(d*x+c)*cos(d*x+c
```

```
)^7+3360*I*cos(d*x+c)^6+6720*sin(d*x+c)*cos(d*x+c)^6+4096*I*cos(d*x+c)^10-1
0080*sin(d*x+c)*cos(d*x+c)^5)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2220 vs. $2(168) = 336$.
time = 0.79, size = 2220, normalized size = 9.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5120*(20*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c) -
 3*sqrt(2)*sin(4*d*x + 4*c) - 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4*d*x
 + 4*c) + 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))
*sqrt(a) + 4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(8*(-I*sqrt(2)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - I*sqrt(2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 - 2*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - I*sqrt(2))*cos(5/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) + 5*(5*I*sqrt(2)*cos(4*d*x + 4*c) + 20*I*sqrt(2)*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*sqrt(2)*sin(4*d*x + 4*c) + 20
*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*I*sqrt(2
))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 8*(sqrt(2)*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sqrt(2)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + sqrt(2))*sin(5/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) - 5*(5*sqrt(2)*cos(4*d*x + 4*c) + 20*sqrt(2)*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*I*sqrt(2)*sin(4*d*x + 4*c) -
 20*I*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*sqr
t(2))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 315*
(2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*a
```

```
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)),
 (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*si
n(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) +
I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))...
```

Fricas [A]

time = 0.37, size = 267, normalized size = 1.20

$$\frac{\left(315\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{64\,d}\right)-315\sqrt{\frac{1}{2}}\,d\sqrt{-\frac{a}{d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-i\,a\right)e^{(-i\,dx-i\,c)}}{64\,d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(10i\,dx+10i\,c)}-64i\,e^{(8i\,dx+8i\,c)}-344i\,e^{(6i\,dx+6i\,c)}-203i\,e^{(4i\,dx+4i\,c)}+95i\,e^{(2i\,dx+2i\,c)}+10i\right)\right)e^{(-4i\,dx-4i\,c)}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(63/64*(sqrt(2)
*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
```

```
rt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I
*d*x + 4*I*c)*log(-63/64*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(10*I*d*x + 10*I*c) - 64*I*
e^(8*I*d*x + 8*I*c) - 344*I*e^(6*I*d*x + 6*I*c) - 203*I*e^(4*I*d*x + 4*I*c)
 + 95*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^5\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d}+\frac{2i(a+ia\tan(c+dx}{17a^7d}$$

```
[Out] -16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/d+24/13*I*(a+I*a*tan(d*x+c))^(13/2)/
a^5/d-4/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^6/d+2/17*I*(a+I*a*tan(d*x+c))^(17/2
)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3568, 45\}$

$$\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}-\frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^4*d) + (((24*I)/13)*(a + I*
a*Tan[c + d*x])^(13/2))/(a^5*d) - (((4*I)/5)*(a + I*a*Tan[c + d*x])^(15/2))
/(a^6*d) + (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{9/2}-12a^2(a+x)^{11/2}+6a(a+x)^{13/2}-\right.\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-$$

Mathematica [A]
time = 1.31, size = 111, normalized size = 0.95

$$\frac{2a\sec^8(c+dx)(\cos(dx)-i\sin(dx))(646\cos(c+dx)+1121\cos(3(c+dx))-11i(34\sin(c+dx)+99\sin(3(c+dx))))(-i\cos(5c+6dx)+\sin(5c+6dx))\sqrt{a+ia\tan(c+dx)}}{12155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^8*(Cos[d*x] - I*Sin[d*x])*(646*Cos[c + d*x] + 1121*Cos[3*
(c + d*x)] - (11*I)*(34*Sin[c + d*x] + 99*Sin[3*(c + d*x)]))*((-I)*Cos[5*c
+ 6*d*x] + Sin[5*c + 6*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(12155*d)
```

Maple [A]
time = 4.12, size = 152, normalized size = 1.30

method	result
default	$-\frac{2\left(2048i\left(\cos^8(dx+c)\right)-2048\sin(dx+c)\left(\cos^7(dx+c)\right)+256i\left(\cos^6(dx+c)\right)-1280\sin(dx+c)\left(\cos^5(dx+c)\right)+112i\left(\cos^4(dx+c)\right)-1008\right.}{12155d\cos(dx+c)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/12155/d*(2048*I*cos(d*x+c)^8-2048*sin(d*x+c)*cos(d*x+c)^7+256*I*cos(d*x+
c)^6-1280*sin(d*x+c)*cos(d*x+c)^5+112*I*cos(d*x+c)^4-1008*sin(d*x+c)*cos(d*
x+c)^3+66*I*cos(d*x+c)^2-858*sin(d*x+c)*cos(d*x+c)-715*I)*(a*(I*sin(d*x+c)+
cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8*a
```

Maxima [A]
time = 0.29, size = 76, normalized size = 0.65

$$\frac{2i\left(715\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}-4862\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a+11220\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a^2-8840\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^3\right)}{12155\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/12155*I*(715*(I*a*tan(d*x + c) + a)^(17/2) - 4862*(I*a*tan(d*x + c) + a)^
(15/2)*a + 11220*(I*a*tan(d*x + c) + a)^(13/2)*a^2 - 8840*(I*a*tan(d*x + c)
 + a)^(11/2)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.38, size = 170, normalized size = 1.45

$$-\frac{512\,\sqrt{2}\,\left(16i\,ae^{(17i\,dx+17i\,c)}+136i\,ae^{(15i\,dx+15i\,c)}+510i\,ae^{(13i\,dx+13i\,c)}+1105i\,ae^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{12155\,\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c)}+8\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -512/12155*sqrt(2)*(16*I*a*e^(17*I*d*x + 17*I*c) + 136*I*a*e^(15*I*d*x + 15
*I*c) + 510*I*a*e^(13*I*d*x + 13*I*c) + 1105*I*a*e^(11*I*d*x + 11*I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x +
 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e
^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) +
8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^8, x)
```

Mupad [B]
time = 16.02, size = 544, normalized size = 4.65

$$-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{12155\,d}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{12155\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3072\mathrm{i}}{12155\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{2431\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,155136\mathrm{i}}{2431\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2413568\mathrm{i}}{12155\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,270336\mathrm{i}}{1105\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11776\mathrm{i}}{85\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}+\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^8,x)
```

```
[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*155136i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(12155*d*(exp(c*2i
+ d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*3072i)/(12155*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a
*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(24
31*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(12155*d) - (a*(a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2413568i)/(12155*d*
(exp(c*2i + d*x*2i) + 1)^5) + (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*270336i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^6)
 - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*11776i)/(85*d*(exp(c*2i + d*x*2i) + 1)^7) + (a*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*(exp(c*2i + d*
x*2i) + 1)^8)
```

3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}$$

```
[Out] -8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d+8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/
d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^3*d) + (((8*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^4*d) - (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a
^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right)\,dx,x,ia\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{2i(a\ }{}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 93, normalized size = 1.06

$$\frac{2a\sec^6(c+dx)(\cos(dx)-i\sin(dx))(52+151\cos(2(c+dx))-135i\sin(2(c+dx)))(-i\cos(4c+5dx)+\sin(4c+5dx))\sqrt{a+ia\tan(c+dx)}}{1287d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^6*(Cos[d*x] - I*Sin[d*x])*(52 + 151*Cos[2*(c + d*x)] - (1
35*I)*Sin[2*(c + d*x)])*((-I)*Cos[4*c + 5*d*x] + Sin[4*c + 5*d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])/(1287*d)
```

Maple [A]
time = 0.78, size = 125, normalized size = 1.42

method	result
default	$-\frac{2(256i(\cos^6(dx+c))-256\sin(dx+c)(\cos^5(dx+c))+32i(\cos^4(dx+c))-160\sin(dx+c)(\cos^3(dx+c))+14i(\cos^2(dx+c))-126\sin(dx+}{1287d\cos(dx+c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1287/d*(256*I*cos(d*x+c)^6-256*sin(d*x+c)*cos(d*x+c)^5+32*I*cos(d*x+c)^4
-160*sin(d*x+c)*cos(d*x+c)^3+14*I*cos(d*x+c)^2-126*sin(d*x+c)*cos(d*x+c)-99
*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6*a
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.66

$$-\frac{2i\left(99\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-468\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a+572\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a^2\right)}{1287\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1287*I*(99*(I*a*tan(d*x + c) + a)^(13/2) - 468*(I*a*tan(d*x + c) + a)^(1
1/2)*a + 572*(I*a*tan(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(64) = 128.
time = 0.40, size = 134, normalized size = 1.52

$$-\frac{128\sqrt{2}\left(8i\,ae^{(13i\,dx+13i\,c)}+52i\,ae^{(11i\,dx+11i\,c)}+143i\,ae^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1287\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -128/1287*sqrt(2)*(8*I*a*e^(13*I*d*x + 13*I*c) + 52*I*a*e^(11*I*d*x + 11*I*
c) + 143*I*a*e^(9*I*d*x + 9*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(1
2*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) +
20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^{6}{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

Mupad [B]
time = 7.31, size = 420, normalized size = 4.77

$$-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1287\,d}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1287\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{429\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,27136\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,58624\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,5120\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^6,x)
```

```
[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*27136i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1287*d*(exp(c*2i + d
*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*128i)/(429*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1287*d)
 - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*58624i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) + (a*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*5120i)/(143*d*(exp(c*2i
+ d*x*2i) + 1)^5) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

```
[Out] -4/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^2/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}-\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-4*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^2*d) + (((2*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 81, normalized size = 1.37

$$-\frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))(\cos(3c+4dx)+i\sin(3c+4dx))(11i+7\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(Cos[3*c + 4*d*x] + I*Sin[3*c
+ 4*d*x])*(11*I + 7*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(63*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.
time = 0.74, size = 98, normalized size = 1.66

method	result
default	$-\frac{2\left(16i\left(\cos^4(dx+c)\right)-16\sin(dx+c)\left(\cos^3(dx+c)\right)+2i\left(\cos^2(dx+c)\right)-10\sin(dx+c)\cos(dx+c)-7i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a}{63d\cos(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63/d*(16*I*cos(d*x+c)^4-16*sin(d*x+c)*cos(d*x+c)^3+2*I*cos(d*x+c)^2-10*s
in(d*x+c)*cos(d*x+c)-7*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/co
s(d*x+c)^4*a
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.68

$$\frac{2i\left(7\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}-18\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a\right)}{63\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*I*(7*(I*a*tan(d*x + c) + a)^(9/2) - 18*(I*a*tan(d*x + c) + a)^(7/2)*a)
/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.
time = 0.36, size = 98, normalized size = 1.66

$$-\frac{32\sqrt{2}\left(2i\,ae^{(9i\,dx+9i\,c)}+9i\,ae^{(7i\,dx+7i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -32/63*sqrt(2)*(2*I*a*e^(9*I*d*x + 9*I*c) + 9*I*a*e^(7*I*d*x + 7*I*c))*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I
*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Mupad [B]
time = 6.10, size = 296, normalized size = 5.02

$$-\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{63\,d}-\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{63\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)}+\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,160i}{21\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,608i}{63\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^3}+\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{9\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*160i)/(21*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*d*(exp(c*2i + d*x*2i)
+ 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*64i)/(63*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*608i)/(63*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*
d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.
time = 0.40, size = 69, normalized size = 2.38

$$\frac{2a\sec^2(c+dx)(\cos(dx)-i\sin(dx))(-i\cos(2c+3dx)+\sin(2c+3dx))\sqrt{a+ia\tan(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^2*(Cos[d*x] - I*Sin[d*x])*((-I)*Cos[2*c + 3*d*x] + Sin[2*
c + 3*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d)
```

Maple [A]
time = 0.21, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5da}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5da}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/a
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.72

$$-\frac{2i\,(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*(I*a*tan(d*x + c) + a)^(5/2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(21) = 42$.
time = 0.41, size = 59, normalized size = 2.03

$$-\frac{8i\,\sqrt{2}\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(5i\,dx+5i\,c)}}{5\,(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -8/5*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(d*e
^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Mupad [B]
time = 1.36, size = 153, normalized size = 5.28

$$-\frac{4\,a\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,7{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,4{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}-5\,\sin\left(2\,c+2\,d\,x\right)-4\,\sin\left(4\,c+4\,d\,x\right)-\sin\left(6\,c+6\,d\,x\right)+4{}\mathrm{i}\right)}{5\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] -(4*a*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(2*c + 2*d*x)*7i + cos(4*c + 4*d*x)*4i + cos(6*c + 6*d*x)*1i
 - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 4i))/(5*d*(
15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}-\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

```
[Out] -1/4*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1
/2)-1/2*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3568, 44, 65, 212}

$$-\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}-\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(S
qrt[2]*d) - ((I/2)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]
))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x,\right.}{4d}$$

$$= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia}\right.}{2d}$$

$$= -\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d} - \frac{ia^2\sqrt{a+ia\tan(c+}}{2d(a-ia\tan(c+d}$$

Mathematica [A]
time = 0.71, size = 97, normalized size = 1.04

$$-\frac{iae^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/4*I)*a*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^((2*I
)*(c + d*x))] + ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^
(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(73) = 146.
time = 1.48, size = 396, normalized size = 4.26

method	result
default	$\frac{\left(i\sqrt{2}\,\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sin(dx+c)+i\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(d}{1+\cos(d}}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(I*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(
1/2))+I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+2^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d
*x+c)-8*I*cos(d*x+c)^4+8*sin(d*x+c)*cos(d*x+c)^3+4*I*cos(d*x+c)^3-4*cos(d*x
+c)^2*sin(d*x+c)+4*I*cos(d*x+c)^2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*a
```

Maxima [A]
time = 0.51, size = 98, normalized size = 1.05

$$\frac{i\left(\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{16\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^3}{4i\,a\tan(dx+c)-4\,a}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 16*sqrt(I*a*tan(d*x + c)
+ a)*a^3/(4*I*a*tan(d*x + c) - 4*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.
time = 0.41, size = 244, normalized size = 2.62

$$-\frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,de^{(2i\,dx+2i\,c)}+i\,d)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,de^{(2i\,dx+2i\,c)}-i\,d)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{2}\left(-i\,ae^{(3i\,dx+3i\,c)}-i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x
+ 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(
2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-
I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\cos^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=166

$$-\frac{15ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}+\frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\,\text{ta}}}$$

```
[Out] -15/64*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^
(1/2)+15/32*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/4*I*a^4/d/(a+I*a*tan(d*x+c))
^(1/2)/(a-I*a*tan(d*x+c))^2-5/16*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*ta
n(d*x+c))
```

Rubi [A]

time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{15ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-15*I)/32)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((15*I)/32)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/4)*a
^4)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/16)*a
^3)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{(5ia^4)\,\text{Subst}\Big($$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{}{16d(a-ia\tan}$$

$$= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan}}$$

$$= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan}}$$

$$= -\frac{15ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c}}$$

Mathematica [A]

time = 0.91, size = 143, normalized size = 0.86

$$\frac{ae^{-2i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-8+9e^{2i(c+dx)}+2e^{4i(c+dx)}\right)+15e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\cos^2(c+dx)(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{32d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-8 + 9*E^((2*I)*(c + d*x)) + 2*E^((4*I)*
(c + d*x))) + 15*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Cos[c + d*x]^2*(
-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(32*d*E^((2*I)*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. 2(133) = 266.
time = 1.10, size = 742, normalized size = 4.47

method	result
default	$-\frac{\left(-45i\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)(\cos^2(dx+c))\sqrt{2}-128i(\cos^7(dx+c))-15\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/512/d*(-128*I*cos(d*x+c)^7+80*I*cos(d*x+c)^5-15*sin(d*x+c)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(7/2)*cos(d*x+c)^3*2^(1/2)-45*I*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))-45*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*
2^(1/2)+256*I*cos(d*x+c)^8-45*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*2^(1
/2)-15*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-240*I*cos(d*x+c)^4+32*I*cos(d*
x+c)^6-256*sin(d*x+c)*cos(d*x+c)^7-15*I*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+128*sin(d*x+c)*cos(d*x+c)^6-15*I*2^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-160*sin(d*x+c)*co
s(d*x+c)^5-45*I*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))+240*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/co
s(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*a
```

Maxima [A]
time = 0.50, size = 158, normalized size = 0.95

$$\frac{i\left(15\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(15\left(i\,a\tan(dx+c)+a\right)^{2}a^{3}-50\left(i\,a\tan(dx+c)+a\right)a^{4}+32\,a^{5}\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}-4\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a+4\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{2}}\right)}{128\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/128*I*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
 c) + a)^2*a^3 - 50*(I*a*tan(d*x + c) + a)*a^4 + 32*a^5)/((I*a*tan(d*x + c)
 + a)^(5/2) - 4*(I*a*tan(d*x + c) + a)^(3/2)*a + 4*sqrt(I*a*tan(d*x + c) +
a)*a^2))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(123) = 246$.
time = 0.39, size = 287, normalized size = 1.73

$$-\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{3}}{d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{\left(2i\,dx+2i\,c\right)}+i\,d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}-a^{2}e^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{a}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{3}}{d^{2}}}\,de^{\left(i\,dx+i\,c\right)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-i\,de^{\left(2i\,dx+2i\,c\right)}-i\,d\right)\sqrt{-\frac{a^{3}}{d^{2}}}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}-a^{2}e^{\left(i\,dx+i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{a}\right)-\sqrt{2}\left(-2i\,ae^{\left(6i\,dx+6i\,c\right)}-11i\,ae^{\left(4i\,dx+4i\,c\right)}-i\,ae^{\left(2i\,dx+2i\,c\right)}+8i\,a\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\right)e^{\left(-i\,dx-i\,c\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1
/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 15*sqrt(1/2)*sqrt(-a
^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*
c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x +
 I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-2*I*a*e^(6*I*d*x + 6*I*c) - 11*I*a*e
^(4*I*d*x + 4*I*c) - I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=239

$$-\frac{105ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,d}+\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\,\mathrm{ta}}$$

```
[Out] -105/512*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*
2^(1/2)+105/256*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)+35/128*I*a^3/d/(a+I*a*tan(
d*x+c))^(3/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(3/2)-3/1
6*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-21/64*I*a^4/d/(a-I*
a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{105ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,d}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}+\frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-105*I)/256)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I
/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) - (((3*I
)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((2
1*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((
105*I)/256)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{(3ia^6)\,\text{Subst}(\ldots}{\ldots}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{\ldots}{16d(a-ia\,\text{ta}\ldots}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{\ldots}{16d(a-ia\,\text{ta}\ldots}$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\,\text{ta}\ldots}$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\,\text{ta}\ldots}$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\,\text{ta}\ldots}$$

$$= -\frac{105ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,d} + \frac{35ia^3}{128d(a+ia\tan(c\ldots}$$

Mathematica [A]
time = 1.18, size = 169, normalized size = 0.71

$$\frac{ae^{-4i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-16-208e^{2i(c+dx)}+165e^{4i(c+dx)}+50e^{6i(c+dx)}+8e^{8i(c+dx)}\right)+315e^{3i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\cos^2(c+dx)(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{768d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-16 - 208*E^((2*I)*(c + d*x)) + 165*E^((
4*I)*(c + d*x)) + 50*E^((6*I)*(c + d*x)) + 8*E^((8*I)*(c + d*x))) + 315*E^(
(3*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Cos[c + d*x]^2*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])/(768*d*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(193) = 386.
time = 1.05, size = 1086, normalized size = 4.54

method	result	size
default	Expression too large to display	1086

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/49152/d*(-315*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d
*x+c)-315*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(d*x+c)^5*sin(d*x
+c)*2^(1/2)-1575*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(d*x+c)^4*
sin(d*x+c)*2^(1/2)-3150*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*cos(d*
x+c)^3*sin(d*x+c)*2^(1/2)-3150*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)
*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-1575*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(11/2)*cos(d*x+c)*sin(d*x+c)*2^(1/2)-9216*cos(d*x+c)^9*sin(d*x+c)-315*2^(1
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)-16384*sin(d*x+c)*cos(d*x+c)^11+10752*sin
(d*x+c)*cos(d*x+c)^8+8192*cos(d*x+c)^10*sin(d*x+c)-315*cos(d*x+c)^5*sin(d*x
+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*2^(1/2))-1575*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))-3150*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-3150
*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-1575*cos(d*x+c)*sin(d*
x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-13440*sin(d*x+c)*cos(d*x+c)^7+20160*sin(d*
x+c)*cos(d*x+c)^6+16384*I*cos(d*x+c)^12-8192*I*cos(d*x+c)^11+1024*I*cos(d*x
+c)^10+1536*I*cos(d*x+c)^9+2688*I*cos(d*x+c)^8+6720*I*cos(d*x+c)^7-20160*I*
cos(d*x+c)^6)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+
cos(d*x+c)-1)/cos(d*x+c)^5*a
```

Maxima [A]

time = 0.51, size = 212, normalized size = 0.89

$$\frac{i\left(315\,\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^{4}a^{3}-1680\,(i\,a\tan(dx+c)+a)^{3}a^{4}+2772\,(i\,a\tan(dx+c)+a)^{2}a^{5}-1152\,(i\,a\tan(dx+c)+a)a^{6}-256\,a^{7}\right)}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+12\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^{2}-8\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^{3}}\right)}{3072\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3072*I*(315*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(d*
x + c) + a)^4*a^3 - 1680*(I*a*tan(d*x + c) + a)^3*a^4 + 2772*(I*a*tan(d*x +
 c) + a)^2*a^5 - 1152*(I*a*tan(d*x + c) + a)*a^6 - 256*a^7)/((I*a*tan(d*x +
 c) + a)^(9/2) - 6*(I*a*tan(d*x + c) + a)^(7/2)*a + 12*(I*a*tan(d*x + c) +
a)^(5/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(3/2)*a^3))/(a*d)
```

Fricas [A]
time = 0.38, size = 311, normalized size = 1.30

$$-\frac{315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^3}{d^2}} d e^{(3 i dx+3 i c)} \log\left(-\frac{4 \left(\sqrt{2} \sqrt{\frac{1}{2}} (i d e^{(2 i dx+2 i c)}+i d) \sqrt{-\frac{a^3}{d^2}} \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}}-a^2 e^{(i dx+i c)}\right) e^{(-i dx-i c)}}{a}\right) - 315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^3}{d^2}} d e^{(3 i dx+3 i c)} \log\left(-\frac{4 \left(\sqrt{2} \sqrt{\frac{1}{2}} (-i d e^{(2 i dx+2 i c)}-i d) \sqrt{-\frac{a^3}{d^2}} \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}}-a^2 e^{(i dx+i c)}\right) e^{(-i dx-i c)}}{a}\right) - \sqrt{2} \left(-8 i a e^{(10 i dx+10 i c)} - 58 i a e^{(8 i dx+8 i c)} - 215 i a e^{(6 i dx+6 i c)} + 43 i a e^{(4 i dx+4 i c)} + 224 i a e^{(2 i dx+2 i c)} + 16 i a\right) \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}}}{1536 d} e^{(-3 i dx-3 i c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1536*(315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)
*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 315*sqrt(1/2)
*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*
I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^
2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-8*I*a*e^(10*I*d*x + 10*I
*c) - 58*I*a*e^(8*I*d*x + 8*I*c) - 215*I*a*e^(6*I*d*x + 6*I*c) + 43*I*a*e^(
4*I*d*x + 4*I*c) + 224*I*a*e^(2*I*d*x + 2*I*c) + 16*I*a)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^6\,{\left (a+a\,\mathrm {tan}\left (c+d\,x\right)\,1{}\mathrm {i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)\sqrt{a}}{11d}$$

```
[Out] 8/33*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+2/11*I*a*sec(d*x+c)^5*(a
+I*a*tan(d*x+c))^(1/2)/d+256/1155*I*a^4*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(
5/2)+64/231*I*a^3*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/1155)*a^4*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((6
4*I)/231)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((8*I)/33
)*a^2*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/11)*a*Sec[c
+ d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d} + \frac{1}{11}(12a)\int \sec^5(c+dx$$

$$= \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+d}}{11d}$$

$$= \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} + \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia}{}$$

$$= \frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}} + \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} +$$

Mathematica [A]
time = 1.13, size = 109, normalized size = 0.74

$$\frac{2a\sec^4(c+dx)(\cos(dx)-i\sin(dx))(i\cos(3c+2dx)+\sin(3c+2dx))(39+494\cos(2(c+dx))+215i\sec(c+dx)\sin(3(c+dx))+110i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{1155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*a*Sec[c + d*x]^4*(Cos[d*x] - I*Sin[d*x])*(I*Cos[3*c + 2*d*x] + Sin[3*c +
 2*d*x])*(39 + 494*Cos[2*(c + d*x)] + (215*I)*Sec[c + d*x]*Sin[3*(c + d*x)]
 + (110*I)*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(1155*d)
```

Maple [A]
time = 0.76, size = 125, normalized size = 0.85

method	result
default	$\frac{2(512i(\cos^6(dx+c))+512\sin(dx+c)(\cos^5(dx+c))-64i(\cos^4(dx+c))+192\sin(dx+c)(\cos^3(dx+c))-20i(\cos^2(dx+c))+140\sin(dx+c}{1155d\cos(dx+c)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/1155/d*(512*I*cos(d*x+c)^6+512*sin(d*x+c)*cos(d*x+c)^5-64*I*cos(d*x+c)^4+
192*sin(d*x+c)*cos(d*x+c)^3-20*I*cos(d*x+c)^2+140*sin(d*x+c)*cos(d*x+c)+105
*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(115) = 230.
time = 10.55, size = 994, normalized size = 6.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 64/1155*(231*I*sqrt(2)*a*cos(6*d*x + 6*c) + 198*I*sqrt(2)*a*cos(4*d*x + 4*c
) + 88*I*sqrt(2)*a*cos(2*d*x + 2*c) - 231*sqrt(2)*a*sin(6*d*x + 6*c) - 198*
sqrt(2)*a*sin(4*d*x + 4*c) - 88*sqrt(2)*a*sin(2*d*x + 2*c) + 16*I*sqrt(2)*a
)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sqrt(a)/(((4*cos(2*d*x + 2*c)^3 + (4*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)
^2 + 4*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(2*
d*x + 2*c)^2 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x
 + 2*c) + I)*sin(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c
)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(6*d*x + 6*c) + 6*(I*cos(2*d*x + 2*c)^2
+ I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x + 4*c) + 4*(I*
cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*c) + 6*cos(2*d
*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (
4*I*cos(2*d*x + 2*c)^3 + (4*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*c)^2 - 4*
sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos
(2*d*x + 2*c) + I)*cos(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x
 + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(6*d*x + 6*c) + 6*(I*cos(2*d*x + 2
*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) +
 9*I*cos(2*d*x + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(8*d*x + 8*c) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 4*(cos(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + 6*I*cos(2*d*x + 2*c)
 + I)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*d)
```

<u>Fricas</u> [A]
time = 0.38, size = 125, normalized size = 0.85

$$-\frac{64\sqrt{2}\left(-231i\,ae^{(6i\,dx+6i\,c)}-198i\,ae^{(4i\,dx+4i\,c)}-88i\,ae^{(2i\,dx+2i\,c)}-16i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1155\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -64/1155*sqrt(2)*(-231*I*a*e^(6*I*d*x + 6*I*c) - 198*I*a*e^(4*I*d*x + 4*I*c
) - 88*I*a*e^(2*I*d*x + 2*I*c) - 16*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/
(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*
c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{3}{2}} \sec^5 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^5, x)
```

Mupad [B]
time = 7.25, size = 293, normalized size = 1.99

$$\frac{a\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,64{}\mathrm{i}}{5\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}-\frac{a\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,192{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}+\frac{a\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,64{}\mathrm{i}}{3\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{a\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,64{}\mathrm{i}}{11\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] (a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*64i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(- c*1i
 - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*192i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*exp(- c*1i - d*x*1i)*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i
)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(
c*2i + d*x*2i) + 1)^5)
```

3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}}+\frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

```
[Out] 16/35*I*a^2*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+2/7*I*a*sec(d*x+c)^3*(a
+I*a*tan(d*x+c))^(1/2)/d+64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A]
time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}}+\frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((64*I)/105)*a^3*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*
I)/35)*a^2*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/7)*a*Se
c[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Integ
erQ[n]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} + \frac{1}{7}(8a)\int \sec^3(c+dx)$$

$$= \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+d}}{7d}$$

$$= \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia}{}$$

Mathematica [A]
time = 0.58, size = 91, normalized size = 0.83

$$\frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))(28+43\cos(2(c+dx))+27i\sin(2(c+dx)))(i\cos(2c+dx)+\sin(2c+dx))\sqrt{a+ia\tan(c+dx)}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(28 + 43*Cos[2*(c + d*x)] + (27
*I)*Sin[2*(c + d*x)])*(I*Cos[2*c + d*x] + Sin[2*c + d*x])*Sqrt[a + I*a*Tan[
c + d*x]])/(105*d)
```

Maple [A]
time = 0.73, size = 98, normalized size = 0.89

method	result
default	$\frac{2(64i(\cos^4(dx+c))+64\sin(dx+c)(\cos^3(dx+c))-8i(\cos^2(dx+c))+24\sin(dx+c)\cos(dx+c)+15i)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a}{105d\cos(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/d*(64*I*cos(d*x+c)^4+64*sin(d*x+c)*cos(d*x+c)^3-8*I*cos(d*x+c)^2+24*s
in(d*x+c)*cos(d*x+c)+15*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/c
os(d*x+c)^3*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(86) = 172.
time = 0.83, size = 580, normalized size = 5.27

$$\frac{16\left(35i\sqrt{2}a\cos(4dx+4c)+28i\sqrt{2}a\cos(2dx+2c)-35\sqrt{2}a\sin(4dx+4c)-28\sqrt{2}a\sin(2dx+2c)+8i\sqrt{2}a\right)\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sqrt{a}}{105\left(\left(2\cos(2dx+2c)^2+(2\cos(2dx+2c)+1)\sin(2dx+2c)^2+2i\sin(2dx+2c)^2+\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)\cos(4dx+4c)+5\cos(2dx+2c)^2+\left(i\cos(2dx+2c)^2+i\sin(2dx+2c)^2+2i\cos(2dx+2c)+i\right)\sin(4dx+4c)+2\left(i\cos(2dx+2c)^2+2i\cos(2dx+2c)+i\right)\sin(2dx+2c)+4\cos(2dx+2c)+1\right)\cos\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),\cos(2dx+2c)+1\right)\right)+\left(2i\cos(2dx+2c)^2+(2i\cos(2dx+2c)+i)\sin(2dx+2c)^2-2\sin(2dx+2c)^2+\left(i\cos(2dx+2c)^2+i\sin(2dx+2c)^2+2i\cos(2dx+2c)+i\right)\cos(4dx+4c)+5i\cos(2dx+2c)^2-\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)\sin(4dx+4c)-2\left(\cos(2dx+2c)^2+2\cos(2dx+2c)+1\right)\sin(2dx+2c)+4i\cos(2dx+2c)+i\right)\sin\left(\frac{1}{2}\arctan\left(\sin(2dx+2c),\cos(2dx+2c)+1\right)\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 16/105*(35*I*sqrt(2)*a*cos(4*d*x + 4*c) + 28*I*sqrt(2)*a*cos(2*d*x + 2*c) -
 35*sqrt(2)*a*sin(4*d*x + 4*c) - 28*sqrt(2)*a*sin(2*d*x + 2*c) + 8*I*sqrt(2
)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a)/(((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2 + (I*cos(2
*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x
+ 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*
c) + 4*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x
 + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*
c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d*x + 2*c)^2
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*
d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c) + 4*I*cos(2*d*x + 2*c) + I)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*d)
```

Fricas [A]
time = 0.48, size = 89, normalized size = 0.81

$$-\frac{16 \sqrt{2} \left(-35 i \, a e^{\left(4 i \, d x + 4 i \, c\right)} - 28 i \, a e^{\left(2 i \, d x + 2 i \, c\right)} - 8 i \, a\right) \sqrt{\frac{a}{e^{\left(2 i \, d x + 2 i \, c\right)} + 1}}}{105 \left(d e^{\left(6 i \, d x + 6 i \, c\right)} + 3 \, d e^{\left(4 i \, d x + 4 i \, c\right)} + 3 \, d e^{\left(2 i \, d x + 2 i \, c\right)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -16/105*sqrt(2)*(-35*I*a*e^(4*I*d*x + 4*I*c) - 28*I*a*e^(2*I*d*x + 2*I*c) -
 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4
*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i a \left(\tan \left(c + d x\right) - i\right)\right)^{\frac{3}{2}} \sec^{3}\left(c + d x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Mupad [B]
time = 6.02, size = 103, normalized size = 0.94

$$\frac{16\,a\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,28{}\mathrm{i}+{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,35{}\mathrm{i}+8{}\mathrm{i}\right)}{105\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] (16*a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*28i + exp(c*4i + d*x*4i)*35i +
 8i))/(105*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] 8/3*I*a^2*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+2/3*I*a*sec(d*x+c)*(a+I*a*t
an(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*
Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= \frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{3}(4a)\int \sec(c+dx)\sqrt{a-}\\ &= \frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 57, normalized size = 0.83

$$-\frac{2a(\cos(c)-i\sin(c))(\cos(dx)-i\sin(dx))(-5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Sin[d*x])*(-5*I + Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(3*d)
```

Maple [A]
time = 0.83, size = 71, normalized size = 1.03

method	result	size
default	$\frac{2\left(4i\left(\cos^{2}(dx+c)\right)+4\sin(dx+c)\cos(dx+c)+i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a}{3d\cos(dx+c)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(4*I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)+I)*(a*(I*sin(d*x+c)+cos(d*x
+c))/cos(d*x+c))^(1/2)/cos(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.77

$$-\frac{4\sqrt{2}\left(-3i\,ae^{(2i\,dx+2i\,c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

[Out] -4/3*sqrt(2)*(-3*I*a*e^(2*I*d*x + 2*I*c) - 2*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{3}{2}}\sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)

Mupad [B]
time = 4.49, size = 98, normalized size = 1.42

$$\frac{2\,a\,\sqrt{\frac{a\,\left(2\cos \left(c+d\,x\right)^2+\sin \left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos \left(c+d\,x\right)^2}}\,\left(\cos \left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 8\mathrm{i}+\cos \left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2 2\mathrm{i}+\sin \left(c+d\,x\right)+\sin \left(3\,c+3\,d\,x\right)-5\mathrm{i}\right)}{3\,d\cos \left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x),x)

[Out] (2*a*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x) + cos(c/2 + (d*x)/2)^2*8i + cos((3*c)/2 + (3*d*x)/2)^2*2i - 5i))/(3*d*cos(c + d*x)^2)

3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=31

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3574}

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A]
time = 0.18, size = 31, normalized size = 1.00

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d

Maple [A]
time = 0.73, size = 42, normalized size = 1.35

method	result	size
default	$-\frac{2 i \sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c) a}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*I/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(25) = 50.
time = 0.57, size = 201, normalized size = 6.48

$$\frac{2\left(i\,a^{\frac{3}{2}}-\frac{2i\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{i\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 2*(I*a^(3/2) - 2*I*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + I*a^(3/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1))

Fricas [A]
time = 0.35, size = 40, normalized size = 1.29

$$\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [B]
time = 0.23, size = 60, normalized size = 1.94

$$-\frac{a\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\sqrt{\frac{a\,\left(2\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos\left(c+d\,x\right)^2}}\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] -(a*(2*cos(c/2 + (d*x)/2)^2 - 1)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^
2))/(2*cos(c + d*x)^2))^(1/2)*2i)/d
```

3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}\,d}-\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] 1/4*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/d*2^(1/2)-1/2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c
)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3571, 3570, 212}

$$\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}\,d}-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/2)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - ((I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{1}{2}a\int \cos(c+dx)\sqrt{a}$$

$$= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\,\mathrm{t}}{3d}$$

$$= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\,\mathrm{t}}{3d}$$

$$= \frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}\,d} - \frac{ia\cos(c+dx)\sqrt{a}}{2}$$

Mathematica [A]
time = 1.09, size = 101, normalized size = 0.83

$$-\frac{iae^{-i(c+dx)}\left(4+5e^{2i(c+dx)}+e^{4i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\,\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/12*I)*a*(4 + 5*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) - 3*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*T
an[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(97) = 194.
time = 0.86, size = 570, normalized size = 4.67

method	result
default	$-\frac{\left(3i\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sqrt{2}\,\sin(dx+c)+6i\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x
```

```
+c)*2^(1/2)+6*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*
x+c)*2^(1/2)-3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+3*I*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-6*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(5/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*cos(d*x+c)*2^(1/2)*sin(d*x+c)-3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+32*
I*cos(d*x+c)^6-16*I*cos(d*x+c)^5-32*sin(d*x+c)*cos(d*x+c)^5+8*I*cos(d*x+c)^
4+16*sin(d*x+c)*cos(d*x+c)^4-24*I*cos(d*x+c)^3-24*sin(d*x+c)*cos(d*x+c)^3)*
(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/
cos(d*x+c)^2*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(91) = 182.
time = 0.63, size = 884, normalized size = 7.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(4*(I*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqr
t(a) + 12*(I*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a) + 3*(2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2
*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a*l
og(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a*log(sqrt(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
```

```
2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*s
qrt(a))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(91) = 182$.
time = 0.38, size = 222, normalized size = 1.82

$$\frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{d}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-i\,a^2\right)e^{(-i\,dx-i\,c)}}{d}\right)+\sqrt{2}\left(-i\,ae^{(4i\,dx+4i\,c)}-5i\,ae^{(2i\,dx+2i\,c)}-4i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log((sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2
*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + I*a^2)*e^(-I*
d*x - I*c)/d) - 3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-(sqrt(2)*sqrt(1/2)*(d*e^(
2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*
a^2)*e^(-I*d*x - I*c)/d) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) - 5*I*a*e^(2*I
*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=192

$$\frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}\,d}+\frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+d}}{16d}$$

```
[Out] 7/32*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)+7/24*I*a^2*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-7/16*I*a*co
s(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-7/30*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c)
)^(1/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$\frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}\,d}+\frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}-\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((7*I)/16)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((7*I)/24)*a^2*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((7*I)/16)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d -
 (((7*I)/30)*a*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c
+ d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}+\frac{1}{10}(7a)\int \cos^3(c+dx)$$

$$= -\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d}-\frac{i\cos^5(c+dx)(a+ia}{5d}$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx}}{30d}$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}$$

$$= \frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}\,d}+\frac{7ia^2\cos(c+}{24d\sqrt{a+ia\tan(}}$$

Mathematica [A]

time = 1.63, size = 160, normalized size = 0.83

$$-\frac{iae^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-15+101e^{2i(c+dx)}+148e^{4i(c+dx)}+38e^{6i(c+dx)}+6e^{8i(c+dx)}-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{240\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/240*I)*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-15
+ 101*E^((2*I)*(c + d*x)) + 148*E^((4*I)*(c + d*x)) + 38*E^((6*I)*(c + d*x)
) + 6*E^((8*I)*(c + d*x)) - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*
x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(155) = 310.
time = 0.82, size = 914, normalized size = 4.76

method	result	size
default	Expression too large to display	914

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7680/d*(420*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d
*x+c)^3*2^(1/2)-1536*I*cos(d*x+c)^9-105*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin
(d*x+c)*2^(1/2)+448*I*cos(d*x+c)^7-420*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(
d*x+c)*2^(1/2)+420*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*c
os(d*x+c)*2^(1/2)-630*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-33
60*I*cos(d*x+c)^5-420*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-105*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+105*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(9/2)*sin(d*x+c)+630*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*ar
ctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)-3072*cos(d*x+c)^9*sin(d*x+c)+256*I*cos(d
*x+c)^8+1536*sin(d*x+c)*cos(d*x+c)^8+1120*I*cos(d*x+c)^6-1792*sin(d*x+c)*co
s(d*x+c)^7+3072*I*cos(d*x+c)^10+2240*sin(d*x+c)*cos(d*x+c)^6+105*I*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
```

```
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)-3360*sin(
d*x+c)*cos(d*x+c)^5)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(
d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^4*a
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.40, size = 274, normalized size = 1.43

$$\frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,de^{(2i\,dx+2i\,c)}\log\left(\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{8\,d}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,de^{(2i\,dx+2i\,c)}\log\left(-\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-i\,a^2\right)e^{(-i\,dx-i\,c)}}{8\,d}\right)+\sqrt{2}\left(-6i\,ae^{(8i\,dx+8i\,c)}-38i\,ae^{(6i\,dx+6i\,c)}-148i\,ae^{(4i\,dx+4i\,c)}-101i\,ae^{(2i\,dx+2i\,c)}+15i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/480*(105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(7/8*(sqrt(2)*
sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)) + I*a^2)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e
^(2*I*d*x + 2*I*c)*log(-7/8*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*
sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*a^2)*e^(-I*d*x - I*c)/
d) + sqrt(2)*(-6*I*a*e^(8*I*d*x + 8*I*c) - 38*I*a*e^(6*I*d*x + 6*I*c) - 148
*I*a*e^(4*I*d*x + 4*I*c) - 101*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}-\frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d}+\frac{2i(a+ia\tan(c+dx}{19a^7d}$$

```
[Out] -16/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^4/d+8/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^
5/d-12/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^6/d+2/19*I*(a+I*a*tan(d*x+c))^(19/2
)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}-\frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^4*d) + (((8*I)/5)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^5*d) - (((12*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))
/(a^6*d) + (((2*I)/19)*(a + I*a*Tan[c + d*x])^(19/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{11/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{11/2} - 12a^2(a+x)^{13/2} + 6a(a+x)^{15/2} - \right.\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d} - \frac{12}{}$$

Mathematica [A]
time = 1.61, size = 113, normalized size = 0.97

$$-\frac{2a^2\sec^8(c+dx)(\cos(6c+8dx)+i\sin(6c+8dx))(-833i+3262i\cos(2(c+dx))+1599\sec(c+dx)\sin(3(c+dx))+494\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{20995d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^8*(Cos[6*c + 8*d*x] + I*Sin[6*c + 8*d*x])*(-833*I + (3
262*I)*Cos[2*(c + d*x)] + 1599*Sec[c + d*x]*Sin[3*(c + d*x)] + 494*Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(20995*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 11.27, size = 171, normalized size = 1.46

method	result
default	$-\frac{2\left(4096i\left(\cos^9(dx+c)\right)-4096\sin(dx+c)\left(\cos^8(dx+c)\right)+512i\left(\cos^7(dx+c)\right)-2560\sin(dx+c)\left(\cos^6(dx+c)\right)+224i\left(\cos^5(dx+c)\right)-2016\right.}{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/20995/d*(4096*I*cos(d*x+c)^9-4096*sin(d*x+c)*cos(d*x+c)^8+512*I*cos(d*x+
c)^7-2560*sin(d*x+c)*cos(d*x+c)^6+224*I*cos(d*x+c)^5-2016*sin(d*x+c)*cos(d*
x+c)^4+132*I*cos(d*x+c)^3-1716*cos(d*x+c)^2*sin(d*x+c)-2535*I*cos(d*x+c)+11
05*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^9*
a^2
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.65

$$\frac{2i\left(1105\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}} - 7410\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}a + 16796\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^2 - 12920\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a^3\right)}{20995\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/20995*I*(1105*(I*a*tan(d*x + c) + a)^(19/2) - 7410*(I*a*tan(d*x + c) + a)
^(17/2)*a + 16796*(I*a*tan(d*x + c) + a)^(15/2)*a^2 - 12920*(I*a*tan(d*x +
c) + a)^(13/2)*a^3)/(a^7*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.
time = 0.47, size = 190, normalized size = 1.62

$$-\frac{1024\,\sqrt{2}\,\left(16i\,a^{2}e^{(19i\,dx+19i\,c)}+152i\,a^{2}e^{(17i\,dx+17i\,c)}+646i\,a^{2}e^{(15i\,dx+15i\,c)}+1615i\,a^{2}e^{(13i\,dx+13i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{20995\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i\,dx+8i\,c)}+84\,de^{(6i\,dx+6i\,c)}+36\,de^{(4i\,dx+4i\,c)}+9\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1024/20995*sqrt(2)*(16*I*a^2*e^(19*I*d*x + 19*I*c) + 152*I*a^2*e^(17*I*d*x
 + 17*I*c) + 646*I*a^2*e^(15*I*d*x + 15*I*c) + 1615*I*a^2*e^(13*I*d*x + 13*
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(1
6*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c)
 + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*
x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

Mupad [**B**]
time = 15.90, size = 626, normalized size = 5.35

$$-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,16384{}\mathrm{i}}{20995\,d}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,8192{}\mathrm{i}}{20995\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,6144{}\mathrm{i}}{20995\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,1024{}\mathrm{i}}{4199\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,536576{}\mathrm{i}}{4199\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,10484736{}\mathrm{i}}{20995\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,17262592{}\mathrm{i}}{20995\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^6}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,1129472{}\mathrm{i}}{1615\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^7}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,98304{}\mathrm{i}}{323\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^8}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,1024{}\mathrm{i}}{19\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^8,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*536576i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(20995*d*(exp(c
*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*6144i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
024i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(20995*d) - (a^2*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*10484
736i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^2*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*17262592i)/(20995*d*(exp(c*
2i + d*x*2i) + 1)^6) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*1129472i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^7) + (
a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*98304i)/(323*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(19*d*(exp(c*2i +
d*x*2i) + 1)^9)
```

3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}$$

```
[Out] -8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/d+8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^
4/d-2/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^5/d
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-8*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^3*d) + (((8*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^4*d) - (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/
(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)\,dx,x,i\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{2i(a\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.85, size = 97, normalized size = 1.10

$$\frac{2a^2\sec^7(c+dx)(60+203\cos(2(c+dx))-187i\sin(2(c+dx)))(-i\cos(5c+7dx)+\sin(5c+7dx))\sqrt{a+ia\tan(c+dx)}}{2145d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^7*(60 + 203*Cos[2*(c + d*x)] - (187*I)*Sin[2*(c + d*x)]
)*((-I)*Cos[5*c + 7*d*x] + Sin[5*c + 7*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(2
145*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(70) = 140.
time = 0.94, size = 144, normalized size = 1.64

method	result
default	$\frac{2\left(-512i\left(\cos^7(dx+c)\right)+512\sin(dx+c)\left(\cos^6(dx+c)\right)-64i\left(\cos^5(dx+c)\right)+320\sin(dx+c)\left(\cos^4(dx+c)\right)-28i\left(\cos^3(dx+c)\right)+252\left(\cos^2(d\right.\right.}{2145d\cos(dx+c)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/2145/d*(-512*I*cos(d*x+c)^7+512*sin(d*x+c)*cos(d*x+c)^6-64*I*cos(d*x+c)^5
+320*sin(d*x+c)*cos(d*x+c)^4-28*I*cos(d*x+c)^3+252*cos(d*x+c)^2*sin(d*x+c)+
341*I*cos(d*x+c)-143*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)/cos(d*x+c)^7*a^2
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.66

$$-\frac{2i\left(143\left(ia\tan\left(dx+c\right)+a\right)^{\frac{15}{2}}-660\left(ia\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}a+780\left(ia\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*I*(143*(I*a*tan(d*x + c) + a)^(15/2) - 660*(I*a*tan(d*x + c) + a)^(
13/2)*a + 780*(I*a*tan(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.
time = 0.40, size = 152, normalized size = 1.73

$$-\frac{256\sqrt{2}\left(8i\,a^2e^{(15i\,dx+15i\,c)}+60i\,a^2e^{(13i\,dx+13i\,c)}+195i\,a^2e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{2145\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -256/2145*sqrt(2)*(8*I*a^2*e^(15*I*d*x + 15*I*c) + 60*I*a^2*e^(13*I*d*x + 1
3*I*c) + 195*I*a^2*e^(11*I*d*x + 11*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x +
10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I
*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

Mupad [B]

time = 11.32, size = 498, normalized size = 5.66

$$-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,2048{}\mathrm{i}}{2145\,d}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,1024{}\mathrm{i}}{2145\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,256{}\mathrm{i}}{715\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,18176{}\mathrm{i}}{429\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,52736{}\mathrm{i}}{429\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,103936{}\mathrm{i}}{715\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,15616{}\mathrm{i}}{195\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^6}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,256{}\mathrm{i}}{15\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^6,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*18176i)/(429*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(2145*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*256i)/(715*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a -
(a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/
(2145*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*52736i)/(429*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^2*(a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*103936i)/(715
*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*15616i)/(195*d*(exp(c*2i + d*x*2i) + 1)
^6) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*256i)/(15*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=59

$$-\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

```
[Out] -4/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^2/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/
d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^2*d) + (((2*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{7/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}\end{aligned}$$

Mathematica [A]
time = 0.65, size = 85, normalized size = 1.44

$$-\frac{2a^2\sec^4(c+dx)(\cos(4c+6dx)+i\sin(4c+6dx))(13i+9\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{99d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^4*(Cos[4*c + 6*d*x] + I*Sin[4*c + 6*d*x])*(13*I + 9*Ta
n[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(99*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.
time = 0.76, size = 117, normalized size = 1.98

method	result
default	$-\frac{2(32i(\cos^5(dx+c))-32\sin(dx+c)(\cos^4(dx+c))+4i(\cos^3(dx+c))-20(\cos^2(dx+c))\sin(dx+c)-23i\cos(dx+c)+9\sin(dx+c))\sqrt{\frac{a(i s}{}}}{99d\cos(dx+c)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/99/d*(32*I*cos(d*x+c)^5-32*sin(d*x+c)*cos(d*x+c)^4+4*I*cos(d*x+c)^3-20*c
os(d*x+c)^2*sin(d*x+c)-23*I*cos(d*x+c)+9*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5*a^2
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.68

$$\frac{2i\left(9\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-22\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/99*I*(9*(I*a*tan(d*x + c) + a)^(11/2) - 22*(I*a*tan(d*x + c) + a)^(9/2)*a
)/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.
time = 0.38, size = 114, normalized size = 1.93

$$-\frac{64\sqrt{2}\left(2i\,a^2e^{(11i\,dx+11i\,c)}+11i\,a^2e^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{99\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -64/99*sqrt(2)*(2*I*a^2*e^(11*I*d*x + 11*I*c) + 11*I*a^2*e^(9*I*d*x + 9*I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*
d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Mupad [**B**]
time = 6.42, size = 370, normalized size = 6.27

$$-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,128{}\mathrm{i}}{99\,d}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,64{}\mathrm{i}}{99\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,512{}\mathrm{i}}{33\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,2944{}\mathrm{i}}{99\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,2176{}\mathrm{i}}{99\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,64{}\mathrm{i}}{11\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*512i)/(33*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*d*(exp(c*2i + d*x*
2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*128i)/(99*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*2944i)/(99*d*(exp(c*2i + d*x*2i) + 1)^3) +
 (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*2176i)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*
x*2i) + 1)^5)
```

3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

```
[Out] -2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^{5/2}\,dx,x,ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.
time = 0.49, size = 73, normalized size = 2.52

$$\frac{2a^2\sec^3(c+dx)(-i\cos(3c+5dx)+\sin(3c+5dx))\sqrt{a+ia\tan(c+dx)}}{7d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3*((-I)*Cos[3*c + 5*d*x] + Sin[3*c + 5*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(7*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.20, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7da}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7da}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*I*(a+I*a*tan(d*x+c))^(7/2)/d/a
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.72

$$-\frac{2i\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}}{7\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7*I*(I*a*tan(d*x + c) + a)^(7/2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.
time = 0.39, size = 73, normalized size = 2.52

$$-\frac{16i\,\sqrt{2}\,a^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(7i\,dx+7i\,c)}}{7\,(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -16/7*I*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(
d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^{\frac{5}{2}}\sec ^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Mupad [B]
time = 6.35, size = 242, normalized size = 8.34

$$-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,16{}\mathrm{i}}{7\,d}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,48{}\mathrm{i}}{7\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,48{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,16{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*48i)/(7*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d) - (a^2*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(7*d*(exp(
c*2i + d*x*2i) + 1)^2) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp
(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

```
[Out] 1/2*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/
2)-I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3568, 43, 65, 212}

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*
d) - (I*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x,\right.}{2d}$$

$$= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\right.}{d}$$

$$= \frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d} - \frac{ia^3\sqrt{a+ia\tan(c+dx}}{d(a-ia\tan(c+dx))}$$

Mathematica [**A**]
time = 0.74, size = 116, normalized size = 1.30

$$-\frac{ie^{-5i(c+dx)}\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{5/2}\left(1+e^{2i(c+dx)}\right)^{5/2}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}-\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(5/2)*(1 + E^((2*
I)*(c + d*x)))^(5/2)*(E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] - ArcSi
nh[E^(I*(c + d*x))]))/(Sqrt[2]*d*E^((5*I)*(c + d*x)))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(73) = 146.
time = 3.00, size = 398, normalized size = 4.47

method	result

default	$-\frac{\left(i\sqrt{2}\,\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sin(dx+c)+\sqrt{2}\,\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arct}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(I*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))+2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+I*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(
d*x+c)+8*I*cos(d*x+c)^4-8*sin(d*x+c)*cos(d*x+c)^3-4*I*cos(d*x+c)^3+4*cos(d*
x+c)^2*sin(d*x+c)-4*I*cos(d*x+c)^2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*a^2
```

Maxima [A]
time = 0.50, size = 98, normalized size = 1.10

$$-\frac{i\left(\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)-\frac{8\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{4}}{2i\,a\tan(dx+c)-2\,a}\right)}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*I*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*sqrt(I*a*tan(d*x + c)
+ a)*a^4/(2*I*a*tan(d*x + c) - 2*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(68) = 136.
time = 0.38, size = 236, normalized size = 2.65

$$\frac{\sqrt{2}\,\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{a^5}{d^2}}\,\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-\sqrt{2}\,\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{a^5}{d^2}}\,\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-2\,\sqrt{2}\,\left(i\,a^2e^{(3i\,dx+3i\,c)}+i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-a^5/d^2)*(
I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x
```

```
 - I*c)/a^2) - sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-
a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/a^2) - 2*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I
*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos{(c+d\,x)}^2\,{\left(a+a\tan(c+d\,x)\,1{\mathrm{i}}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=137

$$-\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}-\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

```
[Out] -3/32*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(
1/2)-1/4*I*a^4*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-3/16*I*a^3*(
a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3568, 44, 65, 212}

$$-\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}-\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-3*I)/16)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/4)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((3*I)/16)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{(3ia^4)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx,\right.}{8d}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))} - \frac{(3ia^3}{}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))} - \frac{(3ia^3}{}$$

$$= -\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d} - \frac{ia^4\sqrt{a+ia\tan(c+}{4d(a-ia\tan(c+}$$

Mathematica [A]
time = 0.87, size = 116, normalized size = 0.85

$$-\frac{ia^2e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(5+2e^{2i(c+dx)}\right)+3\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/32*I)*a^2*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*(5 + 2*E^((2*I)*(c + d*x))) + 3*ArcSinh[E^(I*(c + d*x))])*
Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(110) = 220$.
time = 0.98, size = 744, normalized size = 5.43

method	result
default	$\frac{\left(3i\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\sin(dx+c)+128i\left(\cos^{7}(dx+c)\right)+3\sin(dx+c)\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d*(3*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3*2^(1/2)
*sin(d*x+c)+128*I*cos(d*x+c)^7+3*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3
*2^(1/2)+3*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)+9
*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*2^(1/2)+9*I*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+9*sin(d*x+c)*arctan(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*cos(d*x+c)*2^(1/2)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)-16*I
*cos(d*x+c)^5-256*I*cos(d*x+c)^8+256*sin(d*x+c)*cos(d*x+c)^7+48*I*cos(d*x+c
)^4-128*sin(d*x+c)*cos(d*x+c)^6+96*I*cos(d*x+c)^6+32*sin(d*x+c)*cos(d*x+c)^
5+9*I*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*2^(1/2)*sin(d*x+
c)-48*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/
2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*a^2
```

Maxima [A]
time = 0.50, size = 140, normalized size = 1.02

$$\frac{i\left(3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^{4}-10\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{5}\right)}{(i\,a\tan(dx+c)+a)^{2}-4\,(i\,a\tan(dx+c)+a)a+4\,a^{2}}\right)}{64\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/64*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x + c)
```

```
 + a)^(3/2)*a^4 - 10*sqrt(I*a*tan(d*x + c) + a)*a^5)/((I*a*tan(d*x + c) + a
)^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(102) = 204$.
time = 0.38, size = 263, normalized size = 1.92

$$-\frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-\sqrt{2}\left(-2i\,a^2e^{(5i\,dx+5i\,c)}-7i\,a^2e^{(3i\,dx+3i\,c)}-5i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sq
rt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(
a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2
*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqr
t(2)*(-2*I*a^2*e^(5*I*d*x + 5*I*c) - 7*I*a^2*e^(3*I*d*x + 3*I*c) - 5*I*a^2*
e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=210

$$-\frac{35ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d}+\frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\,\mathrm{t}}}$$

```
[Out] -35/256*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2
^(1/2)+35/128*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)-1/6*I*a^6/d/(a+I*a*tan(d*x+c
))^(1/2)/(a-I*a*tan(d*x+c))^3-7/48*I*a^5/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*
tan(d*x+c))^2-35/192*I*a^4/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{35ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}-\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-35*I)/128)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/6)
*a^6)/(d*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/48)
*a^5)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/19
2)*a^4)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{(7ia^6)\,\text{Subst}}{}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{}{48d(a-ia\,\text{t}}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{}{48d(a-ia\,\text{t}}$$

$$= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\,\text{t}}}$$

$$= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\,\text{t}}}$$

$$= -\frac{35ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d} + \frac{35ia^3}{128d\sqrt{a+ia\,\text{tan}}}$$

Mathematica [A]
time = 1.01, size = 142, normalized size = 0.68

$$-\frac{ia^2e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(\sqrt{1+e^{2i(c+dx)}}\left(-48+87e^{2i(c+dx)}+38e^{4i(c+dx)}+8e^{6i(c+dx)}\right)+105e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/768*I)*a^2*Sqrt[1 + E^((2*I)*(c + d*x))]*(Sqrt[1 + E^((2*I)*(c + d*x))
]*(-48 + 87*E^((2*I)*(c + d*x)) + 38*E^((4*I)*(c + d*x)) + 8*E^((6*I)*(c +
d*x))) + 105*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*E^((2*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(170) = 340.
time = 1.07, size = 1088, normalized size = 5.18

method	result	size

default	Expression too large to display	1088

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24576/d*(-105*I*cos(d*x+c)^5*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(11/2)*sin(d*x+c)-525*I*cos(d*x+c)^4*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(11/2)*sin(d*x+c)-1050*I*cos(d*x+c)^3*2^(1/2)*arctanh(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(11/2)*sin(d*x+c)-1050*I*cos(d*x+c)^2*2^(1/2)*arctanh(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)-525*I*cos(d*x+c)*2^(1/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)-3072*cos(d*x+c)^9*sin(d*x+c)-512
0*I*cos(d*x+c)^10+896*I*cos(d*x+c)^8+2240*I*cos(d*x+c)^7-6720*I*cos(d*x+c)^
6-105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*sin(d*x+c)-16384*sin(d*x+c)*cos(d*x+c)^11
+512*I*cos(d*x+c)^9-105*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2
)*sin(d*x+c)+3584*sin(d*x+c)*cos(d*x+c)^8+8192*cos(d*x+c)^10*sin(d*x+c)-105
*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-525*cos(d*x+c)^4*sin(d
*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-1050*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*2^(1/2))-1050*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-52
5*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-4480*sin(d*x+c)*cos(d*x
+c)^7+6720*sin(d*x+c)*cos(d*x+c)^6+16384*I*cos(d*x+c)^12-8192*I*cos(d*x+c)^
11)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)
-1)/cos(d*x+c)^5*a^2
```

Maxima [A]
time = 0.50, size = 194, normalized size = 0.92

$$\frac{i\left(105\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^{3}a^{4}-560\,(i\,a\tan(dx+c)+a)^{2}a^{5}+924\,(i\,a\tan(dx+c)+a)a^{6}-384\,a^{7}\right)}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+12\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^{2}-8\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{3}}\right)}{1536\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1536*I*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d*
x + c) + a)^3*a^4 - 560*(I*a*tan(d*x + c) + a)^2*a^5 + 924*(I*a*tan(d*x + c
) + a)*a^6 - 384*a^7)/((I*a*tan(d*x + c) + a)^(7/2) - 6*(I*a*tan(d*x + c) +
 a)^(5/2)*a + 12*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(I*a*tan(d*x + c)
 + a)*a^3))/(a*d)
```

Fricas [A]
time = 0.37, size = 309, normalized size = 1.47

$$-\frac{105 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} d e^{\left(i d x + i c\right)} \log\left(\frac{4 \left(a^{3} e^{\left(i d x + i c\right)} - \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} \left(i d e^{\left(2 i d x + 2 i c\right)} + i d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-i d x - i c\right)}}{a^{2}}\right) - 105 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} d e^{\left(i d x + i c\right)} \log\left(\frac{4 \left(a^{3} e^{\left(i d x + i c\right)} - \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} \left(-i d e^{\left(2 i d x + 2 i c\right)} - i d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-i d x - i c\right)}}{a^{2}}\right) - \sqrt{2} \left(-8 i a^{2} e^{\left(8 i d x + 8 i c\right)} - 46 i a^{2} e^{\left(6 i d x + 6 i c\right)} - 125 i a^{2} e^{\left(4 i d x + 4 i c\right)} - 39 i a^{2} e^{\left(2 i d x + 2 i c\right)} + 48 i a^{2}\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}}{768 d} e^{\left(-i d x - i c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/768*(105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x
+ I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 105*sqrt(1/2)*sqr
t(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2
)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqrt(2)*(-8*I*a^2*e^(8*I*d*x + 8*I*c) - 4
6*I*a^2*e^(6*I*d*x + 6*I*c) - 125*I*a^2*e^(4*I*d*x + 4*I*c) - 39*I*a^2*e^(2
*I*d*x + 2*I*c) + 48*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
I*c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3}{}$$

```
[Out] 64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/21*I*a^2*sec(d*x+c)^
3*(a+I*a*tan(d*x+c))^(1/2)/d+256/315*I*a^4*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c)
)^(3/2)+2/9*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/315)*a^4*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((64
*I)/105)*a^3*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((8*I)/21)*a
^2*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/9)*a*Sec[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d} + \frac{1}{3}(4a)\int \sec^3(c+dx)(\\ &= \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{2ia\sec^3(c+dx)(a+ia}{9d} \\ &= \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} + \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+}}{21d} \\ &= \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}} + \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} + \frac{8ia^2}{}\end{aligned}$$

Mathematica [A]
time = 0.94, size = 103, normalized size = 0.70

$$\frac{2a^2\sec^3(c+dx)(i\cos(2c)+\sin(2c))(77+242\cos(2(c+dx))+89i\sec(c+dx)\sin(3(c+dx))+54i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{315d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3*(I*Cos[2*c] + Sin[2*c])*(77 + 242*Cos[2*(c + d*x)] +
(89*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (54*I)*Tan[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(315*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.76, size = 117, normalized size = 0.80

method	result
default	$\frac{2\left(256i\left(\cos^5(dx+c)\right)+256\sin(dx+c)\left(\cos^4(dx+c)\right)-32i\left(\cos^3(dx+c)\right)+96\left(\cos^2(dx+c)\right)\sin(dx+c)+95i\cos(dx+c)-35\sin(dx+c)\right)\sqrt{a(}}{315d\cos(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/d*(256*I*cos(d*x+c)^5+256*sin(d*x+c)*cos(d*x+c)^4-32*I*cos(d*x+c)^3+9
6*cos(d*x+c)^2*sin(d*x+c)+95*I*cos(d*x+c)-35*sin(d*x+c))*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(115) = 230.
time = 280.24, size = 624, normalized size = 4.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 32/315*(105*I*sqrt(2)*a^2*cos(6*d*x + 6*c) + 126*I*sqrt(2)*a^2*cos(4*d*x +
4*c) + 72*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 105*sqrt(2)*a^2*sin(6*d*x + 6*c)
 - 126*sqrt(2)*a^2*sin(4*d*x + 4*c) - 72*sqrt(2)*a^2*sin(2*d*x + 2*c) + 16*
I*sqrt(2)*a^2)*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin
(2*d*x + 2*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2
+ (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*
sin(4*d*x + 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(
2*d*x + 2*c) + 4*cos(2*d*x + 2*c) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)
*sin(2*d*x + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(
2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d*x
 + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(4*d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(
2*d*x + 2*c) + 4*I*cos(2*d*x + 2*c) + I)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))*d)
```

Fricas [**A**]
time = 0.39, size = 121, normalized size = 0.82

$$-\frac{32\,\sqrt{2}\,\left(-105i\,a^{2}e^{(6i\,dx+6i\,c)}-126i\,a^{2}e^{(4i\,dx+4i\,c)}-72i\,a^{2}e^{(2i\,dx+2i\,c)}-16i\,a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{315\,\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*(-105*I*a^2*e^(6*I*d*x + 6*I*c) - 126*I*a^2*e^(4*I*d*x + 4*
I*c) - 72*I*a^2*e^(2*I*d*x + 2*I*c) - 16*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x +
4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Mupad [B]
time = 7.77, size = 301, normalized size = 2.05

$$\frac{a^2\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,32{}\mathrm{i}}{3\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}-\frac{a^2\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,96{}\mathrm{i}}{5\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{a^2\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,96{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}-\frac{a^2\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,32{}\mathrm{i}}{9\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^3,x)
[Out] (a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*32i)/(3*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*96i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) + (a^2*exp(- c*1i - d*x*1i
)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
96i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(- c*1i - d*x*1i)*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d*(
exp(c*2i + d*x*2i) + 1)^4)
```

3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$\frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}+\frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

```
[Out] 64/15*I*a^3*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*a^2*sec(d*x+c)*(a
+I*a*tan(d*x+c))^(1/2)/d+2/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}+\frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/15
)*a^2*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/5)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{1}{5}(8a)\int \sec(c+dx)(a+$$
$$= \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\,\mathrm{t}}{5d}$$
$$= \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx}}{15d}$$

Mathematica [A]
time = 0.47, size = 93, normalized size = 0.89

$$\frac{2a^2\sec^2(c+dx)(i\cos(c-dx)+\sin(c-dx))(20+23\cos(2(c+dx))+7i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^2*(I*Cos[c - d*x] + Sin[c - d*x])*(20 + 23*Cos[2*(c + d
*x)] + (7*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(Cos[d*x]
+ I*Sin[d*x])^2)
```

Maple [A]
time = 0.61, size = 90, normalized size = 0.87

method	result	size
default	$\frac{2\left(32i\left(\cos^3(dx+c)\right)+32\left(\cos^2(dx+c)\right)\sin(dx+c)+11i\cos(dx+c)-3\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{15d\cos(dx+c)^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(32*I*cos(d*x+c)^3+32*cos(d*x+c)^2*sin(d*x+c)+11*I*cos(d*x+c)-3*sin(
d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [A]
time = 0.38, size = 83, normalized size = 0.80

$$-\frac{8\sqrt{2}\left(-15i\,a^{2}e^{(4i\,dx+4i\,c)}-20i\,a^{2}e^{(2i\,dx+2i\,c)}-8i\,a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -8/15*sqrt(2)*(-15*I*a^2*e^(4*I*d*x + 4*I*c) - 20*I*a^2*e^(2*I*d*x + 2*I*c)
 - 8*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\sec{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [B]
time = 6.08, size = 105, normalized size = 1.01

$$\frac{8\,a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,20\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,15\mathrm{i}+8\mathrm{i}\right)}{15\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x),x)
[Out] (8*a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*20i + exp(c*4i + d*x*4i)*15i
+ 8i))/(15*d*(exp(c*2i + d*x*2i) + 1)^2)
```

3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=65

$$-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

```
[Out] -8*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+2*I*a*cos(d*x+c)*(a+I*a*tan(
d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3575, 3574}

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}+(4a)\int \cos(c+dx)(a+$$

$$= -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia}{d}$$

Mathematica [A]
time = 0.35, size = 46, normalized size = 0.71

$$-\frac{2ia^2(3\cos(c+dx)-i\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(3*Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A]
time = 0.90, size = 53, normalized size = 0.82

method	result	size
default	$-\frac{2(3i\cos(dx+c)+\sin(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\, a^2}{d}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(3*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(53) = 106.
time = 0.61, size = 331, normalized size = 5.09

$$\frac{2\left(-3i\,a^{\frac{5}{2}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{4i\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4i\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*(-3*I*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*I*a^(5/2)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 9*I*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 3*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 1)^(5/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)^(5/2)*(4*I*sin(d*x + c)/(cos(d*x + c) + 1) - 5
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 + 1))
```

Fricas [A]
time = 0.42, size = 45, normalized size = 0.69

$$-\frac{2\sqrt{2}\left(i\,a^{2}e^{(2i\,dx+2i\,c)}+2i\,a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [B]
time = 0.39, size = 64, normalized size = 0.98

$$-\frac{2\,a^{2}\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,3\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] -(2*a^2*(cos(c + d*x)*3i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=35

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3574}

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Mathematica [A]
time = 0.38, size = 69, normalized size = 1.97

$$\frac{2a^2\cos^2(c+dx)(-i\cos(c+3dx)+\sin(c+3dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[c + d*x]^2*((-I)*Cos[c + 3*d*x] + Sin[c + 3*d*x])*Sqrt[a + I*a*T
an[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.87, size = 63, normalized size = 1.80

method	result	size
default	$-\frac{2(i\cos(dx+c)-\sin(dx+c))\left(\cos^2(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a^2}{3d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(I*cos(d*x+c)-sin(d*x+c))*cos(d*x+c)^2*(a*(I*sin(d*x+c)+cos(d*x+c))/
cos(d*x+c))^(1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(27) = 54$.
time = 0.63, size = 328, normalized size = 9.37

$$\frac{2\left(i\,a^{\frac{5}{2}}-\frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{i\,a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{2i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(5/2) - 4*I*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*I*a^(5/2
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + I*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x
 + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)^(5/2)*(-6*I*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 18*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*I*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 3))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.
time = 0.36, size = 59, normalized size = 1.69

$$\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-2i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 2*I*a^2*e^(2*I*d*x + 2*I*c) - I*a
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [**B**]
time = 0.91, size = 89, normalized size = 2.54

$$-\frac{a^2\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,3{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] -(a^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*3i - sin(c + d*x) + cos(3*c + 3*d*x)*1i - sin(3*c
+ 3*d*x)))/(6*d)
```

3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{ia\cos^3(c+dx)(a+ia\,\mathrm{tan}}{6d}$$

```
[Out] 1/8*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/d*2^(1/2)-1/4*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/6*I*a*cos(d
*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5
/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{3571, 3570, 212\}$

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/4)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/4)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/6)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/5)*Cos[c
 + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
```

```
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}+\frac{1}{2}a\int \cos^3(c+dx)(a+$$

$$= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}-\frac{i\cos^5(c+dx)(a+i}{5d}$$

$$= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{ia\cos^3(c+dx)(a+i}{6d}$$

$$= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{ia\cos^3(c+dx)(a+i}{6d}$$

$$= \frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d}-\frac{ia^2\cos(c+dx)\sqrt{}}{}$$

Mathematica [A]
time = 1.23, size = 118, normalized size = 0.74

$$-\frac{ia^2e^{-i(c+dx)}\left(23+34e^{2i(c+dx)}+14e^{4i(c+dx)}+3e^{6i(c+dx)}-15\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/120*I)*a^2*(23 + 34*E^((2*I)*(c + d*x)) + 14*E^((4*I)*(c + d*x)) + 3*E
^((6*I)*(c + d*x)) - 15*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(128) = 256.
time = 0.84, size = 916, normalized size = 5.76

method	result	size
default	Expression too large to display	916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1920/d*(15*I*sin(d*x+c)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
```

```
(1/2))*2^(1/2)-768*I*cos(d*x+c)^9-15*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*
x+c)*2^(1/2)+64*I*cos(d*x+c)^7-60*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(9/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c
)*2^(1/2)+60*I*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*2^(1/2)-90*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-480*I*cos
(d*x+c)^5-60*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctan(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*2^(1/2)-15*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(9/2)*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*sin(d*x+c)+90*I*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*2^(1/2)-512*I*cos(d*x+c)^8-1536*cos(d*x+c)^9*sin(d*
x+c)+160*I*cos(d*x+c)^6+768*sin(d*x+c)*cos(d*x+c)^8+1536*I*cos(d*x+c)^10-25
6*sin(d*x+c)*cos(d*x+c)^7+60*I*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)/cos(d*x+c)*2^(1/2))*2^(1/2)+320*sin(d*x+c)*cos(d*x+c)^6+15*I*2^(1/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*sin(d*x+c)-480*sin(d*x+c)*cos(d*x+c)^
5)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-
1)/cos(d*x+c)^4*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(120) = 240$.
time = 0.70, size = 1076, normalized size = 6.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/480*(20*(I*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) - sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/
4)*sqrt(a) + 12*(5*I*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 5*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + (I*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^2*sin(2*d*x
+ 2*c)^2 + 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2)*cos(5/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^2*cos(2*d*x + 2*c)^
2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 1
5*(2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

```
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(
2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a^2*log
(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a^2*log(sqrt(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*s
qrt(a))/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(120) = 240$.
time = 0.41, size = 244, normalized size = 1.53

$$\frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{\left(ia^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{\left(ia^3-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+\sqrt{2}\left(-3i\,a^2e^{(6i\,dx+6i\,c)}-14i\,a^2e^{(4i\,dx+4i\,c)}-34i\,a^2e^{(2i\,dx+2i\,c)}-23i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 + sqrt(2)*sqrt(1/2)*sqr
t(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 - sqrt(2
)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-3*I*a^2*e^(6*I*d*x + 6*I*c)
- 14*I*a^2*e^(4*I*d*x + 4*I*c) - 34*I*a^2*e^(2*I*d*x + 2*I*c) - 23*I*a^2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=231

$$\frac{9ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}\,d}+\frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}$$

```
[Out] 9/64*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)+3/16*I*a^3*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-9/32*I*a^2*
cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-3/20*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*
x+c))^(1/2)/d-9/70*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-1/7*I*cos(d*
x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$\frac{9ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}\,d}+\frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}-\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((9*I)/32)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((3*I)/16)*a^3*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((9*I)/32)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - (((3*I)/20)*a^2*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((9*I)/7
0)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/7)*Cos[c + d*x]^7
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{14}(9a)\int \cos^5(c+dx)$$

$$= -\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}-\frac{i\cos^7(c+dx)(a+ia}{7d}$$

$$= -\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia\cos^5(c+dx)(a+}{70}$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+d}}{20d}$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx}}{32d}$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx}}{32d}$$

$$= \frac{9ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d}+\frac{3ia^3\cos(c+}{16d\sqrt{a+ia\tan(}}$$

Mathematica [A]
time = 1.28, size = 155, normalized size = 0.67

$$-\frac{ia^2e^{-3i(c+dx)}\left(-35+353e^{2i(c+dx)}+544e^{4i(c+dx)}+214e^{6i(c+dx)}+68e^{8i(c+dx)}+10e^{10i(c+dx)}-315e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/2240*I)*a^2*(-35 + 353*E^((2*I)*(c + d*x)) + 544*E^((4*I)*(c + d*x)) +
 214*E^((6*I)*(c + d*x)) + 68*E^((8*I)*(c + d*x)) + 10*E^((10*I)*(c + d*x))
 - 315*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E
^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1259 vs. 2(188) = 376.
time = 1.17, size = 1260, normalized size = 5.45

method	result	size
default	Expression too large to display	1260

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/143360/d*(-315*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*sin(d*x+c)+4725*I*cos(d*x+c)^
4*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+6300*I*cos(
d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+4725
*I*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/
2)-21504*cos(d*x+c)^9*sin(d*x+c)+40960*sin(d*x+c)*cos(d*x+c)^12+3072*I*cos(
d*x+c)^10+81920*I*cos(d*x+c)^14-40960*I*cos(d*x+c)^13-24576*I*cos(d*x+c)^12
+2048*I*cos(d*x+c)^11+5376*I*cos(d*x+c)^9+13440*I*cos(d*x+c)^8-40320*I*cos(
d*x+c)^7-81920*cos(d*x+c)^13*sin(d*x+c)-16384*sin(d*x+c)*cos(d*x+c)^11+315*
I*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2
)+1890*I*cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*2^(1/2)+26880*sin(d*x+c)*cos(d*x+c)^8+18432*cos(d*x+c)^10*sin(d*x+c)-40320
*sin(d*x+c)*cos(d*x+c)^7-315*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^
(1/2)-1890*cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-4725*cos(d*x
```

```
+c)^4*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-6300*cos(d*x+c)^3*sin(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))*2^(1/2)-4725*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*2^(1/2)-1890*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*a
rctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+315*I*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(13/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+1890*I*cos(d*x+c)*
sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2))*(a*(I*sin(d*
x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^6
*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.39, size = 300, normalized size = 1.30

$$-\frac{315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^5}{d^2}} d e^{(2 i d x+2 i c)} \log\left(-\frac{9 \left(-i a^3+\sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^5}{d^2}} \left(d e^{(2 i d x+2 i c)}+d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\right) e^{(-i d x-i c)}}{16 d}\right) - 315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^5}{d^2}} d e^{(2 i d x+2 i c)} \log\left(-\frac{9 \left(-i a^3-\sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^5}{d^2}} \left(d e^{(2 i d x+2 i c)}+d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\right) e^{(-i d x-i c)}}{16 d}\right) - \sqrt{2} \left(-10 i a^2 e^{(10 i d x+10 i c)} - 68 i a^2 e^{(8 i d x+8 i c)} - 214 i a^2 e^{(6 i d x+6 i c)} - 544 i a^2 e^{(4 i d x+4 i c)} - 353 i a^2 e^{(2 i d x+2 i c)} + 35 i a^2\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\right) e^{(-2 i d x-2 i c)}}{2240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2240*(315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a
^3 + sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*sqrt(-a^5/d^2
)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a^3 - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2
)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x
 - I*c)/d) - sqrt(2)*(-10*I*a^2*e^(10*I*d*x + 10*I*c) - 68*I*a^2*e^(8*I*d*x
 + 8*I*c) - 214*I*a^2*e^(6*I*d*x + 6*I*c) - 544*I*a^2*e^(4*I*d*x + 4*I*c) -
 353*I*a^2*e^(2*I*d*x + 2*I*c) + 35*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^7\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d}-\frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d}+\frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}$$

```
[Out] -16/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^4/d+24/17*I*(a+I*a*tan(d*x+c))^(17/2)/
a^5/d-12/19*I*(a+I*a*tan(d*x+c))^(19/2)/a^6/d+2/21*I*(a+I*a*tan(d*x+c))^(21
/2)/a^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}-\frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d}-\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-16*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^4*d) + (((24*I)/17)*(a + I*
a*Tan[c + d*x])^(17/2))/(a^5*d) - (((12*I)/19)*(a + I*a*Tan[c + d*x])^(19/2
))/(a^6*d) + (((2*I)/21)*(a + I*a*Tan[c + d*x])^(21/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{13/2} - 12a^2(a+x)^{15/2} + 6a(a+x)^{17/2} -\right.\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d} + \frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} -$$

Mathematica [A]
time = 1.97, size = 113, normalized size = 0.97

$$-\frac{2a^3\sec^9(c+dx)(\cos(7c+10dx)+i\sin(7c+10dx))(-1311i+4554i\cos(2(c+dx))+2245\sec(c+dx)\sin(3(c+dx))+630\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{33915d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^9*(Cos[7*c + 10*d*x] + I*Sin[7*c + 10*d*x])*(-1311*I +
 (4554*I)*Cos[2*(c + d*x)] + 2245*Sec[c + d*x]*Sin[3*(c + d*x)] + 630*Tan[c
 + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(33915*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 32.62, size = 181, normalized size = 1.55

method	result
default	$-\frac{2(8192i(\cos^{10}(dx+c))-8192(\cos^9(dx+c))\sin(dx+c)+1024i(\cos^8(dx+c))-5120\sin(dx+c)(\cos^7(dx+c))+448i(\cos^6(dx+c))-40}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/33915/d*(8192*I*cos(d*x+c)^10-8192*cos(d*x+c)^9*sin(d*x+c)+1024*I*cos(d*
x+c)^8-5120*sin(d*x+c)*cos(d*x+c)^7+448*I*cos(d*x+c)^6-4032*sin(d*x+c)*cos(
d*x+c)^5+264*I*cos(d*x+c)^4-3432*sin(d*x+c)*cos(d*x+c)^3-8300*I*cos(d*x+c)^
2+5440*sin(d*x+c)*cos(d*x+c)+1615*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c
))^(1/2)/cos(d*x+c)^10*a^3
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.65

$$\frac{2i\left(1615\,(i\,a\tan(dx+c)+a)^{\frac{21}{2}} - 10710\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}}a + 23940\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}a^2 - 18088\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^3\right)}{33915\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")

[Out] 2/33915*I*(1615*(I*a*tan(d*x + c) + a)^(21/2) - 10710*(I*a*tan(d*x + c) + a
)^(19/2)*a + 23940*(I*a*tan(d*x + c) + a)^(17/2)*a^2 - 18088*(I*a*tan(d*x +
 c) + a)^(15/2)*a^3)/(a^7*d)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(85) = 170$.
time = 0.41, size = 202, normalized size = 1.73

$$-\frac{2048\,\sqrt{2}\,\left(16 i\,a^{3} e^{\left(21 i\,d x+21 i\,c\right)}+168 i\,a^{3} e^{\left(19 i\,d x+19 i\,c\right)}+798 i\,a^{3} e^{\left(17 i\,d x+17 i\,c\right)}+2261 i\,a^{3} e^{\left(15 i\,d x+15 i\,c\right)}\right)\sqrt{\frac{a}{e^{\left(2 i\,d x+2 i\,c\right)}+1}}}{33915\,\left(d e^{\left(20 i\,d x+20 i\,c\right)}+10\,d e^{\left(18 i\,d x+18 i\,c\right)}+45\,d e^{\left(16 i\,d x+16 i\,c\right)}+120\,d e^{\left(14 i\,d x+14 i\,c\right)}+210\,d e^{\left(12 i\,d x+12 i\,c\right)}+252\,d e^{\left(10 i\,d x+10 i\,c\right)}+210\,d e^{\left(8 i\,d x+8 i\,c\right)}+120\,d e^{\left(6 i\,d x+6 i\,c\right)}+45\,d e^{\left(4 i\,d x+4 i\,c\right)}+10\,d e^{\left(2 i\,d x+2 i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")

[Out] -2048/33915*sqrt(2)*(16*I*a^3*e^(21*I*d*x + 21*I*c) + 168*I*a^3*e^(19*I*d*x
 + 19*I*c) + 798*I*a^3*e^(17*I*d*x + 17*I*c) + 2261*I*a^3*e^(15*I*d*x + 15*
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(
18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*
c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8
*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10
*d*e^(2*I*d*x + 2*I*c) + d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^8, x)

Mupad [B]
time = 15.04, size = 690, normalized size = 5.90

$$-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32768\mathrm{i}}{33915\,d}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16384\mathrm{i}}{33915\,d\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{11305\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{6783\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,247808\mathrm{i}}{969\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1943552\mathrm{i}}{1615\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,12019712\mathrm{i}}{4845\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,95516672\mathrm{i}}{33915\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}+\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4159488\mathrm{i}}{2261\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}-\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,260096\mathrm{i}}{399\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^9}+\frac{a^3\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{21\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^8,x)
```

```
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*247808i)/(969*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(33915*d*(exp(c
*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*4096i)/(11305*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2
048i)/(6783*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32768i)/(33915*d) - (a^3*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*19435
52i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*12019712i)/(4845*d*(exp(c*2i
+ d*x*2i) + 1)^6) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*95516672i)/(33915*d*(exp(c*2i + d*x*2i) + 1)^7) + (a
^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*4159488i)/(2261*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*260096i)/(399*d*(exp(c*
2i + d*x*2i) + 1)^9) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*2048i)/(21*d*(exp(c*2i + d*x*2i) + 1)^10)
```

3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}$$

```
[Out] -8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^3/d+8/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^
4/d-2/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^5/d
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-8*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^3*d) + (((8*I)/15)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^4*d) - (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/
(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)\,dx, x,\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{2i(a}{}\end{aligned}$$

Mathematica [A]
time = 1.04, size = 97, normalized size = 1.10

$$\frac{2a^3\sec^8(c+dx)(68+263\cos(2(c+dx))-247i\sin(2(c+dx)))(-i\cos(6c+9dx)+\sin(6c+9dx))\sqrt{a+ia\tan(c+dx)}}{3315d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (2*a^3*Sec[c + d*x]^8*(68 + 263*Cos[2*(c + d*x)] - (247*I)*Sin[2*(c + d*x)]
)*((-I)*Cos[6*c + 9*d*x] + Sin[6*c + 9*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3
315*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(70) = 140.
time = 1.00, size = 154, normalized size = 1.75

method	result
default	$\frac{2(-1024i(\cos^8(dx+c))+1024\sin(dx+c)(\cos^7(dx+c))-128i(\cos^6(dx+c))+640\sin(dx+c)(\cos^5(dx+c))-56i(\cos^4(dx+c))+504\sin(}{3315d\cos(dx+c)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/3315/d*(-1024*I*cos(d*x+c)^8+1024*sin(d*x+c)*cos(d*x+c)^7-128*I*cos(d*x+c
)^6+640*sin(d*x+c)*cos(d*x+c)^5-56*I*cos(d*x+c)^4+504*sin(d*x+c)*cos(d*x+c)
^3+1072*I*cos(d*x+c)^2-676*sin(d*x+c)*cos(d*x+c)-195*I)*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8*a^3
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.66

$$-\frac{2i\left(195\,(i\,a\tan\left(dx+c\right)+a)^{\frac{17}{2}}-884\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}a+1020\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] -2/3315*I*(195*(I*a*tan(d*x + c) + a)^(17/2) - 884*(I*a*tan(d*x + c) + a)^(
15/2)*a + 1020*(I*a*tan(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.
time = 0.51, size = 164, normalized size = 1.86

$$-\frac{512\sqrt{2}\left(8i\,a^3e^{(17i\,dx+17i\,c)}+68i\,a^3e^{(15i\,dx+15i\,c)}+255i\,a^3e^{(13i\,dx+13i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3315\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c)}+8\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -512/3315*sqrt(2)*(8*I*a^3*e^(17*I*d*x + 17*I*c) + 68*I*a^3*e^(15*I*d*x + 1
5*I*c) + 255*I*a^3*e^(13*I*d*x + 13*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x +
12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6
*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^6, x)
```

Mupad [B]
time = 15.66, size = 562, normalized size = 6.39

$$-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,4096i}{3315\,d}-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,2048i}{3315\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{1105\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}+\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,56320i}{663\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^3}-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,205312i}{663\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}+\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,540672i}{1105\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1341952i}{3315\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6}+\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,44032i}{255\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^7}-\frac{a^3\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{17\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^6,x)
```

```
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*56320i)/(663*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3315*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*512i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a -
 (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)
/(3315*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*205312i)/(663*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^3*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*540672i)/(1
```

```
105*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1
i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1341952i)/(3315*d*(exp(c*2i + d*x*2i
) + 1)^6) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*44032i)/(255*d*(exp(c*2i + d*x*2i) + 1)^7) - (a^3*(a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*
(exp(c*2i + d*x*2i) + 1)^8)
```

3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=59

$$-\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

```
[Out] -4/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^2/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^
3/d
```

Rubi [A]

time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}-\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^2*d) + (((2*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}\end{aligned}$$

Mathematica [**A**]
time = 0.87, size = 85, normalized size = 1.44

$$-\frac{2a^3\sec^5(c+dx)(\cos(5c+8dx)+i\sin(5c+8dx))(15i+11\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{143d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^5*(Cos[5*c + 8*d*x] + I*Sin[5*c + 8*d*x])*(15*I + 11*T
an[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(143*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(47) = 94.
time = 0.81, size = 127, normalized size = 2.15

method	result
default	$-\frac{2(64i(\cos^6(dx+c))-64\sin(dx+c)(\cos^5(dx+c))+8i(\cos^4(dx+c))-40\sin(dx+c)(\cos^3(dx+c))-68i(\cos^2(dx+c))+40\sin(dx+c)\text{co}}{143d\cos(dx+c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/143/d*(64*I*cos(d*x+c)^6-64*sin(d*x+c)*cos(d*x+c)^5+8*I*cos(d*x+c)^4-40*
sin(d*x+c)*cos(d*x+c)^3-68*I*cos(d*x+c)^2+40*sin(d*x+c)*cos(d*x+c)+11*I)*(a
*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6*a^3
```

Maxima [**A**]
time = 0.27, size = 40, normalized size = 0.68

$$\frac{2i\left(11\,(i\,a\tan{(dx+c)}+a)^{\frac{13}{2}}-26\,(i\,a\tan{(dx+c)}+a)^{\frac{11}{2}}\,a\right)}{143\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/143*I*(11*(I*a*tan(d*x + c) + a)^(13/2) - 26*(I*a*tan(d*x + c) + a)^(11/2
)*a)/(a^3*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(43) = 86.
time = 0.39, size = 126, normalized size = 2.14

$$-\frac{128\sqrt{2}\left(2i\,a^3e^{(13i\,dx+13i\,c)}+13i\,a^3e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{143\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -128/143*sqrt(2)*(2*I*a^3*e^(13*I*d*x + 13*I*c) + 13*I*a^3*e^(11*I*d*x + 11
*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(
10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) +
15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^4, x)
```

Mupad [B]
time = 7.57, size = 434, normalized size = 7.36

$$-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,256i}{143\,d}-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,128i}{143\,d\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,4480i}{143\,d\,{\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,11520i}{143\,d\,{\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}^3}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,12800i}{143\,d\,{\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}^4}-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,6784i}{143\,d\,{\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}^5}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{\mathrm{e}^{c\,2i+d\,x\,2i}+1}}\,128i}{13\,d\,{\left(\mathrm{e}^{c\,2i+d\,x\,2i}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^4,x)
```

```
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*4480i)/(143*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(143*d*(exp(c*2i +
d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*256i)/(143*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*11520i)/(143*d*(exp(c*2i + d*x*2i) +
1)^3) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*12800i)/(143*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6784i)/(143*d*(e
xp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

```
[Out] -2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.
time = 0.54, size = 73, normalized size = 2.52

$$\frac{2a^3\sec^4(c+dx)(-i\cos(4c+7dx)+\sin(4c+7dx))\sqrt{a+ia\tan(c+dx)}}{9d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^4*((-I)*Cos[4*c + 7*d*x] + Sin[4*c + 7*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(9*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.21, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9da}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9da}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*I*(a+I*a*tan(d*x+c))^(9/2)/d/a
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.72

$$-\frac{2i\,(i\,a\tan{(dx+c)}+a)^{\frac{9}{2}}}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/9*I*(I*a*tan(d*x + c) + a)^(9/2)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.
time = 0.37, size = 85, normalized size = 2.93

$$-\frac{32i\,\sqrt{2}\,a^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(9i\,dx+9i\,c)}}{9\,(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -32/9*I*sqrt(2)*a^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(9*I*d*x + 9*I*c)/(
d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) +
 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^2, x)
```

Mupad [B]
time = 6.28, size = 306, normalized size = 10.55

$$-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d}+\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^2,x)
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*
1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d) - (a^3*(a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(3*d*(exp
(c*2i + d*x*2i) + 1)^2) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32
i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$\frac{3i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}-\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}$$

```
[Out] 3*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
-3*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d-I*a^3*(a+I*a*tan(d*x+c))^(3/2)/d/(a-I*a
*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 43, 52, 65, 212}

$$\frac{3i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((3*I)*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((3*I)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (I*a^3*(a + I*a*Tan[c + d
*x])^(3/2))/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(3ia^3)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, i\right.}{2d}$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(3}{}$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(6}{}$$

$$= \frac{3i\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d} - \frac{3ia^3\sqrt{a+ia\tan}}{d}$$

Mathematica [A]
time = 1.55, size = 137, normalized size = 1.18

$$-\frac{i\sqrt{2}\,e^{-4i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3e^{i(c+dx)}+e^{3i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)\right)(a+ia\tan(c+dx))^{7/2}}{d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*E^(I*(c + d*x)) + E^((3*I)*(c + d*x)) - 3*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*(a + I*a*Tan[c + d*x])^(7/2))/(d*E^((4*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(96) = 192$.
time = 1.01, size = 412, normalized size = 3.55

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\sqrt{2}\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\cos(dx+c)\sin(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+3*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)+8*I*cos(d*x+c)^4-4*I*cos(d*x+c)^3-8*sin(d*x+c)*cos(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c)-4*sin(d*x+c)*cos(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)*a^3
```

Maxima [A]
time = 0.50, size = 117, normalized size = 1.01

$$-\frac{i\left(3\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+4\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^4-\frac{4\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^5}{i\,a\tan\left(dx+c\right)-a}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*sqrt(I*a*tan(d*x + c) + a)*a^4 - 4*sqrt(I*a*tan(d*x + c) + a)*a^5/(I*a*tan(d*x + c) - a))/(a*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(89) = 178$.
time = 0.44, size = 235, normalized size = 2.03

$$-\frac{3 \sqrt{2} \sqrt{-\frac{a^{7}}{d^{2}}} d \log\left(\frac{4 \left(a^{4} e^{\left(i d x + i c\right)} + \sqrt{-\frac{a^{7}}{d^{2}}} \left(i d e^{\left(2 i d x + 2 i c\right)} + i d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-i d x - i c\right)}}{a^{3}}\right) - 3 \sqrt{2} \sqrt{-\frac{a^{7}}{d^{2}}} d \log\left(\frac{4 \left(a^{4} e^{\left(i d x + i c\right)} + \sqrt{-\frac{a^{7}}{d^{2}}} \left(-i d e^{\left(2 i d x + 2 i c\right)} - i d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-i d x - i c\right)}}{a^{3}}\right) + 2 \sqrt{2} \left(i a^{3} e^{\left(3 i d x + 3 i c\right)} + 3 i a^{3} e^{\left(i d x + i c\right)}\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^2
)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*
d*x - I*c)/a^3) - 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + s
qrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/a^3) + 2*sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + 3*I*a
^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=137

$$\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d}-\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2}+\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

```
[Out] 1/16*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1
/2)-1/2*I*a^5*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2+1/8*I*a^4*(a+
I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 43, 44, 65, 212}

$$\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d}-\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2}+\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/8)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*d) - ((I/2)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x])^2
) + ((I/8)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx,\right.}{4d}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{(ia^4)}{}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{(ia^4)}{}$$

$$= \frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d} - \frac{ia^5\sqrt{a+ia\tan(c+d}}{2d(a-ia\tan(c+dx}$$

Mathematica [A]
time = 1.60, size = 152, normalized size = 1.11

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}+3e^{3i(c+dx)}+2e^{5i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)(a+ia\tan(c+dx))^{7/2}}{8\sqrt{2}\,d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/8*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(E^(I*(c + d*x))
+ 3*E^((3*I)*(c + d*x)) + 2*E^((5*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*
x))]*ArcSinh[E^(I*(c + d*x))])*(a + I*a*Tan[c + d*x])^(7/2))/(Sqrt[2]*d*E^(
(4*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(110) = 220$.
time = 1.00, size = 742, normalized size = 5.42

method	result
default	$-\frac{\left(i \sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(\cos^3(dx+c)\right)\sqrt{2}+3i\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/d*(-128*I*cos(d*x+c)^7+80*I*cos(d*x+c)^5+sin(d*x+c)*arctan(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/
2)*cos(d*x+c)^3*2^(1/2)+I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x
+c)*cos(d*x+c)^3*2^(1/2)+3*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*2^(1/
2)+3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1
/2)+3*sin(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*2^(1/2)+2^(1/2)*arctan(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)*sin(d*x+c)+256*I*cos(d*x+c)^8+16*I*cos(d*x+c)^4-256*sin(d*x+c)*cos(d
*x+c)^7-224*I*cos(d*x+c)^6+128*sin(d*x+c)*cos(d*x+c)^6+I*2^(1/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(7/2)*sin(d*x+c)+96*sin(d*x+c)*cos(d*x+c)^5+3*I*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)-16
*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I
*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3*a^3
```

Maple [**A**]

time = 0.49, size = 138, normalized size = 1.01

$$-\frac{i\left(\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a^{5}+2\,\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{6}\right)}{\left(i\,a\tan(dx+c)+a\right)^{2}-4\left(i\,a\tan(dx+c)+a\right)a+4\,a^{2}}\right)}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/32*I*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*((I*a*tan(d*x + c) +
a)^(3/2)*a^5 + 2*sqrt(I*a*tan(d*x + c) + a)*a^6)/((I*a*tan(d*x + c) + a)^2
- 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(102) = 204$.
time = 0.37, size = 261, normalized size = 1.91

$$\frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,d\log\left(\frac{4\left(a^{4}e^{(i\,dx+i\,c)}-\sqrt{2}\,\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,(i\,de^{(2i\,dx+2i\,c)}+i\,d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{3}}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,d\log\left(\frac{4\left(a^{4}e^{(i\,dx+i\,c)}-\sqrt{2}\,\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,(-i\,de^{(2i\,dx+2i\,c)}-i\,d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{3}}\right)+\sqrt{2}\left(-2i\,a^{3}e^{(5i\,dx+5i\,c)}-3i\,a^{3}e^{(3i\,dx+3i\,c)}-i\,a^{3}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(
1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e
^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c)
 - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) + sqrt(2)*
(-2*I*a^3*e^(5*I*d*x + 5*I*c) - 3*I*a^3*e^(3*I*d*x + 3*I*c) - I*a^3*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=181

$$-\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}-\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3}-\frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2}-\frac{5ia^4\sqrt{a}}{64d(a-}$$

```
[Out] -5/128*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^
(1/2)-1/6*I*a^6*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^3-5/48*I*a^5*
(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-5/64*I*a^4*(a+I*a*tan(d*x+c
))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A]

time = 0.09, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3568, 44, 65, 212}

$$-\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}-\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3}-\frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2}-\frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-5*I)/64)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/6)*a^6*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^3) - (((5*I)/48)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((5*I)/64)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{(5ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx,\right.}{12d}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{(5ia^5}{}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4}{64d(}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4}{64d(}$$

$$= -\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d} - \frac{ia^6\sqrt{a+ia\tan(c+}}{6d(a-ia\tan(c+d}$$

Mathematica [A]
time = 1.23, size = 129, normalized size = 0.71

$$-\frac{ia^3e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(33+26e^{2i(c+dx)}+8e^{4i(c+dx)}\right)+15\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/384*I)*a^3*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(33 + 26*E^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x))) + 1
5*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(147) = 294.
time = 1.10, size = 1088, normalized size = 6.01

method	result	size
default	Expression too large to display	1088

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12288/d*(-15*I*cos(d*x+c)^5*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(11/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))-75*I*cos(d*x+c)^4*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(11/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))-150*I*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(11/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))-150*I*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(11/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-75*I*cos(d*x+c)*2^(1/2)*sin(d*x+c)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+3072*cos(d*x+c)^9*sin(d*x+c)-15*I*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-15*2^(1/2)*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(11/2)*sin(d*x+c)-11264*I*cos(d*x+c)^10+3584*I*cos(d*x+c)^9+128*I*c
os(d*x+c)^8+320*I*cos(d*x+c)^7-960*I*cos(d*x+c)^6-16384*sin(d*x+c)*cos(d*x+
c)^11+512*sin(d*x+c)*cos(d*x+c)^8+8192*cos(d*x+c)^10*sin(d*x+c)-15*cos(d*x+
c)^5*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-75*cos(d*x+c)^4*sin(d*x+c)*2^(1
/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2))-150*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))-150*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11
/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-75*cos(d*x+c)*
sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)*arctan(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-640*sin(d*x+c)*cos(d*x+c)^7+960*sin(
d*x+c)*cos(d*x+c)^6+16384*I*cos(d*x+c)^12-8192*I*cos(d*x+c)^11)*(a*(I*sin(d
*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^
5*a^3
```

Maple [A]

time = 0.49, size = 176, normalized size = 0.97

$$\frac{i\left(15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(15\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}a^{5}-80\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a^{6}+132\sqrt{i\,a\tan\left(dx+c\right)+a}\,a^{7}\right)}{\left(i\,a\tan(dx+c)+a\right)^{3}-6\left(i\,a\tan(dx+c)+a\right)^{2}a+12\left(i\,a\tan(dx+c)+a\right)a^{2}-8\,a^{3}}\right)}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/768*I*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
 c) + a)^(5/2)*a^5 - 80*(I*a*tan(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(I*a*tan
(d*x + c) + a)*a^7)/((I*a*tan(d*x + c) + a)^3 - 6*(I*a*tan(d*x + c) + a)^2*
a + 12*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(136) = 272$.
time = 0.39, size = 277, normalized size = 1.53

$$-\frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,d\log\left(\frac{4\left(a^{4}e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{3}}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\,d\log\left(\frac{4\left(a^{4}e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^{7}}{d^{2}}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^{3}}\right)-\sqrt{2}\left(-8i\,a^{3}e^{(7i\,dx+7i\,c)}-34i\,a^{3}e^{(5i\,dx+5i\,c)}-59i\,a^{3}e^{(3i\,dx+3i\,c)}-33i\,a^{3}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*
sqrt(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(
4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x
+ 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) -
sqrt(2)*(-8*I*a^3*e^(7*I*d*x + 7*I*c) - 34*I*a^3*e^(5*I*d*x + 5*I*c) - 59*I
*a^3*e^(3*I*d*x + 3*I*c) - 33*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}$$

```
[Out] 256/35*I*a^4*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+64/35*I*a^3*sec(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d+24/35*I*a^2*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
+2/7*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((256*I)/35)*a^4*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((64*I)/3
5)*a^3*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((24*I)/35)*a^2*Sec[c
+ d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/7)*a*Sec[c + d*x]*(a + I*a
*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} + \frac{1}{7}(12a)\int \sec(c+dx)(a$$
$$= \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} + \frac{2ia\sec(c+dx)(a+i}{7d}$$
$$= \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{24ia^2\sec(c+dx)(a+}{35d}$$
$$= \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+a}}{35d}$$

Mathematica [A]
time = 0.87, size = 109, normalized size = 0.78

$$\frac{2a^3\sec^2(c+dx)(i\cos(c-2dx)+\sin(c-2dx))(75+102\cos(2(c+dx))+19i\sec(c+dx)\sin(3(c+dx))+14i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{35d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^2*(I*Cos[c - 2*d*x] + Sin[c - 2*d*x])*(75 + 102*Cos[2*(
c + d*x)] + (19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (14*I)*Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(35*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.60, size = 100, normalized size = 0.72

method	result
default	$\frac{2\left(128i\left(\cos^4(dx+c)\right)+128\sin(dx+c)\left(\cos^3(dx+c)\right)+54i\left(\cos^2(dx+c)\right)-22\sin(dx+c)\cos(dx+c)-5i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\ a}{35d\cos(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(128*I*cos(d*x+c)^4+128*sin(d*x+c)*cos(d*x+c)^3+54*I*cos(d*x+c)^2-22
*sin(d*x+c)*cos(d*x+c)-5*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/
cos(d*x+c)^3*a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Fricas [**A**]
time = 0.39, size = 109, normalized size = 0.78

$$-\frac{16\sqrt{2}\left(-35i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-56i\,a^3e^{(2i\,dx+2i\,c)}-16i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*(-35*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a^3*e^(4*I*d*x + 4*I*c
) - 56*I*a^3*e^(2*I*d*x + 2*I*c) - 16*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Mupad [**B**]
time = 6.00, size = 286, normalized size = 2.06

$$\frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d}-\frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x),x)
```

```
[Out] (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*16i)/d - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2
i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(d*(exp(c*2i
+ d*x*2i) + 1)) + (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(5*d*(exp(c*2i + d*x*2i) + 1
)^2) - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=104

$$-\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

```
[Out] -64/3*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+16/3*I*a^2*cos(d*x+c)*(a+
I*a*tan(d*x+c))^(3/2)/d+2/3*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3575, 3574}

$$-\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-64*I)/3)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((16*I)/3)*a
^2*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/3)*a*Cos[c + d*x]
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d} + \frac{1}{3}(8a)\int \cos(c+dx)(a$$

$$= \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia\cos(c+dx)(a+}{3d}$$

$$= -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{16ia^2\cos(c+dx)(a}{3}$$

Mathematica [**A**]
time = 0.50, size = 59, normalized size = 0.57

$$-\frac{2ia^3\sec(c+dx)(12+11\cos(2(c+dx))-5i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/3)*a^3*Sec[c + d*x]*(12 + 11*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c + d
*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [**A**]
time = 0.96, size = 73, normalized size = 0.70

method	result	size
default	$-\frac{2\left(22i\left(\cos^2(dx+c)\right)+10\sin(dx+c)\cos(dx+c)+i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a^3}{3d\cos(dx+c)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(22*I*cos(d*x+c)^2+10*sin(d*x+c)*cos(d*x+c)+I)*(a*(I*sin(d*x+c)+cos(
d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)*a^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(80) = 160.
time = 0.58, size = 418, normalized size = 4.02

$$\frac{2\left(23i\,a^{\frac{7}{2}}+\frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{130i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{20\,a^{\frac{7}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{23i\,a^{\frac{7}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{7}{2}}}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{6i\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{6i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(23*I*a^(7/2) + 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*a^(7/2)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 130*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 60*a^(7/2)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 88*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*I*a^(7/2
)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-18*I*s
in(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4
2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*I*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18*I*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3))
```

Fricas [**A**]
time = 0.37, size = 71, normalized size = 0.68

$$-\frac{4\sqrt{2}\left(3i\,a^3e^{(4i\,dx+4i\,c)}+12i\,a^3e^{(2i\,dx+2i\,c)}+8i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(2)*(3*I*a^3*e^(4*I*d*x + 4*I*c) + 12*I*a^3*e^(2*I*d*x + 2*I*c) +
8*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c), x)
```

Mupad [B]
time = 4.72, size = 102, normalized size = 0.98

$$-\frac{2\,a^3\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(5\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,35{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,11{}\mathrm{i}\right)}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] -(2*a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
 + 1))^(1/2)*(cos(c + d*x)*35i + 5*sin(c + d*x) + cos(3*c + 3*d*x)*11i + 5*
sin(3*c + 3*d*x)))/(3*d*(cos(2*c + 2*d*x) + 1))
```

3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=71

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

```
[Out] 8/3*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-2*I*a*cos(d*x+c)^3*(a+I*a
*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*Co
s[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}-(4a)\int \cos^3(c+dx)($$

$$=\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+}{d}$$

Mathematica [A]
time = 0.47, size = 86, normalized size = 1.21

$$\frac{2a^3\cos(c+dx)(i\cos(c+dx)+3\sin(c+dx))(\cos(c+4dx)+i\sin(c+4dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]*(I*Cos[c + d*x] + 3*Sin[c + d*x])*(Cos[c + 4*d*x] + I*S
in[c + 4*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A]
time = 0.77, size = 71, normalized size = 1.00

method	result	size
default	$\frac{2\left(-2i\left(\cos^{2}(dx+c)\right)+2\sin(dx+c)\cos(dx+c)+3i\right)\cos(dx+c)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,a^{3}}{3d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)+3*I)*cos(d*x+c)*(a*(I*sin(
d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(57) = 114.
time = 0.60, size = 504, normalized size = 7.10

$$-\frac{2\left(-i\,a^{\frac{7}{2}}-\frac{6\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5i\,a^{\frac{7}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{24\,a^{\frac{7}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\,a^{\frac{7}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{24\,a^{\frac{7}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{5i\,a^{\frac{7}{2}}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6\,a^{\frac{7}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{i\,a^{\frac{7}{2}}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)^{\frac{7}{2}}}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(-\frac{4i\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8i\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{14\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8i\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{3\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{4i\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2*(-I*a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*I*a^(7/2)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 10*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*I*a^(7/2)*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 + I*a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*(-2*I*sin(d*x + c)
/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(7/2)*(12*I*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x
```

```
 + c) + 1)^2 + 24*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 42*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*I*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 12
*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 3*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 3))
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.83

$$\frac{\sqrt{2}\left(-i\,a^3e^{(4i\,dx+4i\,c)}+i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/3*sqrt(2)*(-I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*a
^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^3, x)
```

Mupad [B]
time = 0.85, size = 85, normalized size = 1.20

$$\frac{a^3\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,3{}\mathrm{i}-\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] (a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(c + d*x)*3i + sin(c + d*x) - cos(3*c + 3*d*x)*1i + sin(3*c +
 3*d*x)))/(3*d)
```

3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=35

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

```
[Out] -2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{3574\}$

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 0.65, size = 73, normalized size = 2.09

$$\frac{2a^3\cos^3(c+dx)(-i\cos(2c+5dx)+\sin(2c+5dx))\sqrt{a+ia\tan(c+dx)}}{5d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]^3*((-I)*Cos[2*c + 5*d*x] + Sin[2*c + 5*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(5*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.
time = 0.88, size = 73, normalized size = 2.09

method	result	size
default	$\frac{2\left(-2 i \left(\cos^{2}\left(d x +c \right)\right)+2 \sin \left(d x +c \right) \cos \left(d x +c \right)+i \right) \left(\cos^{3}\left(d x +c \right)\right) \sqrt{\frac{a \left(i \sin \left(d x +c \right)+\cos \left(d x +c \right)\right)}{\cos \left(d x +c \right)}}\, a^{3}}{5 d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(-2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)+I)*cos(d*x+c)^3*(a*(I*sin(
d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(27) = 54$.
time = 0.59, size = 454, normalized size = 12.97

$$\frac{2 \left(i a^{\frac{7}{2}} - \frac{6 i a^{\frac{7}{2}} \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{15 i a^{\frac{7}{2}} \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} - \frac{20 i a^{\frac{7}{2}} \sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} + \frac{15 i a^{\frac{7}{2}} \sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} - \frac{6 i a^{\frac{7}{2}} \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{i a^{\frac{7}{2}} \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right) \left(-\frac{2 i \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} - 1\right)^{\frac{7}{2}}}{-5 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)^{\frac{7}{2}} \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)^{\frac{7}{2}} \left(\frac{2 i \sin(dx+c)}{\cos(dx+c)+1} + \frac{4 \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{10 i \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{5 \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{20 i \sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} + \frac{20 i \sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} - \frac{5 \sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} + \frac{10 i \sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}} - \frac{4 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{2 i \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(7/2) - 6*I*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*I*a^(7/
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*I*a^(7/2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 15*I*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*I*a^(
7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + I*a^(7/2)*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-10*I*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 50*I*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 100*I*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 100*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 25*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 50*I*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 20*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 10*I*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 + 5*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 5))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.
time = 0.36, size = 73, normalized size = 2.09

$$\frac{\sqrt{2} \left(-i a^{3} e^{(6 i d x + 6 i c)} - 3 i a^{3} e^{(4 i d x + 4 i c)} - 3 i a^{3} e^{(2 i d x + 2 i c)} - i a^{3}\right) \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}}}{20 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*(-I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) - 3*
I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^5, x)
```

Mupad [**B**]
time = 5.26, size = 112, normalized size = 3.20

$$-\frac{a^3\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-2\,\sin\left(c+d\,x\right)-3\,\sin\left(3\,c+3\,d\,x\right)-\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,4{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,3{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] -(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*4i - 2*sin(c + d*x) + cos(3*c + 3*d*x)*3i + cos(5*
c + 5*d*x)*1i - 3*sin(3*c + 3*d*x) - sin(5*c + 5*d*x)))/(20*d)
```

3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=196

$$\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{ia^2\cos^3(c+dx)(a+ia\,\mathrm{ta}}{12d}$$

[Out] 1/16*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d*2^(1/2)-1/8*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/10*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2)/d

Rubi [A]
time = 0.22, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3571, 3570, 212}

$$\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}-\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]

[Out] ((I/8)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - ((I/8)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/12)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/10)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/7)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2))/d

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}+\frac{1}{2}a\int \cos^5(c+dx)(a$$

$$= -\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}-\frac{i\cos^7(c+dx)(a+ia}{7d}$$

$$= -\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}-\frac{ia\cos^5(c+dx)(a+}{10}$$

$$= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{ia^2\cos^3(c+dx)(a+}{12d}$$

$$= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{ia^2\cos^3(c+dx)(a+}{12d}$$

$$= \frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{ia^3\cos(c+dx)\sqrt{}}{}$$

Mathematica [A]
time = 2.07, size = 131, normalized size = 0.67

$$-\frac{ia^3e^{-i(c+dx)}\left(176+298e^{2i(c+dx)}+188e^{4i(c+dx)}+81e^{6i(c+dx)}+15e^{8i(c+dx)}-105\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{1680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/1680*I)*a^3*(176 + 298*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x)) +
 81*E^((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) - 105*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]
])/(d*E^(I*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1259 vs. 2(159) = 318.
time = 2.84, size = 1260, normalized size = 6.43

method	result	size
default	Expression too large to display	1260

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/107520/d*(105*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*sin(d*x+c)+7168*cos(d*x+c)^9*si
n(d*x+c)-61440*sin(d*x+c)*cos(d*x+c)^12-122880*I*cos(d*x+c)^14+61440*I*cos(
d*x+c)^13+79872*I*cos(d*x+c)^12-24576*I*cos(d*x+c)^11-1024*I*cos(d*x+c)^10-
1792*I*cos(d*x+c)^9-4480*I*cos(d*x+c)^8+13440*I*cos(d*x+c)^7+122880*cos(d*x
+c)^13*sin(d*x+c)-18432*sin(d*x+c)*cos(d*x+c)^11-8960*sin(d*x+c)*cos(d*x+c)
^8-6144*cos(d*x+c)^10*sin(d*x+c)-105*I*2^(1/2)*cos(d*x+c)^6*sin(d*x+c)*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-630*I*2^(1/2)*cos(d*x+c)^5*sin(d*x+c
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-1575*I*2^(1/2)*cos(d*x+c)^4*si
n(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-2100*I*2^(1/2)*cos(d*x+
c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-1575*I*2^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-630*I*2^(
1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-105*I
*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*sin(d*x+c)+13440*sin(d*
x+c)*cos(d*x+c)^7+105*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+6
30*cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+1575*cos(d*x+c)^4*si
n(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+2100*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*2^(1/2))*2^(1/2)+1575*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(13/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)
+630*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(13/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2))*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^6*a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(149) = 298$.
time = 0.64, size = 1253, normalized size = 6.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/6720*(20*(7*I*sqrt(2)*a^3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - 7*sqrt(2)*a^3*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 3*(I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x +
2*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(7/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(sqrt(2)*a^3*cos(2*d*x + 2*c)^
2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(
2)*a^3)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) + 8
4*(5*I*sqrt(2)*a^3*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - 5*sqrt(2)*a^3*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 (I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*I
*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(5/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + sqrt(2)*a^
3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(2)*a^3)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 105*(2*sqrt(2)*
a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*a^3*arctan
2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a^3*log(sqrt(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + I*sqrt(2)*a^3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [A]

time = 0.40, size = 258, normalized size = 1.32

$$\frac{105 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^7}{d^2}}\, d \log\left(\frac{\left(i a^4 + \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^7}{d^2}} \left(d e^{(2i dx + 2i c)} + d\right) \sqrt{\frac{a}{e^{(2i dx + 2i c)} + 1}}\right) e^{(-i dx - i c)}}{4 d}\right) - 105 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^7}{d^2}}\, d \log\left(\frac{\left(i a^4 - \sqrt{2} \sqrt{\frac{1}{2}} \sqrt{-\frac{a^7}{d^2}} \left(d e^{(2i dx + 2i c)} + d\right) \sqrt{\frac{a}{e^{(2i dx + 2i c)} + 1}}\right) e^{(-i dx - i c)}}{4 d}\right) + \sqrt{2} \left(-15i a^3 e^{(8i dx + 8i c)} - 81i a^3 e^{(6i dx + 6i c)} - 188i a^3 e^{(4i dx + 4i c)} - 298i a^3 e^{(2i dx + 2i c)} - 176i a^3\right) \sqrt{\frac{a}{e^{(2i dx + 2i c)} + 1}}}{1680 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/1680*(105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 + sqrt(2)*sqrt(1/2)*s
qrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 - sqr
t(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-15*I*a^3*e^(8*I*d*x + 8*I
*c) - 81*I*a^3*e^(6*I*d*x + 6*I*c) - 188*I*a^3*e^(4*I*d*x + 4*I*c) - 298*I*
a^3*e^(2*I*d*x + 2*I*c) - 176*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^7\,\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=268

$$\frac{11ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,d}+\frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}-\frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d}$$

```
[Out] 11/128*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d*2^(1/2)+11/96*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-11/64*I*
a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-11/120*I*a^3*cos(d*x+c)^3*(a+I*a*
tan(d*x+c))^(1/2)/d-11/140*I*a^2*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-11
/126*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d-1/9*I*cos(d*x+c)^9*(a+I*a*
tan(d*x+c))^(7/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$\frac{11ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,d}+\frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}-\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d}-\frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d}-\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d}-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}-\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((11*I)/64)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*T
an[c + d*x]])])/(Sqrt[2]*d) + (((11*I)/96)*a^4*Cos[c + d*x])/(d*Sqrt[a + I*
a*Tan[c + d*x]]) - (((11*I)/64)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]
)/d - (((11*I)/120)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((1
1*I)/140)*a^2*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - (((11*I)/126
)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/9)*Cos[c + d*x]^9*
(a + I*a*Tan[c + d*x])^(7/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} + \frac{1}{18}(11a)\int \cos^7(c+d$$

$$= -\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+}{9}$$

$$= -\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)}{}$$

$$= -\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)}{}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+}}{120d}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+}}{64d}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+}}{64d}$$

$$= \frac{11ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} + \frac{11ia^4\cos(c}{96d\sqrt{a+ia\tan}}$$

Mathematica [A]
time = 3.50, size = 188, normalized size = 0.70

$$-\frac{ia^3e^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-315+4303e^{2i(c+dx)}+7034e^{4i(c+dx)}+3754e^{6i(c+dx)}+1798e^{8i(c+dx)}+530e^{10i(c+dx)}+70e^{12i(c+dx)}-3465e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{20160\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/20160*I)*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(
-315 + 4303*E^((2*I)*(c + d*x)) + 7034*E^((4*I)*(c + d*x)) + 3754*E^((6*I)*
(c + d*x)) + 1798*E^((8*I)*(c + d*x)) + 530*E^((10*I)*(c + d*x)) + 70*E^((1
2*I)*(c + d*x)) - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Ar
cTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1603 vs. 2(219) = 438.
time = 2.09, size = 1604, normalized size = 5.99

method	result	size
default	Expression too large to display	1604

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10321920/d*(9175040*I*cos(d*x+c)^18-4587520*I*cos(d*x+c)^17-5570560*I*co
s(d*x+c)^16+1638400*I*cos(d*x+c)^15+65536*I*cos(d*x+c)^14+90112*I*cos(d*x+c
)^13+135168*I*cos(d*x+c)^12+236544*I*cos(d*x+c)^11+591360*I*cos(d*x+c)^10-1
774080*I*cos(d*x+c)^9-9175040*cos(d*x+c)^17*sin(d*x+c)+4587520*cos(d*x+c)^1
6*sin(d*x+c)+983040*cos(d*x+c)^15*sin(d*x+c)-3465*cos(d*x+c)^8*sin(d*x+c)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))*2^(1/2)-27720*cos(d*x+c)^7*sin(d*x+c)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*2^(1/2)-97020*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/
2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-194040*
cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-242550*cos(d*x+c)^4*sin
(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-194040*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2))*2^(1/2)+3465*I*cos(d*x+c)^8*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(17/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))*2^(1/2)+27720*I*cos(d*x+c)^7*sin(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(17/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+97020*I*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(17/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+194040*I*cos(d*x+c)^5*sin(d*x+c)
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+242550*I*cos(d*x+c)^4*s
in(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+194040*I*cos(d
*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctanh(1/2*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)+97020
*I*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/
2)+27720*I*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*2^(1/2)+655360*sin(d*x+c)*cos(d*x+c)^14-1774080*cos(d*x+c)^9*sin(d*x+c)+81
1008*sin(d*x+c)*cos(d*x+c)^12-720896*cos(d*x+c)^13*sin(d*x+c)-97020*cos(d*x
+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-27720*cos(d*x+c)*sin(d*x+c)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
```

```
))^(1/2)*2^(1/2))*2^(1/2)+3465*I*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(17/2)*sin(d*x+c)-946176*sin(d*x+c)*cos(d*x+c)^11+1182720*cos(d*x+c)^10*s
in(d*x+c)-3465*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(17/2)*2^(1/2)*arctan(1/2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c))*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^8*a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.57, size = 314, normalized size = 1.17

$$-\frac{\left(3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,de^{(2i\,dx+2i\,c)}\log\left(-\frac{11\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)-3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,de^{(2i\,dx+2i\,c)}\log\left(-\frac{11\left(-i\,a^4-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)-\sqrt{2}\left(-70i\,a^3e^{(12i\,dx+12i\,c)}-530i\,a^3e^{(10i\,dx+10i\,c)}-1798i\,a^3e^{(8i\,dx+8i\,c)}-3754i\,a^3e^{(6i\,dx+6i\,c)}-7034i\,a^3e^{(4i\,dx+4i\,c)}-4303i\,a^3e^{(2i\,dx+2i\,c)}+315i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-2i\,dx-2i\,c)}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-
I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 3465*sqrt(1/2)*sqrt(-a^7
/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-I*a^4 - sqrt(2)*sqrt(1/2)*sqrt(-a^
7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I
*d*x - I*c)/d) - sqrt(2)*(-70*I*a^3*e^(12*I*d*x + 12*I*c) - 530*I*a^3*e^(10
*I*d*x + 10*I*c) - 1798*I*a^3*e^(8*I*d*x + 8*I*c) - 3754*I*a^3*e^(6*I*d*x +
 6*I*c) - 7034*I*a^3*e^(4*I*d*x + 4*I*c) - 4303*I*a^3*e^(2*I*d*x + 2*I*c) +
 315*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^9, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^9\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=342

$$\frac{195ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d}+\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-19$$

```
[Out] 195/2048*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c)
)^(1/2))/d*2^(1/2)+65/512*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+39/44
8*I*a^4*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-195/1024*I*a^3*cos(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d-13/128*I*a^3*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2
)/d-13/168*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d-65/924*I*a^2*cos(d
*x+c)^7*(a+I*a*tan(d*x+c))^(3/2)/d-5/66*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))
^(5/2)/d-1/11*I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2)/d
```

Rubi [A]
time = 0.40, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$\frac{195ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}+\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}-\frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d}-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}-\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((195*I)/1024)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*
a*Tan[c + d*x]])])/(Sqrt[2]*d) + (((65*I)/512)*a^4*Cos[c + d*x])/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (((39*I)/448)*a^4*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) - (((195*I)/1024)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - (((13*I)/128)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((13
*I)/168)*a^3*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d - (((65*I)/924)*a
^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d - (((5*I)/66)*a*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Ta
n[c + d*x])^(7/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
```

```
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} + \frac{1}{22}(15a)\int \cos^{9}(c+ \\
&= -\frac{5ia\cos^{9}(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a-}{1} \\
&= -\frac{65ia^{2}\cos^{7}(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^{9}(c+dx)}{} \\
&= -\frac{13ia^{3}\cos^{5}(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^{2}\cos^{7}(c+dx}{} \\
&= \frac{39ia^{4}\cos^{3}(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^{3}\cos^{5}(c+dx)\sqrt{a+ia\tan(c}}{168d} \\
&= \frac{39ia^{4}\cos^{3}(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^{3}\cos^{3}(c+dx)\sqrt{a+ia\tan(c}}{128d} \\
&= \frac{65ia^{4}\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^{4}\cos^{3}(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13}{} \\
&= \frac{65ia^{4}\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^{4}\cos^{3}(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{19}{} \\
&= \frac{65ia^{4}\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^{4}\cos^{3}(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{19}{} \\
&= \frac{195ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d} + \frac{65ia^{4}\cos}{512d\sqrt{a+ia}}
\end{aligned}
$$

Mathematica [A]
time = 6.98, size = 194, normalized size = 0.57

$$
-\frac{ia^{3}e^{-5i(c+dx)}\left(-462-7161e^{2i(c+dx)}+47413e^{4i(c+dx)}+78800e^{6i(c+dx)}+38512e^{8i(c+dx)}+19552e^{10i(c+dx)}+7184e^{12i(c+dx)}+1624e^{14i(c+dx)}+168e^{16i(c+dx)}-45045e^{4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{473088d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/473088*I)*a^3*(-462 - 7161*E^((2*I)*(c + d*x)) + 47413*E^((4*I)*(c + d
*x)) + 78800*E^((6*I)*(c + d*x)) + 38512*E^((8*I)*(c + d*x)) + 19552*E^((10
*I)*(c + d*x)) + 7184*E^((12*I)*(c + d*x)) + 1624*E^((14*I)*(c + d*x)) + 16
8*E^((16*I)*(c + d*x)) - 45045*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(
d*E^((5*I)*(c + d*x)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1947 vs. 2(281) = 562.
time = 1.80, size = 1948, normalized size = 5.70

method	result	size
default	Expression too large to display	1948

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/484442112/d*(-352321536*I*cos(d*x+c)^22+176160768*I*cos(d*x+c)^21+2055208
96*I*cos(d*x+c)^20-58720256*I*cos(d*x+c)^19-2097152*I*cos(d*x+c)^18-2621440
*I*cos(d*x+c)^17-3407872*I*cos(d*x+c)^16-4685824*I*cos(d*x+c)^15-7028736*I*
cos(d*x+c)^14+352321536*sin(d*x+c)*cos(d*x+c)^21-176160768*sin(d*x+c)*cos(d
*x+c)^20-29360128*sin(d*x+c)*cos(d*x+c)^19-29360128*sin(d*x+c)*cos(d*x+c)^1
8+450450*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-45045*I*2^(1/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*sin(d*x+c)+31457280*cos(d*x+c)^1
7*sin(d*x+c)-34078720*cos(d*x+c)^16*sin(d*x+c)+37486592*cos(d*x+c)^15*sin(d
*x+c)+45045*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^10*2^(1/2)+450450*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^9*2^(1/2)+2027025*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*sin(d*x+c)*cos(d*x+c)^8*2^(1/2)+5405400*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(2
1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*co
s(d*x+c)^7*2^(1/2)+9459450*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^6*2^(1
/2)+11351340*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^5*2^(1/2)+9459450*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)+5405400*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(21/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2
))*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)+2027025*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
21/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*2^(1/2)-42172416*sin(d*x+c)*cos(d*x+c)^14-9459450*I*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^6*2^(1/2)-11351340*I*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^5*2^(1/2)-
9459450*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^
4*2^(1/2)-5405400*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*c
```

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*c
os(d*x+c)^3*2^(1/2)-2027025*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctanh
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*si
n(d*x+c)*cos(d*x+c)^2*2^(1/2)-450450*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^10*2^(1/2)-450450*I*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^9*2^(1/2)-2027025*I*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^8*2^(1/2)-5405400*I*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)^7*2^(1/2)-
12300288*I*cos(d*x+c)^13-30750720*I*cos(d*x+c)^12+92252160*I*cos(d*x+c)^11-
61501440*sin(d*x+c)*cos(d*x+c)^12+45045*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(21/2)*sin(d*x
+c)+49201152*cos(d*x+c)^13*sin(d*x+c)+92252160*sin(d*x+c)*cos(d*x+c)^11)*(a
*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/co
s(d*x+c)^10*a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.44, size = 342, normalized size = 1.00

$$-\frac{\left(45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,de^{(4i\,dx+4i\,c)}\log\left(-\frac{195\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{512\,d}\right)-45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,de^{(4i\,dx+4i\,c)}\log\left(-\frac{195\left(-i\,a^4-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\,(de^{(2i\,dx+2i\,c)}+d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{512\,d}\right)-\sqrt{2}\left(-168i\,a^3e^{(16i\,dx+16i\,c)}-1624i\,a^3e^{(14i\,dx+14i\,c)}-7184i\,a^3e^{(12i\,dx+12i\,c)}-19552i\,a^3e^{(10i\,dx+10i\,c)}-38512i\,a^3e^{(8i\,dx+8i\,c)}-78800i\,a^3e^{(6i\,dx+6i\,c)}-47413i\,a^3e^{(4i\,dx+4i\,c)}+7161i\,a^3e^{(2i\,dx+2i\,c)}+462i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-4i\,dx-4i\,c)}}{473088\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/473088*(45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/51
2*(-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 45045*sqrt(1/2)*sqrt
(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/512*(-I*a^4 - sqrt(2)*sqrt(1/2)*s
qrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - sqrt(2)*(-168*I*a^3*e^(16*I*d*x + 16*I*c) - 1624*I*
```

```
a^3*e^(14*I*d*x + 14*I*c) - 7184*I*a^3*e^(12*I*d*x + 12*I*c) - 19552*I*a^3*
e^(10*I*d*x + 10*I*c) - 38512*I*a^3*e^(8*I*d*x + 8*I*c) - 78800*I*a^3*e^(6*
I*d*x + 6*I*c) - 47413*I*a^3*e^(4*I*d*x + 4*I*c) + 7161*I*a^3*e^(2*I*d*x +
2*I*c) + 462*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-4*I*d*x - 4*I*c)
/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^11, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{11}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}$$

```
[Out] -16/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d+8/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
-12/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^6/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}-\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^4*d) + (((8*I)/3)*(a + I*a*Ta
n[c + d*x])^(9/2))/(a^5*d) - (((12*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a
^6*d) + (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{5/2}-12a^2(a+x)^{7/2}+6a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx,\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{12i(a+ia\tan(c+}{11a^6d}$$

Mathematica [A]
time = 0.68, size = 95, normalized size = 0.81

$$\frac{2\sec^7(c+dx)(390\cos(c+dx)+445\cos(3(c+dx))-7i(26\sin(c+dx)+59\sin(3(c+dx))))(-i\cos(4(c+dx))+\sin(4(c+dx)))}{3003d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^7*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] - (7*I)*(26*Sin[
c + d*x] + 59*Sin[3*(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])
)/(3003*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.96, size = 127, normalized size = 1.09

method	result
default	$-\frac{2\left(512i\left(\cos^6(dx+c)\right)-512\sin(dx+c)\left(\cos^5(dx+c)\right)+64i\left(\cos^4(dx+c)\right)-320\sin(dx+c)\left(\cos^3(dx+c)\right)+28i\left(\cos^2(dx+c)\right)-252\sin(dx+c\right.}{3003d\cos(dx+c)^6a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3003/d*(512*I*cos(d*x+c)^6-512*sin(d*x+c)*cos(d*x+c)^5+64*I*cos(d*x+c)^4
-320*sin(d*x+c)*cos(d*x+c)^3+28*I*cos(d*x+c)^2-252*sin(d*x+c)*cos(d*x+c)+23
1*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(85) = 170.
time = 0.28, size = 297, normalized size = 2.54

$$-\frac{2i\left(15015\sqrt{ia\tan(dx+c)+a}-\frac{3003\left(3(ia\tan(dx+c)+a)^{\frac{5}{2}}-10(ia\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{ia\tan(dx+c)+a}\,a^2\right)}{a^2}+\frac{143\left(35(ia\tan(dx+c)+a)^{\frac{9}{2}}-180(ia\tan(dx+c)+a)^{\frac{7}{2}}a+378(ia\tan(dx+c)+a)^{\frac{5}{2}}a^2-420(ia\tan(dx+c)+a)^{\frac{3}{2}}a^3+315\sqrt{ia\tan(dx+c)+a}\,a^4\right)}{a^4}-\frac{5\left(231(ia\tan(dx+c)+a)^{\frac{13}{2}}-1638(ia\tan(dx+c)+a)^{\frac{11}{2}}a+5005(ia\tan(dx+c)+a)^{\frac{9}{2}}a^2-8580(ia\tan(dx+c)+a)^{\frac{7}{2}}a^3+9009(ia\tan(dx+c)+a)^{\frac{5}{2}}a^4-6006(ia\tan(dx+c)+a)^{\frac{3}{2}}a^5+3003\sqrt{ia\tan(dx+c)+a}\,a^6\right)}{a^6}\right)}{15015ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15015*I*(15015*sqrt(I*a*tan(d*x + c) + a) - 3003*(3*(I*a*tan(d*x + c) +
a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a
)*a^2)/a^2 + 143*(35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) +
 a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c)
+ a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4 - 5*(231*(I*a*tan(
d*x + c) + a)^(13/2) - 1638*(I*a*tan(d*x + c) + a)^(11/2)*a + 5005*(I*a*tan
(d*x + c) + a)^(9/2)*a^2 - 8580*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 9009*(I*
a*tan(d*x + c) + a)^(5/2)*a^4 - 6006*(I*a*tan(d*x + c) + a)^(3/2)*a^5 + 300
3*sqrt(I*a*tan(d*x + c) + a)*a^6)/a^6)/(a*d)
```

Fricas [A]
time = 0.39, size = 150, normalized size = 1.28

$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(13i\,dx+13i\,c)}+104i\,e^{(11i\,dx+11i\,c)}+286i\,e^{(9i\,dx+9i\,c)}+429i\,e^{(7i\,dx+7i\,c)}\right)}{3003\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(13*I*d*x + 13*
I*c) + 104*I*e^(11*I*d*x + 11*I*c) + 286*I*e^(9*I*d*x + 9*I*c) + 429*I*e^(7
*I*d*x + 7*I*c))/(a*d*e^(12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) +
 15*a*d*e^(8*I*d*x + 8*I*c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*
x + 4*I*c) + 6*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{8}\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**8/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 8.94, size = 434, normalized size = 3.71

$$-\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3003\,a\,d}-\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{3003\,a\,d\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1001\,a\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,640\mathrm{i}}{3003\,a\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{429\,a\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3456\mathrm{i}}{143\,a\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,a\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6
784i)/(429*a*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3003*a*d*(exp(c*2i + d*
x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*256i)/(1001*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*640i)/(3003*a*d*(
exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp
(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3003*a*d) - ((a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3456i)/(143*a*d*(exp(c*2i +
d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x
*2i) + 1))^(1/2)*128i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}$$

```
[Out] -8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d+8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d-
2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^3*d) + (((8*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^4*d) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^5*d
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{3/2} - 4a(a+x)^{5/2} + (a+x)^{7/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3 d} + \frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4 d} - \frac{2i(a+ia\tan(c+d$$

Mathematica [A]
time = 0.41, size = 77, normalized size = 0.88

$$\frac{2\sec^5(c+dx)(36+71\cos(2(c+dx))-55i\sin(2(c+dx)))(-i\cos(3(c+dx))+\sin(3(c+dx)))}{315d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^5*(36 + 71*Cos[2*(c + d*x)] - (55*I)*Sin[2*(c + d*x)])*((-I
)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(315*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.86, size = 100, normalized size = 1.14

method	result
default	$-\frac{2\left(64i\left(\cos^4(dx+c)\right)-64\sin(dx+c)\left(\cos^3(dx+c)\right)+8i\left(\cos^2(dx+c)\right)-40\sin(dx+c)\cos(dx+c)+35i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{315d\cos(dx+c)^4 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(64*I*cos(d*x+c)^4-64*sin(d*x+c)*cos(d*x+c)^3+8*I*cos(d*x+c)^2-40*
sin(d*x+c)*cos(d*x+c)+35*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/
cos(d*x+c)^4/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(64) = 128.
time = 0.29, size = 169, normalized size = 1.92

$$-\frac{2i\left(315\sqrt{ia\tan(dx+c)+a} - \frac{42\left(3(ia\tan(dx+c)+a)^{\frac{5}{2}} - 10(ia\tan(dx+c)+a)^{\frac{3}{2}}a + 15\sqrt{ia\tan(dx+c)+a}\,a^2\right)}{a^2} + \frac{35(ia\tan(dx+c)+a)^{\frac{9}{2}} - 180(ia\tan(dx+c)+a)^{\frac{7}{2}}a + 378(ia\tan(dx+c)+a)^{\frac{5}{2}}a^2 - 420(ia\tan(dx+c)+a)^{\frac{3}{2}}a^3 + 315\sqrt{ia\tan(dx+c)+a}\,a^4}{a^4}\right)}{315ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*I*(315*sqrt(I*a*tan(d*x + c) + a) - 42*(3*(I*a*tan(d*x + c) + a)^(5/
2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)
/a^2 + (35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) + a)^(7/2)*
a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c) + a)^(3/2)
*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4)/(a*d)
```

Fricas [A]
time = 0.43, size = 113, normalized size = 1.28

$$-\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(9i\,dx+9i\,c)}+36i\,e^{(7i\,dx+7i\,c)}+63i\,e^{(5i\,dx+5i\,c)}\right)}{315\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(9*I*d*x + 9*I*c)
+ 36*I*e^(7*I*d*x + 7*I*c) + 63*I*e^(5*I*d*x + 5*I*c))/(a*d*e^(8*I*d*x + 8*
I*c) + 4*a*d*e^(6*I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I
*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 6.38, size = 306, normalized size = 3.48

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{315\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{105\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,320\mathrm{i}}{63\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2)),x)

[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*320i)/(63*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(315*a*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(105*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4)

3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=59

$$-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

```
[Out] -4/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^2/d+2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d
```

Rubi [A]

time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^2*d) + (((2*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 65, normalized size = 1.10

$$-\frac{2\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))(7i+3\tan(c+dx))}{15d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(7*I + 3*Tan[c + 
 d*x]))/(15*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.82, size = 73, normalized size = 1.24

method	result	size
default	$\frac{2\left(-4i\left(\cos^2(dx+c)\right)+4\sin(dx+c)\cos(dx+c)-3i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos(dx+c)^2 a}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(-4*I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-3*I)*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/a
```

Maxima [A]
time = 0.29, size = 79, normalized size = 1.34

$$-\frac{2i\left(15\sqrt{i\,a\tan(dx+c)+a}-\frac{3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{i\,a\tan(dx+c)+a}\,a^2}{a^2}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*I*(15*sqrt(I*a*tan(d*x + c) + a) - (3*(I*a*tan(d*x + c) + a)^(5/2) -
10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^2)
/(a*d)
```

Fricas [A]
time = 0.39, size = 76, normalized size = 1.29

$$-\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(5i\,dx+5i\,c)}+5i\,e^{(3i\,dx+3i\,c)}\right)}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(5*I*d*x + 5*I*c) +
5*I*e^(3*I*d*x + 3*I*c))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4 (c + dx)}{\sqrt{ia (\tan (c + dx) - i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 1.31, size = 155, normalized size = 2.63

$$-\frac{8\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,27{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,9{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}-5\,\sin\left(2\,c+2\,d\,x\right)-4\,\sin\left(4\,c+4\,d\,x\right)-\sin\left(6\,c+6\,d\,x\right)+19{}\mathrm{i}\right)}{15\,a\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] -(8*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*27i + cos(4*c + 4*d*x)*9i + cos(6*c + 6*d*x)*1i
- 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 19i))/(15*a*
d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 32, normalized size = 1.19

$$\frac{2(-i+\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*(-I + Tan[c + d*x]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.26, size = 24, normalized size = 0.89

method	result	size
derivativedivides	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{da}$	24
default	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{da}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/d/a
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.78

$$-\frac{2i\sqrt{i\,a\tan(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(I*a*tan(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.36, size = 37, normalized size = 1.37

$$-\frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(i\,dx+i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.
time = 0.80, size = 55, normalized size = 2.04

$$-\frac{2i\sqrt{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(
1/2*d*x + 1/2*c)^2 - 1))/(a*d)
```

Mupad [B]
time = 0.16, size = 47, normalized size = 1.74

$$-\frac{\sqrt{\frac{a\,\left(2\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos\left(c+d\,x\right)^2}}\,2\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] -(((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2)*2
i)/(a*d)
```

3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=146

$$-\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -5/16*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(
1/2)+5/8*I/d/(a+I*a*tan(d*x+c))^(1/2)+5/12*I*a/d/(a+I*a*tan(d*x+c))^(3/2)-1
/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}+\frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-5*I)/8)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + (((5*I)/12)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*a^2)
/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)/8)/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} - \frac{(5ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\right.}{4d}$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} -$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} +$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} +$$

$$= -\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d} + \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{}{2d(a-}$$

Mathematica [A]

time = 0.74, size = 126, normalized size = 0.86

$$-\frac{ie^{-2i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-2-14e^{2i(c+dx)}+3e^{4i(c+dx)}\right)+15e^{3i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{24d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/24*I)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-2 - 14*E^((2*I)*(c + d*x)) + 3*
E^((4*I)*(c + d*x))) + 15*E^((3*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d
*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x
]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(114) = 228.
time = 5.15, size = 335, normalized size = 2.29

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(32i(\cos^4(dx+c))-15i\cos(dx+c)\arctan\left(\frac{(-i\cos(dx+c)+\sin(dx+c)+i)\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(32*I*cos(d*x+c)^4-15
*I*cos(d*x+c)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2)+32*sin(d*x+c)*cos(d*x+c)^3-15*sin(d*x+c)*arctan(1/2*(-I*cos(d*x+c)+si
n(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-15*I*arctan(1/2*(-I*cos(d*x+c)+sin(d
*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+20*I*cos(d*x+c)^2+60*sin(d*x+c)*cos(d*x
+c))/a
```

Maxima [A]
time = 0.50, size = 138, normalized size = 0.95

$$\frac{i\left(15\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(15\left(i\,a\tan(dx+c)+a\right)^2a-20\left(i\,a\tan(dx+c)+a\right)a^2-8\,a^3\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}-2\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a}\right)}{96\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*I*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
c) + a)^2*a - 20*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3)/((I*a*tan(d*x + c) + a
)^(5/2) - 2*(I*a*tan(d*x + c) + a)^(3/2)*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(105) = 210$.
time = 0.37, size = 271, normalized size = 1.86

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+15i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)}+11i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+2i\right)\right)e^{(-3i\,dx-3i\,c)}}{48\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(-15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2
)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I*sqrt(1/2)*
a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*e^(2
*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) -
a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(-3*I*e^(6*I*d*x + 6*I*c) + 11*I*e^(4*I*d*x + 4*I*c) + 16*I*e^(2*I*d*
x + 2*I*c) + 2*I))*e^(-3*I*d*x - 3*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{63i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}+\frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+}$$

```
[Out] -63/256*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a
^(1/2)+63/128*I/d/(a+I*a*tan(d*x+c))^(1/2)+63/160*I*a^2/d/(a+I*a*tan(d*x+c)
)^(5/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-9/16*I*a^
3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+21/64*I*a/d/(a+I*a*tan(d*x+
c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}}+\frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}}+\frac{63i}{128d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-63*I)/128)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*Sqrt[a]*d) + (((63*I)/160)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - (((9*I)
/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (((21*I
)/64)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((63*I)/128)/(d*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{(9ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)}\right.}{8c}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}$$

$$= -\frac{63i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d} + \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{}{4d(a}$$

Mathematica [A]
time = 0.87, size = 152, normalized size = 0.69

$$-\frac{ie^{-4i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-8-56e^{2i(c+dx)}-288e^{4i(c+dx)}+85e^{6i(c+dx)}+10e^{8i(c+dx)}\right)+315e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{640d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/640*I)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-8 - 56*E^((2*I)*(c + d*x)) - 2
88*E^((4*I)*(c + d*x)) + 85*E^((6*I)*(c + d*x)) + 10*E^((8*I)*(c + d*x))) +
 315*E^((5*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d*E^((4*I)*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(174) = 348.
time = 0.98, size = 362, normalized size = 1.65

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-512i(\cos^6(dx+c))-512\sin(dx+c)(\cos^5(dx+c))+315i\cos(dx+c)\arctan\left(\frac{(-i\cos(dx+c)+\sin(dx}{2\sin(dx+c)\sqrt{-\frac{2}{1+}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2560/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-512*I*cos(d*x+c)
^6-512*sin(d*x+c)*cos(d*x+c)^5+315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)-96*I*cos(d*x+c)^4+315*sin(d*x+c)*ar
ctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+315*I*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-672*sin(d*x+
c)*cos(d*x+c)^3-420*I*cos(d*x+c)^2-1260*sin(d*x+c)*cos(d*x+c))/a
```

Maxima [A]
time = 0.50, size = 192, normalized size = 0.88

$$\frac{i\left(315\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4a-1050\,(i\,a\tan(dx+c)+a)^3a^2+672\,(i\,a\tan(dx+c)+a)^2a^3+192\,(i\,a\tan(dx+c)+a)a^4+128\,a^5\right)}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+4\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2}\right)}{2560\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2560*I*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(d*
x + c) + a)^4*a - 1050*(I*a*tan(d*x + c) + a)^3*a^2 + 672*(I*a*tan(d*x + c)
 + a)^2*a^3 + 192*(I*a*tan(d*x + c) + a)*a^4 + 128*a^5)/((I*a*tan(d*x + c)
+ a)^(9/2) - 4*(I*a*tan(d*x + c) + a)^(7/2)*a + 4*(I*a*tan(d*x + c) + a)^(5
/2)*a^2))/(a*d)
```

Fricas [A]
time = 0.39, size = 293, normalized size = 1.34

$$\frac{\left(-315i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(ade^{(2i\,dx+2i\,c)}+ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+315i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(ade^{(2i\,dx+2i\,c)}+ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-10i\,e^{(10i\,dx+10i\,c)}-95i\,e^{(8i\,dx+8i\,c)}+203i\,e^{(6i\,dx+6i\,c)}+344i\,e^{(4i\,dx+4i\,c)}+64i\,e^{(2i\,dx+2i\,c)}+8i\right)\right)e^{(-5i\,dx-5i\,c)}}{1280\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(-315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqr
t(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
```

```
+ 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 315*I*sqrt(1
/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*
e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)
) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(-10*I*e^(10*I*d*x + 10*I*c) - 95*I*e^(8*I*d*x + 8*I*c) + 203*I*e
^(6*I*d*x + 6*I*c) + 344*I*e^(4*I*d*x + 4*I*c) + 64*I*e^(2*I*d*x + 2*I*c) +
 8*I))*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{\sqrt{a+a\tan(c+d\,x)\ 1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=292

$$-\frac{429i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}\,d}+\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan($$

```
[Out] -429/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)
/a^(1/2)+429/1024*I/d/(a+I*a*tan(d*x+c))^(1/2)+429/896*I*a^3/d/(a+I*a*tan(d
*x+c))^(7/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(7/2)-13/4
8*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-143/192*I*a^4/d/(a-
I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+429/1280*I*a^2/d/(a+I*a*tan(d*x+c)
)^(5/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}}+\frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}}+\frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}}-\frac{429i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-429*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*Sqrt[a]*d) + (((429*I)/896)*a^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - (
(I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) - (((1
3*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (
((143*I)/192)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2))
 + (((429*I)/1280)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((143*I)/512)*a)
/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((429*I)/1024)/(d*Sqrt[a + I*a*Tan[c +
d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(ia^7)\,\text{Subst}\Big(\int \frac{1}{(a-x)^4(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\Big)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{(13ia^6)\,\text{Subst}\Big(\int \frac{1}{(a-x)^3(a\cdots}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{13i\cdots}{48d(a-ia\tan(c+dx))^2\cdots}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{13i\cdots}{48d(a-ia\tan(c+dx))^2\cdots}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/\cdots}}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/\cdots}}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/\cdots}}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/\cdots}}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/\cdots}}$$

$$= -\frac{429i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}\,d} + \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{\cdots}{6d\cdots}$$

Mathematica [A]
time = 1.20, size = 178, normalized size = 0.61

$$-\frac{ie^{-6i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-240-2064e^{2i(c+dx)}-9008e^{4i(c+dx)}-40784e^{6i(c+dx)}+13755e^{8i(c+dx)}+2590e^{10i(c+dx)}+280e^{12i(c+dx)}\right)+45045e^{7i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{107520d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/107520*I)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-240 - 2064*E^((2*I)*(c + d*
x)) - 9008*E^((4*I)*(c + d*x)) - 40784*E^((6*I)*(c + d*x)) + 13755*E^((8*I)
```

```
*(c + d*x)) + 2590*E^((10*I)*(c + d*x)) + 280*E^((12*I)*(c + d*x))) + 45045
*E^((7*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d*E^((6*I)*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.05, size = 389, normalized size = 1.33

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-61440i\left(\cos^8(dx+c)\right)-61440\sin(dx+c)\left(\cos^7(dx+c)\right)-6656i\left(\cos^6(dx+c)\right)-73216\sin(dx+c)\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/430080/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-61440*I*cos(d*
x+c)^8-61440*sin(d*x+c)*cos(d*x+c)^7-6656*I*cos(d*x+c)^6-73216*sin(d*x+c)*c
os(d*x+c)^5+45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)
*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*2^(1/2))-13728*I*cos(d*x+c)^4+45045*sin(d*x+c)*arctan(1/2*(-I
*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+45045*I*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(
d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-96096*sin(d*x+c)*cos(d
*x+c)^3-60060*I*cos(d*x+c)^2-180180*sin(d*x+c)*cos(d*x+c))/a
```

Maxima [A]
time = 0.49, size = 246, normalized size = 0.84

$$\frac{i\left(45045\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)+\frac{4\left(45045\left(i\,a\tan(dx+c)+a\right)^6a-240240\left(i\,a\tan(dx+c)+a\right)^5a^2+396396\left(i\,a\tan(dx+c)+a\right)^4a^3-164736\left(i\,a\tan(dx+c)+a\right)^3a^4-36608\left(i\,a\tan(dx+c)+a\right)^2a^5-19968\left(i\,a\tan(dx+c)+a\right)a^6-15360\,a^7\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{13}{2}}-6\left(i\,a\tan(dx+c)+a\right)^{\frac{11}{2}}a+12\left(i\,a\tan(dx+c)+a\right)^{\frac{9}{2}}a^2-8\left(i\,a\tan(dx+c)+a\right)^{\frac{7}{2}}a^3}\right)}{430080\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/430080*I*(45045*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(45045*(I*a*
tan(d*x + c) + a)^6*a - 240240*(I*a*tan(d*x + c) + a)^5*a^2 + 396396*(I*a*t
an(d*x + c) + a)^4*a^3 - 164736*(I*a*tan(d*x + c) + a)^3*a^4 - 36608*(I*a*t
an(d*x + c) + a)^2*a^5 - 19968*(I*a*tan(d*x + c) + a)*a^6 - 15360*a^7)/((I*
a*tan(d*x + c) + a)^(13/2) - 6*(I*a*tan(d*x + c) + a)^(11/2)*a + 12*(I*a*ta
n(d*x + c) + a)^(9/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(7/2)*a^3))/(a*d)
```

Fricas [A]
time = 0.39, size = 315, normalized size = 1.08

$$\frac{\left(-45045i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+45045i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(7i\,dx+7i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-280i\,e^{(14i\,dx+14i\,c)}-2870i\,e^{(12i\,dx+12i\,c)}-16345i\,e^{(10i\,dx+10i\,c)}+27029i\,e^{(8i\,dx+8i\,c)}+49792i\,e^{(6i\,dx+6i\,c)}+11072i\,e^{(4i\,dx+4i\,c)}+2304i\,e^{(2i\,dx+2i\,c)}+240i\right)\right)e^{(-7i\,dx-7i\,c)}}{215040\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/215040*(-45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(4*(sqrt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-280*I*e^(14*I*d*x + 14*I*c) - 2870*I*e^(12*I*d*x + 12*I*c) - 16345*I*e^(10*I*d*x + 10*I*c) + 27029*I*e^(8*I*d*x + 8*I*c) + 49792*I*e^(6*I*d*x + 6*I*c) + 11072*I*e^(4*I*d*x + 4*I*c) + 2304*I*e^(2*I*d*x + 2*I*c) + 240*I))*e^(-7*I*d*x - 7*I*c)/(a*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^6}{\sqrt{a+a\tan(c+d\,x)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2),x)

[Out] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2), x)

3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+d}{15d(a+ia\tan(c+}$$

```
[Out] 256/6435*I*a^4*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+64/715*I*a^3*sec(d*x
+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+8/65*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c
))^(5/2)+2/15*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((256*I)/6435)*a^4*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((6
4*I)/715)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((8*I)/65
)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/15)*a*Sec[
c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{5}(4a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{65}(32a^2)\int \frac{}{(a+}$$

$$= \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}} + \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^9(c}{15d(a+ia\tan(}$$

$$= \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}} + \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}} + \frac{8ia^2\sec^9}{65d(a+ia\,\mathrm{ta}}$$

Mathematica [A]
time = 0.74, size = 95, normalized size = 0.65

$$\frac{2\sec^8(c+dx)(510\cos(c+dx)+731\cos(3(c+dx))+3i(90\sin(c+dx)+233\sin(3(c+dx))))(i\cos(4(c+dx))+\sin(4(c+dx)))}{6435d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^8*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] + (3*I)*(90*Sin[
c + d*x] + 233*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]))/
(6435*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.48, size = 154, normalized size = 1.05

method	result
default	$\frac{2\left(2048i\left(\cos^8(dx+c)\right)+2048\sin(dx+c)\left(\cos^7(dx+c)\right)-256i\left(\cos^6(dx+c)\right)+768\sin(dx+c)\left(\cos^5(dx+c)\right)-80i\left(\cos^4(dx+c)\right)+560\sin(da\right.}{6435d\cos(dx+c)^7a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/6435/d*(2048*I*cos(d*x+c)^8+2048*sin(d*x+c)*cos(d*x+c)^7-256*I*cos(d*x+c)
^6+768*sin(d*x+c)*cos(d*x+c)^5-80*I*cos(d*x+c)^4+560*sin(d*x+c)*cos(d*x+c)^
3-42*I*cos(d*x+c)^2+462*sin(d*x+c)*cos(d*x+c)-429*I)*(a*(I*sin(d*x+c)+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^7/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(115) = 230.
time = 0.47, size = 608, normalized size = 4.14

$$-\frac{2\left(-1241i\sqrt{a}-\frac{5194\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{6090i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2490\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14430i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33618\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{13442i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{18590\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{18590\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{13442i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{33618\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{14430i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{2490\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{6090i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{5194\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{1241i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{6435\left(a-\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(-1241*I*sqrt(a) - 5194*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 6
090*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2490*sqrt(a)*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 - 14430*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 33618*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13442*I*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18590*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 18590*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13442*I*
sqrt(a)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 33618*sqrt(a)*sin(d*x + c)^
11/(cos(d*x + c) + 1)^11 + 14430*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 2490*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 6090*I*sqrt(a)
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 5194*sqrt(a)*sin(d*x + c)^15/(cos(
d*x + c) + 1)^15 + 1241*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*sq
rt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)/((a - 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*sqrt(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A]
time = 0.50, size = 153, normalized size = 1.04

$$-\frac{256\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(-715i\,e^{(6i\,dx+6i\,c)}-390i\,e^{(4i\,dx+4i\,c)}-120i\,e^{(2i\,dx+2i\,c)}-16i\right)}{6435\,\left(ade^{(14i\,dx+14i\,c)}+7\,ade^{(12i\,dx+12i\,c)}+21\,ade^{(10i\,dx+10i\,c)}+35\,ade^{(8i\,dx+8i\,c)}+35\,ade^{(6i\,dx+6i\,c)}+21\,ade^{(4i\,dx+4i\,c)}+7\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-715*I*e^(6*I*d*x + 6*
I*c) - 390*I*e^(4*I*d*x + 4*I*c) - 120*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a*d*e
^(14*I*d*x + 14*I*c) + 7*a*d*e^(12*I*d*x + 12*I*c) + 21*a*d*e^(10*I*d*x + 1
0*I*c) + 35*a*d*e^(8*I*d*x + 8*I*c) + 35*a*d*e^(6*I*d*x + 6*I*c) + 21*a*d*e
^(4*I*d*x + 4*I*c) + 7*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{9}\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**9/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^9/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 8.98, size = 301, normalized size = 2.05

$$\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{9\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{11\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}+\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{13\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{15\,a\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*256i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4) - (exp(- c*1i
- d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*768i)/(11*a*d*(exp(c*2i + d*x*2i) + 1)^5) + (exp(- c*1i - d*x*1i)*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*768
i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c*1i - d*x*1i)*(a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*a*d*
(exp(c*2i + d*x*2i) + 1)^7)
```

3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 64/693*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+16/99*I*a^2*sec(d*x+c)
^7/d/(a+I*a*tan(d*x+c))^(5/2)+2/11*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(3
/2)
```

Rubi [A]
time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/693)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((16*
I)/99)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/11)*a
*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{11}(8a)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{99}(32a^2)\int \frac{}{(a+}$$

$$= \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}} + \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c}{11d(a+ia\tan(}$$

Mathematica [A]
time = 0.49, size = 77, normalized size = 0.70

$$\frac{2\sec^6(c+dx)(44+107\cos(2(c+dx))+91i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))}{693d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(44 + 107*Cos[2*(c + d*x)] + (91*I)*Sin[2*(c + d*x)])*(I*
Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(693*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.94, size = 127, normalized size = 1.15

method	result
default	$\frac{2(256i(\cos^6(dx+c))+256\sin(dx+c)(\cos^5(dx+c))-32i(\cos^4(dx+c))+96\sin(dx+c)(\cos^3(dx+c))-10i(\cos^2(dx+c))+70\sin(dx+c)\,\mathrm{c}}{693d\cos(dx+c)^5a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693/d*(256*I*cos(d*x+c)^6+256*sin(d*x+c)*cos(d*x+c)^5-32*I*cos(d*x+c)^4+9
6*sin(d*x+c)*cos(d*x+c)^3-10*I*cos(d*x+c)^2+70*sin(d*x+c)*cos(d*x+c)-63*I)*
(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(86) = 172.
time = 0.41, size = 474, normalized size = 4.31

$$-\frac{2\left(-151i\sqrt{a} - \frac{542\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} + \frac{484i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{22\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{627i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{1452\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{1452\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{627i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{22\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{484i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{542\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{151i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{693\left(a - \frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/693*(-151*I*sqrt(a) - 542*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 484*
I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 22*sqrt(a)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 627*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
452*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1452*sqrt(a)*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 + 627*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 22*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 484*I*sqrt(a)*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 - 542*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 151*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*sqrt(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/((a -
 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1)
 + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A]
time = 0.43, size = 116, normalized size = 1.05

$$-\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-99i\,e^{(4i\,dx+4i\,c)}-44i\,e^{(2i\,dx+2i\,c)}-8i\right)}{693\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-99*I*e^(4*I*d*x + 4*I*c
) - 44*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8
*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) +
 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^7(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^7/sqrt(I*a*tan(d*x + c) + a), x)

Mupad [B]
time = 6.16, size = 105, normalized size = 0.95

$$\frac{64\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,44\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,99\mathrm{i}+8\mathrm{i}\right)}{693\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(1/2)),x)

[Out] (64*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*44i + exp(c*4i + d*x*4i)*99i + 8 i))/(693*a*d*(exp(c*2i + d*x*2i) + 1)^5)

3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 8/35*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+2/7*I*a*sec(d*x+c)^5/d/(
a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/
7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}} + \frac{1}{7}(4a)\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$
$$= \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.33, size = 65, normalized size = 0.89

$$-\frac{2\sec^3(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-9i+5\tan(c+dx))}{35d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]^3*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-9*I + 5*Tan[c
+ d*x]))/(35*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.80, size = 100, normalized size = 1.37

method	result	si
default	$\frac{2\left(16i\left(\cos^4(dx+c)\right)+16\sin(dx+c)\left(\cos^3(dx+c)\right)-2i\left(\cos^2(dx+c)\right)+6\sin(dx+c)\cos(dx+c)-5i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{35d\cos(dx+c)^3 a}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(16*I*cos(d*x+c)^4+16*sin(d*x+c)*cos(d*x+c)^3-2*I*cos(d*x+c)^2+6*sin
(d*x+c)*cos(d*x+c)-5*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(
d*x+c)^3/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(57) = 114$.
time = 0.39, size = 340, normalized size = 4.66

$$-\frac{2\left(-9i\sqrt{a}-\frac{26\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{26\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{35\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(-9*I*sqrt(a) - 26*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 14*I*sqr
t(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 14*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt
(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 26*sqrt(a)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*sqrt(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/(
(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A]
time = 0.44, size = 79, normalized size = 1.08

$$-\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(2i\,dx+2i\,c)}-2i\right)}{35\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*
d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^5(c+dx)}{\sqrt{ia\,(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 9.26, size = 91, normalized size = 1.25

$$\frac{16\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,7\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{35\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (16*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*7i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(35*a*d*(exp(c*2i + d*
x*2i) + 1)^3)
```

3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{3574\}$

$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.20, size = 40, normalized size = 1.14

$$\frac{2\sec(c+dx)(i+\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

[Out] (2*Sec[c + d*x]*(I + Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.
time = 0.82, size = 73, normalized size = 2.09

method	result	size
default	$\frac{2\left(2 i \left(\cos^{2}\left(d x +c \right)\right)+2 \sin \left(d x +c \right) \cos \left(d x +c \right)-i \right) \sqrt{\frac{a \left(i \sin \left(d x +c \right)+\cos \left(d x +c \right)\right)}{\cos \left(d x +c \right)}}}{3 d \cos \left(d x +c \right) a}$	73

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3/d*(2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)-I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)/a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.
time = 0.38, size = 206, normalized size = 5.89

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{i\sqrt{a}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{3\left(a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -2/3*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/((a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))

Fricas [A]
time = 0.37, size = 40, normalized size = 1.14

$$\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a*d*e^(2*I*d*x + 2*I*c) +
a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 1.02, size = 98, normalized size = 2.80

$$\frac{2\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}{3\,a\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2)*(cos(c + d*x)*1i + sin(c + d*x) + cos(3*c + 3*d*x)*1i + sin(3*c + 3
*d*x)))/(3*a*d*(cos(2*c + 2*d*x) + 1))
```

3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=52

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2)/
d/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3570, 212}

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(2i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$
$$= \frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.44, size = 70, normalized size = 1.35

$$\frac{2ie^{i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 +
E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(41) = 82.
time = 0.84, size = 137, normalized size = 2.63

method	result	size
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sin(dx+c)\sqrt{2}}{d(i\sin(dx+c)+\cos(dx+c)-1)a}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)/(I*sin(d*x+c)+cos(d*x+c)-1)*2
^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(39) = 78$.
time = 0.44, size = 149, normalized size = 2.87

$$-\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)+\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 1/
2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

```
[Out] 3/8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d*2^
(1/2)/a^(1/2)+1/2*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c)*
(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$,
Rules used = {3583, 3571, 3570, 212}

$$-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}+\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/4)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*
x]]) - (((3*I)/4)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{3\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{4a}$$

$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{3}{8}\int \frac{}{\sqrt{a}}$$

$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{(3i)\mathrm{Subst}}{}$$

$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d} + \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos}{}$$

Mathematica [A]
time = 0.61, size = 96, normalized size = 0.79

$$\frac{\sec(c+dx)\left(3i\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-i(1+\cos(2(c+dx))+3i\sin(2(c+dx)))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*((3*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]] - I*(1 + Cos[2*(c + d*x)] + (3*I)*Sin[2*(c + d*x)])))/(8*d*S
qrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(97) = 194.
time = 1.19, size = 319, normalized size = 2.61

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)+3i\sqrt{2}\,\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)+3*I*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(d*x+c))/a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs. 2(91) = 182.
time = 0.58, size = 837, normalized size = 6.86

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 2*sqrt(2))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4

```
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)
*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(91) = 182$.
time = 0.38, size = 245, normalized size = 2.01

$$\frac{\left(-3i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(2i\,dx+2i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+3i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(2i\,dx+2i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}+i\right)\right)e^{(-2i\,dx-2i\,c)}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(
2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 3*I*sqrt(1/2)*a*d*sqrt(1/(a*
d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*
x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x +
4*I*c) - I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c + dx \right)}}{\sqrt{ia\left(\tan{\left(c + dx \right)} - i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(c+d\,x\right)}{\sqrt{a+a\tan \left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=193

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}+\frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\cos(c-$$

```
[Out] 35/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d
*2^(1/2)/a^(1/2)+35/96*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*cos(d*
x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-35/64*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/
2)/a/d-7/24*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]

time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3583, 3578, 3571, 3570, 212}

$$-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}+\frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((35*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*Sqrt[a]*d) + (((35*I)/96)*Cos[c + d*x])/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) + ((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((3
5*I)/64)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) - (((7*I)/24)*Cos[c
 + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{7\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{8a}$$

$$= \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad} + \frac{35}{48}\int \frac{}{\sqrt{a}}$$

$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{a+}}{24ad}$$

$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+}}{64ad}$$

$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+}}{64ad}$$

$$= \frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,\sqrt{a}\,d} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{}{4d\sqrt{}}$$

Mathematica [A]

time = 0.75, size = 117, normalized size = 0.61

$$\frac{\sec(c+dx)\left(-41i+105i\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-43i\cos(2(c+dx))-2i\cos(4(c+dx))+133\sin(2(c+dx))+14\sin(4(c+dx))\right)}{384d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(-41*I + (105*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]] - (43*I)*Cos[2*(c + d*x)] - (2*I)*Cos[4*(c + d*x)]
 + 133*Sin[2*(c + d*x)] + 14*Sin[4*(c + d*x)]))/(384*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(156) = 312.
time = 0.94, size = 346, normalized size = 1.79

method	result
default	$\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(192i(\cos^5(dx+c))+192\sin(dx+c)(\cos^4(dx+c))+105i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(d}{2\sin(dx}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(192*I*cos(d*x+c)^5+
192*sin(d*x+c)*cos(d*x+c)^4+105*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arct
an(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+105*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+105*I*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+56*I*cos(d*x+c)^3+280*cos(d*x+c)^2*s
in(d*x+c)-420*I*cos(d*x+c))/a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. 2(146) = 292.
time = 0.69, size = 1938, normalized size = 10.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1536*(4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c) -
 3*sqrt(2)*sin(4*d*x + 4*c) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4*d*x
 + 4*c) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))
*sqrt(a) + 12*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c)
- 12*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - sqrt(
2)*sin(4*d*x + 4*c) - 12*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 24*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1)) + (sqrt(2)*cos(4*d*x + 4*c) + 12*sqrt(2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) - I*sqrt(2)*sin(4*d*x + 4*c) - 12*I*sqrt(2)*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 24*sqrt(2))*sin(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 105*(2*sqrt(2)*arctan2((
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1) - I*
sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
```

```
4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a*d)
```

Fricas [**A**]
time = 0.38, size = 267, normalized size = 1.38

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,de^{(2i\,dx+2i\,c)}+i\,d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)+105i\sqrt{\frac{1}{2}}\,ad\sqrt{\frac{1}{ad^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,de^{(2i\,dx+2i\,c)}-i\,d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(8i\,dx+8i\,c)}-88i\,e^{(6i\,dx+6i\,c)}-41i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+6i\right)\right)e^{(-4i\,dx-4i\,c)}}{384\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(-105*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*
(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 105*I*sqrt(1/2)*a*d*sq
rt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*
I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I
)*e^(-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(
8*I*d*x + 8*I*c) - 88*I*e^(6*I*d*x + 6*I*c) - 41*I*e^(4*I*d*x + 4*I*c) + 45
*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}$$

```
[Out] -16/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d+24/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/
d-4/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^6/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^7
/d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}-\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^4*d) + (((24*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^5*d) - (((4*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^6
*d) + (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{3/2}-12a^2(a+x)^{5/2}+6a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d} + \frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d} - \frac{4i(a+ia\tan(c}{3a^6d}$$

Mathematica [A]
time = 0.85, size = 110, normalized size = 0.94

$$\frac{2i\sec^6(c+dx)(\cos(4(c+dx))+i\sin(4(c+dx)))(39i+494i\cos(2(c+dx))+215\sec(c+dx)\sin(3(c+dx))+110\tan(c+dx))}{1155ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (((2*I)/1155)*Sec[c + d*x]^6*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*(39*I
+ (494*I)*Cos[2*(c + d*x)] + 215*Sec[c + d*x]*Sin[3*(c + d*x)] + 110*Tan[c
+ d*x]))/(a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.82, size = 117, normalized size = 1.00

method	result
default	$\frac{2\left(-256i\left(\cos^5(dx+c)\right)+256\sin(dx+c)\left(\cos^4(dx+c)\right)-32i\left(\cos^3(dx+c)\right)+160\left(\cos^2(dx+c)\right)\sin(dx+c)-245i\cos(dx+c)-105\sin(dx+c)\right)}{1155d\cos(dx+c)^5a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/1155/d*(-256*I*cos(d*x+c)^5+256*sin(d*x+c)*cos(d*x+c)^4-32*I*cos(d*x+c)^3
+160*cos(d*x+c)^2*sin(d*x+c)-245*I*cos(d*x+c)-105*sin(d*x+c))*(a*(I*sin(d*x
+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5/a^2
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.65

$$\frac{2i\left(105\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-770\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a+1980\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2-1848\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a^3\right)}{1155\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*I*(105*(I*a*tan(d*x + c) + a)^(11/2) - 770*(I*a*tan(d*x + c) + a)^(9
/2)*a + 1980*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 1848*(I*a*tan(d*x + c) + a)
^(5/2)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.37, size = 149, normalized size = 1.27

$$-\frac{64\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(16i\,e^{(11i\,dx+11i\,c)}+88i\,e^{(9i\,dx+9i\,c)}+198i\,e^{(7i\,dx+7i\,c)}+231i\,e^{(5i\,dx+5i\,c)}\right)}{1155\,\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -64/1155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(11*I*d*x + 11*I
*c) + 88*I*e^(9*I*d*x + 9*I*c) + 198*I*e^(7*I*d*x + 7*I*c) + 231*I*e^(5*I*d
*x + 5*I*c))/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*e^(8*I*d*x + 8*I*c) + 1
0*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x + 4*I*c) + 5*a^2*d*e^(2*I
*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{8}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 7.64, size = 370, normalized size = 3.16

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^2\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1155\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{385\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{33\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2
56i)/(33*a^2*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1155*a^2*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*128i)/(385*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(231*a^2*d
*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(e
xp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1155*a^2*d) - ((a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*a^2*d*(exp(c*2i
 + d*x*2i) + 1)^5)
```

3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}$$

```
[Out] -8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d+8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d-
2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3568, 45\}$

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^3*d) + (((8*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^4*d) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^5*d
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(4a^2\sqrt{a+x} - 4a(a+x)^{3/2} + (a+x)^{5/2}\right) dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3 d} + \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4 d} - \frac{2i(a+ia\tan(c+}{7a^5 d}$$

Mathematica [A]
time = 0.40, size = 92, normalized size = 1.05

$$-\frac{2\sec^5(c+dx)(28+43\cos(2(c+dx))-27i\sin(2(c+dx)))(\cos(3(c+dx))+i\sin(3(c+dx)))}{105ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sec[c + d*x]^5*(28 + 43*Cos[2*(c + d*x)] - (27*I)*Sin[2*(c + d*x)])*(Co
s[3*(c + d*x)] + I*Sin[3*(c + d*x)]))/(105*a*d*(-I + Tan[c + d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.78, size = 90, normalized size = 1.02

method	result	size
default	$-\frac{2\left(32i\left(\cos^3(dx+c)\right)-32\left(\cos^2(dx+c)\right)\sin(dx+c)+39i\cos(dx+c)+15\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{105d\cos(dx+c)^3a^2}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/d*(32*I*cos(d*x+c)^3-32*cos(d*x+c)^2*sin(d*x+c)+39*I*cos(d*x+c)+15*s
in(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/a^2
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.66

$$-\frac{2i\left(15\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}} - 84\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a + 140\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^2\right)}{105\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/105*I*(15*(I*a*tan(d*x + c) + a)^(7/2) - 84*(I*a*tan(d*x + c) + a)^(5/2)
*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.38, size = 108, normalized size = 1.23

$$-\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(7i\,dx+7i\,c)}+28i\,e^{(5i\,dx+5i\,c)}+35i\,e^{(3i\,dx+3i\,c)}\right)}{105\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -16/105*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(7*I*d*x + 7*I*c)
+ 28*I*e^(5*I*d*x + 5*I*c) + 35*I*e^(3*I*d*x + 3*I*c))/(a^2*d*e^(6*I*d*x +
6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 6.68, size = 242, normalized size = 2.75

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{105\,a^2\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{35\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
6i)/(7*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(105*a^2*d*(exp(c*2i + d*x*2
i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
1))^(1/2)*16i)/(35*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(105*a^2*d)
```

3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

```
[Out] -4*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) + (((2*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}} - \sqrt{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= -\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d} + \frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 1.40

$$\frac{2i\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))(5i+\tan(c+dx))}{3ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(5*I + Ta
n[c + d*x]))/(a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.73, size = 61, normalized size = 1.07

method	result	size
default	$-\frac{2(5i\cos(dx+c)+\sin(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{3d\cos(dx+c)a^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(5*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)/cos(d*x+c)/a^2
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.67

$$\frac{2i\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}} - 6\sqrt{i\,a\tan(dx+c)+a}\,a\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*((I*a*tan(d*x + c) + a)^(3/2) - 6*sqrt(I*a*tan(d*x + c) + a)*a)/(a^3*
d)
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.18

$$-\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(3i\,dx+3i\,c)}+3i\,e^{(i\,dx+i\,c)}\right)}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(3*I*d*x + 3*I*c) + 3
*I*e^(I*d*x + I*c))/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 3.66, size = 85, normalized size = 1.49

$$-\frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,5\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{3\,a^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] -(2*(cos(2*c + 2*d*x)*5i + sin(2*c + 2*d*x) + 5i)*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(3*a^2*d*(cos(2*c +
 2*d*x) + 1))
```

3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{ad} \\ &= \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}} \end{aligned}$$

Mathematica [A]
time = 0.20, size = 27, normalized size = 1.00

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.24, size = 24, normalized size = 0.89

method	result	size
derivativedivides	$\frac{2i}{ad\sqrt{a+ia\tan\left(dx+c\right)}}$	24
default	$\frac{2i}{ad\sqrt{a+ia\tan\left(dx+c\right)}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.78

$$\frac{2i}{\sqrt{i\,a\tan\left(dx+c\right)+a}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*I/(sqrt(I*a*tan(d*x + c) + a)*a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.
time = 0.39, size = 49, normalized size = 1.81

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(-I
*d*x - I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 0.24, size = 67, normalized size = 2.48

$$\frac{\left(\cos\left(c+d\,x\right)^2\,2\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{a\,\left(2\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos\left(c+d\,x\right)^2}}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((sin(2*c + 2*d*x) + cos(c + d*x)^2*2i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c
+ d*x)^2))/(2*cos(c + d*x)^2))^(1/2))/(a^2*d)
```

3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}+\frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] -7/32*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(
1/2)+7/16*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+7/20*I*a/d/(a+I*a*tan(d*x+c))^(5/2
)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+7/24*I/d/(a+I*a*t
an(d*x+c))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}}+\frac{7i}{24d(a+ia\tan(c+dx))^{3/2}}+\frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-7*I)/16)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(3/2)*d) + (((7*I)/20)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + ((7*I)/24)/(d*(
a + I*a*Tan[c + d*x])^(3/2)) + ((7*I)/16)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(7ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7}}\right.}{4a}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}$$

$$= -\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{}{2d(a-}$$

Mathematica [A]
time = 1.11, size = 142, normalized size = 0.81

$$-\frac{ie^{-5i(c+dx)}\left(-6-38e^{2i(c+dx)}-148e^{4i(c+dx)}-101e^{6i(c+dx)}+15e^{8i(c+dx)}+105e^{5i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec(c+dx)}{480ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/480*I)*(-6 - 38*E^((2*I)*(c + d*x)) - 148*E^((4*I)*(c + d*x)) - 101*E^
((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) + 105*E^((5*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x])/(a*d*E^((5*
I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(137) = 274.
time = 0.84, size = 362, normalized size = 2.07

method	result
default	$\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(384i(\cos^6(dx+c))+384\sin(dx+c)(\cos^5(dx+c))+32i(\cos^4(dx+c))-105i\cos(dx+c)\arctan\left(\frac{(-i\cos}{2\sin(}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/960/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(384*I*cos(d*x+c)^6+
384*sin(d*x+c)*cos(d*x+c)^5+32*I*cos(d*x+c)^4-105*I*cos(d*x+c)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-105*I*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+224*sin(d*x+c)*cos(d*x
+c)^3-105*sin(d*x+c)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)+140*I*cos(d*x+c)^2+420*sin(d*x+c)*cos(d*x+c))/a^2
```

Maxima [A]
time = 0.50, size = 153, normalized size = 0.87

$$\frac{i\left(\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(105(ia\tan(dx+c)+a)^3-140(ia\tan(dx+c)+a)^2a-56(ia\tan(dx+c)+a)a^2-48a^3\right)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}-2(ia\tan(dx+c)+a)^{\frac{5}{2}}a}\right)}{960ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/960*I*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(105*(I*a*tan(d*x
 + c) + a)^3 - 140*(I*a*tan(d*x + c) + a)^2*a - 56*(I*a*tan(d*x + c) + a)*a
^2 - 48*a^3)/((I*a*tan(d*x + c) + a)^(7/2) - 2*(I*a*tan(d*x + c) + a)^(5/2)
*a))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(126) = 252$.
time = 0.37, size = 294, normalized size = 1.68

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+105i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(8i\,dx+8i\,c)}+101i\,e^{(6i\,dx+6i\,c)}+148i\,e^{(4i\,dx+4i\,c)}+38i\,e^{(2i\,dx+2i\,c)}+6i\right)\right)e^{(-5i\,dx-5i\,c)}}{480\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(
sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 105
*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sq
rt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x + 8*I*c) + 101*I*e^(6*I*d*x
+ 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) + 38*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(
-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{99i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}+\frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+$$

```
[Out] -99/512*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2
^(1/2)+99/256*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+99/224*I*a^2/d/(a+I*a*tan(d*x+
c))^(7/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-11/16*I
*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+99/320*I*a/d/(a+I*a*tan(
d*x+c))^(5/2)+33/128*I/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{99i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}}+\frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}}+\frac{33i}{128d(a+ia\tan(c+dx))^{3/2}}+\frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-99*I)/256)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*a^(3/2)*d) + (((99*I)/224)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (((11*I
)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (((99*
I)/320)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + ((33*I)/128)/(d*(a + I*a*Tan[
c + d*x])^(3/2)) + ((99*I)/256)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(ia^5)\,\text{Subst}\Big(\int \frac{1}{(a-x)^3(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\Big)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{(11ia^4)\,\text{Subst}\Big(\int \frac{1}{(a-x)^2(a+}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{11ia}{16d(a-ia\tan(c+dx))(}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/}}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/}}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/}}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/}}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/}}$$

$$= -\frac{99i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d} + \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{}{4d(}$$

Mathematica [A]
time = 1.38, size = 168, normalized size = 0.68

$$-\frac{ie^{-7i(c+dx)}\left(-40-328e^{2i(c+dx)}-1304e^{4i(c+dx)}-4584e^{6i(c+dx)}-2833e^{8i(c+dx)}+805e^{10i(c+dx)}+70e^{12i(c+dx)}+3465e^{7i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec(c+dx)}{17920ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/17920*I)*(-40 - 328*E^((2*I)*(c + d*x)) - 1304*E^((4*I)*(c + d*x)) - 4
584*E^((6*I)*(c + d*x)) - 2833*E^((8*I)*(c + d*x)) + 805*E^((10*I)*(c + d*x
)) + 70*E^((12*I)*(c + d*x)) + 3465*E^((7*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x])/(a*d*E^((7*I)*(c + d*x))
*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]

time = 1.01, size = 389, normalized size = 1.57

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-10240i\left(\cos^{8}(dx+c)\right)-10240\sin(dx+c)\left(\cos^{7}(dx+c)\right)-512i\left(\cos^{6}(dx+c)\right)-5632\sin(dx+c)\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35840/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-10240*I*cos(d*x
+c)^8-10240*sin(d*x+c)*cos(d*x+c)^7-512*I*cos(d*x+c)^6-5632*sin(d*x+c)*cos(
d*x+c)^5+3465*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x
+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
cos(d*x+c)*2^(1/2)-1056*I*cos(d*x+c)^4+3465*I*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+3465*sin(d*x+c)*arctan(1/2*(-I*cos(d*
x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-7392*sin(d*x+c)*cos(d*x+c)^3-
4620*I*cos(d*x+c)^2-13860*sin(d*x+c)*cos(d*x+c))/a^2
```

Maxima [A]

time = 0.50, size = 207, normalized size = 0.83

$$\frac{i\left(\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(3465\,(ia\tan(dx+c)+a)^{5}-11550\,(ia\tan(dx+c)+a)^{4}a+7392\,(ia\tan(dx+c)+a)^{3}a^{2}+2112\,(ia\tan(dx+c)+a)^{2}a^{3}+1408\,(ia\tan(dx+c)+a)a^{4}+1280\,a^{5}\right)}{(ia\tan(dx+c)+a)^{\frac{11}{2}}-4\,(ia\tan(dx+c)+a)^{\frac{9}{2}}a+4\,(ia\tan(dx+c)+a)^{\frac{7}{2}}a^{2}}\right)}{35840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35840*I*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3465*(I*a*tan
(d*x + c) + a)^5 - 11550*(I*a*tan(d*x + c) + a)^4*a + 7392*(I*a*tan(d*x + c
) + a)^3*a^2 + 2112*(I*a*tan(d*x + c) + a)^2*a^3 + 1408*(I*a*tan(d*x + c) +
 a)*a^4 + 1280*a^5)/((I*a*tan(d*x + c) + a)^(11/2) - 4*(I*a*tan(d*x + c) +
a)^(9/2)*a + 4*(I*a*tan(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

Fricas [A]

time = 0.39, size = 316, normalized size = 1.27

$$\frac{\left(-3465i\sqrt{\frac{1}{2}}a^{2}d\sqrt{\frac{1}{a^{3}d^{2}}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3465i\sqrt{\frac{1}{2}}a^{2}d\sqrt{\frac{1}{a^{3}d^{2}}}e^{(7i\,dx+7i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^{3}d^{2}}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-70i\,e^{(12i\,dx+12i\,c)}-805i\,e^{(10i\,dx+10i\,c)}+2833i\,e^{(8i\,dx+8i\,c)}+4584i\,e^{(6i\,dx+6i\,c)}+1304i\,e^{(4i\,dx+4i\,c)}+328i\,e^{(2i\,dx+2i\,c)}+40i\right)\right)e^{(-7i\,dx-7i\,c)}}{17920\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17920*(-3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(
4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-70*I*e^(12*I*d*x + 12*I*c) - 805*I*e^(10
*I*d*x + 10*I*c) + 2833*I*e^(8*I*d*x + 8*I*c) + 4584*I*e^(6*I*d*x + 6*I*c)
+ 1304*I*e^(4*I*d*x + 4*I*c) + 328*I*e^(2*I*d*x + 2*I*c) + 40*I))*e^(-7*I*d
*x - 7*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{715i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,a^{3/2}d}+\frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan}$$

```
[Out] -715/4096*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d
*2^(1/2)+715/2048*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+715/1152*I*a^3/d/(a+I*a*ta
n(d*x+c))^(9/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(9/2)-5
/16*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)-65/64*I*a^4/d/(a-
I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+715/1792*I*a^2/d/(a+I*a*tan(d*x+c)
)^(7/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+715/3072*I/d/(a+I*a*tan(d*x+
c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{715i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,a^{3/2}d}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}-\frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}}+\frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}}+\frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}}+\frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}}+\frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-715*I)/2048)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(3/2)*d) + (((715*I)/1152)*a^3)/(d*(a + I*a*Tan[c + d*x])^(9/2)) -
((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) - (((
5*I)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (
((65*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) +
(((715*I)/1792)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((143*I)/512)*a)/(
d*(a + I*a*Tan[c + d*x])^(5/2)) + ((715*I)/3072)/(d*(a + I*a*Tan[c + d*x])^
(3/2)) + ((715*I)/2048)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(ia^7)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{(5ia^6)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a\cdots}\right.}{}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5\cdots}{16d(a-ia\tan(c+dx)\cdots}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5\cdots}{16d(a-ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx)\cdots}$$

$$= -\frac{715i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,a^{3/2}d} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \cdots$$

Mathematica [A]

time = 2.26, size = 203, normalized size = 0.63

$$\frac{ie^{-8i(c+dx)}\left(112+1136e^{2i(c+dx)}+5440e^{4i(c+dx)}+17344e^{6i(c+dx)}+57632e^{8i(c+dx)}+33301e^{10i(c+dx)}-13209e^{12i(c+dx)}-1974e^{14i(c+dx)}-168e^{16i(c+dx)}-45045e^{9i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{129024ad\left(1+e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/129024)*(112 + 1136*E^((2*I)*(c + d*x)) + 5440*E^((4*I)*(c + d*x)) + 17
344*E^((6*I)*(c + d*x)) + 57632*E^((8*I)*(c + d*x)) + 33301*E^((10*I)*(c +
d*x)) - 13209*E^((12*I)*(c + d*x)) - 1974*E^((14*I)*(c + d*x)) - 168*E^((16
*I)*(c + d*x)) - 45045*E^((9*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Ar
cSinh[E^(I*(c + d*x))]))/(a*d*E^((8*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))
*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.12, size = 416, normalized size = 1.30

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(114688i\left(\cos^{10}(dx+c)\right)+114688\left(\cos^{9}(dx+c)\right)\sin(dx+c)+4096i\left(\cos^{8}(dx+c)\right)+61440\sin(dx+c)\left(\cos^{7}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/516096/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(114688*I*cos(d*x
+c)^10+114688*cos(d*x+c)^9*sin(d*x+c)+4096*I*cos(d*x+c)^8+61440*sin(d*x+c)*
cos(d*x+c)^7+6656*I*cos(d*x+c)^6+73216*sin(d*x+c)*cos(d*x+c)^5+13728*I*cos(
d*x+c)^4-45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*
x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*cos(d*x+c)*2^(1/2)-45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*ar
ctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))+96096*sin(d*x+c)*cos(d*x+c)^3-45045*sin(d*x+c)*arctan(1
/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+60060*I*cos(d*x+
c)^2+180180*sin(d*x+c)*cos(d*x+c))/a^2
```

Maxima [A]
time = 0.50, size = 261, normalized size = 0.81

$$\frac{i\left(\frac{45045\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i a\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(45045\,(i a\tan(dx+c)+a)^{7}-240240\,(i a\tan(dx+c)+a)^{6}a+396396\,(i a\tan(dx+c)+a)^{5}a^{2}-164736\,(i a\tan(dx+c)+a)^{4}a^{3}-36608\,(i a\tan(dx+c)+a)^{3}a^{4}-19968\,(i a\tan(dx+c)+a)^{2}a^{5}-15360\,(i a\tan(dx+c)+a)a^{6}-14336\,a^{7}\right)}{(i a\tan(dx+c)+a)^{\frac{15}{2}}-6\,(i a\tan(dx+c)+a)^{\frac{13}{2}}a+12\,(i a\tan(dx+c)+a)^{\frac{11}{2}}a^{2}-8\,(i a\tan(dx+c)+a)^{\frac{9}{2}}a^{3}}\right)}{516096\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/516096*I*(45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(45045*(I*a*
tan(d*x + c) + a)^7 - 240240*(I*a*tan(d*x + c) + a)^6*a + 396396*(I*a*tan(d
*x + c) + a)^5*a^2 - 164736*(I*a*tan(d*x + c) + a)^4*a^3 - 36608*(I*a*tan(d
*x + c) + a)^3*a^4 - 19968*(I*a*tan(d*x + c) + a)^2*a^5 - 15360*(I*a*tan(d*
```

```
x + c) + a)*a^6 - 14336*a^7)/((I*a*tan(d*x + c) + a)^(15/2) - 6*(I*a*tan(d*
x + c) + a)^(13/2)*a + 12*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 8*(I*a*tan(d*
x + c) + a)^(9/2)*a^3))/(a*d)
```

Fricas [A]
time = 0.40, size = 338, normalized size = 1.05

$$\frac{\left(-45045 i \sqrt{\frac{1}{2}}\, a^{2} d \sqrt{\frac{1}{a^{3} d^{2}}}\, e^{\left(9 i d x + 9 i c\right)} \log\left(4 \left(\sqrt{2} \sqrt{\frac{1}{2}} \left(a^{2} d e^{\left(2 i d x + 2 i c\right)} + a^{2} d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \sqrt{\frac{1}{a^{3} d^{2}}} + a e^{\left(i d x + i c\right)}\right) e^{\left(-i d x - i c\right)}\right) + 45045 i \sqrt{\frac{1}{2}}\, a^{2} d \sqrt{\frac{1}{a^{3} d^{2}}}\, e^{\left(9 i d x + 9 i c\right)} \log\left(-4 \left(\sqrt{2} \sqrt{\frac{1}{2}} \left(a^{2} d e^{\left(2 i d x + 2 i c\right)} + a^{2} d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \sqrt{\frac{1}{a^{3} d^{2}}} - a e^{\left(i d x + i c\right)}\right) e^{\left(-i d x - i c\right)}\right) + \sqrt{2} \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(-168 i e^{\left(16 i d x + 16 i c\right)} - 1974 i e^{\left(14 i d x + 14 i c\right)} - 13209 i e^{\left(12 i d x + 12 i c\right)} + 33301 i e^{\left(10 i d x + 10 i c\right)} + 57632 i e^{\left(8 i d x + 8 i c\right)} + 17344 i e^{\left(6 i d x + 6 i c\right)} + 5440 i e^{\left(4 i d x + 4 i c\right)} + 1136 i e^{\left(2 i d x + 2 i c\right)} + 112 i\right)\right) e^{\left(-9 i d x - 9 i c\right)}}{258048\, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/258048*(-45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*lo
g(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqr
t(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-168*I*e^(16*I*d*x + 16*I*c) - 1974*I*
e^(14*I*d*x + 14*I*c) - 13209*I*e^(12*I*d*x + 12*I*c) + 33301*I*e^(10*I*d*x
 + 10*I*c) + 57632*I*e^(8*I*d*x + 8*I*c) + 17344*I*e^(6*I*d*x + 6*I*c) + 54
40*I*e^(4*I*d*x + 4*I*c) + 1136*I*e^(2*I*d*x + 2*I*c) + 112*I))*e^(-9*I*d*x
 - 9*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{6}\left(c + dx\right)}{\left(ia\left(\tan\left(c + dx\right) - i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^6}{(a+a\tan (c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 256/12155*I*a^4*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+64/1105*I*a^3*sec
(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+8/85*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan
(d*x+c))^(7/2)+2/17*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/12155)*a^4*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (
((64*I)/1105)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((8*
I)/85)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/17)*
a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{17}(12a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx \\
&= \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{85}\left(32a^2\right)\int \frac{}{(a+} \\
&= \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(}{17d(a+ia\tan} \\
&= \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}} + \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\,\mathrm{s}}{85d(a+i\alpha}
\end{aligned}
$$

Mathematica [A]
time = 1.04, size = 108, normalized size = 0.73

$$\frac{2\sec^9(c+dx)(i\cos(4(c+dx))+\sin(4(c+dx)))(475i-2242i\cos(2(c+dx))+1089\sec(c+dx)\sin(3(c+dx))+374\tan(c+dx))}{12155ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(475*I - (2242*I)
*Cos[2*(c + d*x)] + 1089*Sec[c + d*x]*Sin[3*(c + d*x)] + 374*Tan[c + d*x]))
/(12155*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 3.52, size = 171, normalized size = 1.16

method	result
default	$\frac{2\left(4096i\left(\cos^9(dx+c)\right)+4096\sin(dx+c)\left(\cos^8(dx+c)\right)-512i\left(\cos^7(dx+c)\right)+1536\sin(dx+c)\left(\cos^6(dx+c)\right)-160i\left(\cos^5(dx+c)\right)+1120\sin(}{12155d\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/12155/d*(4096*I*cos(d*x+c)^9+4096*sin(d*x+c)*cos(d*x+c)^8-512*I*cos(d*x+c
)^7+1536*sin(d*x+c)*cos(d*x+c)^6-160*I*cos(d*x+c)^5+1120*sin(d*x+c)*cos(d*x
+c)^4-84*I*cos(d*x+c)^3+924*cos(d*x+c)^2*sin(d*x+c)-1573*I*cos(d*x+c)-715*s
in(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(115) = 230.
time = 0.53, size = 764, normalized size = 5.20

$$-\frac{2\left(-1767i\sqrt{a}-\frac{6854\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{2088i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16438\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5661i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56984\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{13328i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{129336\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7514i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{156468\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{156468\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{7514i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{129336\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{13328i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{56984\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{5661i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{16438\sqrt{a}\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{2088i\sqrt{a}\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}-\frac{6854\sqrt{a}\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}+\frac{1767i\sqrt{a}\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}}{12155\left(a^2-\frac{10a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{120a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{210a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{252a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{210a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{120a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{45a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{10a^2\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}+\frac{a^2\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(-1767*I*sqrt(a) - 6854*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
2088*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16438*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 5661*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 56984*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 13328*I*sqrt(a)
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 129336*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 7514*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15646
8*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 156468*sqrt(a)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - 7514*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 - 129336*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 13328*I*sqrt
(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 56984*sqrt(a)*sin(d*x + c)^15/(
cos(d*x + c) + 1)^15 + 5661*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 16438*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2088*I*sqrt(a)*sin
(d*x + c)^18/(cos(d*x + c) + 1)^18 - 6854*sqrt(a)*sin(d*x + c)^19/(cos(d*x
+ c) + 1)^19 + 1767*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
^(3/2)/((a^2 - 10*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 120*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^2*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^2
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^2*sin(d*x + c)^16/(cos(d*x +
c) + 1)^16 - 10*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^2*sin(d*x + c
)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [**A**]
time = 0.44, size = 184, normalized size = 1.25

$$-\frac{512 \sqrt{2} \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \left(-1105i\,e^{(6i\,dx+6i\,c)}-510i\,e^{(4i\,dx+4i\,c)}-136i\,e^{(2i\,dx+2i\,c)}-16i\right)}{12155 \left(a^2 d e^{(16i\,dx+16i\,c)}+8\,a^2 d e^{(14i\,dx+14i\,c)}+28\,a^2 d e^{(12i\,dx+12i\,c)}+56\,a^2 d e^{(10i\,dx+10i\,c)}+70\,a^2 d e^{(8i\,dx+8i\,c)}+56\,a^2 d e^{(6i\,dx+6i\,c)}+28\,a^2 d e^{(4i\,dx+4i\,c)}+8\,a^2 d e^{(2i\,dx+2i\,c)}+a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -512/12155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1105*I*e^(6*I*d*x +
6*I*c) - 510*I*e^(4*I*d*x + 4*I*c) - 136*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^2
*d*e^(16*I*d*x + 16*I*c) + 8*a^2*d*e^(14*I*d*x + 14*I*c) + 28*a^2*d*e^(12*I
*d*x + 12*I*c) + 56*a^2*d*e^(10*I*d*x + 10*I*c) + 70*a^2*d*e^(8*I*d*x + 8*I
*c) + 56*a^2*d*e^(6*I*d*x + 6*I*c) + 28*a^2*d*e^(4*I*d*x + 4*I*c) + 8*a^2*d
*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 9.86, size = 301, normalized size = 2.05

$$\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{11\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1536i}{13\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6}+\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{5\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^7}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{17\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*512i)/(11*a^2*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*1536i)/(13*a^2*d*(exp(c*2i + d*x*2i) + 1)^6) + (exp(- c*1i - d*x
*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*512i)/(5*a^2*d*(exp(c*2i + d*x*2i) + 1)^7) - (exp(- c*1i - d*x*1i)*(a -
(a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(
17*a^2*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 64/1287*I*a^3*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+16/143*I*a^2*sec(d*x+
c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+2/13*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^
(5/2)
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((64*I)/1287)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((16
*I)/143)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/13)
*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{13}(8a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{143}(32a^2)\int \frac{}{(a}$$

$$= \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}} + \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9}{13d(a+ia\,\mathrm{ta}}$$

Mathematica [A]
time = 0.63, size = 92, normalized size = 0.84

$$\frac{2\sec^8(c+dx)(52+151\cos(2(c+dx))+135i\sin(2(c+dx)))(\cos(3(c+dx))-i\sin(3(c+dx)))}{1287ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^8*(52 + 151*Cos[2*(c + d*x)] + (135*I)*Sin[2*(c + d*x)])*(C
os[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(1287*a*d*(-I + Tan[c + d*x])*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.00, size = 144, normalized size = 1.31

method	result
default	$\frac{2\left(512i\left(\cos^7(dx+c)\right)+512\sin(dx+c)\left(\cos^6(dx+c)\right)-64i\left(\cos^5(dx+c)\right)+192\sin(dx+c)\left(\cos^4(dx+c)\right)-20i\left(\cos^3(dx+c)\right)+140\left(\cos^2(dx+c\right.\right.}{1287d\cos(dx+c)^6a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1287/d*(512*I*cos(d*x+c)^7+512*sin(d*x+c)*cos(d*x+c)^6-64*I*cos(d*x+c)^5+
192*sin(d*x+c)*cos(d*x+c)^4-20*I*cos(d*x+c)^3+140*cos(d*x+c)^2*sin(d*x+c)-2
25*I*cos(d*x+c)-99*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/
2)/cos(d*x+c)^6/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(86) = 172$.
time = 0.47, size = 626, normalized size = 5.69

$$-\frac{2\left(-203i\sqrt{a}-\frac{678\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1802\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3614\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{858i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6578\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{6578\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{858i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{3614\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{26i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{1802\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{2i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{678\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{203i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}}{1287\left(a^2-\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{56a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1287*(-203*I*sqrt(a) - 678*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I
*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1802*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 26*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3
614*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 858*I*sqrt(a)*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 - 6578*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 6578*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 858*I*sqrt(a)*sin(d*x
 + c)^10/(cos(d*x + c) + 1)^10 - 3614*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 + 26*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1802*sqrt(a
)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2*I*sqrt(a)*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 678*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 203*
I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x +
c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [**A**]
time = 0.43, size = 143, normalized size = 1.30

$$-\frac{128\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(-143i\,e^{(4i\,dx+4i\,c)}-52i\,e^{(2i\,dx+2i\,c)}-8i\right)}{1287\,\left(a^2de^{(12i\,dx+12i\,c)}+6\,a^2de^{(10i\,dx+10i\,c)}+15\,a^2de^{(8i\,dx+8i\,c)}+20\,a^2de^{(6i\,dx+6i\,c)}+15\,a^2de^{(4i\,dx+4i\,c)}+6\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -128/1287*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-143*I*e^(4*I*d*x + 4*
I*c) - 52*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2
*d*e^(10*I*d*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d
*x + 6*I*c) + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) +
a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^9\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 8.30, size = 105, normalized size = 0.95

$$\frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,52\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,143\mathrm{i}+8\mathrm{i}\right)}{1287\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] (128*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*52i + exp(c*4i + d*x*4i)*143i +
 8i))/(1287*a^2*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 8/63*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+2/9*I*a*sec(d*x+c)^7/d/(
a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/
9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{9}(4a)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\ &= \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 80, normalized size = 1.10

$$\frac{2\sec^5(c+dx)(i\cos(2(c+dx))+\sin(2(c+dx)))(-11i+7\tan(c+dx))}{63ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^5*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-11*I + 7*Tan[c
+ d*x]))/(63*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.83, size = 117, normalized size = 1.60

method	result
default	$\frac{2\left(32i\left(\cos^5(dx+c)\right)+32\sin(dx+c)\left(\cos^4(dx+c)\right)-4i\left(\cos^3(dx+c)\right)+12\left(\cos^2(dx+c)\right)\sin(dx+c)-17i\cos(dx+c)-7\sin(dx+c)\right)\sqrt{\frac{a(i\sin}{}}}{63d\cos(dx+c)^4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/63/d*(32*I*cos(d*x+c)^5+32*sin(d*x+c)*cos(d*x+c)^4-4*I*cos(d*x+c)^3+12*co
s(d*x+c)^2*sin(d*x+c)-17*I*cos(d*x+c)-7*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(57) = 114.
time = 0.43, size = 488, normalized size = 6.68

$$-\frac{2\left(-11i\sqrt{a}-\frac{30\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{12i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{86\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{108\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{108\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{9i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{86\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{12i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{30\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{11i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}}{63\left(a^2-\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(-11*I*sqrt(a) - 30*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 12*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 86*sqrt(a)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 9*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 108*sqr
t(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 108*sqrt(a)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 86*sqrt
(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 12*I*sqrt(a)*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 - 30*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 11*I
*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 6*a^2*
```

```
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*si
n(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [A]
time = 0.42, size = 102, normalized size = 1.40

$$-\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(2i\,dx+2i\,c)}-2i\right)}{63\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -32/63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e
^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^7\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 6.44, size = 91, normalized size = 1.25

$$\frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,9\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{63\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] (32*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*9i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(63*a^2*d*(exp(c*2i +
d*x*2i) + 1)^4)
```

3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{3574\}$

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.33, size = 59, normalized size = 1.69

$$\frac{2\sec^3(c+dx)(1-i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^3*(1 - I*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(29) = 58$.
time = 0.77, size = 90, normalized size = 2.57

method	result	size
default	$\frac{2\left(4i\left(\cos^{3}\left(dx+c\right)\right)+4\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-3i\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}}{5d\cos\left(dx+c\right)^{2}a^{2}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(4*I*cos(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)-3*I*cos(d*x+c)-sin(d*x+c)
)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(27) = 54$.
time = 0.39, size = 350, normalized size = 10.00

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{2i\sqrt{a}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{6\sqrt{a}\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}-\frac{6\sqrt{a}\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}+\frac{2i\sqrt{a}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}-\frac{2\sqrt{a}\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}+\frac{i\sqrt{a}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}\right)\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)^{\frac{3}{2}}}{5\left(a^{2}-\frac{4a^{2}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{6a^{2}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}-\frac{4a^{2}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+\frac{a^{2}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}\right)d\left(-\frac{2i\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 6*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 -
4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.
time = 0.36, size = 59, normalized size = 1.69

$$\frac{8i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{5\left(a^{2}de^{(4i\,dx+4i\,c)}+2\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 8/5*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(3/2), x)

Mupad [B]
time = 1.70, size = 139, normalized size = 3.97

$$\frac{(\cos(d\,x)-\sin(d\,x)\,1\mathrm{i})\,(\cos(c)-\sin(c)\,1\mathrm{i})\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(2\,\cos(2\,c+2\,d\,x)+\cos(4\,c+4\,d\,x)+1-\sin(2\,c+2\,d\,x)\,2\mathrm{i}-\sin(4\,c+4\,d\,x)\,1\mathrm{i})\,4\mathrm{i}}{5\,a^2\,d\,(4\,\cos(2\,c+2\,d\,x)+\cos(4\,c+4\,d\,x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2)),x)

[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(2*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) - sin(2*c + 2*d*x)*2i - sin(4*c + 4*d*x)*1i + 1)*4i)/(5*a^2*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))

3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(3/2)/d-2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3572, 3570, 212}

$$\frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{2\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(4i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{ad}$$

$$= \frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.92, size = 101, normalized size = 1.17

$$\frac{8e^{3i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (8*E^((3*I)*(c + d*x))*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(71) = 142.
time = 0.81, size = 157, normalized size = 1.83

method	result
default	$\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sin(dx+c)+i\cos(dx+c)}{d(i\sin(dx+c)+\cos(dx+c)-1)a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+I*cos(d*x+c)-I+sin(d
*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(67) = 134$.
time = 0.61, size = 813, normalized size = 9.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a) + 4*(I*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(67) = 134$.
time = 0.37, size = 196, normalized size = 2.28

$$\frac{-i\sqrt{2}\,a^2d\sqrt{\frac{1}{a^3d^2}}\log\left(-\frac{8\left(\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{ad}\right)+i\sqrt{2}\,a^2d\sqrt{\frac{1}{a^3d^2}}\log\left(-\frac{8\left(\left(-i\,ade^{(2i\,dx+2i\,c)}-i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{ad}\right)-2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((I*a*d*e^(2*I*d*x + 2*I*c) + I*
a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I
*c)/(a*d)) + I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((-I*a*d*e^(2*I*d*x +
 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e
^(-I*d*x - I*c)/(a*d)) - 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^
2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 1/4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(3
/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3570, 212}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
```

]

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a}$$

$$= \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{i\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2ad}$$

$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.76, size = 95, normalized size = 1.09

$$\frac{\left(2+\frac{2e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)\sec(c+dx)}{4ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((2 + (2*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[1
 + E^((2*I)*(c + d*x))])*Sec[c + d*x])/(4*a*d*(-I + Tan[c + d*x])*Sqrt[a +
I*a*Tan[c + d*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(68) = 136.
time = 0.73, size = 318, normalized size = 3.66

method	result
default	$\left(i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)+\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx-}{2\sin(dx+c}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+si
n(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+
c)*2^(1/2)+2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*sin(d*x+c)+I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^
(1/2)+8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c))*(a*(I*sin(
d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(64) = 128.
time = 0.39, size = 246, normalized size = 2.83

$$\frac{\left(i \sqrt{\frac{1}{2}} a^{2} d \sqrt{\frac{1}{a^{3} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\frac{\left(\sqrt{2} \sqrt{\frac{1}{2}} \left(i a d e^{\left(2 i d x + 2 i c\right)} + i a d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \sqrt{\frac{1}{a^{3} d^{2}}} + i\right) e^{\left(-i d x - i c\right)}}{a d}\right) - i \sqrt{\frac{1}{2}} a^{2} d \sqrt{\frac{1}{a^{3} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\frac{\left(\sqrt{2} \sqrt{\frac{1}{2}} \left(-i a d e^{\left(2 i d x + 2 i c\right)} - i a d\right) \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \sqrt{\frac{1}{a^{3} d^{2}}} + i\right) e^{\left(-i d x - i c\right)}}{a d}\right) + \sqrt{2} \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(i e^{\left(2 i d x + 2 i c\right)} + i\right)\right) e^{\left(-2 i d x - 2 i c\right)}}{4 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*s
qrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a^3*d^2)) + I)*e^(-I*d*x - I*c)/(a*d)) - I*sqrt(1/2)*a^2*d*sqrt
(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x
 + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + I)
*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(
2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c + dx\right)}{\left(ia \left(\tan\left(c + dx\right) - i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}-\frac{15i\cos(c$$

```
[Out] 15/64*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(3/2)/d*2^(1/2)+5/16*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)-15/32*I*cos(
d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/4*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3583, 3571, 3570, 212}

$$\frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d}-\frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((15*I)/32)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]
)^(3/2)) + (((5*I)/16)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
15*I)/32)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{8a}$$

$$= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{15\int\cos(c+dx)}{}$$

$$= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)}{3}$$

$$= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)}{3}$$

$$= \frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{}{16}$$

Mathematica [A]
time = 1.23, size = 120, normalized size = 0.76

$$\frac{\sec(c+dx)\left(\frac{30e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(-9+6\cos(2(c+dx))+10i\sin(2(c+dx)))\right)}{64ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((30*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))
]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(-9 + 6*Cos[2*(c + d*x)] + (10*I)*Sin
[2*(c + d*x)])))/(64*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(126) = 252$.
time = 0.79, size = 346, normalized size = 2.20

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(64i(\cos^5(dx+c))+15i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(64*I*cos(d*x+c)^5+1
5*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^
(1/2)+64*sin(d*x+c)*cos(d*x+c)^4+15*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*2^(1/2)+8*I*cos(d*x+c)^3+15*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+40*cos(d*x+c)^2*sin(d*x+
c)-60*I*cos(d*x+c))/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(118) = 236$.
time = 0.65, size = 1821, normalized size = 11.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/256*(36*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - s
qrt(2)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + (sqrt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arcta
n2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 4*(cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
```

```
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*((7*I*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 8*I
*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (7*sqrt(2
)*cos(4*d*x + 4*c) - 7*I*sqrt(2)*sin(4*d*x + 4*c) + 8*sqrt(2))*sin(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan2(
(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4
)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1) - I
*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1
/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1
))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
```

```
+ 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(118) = 236$.
time = 0.37, size = 270, normalized size = 1.72

$$\frac{\left(-15i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,ade^{(2i\,dx+2i\,c)}+i\,ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{16\,ad}\right)+15i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,ade^{(2i\,dx+2i\,c)}-i\,ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{16\,ad}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(6i\,dx+6i\,c)}+i\,e^{(4i\,dx+4i\,c)}+11i\,e^{(2i\,dx+2i\,c)}+2i\right)\right)e^{(-4i\,dx-4i\,c)}}{64\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(-15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/1
6*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 15*I*sqrt(
1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/16*(sqrt(2)*sqrt(1
/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-8*I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c) + 11*I*e^
(2*I*d*x + 2*I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c+dx \right)}}{\left(ia\left(\tan{\left(c+dx \right)}-i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(c+dx)}{\sqrt{2}\ \sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}\ a^{3/2}d}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}+\frac{3}{16ad\sqrt{}}$$

```
[Out] 105/512*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/
a^(3/2)/d*2^(1/2)+35/128*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*c
os(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(1/2)-105/256*I*cos(d*x+c)*(a+I*a*tan(d*
x+c))^(1/2)/a^2/d-7/32*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/6*I*
cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.25, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3583, 3578, 3571, 3570, 212}

$$\frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(c+dx)}{\sqrt{2}\ \sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}\ a^{3/2}d}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((105*I)/256)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^(3/2)) + (((35*I)/128)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
 + (((3*I)/16)*Cos[c + d*x]^3)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((105*I)
/256)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) - (((7*I)/32)*Cos[c
+ d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{21\int \cos^3(c+dx)$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{}}{32}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+}{16ad\sqrt{a+ia\tan}}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+}{16ad\sqrt{a+ia\tan}}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+}{16ad\sqrt{a+ia\tan}}$$

$$= \frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}\,a^{3/2}d} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{}{12}$$

Mathematica [A]
time = 1.87, size = 145, normalized size = 0.62

$$\frac{\sec(c+dx)\left(\frac{630e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(158\cos(2(c+dx)) + 8\cos(4(c+dx)) + 3i(55i + 86\sin(2(c+dx)) + 8\sin(4(c+dx))))\right)}{1536ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((630*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(158*Cos[2*(c + d*x)] + 8*Cos[4*(c +
 d*x)] + (3*I)*(55*I + 86*Sin[2*(c + d*x)] + 8*Sin[4*(c + d*x)]))))/(1536*a
*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.84, size = 373, normalized size = 1.60

method	result

default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(1024i(\cos^7(dx+c))+1024\sin(dx+c)(\cos^6(dx+c))+64i(\cos^5(dx+c))+315i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3072/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(1024*I*cos(d*x+c)^7+1024*sin(d*x+c)*cos(d*x+c)^6+64*I*cos(d*x+c)^5+315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+576*sin(d*x+c)*cos(d*x+c)^4+168*I*cos(d*x+c)^3+315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+315*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+840*cos(d*x+c)^2*sin(d*x+c)-1260*I*cos(d*x+c))/a^2

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(178) = 356$.
time = 0.71, size = 2632, normalized size = 11.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/6144*(8*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-4*I*sqrt(2)*cos(6*d*x + 6*c) - 45*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*sqrt(2)*sin(6*d*x + 6*c) - 45*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + (4*sqrt(2)*cos(6*d*x + 6*c) + 45*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*I*sqrt(2)*sin(6*d*x + 6*c) - 45*I*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*(((-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-I*sqrt(2)*cos(6*d*x

```
 + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c)))^2 + 2*(-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c)
)*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - I*sqrt(2)*cos(6*d*
x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))) + 1)) + (I*sqrt(2)*cos(6*d*x + 6*c) + 18*I*sqrt(2)*cos(2/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*I*sqrt(2)*cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + sqrt(2)*sin(6*d*x + 6*c) + 18*sqrt(2)*sin
(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*sqrt(2)*sin(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 64*I*sqrt(2))*cos(1/2*arctan2(si
n(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c))) + 1)) + ((sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2
)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2
+ (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*(sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(5/2*arctan2(s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c))) + 1)) - (sqrt(2)*cos(6*d*x + 6*c) + 18*sqrt(
2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*sqrt(2)*cos(1/
3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - I*sqrt(2)*sin(6*d*x + 6*c)
 - 18*I*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 24*I
*sqrt(2)*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 64*sqrt(2))
*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(
1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 315*(2*sq
rt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), c
os(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*
arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*
x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
+ 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*co
s(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin
```

```
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*
x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2
*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^...
```

Fricas [A]
time = 0.38, size = 292, normalized size = 1.25

$$\frac{\left(-315i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,ade^{(2i\,dx+2i\,c)}+i\,ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{128\,ad}\right)+315i\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,ade^{(2i\,dx+2i\,c)}-i\,ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{128\,ad}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-16i\,e^{(10i\,dx+10i\,c)}-224i\,e^{(8i\,dx+8i\,c)}-43i\,e^{(6i\,dx+6i\,c)}+215i\,e^{(4i\,dx+4i\,c)}+58i\,e^{(2i\,dx+2i\,c)}+8i\right)\right)e^{(-6i\,dx-6i\,c)}}{1536\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(-315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-1
05/128*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 315*I
*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-105/128*(sqrt(2
)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(10*I*d*x + 10*I*c) - 224*I*e^(8*I*d*x + 8
*I*c) - 43*I*e^(6*I*d*x + 6*I*c) + 215*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*
d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{16i(a+ia\tan(c+d}{11a^8d}$$

```
[Out] -32/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d+64/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/
d-16/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/d+16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^8/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}+\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-32*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d) + (((64*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^6*d) - (((16*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^
7*d) + (((16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^8*d) - (((2*I)/13)*(a
 + I*a*Tan[c + d*x])^(13/2))/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^9d}$$
$$= -\frac{i\text{Subst}\left(\int \left(16a^4(a+x)^{3/2}-32a^3(a+x)^{5/2}+24a^2(a+x)^{7/2}-8a(a+x)^{9/}\right.\right.}{a^9d}$$
$$= -\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{16i(a+ia\tan(c}{3a^7d}$$

Mathematica [A]
time = 0.95, size = 116, normalized size = 0.79

$$\frac{2\sec^9(c+dx)(2288i+4264i\cos(2(c+dx))+3131i\cos(4(c+dx))+2600\sin(2(c+dx))+2875\sin(4(c+dx)))(\cos(5(c+dx))+i\sin(5(c+dx)))}{15015a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(2288*I + (4264*I)*Cos[2*(c + d*x)] + (3131*I)*Cos[4*(c +
 d*x)] + 2600*Sin[2*(c + d*x)] + 2875*Sin[4*(c + d*x)])*(Cos[5*(c + d*x)] +
 I*Sin[5*(c + d*x)]))/(15015*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Maple [A]
time = 0.84, size = 127, normalized size = 0.87

method	result
default	$-\frac{2\left(4096i\left(\cos^6(dx+c)\right)-4096\sin(dx+c)\left(\cos^5(dx+c)\right)+512i\left(\cos^4(dx+c)\right)-2560\sin(dx+c)\left(\cos^3(dx+c)\right)+6230i\left(\cos^2(dx+c)\right)+3990\,\mathrm{s}\right.}{15015d\cos(dx+c)^6a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15015/d*(4096*I*cos(d*x+c)^6-4096*sin(d*x+c)*cos(d*x+c)^5+512*I*cos(d*x+
c)^4-2560*sin(d*x+c)*cos(d*x+c)^3+6230*I*cos(d*x+c)^2+3990*sin(d*x+c)*cos(d
*x+c)-1155*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6/a
^3
```

Maxima [A]
time = 0.29, size = 94, normalized size = 0.64

$$-\frac{2i\left(1155\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}-10920\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a+40040\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^2-68640\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^3+48048\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^4\right)}{15015\,a^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15015*I*(1155*(I*a*tan(d*x + c) + a)^(13/2) - 10920*(I*a*tan(d*x + c) +
a)^(11/2)*a + 40040*(I*a*tan(d*x + c) + a)^(9/2)*a^2 - 68640*(I*a*tan(d*x +
 c) + a)^(7/2)*a^3 + 48048*(I*a*tan(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Fricas [A]
time = 0.45, size = 175, normalized size = 1.20

$$-\frac{128\,\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(128i\,e^{(13i\,dx+13i\,c)}+832i\,e^{(11i\,dx+11i\,c)}+2288i\,e^{(9i\,dx+9i\,c)}+3432i\,e^{(7i\,dx+7i\,c)}+3003i\,e^{(5i\,dx+5i\,c)}\right)}{15015\,\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -128/15015*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(13*I*d*x + 1
3*I*c) + 832*I*e^(11*I*d*x + 11*I*c) + 2288*I*e^(9*I*d*x + 9*I*c) + 3432*I*
e^(7*I*d*x + 7*I*c) + 3003*I*e^(5*I*d*x + 5*I*c))/(a^3*d*e^(12*I*d*x + 12*I
*c) + 6*a^3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3
*d*e^(6*I*d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 8.76, size = 434, normalized size = 2.97

$$-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,16384i}{15015\,a^3\,d}-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,8192i}{15015\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,2048i}{5005\,a^3\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{3003\,a^3\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{429\,a^3\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}+\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1792i}{143\,a^3\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}-\frac{\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{13\,a^3\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(5/2)),x)

[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1792i)/(143*a^3*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(15015*a^3*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(5005*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3003*a^3*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(429*a^3*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(15015*a^3*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6)

3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}$$

```
[Out] -16/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d+24/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/
d-12/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/
d
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^4*d) + (((24*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^5*d) - (((12*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
6*d) + (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(8a^3\sqrt{a+x} - 12a^2(a+x)^{3/2} + 6a(a+x)^{5/2} - (a+x)^{7/2}\right)\,dx, x\right.}{a^7d}$$
$$= -\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d} + \frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d} - \frac{12i(a+ia\tan(c}{7a^6d}$$

Mathematica [A]
time = 0.73, size = 108, normalized size = 0.92

$$\frac{2\sec^6(c+dx)(\cos(4(c+dx))+i\sin(4(c+dx)))(77i+242i\cos(2(c+dx))+89\sec(c+dx)\sin(3(c+dx))+54\tan(c+dx))}{315a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^6*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*(77*I + (242*I)*C
os[2*(c + d*x)] + 89*Sec[c + d*x]*Sin[3*(c + d*x)] + 54*Tan[c + d*x]))/(315
*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.79, size = 100, normalized size = 0.85

method	result
default	$-\frac{2\left(128i\left(\cos^4(dx+c)\right)-128\sin(dx+c)\left(\cos^3(dx+c)\right)+226i\left(\cos^2(dx+c)\right)+130\sin(dx+c)\cos(dx+c)-35i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c)}{\cos(dx+c)}}}{315d\cos(dx+c)^4a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/d*(128*I*cos(d*x+c)^4-128*sin(d*x+c)*cos(d*x+c)^3+226*I*cos(d*x+c)^2
+130*sin(d*x+c)*cos(d*x+c)-35*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)/cos(d*x+c)^4/a^3
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.65

$$\frac{2i\left(35\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}} - 270\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}a + 756\,(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}a^2 - 840\,(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}a^3\right)}{315\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/315*I*(35*(I*a*tan(d*x + c) + a)^(9/2) - 270*(I*a*tan(d*x + c) + a)^(7/2)
*a + 756*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 840*(I*a*tan(d*x + c) + a)^(3/2
)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.38, size = 134, normalized size = 1.15

$$-\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(9i\,dx+9i\,c)}+72i\,e^{(7i\,dx+7i\,c)}+126i\,e^{(5i\,dx+5i\,c)}+105i\,e^{(3i\,dx+3i\,c)}\right)}{315\left(a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(9*I*d*x + 9*I*c)
 + 72*I*e^(7*I*d*x + 7*I*c) + 126*I*e^(5*I*d*x + 5*I*c) + 105*I*e^(3*I*d*x
+ 3*I*c))/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*
d*e^(4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^8\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 6.68, size = 306, normalized size = 2.62

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{315\,a^3\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(5/2)),x)

[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3
2i)/(9*a^3*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a^3*d*(exp(c*2i + d*x*
2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*64i)/(105*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*a^3*d*(exp
(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*512i)/(315*a^3*d)
```

3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

```
[Out] -8*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d-2/
5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) + (((8*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^4*d) - (((2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^5d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right)dx, x, ia\tan(c+dx)\right)}{a^5d}$$
$$= -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d} - \frac{2i(a+ia\tan(c+d}{5a^5d}$$

Mathematica [A]
time = 0.49, size = 94, normalized size = 1.09

$$\frac{2\sec^5(c+dx)(20i+23i\cos(2(c+dx))+7\sin(2(c+dx)))(\cos(3(c+dx))+i\sin(3(c+dx)))}{15a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^5*(20*I + (23*I)*Cos[2*(c + d*x)] + 7*Sin[2*(c + d*x)])*(Co
s[3*(c + d*x)] + I*Sin[3*(c + d*x)]))/(15*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.76, size = 73, normalized size = 0.85

method	result	size
default	$-\frac{2\left(46i\left(\cos^2(dx+c)\right)+14\sin(dx+c)\cos(dx+c)-3i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos(dx+c)^2a^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(46*I*cos(d*x+c)^2+14*sin(d*x+c)*cos(d*x+c)-3*I)*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/a^3
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.67

$$-\frac{2i\left(3\left(ia\tan(dx+c)+a\right)^{\frac{5}{2}} - 20\left(ia\tan(dx+c)+a\right)^{\frac{3}{2}}a + 60\sqrt{ia\tan(dx+c)+a}\,a^2\right)}{15\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*I*(3*(I*a*tan(d*x + c) + a)^(5/2) - 20*(I*a*tan(d*x + c) + a)^(3/2)*a
 + 60*sqrt(I*a*tan(d*x + c) + a)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.37, size = 93, normalized size = 1.08

$$-\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(5i\,dx+5i\,c)}+20i\,e^{(3i\,dx+3i\,c)}+15i\,e^{(i\,dx+i\,c)}\right)}{15\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(5*I*d*x + 5*I*c) +
20*I*e^(3*I*d*x + 3*I*c) + 15*I*e^(I*d*x + I*c))/(a^3*d*e^(4*I*d*x + 4*I*c)
 + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 1.23, size = 155, normalized size = 1.80

$$-\frac{4\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,321{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,132{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,23{}\mathrm{i}+35\,\sin\left(2\,c+2\,d\,x\right)+28\,\sin\left(4\,c+4\,d\,x\right)+7\,\sin\left(6\,c+6\,d\,x\right)+212{}\mathrm{i}\right)}{15\,a^3\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] -(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*321i + cos(4*c + 4*d*x)*132i + cos(6*c + 6*d*x)*
23i + 35*sin(2*c + 2*d*x) + 28*sin(4*c + 4*d*x) + 7*sin(6*c + 6*d*x) + 212i
))/(15*a^3*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) +
 10))
```

3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

```
[Out] 4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}+\frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (4*I)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right)dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

Mathematica [A]
time = 0.34, size = 36, normalized size = 0.65

$$\frac{6i - 2\tan(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (6*I - 2*Tan[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.81, size = 65, normalized size = 1.18

method	result	size
default	$\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(2\sin(dx+c)\cos(dx+c)+i+2i(\cos^2(dx+c)))}{d\,a^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*sin(d*x+c)*cos(d*x+c)
+I+2*I*cos(d*x+c)^2)/a^3
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.80

$$\frac{2i\left(\frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{a^2} + \frac{2}{\sqrt{i\,a\tan\left(dx+c\right)+a}\,a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*I*(sqrt(I*a*tan(d*x + c) + a)/a^2 + 2/(sqrt(I*a*tan(d*x + c) + a)*a))/(a*
d)
```

Fricas [A]
time = 0.42, size = 50, normalized size = 0.91

$$-\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{(-i\,dx-i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(2*I*d*x + 2*I*c) - I)
*e^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 0.27, size = 72, normalized size = 1.31

$$\frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+2\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 2i)*((a*(cos(2*c + 2*d*x) + si
n(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(a^3*d)
```

3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 39, normalized size = 1.34

$$\frac{2}{3a^2d(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] 2/(3*a^2*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.23, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24
default	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.72

$$\frac{2i}{3\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2/3*I/((I*a*tan(d*x + c) + a)^(3/2)*a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(21) = 42.
time = 0.38, size = 61, normalized size = 2.10

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 2*I*
e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 3.58, size = 23, normalized size = 0.79

$$\frac{2\mathrm{i}}{3\,a\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] 2i/(3*a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}+\frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+$$

```
[Out] -9/64*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(
1/2)+9/32*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+9/28*I*a/d/(a+I*a*tan(d*x+c))^(7
/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+9/40*I/d/(a+I*a
*tan(d*x+c))^(5/2)+3/16*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}}+\frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-9*I)/32)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(5/2)*d) + (((9*I)/28)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + ((9*I)/40)/(d*(
a + I*a*Tan[c + d*x])^(5/2)) + ((3*I)/16)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)
) + ((9*I)/32)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} - \frac{(9ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x}\right.}{4}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}$$

$$= -\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d} + \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{}{2d(a}$$

Mathematica [A]
time = 1.37, size = 163, normalized size = 0.80

$$-\frac{ie^{-8i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(-10-58e^{2i(c+dx)}-156e^{4i(c+dx)}-388e^{6i(c+dx)}+35e^{8i(c+dx)}\right)+315e^{7i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{4480a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/4480*I)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]
*(-10 - 58*E^((2*I)*(c + d*x)) - 156*E^((4*I)*(c + d*x)) - 388*E^((6*I)*(c
+ d*x)) + 35*E^((8*I)*(c + d*x))) + 315*E^((7*I)*(c + d*x))*ArcSinh[E^(I*(c
 + d*x))])*Sec[c + d*x]^2)/(a^2*d*E^((8*I)*(c + d*x))*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(160) = 320.
time = 0.86, size = 389, normalized size = 1.91

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(2560i\left(\cos^{8}(dx+c)\right)+2560\sin(dx+c)\left(\cos^{7}(dx+c)\right)-768i\left(\cos^{6}(dx+c)\right)+512\sin(dx+c)\left(\cos^{5}(dx+c)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4480/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(2560*I*cos(d*x+c)^
8+2560*sin(d*x+c)*cos(d*x+c)^7-768*I*cos(d*x+c)^6+512*sin(d*x+c)*cos(d*x+c)
^5+96*I*cos(d*x+c)^4-315*I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*2^(1/2))*2^(1/2)-315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2))*2^(1/2)+672*sin(d*x+c)*cos(d*x+c)^3-315*sin(d*x+c)*arct
an(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+420*I*cos(d*
x+c)^2+1260*sin(d*x+c)*cos(d*x+c))/a^3
```

Maxima [A]
time = 0.50, size = 175, normalized size = 0.86

$$\frac{i\left(\frac{4\left(315\,(i\,a\tan(dx+c)+a)^{4}-420\,(i\,a\tan(dx+c)+a)^{3}a-168\,(i\,a\tan(dx+c)+a)^{2}a^{2}-144\,(i\,a\tan(dx+c)+a)a^{3}-160\,a^{4}\right)}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a-2\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^{2}}+\frac{315\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}\right)}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4480*I*(4*(315*(I*a*tan(d*x + c) + a)^4 - 420*(I*a*tan(d*x + c) + a)^3*a
- 168*(I*a*tan(d*x + c) + a)^2*a^2 - 144*(I*a*tan(d*x + c) + a)*a^3 - 160*a
^4)/((I*a*tan(d*x + c) + a)^(9/2)*a - 2*(I*a*tan(d*x + c) + a)^(7/2)*a^2) +
 315*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*s
qrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2))/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(147) = 294.
time = 0.38, size = 305, normalized size = 1.50

$$\frac{\left(-315i\,\sqrt{\frac{1}{2}}\,a^{3}d\sqrt{\frac{1}{a^{5}d^{2}}}\,e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^{5}d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+315i\,\sqrt{\frac{1}{2}}\,a^{3}d\sqrt{\frac{1}{a^{5}d^{2}}}\,e^{(7i\,dx+7i\,c)}\log\left(-4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^{5}d^{2}}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,(-35i\,e^{(10i\,dx+10i\,c)}+353i\,e^{(8i\,dx+8i\,c)}+544i\,e^{(6i\,dx+6i\,c)}+214i\,e^{(4i\,dx+4i\,c)}+68i\,e^{(2i\,dx+2i\,c)}+10i)\right)e^{(-7i\,dx-7i\,c)}}{2240\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(-315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 31
5*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(-35*I*e^(10*I*d*x + 10*I*c) + 353*I*e^(8*I*d
*x + 8*I*c) + 544*I*e^(6*I*d*x + 6*I*c) + 214*I*e^(4*I*d*x + 4*I*c) + 68*I*
e^(2*I*d*x + 2*I*c) + 10*I))*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{143i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d}+\frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c$$

```
[Out] -143/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d
*2^(1/2)+143/512*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+143/288*I*a^2/d/(a+I*a*ta
n(d*x+c))^(9/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)-1
3/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+143/448*I*a/d/(a+I
*a*tan(d*x+c))^(7/2)+143/640*I/d/(a+I*a*tan(d*x+c))^(5/2)+143/768*I/a/d/(a+
I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{143i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}}+\frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}}+\frac{143i}{640d(a+ia\tan(c+dx))^{5/2}}+\frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-143*I)/512)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*a^(5/2)*d) + (((143*I)/288)*a^2)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((
I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (((13
*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (((1
43*I)/448)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + ((143*I)/640)/(d*(a + I*a*
Tan[c + d*x])^(5/2)) + ((143*I)/768)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (
(143*I)/512)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{(13ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+}\right.}{}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia}{16d(a-ia\tan(c+dx))(}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/}}$$

$$= -\frac{143i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}\,a^{5/2}d} + \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{}{4d}$$

Mathematica [A]

time = 1.83, size = 189, normalized size = 0.68

$$-\frac{ie^{-10i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(-280-2200e^{2i(c+dx)}-7944e^{4i(c+dx)}-18808e^{6i(c+dx)}-50584e^{8i(c+dx)}+7875e^{10i(c+dx)}+630e^{12i(c+dx)}\right)+45045e^{9i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^2(c+dx)}{645120a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/645120*I)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x)
)]*(-280 - 2200*E^((2*I)*(c + d*x)) - 7944*E^((4*I)*(c + d*x)) - 18808*E^((
6*I)*(c + d*x)) - 50584*E^((8*I)*(c + d*x)) + 7875*E^((10*I)*(c + d*x)) + 6
```

```
30*E^((12*I)*(c + d*x))) + 45045*E^((9*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x)
)])*Sec[c + d*x]^2)/(a^2*d*E^((10*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.04, size = 416, normalized size = 1.50

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-286720i\left(\cos^{10}(dx+c)\right)-286720\left(\cos^{9}(dx+c)\right)\sin(dx+c)+81920i\left(\cos^{8}(dx+c)\right)-61440\sin(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/645120/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-286720*I*cos(d
*x+c)^10-286720*cos(d*x+c)^9*sin(d*x+c)+81920*I*cos(d*x+c)^8-61440*sin(d*x+
c)*cos(d*x+c)^7-6656*I*cos(d*x+c)^6-73216*sin(d*x+c)*cos(d*x+c)^5+45045*I*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)*arctan(1/2*(-I*cos(d
*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)-13728*I*cos(d*x+c)^4+45045*sin(d*x+c)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c
)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-96096*sin(d*x+c)*cos(d*x+c)^3+45045*I*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c
)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-60060*I*cos(d
*x+c)^2-180180*sin(d*x+c)*cos(d*x+c))/a^3
```

Maxima [A]
time = 0.51, size = 229, normalized size = 0.83

$$\frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^{6}-150150\,(i\,a\tan(dx+c)+a)^{5}a+96096\,(i\,a\tan(dx+c)+a)^{4}a^{2}+27456\,(i\,a\tan(dx+c)+a)^{3}a^{3}+18304\,(i\,a\tan(dx+c)+a)^{2}a^{4}+16640\,(i\,a\tan(dx+c)+a)a^{5}+17920\,a^{6}\right)}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a-4\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^{2}+4\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^{3}}+\frac{45045\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}\right)}{645120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/645120*I*(4*(45045*(I*a*tan(d*x + c) + a)^6 - 150150*(I*a*tan(d*x + c) +
a)^5*a + 96096*(I*a*tan(d*x + c) + a)^4*a^2 + 27456*(I*a*tan(d*x + c) + a)^
3*a^3 + 18304*(I*a*tan(d*x + c) + a)^2*a^4 + 16640*(I*a*tan(d*x + c) + a)*a
^5 + 17920*a^6)/((I*a*tan(d*x + c) + a)^(13/2)*a - 4*(I*a*tan(d*x + c) + a)
^(11/2)*a^2 + 4*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 45045*sqrt(2)*log(-(sqr
t(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(
d*x + c) + a)))/a^(3/2))/(a*d)
```

Fricas [A]
time = 0.42, size = 327, normalized size = 1.18

$$\frac{\left(-45045i\,\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3de^{(2i\,dx+2i\,c)}+a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^5d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+45045i\,\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(9i\,dx+9i\,c)}\log\left(-4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^3de^{(2i\,dx+2i\,c)}+a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^5d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,(-630i\,e^{(14i\,dx+14i\,c)}-8505i\,e^{(12i\,dx+12i\,c)}+42709i\,e^{(10i\,dx+10i\,c)}+69392i\,e^{(8i\,dx+8i\,c)}+26752i\,e^{(6i\,dx+6i\,c)}+10144i\,e^{(4i\,dx+4i\,c)}+2480i\,e^{(2i\,dx+2i\,c)}+280i)\right)e^{(-9i\,dx-9i\,c)}}{322560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/322560*(-45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*lo
g(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqr
t(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-630*I*e^(14*I*d*x + 14*I*c) - 8505*I*
e^(12*I*d*x + 12*I*c) + 42709*I*e^(10*I*d*x + 10*I*c) + 69392*I*e^(8*I*d*x
+ 8*I*c) + 26752*I*e^(6*I*d*x + 6*I*c) + 10144*I*e^(4*I*d*x + 4*I*c) + 2480
*I*e^(2*I*d*x + 2*I*c) + 280*I))*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{1}}{19d(a+ia\tan}$$

```
[Out] 256/20995*I*a^4*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+64/1615*I*a^3*sec
(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+24/323*I*a^2*sec(d*x+c)^13/d/(a+I*a*
tan(d*x+c))^(9/2)+2/19*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/20995)*a^4*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (
((64*I)/1615)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
4*I)/323)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/1
9)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}} + \frac{1}{19}(12a)\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$
$$= \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}} + \frac{1}{323}(96a^2)\int \frac{}{(a}$$
$$= \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} + \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{1}}{19d(a+ia\,t}$$
$$= \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}} + \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} + \frac{24ia^2}{323d(a+}$$

Mathematica [A]
time = 1.11, size = 112, normalized size = 0.76

$$\frac{\sec^{12}(c+dx)(798\cos(c+dx)+1631\cos(3(c+dx))+13i(38\sin(c+dx)+123\sin(3(c+dx))))(-2i\cos(4(c+dx))-2\sin(4(c+dx)))}{20995a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^12*(798*Cos[c + d*x] + 1631*Cos[3*(c + d*x)] + (13*I)*(38*Sin
[c + d*x] + 123*Sin[3*(c + d*x)]))*((-2*I)*Cos[4*(c + d*x)] - 2*Sin[4*(c +
d*x)]))/(20995*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 10.81, size = 181, normalized size = 1.23

method	result
default	$\frac{2(8192i(\cos^{10}(dx+c))+8192(\cos^9(dx+c))\sin(dx+c)-1024i(\cos^8(dx+c))+3072\sin(dx+c)(\cos^7(dx+c))-320i(\cos^6(dx+c))+2240\sin}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/20995/d*(8192*I*cos(d*x+c)^10+8192*cos(d*x+c)^9*sin(d*x+c)-1024*I*cos(d*x
+c)^8+3072*sin(d*x+c)*cos(d*x+c)^7-320*I*cos(d*x+c)^6+2240*sin(d*x+c)*cos(d
*x+c)^5-168*I*cos(d*x+c)^4+1848*sin(d*x+c)*cos(d*x+c)^3-5356*I*cos(d*x+c)^2
-3640*sin(d*x+c)*cos(d*x+c)+1105*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)/cos(d*x+c)^9/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(115) = 230.
time = 0.61, size = 902, normalized size = 6.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/20995*(-2429*I*sqrt(a) - 8850*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
5122*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 45190*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 12924*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 152478*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 40470*I*sqrt(
a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 397594*sqrt(a)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - 50065*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 72
2228*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 19380*I*sqrt(a)*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 - 936700*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c
) + 1)^11 - 936700*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 19380*I*
sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 722228*sqrt(a)*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 + 50065*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 - 397594*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 40470*I*sqr
t(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 152478*sqrt(a)*sin(d*x + c)^19
/(cos(d*x + c) + 1)^19 + 12924*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)
^20 - 45190*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 5122*I*sqrt(a)*
sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 8850*sqrt(a)*sin(d*x + c)^23/(cos(d
*x + c) + 1)^23 + 2429*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(5/2)/((a^3 - 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^3*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 495*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^3*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 924*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*
a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a^3*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 220*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^3*sin
(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12*a^3*sin(d*x + c)^22/(cos(d*x + c) +
 1)^22 + a^3*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(c
os(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [A]
time = 0.58, size = 199, normalized size = 1.35

$$-\frac{1024\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1615i\,e^{(6i\,dx+6i\,c)}-646i\,e^{(4i\,dx+4i\,c)}-152i\,e^{(2i\,dx+2i\,c)}-16i\right)}{20995\,\left(a^3de^{(18i\,dx+18i\,c)}+9\,a^3de^{(16i\,dx+16i\,c)}+36\,a^3de^{(14i\,dx+14i\,c)}+84\,a^3de^{(12i\,dx+12i\,c)}+126\,a^3de^{(10i\,dx+10i\,c)}+126\,a^3de^{(8i\,dx+8i\,c)}+84\,a^3de^{(6i\,dx+6i\,c)}+36\,a^3de^{(4i\,dx+4i\,c)}+9\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1024/20995*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1615*I*e^(6*I*d*x +
 6*I*c) - 646*I*e^(4*I*d*x + 4*I*c) - 152*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^
3*d*e^(18*I*d*x + 18*I*c) + 9*a^3*d*e^(16*I*d*x + 16*I*c) + 36*a^3*d*e^(14*
I*d*x + 14*I*c) + 84*a^3*d*e^(12*I*d*x + 12*I*c) + 126*a^3*d*e^(10*I*d*x +
10*I*c) + 126*a^3*d*e^(8*I*d*x + 8*I*c) + 84*a^3*d*e^(6*I*d*x + 6*I*c) + 36
*a^3*d*e^(4*I*d*x + 4*I*c) + 9*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 11.59, size = 301, normalized size = 2.05

$$\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{13\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^6}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{5\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^7}+\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,3072i}{17\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^8}-\frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{19\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*1024i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c
*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
+ 1))^(1/2)*1024i)/(5*a^3*d*(exp(c*2i + d*x*2i) + 1)^7) + (exp(- c*1i - d*x
*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*3072i)/(17*a^3*d*(exp(c*2i + d*x*2i) + 1)^8) - (exp(- c*1i - d*x*1i)*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i
)/(19*a^3*d*(exp(c*2i + d*x*2i) + 1)^9)
```

3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 64/2145*I*a^3*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+16/195*I*a^2*sec(d*
x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+2/15*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+
c))^(7/2)
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/2145)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((
16*I)/195)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}} + \frac{1}{15}(8a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$
$$= \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}} + \frac{1}{195}(32a^2)\int \frac{}{(a}$$
$$= \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}} + \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{1}}{15d(a+ia\tan}$$

Mathematica [A]
time = 0.88, size = 94, normalized size = 0.85

$$\frac{\sec^{10}(c+dx)(60+203\cos(2(c+dx))+187i\sin(2(c+dx)))(-2i\cos(3(c+dx))-2\sin(3(c+dx)))}{2145a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^10*(60 + 203*Cos[2*(c + d*x)] + (187*I)*Sin[2*(c + d*x)])*((-
2*I)*Cos[3*(c + d*x)] - 2*Sin[3*(c + d*x)]))/(2145*a^2*d*(-I + Tan[c + d*x]
)^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.00, size = 154, normalized size = 1.40

method	result
default	$\frac{2(1024i(\cos^8(dx+c))+1024\sin(dx+c)(\cos^7(dx+c))-128i(\cos^6(dx+c))+384\sin(dx+c)(\cos^5(dx+c))-40i(\cos^4(dx+c))+280\sin(dx+}{2145d\cos(dx+c)^7a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/2145/d*(1024*I*cos(d*x+c)^8+1024*sin(d*x+c)*cos(d*x+c)^7-128*I*cos(d*x+c)
^6+384*sin(d*x+c)*cos(d*x+c)^5-40*I*cos(d*x+c)^4+280*sin(d*x+c)*cos(d*x+c)^
3-736*I*cos(d*x+c)^2-484*sin(d*x+c)*cos(d*x+c)+143*I)*(a*(I*sin(d*x+c)+cos(
d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^7/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(86) = 172.
time = 0.53, size = 764, normalized size = 6.95

$$-\frac{2\left(-263i\sqrt{a} - \frac{830\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{760i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{4270\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1085i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{11576\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{2000i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{23000\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{2470i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{33540\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{33540\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{2470i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{23000\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} + \frac{2000i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} - \frac{11576\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}} + \frac{1085i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} - \frac{4270\sqrt{a}\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}} + \frac{760i\sqrt{a}\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} - \frac{830\sqrt{a}\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}} + \frac{263i\sqrt{a}\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}}{2145\left(a^3 - \frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{45a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{120a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{210a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{252a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{210a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{120a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{45a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} - \frac{10a^3\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} + \frac{a^3\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*(-263*I*sqrt(a) - 830*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 760
*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4270*sqrt(a)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 1085*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 11576*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2000*I*sqrt(a)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 23000*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - 2470*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 33540*sqrt(a
)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 33540*sqrt(a)*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 + 2470*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2
3000*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2000*I*sqrt(a)*sin(d*x
 + c)^14/(cos(d*x + c) + 1)^14 - 11576*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c
) + 1)^15 + 1085*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 4270*sqr
t(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 760*I*sqrt(a)*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 830*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 +
 263*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 -
10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^3*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 120*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^3*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 + 210*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^3*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14 + 45*a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10
*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^3*sin(d*x + c)^20/(cos(d*x +
 c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [**A**]
time = 0.44, size = 158, normalized size = 1.44

$$-\frac{256 \sqrt{2} \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}} \left(-195 i e^{(4 i dx+4 i c)}-60 i e^{(2 i dx+2 i c)}-8 i\right)}{2145 \left(a^3 d e^{(14 i dx+14 i c)}+7 a^3 d e^{(12 i dx+12 i c)}+21 a^3 d e^{(10 i dx+10 i c)}+35 a^3 d e^{(8 i dx+8 i c)}+35 a^3 d e^{(6 i dx+6 i c)}+21 a^3 d e^{(4 i dx+4 i c)}+7 a^3 d e^{(2 i dx+2 i c)}+a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -256/2145*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-195*I*e^(4*I*d*x + 4*
I*c) - 60*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^3*d*e^(14*I*d*x + 14*I*c) + 7*a^3
*d*e^(12*I*d*x + 12*I*c) + 21*a^3*d*e^(10*I*d*x + 10*I*c) + 35*a^3*d*e^(8*I
*d*x + 8*I*c) + 35*a^3*d*e^(6*I*d*x + 6*I*c) + 21*a^3*d*e^(4*I*d*x + 4*I*c)
 + 7*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 8.75, size = 105, normalized size = 0.95

$$\frac{256\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,60\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,195\mathrm{i}+8\mathrm{i}\right)}{2145\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (256*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*60i + exp(c*4i + d*x*4i)*195i +
 8i))/(2145*a^3*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 8/99*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+2/11*I*a*sec(d*x+c)^9/d/
(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A]

time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{11}(4a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.63, size = 80, normalized size = 1.10

$$\frac{2\sec^7(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-13i+9\tan(c+dx))}{99a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^7*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-13*I + 9*Tan[c
+ d*x]))/(99*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(61) = 122.
time = 0.81, size = 127, normalized size = 1.74

method	result
default	$\frac{2\left(64i\left(\cos^6(dx+c)\right)+64\sin(dx+c)\left(\cos^5(dx+c)\right)-8i\left(\cos^4(dx+c)\right)+24\sin(dx+c)\left(\cos^3(dx+c)\right)-52i\left(\cos^2(dx+c)\right)-32\sin(dx+c)\cos(dx\right.}{99d\cos(dx+c)^5a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/99/d*(64*I*cos(d*x+c)^6+64*sin(d*x+c)*cos(d*x+c)^5-8*I*cos(d*x+c)^4+24*si
n(d*x+c)*cos(d*x+c)^3-52*I*cos(d*x+c)^2-32*sin(d*x+c)*cos(d*x+c)+9*I)*(a*(I
*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(57) = 114.
time = 0.48, size = 626, normalized size = 8.58

$$-\frac{2\left(-13i\sqrt{a}-\frac{34\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{46i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{174\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{54i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{394\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{22i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{550\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{550\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{22i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{394\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{54i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{174\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{46i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{34\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{13i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}}{99\left(a^3-\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/99*(-13*I*sqrt(a) - 34*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 46*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 174*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 54*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 394*s
qrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22*I*sqrt(a)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 - 550*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 550*
sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 22*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 394*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 54*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 174*sqrt(a)*sin(d*x +
```

```
c)^13/(cos(d*x + c) + 1)^13 + 46*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) +
 1)^14 - 34*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 13*I*sqrt(a)*si
n(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 8*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*
a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x +
 c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
+ a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x
+ c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.
time = 0.40, size = 117, normalized size = 1.60

$$-\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(2i\,dx+2i\,c)}-2i\right)}{99\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8i\,dx+8i\,c)}+10\,a^3de^{(6i\,dx+6i\,c)}+10\,a^3de^{(4i\,dx+4i\,c)}+5\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -64/99*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 6.38, size = 91, normalized size = 1.25

$$\frac{64\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,11{}\mathrm{i}+2{}\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}}{99\,a^3\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (64*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*11i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(99*a^3*d*(exp(c*2i +
 d*x*2i) + 1)^5)
```

3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 2/7*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{3574\}$

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.51, size = 57, normalized size = 1.63

$$-\frac{2\sec^5(c+dx)(i+\tan(c+dx))}{7a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^5*(I + Tan[c + d*x]))/(7*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.
time = 0.78, size = 100, normalized size = 2.86

method	result	size
default	$\frac{2\left(8i\left(\cos^{4}\left(dx+c\right)\right)+8\sin\left(dx+c\right)\left(\cos^{3}\left(dx+c\right)\right)-8i\left(\cos^{2}\left(dx+c\right)\right)-4\sin\left(dx+c\right)\cos\left(dx+c\right)+i\right)\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}}{7d\cos\left(dx+c\right)^{3}a^{3}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/d*(8*I*cos(d*x+c)^4+8*sin(d*x+c)*cos(d*x+c)^3-8*I*cos(d*x+c)^2-4*sin(d*
x+c)*cos(d*x+c)+I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c
)^3/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(27) = 54$.
time = 0.43, size = 488, normalized size = 13.94

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{4i\sqrt{a}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{10\sqrt{a}\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}-\frac{5i\sqrt{a}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}-\frac{20\sqrt{a}\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-\frac{20\sqrt{a}\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}+\frac{5i\sqrt{a}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}-\frac{10\sqrt{a}\sin\left(dx+c\right)^{9}}{\left(\cos\left(dx+c\right)+1\right)^{9}}+\frac{4i\sqrt{a}\sin\left(dx+c\right)^{10}}{\left(\cos\left(dx+c\right)+1\right)^{10}}-\frac{2\sqrt{a}\sin\left(dx+c\right)^{11}}{\left(\cos\left(dx+c\right)+1\right)^{11}}+\frac{i\sqrt{a}\sin\left(dx+c\right)^{12}}{\left(\cos\left(dx+c\right)+1\right)^{12}}\right)\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)^{\frac{5}{2}}}{7\left(a^{3}-\frac{6a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{15a^{3}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}-\frac{20a^{3}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+\frac{15a^{3}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}-\frac{6a^{3}\sin\left(dx+c\right)^{10}}{\left(\cos\left(dx+c\right)+1\right)^{10}}+\frac{a^{3}\sin\left(dx+c\right)^{12}}{\left(\cos\left(dx+c\right)+1\right)^{12}}\right)d\left(-\frac{2i\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 5*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10*sqrt(a)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - 2*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + I*sqrt(a)*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5
/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 6*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^
3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(27) = 54$.

time = 0.37, size = 74, normalized size = 2.11

$$\frac{16 i \sqrt{2} \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}}}{7 \left(a^3 d e^{(6 i dx+6 i c)}+3 a^3 d e^{(4 i dx+4 i c)}+3 a^3 d e^{(2 i dx+2 i c)}+a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 16/7*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^3*d*e^(6*I*d*x + 6*I*c)
 + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 2.02, size = 50, normalized size = 1.43

$$\frac{e^{-c\,4i-d\,x\,4i}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1i}{\cos(c+d\,x)}}\,2i}{7\,a^3\,d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (exp(- c*4i - d*x*4i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(7*a
^3*d*cos(c + d*x)^3)
```

3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(5/2)/d-4*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*sec(d*x+c)^
3/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3572, 3570, 212}

$$\frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((4*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(5/2)*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*
x])^(3/2)) - ((4*I)*Sec[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
```

```
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{2\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a}$$

$$= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{4\int \frac{\sec(c}{\sqrt{a+ia\,\mathrm{ta}}}}{a}$$

$$= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{(8i)\mathrm{Subst}\left(\int \frac{}{2}\right.}{}$$

$$= \frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.97, size = 82, normalized size = 0.67

$$-\frac{2\sec(c+dx)\left(7i-6i\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+\tan(c+dx)\right)}{3a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (-2*Sec[c + d*x]*(7*I - (6*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]] + Tan[c + d*x]))/(3*a^2*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(102) = 204.
time = 0.85, size = 281, normalized size = 2.28

method	result
default	$\frac{2\left(3\cos(dx+c)\sqrt{2}\sin(dx+c)\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3\sqrt{2}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(d}{2\sin(dx+c)\sqrt{-\frac{2\,c}{1+c}}}\right.\right.}{3d(i\sin(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(3*cos(d*x+c)*2^(1/2)*sin(d*x+c)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c
))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+3*2^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(
d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*sin(d*x+c)+8*I*cos(d*x+c)^2-7*I*cos(d*x+c)+8*sin(d*x+c)*cos(
d*x+c)-I-sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(
d*x+c)+cos(d*x+c)-1)/cos(d*x+c)/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(96) = 192$.
time = 0.64, size = 1074, normalized size = 8.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((-3*I*sqrt(2)*cos(2*d*x + 2*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - 4*I*sqr
t(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*sqrt(2)
*cos(2*d*x + 2*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2))*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - 3*(2*(sqrt(2)*cos(2*
d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*(sqrt(2)*cos(2*
d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - (I*sqrt(2)*cos(2*
d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*cos(2*d*x + 2*c)
+ I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + s
qrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (-I*sqrt(2)*cos(2*d*x + 2*c)^2
 - I*sqrt(2)*sin(2*d*x + 2*c)^2 - 2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))
```

```
*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/((a^3*cos(2*d*x + 2*c)^2 + a^3*sin
(2*d*x + 2*c)^2 + 2*a^3*cos(2*d*x + 2*c) + a^3)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(96) = 192$.
time = 0.40, size = 270, normalized size = 2.20

$$-\frac{2\left(3\sqrt{2}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\frac{16\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)+3\sqrt{2}\left(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\frac{16\left(\left(-i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)+2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{(2i\,dx+2i\,c)}+4i\right)\right)}{3\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(1/(a^5*d^2))*l
og(-16*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*sqrt(2)*(-I*a^3
*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(1/(a^5*d^2))*log(-16*((-I*a^2*d*e^(2
*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^
2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(3*I*e^(2*I*d*x + 2*I*c) + 4*I))/(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3582, 3583, 3570, 212}

$$\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(Sqrt[2]*a^(5/2)*d) + (I*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
```

```
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a}\\ &= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a^2}\\ &= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{i\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^2d}\\ &= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.16, size = 149, normalized size = 1.73

$$\frac{ie^{-\frac{1}{2}i(2c+dx)}\left(-1-e^{2i(c+dx)}+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sec^3(c+dx)\left(\cos\left(c+\frac{dx}{2}\right)+i\sin\left(c+\frac{dx}{2}\right)\right)}{2a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/2)*(-1 - E^((2*I)*(c + d*x)) + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3*(Cos[c + (
d*x)/2] + I*Sin[c + (d*x)/2]))/(a^2*d*E^((I/2)*(2*c + d*x))*(-I + Tan[c + d
*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(71) = 142.
time = 0.91, size = 443, normalized size = 5.15

method	result
default	$-\frac{\sin(dx+c)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-i\sin(dx+c)\cos(dx+c)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c)}{2\sin(dx+c)\sqrt{-\frac{2\cos(d}{1+\cos(}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*sin(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-I*sin(d*
x+c)*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*
cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2))+8*I*sin(d*x+c)*cos(d*x+c)^3-I*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+cos(d*x+c)^2*2^(1/2)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-8*cos(d*x+c)^4-4*I*cos(d*x+c)*si
n(d*x+c)-2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+
8*cos(d*x+c)^2)/(cos(d*x+c)^2-1)/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(67) = 134$.
time = 0.61, size = 827, normalized size = 9.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2*d*x + 2*c) -
 I*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqr
t(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(
```

```
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1
))*sqrt(a))/(a^3*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(67) = 134$.
time = 0.37, size = 245, normalized size = 2.85

$$\frac{i\sqrt{2}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(\frac{2\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)-i\sqrt{2}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(\frac{2\left(\left(-i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)}{4\,a^3d}e^{(-2i\,dx-2i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((I*a^2*d*
e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^
5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) - I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*
e^(2*I*d*x + 2*I*c)*log(2*((-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d))
 - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) - I)
)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\left(ia\left(\tan(c+dx)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}+\frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/32*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+1/4*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+3/16*I*sec(d*x+c
)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3570, 212}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(5/2)) + (((3*I)/16)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
```

```
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+}{\sqrt{a+ia\tan}}}{32a}$$

$$= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{(3i)\text{Subst}\left(\int \frac{}{2-}\right.}{}$$

$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{}{16a}$$

Mathematica [A]
time = 0.96, size = 121, normalized size = 0.99

$$-\frac{i\sec^3(c+dx)\left(7+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+7\cos(2(c+dx))+3i\sin(2(c+dx))\right)}{32a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((-1/32*I)*Sec[c + d*x]^3*(7 + 3*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Cos[2*(c + d*x)] + (3*I)
*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(97) = 194.
time = 0.78, size = 346, normalized size = 2.84

method	result
default	$\left(64i(\cos^5(dx+c))+3i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)+64\sin(dx+c)(\cos^4(dx+c))\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d*(64*I*cos(d*x+c)^5+3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1
/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+64*sin(d*x+c)*cos(d*x+c)^4+3*I*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-24*I*cos(d*x+c)^3+3*2
^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)
+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/co
s(d*x+c))^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(91) = 182.
time = 0.36, size = 267, normalized size = 2.19

$$\frac{-3i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{8\,a^2d}\right)+3i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(-i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{8\,a^2d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+2i\right)}{32\,a^3d}e^{(-4i\,dx-4i\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(-3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(
sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*I*sqr
t(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(sqrt(2)*sqrt(1
/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 7*I*e^(2*I*d*x + 2*I*c) +
2*I))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}+\frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}}+\frac{3}{192a^2d\sqrt{}}$$

```
[Out] 35/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a
^(5/2)/d*2^(1/2)+35/192*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-35/128*
I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+1/6*I*cos(d*x+c)/d/(a+I*a*tan(d
*x+c))^(5/2)+7/48*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3583, 3571, 3570, 212}

$$\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d}-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}+\frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((35*I)/128)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x
])^(5/2)) + (((7*I)/48)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
(((35*I)/192)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/1
28)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{12a}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35\int \frac{\cos(}{\sqrt{a+ia\,\mathrm{t}}}}{96}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c}{192a^2d\sqrt{a+ia}}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c}{192a^2d\sqrt{a+ia}}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c}{192a^2d\sqrt{a+ia}}$$

$$= \frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{}{48}$$

Mathematica [A]
time = 1.52, size = 143, normalized size = 0.74

$$\frac{i\sec^3(c+dx)\left(-125-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-85\cos(2(c+dx))+40\cos(4(c+dx))+7i\sin(2(c+dx))+56i\sin(4(c+dx))\right)}{768a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/768)*Sec[c + d*x]^3*(-125 - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 85*Cos[2*(c + d*x)] + 4
0*Cos[4*(c + d*x)] + (7*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a^
2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(155) = 310.
time = 0.90, size = 373, normalized size = 1.94

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(1024i(\cos^7(dx+c))+1024\sin(dx+c)(\cos^6(dx+c))-320i(\cos^5(dx+c))+105i\sqrt{2}\sqrt{-\frac{2\cos(dx+c}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(1024*I*cos(d*x+c)^
7+1024*sin(d*x+c)*cos(d*x+c)^6-320*I*cos(d*x+c)^5+105*I*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+192*sin(d*x+c)*co
s(d*x+c)^4+105*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
2^(1/2)+56*I*cos(d*x+c)^3+105*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2))*sin(d*x+c)+280*cos(d*x+c)^2*sin(d*x+c)-420*I*cos(d*x+c
))/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2297 vs. 2(145) = 290.
time = 0.65, size = 2297, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3072*(544*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(6*d*x + 6*c) -
sqrt(2)*sin(6*d*x + 6*c))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c),
 cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) +
 1)) + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(3/2*arct
an2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(s
```

```
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + 1)^(1/4)*(29*((I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*co
s(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (I*sqrt(2)*cos(6*d*x
 + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c)))^2 + 2*(I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))
*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + I*sqrt(2)*cos(6*d*x
 + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
+ 6*c))) + 1)) + (19*I*sqrt(2)*cos(6*d*x + 6*c) + 19*sqrt(2)*sin(6*d*x + 6*
c) - 16*I*sqrt(2))*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) -
29*((sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2
(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
^2 + 2*(sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*sin(5/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)
) - (19*sqrt(2)*cos(6*d*x + 6*c) - 19*I*sqrt(2)*sin(6*d*x + 6*c) - 16*sqrt(
2))*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), c
os(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) - 105*(2
*sqrt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 +
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (
cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c),
cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
)) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2
*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x +
```

```
6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(
1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + 2*(cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) + I*
sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))...
```

Fricas [A]
time = 0.44, size = 289, normalized size = 1.51

$$\frac{\left(-105i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{64\,a^2d}\right)+105i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(6i\,dx+6i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{64\,a^2d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-48i\,e^{(8i\,dx+8i\,c)}+39i\,e^{(6i\,dx+6i\,c)}+125i\,e^{(4i\,dx+4i\,c)}+46i\,e^{(2i\,dx+2i\,c)}+8i\right)\right)e^{(-6i\,dx-6i\,c)}}{768\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(-105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35
/64*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 10
5*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35/64*(sqrt(
2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(-48*I*e^(8*I*d*x + 8*I*c) + 39*I*e^(6*I*d*x
 + 6*I*c) + 125*I*e^(4*I*d*x + 4*I*c) + 46*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^
(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{1155i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}+\frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}}+\frac{}{2048a}$$

```
[Out] 1155/8192*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)+385/2048*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+33
/256*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1155/4096*I*cos(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/a^3/d-77/512*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2
)/a^3/d+1/8*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I*cos(d*x+c)^3/
a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.30, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3583, 3578, 3571, 3570, 212}

$$\frac{1155i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d}-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d}+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((1155*I)/4096)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
 + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c
+ d*x])^(5/2)) + (((11*I)/96)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(
3/2)) + (((385*I)/2048)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((33*I)/256)*Cos[c + d*x]^3)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((1155*
I)/4096)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) - (((77*I)/512)*C
os[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{16a} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33\int \frac{\cos^3(c\cdots}{\sqrt{a+ia\tan\cdots}}}{64a^{\cdots}} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33i\cos^3(c\cdots}{256a^2d\sqrt{a+ia\,\mathrm{t}\cdots}} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33i\cos^3(c\cdots}{256a^2d\sqrt{a+ia\,\mathrm{t}\cdots}} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c\cdots}{2048a^2d\sqrt{a+ia\cdots}} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c\cdots}{2048a^2d\sqrt{a+ia\cdots}} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c\cdots}{2048a^2d\sqrt{a+ia\cdots}} \\
&= \frac{1155i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}\,a^{5/2}d} + \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{\cdots}{9\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.46, size = 165, normalized size = 0.61

$$
\frac{i\sec^3(c+dx)\left(-3325-3465e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-1605\cos(2(c+dx))+1800\cos(4(c+dx))+80\cos(6(c+dx))+1111i\sin(2(c+dx))+2552i\sin(4(c+dx))+176i\sin(6(c+dx))\right)}{24576a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/24576)*Sec[c + d*x]^3*(-3325 - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 1605*Cos[2*(c + d*x
)] + 1800*Cos[4*(c + d*x)] + 80*Cos[6*(c + d*x)] + (1111*I)*Sin[2*(c + d*x)
] + (2552*I)*Sin[4*(c + d*x)] + (176*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I + Tan
[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.87, size = 400, normalized size = 1.48

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(24576i\left(\cos^9(dx+c)\right)+24576\sin(dx+c)\left(\cos^8(dx+c)\right)-7168i\left(\cos^7(dx+c)\right)+5120\sin(dx+c)\left(\cos^6(d\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/49152/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(24576*I*cos(d*x+c)^9+24576*sin(d*x+c)*cos(d*x+c)^8-7168*I*cos(d*x+c)^7+5120*sin(d*x+c)*cos(d*x+c)^6+704*I*cos(d*x+c)^5+6336*sin(d*x+c)*cos(d*x+c)^4+3465*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+3465*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+3465*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+1848*I*cos(d*x+c)^3+9240*cos(d*x+c)^2*sin(d*x+c)-13860*I*cos(d*x+c))/a^3

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3789 vs. $2(207) = 414$.
time = 0.74, size = 3789, normalized size = 14.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] -1/98304*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(15*((-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (55*I*sqrt(2)*cos(8*d*x + 8*c) + 960*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 1296*I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 55*sqrt(2)*sin(8*d*x + 8*c) + 960*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 1296*sqrt(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 128*I*sqrt(2))*cos(3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c

```
), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 + 1)) + 15*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)*cos(8*d*x + 8*c
) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
 + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)*cos(8*d*x + 8*c
) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c))) + 1)) - (55*sqrt(2)*cos(8*d*x + 8*c) + 960*sqrt(2)*cos(3/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) - 1296*sqrt(2)*cos(1/2*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c))) - 55*I*sqrt(2)*sin(8*d*x + 8*c) - 960*I*sqrt(2)*
sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1296*I*sqrt(2)*sin(1
/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 128*sqrt(2))*sin(3/2*arct
an2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + 4*(cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ 1)^(1/4)*((73*(-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*co
s(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 73*(-I*sqrt(2)*cos(8
*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c)))^2 + 792*(-I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 - I*sqrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c)))^2 - 2*I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))
) - I*sqrt(2))*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 146*(
-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) - 792*(sqrt(2)*cos(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c)))^2 + sqrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
+ 8*c))) + sqrt(2))*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) -
73*I*sqrt(2)*cos(8*d*x + 8*c) - 73*sqrt(2)*sin(8*d*x + 8*c))*cos(5/2*arctan
2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 3*(-5*I*sqrt(2)*cos(8*d*x + 8*c)
- 120*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 336*
I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 64*I*sqrt(
2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 5*sqrt(2)*sin(8*d
*x + 8*c) - 120*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))
) + 336*sqrt(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 64*s
qrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 640*I*sqrt(2)
)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (73*(sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c)))^2 + 73*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
 + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 792*(sqrt
(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sqrt(2)*sin(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
```

```
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2))*cos(3/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c))) + 146*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin
(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 792*(
-I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 - I*sqrt(
2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 - 2*I*sqrt(2)*cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - I*sqrt(2))*sin(3/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 73*s...
```

Fricas [A]
time = 0.39, size = 311, normalized size = 1.15

$$\frac{\left(-3465i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(8i\,dx+8i\,c)}\log\left(-\frac{1155\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{2048\,a^2d}\right)+3465i\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}\,e^{(8i\,dx+8i\,c)}\log\left(-\frac{1155\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^2de^{(2i\,dx+2i\,c)}-i\,a^2d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{2048\,a^2d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-128i\,e^{(12i\,dx+12i\,c)}-2176i\,e^{(10i\,dx+10i\,c)}+247i\,e^{(8i\,dx+8i\,c)}+3325i\,e^{(6i\,dx+6i\,c)}+1358i\,e^{(4i\,dx+4i\,c)}+376i\,e^{(2i\,dx+2i\,c)}+48i\right)\right)e^{(-8i\,dx-8i\,c)}}{24576\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24576*(-3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(
-1155/2048*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d
)) + 3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(-1155
/2048*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) +
 sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(12*I*d*x + 12*I*c) -
2176*I*e^(10*I*d*x + 10*I*c) + 247*I*e^(8*I*d*x + 8*I*c) + 3325*I*e^(6*I*d*
x + 6*I*c) + 1358*I*e^(4*I*d*x + 4*I*c) + 376*I*e^(2*I*d*x + 2*I*c) + 48*I)
)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{16i(a+ia\tan(c+d$$

```
[Out] -32/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d+64/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/
d-48/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d+16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^8
/d-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d}-\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-32*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d) + (((64*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^6*d) - (((48*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
7*d) + (((16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^8*d) - (((2*I)/11)*(a +
 I*a*Tan[c + d*x])^(11/2))/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\text{Subst}\left(\int (a-x)^4\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^9 d}$$

$$= -\frac{i\text{Subst}\left(\int \left(16a^4\sqrt{a+x} - 32a^3(a+x)^{3/2} + 24a^2(a+x)^{5/2} - 8a(a+x)^{7/2}\right.\right.}{a^9 d}$$

$$= -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5 d} + \frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6 d} - \frac{48i(a+ia\tan(c}{7a^7 d}$$

Mathematica [A]
time = 0.99, size = 114, normalized size = 0.78

$$\frac{2\sec^9(c+dx)(1584+2552\cos(2(c+dx))+1283\cos(4(c+dx))-1144i\sin(2(c+dx))-1027i\sin(4(c+dx)))(\cos(5(c+dx))+i\sin(5(c+dx)))}{3465a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(1584 + 2552*Cos[2*(c + d*x)] + 1283*Cos[4*(c + d*x)] - (
1144*I)*Sin[2*(c + d*x)] - (1027*I)*Sin[4*(c + d*x)])*(Cos[5*(c + d*x)] + I
*Sin[5*(c + d*x)]))/(3465*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [A]
time = 0.83, size = 117, normalized size = 0.80

method	result
default	$\frac{2\left(-2048i\left(\cos^5(dx+c)\right)+2048\sin(dx+c)\left(\cos^4(dx+c)\right)-4876i\left(\cos^3(dx+c)\right)-3340\left(\cos^2(dx+c)\right)\sin(dx+c)+1505i\cos(dx+c)+315\sin(\right.}{3465d\cos(dx+c)^5a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465/d*(-2048*I*cos(d*x+c)^5+2048*sin(d*x+c)*cos(d*x+c)^4-4876*I*cos(d*x+
c)^3-3340*cos(d*x+c)^2*sin(d*x+c)+1505*I*cos(d*x+c)+315*sin(d*x+c))*(a*(I*s
in(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5/a^4
```

Maxima [A]
time = 0.27, size = 94, normalized size = 0.64

$$-\frac{2i\left(315(ia\tan(dx+c)+a)^{\frac{11}{2}} - 3080(ia\tan(dx+c)+a)^{\frac{9}{2}}a + 11880(ia\tan(dx+c)+a)^{\frac{7}{2}}a^2 - 22176(ia\tan(dx+c)+a)^{\frac{5}{2}}a^3 + 18480(ia\tan(dx+c)+a)^{\frac{3}{2}}a^4\right)}{3465a^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3465*I*(315*(I*a*tan(d*x + c) + a)^(11/2) - 3080*(I*a*tan(d*x + c) + a)^
(9/2)*a + 11880*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 22176*(I*a*tan(d*x + c)
+ a)^(5/2)*a^3 + 18480*(I*a*tan(d*x + c) + a)^(3/2)*a^4)/(a^9*d)
```

Fricas [A]
time = 0.44, size = 160, normalized size = 1.10

$$-\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(11i\,dx+11i\,c)}+704i\,e^{(9i\,dx+9i\,c)}+1584i\,e^{(7i\,dx+7i\,c)}+1848i\,e^{(5i\,dx+5i\,c)}+1155i\,e^{(3i\,dx+3i\,c)}\right)}{3465\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -64/3465*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(11*I*d*x + 11*
I*c) + 704*I*e^(9*I*d*x + 9*I*c) + 1584*I*e^(7*I*d*x + 7*I*c) + 1848*I*e^(5
*I*d*x + 5*I*c) + 1155*I*e^(3*I*d*x + 3*I*c))/(a^4*d*e^(10*I*d*x + 10*I*c)
+ 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(
4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 7.63, size = 370, normalized size = 2.53

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{3465\,a^4\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3465\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6
4i)/(11*a^4*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(3465*a^4*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*1024i)/(1155*a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*a^
4*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*a^4*d*(exp(c*2i + d*x*2i) + 1)^4)
 - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*8192i)/(3465*a^4*d)
```

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}$$

```
[Out] -16*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+8*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d-12/
5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-16*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) + ((8*I)*(a + I*a*Tan[c + d*x]
)^(3/2))/(a^5*d) - (((12*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^6*d) + (((2
*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{8a^3}{\sqrt{a+x}} - 12a^2\sqrt{a+x} + 6a(a+x)^{3/2} - (a+x)^{5/2}\right) dx, x, i$$
$$= -\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d} - \frac{12i(a+ia\tan(c+}{5a^6d}$$

Mathematica [A]
time = 0.71, size = 110, normalized size = 0.97

$$\frac{2\sec^7(c+dx)(126\cos(c+dx)+51\cos(3(c+dx))-i(14\sin(c+dx)+19\sin(3(c+dx))))(\cos(4(c+dx))+i\sin(4(c+dx)))}{35a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^7*(126*Cos[c + d*x] + 51*Cos[3*(c + d*x)] - I*(14*Sin[c + d
*x] + 19*Sin[3*(c + d*x)]))*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]))/(35*a^
3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.79, size = 90, normalized size = 0.80

method	result	size
default	$-\frac{2\left(204i\left(\cos^3(dx+c)\right)+76\left(\cos^2(dx+c)\right)\sin(dx+c)-27i\cos(dx+c)-5\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{35d\cos(dx+c)^3a^4}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/d*(204*I*cos(d*x+c)^3+76*cos(d*x+c)^2*sin(d*x+c)-27*I*cos(d*x+c)-5*si
n(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/a^4
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.67

$$\frac{2i\left(5\left(ia\tan(dx+c)+a\right)^{\frac{7}{2}} - 42\left(ia\tan(dx+c)+a\right)^{\frac{5}{2}}a + 140\left(ia\tan(dx+c)+a\right)^{\frac{3}{2}}a^2 - 280\sqrt{ia\tan(dx+c)+a}\,a^3\right)}{35\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 42*(I*a*tan(d*x + c) + a)^(5/2)*a
+ 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 280*sqrt(I*a*tan(d*x + c) + a)*a^3
)/(a^7*d)
```

Fricas [A]
time = 0.37, size = 119, normalized size = 1.05

$$-\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(7i\,dx+7i\,c)}+56i\,e^{(5i\,dx+5i\,c)}+70i\,e^{(3i\,dx+3i\,c)}+35i\,e^{(i\,dx+i\,c)}\right)}{35\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(7*I*d*x + 7*I*c)
+ 56*I*e^(5*I*d*x + 5*I*c) + 70*I*e^(3*I*d*x + 3*I*c) + 35*I*e^(I*d*x + I*c
))/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a^4*d*e^(2*
I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 6.74, size = 242, normalized size = 2.14

$$-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{35\,a^4\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{35\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*256i)/(35*a^4*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*128i)/(35*a^4*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*96i)/(35*a^4*
d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*a^4*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=84

$$\frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

```
[Out] 8*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+8*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-2/3*I
*(a+I*a*tan(d*x+c))^(3/2)/a^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {3568, 45}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (8*I)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((8*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^4*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^{3/2}} - \frac{4a}{\sqrt{a+x}} + \sqrt{a+x}\right) dx, x, ia\tan(c+dx)\right)}{a^5 d}$$
$$= \frac{8i}{a^3 d\sqrt{a+ia\tan(c+dx)}} + \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4 d} - \frac{2i(a+ia\tan(c+dx)}{3a^5 d}$$

Mathematica [A]
time = 0.53, size = 61, normalized size = 0.73

$$\frac{2i\sec^2(c+dx)(12+11\cos(2(c+dx))+5i\sin(2(c+dx)))}{3a^3 d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (((2*I)/3)*Sec[c + d*x]^2*(12 + 11*Cos[2*(c + d*x)] + (5*I)*Sin[2*(c + d*x)
]))/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.88, size = 88, normalized size = 1.05

method	result	size
default	$\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(12i(\cos^3(dx+c))+12(\cos^2(dx+c))\sin(dx+c)+11i\cos(dx+c)+\sin(dx+c))}{3d\cos(dx+c)a^4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(12*I*cos(d*x+c)^3+12*
cos(d*x+c)^2*sin(d*x+c)+11*I*cos(d*x+c)+sin(d*x+c))/cos(d*x+c)/a^4
```

Maxima [A]
time = 0.29, size = 62, normalized size = 0.74

$$\frac{2i\left(\frac{12}{\sqrt{i\,a\tan(dx+c)+a}\,a^2} - \frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}} - 12\sqrt{i\,a\tan(dx+c)+a}\,a}{a^4}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] 2/3*I*(12/(sqrt(I*a*tan(d*x + c) + a)*a^2) - ((I*a*tan(d*x + c) + a)^(3/2)
- 12*sqrt(I*a*tan(d*x + c) + a)*a)/a^4)/(a*d)
```

Fricas [A]
time = 0.39, size = 77, normalized size = 0.92

$$-\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(4i\,dx+4i\,c)}-12i\,e^{(2i\,dx+2i\,c)}-3i\right)}{3\left(a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] -4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(4*I*d*x + 4*I*c) -
12*I*e^(2*I*d*x + 2*I*c) - 3*I)/(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x
 + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 0.74, size = 110, normalized size = 1.31

$$\frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,23\mathrm{i}+\cos(4\,c+4\,d\,x)\,3\mathrm{i}+7\,\sin(2\,c+2\,d\,x)+3\,\sin(4\,c+4\,d\,x)+20\mathrm{i})}{3\,a^4\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] (2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2)*(cos(2*c + 2*d*x)*23i + cos(4*c + 4*d*x)*3i + 7*sin(2*c + 2*d*x) +
3*sin(4*c + 4*d*x) + 20i))/(3*a^4*d*(cos(2*c + 2*d*x) + 1))
```

3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=57

$$\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -2*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+4/3*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((4*I)/3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*I)/(a^3*d*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{5/2}} - \frac{1}{(a+x)^{3/2}}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}$$
$$= \frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.28, size = 80, normalized size = 1.40

$$\frac{2\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))(1+3i\tan(c+dx))}{3a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(1 + (3*I)*Tan[c
+ d*x]))/(3*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.88, size = 88, normalized size = 1.54

method	result	size
default	$\frac{2\cos(dx+c)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(4i(\cos^3(dx+c))+4(\cos^2(dx+c))\sin(dx+c)-5i\cos(dx+c)-3\sin(dx+c))}{3d\,a^4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*cos(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*I*cos(d*
x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)-5*I*cos(d*x+c)-3*sin(d*x+c))/a^4
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.56

$$-\frac{2i\,(3i\,a\tan(dx+c)+a)}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*(3*I*a*tan(d*x + c) + a)/((I*a*tan(d*x + c) + a)^(3/2)*a^3*d)
```

Fricas [A]
time = 0.37, size = 61, normalized size = 1.07

$$\frac{\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(-2i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) - I
*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{4}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 39, normalized size = 1.34

$$-\frac{2\sqrt{a+ia\tan(c+dx)}}{5a^4d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[a + I*a*Tan[c + d*x]])/(5*a^4*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.25, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24
default	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.72

$$\frac{2i}{5\,(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*I/((I*a*tan(d*x + c) + a)^(5/2)*a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(21) = 42$.
time = 0.38, size = 72, normalized size = 2.48

$$\frac{\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)}+3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{20\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*x + 6*I*c) + 3*I
*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a
^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 3.64, size = 23, normalized size = 0.79

$$\frac{2\mathrm{i}}{5\,a\,d\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] 2i/(5*a*d*(a + a*tan(c + d*x)*1i)^(5/2))
```

3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{11i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+d$$

```
[Out] -11/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d*2
^(1/2)+11/64*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+11/36*I*a/d/(a+I*a*tan(d*x+c)
)^(9/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+11/56*I/d/(
a+I*a*tan(d*x+c))^(7/2)+11/80*I/a/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I/a^2/d/
(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{11i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}}+\frac{11i}{56d(a+ia\tan(c+dx))^{7/2}}+\frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-11*I)/64)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*a^(7/2)*d) + (((11*I)/36)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((I/2)*a
^2)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + ((11*I)/56)/(
d*(a + I*a*Tan[c + d*x])^(7/2)) + ((11*I)/80)/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) + ((11*I)/96)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((11*I)/64)/(a^3
*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} - \frac{(11ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x}\right.}{4}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}$$

$$= -\frac{11i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{}{2d(a}$$

Mathematica [A]
time = 1.85, size = 176, normalized size = 0.76

$$-\frac{ie^{-11i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(-70-460e^{2i(c+dx)}-1338e^{4i(c+dx)}-2416e^{6i(c+dx)}-4618e^{8i(c+dx)}+315e^{10i(c+dx)}\right)+3465e^{9i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^3(c+dx)}{161280a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/161280*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*(Sqrt[1 + E^((2*I)*(c + d*x)
)]*(-70 - 460*E^((2*I)*(c + d*x)) - 1338*E^((4*I)*(c + d*x)) - 2416*E^((6*I
)*(c + d*x)) - 4618*E^((8*I)*(c + d*x)) + 315*E^((10*I)*(c + d*x))) + 3465*
E^((9*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^3)/(a^3*d*E^((11
*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(183) = 366$.
time = 1.07, size = 416, normalized size = 1.79

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(71680i(\cos^{10}(dx+c))+71680(\cos^{9}(dx+c))\sin(dx+c)-43520i(\cos^{8}(dx+c))-7680\sin(dx+c)(\cos^{7}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80640/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(71680*I*cos(d*x+c
)^10+71680*cos(d*x+c)^9*sin(d*x+c)-43520*I*cos(d*x+c)^8-7680*sin(d*x+c)*cos
(d*x+c)^7+512*I*cos(d*x+c)^6+5632*sin(d*x+c)*cos(d*x+c)^5+1056*I*cos(d*x+c)
^4-3465*I*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*co
s(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2))*2^(1/2)-3465*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arctan(1/2
*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2))+7392*sin(d*x+c)*cos(d*x+c)^3-3465*sin(d*x+c)*arctan(1/2*(-I*cos
(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+4620*I*cos(d*x+c)^2+13860*
sin(d*x+c)*cos(d*x+c))/a^4
```

Maxima [A]
time = 0.50, size = 195, normalized size = 0.84

$$\frac{i\left(\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^{5}-4620\,(i\,a\tan(dx+c)+a)^{4}a-1848\,(i\,a\tan(dx+c)+a)^{3}a^{2}-1584\,(i\,a\tan(dx+c)+a)^{2}a^{3}-1760\,(i\,a\tan(dx+c)+a)a^{4}-2240\,a^{5}\right)}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^{2}-2\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^{3}}+\frac{3465\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{i\,a\tan\left(dx+c\right)+a}}\right)}{a^{\frac{5}{2}}}\right)}{80640\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/80640*I*(4*(3465*(I*a*tan(d*x + c) + a)^5 - 4620*(I*a*tan(d*x + c) + a)^4
*a - 1848*(I*a*tan(d*x + c) + a)^3*a^2 - 1584*(I*a*tan(d*x + c) + a)^2*a^3
- 1760*(I*a*tan(d*x + c) + a)*a^4 - 2240*a^5)/((I*a*tan(d*x + c) + a)^(11/2
)*a^2 - 2*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 3465*sqrt(2)*log(-(sqrt(2)*sq
rt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c
) + a)))/a^(5/2))/(a*d)
```

Fricas [A]
time = 0.38, size = 316, normalized size = 1.36

$$\frac{\left(-3465i\,\sqrt{\frac{1}{2}}\,a^{4}d\sqrt{\frac{1}{a^{7}d^{2}}}\,e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{4}de^{(2i\,dx+2i\,c)}+a^{4}d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^{7}d^{2}}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3465i\,\sqrt{\frac{1}{2}}\,a^{4}d\sqrt{\frac{1}{a^{7}d^{2}}}\,e^{(9i\,dx+9i\,c)}\log\left(-4\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(a^{4}de^{(2i\,dx+2i\,c)}+a^{4}d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^{7}d^{2}}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,(-315i\,e^{(12i\,dx+12i\,c)}+4303i\,e^{(10i\,dx+10i\,c)}+7034i\,e^{(8i\,dx+8i\,c)}+3754i\,e^{(6i\,dx+6i\,c)}+1798i\,e^{(4i\,dx+4i\,c)}+530i\,e^{(2i\,dx+2i\,c)}+70i)\right)e^{(-9i\,dx-9i\,c)}}{40320\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40320*(-3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(
4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-315*I*e^(12*I*d*x + 12*I*c) + 4303*I*e^(
10*I*d*x + 10*I*c) + 7034*I*e^(8*I*d*x + 8*I*c) + 3754*I*e^(6*I*d*x + 6*I*c
) + 1798*I*e^(4*I*d*x + 4*I*c) + 530*I*e^(2*I*d*x + 2*I*c) + 70*I))*e^(-9*I
*d*x - 9*I*c)/(a^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=306

$$-\frac{195i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\,a^{7/2}d}+\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan$$

```
[Out] -195/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d
*2^(1/2)+195/1024*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+195/352*I*a^2/d/(a+I*a*t
an(d*x+c))^(11/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(11/2
)-15/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(11/2)+65/192*I*a/d/(
a+I*a*tan(d*x+c))^(9/2)+195/896*I/d/(a+I*a*tan(d*x+c))^(7/2)+39/256*I/a/d/(
a+I*a*tan(d*x+c))^(5/2)+65/512*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$-\frac{195i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\,a^{7/2}d}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-195*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(7/2)*d) + (((195*I)/352)*a^2)/(d*(a + I*a*Tan[c + d*x])^(11/2)) -
((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2)) - ((
(15*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) +
(((65*I)/192)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + ((195*I)/896)/(d*(a + I
*a*Tan[c + d*x])^(7/2)) + ((39*I)/256)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) +
 ((65*I)/512)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((195*I)/1024)/(a^3*d*
Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{13/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{(15ia^4)\,\text{Subst}\left(\int \frac{}{(a-x)^2}\right.}{}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{1}{16d(a-ia\tan(c+dx}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}$$

$$= -\frac{195i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\,a^{7/2}d} + \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} -$$

Mathematica [A]
time = 2.71, size = 202, normalized size = 0.66

$$-\frac{ie^{-13i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\left(\sqrt{1+e^{2i(c+dx)}}\left(-168-1456e^{2i(c+dx)}-5728e^{4i(c+dx)}-13824e^{6i(c+dx)}-24688e^{8i(c+dx)}-54112e^{10i(c+dx)}+6699e^{12i(c+dx)}+462e^{14i(c+dx)}\right)+45045e^{11i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sec^3(c+dx)}{1892352a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/1892352*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*(Sqrt[1 + E^((2*I)*(c + d*x
))]*(-168 - 1456*E^((2*I)*(c + d*x)) - 5728*E^((4*I)*(c + d*x)) - 13824*E^(
(6*I)*(c + d*x)) - 24688*E^((8*I)*(c + d*x)) - 54112*E^((10*I)*(c + d*x)) +
 6699*E^((12*I)*(c + d*x)) + 462*E^((14*I)*(c + d*x))) + 45045*E^((11*I)*(c
 + d*x))*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^3)/(a^3*d*E^((13*I)*(c + d*
x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.18, size = 443, normalized size = 1.45

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(688128i\left(\cos^{12}(dx+c)\right)+688128\sin(dx+c)\left(\cos^{11}(dx+c)\right)-401408i\left(\cos^{10}(dx+c)\right)-57344\left(\cos^{9}(dx+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/946176/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(688128*I*cos(d*x
+c)^12+688128*sin(d*x+c)*cos(d*x+c)^11-401408*I*cos(d*x+c)^10-57344*cos(d*x
+c)^9*sin(d*x+c)+4096*I*cos(d*x+c)^8+61440*sin(d*x+c)*cos(d*x+c)^7+6656*I*c
os(d*x+c)^6+73216*sin(d*x+c)*cos(d*x+c)^5+13728*I*cos(d*x+c)^4-45045*I*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)+9
6096*sin(d*x+c)*cos(d*x+c)^3-45045*sin(d*x+c)*arctan(1/2*(-I*cos(d*x+c)+sin
(d*x+c)+I)/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2)*arctan(1/2*(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)/(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+60060*I*cos(d*x+c)^2+180180*sin(d*x+c
)*cos(d*x+c))/a^4
```

Maxima [A]
time = 0.49, size = 249, normalized size = 0.81

$$\frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^{7}-150150\,(i\,a\tan(dx+c)+a)^{6}a+96096\,(i\,a\tan(dx+c)+a)^{5}a^{2}+27456\,(i\,a\tan(dx+c)+a)^{4}a^{3}+18304\,(i\,a\tan(dx+c)+a)^{3}a^{4}+16640\,(i\,a\tan(dx+c)+a)^{2}a^{5}+17920\,(i\,a\tan(dx+c)+a)a^{6}+21504\,a^{7}\right)}{(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^{2}-4\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a^{3}+4\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^{4}}+\frac{45045\,\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{a^{\frac{5}{2}}}\right)}{946176\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/946176*I*(4*(45045*(I*a*tan(d*x + c) + a)^7 - 150150*(I*a*tan(d*x + c) +
a)^6*a + 96096*(I*a*tan(d*x + c) + a)^5*a^2 + 27456*(I*a*tan(d*x + c) + a)^
4*a^3 + 18304*(I*a*tan(d*x + c) + a)^3*a^4 + 16640*(I*a*tan(d*x + c) + a)^2
*a^5 + 17920*(I*a*tan(d*x + c) + a)*a^6 + 21504*a^7)/((I*a*tan(d*x + c) + a
```

```
)^(15/2)*a^2 - 4*(I*a*tan(d*x + c) + a)^(13/2)*a^3 + 4*(I*a*tan(d*x + c) +
a)^(11/2)*a^4) + 45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2))/(a*d)
```

Fricas [A]
time = 0.40, size = 338, normalized size = 1.10

$$\frac{\left(-45045i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(11i\,dx+11i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+45045i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(11i\,dx+11i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-462i\,e^{(16i\,dx+16i\,c)}-7161i\,e^{(14i\,dx+14i\,c)}+47413i\,e^{(12i\,dx+12i\,c)}+78800i\,e^{(10i\,dx+10i\,c)}+38512i\,e^{(8i\,dx+8i\,c)}+19552i\,e^{(6i\,dx+6i\,c)}+7184i\,e^{(4i\,dx+4i\,c)}+1624i\,e^{(2i\,dx+2i\,c)}+168i\right)\right)e^{(-11i\,dx-11i\,c)}}{473088\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/473088*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*
log(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-462*I*e^(16*I*d*x + 16*I*c) - 716
1*I*e^(14*I*d*x + 14*I*c) + 47413*I*e^(12*I*d*x + 12*I*c) + 78800*I*e^(10*I
*d*x + 10*I*c) + 38512*I*e^(8*I*d*x + 8*I*c) + 19552*I*e^(6*I*d*x + 6*I*c)
+ 7184*I*e^(4*I*d*x + 4*I*c) + 1624*I*e^(2*I*d*x + 2*I*c) + 168*I))*e^(-11*
I*d*x - 11*I*c)/(a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=110

$$\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 64/3315*I*a^3*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+16/255*I*a^2*sec(d*
x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+2/17*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x
+c))^(9/2)
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((64*I)/3315)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((
16*I)/255)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}} + \frac{1}{17}(8a)\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx$$
$$= \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}} + \frac{1}{255}(32a^2)\int$$
$$= \frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}} + \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec}{17d(a+ia}$$

Mathematica [A]
time = 1.14, size = 92, normalized size = 0.84

$$-\frac{2\sec^{12}(c+dx)(68+263\cos(2(c+dx))+247i\sin(2(c+dx)))(\cos(3(c+dx))-i\sin(3(c+dx)))}{3315a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sec[c + d*x]^12*(68 + 263*Cos[2*(c + d*x)] + (247*I)*Sin[2*(c + d*x)])*
(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(3315*a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.43, size = 171, normalized size = 1.55

method	result
default	$\frac{2(2048i(\cos^9(dx+c))+2048\sin(dx+c)(\cos^8(dx+c))-256i(\cos^7(dx+c))+768\sin(dx+c)(\cos^6(dx+c))-80i(\cos^5(dx+c))+560\sin(dx+}{3315d\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3315/d*(2048*I*cos(d*x+c)^9+2048*sin(d*x+c)*cos(d*x+c)^8-256*I*cos(d*x+c)
^7+768*sin(d*x+c)*cos(d*x+c)^6-80*I*cos(d*x+c)^5+560*sin(d*x+c)*cos(d*x+c)^
4-2252*I*cos(d*x+c)^3-1748*cos(d*x+c)^2*sin(d*x+c)+871*I*cos(d*x+c)+195*sin
(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8/a^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(86) = 172.
time = 0.60, size = 902, normalized size = 8.20

$$-\frac{2\left(-331i\sqrt{a} - \frac{998\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{1838i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{7522\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{4836i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{27882\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{8954i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{68926\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{12631i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{125052\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{10540i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{168980\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{168980\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} + \frac{10540i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} - \frac{125052\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}} + \frac{12631i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} - \frac{68926\sqrt{a}\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}} + \frac{8954i\sqrt{a}\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} - \frac{27882\sqrt{a}\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}} + \frac{4836i\sqrt{a}\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}} - \frac{7522\sqrt{a}\sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}} + \frac{1838i\sqrt{a}\sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}} - \frac{998\sqrt{a}\sin(dx+c)^{23}}{(\cos(dx+c)+1)^{23}} + \frac{331i\sqrt{a}\sin(dx+c)^{24}}{(\cos(dx+c)+1)^{24}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}}{3315\left(a^4 - \frac{12a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{66a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{220a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{495a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{792a^4\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{924a^4\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{792a^4\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{495a^4\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} - \frac{220a^4\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} + \frac{66a^4\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}} - \frac{12a^4\sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}} + \frac{a^4\sin(dx+c)^{24}}{(\cos(dx+c)+1)^{24}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3315*(-331*I*sqrt(a) - 998*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 183
8*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 7522*sqrt(a)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 4836*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 27882*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8954*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 68926*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 12631*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 125052*sqr
t(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10540*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 168980*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^1
1 - 168980*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 10540*I*sqrt(a)*
sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 125052*sqrt(a)*sin(d*x + c)^15/(cos
(d*x + c) + 1)^15 + 12631*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 -
 68926*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 8954*I*sqrt(a)*sin(d
*x + c)^18/(cos(d*x + c) + 1)^18 - 27882*sqrt(a)*sin(d*x + c)^19/(cos(d*x +
 c) + 1)^19 + 4836*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 7522*s
qrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 1838*I*sqrt(a)*sin(d*x + c)^
22/(cos(d*x + c) + 1)^22 - 998*sqrt(a)*sin(d*x + c)^23/(cos(d*x + c) + 1)^2
3 + 331*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4
 - 12*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^4*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 220*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 495*a^4*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^4*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 924*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*a^4*sin(d*x + c
)^14/(cos(d*x + c) + 1)^14 + 495*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
- 220*a^4*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^4*sin(d*x + c)^20/(c
os(d*x + c) + 1)^20 - 12*a^4*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a^4*si
n(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(86) = 172$.
time = 0.49, size = 173, normalized size = 1.57

$$-\frac{512\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-255i\,e^{(4i\,dx+4i\,c)}-68i\,e^{(2i\,dx+2i\,c)}-8i\right)}{3315\,\left(a^4de^{(16i\,dx+16i\,c)}+8\,a^4de^{(14i\,dx+14i\,c)}+28\,a^4de^{(12i\,dx+12i\,c)}+56\,a^4de^{(10i\,dx+10i\,c)}+70\,a^4de^{(8i\,dx+8i\,c)}+56\,a^4de^{(6i\,dx+6i\,c)}+28\,a^4de^{(4i\,dx+4i\,c)}+8\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -512/3315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-255*I*e^(4*I*d*x + 4*
I*c) - 68*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^4*d*e^(16*I*d*x + 16*I*c) + 8*a^4
*d*e^(14*I*d*x + 14*I*c) + 28*a^4*d*e^(12*I*d*x + 12*I*c) + 56*a^4*d*e^(10*
I*d*x + 10*I*c) + 70*a^4*d*e^(8*I*d*x + 8*I*c) + 56*a^4*d*e^(6*I*d*x + 6*I*
c) + 28*a^4*d*e^(4*I*d*x + 4*I*c) + 8*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [**B**]
time = 9.28, size = 105, normalized size = 0.95

$$\frac{512\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,68\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,255\mathrm{i}+8\mathrm{i}\right)}{3315\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] (512*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*68i + exp(c*4i + d*x*4i)*255i +
 8i))/(3315*a^4*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 8/143*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+2/13*I*a*sec(d*x+c)^1
1/d/(a+I*a*tan(d*x+c))^(9/2)
```

Rubi [A]
time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*
I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{13}(4a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\ &= \frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.84, size = 82, normalized size = 1.12

$$-\frac{2i\sec^9(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-15i+11\tan(c+dx))}{143a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/143)*Sec[c + d*x]^9*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-15*I
 + 11*Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(61) = 122.
time = 0.86, size = 144, normalized size = 1.97

method	result
default	$\frac{2\left(128i\left(\cos^7(dx+c)\right)+128\sin(dx+c)\left(\cos^6(dx+c)\right)-16i\left(\cos^5(dx+c)\right)+48\sin(dx+c)\left(\cos^4(dx+c)\right)-148i\left(\cos^3(dx+c)\right)-108\left(\cos^2(dx+c\right.\right.}{143d\cos(dx+c)^6a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/143/d*(128*I*cos(d*x+c)^7+128*sin(d*x+c)*cos(d*x+c)^6-16*I*cos(d*x+c)^5+4
8*sin(d*x+c)*cos(d*x+c)^4-148*I*cos(d*x+c)^3-108*cos(d*x+c)^2*sin(d*x+c)+51
*I*cos(d*x+c)+11*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
/cos(d*x+c)^6/a^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(57) = 114.
time = 0.53, size = 764, normalized size = 10.47

$$-\frac{2\left(-15i\sqrt{a}-\frac{38\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{88i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{278\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{213i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{920\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{272i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1848\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{182i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2548\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2548\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{182i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{1848\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{272i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{920\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{213i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{278\sqrt{a}\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}+\frac{88i\sqrt{a}\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}-\frac{38\sqrt{a}\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}+\frac{15i\sqrt{a}\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}}{143\left(a^4-\frac{10a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{120a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{210a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{252a^4\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{210a^4\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{120a^4\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{45a^4\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{10a^4\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}+\frac{a^4\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(-15*I*sqrt(a) - 38*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 278*sqrt(a)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - 213*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 920
*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 272*I*sqrt(a)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 1848*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
182*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2548*sqrt(a)*sin(d*x +
```

```
c)^9/(cos(d*x + c) + 1)^9 - 2548*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 182*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1848*sqrt(a)*si
n(d*x + c)^13/(cos(d*x + c) + 1)^13 + 272*I*sqrt(a)*sin(d*x + c)^14/(cos(d*
x + c) + 1)^14 - 920*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 213*I*
sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 278*sqrt(a)*sin(d*x + c)^17
/(cos(d*x + c) + 1)^17 + 88*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18
 - 38*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 + 15*I*sqrt(a)*sin(d*x
+ c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 10*a^4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 45*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 120*a^4*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^4*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 252*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^4*sin(d*x +
c)^12/(cos(d*x + c) + 1)^12 - 120*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + 45*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10*a^4*sin(d*x + c)^18/(c
os(d*x + c) + 1)^18 + a^4*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*si
n(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7
/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.
time = 0.44, size = 132, normalized size = 1.81

$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(2i\,dx+2i\,c)}-2i\right)}{143\left(a^4de^{(12i\,dx+12i\,c)}+6\,a^4de^{(10i\,dx+10i\,c)}+15\,a^4de^{(8i\,dx+8i\,c)}+20\,a^4de^{(6i\,dx+6i\,c)}+15\,a^4de^{(4i\,dx+4i\,c)}+6\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -128/143*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(2*I*d*x + 2*I*
c) - 2*I)/(a^4*d*e^(12*I*d*x + 12*I*c) + 6*a^4*d*e^(10*I*d*x + 10*I*c) + 15
*a^4*d*e^(8*I*d*x + 8*I*c) + 20*a^4*d*e^(6*I*d*x + 6*I*c) + 15*a^4*d*e^(4*I
*d*x + 4*I*c) + 6*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 6.62, size = 91, normalized size = 1.25

$$\frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,13\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{143\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] (128*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*13i + 2i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(143*a^4*d*(exp(c*2i
 + d*x*2i) + 1)^6)
```

3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 2/9*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{3574\}$

$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

Mathematica [A]
time = 0.65, size = 59, normalized size = 1.69

$$\frac{2i\sec^7(c+dx)(i+\tan(c+dx))}{9a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

[Out] (((2*I)/9)*Sec[c + d*x]^7*(I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(29) = 58$.
time = 0.77, size = 115, normalized size = 3.29

method	result
default	$\frac{2(16i(\cos^5(dx+c))+16\sin(dx+c)(\cos^4(dx+c))-20i(\cos^3(dx+c))-12(\cos^2(dx+c))\sin(dx+c)+5i\cos(dx+c)+\sin(dx+c))\sqrt{\frac{a(i\sin(c}{}}}{9d\cos(dx+c)^4a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/9/d*(16*I*cos(d*x+c)^5+16*sin(d*x+c)*cos(d*x+c)^4-20*I*cos(d*x+c)^3-12*co
s(d*x+c)^2*sin(d*x+c)+5*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c
))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/a^4

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(27) = 54$.
time = 0.47, size = 626, normalized size = 17.89

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{6i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{42\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{70\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{70\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{14i\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{42\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{14i\sqrt{a}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{14\sqrt{a}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{6i\sqrt{a}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{2\sqrt{a}\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}+\frac{i\sqrt{a}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}}{9\left(a^4-\frac{8a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{56a^4\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^4\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^4\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^4\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")

[Out] -2/9*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 14*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sqrt(a)*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt(a)*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 - 70*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 70*sqrt(a)*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 14*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x +
 c) + 1)^10 - 42*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 14*I*sqrt(
a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 14*sqrt(a)*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 + 6*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2*sq
rt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + I*sqrt(a)*sin(d*x + c)^16/(co
s(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 8*a^4*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 28*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a^4*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 70*a^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^
4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^4*sin(d*x + c)^12/(cos(d*x +
 c) + 1)^12 - 8*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^4*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.
time = 0.40, size = 89, normalized size = 2.54

$$\frac{32i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{9\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(6i\,dx+6i\,c)}+6\,a^4de^{(4i\,dx+4i\,c)}+4\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 32/9*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^4*d*e^(8*I*d*x + 8*I*c)
 + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x + 4*I*c) + 4*a^4*d*e^(2
*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B]
time = 6.49, size = 50, normalized size = 1.43

$$\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a\,\sin\left(c+d\,x\right)\,1\mathrm{i}}{\cos\left(c+d\,x\right)}}\,2\mathrm{i}}{9\,a^4\,d\,{\cos\left(c+d\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] (exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(9*a
^4*d*cos(c + d*x)^4)
```

3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=160

$$\frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{}{a^3d}$$

```
[Out] 8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(7/2)/d-8*I*sec(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*sec(d*x+c)^
5/a/d/(a+I*a*tan(d*x+c))^(5/2)-4/3*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^
(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3572, 3570, 212}

$$\frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((8*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(7/2)*d) - (((2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*
x])^(5/2)) - (((4*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)
) - ((8*I)*Sec[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
```

```
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} + \frac{2\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{a}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{4\int \frac{\sec^3(c\text{-}}{(a+ia\tan(c}}{a^2}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(}{a^3d\sqrt{a+ia}}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(}{a^3d\sqrt{a+ia}}$$

$$= \frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A]
time = 1.52, size = 130, normalized size = 0.81

$$-\frac{128e^{7i(c+dx)}\left(-23-35e^{2i(c+dx)}-15e^{4i(c+dx)}+15\left(1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{15a^3d\left(1+e^{2i(c+dx)}\right)^6(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-128*E^((7*I)*(c + d*x))*(-23 - 35*E^((2*I)*(c + d*x)) - 15*E^((4*I)*(c +
d*x)) + 15*(1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[Sqrt[1 + E^((2*I)*(c + d
*x))]]))/(15*a^3*d*(1 + E^((2*I)*(c + d*x)))^6*(-I + Tan[c + d*x])^3*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(133) = 266.
time = 1.07, size = 399, normalized size = 2.49

method	result

default	$\frac{2\left(-15(\cos^2(dx+c))\sin(dx+c)\sqrt{2}\,\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-30\cos(dx+c)\sin(dx+c)\sqrt{2}\,\mathrm{arc}\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/15/d*(-15*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-30*cos(d*x+c)*sin(d*x+c)*2^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-15*2^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+92*I*cos(d*x+c)^3+92*cos(d*x+c)^2*sin(d*x+c)-76*I*cos(d*x+c)^2-16*sin(d*x+c)*cos(d*x+c)-19*I*cos(d*x+c)-3*sin(d*x+c)+3*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2/a^4

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1164 vs. $2(125) = 250$.
time = 0.67, size = 1164, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")

[Out] -2/15*(15*(2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + (-I*sqrt(2)*cos(2*d*x + 2*c)^2 - I*sqrt(2)*sin(2*d*x + 2*c)^2 - 2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +

```
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (
I*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*((15*
I*sqrt(2)*cos(4*d*x + 4*c) + 35*I*sqrt(2)*cos(2*d*x + 2*c) - 15*sqrt(2)*sin
(4*d*x + 4*c) - 35*sqrt(2)*sin(2*d*x + 2*c) + 23*I*sqrt(2))*cos(5/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (15*sqrt(2)*cos(4*d*x + 4*c) +
35*sqrt(2)*cos(2*d*x + 2*c) + 15*I*sqrt(2)*sin(4*d*x + 4*c) + 35*I*sqrt(2)*
sin(2*d*x + 2*c) + 23*sqrt(2))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*sqrt(a))/((a^4*cos(2*d*x + 2*c)^2 + a^4*sin(2*d*x + 2*c)^2 +
2*a^4*cos(2*d*x + 2*c) + a^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(125) = 250$.
time = 0.40, size = 326, normalized size = 2.04

$$-\frac{4\left(15\sqrt{2}\left(i a^{4} d e^{(4 i d x+4 i c)}+2 i a^{4} d e^{(2 i d x+2 i c)}+i a^{4} d\right) \sqrt{\frac{1}{a^{7} d^{2}}} \log \left(-\frac{32\left(\left(i a^{3} d e^{(2 i d x+2 i c)}+i a^{3} d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{1}{a^{7} d^{2}}}-i\right) e^{(-i d x-i c)}}{a^{3} d}\right)+15\sqrt{2}\left(-i a^{4} d e^{(4 i d x+4 i c)}-2 i a^{4} d e^{(2 i d x+2 i c)}-i a^{4} d\right) \sqrt{\frac{1}{a^{7} d^{2}}} \log \left(-\frac{32\left(\left(-i a^{3} d e^{(2 i d x+2 i c)}-i a^{3} d\right) \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \sqrt{\frac{1}{a^{7} d^{2}}}-i\right) e^{(-i d x-i c)}}{a^{3} d}\right)+2\sqrt{2} \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\left(15 i e^{(4 i d x+4 i c)}+35 i e^{(2 i d x+2 i c)}+23 i\right)\right)}{15\left(a^{4} d e^{(4 i d x+4 i c)}+2 a^{4} d e^{(2 i d x+2 i c)}+a^{4} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(15*sqrt(2)*(I*a^4*d*e^(4*I*d*x + 4*I*c) + 2*I*a^4*d*e^(2*I*d*x + 2*I
*c) + I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*
a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x -
 I*c)/(a^3*d)) + 15*sqrt(2)*(-I*a^4*d*e^(4*I*d*x + 4*I*c) - 2*I*a^4*d*e^(2*
I*d*x + 2*I*c) - I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((-I*a^3*d*e^(2*I*d*x +
 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)
*e^(-I*d*x - I*c)/(a^3*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(1
5*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 23*I))/(a^4*d*e^(4*I*d
*x + 4*I*c) + 2*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{7}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^7\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -3*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/
2)/a^(7/2)/d-2*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+6*I*sec(d*x+c)/a
^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3582, 3583, 3570, 212}

$$-\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-3*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
 + d*x]])]])/(a^(7/2)*d) - ((2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) + ((6*I)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
```

```
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{a}\\
&= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{12i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{12\int\frac{\sec(c+dx)}{(a+ia\tan(c+d}}{a^2}\\
&= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3\int\frac{\sec(c+}{\sqrt{a+ia\tan}}}{a^3}\\
&= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{(6i)\text{Subst}\left(\int\frac{1}{2-}\right.}{}\\
&= -\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\end{aligned}$$

Mathematica [A]
time = 1.11, size = 126, normalized size = 1.04

$$\frac{16e^{5i(c+dx)}\left(-1-3e^{2i(c+dx)}+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{a^3d\left(1+e^{2i(c+dx)}\right)^4(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (16*E^((5*I)*(c + d*x))*(-1 - 3*E^((2*I)*(c + d*x)) + 3*E^((2*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(a^
```

```
3*d*(1 + E^((2*I)*(c + d*x)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(102) = 204$.
time = 0.82, size = 318, normalized size = 2.63

method	result
default	$-\left(3i\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)+3i\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{(i\,c}{2\sin}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d
*x+c)*2^(1/2)+3*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))
*2^(1/2)+3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*sin(d*x+c)-8*I*cos(d*x+c)^3-8*cos(d*x+c)^2*sin(d*x+c)-4*sin(d*x+c))*(a*(I
*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/a^4
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(96) = 192$.
time = 0.40, size = 245, normalized size = 2.02

$$\frac{\left(-3i\sqrt{2}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(-\frac{12\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+i\right)e^{(-i\,dx-i\,c)}}{a^3d}\right)+3i\sqrt{2}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(2i\,dx+2i\,c)}\log\left(-\frac{12\left(\left(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+i\right)e^{(-i\,dx-i\,c)}}{a^3d}\right)-2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(2i\,dx+2i\,c)}-i\right)\right)e^{(-2i\,dx-2i\,c)}}{2\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((I*a
^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + 3*I*sqrt(2)*a^4*d*sqrt(1/(a^7
*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)
/(a^3*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(2*I*d*x +
2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\,{(a+a\,\tan(c+d\,x)\,1{}\mathrm{i})}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,a^{7/2}d}+\frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -1/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(7/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)-1/8*I*sec(d*x
+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3582, 3583, 3570, 212}

$$-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,a^{7/2}d}-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/8*I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]
])]])/(Sqrt[2]*a^(7/2)*d) + ((I/2)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) - ((I/8)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
```

```
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}} - \frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{3a}$$

$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+d}{\sqrt{a+ia\tan(}}}{16a^3}$$

$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{i\text{Subst}\left(\int \frac{1}{2-ax^2}\right.}{}$$

$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}\,a^{7/2}d} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{}{8a^2}$$

Mathematica [A]
time = 1.05, size = 120, normalized size = 0.96

$$\frac{i\sec^3(c+dx)\left(-3+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-3\cos(2(c+dx))+i\sin(2(c+dx))\right)}{16a^3d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/16)*Sec[c + d*x]^3*(-3 + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 3*Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(100) = 200$.
time = 0.92, size = 346, normalized size = 2.77

method	result
default	$\frac{\left(64i(\cos^5(dx+c))+64\sin(dx+c)(\cos^4(dx+c))-i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(dx+c))\sqrt{2}}{2\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)\cos(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(64*I*cos(d*x+c)^5+64*sin(d*x+c)*cos(d*x+c)^4-I*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)*2^(1/2)-56*I*cos(d*x+c)^3-
2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c
)-I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)-24*co
s(d*x+c)^2*sin(d*x+c)+4*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+
c))^(1/2)/a^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(94) = 188$.
time = 0.63, size = 977, normalized size = 7.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/64*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(4*d*x + 4*c)
- I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*sqrt(a) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x +
4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*c
os(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
```

```
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1))*sqrt(a))/(a^4*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(94) = 188$.
time = 0.40, size = 267, normalized size = 2.14

$$\frac{\left(-i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+i\right)e^{(-i\,dx-i\,c)}}{4\,a^3d}\right)+i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}+i\right)e^{(-i\,dx-i\,c)}}{4\,a^3d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+2i\right)\right)e^{(-4i\,dx-4i\,c)}}{16\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sq
rt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + I*sqrt(1/
2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sqrt(2)*sqrt(1/2)*
(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + 2*I))*
e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}+\frac{5i}{64a^2d(a-}$$

```
[Out] 5/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(7/2)/d*2^(1/2)+1/6*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(7/2)+5/48*I*sec(d*x+
c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+5/64*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^
(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3570, 212}

$$\frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((5*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(7/2)) + (((5*I)/48)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((
(5*I)/64)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
```

```
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{12a}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx}}{32a^2}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c}{64a^2d(a+ia\,\mathrm{ta}}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c}{64a^2d(a+ia\,\mathrm{ta}}$$

$$= \frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{}{48a}$$

Mathematica [A]
time = 1.66, size = 119, normalized size = 0.76

$$-\frac{\sec^3(c+dx)\left(52+\frac{30e^{4i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+82\cos(2(c+dx))+50i\sin(2(c+dx))\right)}{384a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/384*(Sec[c + d*x]^3*(52 + (30*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 82*Cos[2*(c + d*x)] + (50
*I)*Sin[2*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(126) = 252.
time = 0.71, size = 373, normalized size = 2.38

method	result
default	$\frac{\left(1024i(\cos^7(dx+c))+1024\sin(dx+c)(\cos^6(dx+c))-704i(\cos^5(dx+c))+15i\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{(i\cos(dx+c)-i+\sin(d}{2\sin(dx+c)\sqrt{-\frac{2\cos}{1+c}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/d*(1024*I*cos(d*x+c)^7+1024*sin(d*x+c)*cos(d*x+c)^6-704*I*cos(d*x+c)^
5+15*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)
*2^(1/2)-192*sin(d*x+c)*cos(d*x+c)^4+8*I*cos(d*x+c)^3+15*I*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+15*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+40*cos(d*x+c)^2*sin(
d*x+c)-60*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(118) = 236$.
time = 0.38, size = 278, normalized size = 1.77

$$\frac{\left(-15i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,a^3d}\right)+15i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,a^3d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}(33i\,e^{(6i\,dx+6i\,c)}+59i\,e^{(4i\,dx+4i\,c)}+34i\,e^{(2i\,dx+2i\,c)}+8i)\right)e^{(-6i\,dx-6i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(-15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/3
2*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 15*I
*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/32*(sqrt(2)*s
qrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
```

```
c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*(33*I*e^(6*I*d*x + 6*I*c) + 59*I*e^(4*I*d*x + 4*
I*c) + 34*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c + d\,x)\,\left(a + a\tan(c + d\,x)\,1\mathrm{i}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=227

$$\frac{315i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}\,a^{7/2}d}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}+\frac{}{256a^2d}$$

```
[Out] 315/4096*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))
/a^(7/2)/d*2^(1/2)+105/1024*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-315
/2048*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+1/8*I*cos(d*x+c)/d/(a+I*a
*tan(d*x+c))^(7/2)+3/32*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+21/256*I*
cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3583, 3571, 3570, 212}

$$\frac{315i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}\,a^{7/2}d}-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((315*I)/2048)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])]))/(Sqrt[2]*a^(7/2)*d) + ((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d
*x])^(7/2)) + (((3*I)/32)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
+ (((21*I)/256)*Cos[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((105
*I)/1024)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((315*I)/2048
)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{9\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{16a}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21\int \frac{\cos(c+dx)}{(a+ia\tan(c+}}{64a^2}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(}{256a^2d(a+ia\,\mathrm{t}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(}{256a^2d(a+ia\,\mathrm{t}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(}{256a^2d(a+ia\,\mathrm{t}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(}{256a^2d(a+ia\,\mathrm{t}}$$

$$= \frac{315i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}\,a^{7/2}d} + \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{}{3}$$

Mathematica [A]

time = 2.49, size = 141, normalized size = 0.62

$$-\frac{\sec^3(c+dx)\left(420+\frac{630e^{4i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+826\cos(2(c+dx))-224\cos(4(c+dx))+474i\sin(2(c+dx))-288i\sin(4(c+dx))\right)}{4096a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/4096*(Sec[c + d*x]^3*(420 + (630*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 826*Cos[2*(c + d*x)] -
 224*Cos[4*(c + d*x)] + (474*I)*Sin[2*(c + d*x)] - (288*I)*Sin[4*(c + d*x)]
))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(184) = 368.
time = 0.86, size = 400, normalized size = 1.76

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(8192i\left(\cos^9(dx+c)\right)+8192\sin(dx+c)\left(\cos^8(dx+c)\right)-5120i\left(\cos^7(dx+c)\right)-1024\sin(dx+c)\left(\cos^6(dx+c\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8192/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(8192*I*cos(d*x+c)^
9+8192*sin(d*x+c)*cos(d*x+c)^8-5120*I*cos(d*x+c)^7-1024*sin(d*x+c)*cos(d*x+
c)^6+64*I*cos(d*x+c)^5+315*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/
2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2))*cos(d*x+c)*2^(1/2)+576*sin(d*x+c)*cos(d*x+c)^4+315*I*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+168*I*cos(d*x+c)^3+
315*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d
*x+c)+840*cos(d*x+c)^2*sin(d*x+c)-1260*I*cos(d*x+c))/a^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2779 vs. 2(172) = 344.
time = 0.67, size = 2779, normalized size = 12.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/16384*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(325*((-I*sqrt(2)*cos(8*d*x + 8*
c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
+ 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(8*d*x
 + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8
*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(
7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*a
rctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(-I*sqrt(2)*cos(8*d
*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(3/2*arctan2(sin(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))) + 1)) + 325*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8
*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
+ 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c))) + 1)) + 643*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
 + 8*c))*sin(3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
)), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + 4
*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) + 1)^(1/4)*(765*((I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)
*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 +
 (I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(I*sqrt(2)*cos(8*d*x + 8*c) + sqrt
(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)*sin(8*d*x + 8*c))*cos(5/2*arctan2(si
n(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (187*I*sqrt(2)*cos(8*d*x + 8*c) + 187
*sqrt(2)*sin(8*d*x + 8*c) + 128*I*sqrt(2))*cos(1/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c))) + 1)) - 765*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*
x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)
*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8
*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)
*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(5/2*arctan2(sin(1/4*arc
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) + 1)) - (187*sqrt(2)*cos(8*d*x + 8*c) - 187*I*sqrt(2)*
sin(8*d*x + 8*c) + 128*sqrt(2))*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x +
```

```
8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c))) + 1)))*sqrt(a) + 315*(2*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8
*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)
*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1) - 2*
sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c),
 cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + sqrt(
cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8...
```

Fricas [A]
time = 0.37, size = 300, normalized size = 1.32

$$\frac{\left(-315i\,\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(8i\,dx+8i\,c)}\log\left(-\frac{315\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{1024\,a^3d}\right)+315i\,\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(8i\,dx+8i\,c)}\log\left(-\frac{315\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{1024\,a^3d}\right)+\sqrt{2}\,\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,\left(-128i\,e^{(10i\,dx+10i\,c)}+197i\,e^{(8i\,dx+8i\,c)}+535i\,e^{(6i\,dx+6i\,c)}+298i\,e^{(4i\,dx+4i\,c)}+104i\,e^{(2i\,dx+2i\,c)}+16i\right)\right)e^{(-8i\,dx-8i\,c)}}{4096\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(-315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-3
15/1024*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d))
+ 315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-315/1024
*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(10*I*d*x + 10*I*c) + 197*I
*e^(8*I*d*x + 8*I*c) + 535*I*e^(6*I*d*x + 6*I*c) + 298*I*e^(4*I*d*x + 4*I*c
) + 104*I*e^(2*I*d*x + 2*I*c) + 16*I))*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=307

$$\frac{3003i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}\,a^{7/2}d}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+192$$

```
[Out] 3003/32768*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(7/2)/d*2^(1/2)+1001/8192*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+
429/5120*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-3003/16384*I*cos(d*x
+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-1001/10240*I*cos(d*x+c)^3*(a+I*a*tan(d*x
+c))^(1/2)/a^4/d+1/10*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(7/2)+13/160*I*co
s(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+143/1920*I*cos(d*x+c)^3/a^2/d/(a+I*
a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.36, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3583, 3578, 3571, 3570, 212}

$$\frac{3003i\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(c+dx)}{\sqrt{2}\,\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}\,a^{7/2}d}-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}-\frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((3003*I)/16384)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[
c + d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[
c + d*x])^(7/2)) + (((13*I)/160)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x]
)^(5/2)) + (((143*I)/1920)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3
/2)) + (((1001*I)/8192)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((429*I)/5120)*Cos[c + d*x]^3)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((300
3*I)/16384)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) - (((1001*I)/1
0240)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
```

```
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{20a}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143\int \frac{\cos^3(c}{(a+ia\tan(c}}{320a}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos}{1920a^2d(a+ia}$$

$$= \frac{3003i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}\,a^{7/2}d} + \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} +$$

Mathematica [A]
time = 2.83, size = 175, normalized size = 0.57

$$-\frac{\left(42140+20048e^{-2i(c+dx)}+71190e^{2i(c+dx)}+5856e^{-4i(c+dx)}-48640e^{4i(c+dx)}+768e^{-6i(c+dx)}-2560e^{6i(c+dx)}+\frac{90090e^{4i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)\sec^3(c+dx)}{491520a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/491520*((42140 + 20048/E^((2*I)*(c + d*x)) + 71190*E^((2*I)*(c + d*x)) +
 5856/E^((4*I)*(c + d*x)) - 48640*E^((4*I)*(c + d*x)) + 768/E^((6*I)*(c + d
*x)) - 2560*E^((6*I)*(c + d*x)) + (90090*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(a
^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.93, size = 427, normalized size = 1.39

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(786432i(\cos^{11}(dx+c))+786432(\cos^{10}(dx+c))\sin(dx+c)-466944i(\cos^{9}(dx+c))-73728\sin(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/983040/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(786432*I*cos(d*x
+c)^11+786432*cos(d*x+c)^10*sin(d*x+c)-466944*I*cos(d*x+c)^9-73728*sin(d*x+
c)*cos(d*x+c)^8+5120*I*cos(d*x+c)^7+66560*sin(d*x+c)*cos(d*x+c)^6+9152*I*co
s(d*x+c)^5+45045*I*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*cos(d*x+c)*2^(1/2)+82368*sin(d*x+c)*cos(d*x+c)^4+45045*2^(1/2)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+45045*I*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x
+c)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*2^(1/2)+24024*I*cos(d*x+c
)^3+120120*cos(d*x+c)^2*sin(d*x+c)-180180*I*cos(d*x+c))/a^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5821 vs. 2(236) = 472.
time = 0.81, size = 5821, normalized size = 18.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/1966080*(40*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2
+ sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*ar
ctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(3/4)*((79*(-I*sqrt(2)*
cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c)))^2 + 79*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt
(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c)))^2 + 837*(-I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) - I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1
58*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 837*(sqrt(2)*cos(1/5*arctan
```

```
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(4/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c))) - 79*I*sqrt(2)*cos(10*d*x + 10*c) - 79*sqrt(2)*si
n(10*d*x + 10*c))*cos(7/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(1
0*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
 1)) + (-49*I*sqrt(2)*cos(10*d*x + 10*c) - 1155*I*sqrt(2)*cos(4/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sqrt(2)*cos(3/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*I*sqrt(2)*cos(2/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) - 49*sqrt(2)*sin(10*d*x + 10*c) - 1155
*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*sq
rt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*sqrt(2
)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 128*I*sqrt(2))
*cos(3/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))),
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + (79*(sqrt(
2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 79*(sqrt(2)*cos(10*d*x + 10*c) - I*
sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c)))^2 + 837*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
 sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 158*(s
qrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) + 837*(-I*sqrt(2)*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2))*sin(4/5*arctan2(sin(10*d*x +
 10*c), cos(10*d*x + 10*c))) + 79*sqrt(2)*cos(10*d*x + 10*c) - 79*I*sqrt(2)
*sin(10*d*x + 10*c))*sin(7/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) + 1)) + (49*sqrt(2)*cos(10*d*x + 10*c) + 1155*sqrt(2)*cos(4/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))) - 3264*sqrt(2)*cos(3/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 624*sqrt(2)*cos(2/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))) - 49*I*sqrt(2)*sin(10*d*x + 10*c) - 1155*I*s
qrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sq
rt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*I*sqrt
(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 128*sqrt(2))
*sin(3/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))),
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)))*sqrt(a) + 4
*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arct
an2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*(105*((-I*sqrt(2)*cos(10*d*x +
10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c)))^4 + (-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x +
 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^4 + 4*(-I*
```

```
sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c)))^3 + 6*(-I*sqrt(2)*cos(10*d*x + 10*c)
 - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d
*x + 10*c)))^2 + 2*((-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 1
0*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*(-I*sq
rt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(
2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*
c)))^2 + 4*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos
(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1...
```

Fricas [A]
time = 0.45, size = 322, normalized size = 1.05

$$\frac{\left(-45045i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(10i\,dx+10i\,c)}\log\left(-\frac{3003\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{8192\,a^3d}\right)+45045i\sqrt{\frac{1}{2}}\,a^4d\sqrt{\frac{1}{a^7d^2}}\,e^{(10i\,dx+10i\,c)}\log\left(-\frac{3003\left(\sqrt{2}\sqrt{\frac{1}{2}}\,(-i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}-i\right)e^{(-i\,dx-i\,c)}}{8192\,a^3d}\right)+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1280i\,e^{(14i\,dx+14i\,c)}-25600i\,e^{(12i\,dx+12i\,c)}+11275i\,e^{(10i\,dx+10i\,c)}+56665i\,e^{(8i\,dx+8i\,c)}+31094i\,e^{(6i\,dx+6i\,c)}+12952i\,e^{(4i\,dx+4i\,c)}+3312i\,e^{(2i\,dx+2i\,c)}+384i\right)\right)e^{(-10i\,dx-10i\,c)}}{491520\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)*
log(-3003/8192*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a
^3*d)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)*lo
g(-3003/8192*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^
3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1280*I*e^(14*I*d*x + 14
*I*c) - 25600*I*e^(12*I*d*x + 12*I*c) + 11275*I*e^(10*I*d*x + 10*I*c) + 566
65*I*e^(8*I*d*x + 8*I*c) + 31094*I*e^(6*I*d*x + 6*I*c) + 12952*I*e^(4*I*d*x
 + 4*I*c) + 3312*I*e^(2*I*d*x + 2*I*c) + 384*I))*e^(-10*I*d*x - 10*I*c)/(a^
4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=524

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{3/2}e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\text{ArcT}}{\sqrt{2}\,d}$$

```
[Out] I*a*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*a
rctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/
2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+
1/2*I*a^(3/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(
1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(
a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))
*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/4
*I*a^(3/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e
*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (I*a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sq
rt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((I/2)*a^(
3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/S
qrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(S
qrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^
(3/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(ae\sec(c+dx))\int \sqrt{e\sec(c+dx)}\ldots}{2\sqrt{a-ia\tan(c+dx)}\,\sqrt{\ldots}}$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(2ia^2e^3\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{x\ldots}{a^2+\ldots}\right.}{d\sqrt{a-ia\tan(c+dx)}\ldots}$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(ia^2e^2\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{a-e\ldots}{a^2+e^2\ldots}\right.}{d\sqrt{a-ia\tan(c+dx)}\ldots}$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(ia^2e\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{\ldots}{\frac{a}{e}-\sqrt{\ldots}}\right.}{2d\sqrt{a-ia\tan(c+\ldots}}$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{\ldots}}{\sqrt{e\sec\ldots}}\right.}{2\sqrt{2}\,d\sqrt{\ldots}}$$

$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a\ldots}}{\sqrt{a}\sqrt{e\ldots}}\right.}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx\ldots}}$$

Mathematica [A]

time = 1.95, size = 373, normalized size = 0.71

$$\frac{e\sqrt{e\sec(c+dx)}\,(\cos(c)-i\sin(c))\left(\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos(c+dx)\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{1+i\cos(c)-\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}+\sqrt{-1+i\cos(c)+\sin(c)}\left(\sqrt{-1-i\cos(c)-\sin(c)}\,(i\cos(dx)+\sin(dx))\sqrt{i-\tan\left(\frac{dx}{2}\right)}-\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos(c+dx)\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Sec[c + d*x]]*(Cos[c] - I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] - S
in[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(
d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Si
n[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(Sqrt[-1 - I*Co
s[c] - Sin[c]]*(I*Cos[d*x] + Sin[d*x])*Sqrt[I - Tan[(d*x)/2]] - ArcTanh[(Sq
rt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin
[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt
[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[-1 - I*Cos[c] - Si
n[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A]
time = 3.76, size = 309, normalized size = 0.59

method	result
default	$\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)(-1+\cos(dx+c))^2\left(i\cos(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\mathrm{si}}{2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*c
os(d*x+c)*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+co
s(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+co
s(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-2*
cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-2*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^
3/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(3/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1825 vs. 2(372) = 744.
time = 0.66, size = 1825, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -8*(2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arc
tan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*
cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2
*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*sqrt(2)*cos(2*d*x + 2*c) - sqr
t(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (
sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(-2*sqr
t(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - (-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*s
qrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(
```

```
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(
2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-I*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + I*
sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 16*I*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sqrt(a)*e^(3/2)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*d*x + 2*c) -
64*I))
```

Fricas [A]
time = 0.44, size = 386, normalized size = 0.74

$$\frac{d\sqrt{\frac{i\,ae^{3}}{d^{2}}}\log\left(2\left(\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+i\,d\sqrt{\frac{i\,ae^{3}}{d^{2}}}\right)e^{\left(-\frac{3}{2}\right)}\right)-d\sqrt{\frac{i\,ae^{3}}{d^{2}}}\log\left(2\left(\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-i\,d\sqrt{\frac{i\,ae^{3}}{d^{2}}}\right)e^{\left(-\frac{3}{2}\right)}\right)+d\sqrt{-\frac{i\,ae^{3}}{d^{2}}}\log\left(2\left(\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+i\,d\sqrt{-\frac{i\,ae^{3}}{d^{2}}}\right)e^{\left(-\frac{3}{2}\right)}\right)-d\sqrt{-\frac{i\,ae^{3}}{d^{2}}}\log\left(2\left(\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-i\,d\sqrt{-\frac{i\,ae^{3}}{d^{2}}}\right)e^{\left(-\frac{3}{2}\right)}\right)+\frac{4i\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{3}{2}\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(d*sqrt(I*a*e^3/d^2)*log(2*(sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3/2)
+ e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*
c) + 1) + I*d*sqrt(I*a*e^3/d^2))*e^(-3/2)) - d*sqrt(I*a*e^3/d^2)*log(2*(sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3/2) + e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2
*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - I*d*sqrt(I*a*e^3/d^2))*e^
(-3/2)) + d*sqrt(-I*a*e^3/d^2)*log(2*(sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^
(3/2) + e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x
+ 2*I*c) + 1) + I*d*sqrt(-I*a*e^3/d^2))*e^(-3/2)) - d*sqrt(-I*a*e^3/d^2)*lo
g(2*(sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3/2) + e^(2*I*d*x + 2*I*c + 3/2)
)*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - I*d*sqrt(-I*a*e^3
/d^2))*e^(-3/2)) + 4*I*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c + 3/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.395 $\int \sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=323

$$\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+}\right.}$$

```
[Out] -1/2*I*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+1/2*I*ln(a+
2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d
*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+I*arctan(1-2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)*a^(1/2)*
e^(1/2)/d-I*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*se
c(d*x+c))^(1/2))*2^(1/2)*a^(1/2)*e^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{a}\,\sqrt{e}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,d}+\frac{i\sqrt{a}\,\sqrt{e}\,\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/d - (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c
+ d*x]])])/d - (I*Sqrt[a]*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*d) + (I*Sqrt[a]*Sqrt[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
 + d*x])])/(Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(4iae^2)\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= \frac{(2iae)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(2iae}{}$$

$$= -\frac{(ia)\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{i\sqrt{a}\,\sqrt{e}\,\log\left(a-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(\right.}{\sqrt{2}\,d}$$

$$= \frac{i\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i}{}$$

Mathematica [A]
time = 1.70, size = 277, normalized size = 0.86

$$-\frac{2e\left(\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{1+i\cos(c)-\sin(c)}-\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\right)\sqrt{i+\tan\left(\frac{dx}{2}\right)}\,\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}\,\sqrt{1+\cos(2c)+i\sin(2c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*e*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]
]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I
 - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sq
rt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*Sqrt[I + Tan[(d*x)/
2]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[1 + Cos[2*c] +
 I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A]
time = 1.03, size = 230, normalized size = 0.71

method	result

default	$-\frac{\sqrt{\frac{e}{\cos(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(i\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right)-i\,\mathrm{arctanh}\left(\cdots\right.\right.}{d\sin\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*co
s(d*x+c)*(I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
-I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))-arctanh(
1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-arctanh(1/2*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))))*(-1+cos(d*x+c))/sin(d*x+c)/(I
*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(215) = 430$.
time = 0.67, size = 1399, normalized size = 4.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
```

```
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*e^(1/2)/d
```

Fricas [**A**]
time = 0.45, size = 335, normalized size = 1.04

$$\frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(\left(\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{1}{2}}+e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+d\sqrt{\frac{4i\,ae}{d^2}}\right)e^{\left(-\frac{1}{2}\right)}\right)-\frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(\left(\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{1}{2}}+e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-d\sqrt{\frac{4i\,ae}{d^2}}\right)e^{\left(-\frac{1}{2}\right)}\right)-\frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(\left(\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{1}{2}}+e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+d\sqrt{-\frac{4i\,ae}{d^2}}\right)e^{\left(-\frac{1}{2}\right)}\right)+\frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(\left(\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{1}{2}}+e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-d\sqrt{-\frac{4i\,ae}{d^2}}\right)e^{\left(-\frac{1}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(4*I*a*e/d^2)*log((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(1/2) + e
^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c)
+ 1) + d*sqrt(4*I*a*e/d^2))*e^(-1/2)) - 1/2*sqrt(4*I*a*e/d^2)*log((2*sqrt(a
```

```
/(e^(2*I*d*x + 2*I*c) + 1))*(e^(1/2) + e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*
d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - d*sqrt(4*I*a*e/d^2))*e^(-1/2
)) - 1/2*sqrt(-4*I*a*e/d^2)*log((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(1/
2) + e^(2*I*d*x + 2*I*c + 1/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2
*I*c) + 1) + d*sqrt(-4*I*a*e/d^2))*e^(-1/2)) + 1/2*sqrt(-4*I*a*e/d^2)*log((
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(1/2) + e^(2*I*d*x + 2*I*c + 1/2))*e
^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - d*sqrt(-4*I*a*e/d^2)
)*e^(-1/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \sec (c + dx)}\, \sqrt{ia (\tan (c + dx) - i)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{e}{\cos (c + d\,x)}}\, \sqrt{a + a\tan (c + d\,x)\ 1\mathrm{i}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=36

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{3569\}$

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 1.00

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.89, size = 56, normalized size = 1.56

method	result	size
default	$-\frac{2 i \sqrt{\frac{a (i \sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\, \sqrt{\frac{e}{\cos(dx+c)}}\, \cos(dx+c)}{d e}$	56
risch	$-\frac{2 i \sqrt{\frac{a\, e^{2 i (dx+c)}}{e^{2 i (dx+c)}+1}}}{\sqrt{\frac{e\, e^{i (dx+c)}}{e^{2 i (dx+c)}+1}}\, d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(1/2)*
cos(d*x+c)/e
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.53, size = 75, normalized size = 2.08

$$-\frac{2 i \sqrt{a} \sqrt{-\frac{2 i \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}\, e^{\left(-\frac{1}{2}\right)}}{d \sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)*e^(-1/2)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.
time = 0.38, size = 58, normalized size = 1.61

$$\frac{2 \sqrt{\frac{a}{e^{(2 i dx + 2 i c)} + 1}} \left(-i e^{(2 i dx + 2 i c)} - i\right) e^{\left(\frac{1}{2} i dx + \frac{1}{2} i c - \frac{1}{2}\right)}}{d \sqrt{e^{(2 i dx + 2 i c)} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(1/2*I*d
*x + 1/2*I*c - 1/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\,(\tan{(c+dx)}-i)}}{\sqrt{e\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2), x)
```

3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] 4/3*I*a*(e*sec(d*x+c))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*(a+I*a*ta
n(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((
2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}} + \frac{(2a)\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3e^2}$$
$$= \frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.22, size = 48, normalized size = 0.59

$$\frac{2(i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
[Out] (2*(I + 2*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(e*Sec[c + d*x])^(
3/2))
```

Maple [A]
time = 0.85, size = 75, normalized size = 0.93

method	result	size
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))}{3d\,e^3}$	75
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3e\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^
(1/2)*(e/cos(d*x+c))^(3/2)*cos(d*x+c)^2/e^3
```

Maxima [A]
time = 0.58, size = 53, normalized size = 0.65

$$\frac{\sqrt{a}\left(-i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)e^{\left(-\frac{3}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*d
*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))*e^(-3/2)/d
```

Fricas [A]
time = 0.38, size = 69, normalized size = 0.85

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c-\frac{3}{2}\right)}}{3\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*
d*x + 2*I*c) + 3*I)*e^(-1/2*I*d*x - 1/2*I*c - 3/2)/(d*sqrt(e^(2*I*d*x + 2*I
*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 4.63, size = 86, normalized size = 1.06

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+2\,\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}{3\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(3/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 2*sin(2*c + 2*d*x) + 1i
))/(3*d*e^2)
```

3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

```
[Out] 8/15*I*a/d/e^2/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*(a+I*a*t
an(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(5/2)-16/15*I*(a+I*a*tan(d*x+c))^(1/2)/d/
e^2/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3578, 3583, 3569}

$$\frac{8ia}{15de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/15)*a)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) - (((16*I)/
15)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} + \frac{(4a)\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{5e^2}$$

$$= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} + \frac{8\int}{}$$

$$= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} - \frac{16i}{15}$$

Mathematica [A]
time = 0.26, size = 63, normalized size = 0.52

$$\frac{i(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((I/15)*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[
c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.89, size = 85, normalized size = 0.70

method	result	size
default	$\frac{2\left(i\left(\cos^2(dx+c)\right)+4\sin(dx+c)\cos(dx+c)-8i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^3(dx+c)\right)}{15de^5}$	85
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3/e^5
```

Maxima [A]
time = 0.59, size = 129, normalized size = 1.06

$$\frac{\sqrt{a}\left(5i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)-30i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+5\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+30\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right)e^{\left(-\frac{5}{2}\right)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*e^(-5/2)/d
```

Fricas [A]
time = 0.41, size = 80, normalized size = 0.66

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)}-33i\,e^{(4i\,dx+4i\,c)}-25i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{3}{2}\,i\,dx-\frac{3}{2}\,i\,c-\frac{5}{2}\right)}}{30\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(6*I*d*x + 6*I*c) - 33*I*e^(
4*I*d*x + 4*I*c) - 25*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-3/2*I*d*x - 3/2*I*c
- 5/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 4.68, size = 101, normalized size = 0.83

$$\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(4\,\sin(c+d\,x)+4\,\sin(3\,c+3\,d\,x)-\cos(c+d\,x)\,29\mathrm{i}+\cos(3\,c+3\,d\,x)\,1\mathrm{i}\right)}{30\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(4*sin(c + d*x) - cos(c + d*x)*29i + cos(3*c +
 3*d*x)*1i + 4*sin(3*c + 3*d*x)))/(30*d*e^3)
```

3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=164

$$\frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}-\frac{16i\sqrt{\ldots}}{35d\ldots}$$

```
[Out] 12/35*I*a/d/e^2/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/35*I*a*(e*
sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(a+I*a*tan(d*x+c))^(
1/2)/d/(e*sec(d*x+c))^(7/2)-16/35*I*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d
*x+c))^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3578, 3583, 3569}

$$\frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}}+\frac{12ia}{35de^2\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((12*I)/35)*a)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) +
 (((32*I)/35)*a*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
(((2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) - (((16*I
)/35)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} + \frac{(6a)\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{7e^2}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} + \frac{24}{\ldots}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16}{3\ldots}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \ldots$$

Mathematica [A]
time = 0.33, size = 80, normalized size = 0.49

$$\frac{(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{70de^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Sin[c + d*x] + 6*Sin[3*(c +
 d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.89, size = 102, normalized size = 0.62

method	result
default	$\frac{2\left(i\left(\cos^3(dx+c)\right)+6\left(\cos^2(dx+c)\right)\sin(dx+c)+8i\cos(dx+c)+16\sin(dx+c)\right)\left(\cos^4(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}}{35d\,e^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(I*cos(d*x+c)^3+6*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+
c))*cos(d*x+c)^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+
c))^(7/2)/e^7
```

Maxima [A]
time = 0.58, size = 177, normalized size = 1.08

$$\frac{\sqrt{a}\left(7i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5i\cos\left(\frac{7}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-35i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+105i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+7\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{7}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+35\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+105\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)e^{\left(-\frac{7}{2}\right)}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))))*e^(-7/2)/d
```

Fricas [A]
time = 0.35, size = 91, normalized size = 0.55

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(8i\,dx+8i\,c)}-40i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+112i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c-\frac{7}{2}\right)}}{140\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(8*I*d*x + 8*I*c) - 40*I*e^
(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 112*I*e^(2*I*d*x + 2*I*c) +
7*I)*e^(-5/2*I*d*x - 5/2*I*c - 7/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 5.16, size = 109, normalized size = 0.66

$$\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos(2\,c+2\,d\,x)\,36\mathrm{i}+\cos(4\,c+4\,d\,x)\,1\mathrm{i}+76\,\sin(2\,c+2\,d\,x)+6\,\sin(4\,c+4\,d\,x)+35\mathrm{i}\right)}{140\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*36i + cos(4*c + 4*d*x)*1i +
76*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + 35i))/(140*d*e^4)
```

3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=453

$$\frac{7ia^{3/2}e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{7ia^{3/2}e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+d}}\right.}{8\sqrt{2}\,d}$$

```
[Out] 7/16*I*a^(3/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)-7/16*I*a^(3/2)*e^(5/2)*arctan(1+2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)
-7/32*I*a^(3/2)*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/32*I*a^(
3/2)*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/12*I*a^2*(e*sec(d*x
+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*t
an(d*x+c))^(1/2)/d-7/8*I*a*e^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.36, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3579, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{7ia^{3/2}e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{7ia^{3/2}e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{7ia^{3/2}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{16\sqrt{2}\,d}+\frac{7ia^{3/2}e^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{16\sqrt{2}\,d}+\frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{7iae^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{8d}+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((7*I)/8)*a^(3/2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/8)*a^(3/2)*e
^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqr
t[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/16)*a^(3/2)*e^(5/2)*Log[a - (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/16)*a^(3/2)*e^(5/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/12)*
a^2*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/8)*a*e
^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c +
 d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{1}{6}(7a)\int (e\sec($$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan}}{3d}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\,t}}{8d}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\,t}}{8d}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\,t}}{8d}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\,t}}{8d}$$

$$= -\frac{7ia^{3/2}e^{5/2}\log\left(a - \frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} + \mathrm{c}\right.}{16\sqrt{2}\,d}$$

$$= \frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d} - \frac{7ia}{}$$

Mathematica [A]
time = 4.57, size = 376, normalized size = 0.83

$$-\frac{a(e\sec(c+dx))^{5/2}\left(2i\sqrt{1+\cos(2c)+i\sin(2c)}\,(-9+7\cos(2c+2dx)+14i\sin(2c+2dx))\sqrt{i-\tan\left(\frac{dx}{2}\right)}+84\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos^3(c+dx)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}-84\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos^3(c+dx)\sqrt{1-i\cos(c)+\sin(c)}\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}\right)\sqrt{a+ia\tan(c+dx)}}{96d\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/96*(a*(e*Sec[c + d*x])^(5/2)*((2*I)*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(-9
+ 7*Cos[2*c + 2*d*x] + (14*I)*Sin[2*c + 2*d*x])*Sqrt[I - Tan[(d*x)/2]] + 84
*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*
Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] - 84*ArcTanh[
```

```
(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] +
Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[1 - I*Cos[c] + Sin[c]]
*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c +
d*x]])/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A]
time = 0.97, size = 414, normalized size = 0.91

method	result
default	$\frac{\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^{3}\left(21i\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))}{2}\right)(\cos^{3}(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*
(-1+cos(d*x+c))^3*(21*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))*cos(d*x+c)^3-21*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*cos(d*x+c)^3-42*I*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
*sin(d*x+c)-28*I*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-42*(1/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^3+21*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3+21*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3+16*I*(1/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-14*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+44*cos(d*x+c)*(1/(1+cos(d*x+
c)))^(1/2)+16*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(I*sin(d*x+c)+cos(d*x+
c)-1)/(1/(1+cos(d*x+c)))^(5/2)*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2734 vs. 2(304) = 608.
time = 0.83, size = 2734, normalized size = 6.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -192*(42*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*sin(
4*d*x + 4*c) + 3*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*cos(6*d*x
+ 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + I*sq
```

rt(2)*a*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*s
in(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*
x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(6*d*x + 6*c) + 3*
I*sqrt(2)*a*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*
arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt
(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d
*x + 2*c) + I*sqrt(2)*a*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(-I*sqrt(2)*a*cos(6*d*x + 6*c) - 3
*I*sqrt(2)*a*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*
sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2
*c) - I*sqrt(2)*a)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(I*sqrt(2)*a*cos(6*d*x + 6*c)
+ 3*I*sqrt(2)*a*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)
*a*sin(6*d*x + 6*c) - 3*sqrt(2)*a*sin(4*d*x + 4*c) - 3*sqrt(2)*a*sin(2*d*x
+ 2*c) + I*sqrt(2)*a)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 336*a*cos(11/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 288*a*cos(7/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 112*a*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 21*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*sin(
4*d*x + 4*c) + 3*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*log(2*sqrt(2)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + 21*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sq
rt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*sin
(4*d*x + 4*c) + 3*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*log(-2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +

```
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) + 21*(I*sqrt(2)*a*cos(6*d*x + 6*c) + 3*I*sqrt(2)*a*cos(4*d*x + 4*c)
+ 3*I*sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(6*d*x + 6*c) - 3*sqrt(2)*a
*sin(4*d*x + 4*c) - 3*sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2) + 21*(-I*sqrt(2)*a*cos(6*d*x + 6*c) - 3*I*sqrt(2)*a*cos(
4*d*x + 4*c) - 3*I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(6*d*x + 6*c)
+ 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a
)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2) + 21*(I*sqrt(2)*a*cos(6*d*x + 6*c) + 3*I*sq
rt(2)*a*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a*cos(2*...
```

<u>Fricas</u> [A]
time = 0.42, size = 594, normalized size = 1.31

$$\frac{6\sqrt{\frac{49i\,a^3e^5}{64\,d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{7\left(ae^{\frac{5}{2}}+ae^{(2i\,dx+2i\,c+\frac{5}{2})}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+8\sqrt{\frac{49i\,a^3e^5}{64\,d^2}}\,d\right)e^{(-\frac{5}{2})}}{7\,a}\right)-6\sqrt{\frac{49i\,a^3e^5}{64\,d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{7\left(ae^{\frac{5}{2}}+ae^{(2i\,dx+2i\,c+\frac{5}{2})}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-8\sqrt{\frac{49i\,a^3e^5}{64\,d^2}}\,d\right)e^{(-\frac{5}{2})}}{7\,a}\right)-6\sqrt{-\frac{49i\,a^3e^5}{64\,d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{7\left(ae^{\frac{5}{2}}+ae^{(2i\,dx+2i\,c+\frac{5}{2})}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+8\sqrt{-\frac{49i\,a^3e^5}{64\,d^2}}\,d\right)e^{(-\frac{5}{2})}}{7\,a}\right)+6\sqrt{-\frac{49i\,a^3e^5}{64\,d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{7\left(ae^{\frac{5}{2}}+ae^{(2i\,dx+2i\,c+\frac{5}{2})}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-8\sqrt{-\frac{49i\,a^3e^5}{64\,d^2}}\,d\right)e^{(-\frac{5}{2})}}{7\,a}\right)+\frac{\left(-21i\,ae^{(5i\,dx+5i\,c+\frac{5}{2})}+18i\,ae^{(3i\,dx+3i\,c+\frac{5}{2})}+7i\,ae^{(i\,dx+i\,c+\frac{5}{2})}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{12\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(6*sqrt(49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
 2*I*c) + d)*log(2/7*(7*(a*e^(5/2) + a*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
 1) + 8*sqrt(49/64*I*a^3*e^5/d^2)*d)*e^(-5/2)/a) - 6*sqrt(49/64*I*a^3*e^5/d
^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^(
5/2) + a*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 8*sqrt(49/64*I*a^3*e^5/d
^2)*d)*e^(-5/2)/a) - 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) +
2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^(5/2) + a*e^(2*I*d*x + 2*I*c +
 5/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*
I*d*x + 2*I*c) + 1) + 8*sqrt(-49/64*I*a^3*e^5/d^2)*d)*e^(-5/2)/a) + 6*sqrt(
-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
*log(2/7*(7*(a*e^(5/2) + a*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 8*sqrt
(-49/64*I*a^3*e^5/d^2)*d)*e^(-5/2)/a) + (-21*I*a*e^(5*I*d*x + 5*I*c + 5/2)
+ 18*I*a*e^(3*I*d*x + 3*I*c + 5/2) + 7*I*a*e^(I*d*x + I*c + 5/2))*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(5/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=571

$$\frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{5/2}e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\text{A}}{4\sqrt{}}$$

```
[Out] 5/4*I*a^2*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*a^(5/2)*e^(
3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c
))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^
(1/2)+5/8*I*a^(5/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/
2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1
/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(5/2)*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^
(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d
*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1
/2)-5/16*I*a^(5/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^
(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1
/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a*(e*sec(d*x+c)
)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.38, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((5*I)/4)*a^2*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
5*I)/4)*a^(5/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*
x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/4)*a^(5/2)*e^(3/2)*ArcT
an[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c +
 d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + ((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
```

```
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} + \frac{1}{4}(5a)\int (e\sec(c$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(}}{2d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(}}{2d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(}}{2d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(}}{2d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(}}{2d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}}{\sqrt{e}}\right.}{8\sqrt{2}\,d}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{5/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{}}{\sqrt{a}\sqrt{}}\right.}{4\sqrt{2}\,d\sqrt{a-ia\tan(c+}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11319 vs. 2(571) = 1142.
time = 58.46, size = 11319, normalized size = 19.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 0.96, size = 363, normalized size = 0.64

method	result
default	$\frac{\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(10i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\cos(dx+c)\sin(dx+c)-5i(\cos^2(dx+c))\,\mathrm{arct}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(
-1+cos(d*x+c))^2*(10*I*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-5*I*c
os(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
+5*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))-4*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+5*cos(d*x+c)^2*arctanh(1/2*(
1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+5*cos(d*x+c)^2*arctanh(1
/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-10*(1/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^2-14*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-4*(1/(1+cos(d*x
+c)))^(1/2))/sin(d*x+c)^3/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(3
/2)*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(403) = 806$.
time = 0.73, size = 2302, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -32*(10*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt
(2)*a*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 10*(sqrt(2)*a*
cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(4*d*x + 4
*c) + 2*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 10*(sqrt(2)*a*cos(4*d*x + 4*c) +
 2*sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(4*d*x + 4*c) + 2*I*sqrt(2)*
a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 10*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*
d*x + 2*c) + I*sqrt(2)*a*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*sin(2*d*x + 2*c)
```

```
+ sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) - 10*(-I*sqrt(2)*a*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*cos(2*d*x + 2*c) +
sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*ar
ctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 10*(I*sqrt(2)*a*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a*cos(2
*d*x + 2*c) - sqrt(2)*a*sin(4*d*x + 4*c) - 2*sqrt(2)*a*sin(2*d*x + 2*c) + I
*sqrt(2)*a)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) - 144*a*cos(5/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 80*a*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 5*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + I*sq
rt(2)*a*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*log(
2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 5*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x +
2*c) + I*sqrt(2)*a*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt
(2)*a)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 5*(-I*sqrt(2)*a*cos(4*d*x + 4*c) - 2*I*sqrt(
2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x
+ 2*c) - I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a*cos(4*
d*x + 4*c) + 2*I*sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(4*d*x + 4*c) -
2*sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) -
 5*(-I*sqrt(2)*a*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2
)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*log(2*co
```

```
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a*cos
(2*d*x + 2*c) - sqrt(2)*a*sin(4*d*x + 4*c) - 2*sqrt(2)*a*sin(2*d*x + 2*c) +
 I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1...
```

<u>Fricas</u> [A]
time = 0.45, size = 511, normalized size = 0.89

$$\frac{\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{5\left(ae^{\frac{3}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+4i\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\,d\right)e^{\left(-\frac{3}{2}\right)}}{5a}\right)-\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{5\left(ae^{\frac{3}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-4i\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\,d\right)e^{\left(-\frac{3}{2}\right)}}{5a}\right)+\sqrt{-\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{5\left(ae^{\frac{3}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+4i\sqrt{-\frac{25i\,a^3e^3}{16\,d^2}}\,d\right)e^{\left(-\frac{3}{2}\right)}}{5a}\right)-\sqrt{-\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left(\frac{5\left(ae^{\frac{3}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-4i\sqrt{-\frac{25i\,a^3e^3}{16\,d^2}}\,d\right)e^{\left(-\frac{3}{2}\right)}}{5a}\right)+\frac{\left(5i\,ae^{\frac{3}{2}}+9i\,ae^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{2\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e^
(3/2) + a*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1
/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 4*I*sqrt(25/16*I*a^3*e^
3/d^2)*d)*e^(-3/2)/a) - sqrt(25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) +
d)*log(2/5*(5*(a*e^(3/2) + a*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 4*I*
sqrt(25/16*I*a^3*e^3/d^2)*d)*e^(-3/2)/a) + sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^
(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e^(3/2) + a*e^(2*I*d*x + 2*I*c + 3/2))
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x
+ 2*I*c) + 1) + 4*I*sqrt(-25/16*I*a^3*e^3/d^2)*d)*e^(-3/2)/a) - sqrt(-25/16
*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e^(3/2) + a*e^(2*
I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 4*I*sqrt(-25/16*I*a^3*e^3/d^2)*d)*e^(-
3/2)/a) + (5*I*a*e^(3/2) + 9*I*a*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(
d*e^(2*I*d*x + 2*I*c) + d)
```

<u>Sympy</u> [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.402 $\int \sqrt{e \sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=364

$$\frac{3ia^{3/2}\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{3ia^{3/2}\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\,d}$$

```
[Out] 3/2*I*a^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-3/2*I*a^(3/2)*arctan(1+2^(1/2)*e^(1/2)
*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-3
/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+3/4*I*a^(3/2)
*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)
+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+I*a*(e*sec(d*x+c))^(1/2)*
(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.21, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3579, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{3ia^{3/2}\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{3ia^{3/2}\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d}-\frac{3ia^{3/2}\sqrt{e}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d}+\frac{3ia^{3/2}\sqrt{e}\,\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d}+\frac{ia\sqrt{a+ia\tan(c+dx)}\,\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((3*I)*a^(3/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - ((3*I)*a^(3/2)*Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])])/(Sqrt[2]*d) - (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt
[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]
*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a +
(Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] +
 Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (I*a*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rubi steps

$$\int \sqrt{e \sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} + \frac{1}{2}(3a)\int \sqrt{e\sec$$

$$= \frac{ia\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(6ia^2e^2)\operatorname{Subst}\Big(}{}$$

$$= \frac{ia\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} + \frac{(3ia^2e)\operatorname{Subst}\Big(}{}$$

$$= \frac{ia\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(3ia^2)\operatorname{Subst}\Bigg(\int}{}$$

$$= -\frac{3ia^{3/2}\sqrt{e}\,\log\left(a-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\right.}{2\sqrt{2}\,d}$$

$$= \frac{3ia^{3/2}\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d} - \frac{3}{}$$

Mathematica [A]
time = 3.57, size = 338, normalized size = 0.93

$$\frac{ae\left(i\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}-3\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{1+i\cos(c)-\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}+3\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i+\tan\left(\frac{dx}{2}\right)}\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}\,\sqrt{1+\cos(2c)+i\sin(2c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e*(I*Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]
 - 3*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*
Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + 3*ArcTanh[(Sqrt[1 + I*
Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt
[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*
x]]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A]
time = 0.98, size = 304, normalized size = 0.84

method	result
default	$-\frac{\sqrt{\frac{e}{\cos(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*
(-1+cos(d*x+c))*(3*I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))-3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))*cos(d*x+c)-2*I*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*cos(d*x+
c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-3*cos(d*
x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-2*cos(
d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-2*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)/(1/(
1+cos(d*x+c)))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)*a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1870 vs. 2(244) = 488.
time = 0.67, size = 1870, normalized size = 5.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -8*(6*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*
a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sq
rt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*
cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*cos(2*d*x
 + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*arctan2(sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*
sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*arct
an2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 16*a*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 3*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2
)*a)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin
(2*d*x + 2*c) + sqrt(2)*a)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(I*sqrt(2)*a*cos(2*d*x
 + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) + 3*(-I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*
sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(I*sqrt(2)*a*cos(2*d*x + 2*c)
- sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
 3*(-I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*
a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 16*I*a*sin(3/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*sqrt(a)*e^(1/2)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*
d*x + 2*c) - 64*I))
```

Fricas [**A**]

time = 0.39, size = 433, normalized size = 1.19

$$\frac{\sqrt{\frac{9i\,a^3e}{d^2}}\,d\log\left(\frac{2\left(\frac{3\left(ae^{\frac{1}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{\frac{9i\,a^3e}{d^2}}\,d\right)e^{\left(-\frac{1}{2}\right)}}{3\,a}\right)-\sqrt{\frac{9i\,a^3e}{d^2}}\,d\log\left(\frac{2\left(\frac{3\left(ae^{\frac{1}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{\frac{9i\,a^3e}{d^2}}\,d\right)e^{\left(-\frac{1}{2}\right)}}{3\,a}\right)-\sqrt{-\frac{9i\,a^3e}{d^2}}\,d\log\left(\frac{2\left(\frac{3\left(ae^{\frac{1}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}+\sqrt{-\frac{9i\,a^3e}{d^2}}\,d\right)e^{\left(-\frac{1}{2}\right)}}{3\,a}\right)+\sqrt{-\frac{9i\,a^3e}{d^2}}\,d\log\left(\frac{2\left(\frac{3\left(ae^{\frac{1}{2}}+ae^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{9i\,a^3e}{d^2}}\,d\right)e^{\left(-\frac{1}{2}\right)}}{3\,a}\right)+\frac{4i\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c+\frac{1}{2}\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(1/2) + a*e^(2*I*d*x + 2*I*c + 1
/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*
d*x + 2*I*c) + 1) + sqrt(9*I*a^3*e/d^2)*d)*e^(-1/2)/a) - sqrt(9*I*a^3*e/d^2
)*d*log(2/3*(3*(a*e^(1/2) + a*e^(2*I*d*x + 2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - sqr
t(9*I*a^3*e/d^2)*d)*e^(-1/2)/a) - sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(1
/2) + a*e^(2*I*d*x + 2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + sqrt(-9*I*a^3*e/d^2)*d)*e
^(-1/2)/a) + sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(1/2) + a*e^(2*I*d*x +
2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqr
t(e^(2*I*d*x + 2*I*c) + 1) - sqrt(-9*I*a^3*e/d^2)*d)*e^(-1/2)/a) + 4*I*a*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c + 1/2)/sqrt(e^(2*I*d
*x + 2*I*c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\sec\left(c+dx\right)}\,\left(ia(\tan\left(c+dx\right)-i)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=520

$$\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia}}{\sqrt{a}\,\sqrt{e\sec(}}\right.}{d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a}}$$

```
[Out] -1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a^(5/2)*ln(a+2^(1/2
)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*
(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c
))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*a^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2
)/d/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4*I*a*(a+I*a*
tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3577, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\sqrt{2}\,a^{5/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}\,a^{5/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{5/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}\,d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{5/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}\,d\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*a^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
(I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqr
t[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqr
t[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4
*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{(a^2\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\,\mathrm{t}}}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{(4ia^3e\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \sqrt{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{(2ia^3\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{tan}}}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{(ia^3\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}$$

$$= -\frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\,\mathrm{tan}\right.}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}$$

$$= \frac{i\sqrt{2}\,a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}\,a}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11314 vs. 2(520) = 1040.
time = 6.20, size = 11314, normalized size = 21.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 0.93, size = 286, normalized size = 0.55

method	result

default	$-\frac{\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\ (\cos(dx+c)+1+\sin(dx+c))}{2}\right)\sin(dx+c)-i\sqrt{\frac{1}{1+\cos(dx+c)}}\ \operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(I*(1/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-I*(1/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)+arctanh(1/2*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(1/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(
1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*I*cos(d*x+c)-4*I-4*sin(d*x+c))*(a*(I*s
in(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/(e/cos(
d*x+c))^(1/2)*a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1461 vs. $2(372) = 744$.
time = 0.73, size = 1461, normalized size = 2.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 2*sqrt(2)*a*arctan2(sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)
*a*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + I*sqrt(2)*a*log(2*sqrt(2)*sin(1/2*arctan2(sin(2
```

```
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - I*sqrt(2)*a*
log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + sqrt(2)*a
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*I*a*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sqrt(a)*e^(-1/2)/d
```

Fricas [**A**]
time = 0.38, size = 439, normalized size = 0.84

$$-\frac{\left(\sqrt{\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}\log\left(\frac{i\sqrt{\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}+\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a}\right)-\sqrt{\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}\log\left(\frac{-i\sqrt{\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}+\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a}\right)+\sqrt{-\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}\log\left(\frac{i\sqrt{-\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}+\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a}\right)-\sqrt{-\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}\log\left(\frac{-i\sqrt{-\frac{4i\,a^3e^{(-1)}}{d^2}}\,de^{\frac{1}{2}}+\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a}\right)+\frac{8\left(i\,ae^{(2i\,dx+2i\,c)}+i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(4*I*a^3*e^(-1)/d^2)*d*e^(1/2)*log((I*sqrt(4*I*a^3*e^(-1)/d^2)*d*
e^(1/2) + 2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e
^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a) - sqrt(4*I*a^3*e^(
-1)/d^2)*d*e^(1/2)*log((-I*sqrt(4*I*a^3*e^(-1)/d^2)*d*e^(1/2) + 2*(a*e^(2*I
```

```
*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a) + sqrt(-4*I*a^3*e^(-1)/d^2)*d*e^(1/2)*l
og((I*sqrt(-4*I*a^3*e^(-1)/d^2)*d*e^(1/2) + 2*(a*e^(2*I*d*x + 2*I*c) + a)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x +
2*I*c) + 1))/a) - sqrt(-4*I*a^3*e^(-1)/d^2)*d*e^(1/2)*log((-I*sqrt(-4*I*a^3
*e^(-1)/d^2)*d*e^(1/2) + 2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a) + 8
*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{\sqrt{e\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(e*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2), x)
```

3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3569}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.09, size = 38, normalized size = 1.00

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.
time = 0.78, size = 76, normalized size = 2.00

method	result	size
risch	$-\frac{2 i a \sqrt{\frac{a\,{\mathrm e}^{2 i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\,{\mathrm e}^{i (d x +c)}}{3 e \sqrt{\frac{e\,{\mathrm e}^{i (d x +c)}}{{\mathrm e}^{2 i (d x +c)}+1}}\, d}$	72
default	$-\frac{2 \left(i \cos \left(d x +c \right)-\sin \left(d x +c \right)\right) \sqrt{\frac{a \left(i \sin \left(d x +c \right)+\cos \left(d x +c \right)\right)}{\cos \left(d x +c \right)}}\, \left(\frac{e}{\cos \left(d x +c \right)}\right)^{\frac{3}{2}} \left(\cos^{2}\left(d x +c \right)\right) a}{3 d \,e^{3}}$	76

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d*(I*cos(d*x+c)-sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(3/2)*cos(d*x+c)^2/e^3*a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(28) = 56$.
time = 0.52, size = 75, normalized size = 1.97

$$-\frac{2 i \, a^{\frac{3}{2}} {\left(-\frac{2 i \, \sin\left(d x + c\right)}{\cos\left(d x + c\right) + 1} + \frac{\sin\left(d x + c\right)^{2}}{{\left(\cos\left(d x + c\right) + 1\right)}^{2}} - 1\right)}^{\frac{3}{2}} e^{\left(-\frac{3}{2}\right)}}{3 \, d {\left(-\frac{\sin\left(d x + c\right)^{2}}{{\left(\cos\left(d x + c\right) + 1\right)}^{2}} - 1\right)}^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -2/3*I*a^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)*e^(-3/2)/(d*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.
time = 0.39, size = 70, normalized size = 1.84

$$\frac{2 \, {\left(-i \, a e^{\left(3 i \, d x + 3 i \, c\right)} - i \, a e^{\left(i \, d x + i \, c\right)}\right)} \sqrt{\frac{a}{e^{\left(2 i \, d x + 2 i \, c\right)} + 1}} e^{\left(\frac{1}{2} i \, d x + \frac{1}{2} i \, c - \frac{3}{2}\right)}}{3 \, d \sqrt{e^{\left(2 i \, d x + 2 i \, c\right)} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c - 3/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan (c + dx) - i))^{\frac{3}{2}}}{(e \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a + a\,\tan(c + d\,x)\,1\mathrm{i}\right)}^{3/2}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2), x)
```

3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] -4/5*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(1/2)-2/5*I*(a+I*a*t
an(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - ((
(2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}} + \frac{(2a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$
$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.43, size = 84, normalized size = 1.04

$$-\frac{2a(\cos(dx)-i\sin(dx))(\cos(c+2dx)+i\sin(c+2dx))(3i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{5de(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*(3*I + 2*
Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*e*(e*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 0.79, size = 86, normalized size = 1.06

method	result	size
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+5\right)}{5e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	74
default	$-\frac{2\left(i\left(\cos^2(dx+c)\right)-\sin(dx+c)\cos(dx+c)+2i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^3(dx+c)\right)a}{5d\,e^5}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(I*cos(d*x+c)^2-sin(d*x+c)*cos(d*x+c)+2*I)*(a*(I*sin(d*x+c)+cos(d*x+
c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3/e^5*a
```

Maxima [A]
time = 0.55, size = 58, normalized size = 0.72

$$\frac{\left(-i\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\,e^{\left(-\frac{5}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*(-I*a*cos(5/2*d*x + 5/2*c) - 5*I*a*cos(1/2*d*x + 1/2*c) + a*sin(5/2*d*x
 + 5/2*c) + 5*a*sin(1/2*d*x + 1/2*c))*sqrt(a)*e^(-5/2)/d
```

Fricas [A]
time = 0.38, size = 73, normalized size = 0.90

$$\frac{\left(-i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-5i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c-\frac{5}{2}\right)}}{5\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) - 5*I*a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c - 5/2)/(d*sqrt(e^(2*I*d*x
+ 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 4.71, size = 102, normalized size = 1.26

$$-\frac{a\,\sqrt{\frac{e}{\cos{(c+d\,x)}}}\,\sqrt{\frac{a\,\left(\cos{(2\,c+2\,d\,x)}+1+\sin{(2\,c+2\,d\,x)}\,1\mathrm{i}\right)}{\cos{(2\,c+2\,d\,x)}+1}}\,\left(-\sin{(c+d\,x)}-\sin{(3\,c+3\,d\,x)}+\cos{(c+d\,x)}\,11\mathrm{i}+\cos{(3\,c+3\,d\,x)}\,1\mathrm{i}\right)}{10\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] -(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*11i - sin(c + d*x) + cos(3*c
+ 3*d*x)*1i - sin(3*c + 3*d*x)))/(10*d*e^3)
```

3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] 16/21*I*a^2*(e*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-8/21*I*a*(a
+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*tan(d*x+c))^
(3/2)/d/(e*sec(d*x+c))^(7/2)
```

Rubi [A]

time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$\frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((16*I)/21)*a^2*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((8*I)/21)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2)) -
 (((2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}} + \frac{(4a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$

$$= -\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}} + \frac{(8a^2)\int \frac{\sqrt{e\operatorname{se}}}{\sqrt{a+ia}}}{21}$$

$$= \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}} - \frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c}{7d(e\sec(c+}$$

Mathematica [A]
time = 0.49, size = 98, normalized size = 0.78

$$\frac{a(\cos(dx)-i\sin(dx))(-7i+9i\cos(2(c+dx))+12\sin(2(c+dx)))(\cos(c+2dx)+i\sin(c+2dx))\sqrt{a+ia\tan(c+dx)}}{21de^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
[Out] (a*(Cos[d*x] - I*Sin[d*x])*(-7*I + (9*I)*Cos[2*(c + d*x)] + 12*Sin[2*(c + d
*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(21*d
*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.81, size = 103, normalized size = 0.82

method	result
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(3\,e^{3i(dx+c)}-7\cos(dx+c)+35i\sin(dx+c))}{42e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$
default	$-\frac{2(3i(\cos^3(dx+c))-3(\cos^2(dx+c))\sin(dx+c)-4i\cos(dx+c)-8\sin(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}(\cos^4(dx}{21d\,e^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/21/d*(3*I*cos(d*x+c)^3-3*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c)-8*sin(d*
x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(7/2)*c
os(d*x+c)^4/e^7*a
```

Maxima [A]
time = 0.55, size = 83, normalized size = 0.66

$$\frac{\left(-3i\,a\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-14i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+14\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\,e^{\left(-\frac{7}{2}\right)}}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/42*(-3*I*a*cos(7/2*d*x + 7/2*c) - 14*I*a*cos(3/2*d*x + 3/2*c) + 21*I*a*co
s(1/2*d*x + 1/2*c) + 3*a*sin(7/2*d*x + 7/2*c) + 14*a*sin(3/2*d*x + 3/2*c) +
 21*a*sin(1/2*d*x + 1/2*c))*sqrt(a)*e^(-7/2)/d
```

Fricas [A]
time = 0.38, size = 85, normalized size = 0.68

$$\frac{\left(-3i\,ae^{(6i\,dx+6i\,c)} - 17i\,ae^{(4i\,dx+4i\,c)} + 7i\,ae^{(2i\,dx+2i\,c)} + 21i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c-\frac{7}{2}\right)}}{42\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/42*(-3*I*a*e^(6*I*d*x + 6*I*c) - 17*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*
I*d*x + 2*I*c) + 21*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x -
1/2*I*c - 7/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 4.92, size = 110, normalized size = 0.88

$$\frac{a\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,4\mathrm{i}-\cos(4\,c+4\,d\,x)\,3\mathrm{i}+38\,\sin(2\,c+2\,d\,x)+3\,\sin(4\,c+4\,d\,x)+7\mathrm{i})}{84\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] (a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))
/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*4i - cos(4*c + 4*d*x)*3i +
 38*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) + 7i))/(84*d*e^4)
```

3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=167

$$\frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{2i(a+}{9d(e}$$

```
[Out] 16/45*I*a^2/d/e^4/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/15*I*a*(a
+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(5/2)-32/45*I*a*(a+I*a*tan(d*x+
c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*se
c(d*x+c))^(9/2)
```

Rubi [A]

time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3578, 3583, 3569}

$$\frac{16ia^2}{45de^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((16*I)/45)*a^2)/(d*e^4*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((4*I)/15)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(5/2)) -
 (((32*I)/45)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]]) -
(((2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(2a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(8a^2)\int \frac{}{\sqrt{e\sec(c}}}{}$$

$$= \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} -$$

$$= \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} -$$

Mathematica [A]
time = 0.65, size = 113, normalized size = 0.68

$$\frac{a(\cos(dx)-i\sin(dx))(-81i\cos(c+dx)+5i\cos(3(c+dx))-54\sin(c+dx)+10\sin(3(c+dx)))(\cos(c+2dx)+i\sin(c+2dx))\sqrt{a+ia\tan(c+dx)}}{90de^4\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
[Out] (a*(Cos[d*x] - I*Sin[d*x])*((-81*I)*Cos[c + d*x] + (5*I)*Cos[3*(c + d*x)] -
 54*Sin[c + d*x] + 10*Sin[3*(c + d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])
*Sqrt[a + I*a*Tan[c + d*x]])/(90*d*e^4*Sqrt[e*Sec[c + d*x]])
```

Maple [A]
time = 0.84, size = 113, normalized size = 0.68

method	result
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\left(5\,e^{4i(dx+c)}+135+12\cos(2dx+2c)+42i\sin(2dx+2c)\right)}{180e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$

default	$-\frac{2\left(5i\left(\cos^4(dx+c)\right)-5\sin(dx+c)\left(\cos^3(dx+c)\right)-2i\left(\cos^2(dx+c)\right)-8\sin(dx+c)\cos(dx+c)+16i\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(d}\right.}{45d\,e^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(5*I*cos(d*x+c)^4-5*sin(d*x+c)*cos(d*x+c)^3-2*I*cos(d*x+c)^2-8*sin(
d*x+c)*cos(d*x+c)+16*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/c
os(d*x+c))^(9/2)*cos(d*x+c)^5/e^9*a
```

Maxima [A]
time = 0.56, size = 159, normalized size = 0.95

$$\frac{\left(-5i\,a\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+15i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-27i\,a\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)-135i\,a\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+5\,a\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+15\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+27\,a\sin\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+135\,a\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right)\sqrt{a}\,e^{\left(-\frac{9}{2}\right)}}{180\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/180*(-5*I*a*cos(9/2*d*x + 9/2*c) + 15*I*a*cos(3/2*d*x + 3/2*c) - 27*I*a*c
os(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 135*I*a*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*sin(9/2*d*x +
 9/2*c) + 15*a*sin(3/2*d*x + 3/2*c) + 27*a*sin(5/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 135*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sqrt(a)*e^(-9/2)/d
```

Fricas [A]
time = 0.39, size = 97, normalized size = 0.58

$$\frac{\left(-5i\,ae^{(8i\,dx+8i\,c)}-32i\,ae^{(6i\,dx+6i\,c)}-162i\,ae^{(4i\,dx+4i\,c)}-120i\,ae^{(2i\,dx+2i\,c)}+15i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c-\frac{9}{2}\right)}}{180\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/180*(-5*I*a*e^(8*I*d*x + 8*I*c) - 32*I*a*e^(6*I*d*x + 6*I*c) - 162*I*a*e^
(4*I*d*x + 4*I*c) - 120*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c - 9/2)/(d*sqrt(e^(2*I*d*x + 2*I*c)
 + 1))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*e^(-9/2)/sec(d*x + c)^(9/2), x)
```

Mupad [B]
time = 5.58, size = 125, normalized size = 0.75

$$-\frac{a\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-42\,\sin\left(c+d\,x\right)-47\,\sin\left(3\,c+3\,d\,x\right)-5\,\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,282{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,17{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,5{}\mathrm{i}\right)}{360\,d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] -(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*282i - 42*sin(c + d*x) + cos(
3*c + 3*d*x)*17i + cos(5*c + 5*d*x)*5i - 47*sin(3*c + 3*d*x) - 5*sin(5*c +
5*d*x)))/(360*d*e^5)
```

3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=612

$$\frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^{7/2}e^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e^{3/2}A}{8\sqrt{}}$$

```
[Out] 15/8*I*a^3*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-15/16*I*a^(7/2)*
e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*
x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c
))^(1/2)+15/16*I*a^(7/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c)
)^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c
))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+15/32*I*a^(7/2)*e^(3/2)*ln(a-2^(1/2)*a^(1
/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a
*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+
c))^(1/2)-15/32*I*a^(7/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d
*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)
/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*a^2*(e*s
ec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*(e*sec(d*x+c))^(3/2)*(a
+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.48, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}+\frac{3ia^2\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}{4d}+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((15*I)/8)*a^3*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(15*I)/8)*a^(7/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((15*I)/8)*a^(7/2)*e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) - (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqr
t[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x
]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*a^2*(e*Sec[c + d*x])^(3/2)*Sqrt
```

```
[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c
+ d*x])^(3/2))/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
```

```
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{1}{2}(3a)\int (e\,\mathrm{se}$$

$$= \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia}}{4d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia}}{4d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia}}{4d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia}}{4d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia}}{4d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{}}{}\right.}{16\sqrt{}}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{15ia^{7/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}}{\sqrt{a}}\right.}{8\sqrt{2}\,d\sqrt{a-ia\tan(c-}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11411 vs. 2(612) = 1224.
time = 58.71, size = 11411, normalized size = 18.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] Result too large to show
```

Maple [A]
time = 1.00, size = 424, normalized size = 0.69

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(45i\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right)(\cos^3(dx+c))-45i\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(-1+cos(d*x+c))^2*(45*I*cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-45*I*cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+90*I*cos(d*x+c)^2*sin(d*x+c)*(1/(1+
cos(d*x+c)))^(1/2)-68*I*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+45*a
rctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3
+45*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x
+c)^3-90*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-16*I*sin(d*x+c)*(1/(1+cos(d*
x+c)))^(1/2)-158*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-52*cos(d*x+c)*(1/(1+
cos(d*x+c)))^(1/2)+16*(1/(1+cos(d*x+c)))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c)
)/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*
x+c)/sin(d*x+c)^3/(1/(1+cos(d*x+c)))^(3/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2914 vs. 2(434) = 868.
time = 0.81, size = 2914, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 192*(1808*a^2*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016*a
^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*a^2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1808*I*a^2*sin(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016*I*a^2*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 720*I*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 90*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(6*d*x + 6*c)
+ 3*I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqr
t(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
```

```
- 90*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqr
t(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^
2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a
^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*
d*x + 2*c) + I*sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 3*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*cos(6*d*x + 6
*c) + 3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*s
qrt(2)*a^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 3*I*sqrt(2
)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) + 90*(-I*sqrt(2)*a^2*cos(6*d*x + 6*c) - 3*I*sq
rt(2)*a^2*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2
*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4*d*x + 4*c) + 3*sqrt(2)*a^2*sin(2*d*
x + 2*c) - I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))),
 sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 90*(I*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 3*I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*cos(2*d*x + 2
*c) - sqrt(2)*a^2*sin(6*d*x + 6*c) - 3*sqrt(2)*a^2*sin(4*d*x + 4*c) - 3*sqr
t(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 45*(sqr
t(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*
cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*sin(4*d
*x + 4*c) + 3*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(2*sqrt(2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) + 45*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4*d*x + 4*c) +
 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*I*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*
log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
```

```
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 45*(-I*sqrt(2)*a^2*cos(6*d*x + 6*c) - 3*I*sqrt(2)*a
^2*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(6*
d*x + 6*c) + 3*sqrt(2)*a^2*sin(4*d*x + 4*c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c
) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 45*(I*sqrt(2)*a^2*cos(6*
d*x + 6*c) + 3*I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*cos(2*d*x +
 2*c) - sqrt(2)*a^2*sin(6*d*x + 6*c) - 3*sqrt(2)*a^2*sin(4*d*x + 4*c) - 3*s
qrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*lo...
```

Fricas [A]
time = 0.40, size = 608, normalized size = 0.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
+ 2*I*c) + d)*log(2/15*(15*(a^2*e^(3/2) + a^2*e^(2*I*d*x + 2*I*c + 3/2))*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2
*I*c) + 1) + 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)*e^(-3/2)/a^2) - 6*sqrt(225/6
4*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(
2/15*(15*(a^2*e^(3/2) + a^2*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 8*I*s
qrt(225/64*I*a^5*e^3/d^2)*d)*e^(-3/2)/a^2) + 6*sqrt(-225/64*I*a^5*e^3/d^2)*
(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e^(
3/2) + a^2*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 8*I*sqrt(-225/64*I*a^5
*e^3/d^2)*d)*e^(-3/2)/a^2) - 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x +
4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e^(3/2) + a^2*e^(2*
I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 8*I*sqrt(-225/64*I*a^5*e^3/d^2)*d)*e^(
-3/2)/a^2) + (45*I*a^2*e^(3/2) + 113*I*a^2*e^(4*I*d*x + 4*I*c + 3/2) + 126*
I*a^2*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.409 $\int \sqrt{e \sec(c+dx)}\, (a+ia\tan(c+dx))^{5/2}\, dx$

Optimal. Leaf size=411

$$\frac{21ia^{5/2}\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{21ia^{5/2}\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d}$$

```
[Out] 21/8*I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-21/8*I*a^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)
-21/16*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*s
ec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+21/16*I*a
^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+7/4*I*a^2*(e*sec(d*
x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*
tan(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3579, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{21ia^{5/2}\sqrt{e}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{21ia^{5/2}\sqrt{e}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d}-\frac{21ia^{5/2}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d}+\frac{21ia^{5/2}\sqrt{e}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d}+\frac{7ia^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{4d}+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((21*I)/4)*a^(5/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/4)*a^(5/2)
*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/8)*a^(5/2)*Sqrt[e]*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((21*I)/8)*a^(5/2)*Sqrt
[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/4)
*a^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/2)*a*Sqrt[e*S
ec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
```

```
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}{2d} + \frac{1}{4}(7a)\int \sqrt{e\sec$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+d}{}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+d}{}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+d}{}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+d}{}$$

$$= -\frac{21ia^{5/2}\sqrt{e}\,\log\left(a-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+c\right.}{8\sqrt{2}\,d}$$

$$= \frac{21ia^{5/2}\sqrt{e}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d} - \frac{21}{}$$

Mathematica [A]
time = 4.51, size = 387, normalized size = 0.94

$$-\frac{a^2\sqrt{e\sec(c+dx)}\,(\cos(2dx)+i\sin(2dx))\sqrt{a+ia\tan(c+dx)}\left(21\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos(c+dx)\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}-21\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\cos(c+dx)\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}+\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}(-9i+2\tan(c+dx))\right)}{4d\sqrt{1+\cos(2c)+i\sin(2c)}\,(\cos(dx)+i\sin(dx))^2\sqrt{i-\tan\left(\frac{dx}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^2*Sqrt[e*Sec[c + d*x]]*(Cos[2*d*x] + I*Sin[2*d*x])*Sqrt[a + I*a*Tan
[c + d*x]]*(21*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] -
```

```
21*ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 +
I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c]
+ Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[1 + C
os[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*(-9*I + 2*Tan[c + d*x])))/(d*S
qrt[1 + Cos[2*c] + I*Sin[2*c]]*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I - Tan[(d*x)
/2]])
```

Maple [A]
time = 1.17, size = 371, normalized size = 0.90

method	result
default	$-\frac{(-1+\cos(dx+c))\left(21i\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right)-21i\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\mathrm{co}}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(21*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)+1+sin(d*x+c)))*cos(d*x+c)^2-21*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-22*I*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)*sin(d*x+c)-4*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-21*cos(d*x+c)^2*arct
anh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-21*cos(d*x+c)^2
*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-22*(1/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)^2-18*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+4*(1/
(1+cos(d*x+c)))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/co
s(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c)/sin(d*x+c)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2430 vs. 2(275) = 550.
time = 0.71, size = 2430, normalized size = 5.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 32*(176*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*a^2*
cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*I*a^2*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*I*a^2*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqr
```

t(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 42*(-I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 42*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 21*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 21*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 21*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*

```
cos(2*d*x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d*x +
 2*c) + I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 21*(-I*sqrt(2)*a^2*c
os(4*d*x + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2) - 21*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*cos(2*d*
x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d*x + 2*c) +
I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 21*(-I*sqrt(2)*a^2*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(4*d*x + 4*c) +
 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^2...
```

Fricas [A]

time = 0.40, size = 539, normalized size = 1.31

$$\frac{\sqrt{\frac{441 i a^5 e}{16 d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{2\left(\frac{21\left(a^2e^{\frac{1}{2}}+a^2e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+4\sqrt{\frac{441 i a^5 e}{16 d^2}}d\right)e^{\left(-\frac{1}{2}\right)}}{21a^2}\right)-\sqrt{\frac{441 i a^5 e}{16 d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{2\left(\frac{21\left(a^2e^{\frac{1}{2}}+a^2e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}-4\sqrt{\frac{441 i a^5 e}{16 d^2}}d\right)e^{\left(-\frac{1}{2}\right)}}{21a^2}\right)-\sqrt{-\frac{441 i a^5 e}{16 d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{2\left(\frac{21\left(a^2e^{\frac{1}{2}}+a^2e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}+4\sqrt{-\frac{441 i a^5 e}{16 d^2}}d\right)e^{\left(-\frac{1}{2}\right)}}{21a^2}\right)+\sqrt{-\frac{441 i a^5 e}{16 d^2}}\left(de^{(2idx+2ic)}+d\right)\log\left(\frac{2\left(\frac{21\left(a^2e^{\frac{1}{2}}+a^2e^{\left(2idx+2ic+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}-4\sqrt{-\frac{441 i a^5 e}{16 d^2}}d\right)e^{\left(-\frac{1}{2}\right)}}{21a^2}\right)+\frac{\left(11ia^2e^{\left(3idx+3ic+\frac{1}{2}\right)}+7ia^2e^{\left(idx+ic+\frac{1}{2}\right)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{\sqrt{e^{(2idx+2ic)}+1}}}{2\left(de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*(a^2
*e^(1/2) + a^2*e^(2*I*d*x + 2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) + 4*sqrt(441/16*I*a^
5*e/d^2)*d)*e^(-1/2)/a^2) - sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c)
 + d)*log(2/21*(21*(a^2*e^(1/2) + a^2*e^(2*I*d*x + 2*I*c + 1/2))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
1) - 4*sqrt(441/16*I*a^5*e/d^2)*d)*e^(-1/2)/a^2) - sqrt(-441/16*I*a^5*e/d^2
)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*(a^2*e^(1/2) + a^2*e^(2*I*d*x +
2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqr
t(e^(2*I*d*x + 2*I*c) + 1) + 4*sqrt(-441/16*I*a^5*e/d^2)*d)*e^(-1/2)/a^2) +
 sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*(a^2*e^
(1/2) + a^2*e^(2*I*d*x + 2*I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^
(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1) - 4*sqrt(-441/16*I*a^5*
e/d^2)*d)*e^(-1/2)/a^2) + (11*I*a^2*e^(3*I*d*x + 3*I*c + 1/2) + 7*I*a^2*e^(
I*d*x + I*c + 1/2))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=563

$$\frac{5ia^{7/2}\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+d}}\right.}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}$$

```
[Out] 5/2*I*a^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)-5/2*I*a^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*
x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/4*I*a^(7/2)*ln(a-2^(1/2)*a^
(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I
*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*
a*tan(d*x+c))^(1/2)+5/4*I*a^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d
*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)
/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-10*I*a
^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)+I*a*(a+I*a*tan(d*x+c))^(
3/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3579, 3577, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{5ia^{7/2}\sec(c+dx)\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\sec(c+dx)\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((5*I)*a^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqr
t[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((5*I)*a^(7/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((5*I)/2)*a^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/2)*a^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
- I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - ((10*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[
c + d*x]]) + (I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
```

```
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} + \frac{1}{2}(5a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} - \frac{(5a^3)\int \frac{(e\operatorname{se}}{\sqrt{a+i\dots}}}{2\dots}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} - \frac{(5a^3\sec(c+dx)\dots}{2e\sqrt{a-\dots}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} - \frac{(10ia^4e\sec(c+\dots}{d\sqrt{\dots}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} + \frac{(5ia^4\sec(c+dx\dots}{d\sqrt{a\dots}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}} - \frac{(5ia^4\sec(c+dx\dots}{2a\dots}$$

$$= -\frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan\dots\right.}{2\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx\dots}}$$

$$= \frac{5ia^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\,d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{7/2}\tan^{-\dots}}{\sqrt{2}\,d\sqrt{\dots}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11357 vs. $2(563) = 1126$.
time = 6.36, size = 11357, normalized size = 20.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [**A**]
time = 1.14, size = 347, normalized size = 0.62

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(5i\cos(dx+c)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*I*cos(d*x+c)*sin(d
*x+c)*(1/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))-5*I*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*arct
anh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+5*cos(d*x+c)*si
n(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))+5*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*arc
tanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+16*I*cos(d*x+c
)^2-18*I*cos(d*x+c)-16*sin(d*x+c)*cos(d*x+c)+2*I-2*sin(d*x+c))/(I*sin(d*x+c
)+cos(d*x+c)-1)/cos(d*x+c)/(e/cos(d*x+c))^(1/2)*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2008 vs. 2(403) = 806.
time = 0.66, size = 2008, normalized size = 3.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 8*(10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt
(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2
)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)
*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 10
*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a
^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 10*
(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)
*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 10*(I*sqrt(2)*a^2*cos(2*d*x + 2*c) - sqrt(2)*a
^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 16*(4*a^2*cos
(2*d*x + 2*c) + 4*I*a^2*sin(2*d*x + 2*c) + 5*a^2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 5*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a
^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 5*(sqrt(2)*a^2*co
s(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(-2*sqrt(
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1) - 5*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*
c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a^2*cos(2*
d*x + 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 5*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x
+ 2*c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a^2*co
s(2*d*x + 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 16*(4*I*a^2*cos(2*d*x + 2*c) - 4*a^2*sin(2*d*x + 2*c) +
 5*I*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*e^(
```

```
-1/2)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*d*x + 2*c) - 64*I))
```

Fricas [A]
time = 0.43, size = 463, normalized size = 0.82

$$\frac{\left(\sqrt{\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} \log\left(-\frac{2\left(i\sqrt{\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} - \frac{5\left(a^2 e^{(2 i d x+2 i c)}+a^2\right)\sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\, e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right)}{5 a^2}\right) - \sqrt{\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} \log\left(-\frac{2\left(-i\sqrt{\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} - \frac{5\left(a^2 e^{(2 i d x+2 i c)}+a^2\right)\sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\, e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right)}{5 a^2}\right) + \sqrt{-\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} \log\left(-\frac{2\left(i\sqrt{-\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} - \frac{5\left(a^2 e^{(2 i d x+2 i c)}+a^2\right)\sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\, e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right)}{5 a^2}\right) - \sqrt{-\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} \log\left(-\frac{2\left(-i\sqrt{-\frac{25 i a^5 e^{(-1)}}{d^2}}\, d e^{\frac{1}{2}} - \frac{5\left(a^2 e^{(2 i d x+2 i c)}+a^2\right)\sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\, e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right)}{5 a^2}\right) - \frac{4\left(4 i a^2 e^{(2 i d x+2 i c)}+5 i a^2\right)\sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}}\, e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{\left(-\frac{1}{2}\right)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(25*I*a^5*e^(-1)/d^2)*d*e^(1/2)*log(-2/5*(I*sqrt(25*I*a^5*e^(-1)/d
^2)*d*e^(1/2) - 5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) - sqrt
(25*I*a^5*e^(-1)/d^2)*d*e^(1/2)*log(-2/5*(-I*sqrt(25*I*a^5*e^(-1)/d^2)*d*e^
(1/2) - 5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) + sqrt(-25*I*a
^5*e^(-1)/d^2)*d*e^(1/2)*log(-2/5*(I*sqrt(-25*I*a^5*e^(-1)/d^2)*d*e^(1/2) -
 5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) - sqrt(-25*I*a^5*e^(-
1)/d^2)*d*e^(1/2)*log(-2/5*(-I*sqrt(-25*I*a^5*e^(-1)/d^2)*d*e^(1/2) - 5*(a^
2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) - 4*(4*I*a^2*e^(2*I*d*x + 2
*I*c) + 5*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/
sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2), x)
```

3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}$$

```
[Out] 1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-1/2*I*a^(5/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-I*a^(5/2)*arctan(1-2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d
/e^(3/2)+I*a^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d/e^(3/2)-4/3*I*a*(a+I*a*tan(d*x+c))^(3/2)/
d/(e*sec(d*x+c))^(3/2)
```

Rubi [A]

time = 0.21, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3577, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{i\sqrt{2}\,a^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{ia^{5/2}\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,de^{3/2}}-\frac{ia^{5/2}\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,de^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(d*e^(3/2)) + (I*Sqrt[2]*a^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])])/(d*e^(3/2)) + (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*d*e^(3/2)) - (I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*d*e^(3/2)) - (((4*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/
(d*(e*Sec[c + d*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
```

```
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{a^2\int\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,d}{e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(4ia^3)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c}}{\sqrt{e\sec(c+}}\right.}{d}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{(2ia^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c}}{\sqrt{e\sec(c+}}\right.}{de}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(ia^3)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+}}{\sqrt{e}}\right.}{de^2}$$

$$= \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(\right.}{\sqrt{2}\,de^{3/2}}$$

$$= -\frac{i\sqrt{2}\,a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{i\sqrt{2}\,a^{5/2}\tan^{-1}\left(\right.}{}$$

Mathematica [A]

time = 3.87, size = 343, normalized size = 0.95

$$\frac{e\left(-\frac{4}{3}i\cos(dx)(\cos(c)-i\sin(c))+\frac{4}{3}(\cos(c)-i\sin(c))\sin(dx)+\frac{2\left(\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\sqrt{-1+i\cos(c)+\sin(c)}\right)(\cos(2c)-i\sin(2c))\sqrt{i+\tan\left(\frac{dx}{2}\right)}}{\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}\right)(a+ia\tan(c+dx))^{5/2}}{d(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (e*(((-4*I)/3)*Cos[d*x]*(Cos[c] - I*Sin[c]) + (4*(Cos[c] - I*Sin[c])*Sin[d*
x])/3 + (2*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(S
```

```
qrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - S
in[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*S
qrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]]
)]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] - I*
Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I -
 Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2)*(C
os[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.96, size = 323, normalized size = 0.89

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right)\sin(dx+c)-3i\sqrt{\frac{1}{1+\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*I*(1/(1+cos(d*x+c)
))^(1/2)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*si
n(d*x+c)-3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)*(1/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8*I*cos(d*x+c)^2-3*arctanh(1/2*(1/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(1/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(1
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-8*sin(d*x+c)*cos(d*x+c)-4*I*cos(d*x+c)+4*
sin(d*x+c)-4*I)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)/(e/cos(d*x+c))^(3/2)
*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1491 vs. 2(244) = 488.
time = 0.63, size = 1491, normalized size = 4.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/12*(-6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(si
```

```
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*sqrt(2)*a^2*arctan2(sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 6*sqrt(2)*a^2*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*I*sqrt(2)*a^2*log(2*sqrt(2)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 3*I*sqrt(2)*a^2*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*sqrt(2)*a^2*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*sqrt(2)*a
^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + 16*I*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*a^
2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*e^(-3/2)/d
```

Fricas [A]

time = 0.43, size = 468, normalized size = 1.29

$$-\frac{\left(3\sqrt{\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}\log\left(\frac{\sqrt{\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}+\frac{2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a^2}\right)-3\sqrt{\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}\log\left(-\frac{\sqrt{\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}-\frac{2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a^2}\right)-3\sqrt{-\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}\log\left(\frac{\sqrt{-\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}+\frac{2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a^2}\right)+3\sqrt{-\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}\log\left(-\frac{\sqrt{-\frac{4i\,a^5e^{(-3)}}{d^2}}\,de^{\frac{3}{2}}-\frac{2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}}{a^2}\right)+\frac{8\left(i\,a^2e^{(3i\,dx+3i\,c)}+i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{\left(-\frac{3}{2}\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*sqrt(4*I*a^5*e^(-3)/d^2)*d*e^(3/2)*log((sqrt(4*I*a^5*e^(-3)/d^2)*d*
e^(3/2) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) - 3*sqrt(4*I
*a^5*e^(-3)/d^2)*d*e^(3/2)*log(-(sqrt(4*I*a^5*e^(-3)/d^2)*d*e^(3/2) - 2*(a^
2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/a^2) - 3*sqrt(-4*I*a^5*e^(-3)/d^
2)*d*e^(3/2)*log((sqrt(-4*I*a^5*e^(-3)/d^2)*d*e^(3/2) + 2*(a^2*e^(2*I*d*x +
 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sq
rt(e^(2*I*d*x + 2*I*c) + 1))/a^2) + 3*sqrt(-4*I*a^5*e^(-3)/d^2)*d*e^(3/2)*l
og(-(sqrt(-4*I*a^5*e^(-3)/d^2)*d*e^(3/2) - 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x
 + 2*I*c) + 1))/a^2) + 8*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c)
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x
 + 2*I*c) + 1))*e^(-3/2)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2), x)
```

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3569}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.15, size = 38, normalized size = 1.00

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.
time = 0.79, size = 88, normalized size = 2.32

method	result	size
risch	$-\frac{2 i a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 i (d x+c)}}{{\mathrm e}^{2 i (d x+c)}+1}}\, {\mathrm e}^{2 i (d x+c)}}{5 e^{2} \sqrt{\frac{e\,{\mathrm e}^{i (d x+c)}}{{\mathrm e}^{2 i (d x+c)}+1}}\, d}$	74
default	$-\frac{2\left(2 i \left(\cos^{2}\left(d x+c\right)\right)-2 \sin \left(d x+c\right) \cos \left(d x+c\right)-i\right) \sqrt{\frac{a \left(i \sin \left(d x+c\right)+\cos \left(d x+c\right)\right)}{\cos \left(d x+c\right)}}\, \left(\frac{e}{\cos \left(d x+c\right)}\right)^{\frac{5}{2}} \left(\cos^{3}\left(d x+c\right)\right) a^{2}}{5 d\, e^{5}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(2*I*cos(d*x+c)^2-2*sin(d*x+c)*cos(d*x+c)-I)*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3/e^5*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(28) = 56$.
time = 0.50, size = 75, normalized size = 1.97

$$-\frac{2 i\, a^{\frac{5}{2}} \left(-\frac{2 i\, \sin\left(d x+c\right)}{\cos\left(d x+c\right)+1}+\frac{\sin\left(d x+c\right)^{2}}{\left(\cos\left(d x+c\right)+1\right)^{2}}-1\right)^{\frac{5}{2}} e^{\left(-\frac{5}{2}\right)}}{5\, d \left(-\frac{\sin\left(d x+c\right)^{2}}{\left(\cos\left(d x+c\right)+1\right)^{2}}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*I*a^(5/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2)*e^(-5/2)/(d*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1)^(5/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(28) = 56$.
time = 0.37, size = 74, normalized size = 1.95

$$\frac{2 \left(-i\, a^{2} e^{\left(4 i\, d x+4 i\, c\right)}-i\, a^{2} e^{\left(2 i\, d x+2 i\, c\right)}\right) \sqrt{\frac{a}{e^{\left(2 i\, d x+2 i\, c\right)}+1}}\, e^{\left(\frac{1}{2} i\, d x+\frac{1}{2} i\, c-\frac{5}{2}\right)}}{5\, d \sqrt{e^{\left(2 i\, d x+2 i\, c\right)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(-I*a^2*e^(4*I*d*x + 4*I*c) - I*a^2*e^(2*I*d*x + 2*I*c))*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c - 5/2)/(d*sqrt(e^(2*I*d*x + 2*I*
c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 4.55, size = 104, normalized size = 2.74

$$-\frac{a^2\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}{5\,d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*1i - sin(c + d*x) + cos(3*c
 + 3*d*x)*1i - sin(3*c + 3*d*x)))/(5*d*e^3)
```

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] -4/21*I*a*(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*
tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(7/2)
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2))
 - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(2a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\ &= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 92, normalized size = 1.14

$$-\frac{2a^2(\cos(2(c+2dx))+i\sin(2(c+2dx)))(5i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^2*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*(5*I + 2*Tan[c + d*x])*
Sqrt[a + I*a*Tan[c + d*x]])/(21*d*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*
Sin[d*x])^2)
```

Maple [A]
time = 0.81, size = 105, normalized size = 1.30

method	result
risch	$-\frac{i a^2 \sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(3\,e^{3i(dx+c)}+7\,e^{i(dx+c)}\right)}{21 e^3 \sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$
default	$-\frac{2\left(6i\left(\cos^3(dx+c)\right)-6\left(\cos^2(dx+c)\right)\sin(dx+c)-i\cos(dx+c)-2\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^4(dx+c\right.}{21d\,e^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(6*I*cos(d*x+c)^3-6*cos(d*x+c)^2*sin(d*x+c)-I*cos(d*x+c)-2*sin(d*x+
c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(7/2)*cos
(d*x+c)^4/e^7*a^2
```

Maxima [A]
time = 0.55, size = 93, normalized size = 1.15

$$\frac{\left(-7i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,a^2\cos\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+7\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,a^2\sin\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right)\sqrt{a}\,e^{\left(-\frac{7}{2}\right)}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(-7*I*a^2*cos(3/2*d*x + 3/2*c) - 3*I*a^2*cos(7/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*a^2*sin(3/2*d*x + 3/2*c) + 3*a^2*sin(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*e^(-7/2)/d
```

Fricas [A]
time = 0.38, size = 88, normalized size = 1.09

$$\frac{\left(-3i\,a^2e^{(5i\,dx+5i\,c)}-10i\,a^2e^{(3i\,dx+3i\,c)}-7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c-\frac{7}{2}\right)}}{21\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/21*(-3*I*a^2*e^(5*I*d*x + 5*I*c) - 10*I*a^2*e^(3*I*d*x + 3*I*c) - 7*I*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
- 7/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-7/2)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 4.99, size = 112, normalized size = 1.38

$$-\frac{a^2\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,10{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,3{}\mathrm{i}-10\,\sin\left(2\,c+2\,d\,x\right)-3\,\sin\left(4\,c+4\,d\,x\right)+7{}\mathrm{i}\right)}{42\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*10i + cos(4*c + 4*d*x)*
3i - 10*sin(2*c + 2*d*x) - 3*sin(4*c + 4*d*x) + 7i))/(42*d*e^4)
```

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] -16/45*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-8/45*I*a*(
a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(5/2)-2/9*I*(a+I*a*tan(d*x+c))
^(5/2)/d/(e*sec(d*x+c))^(9/2)
```

Rubi [A]
time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((-16*I)/45)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]])
- (((8*I)/45)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(5/2)
) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(4a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}$$

$$= -\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(8a^2)\int \frac{\sqrt{a+i}}{\sqrt{e\,s}}}{4}$$

$$= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan}{9d(e\sec(c}$$

Mathematica [A]
time = 0.54, size = 104, normalized size = 0.83

$$\frac{a^2(9+25\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(2(c+2dx))+\sin(2(c+2dx)))\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}\,(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (a^2*(9 + 25*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-I)*Cos[2*(c + 2
*d*x)] + Sin[2*(c + 2*d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(45*d*e^4*Sqrt[e*S
ec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.82, size = 115, normalized size = 0.92

method	result
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(5\,e^{4i(dx+c)}+18\,e^{2i(dx+c)}+45)}{90e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$
default	$-\frac{2(10i(\cos^4(dx+c))-10\sin(dx+c)(\cos^3(dx+c))-i(\cos^2(dx+c))-4\sin(dx+c)\cos(dx+c)+8i)(\cos^5(dx+c))\sqrt{\frac{a(i\sin(dx+c)+\cos}{\cos(dx+c)}}}{45d\,e^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(10*I*cos(d*x+c)^4-10*sin(d*x+c)*cos(d*x+c)^3-I*cos(d*x+c)^2-4*sin(
d*x+c)*cos(d*x+c)+8*I)*cos(d*x+c)^5*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)*(e/cos(d*x+c))^(9/2)/e^9*a^2
```

Maxima [A]
time = 0.57, size = 95, normalized size = 0.76

$$\frac{\left(-5i\,a^2\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)-18i\,a^2\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-45i\,a^2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+18\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+45\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\,e^{\left(-\frac{9}{2}\right)}}{90\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*(-5*I*a^2*cos(9/2*d*x + 9/2*c) - 18*I*a^2*cos(5/2*d*x + 5/2*c) - 45*I*
a^2*cos(1/2*d*x + 1/2*c) + 5*a^2*sin(9/2*d*x + 9/2*c) + 18*a^2*sin(5/2*d*x
+ 5/2*c) + 45*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)*e^(-9/2)/d
```

Fricas [**A**]
time = 0.38, size = 93, normalized size = 0.74

$$\frac{\left(-5i\,a^2e^{(6i\,dx+6i\,c)}-23i\,a^2e^{(4i\,dx+4i\,c)}-63i\,a^2e^{(2i\,dx+2i\,c)}-45i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c-\frac{9}{2}\right)}}{90\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/90*(-5*I*a^2*e^(6*I*d*x + 6*I*c) - 23*I*a^2*e^(4*I*d*x + 4*I*c) - 63*I*a^
2*e^(2*I*d*x + 2*I*c) - 45*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*
I*d*x + 1/2*I*c - 9/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-9/2)/sec(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 5.52, size = 127, normalized size = 1.02

$$-\frac{a^2\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-18\,\sin\left(c+d\,x\right)-23\,\sin\left(3\,c+3\,d\,x\right)-5\,\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,108{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,23{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,5{}\mathrm{i}\right)}{180\,d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*108i - 18*sin(c + d*x) + co
s(3*c + 3*d*x)*23i + cos(5*c + 5*d*x)*5i - 23*sin(3*c + 3*d*x) - 5*sin(5*c
+ 5*d*x)))/(180*d*e^5)
```

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=169

$$\frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] 32/77*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^6/(a+I*a*tan(d*x+c))^(1/2)-16/77*I*a^2
*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(3/2)-12/77*I*a*(a+I*a*tan(d
*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(7/2)-2/11*I*(a+I*a*tan(d*x+c))^(5/2)/d/(
e*sec(d*x+c))^(11/2)
```

Rubi [A]
time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3578, 3569}

$$\frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (((32*I)/77)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^6*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((16*I)/77)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*(e*Sec[c + d*x])^(3/2)
) - (((12*I)/77)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(7
/2)) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)
)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}} + \frac{(6a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}$$

$$= -\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}} + \frac{(24a^2)\int \frac{\sqrt{a-}}{(}}{}$$

$$= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan}{11d(e\sec(}$$

$$= \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}} - \frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan}{77de^2(e\sec(}$$

Mathematica [A]
time = 0.81, size = 121, normalized size = 0.72

$$\frac{a^2(-55i\cos(c+dx)+35i\cos(3(c+dx))-22\sin(c+dx)+42\sin(3(c+dx)))(\cos(2(c+2dx))+i\sin(2(c+2dx)))\sqrt{a+ia\tan(c+dx)}}{154de^5\sqrt{e\sec(c+dx)}\,(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*((-55*I)*Cos[c + d*x] + (35*I)*Cos[3*(c + d*x)] - 22*Sin[c + d*x] + 42
*Sin[3*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*Sqrt[a + I*a
*Tan[c + d*x]])/(154*d*e^5*Sqrt[e*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A]
time = 0.94, size = 132, normalized size = 0.78

method	result
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(7\,e^{5i(dx+c)}+33\,e^{3i(dx+c)}+154i\sin(dx+c)\right)}{308e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$
default	$-\frac{2\left(14i\left(\cos^5(dx+c)\right)-14\sin(dx+c)\left(\cos^4(dx+c)\right)-i\left(\cos^3(dx+c)\right)-6\left(\cos^2(dx+c)\right)\sin(dx+c)-8i\cos(dx+c)-16\sin(dx+c)\right)\left(\cos^6(d\right.}{77d\,e^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/d*(14*I*cos(d*x+c)^5-14*sin(d*x+c)*cos(d*x+c)^4-I*cos(d*x+c)^3-6*cos(
d*x+c)^2*sin(d*x+c)-8*I*cos(d*x+c)-16*sin(d*x+c))*cos(d*x+c)^6*(a*(I*sin(d*
x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(11/2)/e^11*a^2
```

Maxima [A]
time = 0.55, size = 123, normalized size = 0.73

$$\frac{\left(-7i\,a^2\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)-33i\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-77i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+77i\,a^2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^2\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+77\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\,e^{\left(-\frac{11}{2}\right)}}{308\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] 1/308*(-7*I*a^2*cos(11/2*d*x + 11/2*c) - 33*I*a^2*cos(7/2*d*x + 7/2*c) - 77
*I*a^2*cos(3/2*d*x + 3/2*c) + 77*I*a^2*cos(1/2*d*x + 1/2*c) + 7*a^2*sin(11/
2*d*x + 11/2*c) + 33*a^2*sin(7/2*d*x + 7/2*c) + 77*a^2*sin(3/2*d*x + 3/2*c)
 + 77*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)*e^(-11/2)/d
```

Fricas [A]
time = 0.39, size = 93, normalized size = 0.55

$$\frac{\left(-7i\,a^2e^{(8i\,dx+8i\,c)}-40i\,a^2e^{(6i\,dx+6i\,c)}-110i\,a^2e^{(4i\,dx+4i\,c)}+77i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c-\frac{11}{2}\right)}}{308\,d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/308*(-7*I*a^2*e^(8*I*d*x + 8*I*c) - 40*I*a^2*e^(6*I*d*x + 6*I*c) - 110*I*
a^2*e^(4*I*d*x + 4*I*c) + 77*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1
/2*I*d*x - 1/2*I*c - 11/2)/(d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*e^(-11/2)/sec(d*x + c)^(11/2), x)
```

Mupad [B]
time = 6.07, size = 133, normalized size = 0.79

$$-\frac{a^2\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(-187\,\sin\left(2\,c+2\,d\,x\right)-40\,\sin\left(4\,c+4\,d\,x\right)-7\,\sin\left(6\,c+6\,d\,x\right)+\cos\left(2\,c+2\,d\,x\right)\,33{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,40{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,7{}\mathrm{i}\right)}{616\,d\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(11/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*33i + cos(4*c + 4*d*x)*
40i + cos(6*c + 6*d*x)*7i - 187*sin(2*c + 2*d*x) - 40*sin(4*c + 4*d*x) - 7*
sin(6*c + 6*d*x)))/(616*d*e^6)
```

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=369

$$\frac{ie^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{ie^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{ie^{5/}}{}$$

```
[Out] 1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)
*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1
/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)+1/4*I*e^(5/2)
*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)
+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)-I*e^2*(e*sec(d*x+c))^(1/2
)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{ie^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{ie^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d}+\frac{ie^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sq
rt[2]*Sqrt[a]*d) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
```

```
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{e^2\int\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\,\mathrm{ta}}}{2a}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(2ie^4)\,\mathrm{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\sqrt{}\right.}{d}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{(ie^3)\,\mathrm{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\sqrt{c}\right.}{d}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(ie^2)\,\mathrm{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\right.}{2a}$$

$$= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c\right.}{2\sqrt{2}\sqrt{a}\,d}$$

$$= \frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}}{\sqrt{a}}\right.}{\sqrt{2}\sqrt{}}$$

Mathematica [A]
time = 3.93, size = 350, normalized size = 0.95

$$\frac{e^3\left(\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-i\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}+i\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}\right)(-i+\tan(c+dx))}{d\sqrt{e\sec(c+dx)}\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e^3*(Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]] -
 I*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 -
I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sq
rt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + I*ArcTanh[(Sqrt[1 + I*Co
s[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I
```

```
 + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]
*Sqrt[I + Tan[(d*x)/2]])*(-I + Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[
1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [A]
time = 3.92, size = 316, normalized size = 0.86

method	result
default	$-\frac{\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}} \left(\cos^2(dx+c)\right)\left(-1+\cos(dx+c)\right)^3 \sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}} \left(i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\ (\cos(dx+}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(-1+cos(d*x+c))^3*(a*(I*sin(d*x+c)
+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)
-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
+2*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+2*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+
c)^5/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(5/2)/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2137 vs. 2(246) = 492.
time = 0.63, size = 2137, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -8*(2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3
```

```
/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - I*sqrt(2))*arct
an2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(I*sqrt(2)*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - sqrt(2)*sin(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2))*arctan2(-sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (sqrt(2)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(2*sqrt(2)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (sqrt(2)*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(-2
*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 1) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
```

```
c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2) + (-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c...
```

Fricas [A]
time = 0.42, size = 411, normalized size = 1.11

$$\frac{a d \sqrt{\frac{i e^{5}}{a d^{2}}} \log\left(2 \left(a d \sqrt{\frac{i e^{5}}{a d^{2}}} + \frac{\sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{5}{2}} + e^{\left(2 i d x + 2 i c + \frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{5}{2}\right)}\right) - a d \sqrt{\frac{i e^{5}}{a d^{2}}} \log\left(-2 \left(a d \sqrt{\frac{i e^{5}}{a d^{2}}} - \frac{\sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{5}{2}} + e^{\left(2 i d x + 2 i c + \frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{5}{2}\right)}\right) - a d \sqrt{-\frac{i e^{5}}{a d^{2}}} \log\left(2 \left(a d \sqrt{-\frac{i e^{5}}{a d^{2}}} + \frac{\sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{5}{2}} + e^{\left(2 i d x + 2 i c + \frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{5}{2}\right)}\right) + a d \sqrt{-\frac{i e^{5}}{a d^{2}}} \log\left(-2 \left(a d \sqrt{-\frac{i e^{5}}{a d^{2}}} - \frac{\sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{5}{2}} + e^{\left(2 i d x + 2 i c + \frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{5}{2}\right)}\right) - \frac{4 i \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} e^{\left(\frac{3}{2} i d x + \frac{3}{2} i c + \frac{5}{2}\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}}{2 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a*d*sqrt(I*e^5/(a*d^2))*log(2*(a*d*sqrt(I*e^5/(a*d^2)) + sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1
/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)) - a*d*sqrt(I*e^5/(a*d^2))*
log(-2*(a*d*sqrt(I*e^5/(a*d^2)) - sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2
) + e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*
I*c) + 1))*e^(-5/2)) - a*d*sqrt(-I*e^5/(a*d^2))*log(2*(a*d*sqrt(-I*e^5/(a*d
^2)) + sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x + 2*I*c + 5/
2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)) + a*d*
sqrt(-I*e^5/(a*d^2))*log(-2*(a*d*sqrt(-I*e^5/(a*d^2)) - sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x + 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c
)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2)) - 4*I*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*e^(3/2*I*d*x + 3/2*I*c + 5/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=483

$$-\frac{i\sqrt{2}\,\sqrt{a}\,e^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\,\sqrt{a}\,e^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{}}{\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+d}}$$

```
[Out] 1/2*I*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)/(
a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*e^(3/2)*ln(a+2^(1/2)
*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(
a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)-I*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c)
)^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)*a^(1/2)/d/(a-I*a*t
an(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*e^(3/2)*arctan(1+2^(1/2)*e^(1/2
)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)
*a^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{a}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{a}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*Sqrt[a]*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
 d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c +
 d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(e\sec(c+dx))\int \sqrt{e\sec(c+dx)}\,\sqrt{a-ia\tan(c+dx)}\,dx}{\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(4iae^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{\left(2iae^2\sec(c+dx)\right)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{(2iae}{}$$

$$= \frac{(iae\sec(c+dx))\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} +$$

$$= \frac{i\sqrt{a}\,e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\,t\right.}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i\sqrt{2}\,\sqrt{a}\,e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{}}{}$$

Mathematica [A]
time = 1.45, size = 302, normalized size = 0.63

$$\frac{2e\sqrt{e\sec(c+dx)}\left(\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{1+i\cos(c)-\sin(c)}-\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\right)(\cos(dx)+i\sin(dx))\sqrt{i+\tan\left(\frac{dx}{2}\right)}}{d\sqrt{-1-i\cos(c)-\sin(c)}\,\sqrt{-1+i\cos(c)+\sin(c)}\,\sqrt{i-\tan\left(\frac{dx}{2}\right)}\,\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*e*Sqrt[e*Sec[c + d*x]]*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Ta
n[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
```

```
 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos
[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
+ Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])
*(Cos[d*x] + I*Sin[d*x])*Sqrt[I + Tan[(d*x)/2]])/(d*Sqrt[-1 - I*Cos[c] - Si
n[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[
c + d*x]])
```

Maple [A]
time = 1.12, size = 232, normalized size = 0.48

method	result
default	$\frac{\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}} (\cos^2(dx+c))(-1+\cos(dx+c))^2 \sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}} \left(i \operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c)}{2}\right.\right.}{d\sin(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(e/cos(d*x+c))^(3/2)*cos(d*x+c)^2*(-1+cos(d*x+c))^2*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)*(I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))-I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))+arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+
arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))))/sin(d*x+c)
^3/(1/(1+cos(d*x+c)))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(343) = 686$.
time = 0.58, size = 713, normalized size = 1.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*sqrt(
2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
 + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + I*
sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/
```

```
2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^
2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*e^(3/2)/(sqrt(a)*d)
```

Fricas [**A**]
time = 0.38, size = 357, normalized size = 0.74

$$\frac{1}{2}\sqrt{\frac{4i\,e^{3}}{ad^{2}}}\log\left(\left(i\,ad\sqrt{\frac{4i\,e^{3}}{ad^{2}}}+\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{(2i\,dx+2i\,c+\frac{3}{2})}\right)e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-\frac{3}{2})}\right)-\frac{1}{2}\sqrt{\frac{4i\,e^{3}}{ad^{2}}}\log\left(\left(-i\,ad\sqrt{\frac{4i\,e^{3}}{ad^{2}}}+\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{(2i\,dx+2i\,c+\frac{3}{2})}\right)e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-\frac{3}{2})}\right)+\frac{1}{2}\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}\log\left(\left(i\,ad\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}+\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{(2i\,dx+2i\,c+\frac{3}{2})}\right)e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-\frac{3}{2})}\right)-\frac{1}{2}\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}\log\left(\left(-i\,ad\sqrt{-\frac{4i\,e^{3}}{ad^{2}}}+\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{\frac{3}{2}}+e^{(2i\,dx+2i\,c+\frac{3}{2})}\right)e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}}{\sqrt{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-\frac{3}{2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(4*I*e^3/(a*d^2))*log((I*a*d*sqrt(4*I*e^3/(a*d^2)) + 2*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(e^(3/2) + e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x +
1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2)) - 1/2*sqrt(4*I*e^3/(a*d^2
))*log((-I*a*d*sqrt(4*I*e^3/(a*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
(e^(3/2) + e^(2*I*d*x + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d
*x + 2*I*c) + 1))*e^(-3/2)) + 1/2*sqrt(-4*I*e^3/(a*d^2))*log((I*a*d*sqrt(-4
*I*e^3/(a*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3/2) + e^(2*I*d*x
 + 2*I*c + 3/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(
-3/2)) - 1/2*sqrt(-4*I*e^3/(a*d^2))*log((-I*a*d*sqrt(-4*I*e^3/(a*d^2)) + 2*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(3/2) + e^(2*I*d*x + 2*I*c + 3/2))*e^(
1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.418 $\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3569}

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 1.00

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(30) = 60$.
time = 0.88, size = 74, normalized size = 2.06

method	result	size
risch	$\frac{2i\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	59
default	$\frac{2i\sqrt{\frac{e}{\cos(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)}{d(i\sin(dx+c)+\cos(dx+c))a}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*c
os(d*x+c)/(I*sin(d*x+c)+cos(d*x+c))/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(28) = 56$.
time = 0.49, size = 75, normalized size = 2.08

$$\frac{2i\sqrt{-\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1}\,e^{\frac{1}{2}}}{\sqrt{a}\,d\sqrt{-\frac{2i\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*e^(1/2)/(sqrt(a)*d*sqrt(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.
time = 0.35, size = 64, normalized size = 1.78

$$\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{\frac{1}{2}}+i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(1/2) + I*e^(2*I*d*x + 2*I*c + 1/2
))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 4.31, size = 40, normalized size = 1.11

$$\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,2\mathrm{i}}{d\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*2i)/(d*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)
)
```

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

```
[Out] 2/3*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(a+I*a*tan(d*x+
c))^(1/2)/a/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{3d\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} + \frac{2\int \frac{\sqrt{a+ia\tan}}{\sqrt{e\sec(c}}}{3a}$$
$$= \frac{2i}{3d\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(}}{3ad\sqrt{e\sec(c-}}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 0.60

$$\frac{-2i+4\tan(c+dx)}{3d\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (-2*I + 4*Tan[c + d*x])/(3*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [A]
time = 0.90, size = 85, normalized size = 1.06

method	result	size
default	$-\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(i\cos(dx+c)-2\sin(dx+c))}{3d(i\sin(dx+c)+\cos(dx+c))\sqrt{\frac{e}{\cos(dx+c)}}\,a}$	85
risch	$-\frac{i(3\,e^{2i(dx+c)}-1)}{3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/3/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*cos(d*x+c)-2*sin(d
*x+c))/(I*sin(d*x+c)+cos(d*x+c))/(e/cos(d*x+c))^(1/2)/a
```

Maxima [A]
time = 0.54, size = 79, normalized size = 0.99

$$\frac{\left(i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right)e^{\left(-\frac{1}{2}\right)}}{3\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*e^(-1/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.90

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c-\frac{1}{2}\right)}}{3\,ad\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*
I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I*c - 1/2)/(a*d*sqrt(e^(2*I*d*x + 2
*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\sec\left(c+dx\right)}\,\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-1/2)/(sqrt(I*a*tan(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 0.78, size = 78, normalized size = 0.98

$$-\frac{2\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(-2\,\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}\right)}{3\,d\,e\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] -(2*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*1i - 2*sin(c + d*x)))/(3*d*e*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=121

$$\frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*sec(d*x+c)
)^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/
(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/
15)*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((8*I)/15)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{4\int \frac{\sqrt{a+ia\,\mathrm{t}}}{(e\sec(c}}{5}$$

$$= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{8i\sqrt{a+ia\,\mathrm{tan}}}{15ad(e\sec(c}$$

$$= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i\sqrt{e\sec}}{15de^2\sqrt{a+ia}}$$

Mathematica [A]
time = 0.31, size = 68, normalized size = 0.56

$$-\frac{i\sec^2(c+dx)(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] ((-1/15*I)*Sec[c + d*x]^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)])
)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.87, size = 105, normalized size = 0.87

method	result
default	$\frac{2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^2(dx+c))\,(3i(\cos^3(dx+c))+3(\cos^2(dx+c))\sin(dx+c)+4i\cos(dx+c)+8\sin(dx+c}{15d\,e^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
[Out] 2/15/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)^2*(3*I*cos(d*x+c)^3+3*cos(d*x+c)^2*sin(d*x+c)+4*I*cos(d*x+c)+8*s
in(d*x+c))/e^3/a
```

Maxima [A]
time = 0.55, size = 129, normalized size = 1.07

$$\frac{\left(3i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+3\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)e^{\left(-\frac{3}{2}\right)}}{30\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))*e^(-3/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.69

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(6i\,dx+6i\,c)}+25i\,e^{(4i\,dx+4i\,c)}+33i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c-\frac{3}{2}\right)}}{30\,ad\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(6*I*d*x + 6*I*c) + 25*I*e^(
4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5/2*I*d*x - 5/2*I*c
- 3/2)/(a*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-3/2)/(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 4.03, size = 86, normalized size = 0.71

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(4\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+15{}\mathrm{i}\right)}{15\,d\,e^2\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(4*sin(2*c + 2*d*x) - cos(2*c + 2*d*x)*1i + 15i))/(
15*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x
) + 1))^(1/2))
```

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=165

$$\frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}+\frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{12i\sqrt{a+ia\tan(c-}}{35ad(e\sec(c+dx}$$

```
[Out] 2/7*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+16/35*I/d/e^2/(e*sec(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
/(e*sec(d*x+c))^(5/2)-32/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c
))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}+\frac{16i}{35de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((16*I)/3
5)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((12*I)/35)*S
qrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(5/2)) - (((32*I)/35)*Sqrt
[a + I*a*Tan[c + d*x]])/(a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{6\int \frac{\sqrt{a+ia\,t}}{(e\sec(c}}{7}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{12i\sqrt{a+ia\,ta}}{35ad(e\sec(c}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{}{35de^2\sqrt{e\sec(}}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{}{35de^2\sqrt{e\sec(}}$$

Mathematica [A]
time = 0.45, size = 79, normalized size = 0.48

$$-\frac{i(17+\cos(2(c+dx))+3i\sec(c+dx)\sin(3(c+dx))+35i\tan(c+dx))}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-1/35*I)*(17 + Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (
35*I)*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [A]
time = 0.89, size = 115, normalized size = 0.70

method	result
default	$\frac{2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^3(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(5i\left(\cos^4(dx+c)\right)+5\sin(dx+c)\left(\cos^3(dx+c)\right)+2i\left(\cos^2(dx+c)\right)+8\sin(dx\right.}{35d\,e^5a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/35/d*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d
*x+c))^(1/2)*(5*I*cos(d*x+c)^4+5*sin(d*x+c)*cos(d*x+c)^3+2*I*cos(d*x+c)^2+8
*sin(d*x+c)*cos(d*x+c)-16*I)/e^5/a
```

Maxima [A]
time = 0.56, size = 177, normalized size = 1.07

$$\frac{\left(5i\,\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-7i\,\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+35i\,\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)-105i\,\cos\left(\frac{1}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+5\,\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7\,\sin\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+35\,\sin\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+105\,\sin\left(\frac{1}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\right)e^{\left(-\frac{5}{2}\right)}}{140\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*e^(-5/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.39, size = 94, normalized size = 0.57

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(8i\,dx+8i\,c)}-112i\,e^{(6i\,dx+6i\,c)}-70i\,e^{(4i\,dx+4i\,c)}+40i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c-\frac{5}{2}\right)}}{140\,ad\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(8*I*d*x + 8*I*c) - 112*I*e
^(6*I*d*x + 6*I*c) - 70*I*e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x + 2*I*c) +
5*I)*e^(-7/2*I*d*x - 7/2*I*c - 5/2)/(a*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(5/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(e^(-5/2)/(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Mupad [B]
time = 4.21, size = 101, normalized size = 0.61

$$-\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(-\sin\left(c+d\,x\right)-\frac{3\,\sin(3\,c+3\,d\,x)}{35}+\frac{\cos(c+d\,x)\,1\mathrm{i}}{2}+\frac{\cos(3\,c+3\,d\,x)\,1\mathrm{i}}{70}\right)}{d\,e^3\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] -((e/cos(c + d*x))^(1/2)*((cos(c + d*x)*1i)/2 - sin(c + d*x) + (cos(3*c + 3
*d*x)*1i)/70 - (3*sin(3*c + 3*d*x))/35))/(d*e^3*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{256i\sqrt{e\sec(c}}{315de^4\sqrt{a+ia\,\mathrm{t}}}$$

```
[Out] 2/9*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+32/105*I/d/e^2/(e*sec
(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/315*I*(e*sec(d*x+c))^(1/2)/d/e^
4/(a+I*a*tan(d*x+c))^(1/2)-16/63*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*sec(d*x+
c))^(7/2)-128/315*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.25, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$\frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}+\frac{32i}{105de^2\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((32*I)/1
05)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/3
15)*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/63)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(7/2)) - (((128*I)/315)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{8\int \frac{\sqrt{a+ia\,t\cdots}}{(e\sec(c\cdots}}{9\cdots}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\,ta\cdots}}{63ad(e\sec(c\cdots}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{\cdots}{105de^2(e\sec(c\cdots}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{\cdots}{105de^2(e\sec(c\cdots}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{\cdots}{105de^2(e\sec(c\cdots}$$

Mathematica [A]
time = 0.47, size = 87, normalized size = 0.42

$$\frac{\sqrt{e\sec(c+dx)}\,(945i-84i\cos(2(c+dx))-5i\cos(4(c+dx))+336\sin(2(c+dx))+40\sin(4(c+dx)))}{1260de^4\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[e*Sec[c + d*x]]*(945*I - (84*I)*Cos[2*(c + d*x)] - (5*I)*Cos[4*(c + d
*x)] + 336*Sin[2*(c + d*x)] + 40*Sin[4*(c + d*x)]))/(1260*d*e^4*Sqrt[a + I*
a*Tan[c + d*x]])
```

Maple [A]
time = 1.14, size = 132, normalized size = 0.64

method	result
default	$\frac{2(\cos^4(dx+c))\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(35i(\cos^5(dx+c))+35\sin(dx+c)(\cos^4(dx+c))+8i(\cos^3(dx+c))+48(\cos^2}{315d\,e^7a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)

[Out] 2/315/d*cos(d*x+c)^4*(e/cos(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(
d*x+c))^(1/2)*(35*I*cos(d*x+c)^5+35*sin(d*x+c)*cos(d*x+c)^4+8*I*cos(d*x+c)^
3+48*cos(d*x+c)^2*sin(d*x+c)+64*I*cos(d*x+c)+128*sin(d*x+c))/e^7/a

Maxima [A]
time = 0.56, size = 225, normalized size = 1.09

$$\frac{\left(35i\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)-45i\cos\left(\frac{7}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+252i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-420i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+1890i\cos\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+35\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+45\sin\left(\frac{7}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+252\sin\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+420\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+1890\sin\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\right)e^{\left(-\frac{7}{2}\right)}}{2520\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")

[Out] 1/2520*(35*I*cos(9/2*d*x + 9/2*c) - 45*I*cos(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))) - 420*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*
d*x + 9/2*c))) + 1890*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 35*sin(9/2*d*x + 9/2*c) + 45*sin(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))))*e^(-7/2)/(sqrt(a)*d)

Fricas [A]
time = 0.36, size = 105, normalized size = 0.51

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-45i\,e^{(10i\,dx+10i\,c)}-465i\,e^{(8i\,dx+8i\,c)}+1470i\,e^{(6i\,dx+6i\,c)}+2142i\,e^{(4i\,dx+4i\,c)}+287i\,e^{(2i\,dx+2i\,c)}+35i\right)e^{\left(-\frac{9}{2}i\,dx-\frac{9}{2}i\,c-\frac{7}{2}\right)}}{2520\,ad\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")

[Out] 1/2520*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-45*I*e^(10*I*d*x + 10*I*c) - 465
*I*e^(8*I*d*x + 8*I*c) + 1470*I*e^(6*I*d*x + 6*I*c) + 2142*I*e^(4*I*d*x + 4

```
*I*c) + 287*I*e^(2*I*d*x + 2*I*c) + 35*I)*e^(-9/2*I*d*x - 9/2*I*c - 7/2)/(a
*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(e^(-7/2)/(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

Mupad [B]
time = 4.31, size = 109, normalized size = 0.53

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(336\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(4\,c+4\,d\,x\right)\,5{}\mathrm{i}-\cos\left(2\,c+2\,d\,x\right)\,84{}\mathrm{i}+40\,\sin\left(4\,c+4\,d\,x\right)+945{}\mathrm{i}\right)}{1260\,d\,e^4\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] ((e/cos(c + d*x))^(1/2)*(336*sin(2*c + 2*d*x) - cos(4*c + 4*d*x)*5i - cos(2
*c + 2*d*x)*84i + 40*sin(4*c + 4*d*x) + 945i))/(1260*d*e^4*((a*(cos(2*c + 2
*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=529

$$-\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}-\frac{3ie^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{3ie^{7/2}\text{ArcTan}\left(\right.}{\sqrt{2}\sqrt{a}\,d}$$

```
[Out] -I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-3/2*I*e^(7/2)*arct
an(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))
*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(
1/2)+3/2*I*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I
*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*se
c(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2
)-3/4*I*e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*se
c(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/a^(1/2)
/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3581, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3ie^{7/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{3ie^{7/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{3ie^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{3ie^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - ((3*I)
*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((3*I)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*
x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
]) + (((3*I)/2)*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((3*I)/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*
Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{a^2}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^3\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\,t}}{2a\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(6ie^5\sec(c+dx))\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a}}{\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(3ie^4\sec(c+dx))\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a}}{\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^3\sec(c+dx))\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,d\right.}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}\right.}{2\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{3ie^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+d}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,t}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11282 vs. 2(529) = 1058.
time = 57.15, size = 11282, normalized size = 21.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1021 vs. 2(421) = 842.
time = 1.05, size = 1022, normalized size = 1.93

method	result	size
default	Expression too large to display	1022

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^3*(-6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)-3*I*arctanh(1/2*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-6*I*arctanh(1/2*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+3*I*arct
anh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(
d*x+c)+3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*
cos(d*x+c)*sin(d*x+c)-3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1-sin(d*x+c)))*cos(d*x+c)+6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^3-6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3+3*I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-6*arctanh(1/2*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3-6*arctanh(1/2*(1/(1+cos(d*x+c
)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3-6*arctanh(1/2*(1/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+6*arctanh(
1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*
x+c)-4*I*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+3*I*arctanh(1/2*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+3*cos(d*x+c)^
2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3*cos(d*x
+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-4*(1/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-3*arctanh(1/2*(1/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c)*a
rctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3*cos(d*x+c)
*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+4*(1/(1+co
s(d*x+c)))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)
^3*(e/cos(d*x+c))^(7/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/sin(d*
x+c)^7/(1/(1+cos(d*x+c)))^(7/2)/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1700 vs. 2(374) = 748.
time = 0.62, size = 1700, normalized size = 3.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 8*(6*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*a
rctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1
/2*c) + cos(d*x + c) + 1) + 6*(I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d
*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c)
, -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - 3*(2*sqrt(2)*arctan2(
sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*sqr
t(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c
) + 1) + 2*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 2*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -
sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x
 + 2*c) - 3*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2
))*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + s
in(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 1) + 3*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*l
og(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x +
1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(
d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)
 + 3*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
 -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d
*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*I*sqrt(2)*arctan2(sq
rt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + sqrt(2
)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(2*d*x + 2*c) - 6*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x +
1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 6*sqrt(2)*arctan2(sqrt(2)*c
os(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 6*sqrt(2)*arc
tan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) -
6*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x +
1/2*c) + 1) - 3*I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
```

```
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*I*
sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*I*sqrt(2)*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 16*cos(1/2*d*x + 1/2*c
) - 16*I*sin(1/2*d*x + 1/2*c))*sqrt(a)*e^(7/2)/((-64*I*a^2*cos(2*d*x + 2*c)
 + 64*a^2*sin(2*d*x + 2*c) - 64*I*a^2)*d)
```

Fricas [**A**]
time = 0.39, size = 433, normalized size = 0.82

$$-\frac{a^{2} d \sqrt{\frac{9 i e^{7}}{a^{3} d^{2}}} \log\left(-\frac{2}{3} \left(i a^{2} d \sqrt{\frac{9 i e^{7}}{a^{3} d^{2}}} - \frac{3 \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x + 2 i c)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right) - a^{2} d \sqrt{\frac{9 i e^{7}}{a^{3} d^{2}}} \log\left(-\frac{2}{3} \left(-i a^{2} d \sqrt{\frac{9 i e^{7}}{a^{3} d^{2}}} - \frac{3 \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x + 2 i c)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right) + a^{2} d \sqrt{-\frac{9 i e^{7}}{a^{3} d^{2}}} \log\left(-\frac{2}{3} \left(i a^{2} d \sqrt{-\frac{9 i e^{7}}{a^{3} d^{2}}} - \frac{3 \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x + 2 i c)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right) - a^{2} d \sqrt{-\frac{9 i e^{7}}{a^{3} d^{2}}} \log\left(-\frac{2}{3} \left(-i a^{2} d \sqrt{-\frac{9 i e^{7}}{a^{3} d^{2}}} - \frac{3 \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{(2 i d x + 2 i c)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right) + \frac{4 i \sqrt{\frac{a}{e^{(2 i d x + 2 i c)} + 1}} e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c + \frac{7}{2}\right)}}{\sqrt{e^{(2 i d x + 2 i c)} + 1}}}{2 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a^2*d*sqrt(9*I*e^7/(a^3*d^2))*log(-2/3*(I*a^2*d*sqrt(9*I*e^7/(a^3*d^2
)) - 3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/
2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)) - a^2*
d*sqrt(9*I*e^7/(a^3*d^2))*log(-2/3*(-I*a^2*d*sqrt(9*I*e^7/(a^3*d^2)) - 3*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/2))*e^(1/
2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)) + a^2*d*sqrt(-9
*I*e^7/(a^3*d^2))*log(-2/3*(I*a^2*d*sqrt(-9*I*e^7/(a^3*d^2)) - 3*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x
+ 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)) - a^2*d*sqrt(-9*I*e^7/(
a^3*d^2))*log(-2/3*(-I*a^2*d*sqrt(-9*I*e^7/(a^3*d^2)) - 3*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I
*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)) + 4*I*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c + 7/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1))/(a^2
*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=365

$$-\frac{i\sqrt{2}\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{i\sqrt{2}\,e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}$$

```
[Out] 1/2*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-1/2*I*e^(5/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-I*e^(5/2)*arctan(1-2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a
^(3/2)/d+I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+4*I*e^2*(e*sec(d*x+c))^(1/2)/a/d/
(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3581, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\sqrt{2}\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{i\sqrt{2}\,e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,a^{3/2}d}-\frac{ie^{5/2}\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}\,a^{3/2}d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(a^(3/2)*d) + (I*Sqrt[2]*e^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]])/(a^(3/2)*d) + (I*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])]])/(Sqrt[2]*a^(3/2)*d) - (I*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])]])/(Sqrt[2]*a^(3/2)*d) + ((4*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(4ie^4)\,\mathrm{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx}}{\sqrt{e\sec(c+dx)}}\right.}{ad}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(2ie^3)\,\mathrm{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx}}{\sqrt{e\sec(c+dx)}}\right.}{ad}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(ie^2)\,\mathrm{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan}}{\sqrt{e\sec(c}}\right.}{ad}$$

$$= \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c\right.}{\sqrt{2}\,a^{3/2}d}$$

$$= -\frac{i\sqrt{2}\,e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{i\sqrt{2}\,e^{5/2}\tan^{-1}\left(1\right.}{}$$

Mathematica [A]
time = 4.26, size = 338, normalized size = 0.93

$$\frac{e(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2\left(\cos(dx)(4i\cos(c)-4\sin(c))+4(\cos(c)+i\sin(c))\sin(dx)+\frac{2\left(\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\sqrt{-1+i\cos(c)+\sin(c)}\right)(\cos(2c)+i\sin(2c))\sqrt{i+\tan\left(\frac{dx}{2}\right)}}{\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}\right)}{d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*(Cos[d*x]*((4*I)*Cos[c]
 - 4*Sin[c]) + 4*(Cos[c] + I*Sin[c])*Sin[d*x] + (2*(ArcTanh[(Sqrt[1 - I*Cos
```

```
[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
+ Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]
- ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I
*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt
[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(
Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]))/(d*(a + I*a*Tan[c
 + d*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(285) = 570$.
time = 1.12, size = 957, normalized size = 2.62

method	result	size
default	Expression too large to display	957

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))^2*(-I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))-I*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))+2*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))+2*I*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*I*cos(d*x+c)*sin(d*x+c)*arctanh(
1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*cos(d*x+c)*arctan
h(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+I*cos(d*x+c)*arct
anh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+I*arctanh(1/2*(
1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*sin(d*x+c)*arctanh(1/2
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*cos(d*x+c)^2*arctanh
(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*cos(d*x+c)^2*arc
tanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-8*(1/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)^2-2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-2*I*cos(d*x+c)^2*arctan
h(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-8*I*cos(d*x+c)*si
n(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)+1-sin(d*x+c)))-arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))+8*(1/(1+cos(d*x+c)))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*
x+c))^(1/2)*cos(d*x+c)^3*(e/cos(d*x+c))^(5/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*
cos(d*x+c)^2-1)/(1/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^5/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(246) = 492$.

time = 0.60, size = 735, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*sqrt(
2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
 + 1/2*c) + cos(d*x + c) + 1) - 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + I*
sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^
2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 16*I*cos(1/2*d*x + 1/2*c) -
 16*sin(1/2*d*x + 1/2*c))*e^(5/2)/(a^(3/2)*d)
```

Fricas [A]
time = 0.40, size = 488, normalized size = 1.34

$$-\frac{a^{2} d \sqrt{\frac{4 i e^{5}}{a^{3} d^{2}}} e^{\left(i d x+i c\right)} \log\left(\left(a^{2} d \sqrt{\frac{4 i e^{5}}{a^{3} d^{2}}}+\frac{2 \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} \left(e^{\frac{5}{2}}+e^{\left(2 i d x+2 i c+\frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x+2 i c\right)}+1}}\right) e^{\left(-\frac{5}{2}\right)}\right)-a^{2} d \sqrt{\frac{4 i e^{5}}{a^{3} d^{2}}} e^{\left(i d x+i c\right)} \log\left(-\left(a^{2} d \sqrt{\frac{4 i e^{5}}{a^{3} d^{2}}}-\frac{2 \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} \left(e^{\frac{5}{2}}+e^{\left(2 i d x+2 i c+\frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x+2 i c\right)}+1}}\right) e^{\left(-\frac{5}{2}\right)}\right)-a^{2} d \sqrt{-\frac{4 i e^{5}}{a^{3} d^{2}}} e^{\left(i d x+i c\right)} \log\left(\left(a^{2} d \sqrt{-\frac{4 i e^{5}}{a^{3} d^{2}}}+\frac{2 \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} \left(e^{\frac{5}{2}}+e^{\left(2 i d x+2 i c+\frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x+2 i c\right)}+1}}\right) e^{\left(-\frac{5}{2}\right)}\right)+a^{2} d \sqrt{-\frac{4 i e^{5}}{a^{3} d^{2}}} e^{\left(i d x+i c\right)} \log\left(-\left(a^{2} d \sqrt{-\frac{4 i e^{5}}{a^{3} d^{2}}}-\frac{2 \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} \left(e^{\frac{5}{2}}+e^{\left(2 i d x+2 i c+\frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x+2 i c\right)}+1}}\right) e^{\left(-\frac{5}{2}\right)}\right)+\frac{8 \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} \left(-i e^{\frac{5}{2}}-i e^{\left(2 i d x+2 i c+\frac{5}{2}\right)}\right) e^{\left(\frac{1}{2} i d x+\frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x+2 i c\right)}+1}}\right) e^{\left(-i d x-i c\right)}}{2 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(4*I*e^5
/(a^3*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x + 2
*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2
)) - a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(4*I*e^5
```

```
/(a^3*d^2)) - 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x + 2
*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2
)) - a^2*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(-4*I*e^
5/(a^3*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x +
2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/
2)) + a^2*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(-4*I*
e^5/(a^3*d^2)) - 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(5/2) + e^(2*I*d*x
+ 2*I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-
5/2)) + 8*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(5/2) - I*e^(2*I*d*x + 2*
I*c + 5/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e^(-I*d*
x - I*c)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3569}

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.12, size = 38, normalized size = 1.00

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(30) = 60.
time = 0.81, size = 85, normalized size = 2.24

method	result	size
risch	$\frac{2ie\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{3a\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72
default	$\frac{2i\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^2(dx+c))\left(-2i\cos(dx+c)\sin(dx+c)+2(\cos^2(dx+c))-1\right)}{3d\,a^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)^2*(-2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.51, size = 75, normalized size = 1.97

$$\frac{2i\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}e^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*I*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)*e^(3/2)/(a^(3/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(3/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.
time = 0.37, size = 64, normalized size = 1.68

$$\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{\frac{3}{2}}+i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^2d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(3/2) + I*e^(2*I*d*x + 2*I*c + 3
/2))*e^(-3/2*I*d*x - 3/2*I*c)/(a^2*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sec{(c + dx)})^{\frac{3}{2}}}{(ia (\tan{(c + dx)} - i))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.426 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 4/5*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+2/5*I*(e*sec(d*x+c)
)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)
/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}} + \frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{5a}$$
$$= \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}} + \frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.21, size = 63, normalized size = 0.79

$$\frac{2\sqrt{e\sec(c+dx)}\,(3+2i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[e*Sec[c + d*x]]*(3 + (2*I)*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x]
)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.86, size = 101, normalized size = 1.26

method	result
default	$-\frac{2i\sqrt{\frac{e}{\cos(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(2i\sin(dx+c)\left(\cos^2(dx+c)\right)-2\left(\cos^3(dx+c)\right)+2i\sin(dx+c)-\cos(dx+c)\right.}{5d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*cos(d*x+c)*(2*I*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)^3+2*I*sin(d*x+c)-cos
(d*x+c))/a^2
```

Maxima [A]
time = 0.56, size = 79, normalized size = 0.99

$$\frac{\left(i\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\right)e^{\frac{1}{2}}}{5\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*(I*cos(5/2*d*x + 5/2*c) + 5*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c))) + sin(5/2*d*x + 5/2*c) + 5*sin(1/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))))*e^(1/2)/(a^(3/2)*d)
```

Fricas [A]
time = 0.38, size = 76, normalized size = 0.95

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{\frac{1}{2}}+5i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+6i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^2d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(1/2) + 5*I*e^(4*I*d*x + 4*I*c +
 1/2) + 6*I*e^(2*I*d*x + 2*I*c + 1/2))*e^(-5/2*I*d*x - 5/2*I*c)/(a^2*d*sqrt
(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{e\sec\left(c+dx\right)}}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B]
time = 3.94, size = 84, normalized size = 1.05

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5{}\mathrm{i}\right)}{5\,a\,d\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 5i))/(5*a
*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2))
```

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2i}{7d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{8i}{21ad\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{16i\sqrt{a+ia\tan(c}{21a^2d\sqrt{e\sec(c+}}$$

```
[Out] 8/21*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-16/21*I*(a+I*a*tan
(d*x+c))^(1/2)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*I/d/(e*sec(d*x+c))^(1/2)/(a+I
*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}+\frac{8i}{21ad\sqrt{a+ia\tan(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((8*I)/21
)/(a*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/21)*Sqrt
[a + I*a*Tan[c + d*x]])/(a^2*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{7d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{4\int \frac{}{\sqrt{e\sec(c+}}}{}$$

$$= \frac{2i}{7d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{}{21ad\sqrt{e\sec(c-}}$$

$$= \frac{2i}{7d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}} + \frac{}{21ad\sqrt{e\sec(c-}}$$

Mathematica [A]
time = 0.37, size = 83, normalized size = 0.69

$$-\frac{\sec^2(c+dx)(-7+9\cos(2(c+dx))+12i\sin(2(c+dx)))}{21ad\sqrt{e\sec(c+dx)}\,(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -1/21*(Sec[c + d*x]^2*(-7 + 9*Cos[2*(c + d*x)] + (12*I)*Sin[2*(c + d*x)]))/
(a*d*Sqrt[e*Sec[c + d*x]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.17, size = 106, normalized size = 0.88

method	result	size
default	$-\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(9i(\cos^2(dx+c))-12\sin(dx+c)\cos(dx+c)-8i)}{21d(2i\cos(dx+c)\sin(dx+c)+2(\cos^2(dx+c))-1)\sqrt{\frac{e}{\cos(dx+c)}}\,a^2}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/21/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(9*I*cos(d*x+c)^2-12
*sin(d*x+c)*cos(d*x+c)-8*I)/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/(e
/cos(d*x+c))^(1/2)/a^2
```

Maxima [A]
time = 0.56, size = 129, normalized size = 1.07

$$\frac{\left(3i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)+14i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-21i\cos\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+3\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+14\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+21\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)e^{\left(-\frac{1}{2}\right)}}{42\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/42*(3*I*cos(7/2*d*x + 7/2*c) + 14*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) - 21*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 14*sin(3/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 21*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))))*e^(-1/2)/(a^(3/2)*d)
```

Fricas [A]
time = 0.37, size = 83, normalized size = 0.69

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-21i\,e^{(6i\,dx+6i\,c)}-7i\,e^{(4i\,dx+4i\,c)}+17i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c-\frac{1}{2}\right)}}{42\,a^2d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/42*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-21*I*e^(6*I*d*x + 6*I*c) - 7*I*e^(
4*I*d*x + 4*I*c) + 17*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-7/2*I*d*x - 7/2*I*c
- 1/2)/(a^2*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\sec\left(c+dx\right)}\,\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 4.16, size = 104, normalized size = 0.86

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(35\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)-\cos\left(c+d\,x\right)\,7\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,3\mathrm{i}\right)}{42\,a\,d\,e\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] ((e/cos(c + d*x))^(1/2)*(35*sin(c + d*x) - cos(c + d*x)*7i + cos(3*c + 3*d*
x)*3i + 3*sin(3*c + 3*d*x)))/(42*a*d*e*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*
d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=165

$$\frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}+\frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{32i\sqrt{e\sec(}}{45ade^2\sqrt{a+ia}}$$

```
[Out] 4/15*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/45*I*(e*sec(d*x
+c))^(1/2)/a/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-16/45*I*(a+I*a*tan(d*x+c))^(1/2
)/a^2/d/(e*sec(d*x+c))^(3/2)+2/9*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)
)^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}+\frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}}+\frac{4i}{15ad\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((4*I)/
15)/(a*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((32*I)/45)*
Sqrt[e*Sec[c + d*x]])/(a*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/45)*S
qrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{2\int \frac{\quad}{(e\sec(c+dx)}}{\quad}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{\quad}{15ad(e\sec(c}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{\quad}{15ad(e\sec(c}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{\quad}{15ad(e\sec(c}$$

Mathematica [A]
time = 0.57, size = 100, normalized size = 0.61

$$-\frac{\sec^3(c+dx)(-81\cos(c+dx)+5\cos(3(c+dx))-54i\sin(c+dx)+10i\sin(3(c+dx)))}{90ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -1/90*(Sec[c + d*x]^3*(-81*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - (54*I)*Sin[c
 + d*x] + (10*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c
 + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.89, size = 132, normalized size = 0.80

method	result
default	$\frac{2(\cos^2(dx+c))\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(10i(\cos^5(dx+c))+10\sin(dx+c)(\cos^4(dx+c))+i(\cos^3(dx+c))+6(\cos^2(}{45d\,e^3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/45/d*cos(d*x+c)^2*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d
*x+c))^(1/2)*(10*I*cos(d*x+c)^5+10*sin(d*x+c)*cos(d*x+c)^4+I*cos(d*x+c)^3+6
*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+c))/e^3/a^2
```

Maxima [A]
time = 0.55, size = 177, normalized size = 1.07

$$\frac{\left(5i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+27i\,\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)-15i\,\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)+135i\,\cos\left(\frac{1}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)+5\,\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+27\,\sin\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)+15\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)+135\,\sin\left(\frac{1}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\right)e^{\left(-\frac{3}{2}\right)}}{180\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/180*(5*I*cos(9/2*d*x + 9/2*c) + 27*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) - 15*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 135*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 27*sin(5/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) + 15*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 135*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))))*e^(-3/2)/(a^(3/2)*d)
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.57

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(8i\,dx+8i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+162i\,e^{(4i\,dx+4i\,c)}+32i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{9}{2}i\,dx-\frac{9}{2}i\,c-\frac{3}{2}\right)}}{180\,a^2d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/180*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x + 8*I*c) + 120*I*
e^(6*I*d*x + 6*I*c) + 162*I*e^(4*I*d*x + 4*I*c) + 32*I*e^(2*I*d*x + 2*I*c)
+ 5*I)*e^(-9/2*I*d*x - 9/2*I*c - 3/2)/(a^2*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 4.20, size = 112, normalized size = 0.68

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,12\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+42\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(4\,c+4\,d\,x\right)+135\mathrm{i}\right)}{180\,a\,d\,e^2\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*12i + cos(4*c + 4*d*x)*5i + 42*si
n(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 135i))/(180*a*d*e^2*((a*(cos(2*c + 2*
d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}+\frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}+\frac{\quad}{385ade^2\sqrt{e\sec}}$$

```
[Out] 16/77*I/a/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+128/385*I/a/d/e^2
/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-96/385*I*(a+I*a*tan(d*x+c))^
(1/2)/a^2/d/(e*sec(d*x+c))^(5/2)-256/385*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/e
^2/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3
/2)
```

Rubi [A]
time = 0.26, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$-\frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}-\frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}}+\frac{128i}{385ade^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{16i}{77ad\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I
)/77)/(a*d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((128*I)/38
5)/(a*d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((96*I)/385
)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(5/2)) - (((256*I)/38
5)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{8\int \frac{}{(e\sec(c+dx}}{}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{}{77ad(e\sec(c}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{}{77ad(e\sec(c}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{}{77ad(e\sec(c}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{}{77ad(e\sec(c}$$

Mathematica [A]
time = 0.61, size = 100, normalized size = 0.48

$$-\frac{(e\sec(c+dx))^{3/2}(-385+660\cos(2(c+dx))+21\cos(4(c+dx))+880i\sin(2(c+dx))+56i\sin(4(c+dx)))}{1540ade^4(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -1/1540*((e*Sec[c + d*x])^(3/2)*(-385 + 660*Cos[2*(c + d*x)] + 21*Cos[4*(c
+ d*x)] + (880*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a*d*e^4*(-I
 + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.90, size = 142, normalized size = 0.68

method	result

default	$\frac{2\left(\cos^3(dx+c)\right)\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\left(70i\left(\cos^6(dx+c)\right)+70\sin(dx+c)\left(\cos^5(dx+c)\right)+5i\left(\cos^4(dx+c)\right)+40\sin\right.}{385d\,e^5a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/385/d*cos(d*x+c)^3*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(
d*x+c))^(1/2)*(70*I*cos(d*x+c)^6+70*sin(d*x+c)*cos(d*x+c)^5+5*I*cos(d*x+c)^
4+40*sin(d*x+c)*cos(d*x+c)^3+16*I*cos(d*x+c)^2+64*sin(d*x+c)*cos(d*x+c)-128
*I)/e^5/a^2
```

Maxima [A]
time = 0.57, size = 225, normalized size = 1.08

$$\frac{\left(35i\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)+220i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)-77i\cos\left(\frac{5}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+770i\cos\left(\frac{3}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)-1540i\cos\left(\frac{1}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+35\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right)+220\sin\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+77\sin\left(\frac{5}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+770\sin\left(\frac{3}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+1540\sin\left(\frac{1}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)\right)e^{\left(-\frac{5}{2}\right)}}{3080\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/3080*(35*I*cos(11/2*d*x + 11/2*c) + 220*I*cos(7/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(5/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c))) + 770*I*cos(3/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 1540*I*cos(1/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 35*sin(11/2*d*x + 11/2*c) + 220*sin(7/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(5/11*arct
an2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*sin(3/11*arctan2
(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1540*sin(1/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*e^(-5/2)/(a^(3/2)*d)
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.50

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(10i\,dx+10i\,c)}-1617i\,e^{(8i\,dx+8i\,c)}-770i\,e^{(6i\,dx+6i\,c)}+990i\,e^{(4i\,dx+4i\,c)}+255i\,e^{(2i\,dx+2i\,c)}+35i\right)e^{\left(-\frac{11}{2}i\,dx-\frac{11}{2}i\,c-\frac{5}{2}\right)}}{3080\,a^2d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/3080*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-77*I*e^(10*I*d*x + 10*I*c) - 161
7*I*e^(8*I*d*x + 8*I*c) - 770*I*e^(6*I*d*x + 6*I*c) + 990*I*e^(4*I*d*x + 4*
```

```
I*c) + 255*I*e^(2*I*d*x + 2*I*c) + 35*I)*e^(-11/2*I*d*x - 11/2*I*c - 5/2)/(
a^2*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
[Out] integrate(e^(-5/2)/((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [B]
time = 4.57, size = 127, normalized size = 0.61

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(2310\,\sin\left(c+d\,x\right)+297\,\sin\left(3\,c+3\,d\,x\right)+35\,\sin\left(5\,c+5\,d\,x\right)-\cos\left(c+d\,x\right)\,770{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,143{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,35{}\mathrm{i}\right)}{3080\,a\,d\,e^3\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] ((e/cos(c + d*x))^(1/2)*(2310*sin(c + d*x) - cos(c + d*x)*770i + cos(3*c +
3*d*x)*143i + cos(5*c + 5*d*x)*35i + 297*sin(3*c + 3*d*x) + 35*sin(5*c + 5*
d*x)))/(3080*a*d*e^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos
(2*c + 2*d*x) + 1))^(1/2))
```

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=411

$$-\frac{5ie^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{5ie^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}$$

```
[Out] -5/2*I*e^(9/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+5/2*I*e^(9/2)*arctan(1+2^(1/2)*e^(1/2
)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+
5/4*I*e^(9/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)-5/4*I*e^(9/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)+5*I*e^4*(e*sec(d*x+c))^(
1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+4*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a
*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3581, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5ie^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{5ie^{9/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}+\frac{5ie^{9/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\,a^{5/2}d}-\frac{5ie^{9/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\,a^{5/2}d}+\frac{5ie^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{a^3d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-5*I)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((5*I)*e^(9/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])])/(Sqrt[2]*a^(5/2)*d) + (((5*I)/2)*e^(9/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) - (((5*I)/2)*e^(9/2)*Log[a + (Sqrt[2
]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c
+ d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) + ((4*I)*e^2*(e*Sec[c +
 d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)*e^4*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
```

```
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{(5e^2)\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \text{(}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \cdots$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \cdots$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \cdots$$

$$= \frac{5ie^{9/2}\log\left(a - \frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} + \cos(c+dx)(a+ia\tan\cdots\right.}{2\sqrt{2}\,a^{5/2}d}$$

$$= -\frac{5ie^{9/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d} + \frac{5ie^{9/2}\tan^{-1}\left(1 + \cdots\right.}{\cdots}$$

Mathematica [A]

time = 6.25, size = 370, normalized size = 0.90

$$\frac{e^2(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\cos(dx)(8i\cos(2c)-8\sin(2c))+\sec(c+dx)(i\cos(3c)-\sin(3c))+8(\cos(2c)+i\sin(2c))\sin(dx)+\frac{5\left(\tanh^{-1}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\tanh^{-1}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}\sqrt{-1+i\cos(c)+\sin(c)}\right)(\cos(3c)+i\sin(3c))\sqrt{i+\tan\left(\frac{dx}{2}\right)}}{\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}\right)}{d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(Cos[d*x]*((8*I)*Cos[
2*c] - 8*Sin[2*c]) + Sec[c + d*x]*(I*Cos[3*c] - Sin[3*c]) + 8*(Cos[2*c] + I
*Sin[2*c])*Sin[d*x] + (5*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan
[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[
c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I +
 Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*
(Cos[3*c] + I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[
2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1438 vs. 2(321) = 642.
time = 1.02, size = 1439, normalized size = 3.50

method	result	size
default	Expression too large to display	1439

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^4*(-80*cos(d*x+c)^4*(1/(1+cos(d*x+c)))^(1/2)+20*cos(
d*x+c)^4*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+20
*cos(d*x+c)^4*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))-15*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d
*x+c)))-15*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))-10*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*cos(d*x+c)^3-10*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c)))*cos(d*x+c)^3+84*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-20*cos(d*x+
c)^3*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))+20*cos(d*x+c)^3*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))+20*I*cos(d*x+c)^4*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)+1-sin(d*x+c)))-20*I*cos(d*x+c)^4*arctanh(1/2*(1/(1+cos(d*x+c))
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-10*I*cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(
d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+10*I*cos(d*x+c)^3*arctanh(1/2*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-15*I*cos(d*x+c)^2*arctanh(
1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+15*I*cos(d*x+c)^2*a
```

```
rctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+5*I*cos(d*x+
c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-5*I*cos(
d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+10*a
rctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2
*sin(d*x+c)-10*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
)))*cos(d*x+c)^2*sin(d*x+c)+5*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-5*arctanh(1/2*(1/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-4*(1/(1+cos(d*x+c)))^
(1/2)+5*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d
*x+c)))+5*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))+20*I*cos(d*x+c)^3*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)
*(cos(d*x+c)+1-sin(d*x+c)))+20*I*cos(d*x+c)^3*sin(d*x+c)*arctanh(1/2*(1/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-80*I*cos(d*x+c)^3*sin(d*x+c)*
(1/(1+cos(d*x+c)))^(1/2)-10*I*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-10*I*cos(d*x+c)^2*sin(d*x+c)*arc
tanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-5*I*cos(d*x+c)
*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
-5*I*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
+1+sin(d*x+c)))+44*I*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2))*(a*(I*
sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(e/cos(d*x+c))^(9/2)/
(4*I*sin(d*x+c)*cos(d*x+c)^2+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))/(1/(
1+cos(d*x+c)))^(9/2)/sin(d*x+c)^9/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(277) = 554$.
time = 0.64, size = 2281, normalized size = 5.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (10*(-I*sqrt(2)*cos(3/2*d*x + 3/2*c) - I*sqrt(2)*cos(1/2*d*x + 1/2*c) - sqr
t(2)*sin(3/2*d*x + 3/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*arctan2(sqrt(2)*s
in(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x
+ c) + 1) + 10*(I*sqrt(2)*cos(3/2*d*x + 3/2*c) + I*sqrt(2)*cos(1/2*d*x + 1/
2*c) + sqrt(2)*sin(3/2*d*x + 3/2*c) - sqrt(2)*sin(1/2*d*x + 1/2*c))*arctan2
(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c
) + cos(d*x + c) + 1) + (10*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) +
1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d
*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*arctan2(sq
rt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt
(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 1) - 5*I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
```

```
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5
*I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2
)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 64*cos(1/2*d*x + 1/2*c) + 64
*I*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 5*(2*sqrt(2)*arctan2(sqrt(2
)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*sqrt(2)*a
rctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1)
 + 2*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2
)*sin(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - I*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + I*s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1
/2*c) - 64*cos(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*cos(3/2*d*x + 3/2*c) + sqrt(
2)*cos(1/2*d*x + 1/2*c) - I*sqrt(2)*sin(3/2*d*x + 3/2*c) + I*sqrt(2)*sin(1/
2*d*x + 1/2*c))*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2
)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x +
 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 1) - 5*(sqrt(2)*cos(3/2*d*x + 3/2*c) + sqrt(2)*cos(1/2*d*x +
1/2*c) - I*sqrt(2)*sin(3/2*d*x + 3/2*c) + I*sqrt(2)*sin(1/2*d*x + 1/2*c))*l
og(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x +
1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(
d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)
 - (10*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*
d*x + 1/2*c) + 1) + 10*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d
*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*arctan2(s
qrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 5*sqr
t(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 64*I*cos(1/2*d*x + 1/2*c) - 64*sin(1/2*d*x + 1/2*
c))*sin(3/2*d*x + 3/2*c) + 5*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*co
```

```
s(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*ar
ctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) +
 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 1) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s...
```

Fricas [A]
time = 0.42, size = 490, normalized size = 1.19

$$-\frac{a^{3} d \sqrt{\frac{25 i e^{9}}{a^{5} d^{2}}} e^{(i d x+i c)} \log\left(\frac{2}{5} \left(a^{3} d \sqrt{\frac{25 i e^{9}}{a^{5} d^{2}}} + \frac{5 \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \left(e^{\frac{9}{2}}+e^{(2 i d x+2 i c+\frac{9}{2})}\right) e^{(\frac{1}{2} i d x+\frac{1}{2} i c)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{(-\frac{9}{2})}\right) - a^{3} d \sqrt{\frac{25 i e^{9}}{a^{5} d^{2}}} e^{(i d x+i c)} \log\left(-\frac{2}{5} \left(a^{3} d \sqrt{\frac{25 i e^{9}}{a^{5} d^{2}}} - \frac{5 \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \left(e^{\frac{9}{2}}+e^{(2 i d x+2 i c+\frac{9}{2})}\right) e^{(\frac{1}{2} i d x+\frac{1}{2} i c)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{(-\frac{9}{2})}\right) - a^{3} d \sqrt{-\frac{25 i e^{9}}{a^{5} d^{2}}} e^{(i d x+i c)} \log\left(\frac{2}{5} \left(a^{3} d \sqrt{-\frac{25 i e^{9}}{a^{5} d^{2}}} + \frac{5 \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \left(e^{\frac{9}{2}}+e^{(2 i d x+2 i c+\frac{9}{2})}\right) e^{(\frac{1}{2} i d x+\frac{1}{2} i c)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{(-\frac{9}{2})}\right) + a^{3} d \sqrt{-\frac{25 i e^{9}}{a^{5} d^{2}}} e^{(i d x+i c)} \log\left(-\frac{2}{5} \left(a^{3} d \sqrt{-\frac{25 i e^{9}}{a^{5} d^{2}}} - \frac{5 \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \left(e^{\frac{9}{2}}+e^{(2 i d x+2 i c+\frac{9}{2})}\right) e^{(\frac{1}{2} i d x+\frac{1}{2} i c)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{(-\frac{9}{2})}\right) + \frac{4 \sqrt{\frac{a}{e^{(2 i d x+2 i c)}+1}} \left(-4 i e^{\frac{9}{2}} - 5 i e^{(2 i d x+2 i c+\frac{9}{2})}\right) e^{(\frac{1}{2} i d x+\frac{1}{2} i c)}}{\sqrt{e^{(2 i d x+2 i c)}+1}}\right) e^{(-i d x-i c)}}{2 a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a^3*d*sqrt(25*I*e^9/(a^5*d^2))*e^(I*d*x + I*c)*log(2/5*(a^3*d*sqrt(25
*I*e^9/(a^5*d^2)) + 5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(9/2) + e^(2*I*d
*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))*e
^(-9/2)) - a^3*d*sqrt(25*I*e^9/(a^5*d^2))*e^(I*d*x + I*c)*log(-2/5*(a^3*d*s
qrt(25*I*e^9/(a^5*d^2)) - 5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(9/2) + e^
(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) +
 1))*e^(-9/2)) - a^3*d*sqrt(-25*I*e^9/(a^5*d^2))*e^(I*d*x + I*c)*log(2/5*(a
^3*d*sqrt(-25*I*e^9/(a^5*d^2)) + 5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(9/
2) + e^(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2
*I*c) + 1))*e^(-9/2)) + a^3*d*sqrt(-25*I*e^9/(a^5*d^2))*e^(I*d*x + I*c)*log
(-2/5*(a^3*d*sqrt(-25*I*e^9/(a^5*d^2)) - 5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*(e^(9/2) + e^(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I
*d*x + 2*I*c) + 1))*e^(-9/2)) + 4*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-4*I*e
^(9/2) - 5*I*e^(2*I*d*x + 2*I*c + 9/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I
*d*x + 2*I*c) + 1))*e^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(9/2)*sec(d*x + c)^(9/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=527

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\sqrt{2}\,e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}\,e^{7/2}\text{A}}{a}$$

```
[Out] -1/2*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*e^(7/2)*ln(a+2^(1/2
)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*
(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+I*e^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c
))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/a^(3/2)/d/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*e^(7/2)*arctan(1+2^(1/2)*e^(1/
2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2
)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+4/3*I*e^2*(e*
sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3581, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\sqrt{2}\,e^{7/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}\,e^{7/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}-\frac{ie^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}\,a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{ie^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}\,a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
 (I*Sqrt[2]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*e^(7/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec
[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
) - (I*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/
(Sqrt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
 (I*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sq
rt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sq
rt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{(e^3\sec(c+dx))\int \sqrt{e\sec(c+dx)}\,\sqrt{a-ia\tan}}{a^2\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c-}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{(4ie^5\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-}}{\sqrt{}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{(2ie^4\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-}}{\sqrt{}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{(ie^3\sec(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}\,x}{\sqrt{e}}+x^2}\,dx,\right.}{ad\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\,\sqrt{a-ia\tan(c+dx}}{\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\,a^{3/2}d\sqrt{a-ia\tan(c}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{2}\,e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,\sqrt{a-ia\tan(c+}}{\sqrt{a}\,\sqrt{e\sec(c+dx)}}\right.}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan}}$$

<u>Mathematica</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 11295 vs. $2(527) = 1054$.
time = 30.19, size = 11295, normalized size = 21.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

<u>Maple</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(421) = 842$.
time = 0.85, size = 1324, normalized size = 2.51

method	result	size
default	Expression too large to display	1324

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-1+cos(d*x+c))^3*(6*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)+1-sin(d*x+c)))+6*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-12*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)-12*I*arctanh(1/2*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+6*I*ar
ctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*si
n(d*x+c)+6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*cos(d*x+c)*sin(d*x+c)-12*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
+1-sin(d*x+c)))*cos(d*x+c)^3-12*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3-8*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3
*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+
c)+3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(
d*x+c)+12*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*cos(d*x+c)^3-12*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d
*x+c)))*cos(d*x+c)^3-6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))*cos(d*x+c)^2+6*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-9*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+9*I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d
*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-8*I*cos(d*x+c)*sin(d*x+c)*(1/(1+co
s(d*x+c)))^(1/2)-12*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(
d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+12*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+6*arctanh(1/2*(1/(1+cos(d*
x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-6*arctanh(1/2
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+
3*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)-3*sin(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c
)))-3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+8*(1/
(1+cos(d*x+c)))^(1/2)-3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))+9*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))+9*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))+3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))-3*I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
)))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(e/cos(d*x+
c))^(7/2)/(4*I*sin(d*x+c)*cos(d*x+c)^2+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*
x+c))/(1/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^7/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1423 vs. $2(374) = 748$.

time = 0.64, size = 1423, normalized size = 2.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/12*(6*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 6*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 6*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*I*s
qrt(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sq
rt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos
(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1) - 3*I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 3*sqrt(2)*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*a
```

```
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*log(2*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 16*I*cos(3/2*d*x +
3/2*c) + 16*sin(3/2*d*x + 3/2*c))*e^(7/2)/(a^(5/2)*d)
```

Fricas [A]
time = 0.40, size = 492, normalized size = 0.93

$$-\frac{3 a^{3} d \sqrt{\frac{4 i e^{7}}{a^{5} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\left(\left(i a^{3} d \sqrt{\frac{4 i e^{7}}{a^{5} d^{2}}} + \frac{2 \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right)\right) - 3 a^{3} d \sqrt{\frac{4 i e^{7}}{a^{5} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\left(\left(-i a^{3} d \sqrt{\frac{4 i e^{7}}{a^{5} d^{2}}} + \frac{2 \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right)\right) + 3 a^{3} d \sqrt{-\frac{4 i e^{7}}{a^{5} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\left(\left(i a^{3} d \sqrt{-\frac{4 i e^{7}}{a^{5} d^{2}}} + \frac{2 \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right)\right) - 3 a^{3} d \sqrt{-\frac{4 i e^{7}}{a^{5} d^{2}}} e^{\left(2 i d x + 2 i c\right)} \log\left(\left(\left(-i a^{3} d \sqrt{-\frac{4 i e^{7}}{a^{5} d^{2}}} + \frac{2 \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(e^{\frac{7}{2}} + e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-\frac{7}{2}\right)}\right)\right) + \frac{8 \sqrt{\frac{a}{e^{\left(2 i d x + 2 i c\right)} + 1}} \left(-i e^{\frac{7}{2}} - i e^{\left(2 i d x + 2 i c + \frac{7}{2}\right)}\right) e^{\left(\frac{1}{2} i d x + \frac{1}{2} i c\right)}}{\sqrt{e^{\left(2 i d x + 2 i c\right)} + 1}}\right) e^{\left(-2 i d x - 2 i c\right)}}{6 a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^3*d*sqrt
(4*I*e^7/(a^5*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(7/2) + e^(2*I
*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*c) + 1))
*e^(-7/2)) - 3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^
3*d*sqrt(4*I*e^7/(a^5*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(7/2)
+ e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d*x + 2*I*
c) + 1))*e^(-7/2)) + 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*l
og((I*a^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^(2*I*d
*x + 2*I*c) + 1))*e^(-7/2)) - 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x +
 2*I*c)*log((-I*a^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e^(7/2) + e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqr
t(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2)) + 8*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*(-I*e^(7/2) - I*e^(2*I*d*x + 2*I*c + 7/2))*e^(1/2*I*d*x + 1/2*I*c)/sqrt(e^
(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(7/2)*sec(d*x + c)^(7/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{3569\}$

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.28, size = 38, normalized size = 1.00

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(30) = 60.
time = 0.92, size = 103, normalized size = 2.71

method	result
default	$\frac{2i\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^3(dx+c))\,(-4i\sin(dx+c)(\cos^2(dx+c))+4(\cos^3(dx+c))+i\sin(dx+c)-3\cos(dx+c}{5d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*I/d*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)^3*(-4*I*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^3+I*sin(d*x+c)-3*c
os(d*x+c))/a^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.50, size = 75, normalized size = 1.97

$$\frac{2i\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}e^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 2/5*I*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)*e^(5/2)/(a^(5/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(5/2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.
time = 0.41, size = 64, normalized size = 1.68

$$\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{\frac{5}{2}}+i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^3d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

[Out] 2/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(5/2) + I*e^(2*I*d*x + 2*I*c + 5/2))*e^(-5/2*I*d*x - 5/2*I*c)/(a^3*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(e^(5/2)*sec(d*x + c)^(5/2)/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)

[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}+\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/7*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(5/2)+4/21*I*(e*sec(d*x+c))
^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4*
I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}} + \frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{7a}$$
$$= \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}} + \frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.26, size = 63, normalized size = 0.79

$$\frac{2(e\sec(c+dx))^{3/2}(-5i+2\tan(c+dx))}{21a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (2*(e*Sec[c + d*x])^(3/2)*(-5*I + 2*Tan[c + d*x]))/(21*a^2*d*(-I + Tan[c +
d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.07, size = 112, normalized size = 1.40

method	result
default	$\frac{2i\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^2(dx+c))\left(-12i\sin(dx+c)(\cos^3(dx+c))+12(\cos^4(dx+c))-i\cos(dx+c)\sin(dx+\right.}{21d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/21*I/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*cos(d*x+c)^2*(-12*I*cos(d*x+c)^3*sin(d*x+c)+12*cos(d*x+c)^4-I*cos(d*x+c)*
sin(d*x+c)-5*cos(d*x+c)^2-2)/a^3
```

Maxima [A]
time = 0.55, size = 81, normalized size = 1.01

$$\frac{\left(3i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+3\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)e^{\frac{3}{2}}}{21a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(3*I*cos(7/2*d*x + 7/2*c) + 7*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 7*sin(3/7*arctan2(sin(7/2
*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*e^(3/2)/(a^(5/2)*d)
```

Fricas [A]
time = 0.40, size = 76, normalized size = 0.95

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{\frac{3}{2}}+7i\,e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+10i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c\right)}}{21\,a^{3}d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
[Out] 1/21*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(3/2) + 7*I*e^(4*I*d*x + 4*I*
c + 3/2) + 10*I*e^(2*I*d*x + 2*I*c + 3/2))*e^(-7/2*I*d*x - 7/2*I*c)/(a^3*d*
sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate(e^(3/2)*sec(d*x + c)^(3/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 4.23, size = 102, normalized size = 1.28

$$\frac{e\,\sqrt{\frac{e}{\cos{(c+d\,x)}}}\,\left(7\,\sin{(c+d\,x)}+3\,\sin{(3\,c+3\,d\,x)}+\cos{(c+d\,x)}\,7\mathrm{i}+\cos{(3\,c+3\,d\,x)}\,3\mathrm{i}\right)}{21\,a^2\,d\,\sqrt{\frac{a\,\left(\cos{(2\,c+2\,d\,x)}+1+\sin{(2\,c+2\,d\,x)}\,1\mathrm{i}\right)}{\cos{(2\,c+2\,d\,x)}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] (e*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*7i + 7*sin(c + d*x) + cos(3*c + 3*d
*x)*3i + 3*sin(3*c + 3*d*x)))/(21*a^2*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2
*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.434 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 16/45*I*(e*sec(d*x+c))^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2/9*I*(e*sec(d*
x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+8/45*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I
*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((8*I)
/45)*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*I)/45
)*Sqrt[e*Sec[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{9a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}} + \frac{8\int \frac{\sqrt{e\sec(c}}{\sqrt{a+ia\tan}}}{45a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}} + \frac{16i\sqrt{e\sec(}}{45a^2d\sqrt{a+ia}}$$

Mathematica [A]
time = 0.38, size = 85, normalized size = 0.70

$$-\frac{i\sec^2(c+dx)\sqrt{e\sec(c+dx)}\,(9+25\cos(2(c+dx))+20i\sin(2(c+dx)))}{45a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/45*I)*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(9 + 25*Cos[2*(c + d*x)] + (
20*I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [A]
time = 0.90, size = 128, normalized size = 1.06

method	result
default	$\frac{2i\cos(dx+c)\sqrt{\frac{e}{\cos(dx+c)}}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,\left(-20i\left(\cos^4(dx+c)\right)\sin(dx+c)+20\left(\cos^5(dx+c)\right)-3i\sin(dx+c)\left(\cos^2(dx\right.\right.}{45d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*I/d*cos(d*x+c)*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d
*x+c))^(1/2)*(-20*I*cos(d*x+c)^4*sin(d*x+c)+20*cos(d*x+c)^5-3*I*cos(d*x+c)^
2*sin(d*x+c)-7*cos(d*x+c)^3-8*I*sin(d*x+c)+4*cos(d*x+c))/a^3
```

Maxima [A]
time = 0.54, size = 129, normalized size = 1.07

$$\frac{\left(5i\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)+18i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+45i\cos\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+5\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+18\sin\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+45\sin\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)\right)e^{\frac{1}{2}}}{90a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*(5*I*cos(9/2*d*x + 9/2*c) + 18*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 45*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/
2*d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 18*sin(5/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))))*e^(1/2)/(a^(5/2)*d)
```

Fricas [A]
time = 0.38, size = 88, normalized size = 0.73

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{\frac{1}{2}}+45i\,e^{\left(6i\,dx+6i\,c+\frac{1}{2}\right)}+63i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+23i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)e^{\left(-\frac{9}{2}i\,dx-\frac{9}{2}i\,c\right)}}{90\,a^3d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/90*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(1/2) + 45*I*e^(6*I*d*x + 6*I
*c + 1/2) + 63*I*e^(4*I*d*x + 4*I*c + 1/2) + 23*I*e^(2*I*d*x + 2*I*c + 1/2)
)*e^(-9/2*I*d*x - 9/2*I*c)/(a^3*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sec{(c+dx)}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(1/2)*sqrt(sec(d*x + c))/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 4.22, size = 109, normalized size = 0.90

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,18\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+18\,\sin\left(2\,c+2\,d\,x\right)+5\,\sin\left(4\,c+4\,d\,x\right)+45\mathrm{i}\right)}{90\,a^2\,d\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*18i + cos(4*c + 4*d*x)*5i + 18*si
n(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 45i))/(90*a^2*d*((a*(cos(2*c + 2*d*x)
 + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{2i}{11d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}+\frac{12i}{77ad\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}+\frac{\quad}{77a^2d\sqrt{e\sec(c+}$$

```
[Out] 16/77*I/a^2/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-32/77*I*(a+I*a*
tan(d*x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(1/2)/
(a+I*a*tan(d*x+c))^(5/2)+12/77*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)
)^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3583, 3569}

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{16i}{77a^2d\sqrt{a+ia\tan(c+dx)}\,\sqrt{e\sec(c+dx)}}+\frac{12i}{77ad(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + ((12*I)/
77)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I)/77)/(
a^2*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((32*I)/77)*Sqrt[
a + I*a*Tan[c + d*x]])/(a^3*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{11d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{6\int \frac{}{\sqrt{e\sec(c}}}{}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{77ad\sqrt{e\sec(}}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{77ad\sqrt{e\sec(}}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{5/2}} + \frac{}{77ad\sqrt{e\sec(}}$$

Mathematica [A]
time = 0.50, size = 102, normalized size = 0.63

$$\frac{i\sec^3(c+dx)(-55\cos(c+dx)+35\cos(3(c+dx))-22i\sin(c+dx)+42i\sin(3(c+dx)))}{154a^2d\sqrt{e\sec(c+dx)}\,(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
[Out] ((I/154)*Sec[c + d*x]^3*(-55*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - (22*I)*Si
n[c + d*x] + (42*I)*Sin[3*(c + d*x)]))/(a^2*d*Sqrt[e*Sec[c + d*x]]*(-I + Ta
n[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.88, size = 140, normalized size = 0.86

method	result
default	$\frac{2\sqrt{\frac{e}{\cos(dx+c)}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(28i(\cos^6(dx+c))+28\sin(dx+c)(\cos^5(dx+c))-9i(\cos^4(dx+c))+5\sin(}{77de\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE
)
[Out] 2/77/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)*(28*I*cos(d*x+c)^6+28*sin(d*x+c)*cos(d*x+c)^5-9*I*cos(d*x+c)^4+5
*sin(d*x+c)*cos(d*x+c)^3+2*I*cos(d*x+c)^2+8*sin(d*x+c)*cos(d*x+c)-16*I)/e/a
^3
```

Maxima [A]
time = 0.56, size = 177, normalized size = 1.09

$$\frac{\left(7i\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)+77i\cos\left(\frac{3}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)-77i\cos\left(\frac{1}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)+7\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33\sin\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)+77\sin\left(\frac{3}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)+77\sin\left(\frac{1}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right),\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right)\right)\right)e^{\left(-\frac{1}{2}\right)}}{308\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/308*(7*I*cos(11/2*d*x + 11/2*c) + 33*I*cos(7/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 77*I*cos(3/11*arctan2(sin(11/2*d*x + 11/2
*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(1/11*arctan2(sin(11/2*d*x + 11/2*c
), cos(11/2*d*x + 11/2*c))) + 7*sin(11/2*d*x + 11/2*c) + 33*sin(7/11*arctan
2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(3/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(1/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*e^(-1/2)/(a^(5/2)*d)
```

Fricas [A]
time = 0.38, size = 83, normalized size = 0.51

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(8i\,dx+8i\,c)}+110i\,e^{(4i\,dx+4i\,c)}+40i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(-\frac{11}{2}\,i\,dx-\frac{11}{2}\,i\,c-\frac{1}{2}\right)}}{308\,a^{3}d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/308*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-77*I*e^(8*I*d*x + 8*I*c) + 110*I*
e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-11/2*I*d*x - 11/2
*I*c - 1/2)/(a^3*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\,\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 4.51, size = 118, normalized size = 0.73

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(154\,\sin\left(c+d\,x\right)+33\,\sin\left(3\,c+3\,d\,x\right)+7\,\sin\left(5\,c+5\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)\,33{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,7{}\mathrm{i}\right)}{308\,a^2\,d\,e\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(154*sin(c + d*x) + cos(3*c + 3*d*x)*33i + cos(5*c
+ 5*d*x)*7i + 33*sin(3*c + 3*d*x) + 7*sin(5*c + 5*d*x)))/(308*a^2*d*e*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=206

$$\frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}+\frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}+\frac{}{195a^2d(e\sec(c}$$

```
[Out] 32/195*I/a^2/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/585*I*(e*s
ec(d*x+c))^(1/2)/a^2/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-128/585*I*(a+I*a*tan(d*
x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(3/2)+2/13*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a
*tan(d*x+c))^(5/2)+16/117*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A]
time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$-\frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}+\frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}}+\frac{32i}{195a^2d\sqrt{a+ia\tan(c+dx)}\,(e\sec(c+dx))^{3/2}}+\frac{16i}{117ad(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + ((16*I
)/117)/(a*d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((32*I)/
195)/(a^2*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/
585)*Sqrt[e*Sec[c + d*x]])/(a^2*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((128*
I)/585)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{8\int \frac{}{(e\sec(c+}}{}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{}{117ad(e\,\mathrm{se}}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{}{117ad(e\,\mathrm{se}}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{}{117ad(e\,\mathrm{se}}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{}{117ad(e\,\mathrm{se}}$$

Mathematica [A]
time = 0.67, size = 107, normalized size = 0.52

$$\frac{\sec^4(c+dx)(-351i-1300i\cos(2(c+dx))+75i\cos(4(c+dx))+1040\sin(2(c+dx))-120\sin(4(c+dx)))}{2340a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[c + d*x]^4*(-351*I - (1300*I)*Cos[2*(c + d*x)] + (75*I)*Cos[4*(c + d*x
)] + 1040*Sin[2*(c + d*x)] - 120*Sin[4*(c + d*x)]))/(2340*a^2*d*(e*Sec[c +
d*x])^(3/2)*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.89, size = 159, normalized size = 0.77

method	result

default	$\frac{2\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^2(dx+c))\,(180i(\cos^7(dx+c))+180\sin(dx+c)(\cos^6(dx+c))-55i(\cos^5(dx+c))+35\,\mathrm{s}}{585d\,e^3a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/585/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)^2*(180*I*cos(d*x+c)^7+180*sin(d*x+c)*cos(d*x+c)^6-55*I*cos(d*x+
c)^5+35*sin(d*x+c)*cos(d*x+c)^4+8*I*cos(d*x+c)^3+48*cos(d*x+c)^2*sin(d*x+c)
+64*I*cos(d*x+c)+128*sin(d*x+c))/e^3/a^3
```

Maxima [A]
time = 0.56, size = 225, normalized size = 1.09

$$\frac{\left(45i\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)+260i\cos\left(\frac{9}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+702i\cos\left(\frac{5}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)-195i\cos\left(\frac{3}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+2340i\cos\left(\frac{1}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+45\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right)+260\sin\left(\frac{9}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+702\sin\left(\frac{5}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+195\sin\left(\frac{3}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)+2340\sin\left(\frac{1}{13}\arctan\left(\sin\left(\frac{13}{2}dx+\frac{13}{2}c\right),\cos\left(\frac{13}{2}dx+\frac{13}{2}c\right)\right)\right)\right)e^{\left(-\frac{3}{2}\right)}}{4680\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/4680*(45*I*cos(13/2*d*x + 13/2*c) + 260*I*cos(9/13*arctan2(sin(13/2*d*x +
 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*I*cos(5/13*arctan2(sin(13/2*d*x +
13/2*c), cos(13/2*d*x + 13/2*c))) - 195*I*cos(3/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 2340*I*cos(1/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 45*sin(13/2*d*x + 13/2*c) + 260*sin(9/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*sin(5/13*ar
ctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 195*sin(3/13*arcta
n2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*sin(1/13*arctan2
(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))))*e^(-3/2)/(a^(5/2)*d)
```

Fricas [A]
time = 0.42, size = 105, normalized size = 0.51

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(10i\,dx+10i\,c)}+2145i\,e^{(8i\,dx+8i\,c)}+3042i\,e^{(6i\,dx+6i\,c)}+962i\,e^{(4i\,dx+4i\,c)}+305i\,e^{(2i\,dx+2i\,c)}+45i\right)e^{\left(-\frac{13}{2}i\,dx-\frac{13}{2}i\,c-\frac{3}{2}\right)}}{4680\,a^3d\sqrt{e^{(2i\,dx+2i\,c)}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/4680*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-195*I*e^(10*I*d*x + 10*I*c) + 21
45*I*e^(8*I*d*x + 8*I*c) + 3042*I*e^(6*I*d*x + 6*I*c) + 962*I*e^(4*I*d*x +
```

```
4*I*c) + 305*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-13/2*I*d*x - 13/2*I*c - 3/2)
/(a^3*d*sqrt(e^(2*I*d*x + 2*I*c) + 1))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [**B**]
time = 4.72, size = 135, normalized size = 0.66

$$\frac{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,507{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,260{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,45{}\mathrm{i}+897\,\sin\left(2\,c+2\,d\,x\right)+260\,\sin\left(4\,c+4\,d\,x\right)+45\,\sin\left(6\,c+6\,d\,x\right)+2340{}\mathrm{i}\right)}{4680\,a^2\,d\,e^2\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*507i + cos(4*c + 4*d*x)*260i + co
s(6*c + 6*d*x)*45i + 897*sin(2*c + 2*d*x) + 260*sin(4*c + 4*d*x) + 45*sin(6
*c + 6*d*x) + 2340i))/(4680*a^2*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d
*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=86

$$\frac{3i2^{2/3}a\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{13}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}\sqrt[3]{1+i\tan(c+dx)}}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/7*I*2^(2/3)*a*hypergeom([1/3, 7/6],[13/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(7/3)*(1+I*tan(d*x+c))^(1/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}\,(e\sec(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{13}{6};\frac{1}{2}(1-i\tan(c+dx))\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(e\sec(c+dx))^{7/3}\int(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{2/3}\,dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{7/3}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a-iax}}{\sqrt[3]{a+iax}}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{7/3}\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a-iax}}{\sqrt[3]{\frac{1}{2}+\frac{ix}{2}}}\,dx,x,\tan\right.}{\sqrt[3]{2}\,d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{3i2^{2/3}a\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{13}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}\sqrt[3]{1+i\tan(c+}}{7d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.79, size = 118, normalized size = 1.37

$$-\frac{3i\sqrt[3]{2}\,ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{4/3}\left(4+\left(1+e^{2i(c+dx)}\right)^{5/6}{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};-e^{2i(c+dx)}\right)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (((-3*I)/5)*2^(1/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c
 + d*x))))^(4/3)*(4 + (1 + E^((2*I)*(c + d*x)))^(5/6)*Hypergeometric2F1[2/3
, 5/6, 5/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(7/3)*integrate(sec(d*x + c)^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(5*a*d*integral(-2/5*I*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/3
*I*d*x + 1/3*I*c + 7/3)/(a*d*(e^(2*I*d*x + 2*I*c) + 1)^(1/3)), x) - 6*I*2^(
5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(4/3*I*d*x + 4/3*I*c + 7/3)/(e^(2*
I*d*x + 2*I*c) + 1)^(1/3))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=86

$$\frac{3i\sqrt[3]{2}\,a\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(1+i\tan(c+dx))^{2/3}}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/5*I*2^(1/3)*a*hypergeom([2/3, 5/6],[11/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(5/3)*(1+I*tan(d*x+c))^(2/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i\sqrt[3]{2}\,a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(c+dx))\right)}{5d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{a+ia\tan(c+dx)}\,dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}\,(a+iax)^{2/3}}\,dx, x, \tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/3}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{2/3}\sqrt[6]{a-iax}}\,dx, x\right.}{2^{2/3}d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{3i\sqrt[3]{2}\,a\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(1+i\tan(c+dx}{5d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.67, size = 116, normalized size = 1.35

$$\frac{3i2^{2/3}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(-2+\sqrt[6]{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{4}{3};-e^{2i(c+dx)}\right)\right)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((3*I)*2^(2/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*
x))))^(2/3)*(-2 + (1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[1/6, 1/
3, 4/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(5/3)*integrate(sec(d*x + c)^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(5/3) + I*e^(2
*I*d*x + 2*I*c + 5/3))*e^(-4/3*I*d*x - 4/3*I*c)/(a*d*(e^(2*I*d*x + 2*I*c) +
 1)^(2/3)), x) - 6*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(5/3) + I
*e^(2*I*d*x + 2*I*c + 5/3))*e^(2/3*I*d*x + 2/3*I*c)/(e^(2*I*d*x + 2*I*c) +
1)^(2/3))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{3i\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i\tan(c+dx)}}{2\sqrt[6]{2}\,d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/4*I*hypergeom([1/3, 7/6],[4/3],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(2/3)
*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i\sqrt[6]{1+i\tan(c+dx)}\,(e\sec(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-i\tan(c+dx))\right)}{2\sqrt[6]{2}\,d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec[
c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{a+ia\tan(c+dx)}}\,dx}{\sqrt[3]{a-ia\tan(c+dx)}\,\sqrt[3]{a+ia\tan(c+dx)}}\\ &= \frac{\left(a^2(e\sec(c+dx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{2/3}(a+iax)^{7/6}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[3]{a-ia\tan(c+dx)}\,\sqrt[3]{a+ia\tan(c+dx)}}\\ &= \frac{\left(a(e\sec(c+dx))^{2/3}\sqrt[6]{\frac{a+ia\tan(c+dx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}(a-iax)^{2/3}}\,dx,\right.}{2\sqrt[6]{2}\,d\sqrt[3]{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\\ &= \frac{3i\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i\tan(c+dx)}}{2\sqrt[6]{2}\,d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 116, normalized size = 1.36

$$\frac{3i\sqrt[6]{2}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\sqrt[6]{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-e^{2i(c+dx)}\right)}{d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((3*I)*2^(1/6)*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E
^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c +
d*x))])/(d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
[Out] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] e^(2/3)*integrate(sec(d*x + c)^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
[Out] -(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2/3) - I*e^(4*I*d*x +
4*I*c + 2/3) - 2*I*e^(2*I*d*x + 2*I*c + 2/3))*e^(2/3*I*d*x + 2/3*I*c)/(e^(2
*I*d*x + 2*I*c) + 1)^(2/3) - (a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x +
2*I*c) + a*d*e^(I*d*x + I*c))*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(4*I*e^(2/3) + I*e^(4*I*d*x + 4*I*c + 2/3) + 7*I*e^(3*I*d*x + 3*I*c +
 2/3) + 5*I*e^(2*I*d*x + 2*I*c + 2/3) + 7*I*e^(I*d*x + I*c + 2/3))*e^(2/3*I
*d*x + 2/3*I*c)/((a*d*e^(4*I*d*x + 4*I*c) - 3*a*d*e^(3*I*d*x + 3*I*c) + 3*a
*d*e^(2*I*d*x + 2*I*c) - a*d*e^(I*d*x + I*c))*(e^(2*I*d*x + 2*I*c) + 1)^(2/
3)), x))/(a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x + 2*I*c) + a*d*e^(I*d*
x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.440 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=83

$$\frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{6};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\,\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}\,d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/2*I*hypergeom([1/6, 4/3],[7/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(1/3)
*(1+I*tan(d*x+c))^(1/3)*2^(2/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{3i\sqrt[3]{1+i\tan(c+dx)}\,\sqrt[3]{e\sec(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{6};\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}\,d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{\sqrt[6]{a-ia\tan(c+dx)}\,\sqrt[6]{a+ia\tan(c+dx)}}\\ &= \frac{\left(a^2\sqrt[3]{e\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{4/3}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[6]{a-ia\tan(c+dx)}\,\sqrt[6]{a+ia\tan(c+dx)}}\\ &= \frac{\left(a\sqrt[3]{e\sec(c+dx)}\,\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{4/3}(a-iax)^{5/6}}\,dx, x\right.}{2\sqrt[3]{2}\,d\sqrt[6]{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\\ &= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{6};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\,\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}\,d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 95, normalized size = 1.14

$$\frac{3\left(8i-\frac{2ie^{2i(c+dx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};-e^{2i(c+dx)}\right)}{\sqrt[6]{1+e^{2i(c+dx)}}}\right)\sqrt[3]{e\sec(c+dx)}}{16d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (3*(8*I - ((2*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 5/3, -E^((
2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(1/6))*(e*Sec[c + d*x])^(1/3))/
(16*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(e \sec (dx+c))^{\frac{1}{3}}}{\sqrt{a+ia \tan (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(1/3)*integrate(sec(d*x + c)^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a*d*e^(I*d*x + I*c)*integral(-1/4*I*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/3*I*d*x + 1/3*I*c + 1/3)/(a*d*(e^(2*I*d*x + 2*I*c) + 1)^(1/3
)), x) - 3*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(1/3) - I*e^(2*I
*d*x + 2*I*c + 1/3))*e^(1/3*I*d*x + 1/3*I*c)/(e^(2*I*d*x + 2*I*c) + 1)^(1/3
))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{e \sec (c+dx)}}{\sqrt{ia (\tan (c+dx)-i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=83

$$-\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3/2*I*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(
2/3)*2^(1/3)/d/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i(1+i\tan(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*Ta
n[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(c+dx)}\,\sqrt[6]{a+ia\tan(c+dx)}\right)\int \frac{}{\sqrt[6]{a-ia\,\mathrm{ta}}}}{\sqrt[3]{e\sec(c+dx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(c+dx)}\,\sqrt[6]{a+ia\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{(a}\right.}{d\sqrt[3]{e\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt[6]{a-ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{}{\left(\frac{1}{2}+\frac{ix}{2}\right.}\right.}{2\,2^{2/3}d\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+ia\tan(}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.86, size = 95, normalized size = 1.14

$$\frac{12i-\frac{30ie^{2i(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{3};\frac{4}{3};-e^{2i(c+dx)}\right)}{\left(1+e^{2i(c+dx)}\right)^{5/6}}}{16d\sqrt[3]{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (12*I - ((30*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/6, 1/3, 4/3, -E^((2
*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(5/6))/(16*d*(e*Sec[c + d*x])^(1
/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{a+ia\tan\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] e^(-1/3)*integrate(1/(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/8*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(4*I*e^(6*I*d*x + 6*I*c)
+ 9*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3
*I*c)/(e^(2*I*d*x + 2*I*c) + 1)^(2/3) - 8*(a*d*e^(4*I*d*x + 4*I*c + 1/3) -
a*d*e^(2*I*d*x + 2*I*c + 1/3))*integral(-15/16*2^(1/6)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(3*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(2/
3*I*d*x + 2/3*I*c)/((a*d*e^(6*I*d*x + 6*I*c + 1/3) - 2*a*d*e^(4*I*d*x + 4*I
*c + 1/3) + a*d*e^(2*I*d*x + 2*I*c + 1/3))*(e^(2*I*d*x + 2*I*c) + 1)^(2/3))
, x))/(a*d*e^(4*I*d*x + 4*I*c + 1/3) - a*d*e^(2*I*d*x + 2*I*c + 1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{e \sec{\left(c + dx \right)}}\,\sqrt{ia \left(\tan{\left(c + dx \right)} - i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(1/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{3i\,{}_2F_1\left(-\frac{2}{3},\frac{13}{6};\frac{1}{3};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{8\sqrt[6]{2}\,ad(e\sec(c+dx))^{4/3}}$$

```
[Out] -3/16*I*hypergeom([-2/3, 13/6],[1/3],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c
))^(1/2)*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/a/d/(e*sec(d*x+c))^(4/3)
```

Rubi [A]
time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$-\frac{3i\sqrt[6]{1+i\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,{}_2F_1\left(-\frac{2}{3},\frac{13}{6};\frac{1}{3};\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}\,ad(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1 +
 I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c +
d*x])^(4/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{((a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3})\int \frac{}{(a-ia\tan(}}{(e\sec(c+dx))^{4/3}}$$

$$= \frac{(a^2(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3})\,\mathrm{Subst}\Big(\int}{d(e\sec(c+dx))^{4/3}}$$

$$= \frac{\Bigg((a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\,\sqrt[6]{\frac{a+ia\tan}{a}}}{4\sqrt[6]{2}\,d(e\sec}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{2}{3},\frac{13}{6};\frac{1}{3};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx}}{8\sqrt[6]{2}\,ad(e\sec(c+dx))^{4/3}}$$

Mathematica [A]
time = 0.78, size = 112, normalized size = 1.27

$$-\frac{3i\sec^2(c+dx)\left(3+3\cos(2(c+dx))-55\sqrt[6]{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-e^{2i(c+dx)}\right)+11i\sin(2(c+dx))\right)}{112d(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((-3*I)/112)*Sec[c + d*x]^2*(3 + 3*Cos[2*(c + d*x)] - 55*(1 + E^((2*I)*(c
+ d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c + d*x))] + (1
1*I)*Sin[2*(c + d*x)]))/(d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] e^(-4/3)*integrate(1/(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/112*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(7*I*e^(8*I*d*x + 8*I*c
) - 14*I*e^(7*I*d*x + 7*I*c) - 38*I*e^(6*I*d*x + 6*I*c) - 20*I*e^(5*I*d*x +
 5*I*c) - 101*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(3*I*d*x + 3*I*c) - 60*I*e^(2*I
*d*x + 2*I*c) + 8*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I*d*x + 2/3*I*c)/(e^(2*I*
d*x + 2*I*c) + 1)^(2/3) - 112*(a*d*e^(5*I*d*x + 5*I*c + 4/3) - 2*a*d*e^(4*I
*d*x + 4*I*c + 4/3) + a*d*e^(3*I*d*x + 3*I*c + 4/3))*integral(-55/112*2^(1/
6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c) - 7*I*e^(3*I*d
*x + 3*I*c) - 5*I*e^(2*I*d*x + 2*I*c) - 7*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I
*d*x + 2/3*I*c)/((a*d*e^(4*I*d*x + 4*I*c + 4/3) - 3*a*d*e^(3*I*d*x + 3*I*c
+ 4/3) + 3*a*d*e^(2*I*d*x + 2*I*c + 4/3) - a*d*e^(I*d*x + I*c + 4/3))*(e^(2
*I*d*x + 2*I*c) + 1)^(2/3)), x))/(a*d*e^(5*I*d*x + 5*I*c + 4/3) - 2*a*d*e^(
4*I*d*x + 4*I*c + 4/3) + a*d*e^(3*I*d*x + 3*I*c + 4/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \sec \left(c + dx\right)\right)^{\frac{4}{3}} \sqrt{ia \left(\tan \left(c + dx\right) - i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(4/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{4/3}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=437

$$\frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}-\frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}+\frac{5i\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a\ldots}}{\ldots}\right.}{12\ 2^{2/3}\sqrt{3}\,a^{5/3}f\sqrt[3]{a-\ldots}}$$

```
[Out] 1/4*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(7/3)-5/144*x*(d*sec(f*x+e)
)^(2/3)*2^(1/3)/a^(5/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5
/144*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(f*x
+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5/48*I*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f*
x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(f*x+e))^(1/3
)/(a+I*a*tan(f*x+e))^(1/3)+5/72*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*
x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f*3^(1/2
)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+5/24*I*(d*sec(f*x+e))^(
2/3)/f/(a+I*a*tan(f*x+e))^(1/3)/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A]

time = 0.28, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3586, 3603, 3568, 44, 59, 631, 210, 31}

$$\frac{5i(d\sec(e+fx))^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{12\ 2^{2/3}\sqrt{3}\,a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5i(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5i(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}+\frac{5i(d\sec(e+fx))^{2/3}}{24f\sqrt[3]{a+ia\tan(e+fx)}\,(a^2+ia^2\tan(e+fx))}+\frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((I/4)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(7/3)) - (5*x*(d*S
ec[e + f*x])^(2/3))/(72*2^(2/3)*a^(5/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I
*a*Tan[e + f*x])^(1/3)) + (((5*I)/12)*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Ta
n[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt
[3]*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
(((5*I)/72)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(5/3)*f*(a
 - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((5*I)/24)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
+ (((5*I)/24)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)*(a^2
+ I*a^2*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{(d\sec(e+fx))^{2/3}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{7/3}\,dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{\left(5i(d\sec(e+fx))^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{2/3}}\,dx \ldots\right.}{12f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\,\mathrm{ta}\ldots}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} + \frac{\left(5i(d\sec(e+f\ldots\right.}{36af\sqrt[3]{\ldots}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{\ldots}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a\ldots}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{\ldots}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a\ldots}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{\ldots}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a\ldots}}$$

Mathematica [A]
time = 1.99, size = 240, normalized size = 0.55

$$\frac{e^{-2i(e+fx)}\left(9i+33ie^{2i(e+fx)}+24ie^{4i(e+fx)}-10e^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,fx-10i\sqrt{3}\,e^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\mathrm{ArcTan}\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)-15ie^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)\sec^2(e+fx)(d\sec(e+fx))^{2/3}}{144f(a+ia\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((9*I + (33*I)*E^((2*I)*(e + f*x)) + (24*I)*E^((4*I)*(e + f*x)) - 10*E^((4*
I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*x - (10*I)*Sqrt[3]*E^((4*I)
*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e
+ f*x)))^(1/3))/Sqrt[3]] - (15*I)*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e + f*
x)))^(1/3)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)])*Sec[e + f*x]^2*(d*Sec[
e + f*x])^(2/3))/(144*E^((2*I)*(e + f*x))*f*(a + I*a*Tan[e + f*x])^(7/3))
```

Maple [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx+e))^{\frac{2}{3}}}{(a+ia\tan (fx+e))^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4194 vs. 2(344) = 688.
time = 0.70, size = 4194, normalized size = 9.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] 1/288*(48*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f
*x + 4*e), cos(4*f*x + 4*e))) + 1)^(5/6)*((I*2^(1/3)*cos(4*f*x + 4*e) + 2^(
1/3)*sin(4*f*x + 4*e))*cos(5/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), co
s(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)
) - (2^(1/3)*cos(4*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(5/3*arctan2
(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(
4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)))*d^(2/3) + 30*(cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
1)^(1/3)*((-I*2^(1/3)*cos(4*f*x + 4*e) - 2^(1/3)*sin(4*f*x + 4*e))*cos(2/3*
arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (2^(1/3)*cos(4*f*x + 4*e) -
 I*2^(1/3)*sin(4*f*x + 4*e))*sin(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*
e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))
) + 1)))*d^(2/3) + 5*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(1/2*arc
```

tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + 4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) - 4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))) + 4/3) - 2*2^(1/3)*arctan2((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*a

```
rctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*sin(2/3*arctan2(sin(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))) + 1)) + (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 +
 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*
arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4...
```

Fricas [A]
time = 0.46, size = 556, normalized size = 1.27

$$\frac{48\,a^3 f\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(6 i f x+6 i e)}\log\left(-\frac{2}{5}\left(72 i\,a^3 f\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(2 i f x+2 i e)}-5\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(e^{(2 i f x+2 i e)}+1\right)e^{(2 i f x+2 i e)}\right)e^{(-2 i f x-2 i e)}\right)+2^{\frac{1}{3}}\left(\frac{a}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(8 i\,e^{(6 i f x+6 i e)}+19 i\,e^{(4 i f x+4 i e)}+14 i\,e^{(2 i f x+2 i e)}+3 i\right)e^{(2 i f x+2 i e)}-24\left(-i\sqrt{3}\,a^3 f+a^3 f\right)\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(6 i f x+6 i e)}\log\left(\frac{2}{5}\left(5\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(e^{(2 i f x+2 i e)}+1\right)e^{(2 i f x+2 i e)}+36\left(\sqrt{3}\,a^3 f+i\,a^3 f\right)\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(2 i f x+2 i e)}\right)e^{(-2 i f x-2 i e)}\right)-24\left(i\sqrt{3}\,a^3 f+a^3 f\right)\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(6 i f x+6 i e)}\log\left(\frac{2}{5}\left(5\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2 i f x+2 i e)}+1}\right)^{\frac{2}{3}}\left(e^{(2 i f x+2 i e)}+1\right)e^{(2 i f x+2 i e)}-36\left(\sqrt{3}\,a^3 f-i\,a^3 f\right)\left(\frac{125 i d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(2 i f x+2 i e)}\right)e^{(-2 i f x-2 i e)}\right)\right)e^{(-6 i f x-6 i e)}}{48\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(-
2/5*(72*I*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 5*
2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) +
 2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*(8*I*e^(6*I*f*x + 6*I*e) + 19*I*e^(4*I*f*x + 4*I*e) + 14*I*e^(2*I*f*x
 + 2*I*e) + 3*I)*e^(2*I*f*x + 2*I*e) - 24*(-I*sqrt(3)*a^3*f + a^3*f)*(125/1
86624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*
x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 36*(sqrt(3)*a^3*f + I*a^3*f)*(125/186
624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 24*
(I*sqrt(3)*a^3*f + a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6
*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 36*(s
qrt(3)*a^3*f - I*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I
*e))*e^(-2*I*f*x - 2*I*e)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(7/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{7/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3),x)
```

```
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3), x)
```

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=378

$$\frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}}-\frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\ \sqrt[3]{a+ia\tan(e+fx)}}+\frac{i\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a}}{\sqrt{}}\right.}{2^{2/3}\sqrt{3}\ a^{2/3}f\sqrt[3]{a-i}}$$

```
[Out] 1/2*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(4/3)-1/12*x*(d*sec(f*x+e))
^(2/3)*2^(1/3)/a^(2/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/
12*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*x+e
))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/4*I*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f*x+e
))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(
a+I*a*tan(f*x+e))^(1/3)+1/6*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*x+e)
)^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f*3^(1/2)/(a
-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A]
time = 0.25, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3586, 3603, 3568, 44, 59, 631, 210, 31}

$$\frac{i(d\sec(e+fx))^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}\sqrt{3}\ a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\ \sqrt[3]{a+ia\tan(e+fx)}}-\frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\ \sqrt[3]{a+ia\tan(e+fx)}}-\frac{i(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\ \sqrt[3]{a+ia\tan(e+fx)}}-\frac{i(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\ \sqrt[3]{a+ia\tan(e+fx)}}+\frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

```
[Out] ((I/2)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(4/3)) - (x*(d*Sec
[e + f*x])^(2/3))/(6*2^(2/3)*a^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*
Tan[e + f*x])^(1/3)) + (I*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Tan[e + f*x])^
(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt[3]*a^(2/3)*
f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/6)*Log[C
os[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I*a*Tan[e + f*
x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/2)*Log[2^(1/3)*a^(1/3) - (a -
 I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I
*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
```

```
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{(d\sec(e+fx))^{2/3}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{4/3}\,dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} + \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx\right.}{3f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\,\text{ta}}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(}}$$

Mathematica [A]
time = 1.39, size = 220, normalized size = 0.58

$$\frac{e^{-i(e+fx)}\left(3i+3ie^{2i(e+fx)}-2e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,fx-2i\sqrt{3}\,e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)-3ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)(d\sec(e+fx))^{5/3}}{12df(a+ia\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

```
[Out] ((3*I + (3*I)*E^((2*I)*(e + f*x)) - 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e
+ f*x)))^(1/3)*f*x - (2*I)*Sqrt[3]*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f
*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))^(1/3))/Sqrt[3]] - (3*I)
*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Log[1 - (1 + E^((2*I)*
```

```
(e + f*x)))^(1/3)])*(d*Sec[e + f*x])^(5/3))/(12*d*E^(I*(e + f*x))*f*(a + I*
a*Tan[e + f*x])^(4/3))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2045 vs. 2(296) = 592.
time = 0.60, size = 2045, normalized size = 5.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] -1/24*(6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/3)*((-I*2^(1/3)*cos(2*f*x + 2*e) - 2^(1/3)*sin(2*f*x + 2*e))*cos(2/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*cos(2*f*x + 2*e)
- I*2^(1/3)*sin(2*f*x + 2*e))*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)))*d^(2/3) - (-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*si
n(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sq
rt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*
```

```
e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))^2) - 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/
3) - 2*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/3)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - 1) - 2*I*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*
cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)*l
og((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/3)
*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1))^2) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/6)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1
))*d^(2/3))/(a^(4/3)*f)
```

Fricas [A]
time = 0.41, size = 542, normalized size = 1.43

$$\frac{\left(4\,a^{2} f \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(4i\,f x+4i\,e\right)} \log\left(-2 \left(6i\,a^{2} f \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(2i\,f x+2i\,e\right)} - 2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(e^{\left(2i\,f x+2i\,e\right)}+1\right) e^{\left(2i\,f x+2i\,e\right)}\right) e^{\left(-2i\,f x-2i\,e\right)}\right) + 2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(i\,e^{\left(4i\,f x+4i\,e\right)} + 2i\,e^{\left(2i\,f x+2i\,e\right)} + i\right) e^{\left(2i\,f x+2i\,e\right)} - 2 \left(-i\,\sqrt{3}\,a^{2} f + a^{2} f\right) \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(4i\,f x+4i\,e\right)} \log\left(2 \left(2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(e^{\left(2i\,f x+2i\,e\right)}+1\right) e^{\left(2i\,f x+2i\,e\right)} + 3 \left(\sqrt{3}\,a^{2} f + i\,a^{2} f\right) \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(2i\,f x+2i\,e\right)}\right) e^{\left(-2i\,f x-2i\,e\right)}\right) - 2 \left(i\,\sqrt{3}\,a^{2} f + a^{2} f\right) \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(4i\,f x+4i\,e\right)} \log\left(2 \left(2^{\frac{1}{3}} \left(\frac{a}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2i\,f x+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(e^{\left(2i\,f x+2i\,e\right)}+1\right) e^{\left(2i\,f x+2i\,e\right)} - 3 \left(\sqrt{3}\,a^{2} f - i\,a^{2} f\right) \left(\frac{i\,d^{2}}{108\,a^{4} f^{3}}\right)^{\frac{1}{3}} e^{\left(2i\,f x+2i\,e\right)}\right) e^{\left(-2i\,f x-2i\,e\right)}\right)\right) e^{\left(-4i\,f x-4i\,e\right)}}{4\,a^{2} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(-2*(6*I*
a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 2^(1/3)*(a/(e^(2*
I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x
+ 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 2^(1/3)*(a/(e^(2
*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(4*I*f
*x + 4*I*e) + 2*I*e^(2*I*f*x + 2*I*e) + I)*e^(2*I*f*x + 2*I*e) - 2*(-I*sqrt
(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(2*
(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 3*(sqrt(3)*a^2*f + I*
a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I
*e)) - 2*(I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x
 + 4*I*e)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 3*(sq
rt(3)*a^2*f - I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e
^(-2*I*f*x - 2*I*e)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{2}{3}}}{(ia(\tan(e+fx)-i))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(4/3),x)
[Out] Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="giac"
)
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3),x)
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3), x)
```

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt[3]{a}\,x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}+\frac{i\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)(d}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e-}$$

```
[Out] -1/4*a^(1/3)*x*(d*sec(f*x+e))^(2/3)*2^(1/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a
*tan(f*x+e))^(1/3)-1/4*I*a^(1/3)*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3
)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-3/4*I*a^(1/3)*ln(2^(1
/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/f/(a-I*a
*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+1/2*I*a^(1/3)*arctan(1/3*(a^(1/
3)+2^(2/3)*(a-I*a*tan(f*x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*
3^(1/2)*2^(1/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A]

time = 0.13, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3573, 3562, 59, 631, 210, 31}

$$\frac{i\sqrt{3}\,\sqrt[3]{a}\,(d\sec(e+fx))^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{\sqrt[3]{a}\,x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{3i\sqrt[3]{a}\,(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}-\frac{i\sqrt[3]{a}\,(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/2*(a^(1/3)*x*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*(a - I*a*Tan[e + f*x])^(1/
3)*(a + I*a*Tan[e + f*x])^(1/3)) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(
2/3)*(a - I*a*Tan[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3
))/(2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
((I/2)*a^(1/3)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*f*(a - I*
a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((3*I)/2)*a^(1/3)*Lo
g[2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(
2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \sqrt[3]{a-ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= -\frac{\sqrt[3]{a}\,x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\,\log(\cos(e}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(}$$

$$= -\frac{\sqrt[3]{a}\,x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\,\log(\cos(e}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(}$$

$$= -\frac{\sqrt[3]{a}\,x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\,\sqrt[3]{a+ia\tan(e+fx)}} + \frac{i\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+}{}\right.}{2^{2/3}f\sqrt[3]{a-}}$$

Mathematica [A]
time = 0.69, size = 161, normalized size = 0.47

$$-\frac{\left(\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{2/3}\sqrt[3]{1+e^{2i(e+fx)}}\left(2fx+2i\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)+3i\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)}{2\,2^{2/3}\sqrt[3]{\frac{ae^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/2*(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(2/3)*(1 + E^((2*I)*(
e + f*x)))^(1/3)*(2*f*x + (2*I)*Sqrt[3]*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*
x)))^(1/3))/Sqrt[3]] + (3*I)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)]))/(2^
(2/3)*((a*E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*f)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1881 vs. $2(266) = 532$.
time = 0.59, size = 1881, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="maxim
a")
```

```
[Out] 1/8*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*arcta
n2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3
*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3)
 - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) - 4
/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)
*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3) - 2*2^(1/3)*arct
an2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3
)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*I
```

```
*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*sin(1
/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 - 2*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)*log((cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/3)*(cos(2/3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6
)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/3*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 2*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(
1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1))*d^(2/3)/(a^(1/3)
*f)
```

Fricas [A]
time = 0.39, size = 385, normalized size = 1.13

$$\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}(e^{(2ifx+2ie)}+1)e^{(2ifx+2ie)}+\left(\sqrt{3}af+iaf\right)\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}(e^{(2ifx+2ie)}+1)e^{(2ifx+2ie)}-\left(\sqrt{3}af-iaf\right)\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)+\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}(e^{(2ifx+2ie)}+1)e^{(2ifx+2ie)}-2iaf\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}e^{(2ifx+2ie)}\right)e^{(-2ifx-2ie)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="frica
s")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*(1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I
*e) + 1)*e^(2*I*f*x + 2*I*e) + (sqrt(3)*a*f + I*a*f)*(1/4*I*d^2/(a*f^3))^(1
/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 1/2*(-I*sqrt(3) - 1)*(1/4*
I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I
*e) - (sqrt(3)*a*f - I*a*f)*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*
e^(-2*I*f*x - 2*I*e)) + (1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I
*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x +
 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 2*I*a*f*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I
*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{2}{3}}}{\sqrt[3]{ia(\tan(e+fx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3),x)
```

```
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3), x)
```

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$

Optimal. Leaf size=37

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

```
[Out] 3*I*a*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{3574\}$

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

```
[Out] ((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A]
time = 0.39, size = 47, normalized size = 1.27

$$\frac{3d^2(i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{f(d\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

```
[Out] (3*d^2*(I + Tan[e + f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(f*(d*Sec[e + f*x])
^(4/3))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}} \left(a+ia\tan\left(fx+e\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(31) = 62.
time = 0.54, size = 114, normalized size = 3.08

$$-\frac{3\left(-i\cdot 2^{\frac{1}{3}}\cos\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}\sin\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)\right)a^{\frac{2}{3}}d^{\frac{2}{3}}}{\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{6}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="maxim
a")
```

```
[Out] -3*(-I*2^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2
^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^
(2/3)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/6)*f)
```

Fricas [A]
time = 0.36, size = 58, normalized size = 1.57

$$-\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,fx+2i\,e)}-i\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="frica
s")
```

```
[Out] -3*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(-I*e^(2*I*f*x + 2*I*e) - I)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(2/3),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)*(I*a*(tan(e + f*x) - I))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(2/3), x)
```

Mupad [B]
time = 4.81, size = 81, normalized size = 2.19

$$\frac{3\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\left(\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}+\sin\left(2\,e+2\,f\,x\right)+1\mathrm{i}\right)\left(\frac{a\,\left(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}\right)^{2/3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(2/3),x)
```

```
[Out] (3*(d/cos(e + f*x))^(2/3)*(cos(2*e + 2*f*x)*1i + sin(2*e + 2*f*x) + 1i)*((a
*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(2/3
))/(2*f)
```

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$

Optimal. Leaf size=81

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}$$

```
[Out] 9/2*I*a^2*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+3/4*I*a*(d*sec(f*
x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f
```

Rubi [A]

time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

```
[Out] (((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) + (
((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}+\frac{1}{2}(3a)\int (d\mathrm{s}\\ &= \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan}{4f}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 70, normalized size = 0.86

$$-\frac{3ad(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(-7i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{4f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

```
[Out] (-3*a*d*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(-7*I + Tan[e + f*x])*(
a + I*a*Tan[e + f*x])^(2/3))/(4*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(65) = 130.
time = 0.56, size = 336, normalized size = 4.15

$$\frac{3\left(\left(-i\cdot 2^{\frac{1}{3}}a\cos\left(\frac{4}{3}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right)-2^{\frac{1}{3}}a\sin\left(\frac{4}{3}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right)\right)\sqrt{\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1}\,a^{\frac{2}{3}}d^{\frac{2}{3}}+4\left(\left(i\cdot 2^{\frac{1}{3}}a\cos(2fx+2e)^2+i\cdot 2^{\frac{1}{3}}a\sin(2fx+2e)^2+2i\cdot 2^{\frac{1}{3}}a\cos(2fx+2e)+i\cdot 2^{\frac{1}{3}}a\right)\cos\left(\frac{1}{3}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right)+\left(2^{\frac{1}{3}}a\cos(2fx+2e)^2+2^{\frac{1}{3}}a\sin(2fx+2e)^2+2\cdot 2^{\frac{1}{3}}a\cos(2fx+2e)+2^{\frac{1}{3}}a\right)\sin\left(\frac{1}{3}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)+1\right)\right)\right)a^{\frac{2}{3}}d^{\frac{2}{3}}\right)}{2\left(\cos(2fx+2e)^2+\sin(2fx+2e)^2+2\cos(2fx+2e)+1\right)^{\frac{7}{6}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="maxim
a")
```

```
[Out] 3/2*((-I*2^(1/3)*a*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 - 2^(1/3)*a*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/3)*
d^(2/3) + 4*((I*2^(1/3)*a*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a*sin(2*f*x + 2*e)
^2 + 2*I*2^(1/3)*a*cos(2*f*x + 2*e) + I*2^(1/3)*a)*cos(1/3*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*a*cos(2*f*x + 2*e)^2 + 2^(1/3)*
a*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a*cos(2*f*x + 2*e) + 2^(1/3)*a)*sin(1/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(7/6)*f)
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.75

$$-\frac{3\cdot 2^{\frac{1}{3}}\left(-4i\,ae^{(2i\,fx+2i\,e)}-3i\,a\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="frica
s")
```

```
[Out] -3/2*2^(1/3)*(-4*I*a*e^(2*I*f*x + 2*I*e) - 3*I*a)*(a/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(5/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(5/3), x)
```

Mupad [B]
time = 4.13, size = 90, normalized size = 1.11

$$\frac{3\,a\,\left(\frac{d}{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}\right)^{2/3}\,\left({\cos\left(e+f\,x\right)}^2\,6{}\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+1{}\mathrm{i}\right)\,{\left(\frac{a\,\left(2\,{\cos\left(e+f\,x\right)}^2+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)}^{2/3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(5/3),x)
```

```
[Out] (3*a*(d/(2*cos(e/2 + (f*x)/2)^2 - 1))^(2/3)*(3*sin(2*e + 2*f*x) + cos(e + f
*x)^2*6i + 1i)*((a*(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f*x
)^2))^(2/3))/(4*f)
```

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

Optimal. Leaf size=122

$$\frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}$$

```
[Out] 54/7*I*a^3*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+9/7*I*a^2*(d*sec
(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+3/7*I*a*(d*sec(f*x+e))^(2/3)*(a+I
*a*tan(f*x+e))^(5/3)/f
```

Rubi [A]
time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{9ia^2(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{7f}+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (((54*I)/7)*a^3*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((9*I)/7)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f + (((
3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f} + \frac{1}{7}(12a)\int (d \cdots \\
&= \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} + \frac{3ia(d\sec(e \cdots}{} \\
&= \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\,\mathrm{ta}\cdots}{7f}
\end{aligned}
$$

Mathematica [**A**]
time = 0.69, size = 100, normalized size = 0.82

$$\frac{3a^2(d\sec(e+fx))^{5/3}(i\cos(e-fx)+\sin(e-fx))(21+23\cos(2(e+fx))+5i\sin(2(e+fx)))(a+ia\tan(e+fx))^{2/3}}{14df(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (3*a^2*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - f*x] + Sin[e - f*x])*(21 + 23*Cos[
2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*(a + I*a*Tan[e + f*x])^(2/3))/(14*d*
f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(98) = 196.
time = 0.54, size = 426, normalized size = 3.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] 6/7*(7*(-I*2^(1/3)*a^2*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) - 2^(1/3)*a^2*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
```

```
)*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^
(2/3)*d^(2/3) + 2*(I*2^(1/3)*a^2*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) + 2^(1/3)*a^2*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) + 7*(I*2^(1/3)*a^2*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^2*sin(2*f*x
+ 2*e)^2 + 2*I*2^(1/3)*a^2*cos(2*f*x + 2*e) + I*2^(1/3)*a^2)*cos(1/3*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^2*cos(2*f*x + 2*e
)^2 + 2^(1/3)*a^2*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^2*cos(2*f*x + 2*e) + 2^(
1/3)*a^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)
*d^(2/3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(7/6)*f)
```

Fricas [**A**]
time = 0.40, size = 114, normalized size = 0.93

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(-14i\,a^{2}e^{(4i\,fx+4i\,e)}-21i\,a^{2}e^{(2i\,fx+2i\,e)}-9i\,a^{2}\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{(2i\,fx+2i\,e)}}{7\left(fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="frica
s")
```

```
[Out] -6/7*2^(1/3)*(-14*I*a^2*e^(4*I*f*x + 4*I*e) - 21*I*a^2*e^(2*I*f*x + 2*I*e)
- 9*I*a^2)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(8/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(8/3), x)
```

Mupad [B]
time = 5.27, size = 122, normalized size = 1.00

$$\frac{3\,a^2\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{2/3}\,{\left(\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1}\right)}^{2/3}\,\left(\cos\left(2\,e+2\,f\,x\right)\,44{}\mathrm{i}+\cos\left(4\,e+4\,f\,x\right)\,9{}\mathrm{i}+16\,\sin\left(2\,e+2\,f\,x\right)+9\,\sin\left(4\,e+4\,f\,x\right)+35{}\mathrm{i}\right)}{14\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(8/3),x)
```

```
[Out] (3*a^2*(d/cos(e + f*x))^(2/3)*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i +
 1))/(cos(2*e + 2*f*x) + 1))^(2/3)*(cos(2*e + 2*f*x)*44i + cos(4*e + 4*f*x)
*9i + 16*sin(2*e + 2*f*x) + 9*sin(4*e + 4*f*x) + 35i))/(14*f*(cos(2*e + 2*f
*x) + 1))
```

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

Optimal. Leaf size=163

$$\frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f}+\frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia}{35f}$$

```
[Out] 486/35*I*a^4*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+81/35*I*a^3*(d
*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+27/35*I*a^2*(d*sec(f*x+e))^(2
/3)*(a+I*a*tan(f*x+e))^(5/3)/f+3/10*I*a*(d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x
+e))^(8/3)/f
```

Rubi [A]
time = 0.23, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{35f}+\frac{27ia^2(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{35f}+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (((486*I)/35)*a^4*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
+ (((81*I)/35)*a^3*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f +
 (((27*I)/35)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f +
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}+\frac{1}{5}(9a)\int (d\\ &= \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f}+\frac{3ia(d\sec}{}\\ &= \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f}+\frac{27ia^2(d\,\mathrm{s}}{}\\ &= \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(d\sec(e+fx))^{2/3}(a+i}{35f}\end{aligned}$$

Mathematica [A]
time = 1.27, size = 116, normalized size = 0.71

$$\frac{3a^3(d\sec(e+fx))^{5/3}(i\cos(e-2fx)+\sin(e-2fx))(364+442\cos(2(e+fx))+59i\sec(e+fx)\sin(3(e+fx))+45i\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{140df(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (3*a^3*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(364 + 44
2*Cos[2*(e + f*x)] + (59*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (45*I)*Tan[e +
f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(140*d*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(131) = 262.
time = 0.56, size = 1041, normalized size = 6.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="maxi
ma")
```

```
[Out] 6/35*(7*(-2*I*2^(1/3)*a^3*cos(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - 2*2^(1/3)*a^3*sin(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) + 15*(-I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 - I*2^(1/3)*a^3*sin(2*f*x +
 2*e)^2 - 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - I*2^(1/3)*a^3)*cos(4/3*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 15*(2^(1/3)*a^3*cos(2*f*x + 2*e
)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(
1/3)*a^3)*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/3)*d^(
2/3) + 20*(3*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^3*sin(2*f*x +
2*e)^2 + 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*cos(7/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^3*cos(2*f*x + 2*e
)^4 + I*2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^3
 + 6*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*I*2^(1/3)*a^3*cos(2*f*x + 2*e) +
I*2^(1/3)*a^3 + 2*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^3*cos(2
*f*x + 2*e) + I*2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + 3*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2^(1/3
)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(1/3)*a^3)*si
n(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^3*cos
(2*f*x + 2*e)^4 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*2^(1/3)*a^3*cos(2*f*x
+ 2*e)^3 + 6*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*2^(1/3)*a^3*cos(2*f*x + 2*e
) + 2^(1/3)*a^3 + 2*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f
*x + 2*e) + 2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*sin(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^4 + sin(2*f
*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) +
1)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)
*f)
```

Fricas [A]
time = 0.39, size = 142, normalized size = 0.87

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(-140i\,a^{3}e^{(6i\,fx+6i\,e)}-315i\,a^{3}e^{(4i\,fx+4i\,e)}-270i\,a^{3}e^{(2i\,fx+2i\,e)}-81i\,a^{3}\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{(2i\,fx+2i\,e)}}{35\left(fe^{(6i\,fx+6i\,e)}+2\,fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="fric
as")
```

```
[Out] -6/35*2^(1/3)*(-140*I*a^3*e^(6*I*f*x + 6*I*e) - 315*I*a^3*e^(4*I*f*x + 4*I*
e) - 270*I*a^3*e^(2*I*f*x + 2*I*e) - 81*I*a^3)*(a/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(6*I*f
*x + 6*I*e) + 2*f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(11/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(11/3), x)
```

Mupad [B]
time = 7.88, size = 303, normalized size = 1.86

$$\frac{\left(-\frac{d}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}\left(2\sin\left(2e+2fx\right)^2+\sin\left(4e+4fx\right)1\mathrm{i}-1\right)\left(\frac{a^3\left(a-\frac{a\sin(e+fx)1\mathrm{i}}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}243\mathrm{i}}{35f}+\frac{a^3\left(a-\frac{a\sin(e+fx)1\mathrm{i}}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}\left(-2\sin(e+fx)^2+\sin(2e+2fx)1\mathrm{i}+1\right)162\mathrm{i}}{7f}+\frac{a^3\left(a-\frac{a\sin(e+fx)1\mathrm{i}}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}\left(-2\sin(2e+2fx)^2+\sin(4e+4fx)1\mathrm{i}+1\right)27\mathrm{i}}{f}+\frac{a^3\left(a-\frac{a\sin(e+fx)1\mathrm{i}}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}\left(-2\sin(3e+3fx)^2+\sin(6e+6fx)1\mathrm{i}+1\right)12\mathrm{i}}{f}\right)}{4\left(\sin\left(e+fx\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(11/3),x)
```

```
[Out] ((-d/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4*f*x)*1i + 2*sin(2*e +
 2*f*x)^2 - 1)*((a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))
^(2/3)*243i)/(35*f) + (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2
 - 1))^(2/3)*(sin(2*e + 2*f*x)*1i - 2*sin(e + f*x)^2 + 1)*162i)/(7*f) + (a^
3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4
*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*27i)/f + (a^3*(a - (a*sin(e + f*x)*1i)
/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(6*e + 6*f*x)*1i - 2*sin(3*e + 3*f
*x)^2 + 1)*12i)/f))/(4*(sin(e + f*x)^2 - 1))
```

3.450 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=86

$$\frac{i2^{5+\frac{m}{2}}a^5\,{}_2F_1\left(-4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(5+1/2*m)*a^5*hypergeom([1/2*m, -4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(-\frac{m}{2}-4,\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (I*2^(5 + m/2)*a^5*Hypergeometric2F1[-4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = ((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\cdots}$$

$$= \frac{(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx\cdots}{\cdots}$$

$$= \frac{\left(2^{4+\frac{m}{2}}a^6(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+d\cdots}{a}\right.\right.}{\cdots}$$

$$= \frac{i2^{5+\frac{m}{2}}a^5\,{}_2F_1\left(-4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+\cdots}{dm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1214 vs. 2(86) = 172.
time = 12.26, size = 1214, normalized size = 14.12

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I)*2^(5 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)
*(c + d*x)))*Hypergeometric2F1[1, 1 - m/2, (2 + m)/2, -E^((2*I)*(c + d*x))]
*Sec[c + d*x]^(-5 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5)/(d*(E^(
(3*I)*c) + E^((5*I)*c))*m*(Cos[d*x] + I*Sin[d*x])^5) + (I*2^(5 + m)*(E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d*x)))^m*(E^(I*d*
m*x)*(2 + m)*Hypergeometric2F1[m/2, 1 + m, (2 + m)/2, -E^((2*I)*(c + d*x))]
 - E^(I*d*(2 + m)*x)*m*Hypergeometric2F1[1 + m, (2 + m)/2, (4 + m)/2, -E^((
2*I)*(c + d*x))])*Sec[c + d*x]^(-5 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c +
 d*x])^5)/(d*E^(I*(3*c + d*m*x))*(1 + E^((2*I)*c))*m*(2 + m)*(Cos[d*x] + I*
Sin[d*x])^5) + (I*2^(5 + m)*E^((-3*I)*c + (2*I)*d*x)*(1 + 4*E^((2*I)*c))*(E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d*x)))^m*Hyp
ergeometric2F1[(2 + m)/2, 2 + m, (4 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d
```

```
*x]^(-5 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5)/(d*(1 + E^((2*I)*
c))*(2 + m)*(Cos[d*x] + I*Sin[d*x])^5) - ((3*I)*2^(5 + m)*(E^(I*(c + d*x))/
(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d*x)))^m*(E^(I*d*(2 + m)*x)
*(4 + m)*Hypergeometric2F1[(2 + m)/2, 3 + m, (4 + m)/2, -E^((2*I)*(c + d*x)
)] - E^(I*d*(4 + m)*x)*(2 + m)*Hypergeometric2F1[3 + m, (4 + m)/2, (6 + m)/
2, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(-5 - m)*(e*Sec[c + d*x])^m*(a + I*a
*Tan[c + d*x])^5)/(d*E^(I*(c + d*m*x))*(1 + E^((2*I)*c))*(2 + m)*(4 + m)*(C
os[d*x] + I*Sin[d*x])^5) - (I*2^(5 + m)*(2 + 3*E^((2*I)*c))*(E^(I*(c + d*x)
)/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F
1[(4 + m)/2, 4 + m, (6 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 - m)*
(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5)/(d*E^(I*(c - 4*d*x))*(1 + E^((
2*I)*c))*(4 + m)*(Cos[d*x] + I*Sin[d*x])^5) + (I*2^(5 + m)*E^(I*(c - d*m*x)
)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c + d*x)))^m
*(E^(I*d*(4 + m)*x)*(6 + m)*Hypergeometric2F1[(4 + m)/2, 5 + m, (6 + m)/2,
-E^((2*I)*(c + d*x))] - E^(I*d*(6 + m)*x)*(4 + m)*Hypergeometric2F1[5 + m,
(6 + m)/2, (8 + m)/2, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(-5 - m)*(e*Sec[c
 + d*x])^m*(a + I*a*Tan[c + d*x])^5)/(d*(1 + E^((2*I)*c))*(4 + m)*(6 + m)*(
Cos[d*x] + I*Sin[d*x])^5)
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(e \sec \left(d x +c \right)\right)^{m} \left(a +i a \tan \left(d x +c \right)\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] integral(32*a^5*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(10*I
*d*x + 10*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia^5\left(\int\left(-i(e\sec\left(c+dx\right))^m\right)dx+\int 5(e\sec\left(c+dx\right))^m\tan\left(c+dx\right)dx+\int\left(-10(e\sec\left(c+dx\right))^m\tan^3\left(c+dx\right)\right)dx+\int(e\sec\left(c+dx\right))^m\tan^5\left(c+dx\right)dx+\int 10i(e\sec\left(c+dx\right))^m\tan^2\left(c+dx\right)dx+\int\left(-5i(e\sec\left(c+dx\right))^m\tan^4\left(c+dx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral(5*(e*sec(c + d*x))**
m*tan(c + d*x), x) + Integral(-10*(e*sec(c + d*x))**m*tan(c + d*x)**3, x) +
 Integral((e*sec(c + d*x))**m*tan(c + d*x)**5, x) + Integral(10*I*(e*sec(c
+ d*x))**m*tan(c + d*x)**2, x) + Integral(-5*I*(e*sec(c + d*x))**m*tan(c +
d*x)**4, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^5 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5, x)
```

3.451 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{i2^{3+\frac{m}{2}}a^3\,{}_2F_1\left(-2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(3+1/2*m)*a^3*hypergeom([1/2*m, -2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(-\frac{m}{2}-2,\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I*2^(3 + m/2)*a^3*Hypergeometric2F1[-2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = ((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))$$

$$= \frac{(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx}{}$$

$$= \frac{\left(2^{2+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+d}{a}\right.\right.}{}$$

$$= \frac{i2^{3+\frac{m}{2}}a^3\,{}_2F_1\left(-2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+}{dm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 438 vs. 2(86) = 172.
time = 6.43, size = 438, normalized size = 5.09

$$-\frac{2^{3+m}a^3e^{-i(c+dmx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(\left(e^{idmx}+e^{i(2c+d(2+m)x)}\right)(8+6m+m^2)\,{}_2F_1\left(1,1-\frac{m}{2};\frac{2+m}{2};-e^{2i(c+dx)}\right)-e^{idmx}\left(1+e^{2i(c+dx)}\right)^m\left((8+6m+m^2)\,{}_2F_1\left(\frac{m}{2},1+m;\frac{2+m}{2};-e^{2i(c+dx)}\right)+e^{2idx}m\left(-\left((4+m)\,{}_2F_1\left(1+m,\frac{2+m}{2};\frac{4+m}{2};-e^{2i(c+dx)}\right)\right)+(1+2e^{2ic})(4+m)\,{}_2F_1\left(\frac{2+m}{2},2+m;\frac{4+m}{2};-e^{2i(c+dx)}\right)+e^{2ic}\left(-\left((4+m)\,{}_2F_1\left(\frac{2+m}{2},3+m;\frac{4+m}{2};-e^{2i(c+dx)}\right)\right)+e^{2idx}(2+m)\,{}_2F_1\left(3+m,\frac{4+m}{2};\frac{6+m}{2};-e^{2i(c+dx)}\right)\right)\right)\right)\right)\sec^{-3-m}(c+dx)(e\sec(c+dx))^m(-i+\tan(c+dx))^3}{d\left(1+e^{2ic}\right)m(2+m)(4+m)(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -((2^(3 + m)*a^3*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*((E^(I*d*m*x
) + E^(I*(2*c + d*(2 + m)*x)))*(8 + 6*m + m^2)*Hypergeometric2F1[1, 1 - m/2
, (2 + m)/2, -E^((2*I)*(c + d*x))] - E^(I*d*m*x)*(1 + E^((2*I)*(c + d*x)))^
m*((8 + 6*m + m^2)*Hypergeometric2F1[m/2, 1 + m, (2 + m)/2, -E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*m*(-((4 + m)*Hypergeometric2F1[1 + m, (2 + m)/2, (4
+ m)/2, -E^((2*I)*(c + d*x))]) + (1 + 2*E^((2*I)*c))*(4 + m)*Hypergeometric
2F1[(2 + m)/2, 2 + m, (4 + m)/2, -E^((2*I)*(c + d*x))] + E^((2*I)*c)*(-((4
+ m)*Hypergeometric2F1[(2 + m)/2, 3 + m, (4 + m)/2, -E^((2*I)*(c + d*x))])
+ E^((2*I)*d*x)*(2 + m)*Hypergeometric2F1[3 + m, (4 + m)/2, (6 + m)/2, -E^(
(2*I)*(c + d*x))]))))*Sec[c + d*x]^(-3 - m)*(e*Sec[c + d*x])^m*(-I + Tan[c
+ d*x])^3)/(d*E^(I*(c + d*m*x))*(1 + E^((2*I)*c))*m*(2 + m)*(4 + m)*(Cos[d*
x] + I*Sin[d*x])^3))
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{m} (a+ia\tan(dx+c))^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(6*I*d
*x + 6*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2
*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ia^{3}\left(\int i(e\sec(c+dx))^{m}\, dx + \int \left(-3(e\sec(c+dx))^{m}\tan(c+dx)\right) dx + \int (e\sec(c+dx))^{m}\tan^{3}(c+dx)\, dx + \int \left(-3i(e\sec(c+dx))^{m}\tan^{2}(c+dx)\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(e*sec(c + d*x))**m, x) + Integral(-3*(e*sec(c + d*x))*
*m*tan(c + d*x), x) + Integral((e*sec(c + d*x))**m*tan(c + d*x)**3, x) + In
tegral(-3*I*(e*sec(c + d*x))**m*tan(c + d*x)**2, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3, x)
```

3.452 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{i2^{2+\frac{m}{2}}a^2\,{}_2F_1\left(-1-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(2+1/2*m)*a^2*hypergeom([1/2*m, -1-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]

time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^2 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(-\frac{m}{2}-1,\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (I*2^(2 + m/2)*a^2*Hypergeometric2F1[-1 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = ((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))$$

$$= \frac{(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx}{}$$

$$= \frac{\left(2^{1+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+d}{a}\right.\right.}{}$$

$$= \frac{i2^{2+\frac{m}{2}}a^2\,{}_2F_1\left(-1-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+}{dm}$$

Mathematica [A]
time = 1.01, size = 156, normalized size = 1.81

$$-\frac{i2^{2+m}e^{2i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m{}_2F_1\left(2+m,\frac{4+m}{2};\frac{6+m}{2};-e^{2i(c+dx)}\right)\sec^{-2-m}(c+dx)(e\sec(c+dx))^m(a+ia\tan(c+dx))^2}{d(4+m)(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
[Out] ((-I)*2^(2 + m)*E^((2*I)*(c + 2*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x))))^m*(1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[2 + m, (4 + m)/2, (6
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-2 - m)*(e*Sec[c + d*x])^m*(a
+ I*a*Tan[c + d*x])^2)/(d*(4 + m)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] integral(4*a^2*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(4*I*d
*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\left(-(e\sec(c+dx))^m\right)dx+\int(e\sec(c+dx))^m\tan^2(c+dx)\,dx+\int\left(-2i(e\sec(c+dx))^m\tan(c+dx)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
[Out] -a**2*(Integral(-(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*tan
(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**m*tan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{i2^{1+\frac{m}{2}}a\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(1+1/2*m)*a*hypergeom([1/2*m, -1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [**A**]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{\frac{m}{2}+1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(1 + m/2)*a*Hypergeometric2F1[-1/2*m, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-}\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx)\right.}{d}$$

$$= \frac{\left(2^{m/2}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right.\right.}{}$$

$$= \frac{i2^{1+\frac{m}{2}}a\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m}{dm}$$

Mathematica [A]
time = 0.70, size = 147, normalized size = 1.79

$$\frac{2^{1+m}ae^{i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\,{}_2F_1\left(1+m,\frac{2+m}{2};\frac{4+m}{2};-e^{2i(c+dx)}\right)\sec^{-1-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)-i\sin(dx))(-i+\tan(c+dx))}{d(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*a*E^(I*(c + 2*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
m*(1 + E^((2*I)*(c + d*x)))^m*Hypergeometric2F1[1 + m, (2 + m)/2, (4 + m)/2
, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x]
- I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*(2 + m))
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^m\left(a+ia\tan\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(2*a*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(2*I*d*x
 + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i(e\sec\left(c+dx\right))^{m}\right)\,dx+\int\left(e\sec\left(c+dx\right)\right)^{m}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*tan
(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{-1+\frac{m}{2}}\,{}_2F_1\left(2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{adm}$$

```
[Out] I*2^(-1+1/2*m)*hypergeom([1/2*m, 2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(2-\frac{m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(-1 + m/2)*Hypergeometric2F1[2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a\right.}{d}$$

$$= \frac{\left(2^{-2+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\right.}{d}$$

$$= \frac{i2^{-1+\frac{m}{2}}\,{}_2F_1\left(2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+d}{adm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 212 vs. 2(86) = 172.
time = 1.02, size = 212, normalized size = 2.47

$$-\frac{i2^{-1+m}e^{-i(c+dmx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\left(e^{id(-2+m)x}m\,{}_2F_1\left(\frac{1}{2}(-2+m),m;\frac{m}{2};-e^{2i(c+dx)}\right)+e^{i(2c+dmx)}(-2+m)\,{}_2F_1\left(\frac{m}{2},m;\frac{2+m}{2};-e^{2i(c+dx)}\right)\right)\sec^{1-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))}{d(-2+m)m(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^m*(E^(I*d*(-2 + m)*x)*m*Hypergeometric2F1[(-2 + m)/2, m, m/2,
 -E^((2*I)*(c + d*x))] + E^(I*(2*c + d*m*x))*(-2 + m)*Hypergeometric2F1[m/2
, m, (2 + m)/2, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(1 - m)*(e*Sec[c + d*x]
)^m*(Cos[d*x] + I*Sin[d*x]))/(d*E^(I*(c + d*m*x))*(-2 + m)*m*(a + I*a*Tan[c
 + d*x]))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(2*I*d*
x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{(e\sec(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral((e*sec(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{-2+\frac{m}{2}}\,{}_2F_1\left(3-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^2dm}$$

```
[Out] I*2^(-2+1/2*m)*hypergeom([1/2*m, 3-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a^2/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]

time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(3-\frac{m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (I*2^(-2 + m/2)*Hypergeometric2F1[3 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx &= \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\\ &= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}(}{d}\\ &= \frac{\left(2^{-3+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}(}{ad}\\ &= \frac{i2^{-2+\frac{m}{2}}\,{}_2F_1\left(3-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c}{a^2dm}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 279 vs. 2(86) = 172.
time = 16.72, size = 279, normalized size = 3.24

$$-\frac{i2^{-2+m}e^{-i(2c+dmx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\left(e^{id(-4+m)x}(-2+m)m\,{}_2F_1\left(\frac{1}{2}(-4+m),m;\frac{1}{2}(-2+m);-e^{2i(c+dx)}\right)+e^{2ic}(-4+m)\left(2e^{id(-2+m)x}m\,{}_2F_1\left(\frac{1}{2}(-2+m),m;\frac{m}{2};-e^{2i(c+dx)}\right)+e^{i(2c+dmx)}(-2+m)\,{}_2F_1\left(\frac{m}{2},m;\frac{2+m}{2};-e^{2i(c+dx)}\right)\right)\right)\sec^{2-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))^2}{d(-4+m)(-2+m)m(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^m*(E^(I*d*(-4 + m)*x)*(-2 + m)*m*Hypergeometric2F1[(-4 + m)/2
, m, (-2 + m)/2, -E^((2*I)*(c + d*x))] + E^((2*I)*c)*(-4 + m)*(2*E^(I*d*(-2
 + m)*x)*m*Hypergeometric2F1[(-2 + m)/2, m, m/2, -E^((2*I)*(c + d*x))] + E^
(I*(2*c + d*m*x))*(-2 + m)*Hypergeometric2F1[m/2, m, (2 + m)/2, -E^((2*I)*(
c + d*x))]))*Sec[c + d*x]^(2 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]
)^2)/(d*E^(I*(2*c + d*m*x))*(-4 + m)*(-2 + m)*m*(a + I*a*Tan[c + d*x])^2)
```

Maple [F]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*
x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(e\sec{(c+dx)})^m}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral((e*sec(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/
a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.456 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{-3+\frac{m}{2}}\,{}_2F_1\left(4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^3dm}$$

```
[Out] I*2^(-3+1/2*m)*hypergeom([1/2*m, 4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a^3/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,{}_2F_1\left(4-\frac{m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I*2^(-3 + m/2)*Hypergeometric2F1[4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\operatorname{Subst}(}{d}$$

$$= \frac{\left(2^{-4+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\operatorname{Subst}(}{a^2d}$$

$$= \frac{i2^{-3+\frac{m}{2}}\,{}_2F_1\left(4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c}{a^3dm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 347 vs. 2(86) = 172.
time = 10.14, size = 347, normalized size = 4.03

$$-\frac{i2^{-3+m}e^{-i(3c+dmx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\left(e^{id(-6+m)x}m(8-6m+m^2)\,{}_2F_1\left(\frac{1}{2}(-6+m),m;\frac{1}{2}(-4+m);-e^{2i(c+dx)}\right)+e^{2ic}(-6+m)\left(3e^{id(-4+m)x}(-2+m)m\,{}_2F_1\left(\frac{1}{2}(-4+m),m;\frac{1}{2}(-2+m);-e^{2i(c+dx)}\right)+e^{2ic}(-4+m)\left(3e^{id(-2+m)x}m\,{}_2F_1\left(\frac{1}{2}(-2+m),m;\frac{m}{2};-e^{2i(c+dx)}\right)+e^{i(2c+dmx)}(-2+m)\,{}_2F_1\left(\frac{m}{2},m;\frac{2+m}{2};-e^{2i(c+dx)}\right)\right)\right)\right)\sec^{3-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))^3}{d(-6+m)(-4+m)(-2+m)m(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I)*2^(-3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^m*(E^(I*d*(-6 + m)*x)*m*(8 - 6*m + m^2)*Hypergeometric2F1[(-6
 + m)/2, m, (-4 + m)/2, -E^((2*I)*(c + d*x))] + E^((2*I)*c)*(-6 + m)*(3*E^(
I*d*(-4 + m)*x)*(-2 + m)*m*Hypergeometric2F1[(-4 + m)/2, m, (-2 + m)/2, -E^
((2*I)*(c + d*x))] + E^((2*I)*c)*(-4 + m)*(3*E^(I*d*(-2 + m)*x)*m*Hypergeom
etric2F1[(-2 + m)/2, m, m/2, -E^((2*I)*(c + d*x))] + E^(I*(2*c + d*m*x))*(-
2 + m)*Hypergeometric2F1[m/2, m, (2 + m)/2, -E^((2*I)*(c + d*x))])))*Sec[c
+ d*x]^(3 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^3)/(d*E^(I*(3*c +
 d*m*x))*(-6 + m)*(-4 + m)*(-2 + m)*m*(a + I*a*Tan[c + d*x])^3)
```

Maple [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(6*I*d*
x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-6*I*d*x
 - 6*I*c)/a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{(e\sec(c+dx))^m}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral((e*sec(c + d*x))**m/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*t
an(c + d*x) + I), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3, x)
```

3.457 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{7+m}{2}}a^3\,{}_2F_1\left(\frac{1}{2}(-5-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(-1-m)}\sqrt{a+ia}}{dm}$$

```
[Out] I*2^(7/2+1/2*m)*a^3*hypergeom([1/2*m, -5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d
*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2
*m)/d/m
```

Rubi [A]
time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^3 2^{\frac{m+7}{2}}\sqrt{a+ia\tan(c+dx)}\,(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{1}{2}(-m-5),\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (I*2^((7 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx)\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+\right.}{}$$

$$= \frac{\left(2^{\frac{5}{2}+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\,\mathrm{t}}\right.}{}$$

$$= \frac{i2^{\frac{7+m}{2}}a^3\,{}_2F_1\left(\frac{1}{2}(-5-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec}{dm}$$

Mathematica [A]
time = 2.95, size = 186, normalized size = 1.71

$$-\frac{i2^{\frac{7}{2}+m}e^{3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}{}_2F_1\left(\frac{7}{2}+m,\frac{7+m}{2};\frac{9+m}{2};-e^{2i(c+dx)}\right)\sec^{-\frac{7}{2}-m}(c+dx)(e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}}{d(7+m)(\cos(dx)+i\sin(dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*2^(7/2 + m)*E^((3*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hype
rgeometric2F1[7/2 + m, (7 + m)/2, (9 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c +
d*x]^(-7/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2))/(d*(7 + m)
*(Cos[d*x] + I*Sin[d*x])^(7/2))
```

Maple [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(8*sqrt(2)*a^3*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(e^(6*I*d*x + 6*I*c)
+ 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos \left(c+d\,x\right)}\right)^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.458 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{5+m}{2}}a^2\,{}_2F_1\left(\frac{1}{2}(-3-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(-1-m)}\sqrt{a+ia}}{dm}$$

```
[Out] I*2^(5/2+1/2*m)*a^2*hypergeom([1/2*m, -3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d
*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2
*m)/d/m
```

Rubi [A]

time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^2 2^{\frac{m+5}{2}}\sqrt{a+ia\tan(c+dx)}\,(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{1}{2}(-m-3),\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((5 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = ((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx)$$

$$= \frac{(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+}{}$$

$$= \frac{\left(2^{\frac{3}{2}+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\,\mathrm{t}}\right.}{}$$

$$= \frac{i2^{\frac{5+m}{2}}a^2\,{}_2F_1\left(\frac{1}{2}(-3-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec}{dm}$$

Mathematica [A]
time = 3.70, size = 186, normalized size = 1.71

$$-\frac{i2^{\frac{5}{2}+m}e^{2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{5}{2}+m,\frac{5+m}{2};\frac{7+m}{2};-e^{2i(c+dx)}\right)\sec^{-\frac{5}{2}-m}(c+dx)(e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}}{d(5+m)(\cos(dx)+i\sin(dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(5/2 + m)*E^((2*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hype
rgeometric2F1[5/2 + m, (5 + m)/2, (7 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c +
d*x]^(-5/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2))/(d*(5 + m)
*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*a^2*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(e^(4*I*d*x + 4*I*c)
+ 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.459 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{i2^{\frac{3+m}{2}} a\, {}_2F_1\left(\frac{1}{2}(-1-m), \frac{m}{2}; \frac{2+m}{2}; \frac{1}{2}(1-i\tan(c+dx))\right) (e\sec(c+dx))^m (1+i\tan(c+dx))^{\frac{1}{2}(-1-m)} \sqrt{a+ia\,\mathrm{t}\cdots}}{dm}$$

```
[Out] I*2^(3/2+1/2*m)*a*hypergeom([1/2*m, -1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x
+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2*m
)/d/m
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{\frac{m+3}{2}}\sqrt{a+ia\tan(c+dx)}\,(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\, {}_2F_1\left(\frac{1}{2}(-m-1), \frac{m}{2}; \frac{m+2}{2}; \frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((3 + m)/2)*a*Hypergeometric2F1[(-1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a +
 I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx)\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+d\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\,\mathrm{t}}\right.}{}$$

$$= \frac{i2^{\frac{3+m}{2}}a\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(}{dm}$$

Mathematica [A]
time = 1.42, size = 186, normalized size = 1.74

$$-\frac{i2^{\frac{3}{2}+m}e^{i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}{}_2F_1\left(\frac{3}{2}+m,\frac{3+m}{2};\frac{5+m}{2};-e^{2i(c+dx)}\right)\sec^{-\frac{3}{2}-m}(c+dx)(e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}}{d(3+m)(\cos(dx)+i\sin(dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(3/2 + m)*E^(I*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeo
metric2F1[3/2 + m, (3 + m)/2, (5 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
^(-3/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2))/(d*(3 + m)*(Co
s[d*x] + I*Sin[d*x])^(3/2))
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^m\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*a*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(e^(2*I*d*x + 2*I*c) +
1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec(c+dx))^m\,(ia(\tan(c+dx)-i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.460 $\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{i2^{\frac{1+m}{2}}a\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(1/2+1/2*m)*a*hypergeom([1/2*m, 1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{\frac{m+1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((1 + m)/2)*a*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = ((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))$$

$$= \frac{(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c}{a}\right.\right.}{d\sqrt{a+}}$$

$$= \frac{i2^{\frac{1+m}{2}}a\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.80, size = 174, normalized size = 1.63

$$-\frac{i2^{\frac{1}{2}+m}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}{}_2F_1\left(\frac{1}{2}+m,\frac{1+m}{2};\frac{3+m}{2};-e^{2i(c+dx)}\right)\sec^{-\frac{1}{2}-m}(c+dx)(e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}}{d(1+m)\sqrt{\cos(dx)+i\sin(dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeometric2F1[1/2 + m,
 (1 + m)/2, (3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - m)*(e*Sec
[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*(1 + m)*Sqrt[Cos[d*x] + I*Sin[d
*x]])
```

Maple [F]
time = 1.14, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^m\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec{(c+dx)})^m\sqrt{ia(\tan{(c+dx)}-i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{i2^{\frac{1}{2}(-1+m)}\,{}_2F_1\left(\frac{3-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-1/2+1/2*m)*hypergeom([1/2*m, 3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m-1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{3-m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((-1 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\operatorname{Subst}\left(\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+\frac{m}{2}}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subs}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-1+m)}\,{}_2F_1\left(\frac{3-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 1.30, size = 174, normalized size = 1.64

$$-\frac{i2^{-\frac{1}{2}+m}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+m}{}_2F_1\left(\frac{1}{2}(-1+m),-\frac{1}{2}+m;\frac{1+m}{2};-e^{2i(c+dx)}\right)\sec^{\frac{1}{2}-m}(c+dx)(e\sec(c+dx))^m\sqrt{\cos(dx)+i\sin(dx)}}{d\sqrt{e^{idx}}(-1+m)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-I)*2^(-1/2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + m)*(
1 + E^((2*I)*(c + d*x)))^(-1/2 + m)*Hypergeometric2F1[(-1 + m)/2, -1/2 + m,
 (1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - m)*(e*Sec[c + d*x])^m
*Sqrt[Cos[d*x] + I*Sin[d*x]])/(d*Sqrt[E^(I*d*x)]*(-1 + m)*Sqrt[a + I*a*Tan[
c + d*x]])
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/
a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sec{(c+dx)})^m}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{1}{2}(-3+m)}\,{}_2F_1\left(\frac{5-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{adm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-3/2+1/2*m)*hypergeom([1/2*m, 5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/a/d/m/(a+I*a*tan(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{5-m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{adm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((-3 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a*d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-i$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subs}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-3+m)}\,{}_2F_1\left(\frac{5-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(}{adm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 1.86, size = 178, normalized size = 1.63

$$-\frac{i2^{-\frac{3}{2}+m}e^{-2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^3\,{}_2F_1\left(1,1-\frac{m}{2};\frac{1}{2}(-1+m);-e^{2i(c+dx)}\right)\sec^{\frac{3}{2}-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))^{3/2}}{d(-3+m)(a+ia\tan(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(-3/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (-1
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(3/2))/(d*E^((2*I)*(c + 2*d*x))*(-3 + m)*(a + I*a*Tan
[c + d*x])^(3/2))
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c
) + 1)*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(c+dx))^m}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{1}{2}(-5+m)}\,{}_2F_1\left(\frac{7-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-5/2+1/2*m)*hypergeom([1/2*m, 7/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/a^2/d/m/(a+I*a*tan(d*x+c))
^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,{}_2F_1\left(\frac{7-m}{2},\frac{m}{2};\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((-5 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a^2*d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-i$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subs}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-5+m)}\,{}_2F_1\left(\frac{7-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 3.64, size = 178, normalized size = 1.63

$$-\frac{i2^{-\frac{5}{2}+m}e^{-3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^4\,{}_2F_1\left(1,1-\frac{m}{2};\frac{1}{2}(-3+m);-e^{2i(c+dx)}\right)\sec^{\frac{5}{2}-m}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))^{5/2}}{d(-5+m)(a+ia\tan(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (-3
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(5/2))/(d*E^((3*I)*(c + 2*d*x))*(-5 + m)*(a + I*a*Tan
[c + d*x])^(5/2))
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/8*sqrt(2)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c
) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sec{(c + dx)})^m}{(ia(\tan{(c + dx)} - i))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=105

$$\frac{i2^{\frac{m}{2}+n}\,{}_2F_1\left(\frac{m}{2},1-\frac{m}{2}-n;\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-\frac{m}{2}-n}(a+ia\tan(c+dx))^n}{dm}$$

```
[Out] I*2^(1/2*m+n)*hypergeom([1/2*m, 1-1/2*m-n],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

Rubi [A]

time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{m}{2}+n}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m(1+i\tan(c+dx))^{-\frac{m}{2}-n}\,{}_2F_1\left(\frac{m}{2},-\frac{m}{2}-n+1;\frac{m+2}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(m/2 + n)*Hypergeometric2F1[m/2, 1 - m/2 - n, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^(-1/2*m - n)*(a + I*a*Ta
n[c + d*x])^n)/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx\right.}{}$$

$$= \frac{\left(2^{-1+\frac{m}{2}+n}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\,\mathrm{ta}\right.}{}$$

$$= \frac{i2^{\frac{m}{2}+n}\,{}_2F_1\left(\frac{m}{2},1-\frac{m}{2}-n;\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+}{dm}$$

Mathematica [A]
time = 9.42, size = 165, normalized size = 1.57

$$-\frac{i2^{m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}\left(1+e^{2i(c+dx)}\right)^{m+n}{}_2F_1\left(\frac{m}{2}+n,m+n;1+\frac{m}{2}+n;-e^{2i(c+dx)}\right)\sec^{-m-n}(c+dx)(e\sec(c+dx))^m(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(m+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(
m + n)*(1 + E^((2*I)*(c + d*x)))^(m + n)*Hypergeometric2F1[m/2 + n, m + n,
1 + m/2 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-m - n)*(e*Sec[c + d*x])^m
*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.69, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^m\,(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(I*d*n*x + I
*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) +
 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \sec{(c + dx)})^m (ia(\tan{(c + dx)} - i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos(c + d\,x)}\right)^m (a + a\tan(c + d\,x)\ 1\mathrm{i})^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}+\frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)}-\frac{i(a+ia\tan(c+dx))^{5+n}}{a^5d(5+n)}$$

```
[Out] -4*I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/(3+n)+4*I*(a+I*a*tan(d*x+c))^(4+n)/a^4/
d/(4+n)-I*(a+I*a*tan(d*x+c))^(5+n)/a^5/d/(5+n)
```

Rubi [A]

time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$-\frac{i(a+ia\tan(c+dx))^{n+5}}{a^5d(n+5)}+\frac{4i(a+ia\tan(c+dx))^{n+4}}{a^4d(n+4)}-\frac{4i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-4*I)*(a + I*a*Tan[c + d*x])^(3 + n))/(a^3*d*(3 + n)) + ((4*I)*(a + I*a*T
an[c + d*x])^(4 + n))/(a^4*d*(4 + n)) - (I*(a + I*a*Tan[c + d*x])^(5 + n))/
(a^5*d*(5 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{2+n}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{2+n}-4a(a+x)^{3+n}+(a+x)^{4+n}\right)\,dx,x,i\right.}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}+\frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)}-\frac{i(a+}{}\end{aligned}$$

Mathematica [A]
time = 14.43, size = 171, normalized size = 1.76

$$-\frac{i2^{5+n}e^{6i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(20+2e^{4i(c+dx)}+9n+n^2+2e^{2i(c+dx)}(5+n)\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d\left(1+e^{2i(c+dx)}\right)^5(3+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5 + n)*E^((6*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(20 + 2*E^((4*I)*(c + d*x)) + 9*n + n^2 + 2*E^((2*I)*(
c + d*x))*(5 + n))*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^5
*(3 + n)*(4 + n)*(5 + n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.85, size = 3316, normalized size = 34.19

method	result	size
risch	Expression too large to display	3316

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -32*I/(exp(2*I*(d*x+c))+1)^5/(4+n)/(5+n)/d/(3+n)*(2/((exp(2*I*(d*x+c))+1)^n
)*(exp(I*(Re(d*x)+Re(c)))^n)^2*a^n*2^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*
I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(
2*I*(d*x+c)))^3*n)*exp(10*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn
(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(
2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+
c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(10*I*c)*exp(-
1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(I*Pi*csgn
(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2
*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(-1/2*I*
Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*c
sgn(I/(exp(2*I*(d*x+c))+1))*n)+2*n/((exp(2*I*(d*x+c))+1)^n)*(exp(I*(Re(d*x)
+Re(c)))^n)^2*a^n*2^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I/(exp(
2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n
)*exp(8*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*ex
p(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
```

```
+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(8*I*c)*exp(-1/2*I*Pi*csgn(I*exp
(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c))
)^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*e
xp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*
I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(
d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x
+c))+1))*n)+n^2/((exp(2*I*(d*x+c))+1)^n)*(exp(I*(Re(d*x)+Re(c)))^n)^2*a^n*2
^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp
(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/
2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(6*I*d*x)*exp(
1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n
)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*
n)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp
(2*I*(d*x+c))+1))*n)*exp(6*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn
(I*exp(I*(d*x+c)))^2*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d
*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2
*I*(d*x+c)))*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(ex
p(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n)+10/((ex
p(2*I*(d*x+c))+1)^n)*(exp(I*(Re(d*x)+Re(c)))^n)^2*a^n*2^n*exp(-2*n*Im(d*x)-
2*n*Im(c))*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(
I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(8*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(
2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I
*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(
I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(1/2*I*Pi*csgn
(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)
*exp(8*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2
*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*
I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*
a)*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*ex
p(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n)+9*n/((exp(2*I*(d*x+c))+1)^n
)*(exp(I*(Re(d*x)+Re(c)))^n)^2*a^n*2^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*
I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(
2*I*(d*x+c)))^3*n)*exp(6*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(
I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(6*I*c)*exp(-1/
2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(I*Pi*csgn(I
*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I
*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(85) = 170$.
time = 0.37, size = 247, normalized size = 2.55

$$-\frac{32 \, {\left(2 \, {\left(i \, n + 5 i\right)} e^{\left(8 i \, d x + 8 i \, c\right)} + {\left(i \, n^{2} + 9 i \, n + 20 i\right)} e^{\left(6 i \, d x + 6 i \, c\right)} + 2 i \, e^{\left(10 i \, d x + 10 i \, c\right)}\right)} \left(\frac{2 \, a e^{\left(2 i \, d x + 2 i \, c\right)}}{e^{\left(2 i \, d x + 2 i \, c\right)} + 1}\right)^{n}}{d n^{3} + 12 \, d n^{2} + 47 \, d n + {\left(d n^{3} + 12 \, d n^{2} + 47 \, d n + 60 \, d\right)} e^{\left(10 i \, d x + 10 i \, c\right)} + 5 \, {\left(d n^{3} + 12 \, d n^{2} + 47 \, d n + 60 \, d\right)} e^{\left(8 i \, d x + 8 i \, c\right)} + 10 \, {\left(d n^{3} + 12 \, d n^{2} + 47 \, d n + 60 \, d\right)} e^{\left(6 i \, d x + 6 i \, c\right)} + 10 \, {\left(d n^{3} + 12 \, d n^{2} + 47 \, d n + 60 \, d\right)} e^{\left(4 i \, d x + 4 i \, c\right)} + 5 \, {\left(d n^{3} + 12 \, d n^{2} + 47 \, d n + 60 \, d\right)} e^{\left(2 i \, d x + 2 i \, c\right)} + 60 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -32*(2*(I*n + 5*I)*e^(8*I*d*x + 8*I*c) + (I*n^2 + 9*I*n + 20*I)*e^(6*I*d*x
+ 6*I*c) + 2*I*e^(10*I*d*x + 10*I*c))*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x
+ 2*I*c) + 1))^n/(d*n^3 + 12*d*n^2 + 47*d*n + (d*n^3 + 12*d*n^2 + 47*d*n +
60*d)*e^(10*I*d*x + 10*I*c) + 5*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(8*I*d
*x + 8*I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(6*I*d*x + 6*I*c) + 1
0*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(4*I*d*x + 4*I*c) + 5*(d*n^3 + 12*d*
n^2 + 47*d*n + 60*d)*e^(2*I*d*x + 2*I*c) + 60*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia(\tan \left(c + dx\right) - i)\right)^{n} \sec^{6} \left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

Mupad [B]
time = 8.13, size = 168, normalized size = 1.73

$$\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,{\left(a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}\right)}^n\,\left(\frac{64\,\mathrm{e}^{c\,10\mathrm{i}+d\,x\,10\mathrm{i}}}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}+\frac{\mathrm{e}^{c\,6\mathrm{i}+d\,x\,6\mathrm{i}}\,\left(32\,n^2+288\,n+640\right)}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}+\frac{\mathrm{e}^{c\,8\mathrm{i}+d\,x\,8\mathrm{i}}\,\left(64\,n+320\right)}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}\right)}{32\,{\cos\left(c+d\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^6,x)
```

```
[Out] (exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*((64*exp(c*1
0i + d*x*10i))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*6i + d*x*6i)*(
288*n + 32*n^2 + 640))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*8i + d
*x*8i)*(64*n + 320))/(d*(n*47i + n^2*12i + n^3*1i + 60i))))/(32*cos(c + d*x
)^5)
```

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=65

$$-\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)}+\frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2+n)+I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/
(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\frac{i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}-\frac{2i(a+ia\tan(c+dx))^{n+2}}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-2*I)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*(2 + n)) + (I*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{1+n}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{1+n}-(a+x)^{2+n}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)}+\frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(65) = 130.
time = 13.76, size = 143, normalized size = 2.20

$$-\frac{i2^{3+n}e^{4i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(3+e^{2i(c+dx)}+n\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d\left(1+e^{2i(c+dx)}\right)^3(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3 + n)*E^((4*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(3 + E^((2*I)*(c + d*x)) + n)*(a + I*a*Tan[c + d*x])^n
)/(d*(1 + E^((2*I)*(c + d*x)))^3*(2 + n)*(3 + n)*Sec[c + d*x]^n*(Cos[d*x] +
 I*Sin[d*x])^n)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.54, size = 1668, normalized size = 25.66

method	result	size
risch	Expression too large to display	1668

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -8*I/(exp(2*I*(d*x+c))+1)^3/(3+n)/d/(2+n)*(1/((exp(2*I*(d*x+c))+1)^n)*(exp(
I*(Re(d*x)+Re(c)))^n)^2*a^n*2^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^3*n)*exp(6*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(
2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*ex
p(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(6*I*c)*exp(-1/2*I*Pi*
csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(I*Pi*csgn(I*exp(2*
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c
)))^3*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I
*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I
*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(ex
p(2*I*(d*x+c))+1))*n)+n/((exp(2*I*(d*x+c))+1)^n)*(exp(I*(Re(d*x)+Re(c)))^n)
^2*a^n*2^n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)
))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(4*I*d*x)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d
*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(-1/2*I*Pi
*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(
exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*e
xp(2*I*(d*x+c)))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x
```

```
+c)))*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^2*n)*exp(4*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n)
*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn
(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)
+3/((exp(2*I*(d*x+c))+1)^n)*(exp(I*(Re(d*x)+Re(c)))^n)^2*a^n*2^n*exp(-2*n*I
m(d*x)-2*n*Im(c))*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+
c)))^2*n)*exp(4*I*d*x)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)
*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(1/2*I*Pi
*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n
)*exp(4*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*
I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c))
)*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(-1/2*I*P
i*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(57) = 114.
time = 0.39, size = 142, normalized size = 2.18

$$-\frac{8\left((i\,n+3i)e^{(4i\,dx+4i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}}{dn^2+5\,dn+(dn^2+5\,dn+6\,d)e^{(6i\,dx+6i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(4i\,dx+4i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(2i\,dx+2i\,c)}+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -8*((I*n + 3*I)*e^(4*I*d*x + 4*I*c) + I*e^(6*I*d*x + 6*I*c))*(2*a*e^(2*I*d*
x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n^2 + 5*d*n + (d*n^2 + 5*d*n + 6
*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^2 + 5*d*n + 6*d)*e^(4*I*d*x + 4*I*c) + 3*(
d*n^2 + 5*d*n + 6*d)*e^(2*I*d*x + 2*I*c) + 6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^n \sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

Mupad [B]
time = 2.44, size = 216, normalized size = 3.32

$$-\frac{4\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}\right)^n\,(n\,3i+\cos(2\,c+2\,d\,x)\,15i+\cos(4\,c+4\,d\,x)\,6i+\cos(6\,c+6\,d\,x)\,1i-9\,\sin(2\,c+2\,d\,x)-6\,\sin(4\,c+4\,d\,x)-\sin(6\,c+6\,d\,x)+n\,\cos(2\,c+2\,d\,x)\,4i+n\,\cos(4\,c+4\,d\,x)\,1i-2\,n\,\sin(2\,c+2\,d\,x)-n\,\sin(4\,c+4\,d\,x)+10i)}{d\,(n^2+5\,n+6)\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+\cos(6\,c+6\,d\,x)+10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^4,x)
```

```
[Out] -(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^n*(n*3i + cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos(6*c + 6*d*x)*
1i - 9*sin(2*c + 2*d*x) - 6*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + n*cos(2*c
 + 2*d*x)*4i + n*cos(4*c + 4*d*x)*1i - 2*n*sin(2*c + 2*d*x) - n*sin(4*c + 4
*d*x) + 10i))/(d*(5*n + n^2 + 6)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x)
+ cos(6*c + 6*d*x) + 10))
```

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\text{Subst}\left(\int (a+x)^n\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 111 vs. 2(32) = 64.
time = 13.20, size = 111, normalized size = 3.47

$$-\frac{i2^{1+n}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1+n}\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x))))^(1 + n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 + n)*Sec[c + d*x]^n*
(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [A]
time = 0.10, size = 31, normalized size = 0.97

method	result
derivativedivides	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
default	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
risch	$-\frac{2ie^{\frac{i\pi\,\mathrm{csgn}\left(\frac{ie^{2i(dx+c)}}{e^{2i(dx+c)}+1}\right)\mathrm{csgn}\left(\frac{ia\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}\right)^{2}n}{2}-\frac{i\pi\mathrm{csgn}\left(\frac{ie^{2i(dx+c)}}{e^{2i(dx+c)}+1}\right)^{3}n}{2}+2idx-\frac{i\pi\mathrm{csgn}\left(ie^{2i(dx+c)}\right)^{3}n}{2}+\frac{i\pi\mathrm{csgn}\left(\frac{ie^{2i(d}}{e^{2i(dx-}}\right.}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.88

$$-\frac{i\,(i\,a\tan\left(dx+c\right)+a)^{n+1}}{ad(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*d*(n + 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.
time = 0.38, size = 60, normalized size = 1.88

$$-\frac{2i\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}e^{(2i\,dx+2i\,c)}}{dn+(dn+d)e^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -2*I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(2*I*d*x + 2*I
*c)/(d*n + (d*n + d)*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^n \sec^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(28) = 56.
time = 0.73, size = 62, normalized size = 1.94

$$-\frac{i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{n+1}}{ad(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d*
x + 1/2*c)^2 - 1))^(n + 1)/(a*d*(n + 1))
```

Mupad [B]
time = 0.42, size = 104, normalized size = 3.25

$$\frac{2\,\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)}^n}{d\,\left(n+1\right)\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}-\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^2,x)
```

```
[Out] (2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*((a*(cos(2*c + 2*d
*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(n + 1)*(cos(
2*c + 2*d*x)*1i - sin(2*c + 2*d*x) + 1i))
```

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=56

$$\frac{ia\,{}_2F_1\left(2,-1+n;n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}$$

[Out] 1/4*I*a*hypergeom([2, -1+n],[n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(-1+n)/d/(1-n)

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 70}

$$\frac{ia(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(2,n-1;n;\frac{1}{2}(i\tan(c+dx)+1)\right)}{4d(1-n)}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n))

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^{-2+n}}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{ia\,{}_2F_1\left(2,-1+n;n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 256 vs. 2(56) = 112.
time = 14.46, size = 256, normalized size = 4.57

$$-\frac{i2^{-3+n}e^{-2i(c+dnx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(\left(e^{2id(-1+n)x}+e^{2i(c+dnx)}\right)n(1+n)+2e^{2i(c+dnx)}\left(1+e^{2i(c+dx)}\right)^n(-1+n^2)\,{}_2F_1\left(n,n;1+n;-e^{2i(c+dx)}\right)+e^{2i(2c+dx+dnx)}\left(1+e^{2i(c+dx)}\right)^n(-1+n)n\,{}_2F_1\left(n,1+n;2+n;-e^{2i(c+dx)}\right)\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{dn\,(-1+n^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*((E^((2*I)*d*(-1 + n)*x) + E^((2*I)*(c + d*n*x)))*n*(1 + n) + 2*E^((2*I)*
(c + d*n*x))*(1 + E^((2*I)*(c + d*x)))^n*(-1 + n^2)*Hypergeometric2F1[n, n,
 1 + n, -E^((2*I)*(c + d*x))] + E^((2*I)*(2*c + d*x + d*n*x))*(1 + E^((2*I)
*(c + d*x)))^n*(-1 + n)*n*Hypergeometric2F1[n, 1 + n, 2 + n, -E^((2*I)*(c +
 d*x))])*(a + I*a*Tan[c + d*x])^n)/(d*E^((2*I)*(c + d*n*x))*n*(-1 + n^2)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.97, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(4*I*
d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan{(c+dx)}-i))^n\cos^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos{(c+d\,x)}^2\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n, x)
```

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=60

$$\frac{ia^2\,{}_2F_1\left(3,-2+n;-1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}$$

```
[Out] 1/8*I*a^2*hypergeom([3, -2+n],[-1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c
))^(-2+n)/d/(2-n)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 70}

$$\frac{ia^2(a+ia\tan(c+dx))^{n-2}\,{}_2F_1\left(3,n-2;n-1;\frac{1}{2}(i\tan(c+dx)+1)\right)}{8d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]*(a
+ I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{(a+x)^{-3+n}}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{ia^2\,{}_2F_1\left(3,-2+n;-1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+d\end{aligned}$$

$$8d(2-n)$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(60) = 120.
time = 8.33, size = 143, normalized size = 2.38

$$-\frac{i2^{-5+n}e^{-4i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^5{}_2F_1\left(1,3;-1+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^5*Hypergeometric2F1[1, 3, -1 + n, -E^((2*I)*(c
+ d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((4*I)*(c + d*x))*(-2 + n)*Sec[c +
d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/16*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(8*I
*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*
x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c + dx) - i))^n \cos^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos (c + d\,x)^4\, (a + a\tan(c + d\,x)\ 1\mathrm{i})^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n, x)
```

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=60

$$\frac{ia^3\,{}_2F_1\left(4,-3+n;-2+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}$$

```
[Out] 1/16*I*a^3*hypergeom([4, -3+n],[-2+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+
c))^(-3+n)/d/(3-n)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 70}

$$\frac{ia^3(a+ia\tan(c+dx))^{n-3}\,{}_2F_1\left(4,n-3;n-2;\frac{1}{2}(i\tan(c+dx)+1)\right)}{16d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2]*(a
 + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{(a+x)^{-4+n}}{(a-x)^4}\,dx,x,ia\tan(c+dx)\right)}{d}$$
$$= \frac{ia^3\,{}_2F_1\left(4,-3+n;-2+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+$$
$$16d(3-n)$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(60) = 120.
time = 10.84, size = 143, normalized size = 2.38

$$-\frac{i2^{-7+n}e^{-6i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^7\,{}_2F_1\left(1,4;-2+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(-3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-7 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^7*Hypergeometric2F1[1, 4, -2 + n, -E^((2*I)*(c
+ d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((6*I)*(c + d*x))*(-3 + n)*Sec[c +
d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\cos^6(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/64*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(12*
I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(
```

```
6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*e^(-
6*I*d*x - 6*I*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n, x)
```

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{i2^{\frac{5}{2}+n}a^2\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-n;\frac{7}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+ia\tan(c+dx))^{-2+n}}{5d}$$

```
[Out] 1/5*I*2^(5/2+n)*a^2*hypergeom([5/2, -3/2-n],[7/2],1/2-1/2*I*tan(d*x+c))*sec
(d*x+c)^5*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-2+n)/d
```

Rubi [A]
time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia^2 2^{n+\frac{5}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\,{}_2F_1\left(\frac{5}{2},-n-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c +
 d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-2 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{\sec^5(c+dx)\int(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{\frac{5}{2}+n}\,dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$
$$= \frac{(a^2\sec^5(c+dx))\,\text{Subst}\left(\int(a-iax)^{3/2}(a+iax)^{\frac{3}{2}+n}\,dx,x,\tan(c+\right.}{d(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$
$$= \frac{\left(2^{\frac{3}{2}+n}a^3\sec^5(c+dx)(a+ia\tan(c+dx))^{-2+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}}\right.}{d(a-ia\tan(c-}$$
$$= \frac{i2^{\frac{5}{2}+n}a^2\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-n;\frac{7}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+}{5d}$$

Mathematica [A]
time = 14.34, size = 153, normalized size = 1.63

$$-\frac{i2^{5+n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n\,{}_2F_1\left(\frac{5}{2}+n,5+n;\frac{7}{2}+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(5+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5 + n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*Hypergeometric2F1[5/2 + n,
 5 + n, 7/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 2*
n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(5*I*d*
x + 5*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^n \sec^5 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5, x)
```

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=92

$$\frac{i2^{\frac{3}{2}+n}a\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-n;\frac{5}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+ia\tan(c+dx))^{-1+n}}{3d}$$

```
[Out] 1/3*I*2^(3/2+n)*a*hypergeom([3/2, -1/2-n],[5/2],1/2-1/2*I*tan(d*x+c))*sec(d
*x+c)^3*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A]
time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(\frac{3}{2},-n-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c + d
*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{\sec^3(c+dx)\int (a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{\frac{3}{2}+n}\,dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$
$$= \frac{(a^2\sec^3(c+dx))\,\text{Subst}\left(\int \sqrt{a-iax}\,(a+iax)^{\frac{1}{2}+n}\,dx, x, \tan(c+ \right.}{d(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$
$$= \frac{\left(2^{\frac{1}{2}+n}a^2\sec^3(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}}\right.}{d(a-ia\tan(c+}$$
$$= \frac{i2^{\frac{3}{2}+n}a\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-n;\frac{5}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+}{3d}$$

Mathematica [A]
time = 13.54, size = 153, normalized size = 1.66

$$-\frac{i2^{3+n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n{}_2F_1\left(\frac{3}{2}+n,3+n;\frac{5}{2}+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(3+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3 + n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*Hypergeometric2F1[3/2 + n,
 3 + n, 5/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 2*
n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\sec^3(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(3*I*d*x
 + 3*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^n \sec^3 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3, x)
```

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=88

$$\frac{i2^{\frac{1}{2}+n}a\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{-1+n}}{d}$$

```
[Out] I*2^(1/2+n)*a*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*I*tan(d*x+c))*sec(d*x+c)
*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A]
time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])/
2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1 +
 n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx &= \frac{\sec(c+dx)\int\sqrt{a-ia\tan(c+dx)}\,(a+ia\tan(c+dx))^{\frac{1}{2}+n}\,dx}{\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\\ &= \frac{(a^2\sec(c+dx))\,\mathrm{Subst}\left(\int\frac{(a+iax)^{-\frac{1}{2}+n}}{\sqrt{a-iax}}\,dx,x,\tan(c+dx)\right)}{d\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}\\ &= \frac{\left(2^{-\frac{1}{2}+n}a^2\sec(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right.}{d\sqrt{a-ia\tan(c+dx)}}\\ &= \frac{i2^{\frac{1}{2}+n}a\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan}{d}\end{aligned}$$

Mathematica [A]
time = 8.82, size = 146, normalized size = 1.66

$$-\frac{i2^{1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1+n}\left(1+e^{2i(c+dx)}\right)^{1+n}{}_2F_1\left(\frac{1}{2}+n,1+n;\frac{3}{2}+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(
1 + n)*(1 + E^((2*I)*(c + d*x)))^(1 + n)*Hypergeometric2F1[1/2 + n, 1 + n,
3/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*Sec[c
 + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \sec\left(dx+c\right)\left(a+ia\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*x +
 I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan(c+dx)-i))^n \sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x), x)
```

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-n;\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n}{d}$$

```
[Out] -I*2^(-1/2+n)*cos(d*x+c)*hypergeom([-1/2, 3/2-n],[1/2],1/2-1/2*I*tan(d*x+c)
)*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-n;\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)
/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\right)\int \frac{(a+}{\sqrt{}}$$

$$= \frac{\left(a^2\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\right)\text{Subs}}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+n}a\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\,(a+ia\tan(c+dx))^n\,\right(}{d}$$

$$= -\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-n;\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+}{d}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 195 vs. 2(85) = 170.
time = 13.56, size = 195, normalized size = 2.29

$$-\frac{i2^{-1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1+n}\left(1+e^{2i(c+dx)}\right)^{-1+n}\left((1+2n)\,{}_2F_1\left(-\frac{1}{2}+n,n;\frac{1}{2}+n;-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(-1+2n)\,{}_2F_1\left(n,\frac{1}{2}+n;\frac{3}{2}+n;-e^{2i(c+dx)}\right)\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d\left(-1+4n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(-1 + n)*(1 + E^((2*I)*(c + d*x)))^(-1 + n)*((1 + 2*n)*Hypergeometric2F1[-1
/2 + n, n, 1/2 + n, -E^((2*I)*(c + d*x))] + E^((2*I)*(c + d*x))*(-1 + 2*n)*
Hypergeometric2F1[n, 1/2 + n, 3/2 + n, -E^((2*I)*(c + d*x))])*(a + I*a*Tan[
c + d*x])^n)/(d*(-1 + 4*n^2)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(a+ia\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(2*I*
d*x + 2*I*c) + 1)*e^(-I*d*x - I*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\tan (c+dx)-i))^n \cos (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)\,\left(a+a\tan(c+dx)\,1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$-\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-n;-\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{1+n}}{3ad}$$

[Out] -1/3*I*2^(-3/2+n)*cos(d*x+c)^3*hypergeom([-3/2, 5/2-n],[-1/2],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d

Rubi [A]
time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-n;-\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((-1/3*I)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d)

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx &= \left(\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\int \frac{(\\ &= \frac{\left(a^2\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\mathrm{S}}{d}\\ &= \frac{\left(2^{-\frac{5}{2}+n}\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{1}\right.}{d}\\ &= -\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-n;-\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(}{3ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 321 vs. $2(94) = 188$.
time = 14.75, size = 321, normalized size = 3.41

$$-\frac{i2^{-3+n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^n\left(\left(-3-2n+12n^2+8n^3\right){}_2F_1\left(-\frac{3}{2}+n,n;-\frac{1}{2}+n;-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(-3+2n)\left(3\left(3+8n+4n^2\right){}_2F_1\left(-\frac{1}{2}+n,n;\frac{1}{2}+n;-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(-1+2n)\left((9+6n)\,{}_2F_1\left(n,\frac{1}{2}+n;\frac{3}{2}+n;-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(1+2n)\,{}_2F_1\left(n,\frac{3}{2}+n;\frac{5}{2}+n;-e^{2i(c+dx)}\right)\right)\right)\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d\left(9-40n^2+16n^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^n*((-3 - 2*n + 12*n^2 + 8*n^3)*Hypergeometric2F
1[-3/2 + n, n, -1/2 + n, -E^((2*I)*(c + d*x))] + E^((2*I)*(c + d*x))*(-3 +
2*n)*(3*(3 + 8*n + 4*n^2)*Hypergeometric2F1[-1/2 + n, n, 1/2 + n, -E^((2*I)
*(c + d*x))] + E^((2*I)*(c + d*x))*(-1 + 2*n)*((9 + 6*n)*Hypergeometric2F1[
n, 1/2 + n, 3/2 + n, -E^((2*I)*(c + d*x))] + E^((2*I)*(c + d*x))*(1 + 2*n)*
Hypergeometric2F1[n, 3/2 + n, 5/2 + n, -E^((2*I)*(c + d*x))])))*(a + I*a*Ta
n[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(9 - 40*n^2 + 16*n^4)*Sec[c + d*x]^n*
(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.69, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(6*I*
d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-3*I*d
*x - 3*I*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\,\,1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n, x)
```

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$-\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-n;-\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{2+n}}{5a^2d}$$

```
[Out] -1/5*I*2^(-5/2+n)*cos(d*x+c)^5*hypergeom([-5/2, 7/2-n],[-3/2],1/2-1/2*I*tan
(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(2+n)/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-n;-\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/2
, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d
*x])^(2 + n))/(a^2*d)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\int \frac{(}{}$$

$$= \frac{\left(a^2\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\mathrm{S}}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+n}\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{2}\right.}{a}$$

$$= -\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-n;-\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(}{5a^2d}$$

Mathematica [A]
time = 10.80, size = 149, normalized size = 1.59

$$-\frac{i2^{-5+n}e^{-5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^6\,{}_2F_1\left(1,\frac{7}{2};-\frac{3}{2}+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(-5+2n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^6*Hypergeometric2F1[1, 7/2, -3/2 + n, -E^((2*I)
*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((5*I)*(c + d*x))*(-5 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(10*
I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*e^(4*
I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,\left(a+a\,\tan(c+d\,x)\,\,1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n, x)
```

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=96

$$\frac{i2^{\frac{9}{4}+n}a\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-n;\frac{9}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-\frac{1}{4}-n}(a+ia\tan(c+dx))}{5d}$$

```
[Out] 1/5*I*2^(9/4+n)*a*hypergeom([5/4, -1/4-n],[9/4],1/2-1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(5/2)*(1+I*tan(d*x+c))^(-1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(\frac{5}{4},-n-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/2}\right)\operatorname{Subst}\left(\int \sqrt[4]{a-iax}\,(a+iax)^{\frac{1}{4}+n}\,dx, x\right.}{d(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/}}$$

$$= \frac{\left(2^{\frac{1}{4}+n}a^2(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(}{a}\right.\right.}{d(a-ia\,t}$$

$$= \frac{i2^{\frac{9}{4}+n}a\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-n;\frac{9}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx}{5d}$$

Mathematica [A]
time = 9.63, size = 181, normalized size = 1.89

$$-\frac{i2^{\frac{7}{2}+n}e^{2i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}{}_2F_1\left(\frac{5}{4}+n,\frac{5}{2}+n;\frac{9}{4}+n;-e^{2i(c+dx)}\right)\sec^{-\frac{5}{2}-n}(c+dx)(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(5+4n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(7/2 + n)*E^((2*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeo
metric2F1[5/4 + n, 5/2 + n, 9/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5
/2 - n)*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 4*n)*(Cos[
d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{\frac{5}{2}}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^
(5/2*I*d*x + 5/2*I*c + 5/2)/((e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) +
 1)*sqrt(e^(2*I*d*x + 2*I*c) + 1)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(5/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=96

$$\frac{i2^{\frac{7}{4}+n}a\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-n;\frac{7}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{-1+}}{3d}$$

[Out] 1/3*I*2^(7/4+n)*a*hypergeom([3/4, 1/4-n],[7/4],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(3/2)*(1+I*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-n;\frac{7}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(-1 + n))/d

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \frac{(e\sec(c+dx))^{3/2}\int(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{(a+iax)^{-\frac{1}{4}+n}}{\sqrt[4]{a-iax}}\,dx,x,\tan(c+dx}{d(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+n}a^2(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan}{}\right.\right.}{d(a-ia\tan(c}$$

$$= \frac{i2^{\frac{7}{4}+n}a\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-n;\frac{7}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))}{3d}$$

Mathematica [A]
time = 9.36, size = 170, normalized size = 1.77

$$-\frac{i2^{\frac{5}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{3}{2}+n}{}_2F_1\left(\frac{3}{4}+n,\frac{3}{2}+n;\frac{7}{4}+n;-e^{2i(c+dx)}\right)\sec^{-\frac{3}{2}-n}(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(3+4n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(3/2 + n)*(1 + E^((2*I)*(c + d*x)))^(3/2 + n)*Hypergeometric2F1[3/4 + n, 3
/2 + n, 7/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3/2 - n)*(e*Sec[c + d
*x])^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n
)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(a+ia\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^
(3/2*I*d*x + 3/2*I*c + 3/2)/(e^(2*I*d*x + 2*I*c) + 1)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec{(c+dx)})^{\frac{3}{2}}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.479 $\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{i2^{\frac{5}{4}+n}a\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-n;\frac{5}{4};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}\,(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{-1+n}}{d}$$

```
[Out] I*2^(5/4+n)*a*hypergeom([1/4, 3/4-n],[5/4],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(1/2)*(1+I*tan(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A]
time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}\,(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-n;\frac{5}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])/
2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^n\,dx = \frac{\sqrt{e\sec(c+dx)}\int \sqrt[4]{a-ia\tan(c+dx)}\,(a+ia\tan(c+dx))}{\sqrt[4]{a-ia\tan(c+dx)}\,\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2\sqrt{e\sec(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{3}{4}+n}}{(a-iax)^{3/4}}\,dx,x,\tan(c+dx)\right)}{d\sqrt[4]{a-ia\tan(c+dx)}\,\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+n}a^2\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(}{a}\right.\right.}{d\sqrt[4]{a-ia\tan(c}}$$

$$= \frac{i2^{\frac{5}{4}+n}a\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-n;\frac{5}{4};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}}{d}$$

Mathematica [A]
time = 8.82, size = 170, normalized size = 1.81

$$-\frac{i2^{\frac{3}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}{}_2F_1\left(\frac{1}{4}+n,\frac{1}{2}+n;\frac{5}{4}+n;-e^{2i(c+dx)}\right)\sec^{-\frac{1}{2}-n}(c+dx)\sqrt{e\sec(c+dx)}\,(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(1+4n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[1/4 + n, 1
/2 + n, 5/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - n)*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \sqrt{e\sec(dx+c)}\,(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
[Out] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)^n*sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral(sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(1
/2*I*d*x + 1/2*I*c + 1/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\sec\left(c+dx\right)}\left(ia(\tan\left(c+dx\right)-i)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(1/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{i2^{\frac{3}{4}+n}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-n;\frac{3}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] -I*2^(3/4+n)*hypergeom([-1/4, 5/4-n],[3/4],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d
*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-n;\frac{3}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x]
)/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*Sec
[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \frac{\left(\sqrt[4]{a-ia\tan(c+dx)}\,\sqrt[4]{a+ia\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{1}{4}+n}}{\sqrt[4]{a-ia\tan(c+dx)}}\,dx}{\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(a^2\sqrt[4]{a-ia\tan(c+dx)}\,\sqrt[4]{a+ia\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{5}{4}+n}}{(a-iax)^{5/4}}\,dx, x, t\right.}{d\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(2^{-\frac{5}{4}+n}a\sqrt[4]{a-ia\tan(c+dx)}\,(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\operatorname{Su}}{d\sqrt{e\sec(c+dx)}}$$

$$= -\frac{i2^{\frac{3}{4}+n}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-n;\frac{3}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+i}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A]
time = 10.09, size = 143, normalized size = 1.57

$$-\frac{i2^{\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+n}{}_2F_1\left(-\frac{1}{2}+n,-\frac{1}{4}+n;\frac{3}{4}+n;-e^{2i(c+dx)}\right)\sec^{\frac{1}{2}-n}(c+dx)(a+ia\tan(c+dx))^n}{d(-1+4n)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
[Out] ((-I)*2^(1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^(-1/2 + n)*Hypergeometric2F1[-1/2 + n, -1/4 + n, 3/
4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/(d*(-1 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^n}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)^n/sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c - 1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^n}{\sqrt{e\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n/sqrt(e*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(-1/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2), x)
```

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{i2^{\frac{1}{4}+n}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-n;\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -1/3*I*2^(1/4+n)*hypergeom([-3/4, 7/4-n],[1/4],1/2-1/2*I*tan(d*x+c))*(1+I*t
an(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-n;\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c +
d*x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec
[c + d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\left((a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{3}{4}+n}}{(a-ia\tan(c+dx))^{3/4}}\,dx}{(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{7}{4}+n}}{(a-iax)^{7/4}}\,dx, x\right.}{d(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+n}a(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{3}{4}-n}\right)\operatorname{Su}}{d(e\sec(c+dx))^{3/2}}$$

$$= -\frac{i2^{\frac{1}{4}+n}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-n;\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+i}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A]
time = 10.84, size = 143, normalized size = 1.54

$$-\frac{i2^{-\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{3}{2}+n}\,{}_2F_1\left(-\frac{3}{2}+n,-\frac{3}{4}+n;\frac{1}{4}+n;-e^{2i(c+dx)}\right)\sec^{\frac{3}{2}-n}(c+dx)(a+ia\tan(c+dx))^n}{d(-3+4n)(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
[Out] ((-I)*2^(-1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-3/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(-3/2 + n)*Hypergeometric2F1[-3/2 + n, -3/4 + n, 1
/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - n)*(a + I*a*Tan[c + d*x])
^n)/(d*(-3 + 4*n)*(e*Sec[c + d*x])^(3/2))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)^n/sec(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c -
 3/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(-3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \tan(c + d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2), x)
```

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{i2^{-\frac{1}{4}+n}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4}-n;-\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{1+n}}{5ad(e\sec(c+dx))^{5/2}}$$

```
[Out] -1/5*I*2^(-1/4+n)*hypergeom([-5/4, 9/4-n],[-1/4],1/2-1/2*I*tan(d*x+c))*(1+I
*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$-\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4}-n;-\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)}{5ad(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c
+ d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(
a*d*(e*Sec[c + d*x])^(5/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left((a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{5}{4}+n}}{(a-ia\tan(c+dx))^{5/4}}\,dx}{(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{9}{4}+n}}{(a-iax)^{9/4}}\,dx, x\right.}{d(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(2^{-\frac{9}{4}+n}(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\operatorname{S}}{d(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i2^{-\frac{1}{4}+n}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4}-n;-\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a}{5ad(e\sec(c+dx))^{5/2}}$$

Mathematica [A]
time = 11.15, size = 157, normalized size = 1.60

$$-\frac{i2^{-\frac{3}{2}+n}e^{-3i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}{}_2F_1\left(-\frac{5}{2}+n,-\frac{5}{4}+n;-\frac{1}{4}+n;-e^{2i(c+dx)}\right)\sec^{\frac{1}{2}-n}(c+dx)(a+ia\tan(c+dx))^n}{de^2(-5+4n)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
[Out] ((-I)*2^(-3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[-5/2 + n, -5/4 + n, -1/
4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/(d*e^2*E^((3*I)*(c + d*x))*(-5 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)^n/sec(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/8*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e
^(-5/2*I*d*x - 5/2*I*c - 5/2)/sqrt(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

[Out] integrate((I*a*tan(d*x + c) + a)^n*e^(-5/2)/sec(d*x + c)^(5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2),x)

[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2), x)

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=269

$$\frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)}+\frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)}-\frac{12i(e\sec(c+dx))}{a^2d(}$$

```
[Out] I*(e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n/d/(4-n)+4*I*(e*sec(d*x+c))^(-4
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-6*n+8)-12*I*(e*sec(d*x+c))^(-4-n)*(a+
I*a*tan(d*x+c))^(2+n)/a^2/d/(2-n)/(4-n)/n+24*I*(e*sec(d*x+c))^(-4-n)*(a+I*a
*tan(d*x+c))^(3+n)/a^3/d/(4-n)/n/(-n^2+4)-24*I*(e*sec(d*x+c))^(-4-n)*(a+I*a
*tan(d*x+c))^(4+n)/a^4/d/n/(n^4-20*n^2+64)
```

Rubi [A]
time = 0.29, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3585, 3569}

$$-\frac{24i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{a^4dn\,(n^4-20n^2+64)}+\frac{24i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{a^3d(4-n)n\,(4-n^2)}-\frac{12i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{a^2d(2-n)(4-n)n}+\frac{4i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{ad\,(n^2-6n+8)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + ((4*I)
*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(8 - 6*n +
n^2)) - ((12*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(
a^2*d*(2 - n)*(4 - n)*n) + ((24*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(4 - n)*n*(4 - n^2)) - ((24*I)*(e*Sec[c + d*x])^(-
4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a^4*d*n*(64 - 20*n^2 + n^4))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4\int(e\sec(c+ \cdots}{\cdots}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+d \cdots}{\cdots}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+d \cdots}{\cdots}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+d \cdots}{\cdots}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+d \cdots}{\cdots}$$

Mathematica [A]
time = 0.73, size = 165, normalized size = 0.61

$$-\frac{i(e\sec(c+dx))^{-n}\left(192-60n^2+3n^4+4n^2(-16+n^2)\cos(2(c+dx))+n^2(-4+n^2)\cos(4(c+dx))+128in\sin(2(c+dx))-8in^3\sin(2(c+dx))+16in\sin(4(c+dx))-4in^3\sin(4(c+dx))\right)(a+ia\tan(c+dx))^n}{8de^4(-4+n)(-2+n)n(2+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/8*I)*(192 - 60*n^2 + 3*n^4 + 4*n^2*(-16 + n^2)*Cos[2*(c + d*x)] + n^2*
(-4 + n^2)*Cos[4*(c + d*x)] + (128*I)*n*Sin[2*(c + d*x)] - (8*I)*n^3*Sin[2*
(c + d*x)] + (16*I)*n*Sin[4*(c + d*x)] - (4*I)*n^3*Sin[4*(c + d*x)])*(a + I
*a*Tan[c + d*x])^n)/(d*e^4*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(e*Sec[c + d
*x])^n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 5823, normalized size = 21.65

method	result	size
risch	Expression too large to display	5823

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.60, size = 430, normalized size = 1.60

$$\frac{\left((-ia^nn^4+4ia^nn^3+4ia^nn^2-16ia^nn)\cos((dx+c)(n+4))-4(ia^nn^4-2ia^nn^3-16ia^nn^2+32ia^nn)\cos((dx+c)(n+2))-4(ia^nn^4+2ia^nn^3-16ia^nn^2-32ia^nn)\cos((dx+c)(n-2))+(-ia^nn^4-4ia^nn^3+4ia^nn^2+16ia^nn)\cos((dx+c)(n-4))-6(ia^nn^4-20ia^nn^2+64ia^n)\cos((dx+c)n)+(a^nn^4-4a^nn^3-4a^nn^2+16a^nn)\sin((dx+c)(n+4))+4(a^nn^4-2a^nn^3-16a^nn^2+32a^nn)\sin((dx+c)(n+2))+4(a^nn^4+2a^nn^3-16a^nn^2-32a^nn)\sin((dx+c)(n-2))+(a^nn^4+4a^nn^3-4a^nn^2-16a^nn)\sin((dx+c)(n-4))+6(a^nn^4-20a^nn^2+64a^n)\sin((dx+c)n)\right)e^{(-n)}}{16(n^5e^4-20n^3e^4+64ne^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/16*((-I*a^n*n^4 + 4*I*a^n*n^3 + 4*I*a^n*n^2 - 16*I*a^n*n)*cos((d*x + c)*(
n + 4)) - 4*(I*a^n*n^4 - 2*I*a^n*n^3 - 16*I*a^n*n^2 + 32*I*a^n*n)*cos((d*x
+ c)*(n + 2)) - 4*(I*a^n*n^4 + 2*I*a^n*n^3 - 16*I*a^n*n^2 - 32*I*a^n*n)*cos
((d*x + c)*(n - 2)) + (-I*a^n*n^4 - 4*I*a^n*n^3 + 4*I*a^n*n^2 + 16*I*a^n*n)
*cos((d*x + c)*(n - 4)) - 6*(I*a^n*n^4 - 20*I*a^n*n^2 + 64*I*a^n)*cos((d*x
+ c)*n) + (a^n*n^4 - 4*a^n*n^3 - 4*a^n*n^2 + 16*a^n*n)*sin((d*x + c)*(n + 4
)) + 4*(a^n*n^4 - 2*a^n*n^3 - 16*a^n*n^2 + 32*a^n*n)*sin((d*x + c)*(n + 2))
 + 4*(a^n*n^4 + 2*a^n*n^3 - 16*a^n*n^2 - 32*a^n*n)*sin((d*x + c)*(n - 2)) +
 (a^n*n^4 + 4*a^n*n^3 - 4*a^n*n^2 - 16*a^n*n)*sin((d*x + c)*(n - 4)) + 6*(a
^n*n^4 - 20*a^n*n^2 + 64*a^n)*sin((d*x + c)*n))*e^(-n)/((n^5*e^4 - 20*n^3*e
^4 + 64*n*e^4)*d)
```

Fricas [A]
time = 0.37, size = 334, normalized size = 1.24

$$\frac{\left(-i\,n^4-4i\,n^3+4i\,n^2+\left(-i\,n^4+4i\,n^3+4i\,n^2-16i\,n\right)e^{(8i\,dx+8i\,c)}-4\left(i\,n^4-2i\,n^3-16i\,n^2+32i\,n\right)e^{(6i\,dx+6i\,c)}-6\left(i\,n^4-20i\,n^2+64i\right)e^{(4i\,dx+4i\,c)}-4\left(i\,n^4+2i\,n^3-16i\,n^2-32i\,n\right)e^{(2i\,dx+2i\,c)}+16i\,n\right)\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-4}e^{\left(i\,dnx+i\,cn+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)\right)}}{dn^5-20\,dn^3+64\,dn+\left(dn^5-20\,dn^3+64\,dn\right)e^{(8i\,dx+8i\,c)}+4\left(dn^5-20\,dn^3+64\,dn\right)e^{(6i\,dx+6i\,c)}+6\left(dn^5-20\,dn^3+64\,dn\right)e^{(4i\,dx+4i\,c)}+4\left(dn^5-20\,dn^3+64\,dn\right)e^{(2i\,dx+2i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^4 - 4*I*n^3 + 4*I*n^2 + (-I*n^4 + 4*I*n^3 + 4*I*n^2 - 16*I*n)*e^(8*I*
d*x + 8*I*c) - 4*(I*n^4 - 2*I*n^3 - 16*I*n^2 + 32*I*n)*e^(6*I*d*x + 6*I*c)
- 6*(I*n^4 - 20*I*n^2 + 64*I)*e^(4*I*d*x + 4*I*c) - 4*(I*n^4 + 2*I*n^3 - 16
*I*n^2 - 32*I*n)*e^(2*I*d*x + 2*I*c) + 16*I*n)*(2*e^(I*d*x + I*c + 1)/(e^(2
*I*d*x + 2*I*c) + 1))^(-n - 4)*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log
(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*n^5 - 20*d*n^3 + 64*d
*n + (d*n^5 - 20*d*n^3 + 64*d*n)*e^(8*I*d*x + 8*I*c) + 4*(d*n^5 - 20*d*n^3
+ 64*d*n)*e^(6*I*d*x + 6*I*c) + 6*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(4*I*d*x +
4*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(c+dx\right)\right)^{-n-4}\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-4-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(-n - 4)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 9.90, size = 511, normalized size = 1.90

$$\frac{\left(2\sin\left(2c+2dx\right)^2+\sin\left(4c+4dx\right)\,1\mathrm{i}-1\right)\left(\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-n^3-4n^2+4n+16\right)}{d\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}+\frac{4\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-2\sin(c+d\,x)^2+\sin(2c+2d\,x)\,1\mathrm{i}+1\right)\left(-n^3-2n^2+16n+32\right)}{d\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}+\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-2\sin(4c+4d\,x)^2+\sin(8c+8d\,x)\,1\mathrm{i}+1\right)\left(-n^3+4n^2+4n-16\right)}{d\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}+\frac{4\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-2\sin(3c+3d\,x)^2+\sin(6c+6d\,x)\,1\mathrm{i}+1\right)\left(-n^3+2n^2+16n-32\right)}{d\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}-\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-2\sin(2c+2d\,x)^2+\sin(4c+4d\,x)\,1\mathrm{i}+1\right)\left(6n^4-120n^2+384\right)}{d\,n\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}\right)}{16\left(-\frac{e}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^{n+4}\left(\sin\left(c+d\,x\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 4),x)
```

```
[Out] ((sin(4*c + 4*d*x)*1i + 2*sin(2*c + 2*d*x)^2 - 1)*(((a - (a*sin(c + d*x)*1i
)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(4*n - 4*n^2 - n^3 + 16))/(d*(n^4*1i - n^
2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n
*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1)*(16*n - 2*n^2 - n^3 + 32))/(d
*(n^4*1i - n^2*20i + 64i)) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2
)^2 - 1))^n*(sin(8*c + 8*d*x)*1i - 2*sin(4*c + 4*d*x)^2 + 1)*(4*n + 4*n^2 -
 n^3 - 16))/(d*(n^4*1i - n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*s
in(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*x)^2 + 1
)*(16*n + 2*n^2 - n^3 - 32))/(d*(n^4*1i - n^2*20i + 64i)) - ((a - (a*sin(c
+ d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i - 2*sin(2*c
 + 2*d*x)^2 + 1)*(6*n^4 - 120*n^2 + 384))/(d*n*(n^4*1i - n^2*20i + 64i))))/
(16*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 4)*(sin(c + d*x)^2 - 1)^2)
```

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=205

$$\frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)}+\frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(3-4n+n^2)}-\frac{6i(e\sec(c+dx))^{-}}{a^2d(3}$$

```
[Out] I*(e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n/d/(3-n)+3*I*(e*sec(d*x+c))^(-3
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-4*n+3)-6*I*(e*sec(d*x+c))^(-3-n)*(a+I
*a*tan(d*x+c))^(2+n)/a^2/d/(3-n)/(-n^2+1)+6*I*(e*sec(d*x+c))^(-3-n)*(a+I*a*
tan(d*x+c))^(3+n)/a^3/d/(n^4-10*n^2+9)
```

Rubi [A]

time = 0.23, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3585, 3569}

$$\frac{6i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{a^3d\,(n^4-10n^2+9)}-\frac{6i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{a^2d(3-n)\,(1-n^2)}+\frac{3i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{ad\,(n^2-4n+3)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + ((3*I)
*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(3 - 4*n +
n^2)) - ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a
^2*d*(3 - n)*(1 - n^2)) + ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c +
 d*x])^(3 + n))/(a^3*d*(9 - 10*n^2 + n^4))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3\int(e\sec(c+}{}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+d}{}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+d}{}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+d}{}$$

Mathematica [A]
time = 0.75, size = 119, normalized size = 0.58

$$\frac{(e\sec(c+dx))^{-n}\left(-3in(-9+n^2)\cos(c+dx)-in(-1+n^2)\cos(3(c+dx))-6(-5+n^2+(-1+n^2)\cos(2(c+dx)))\sin(c+dx)\right)(a+ia\tan(c+dx))^n}{4de^3(-3+n)(-1+n)(1+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (((-3*I)*n*(-9 + n^2)*Cos[c + d*x] - I*n*(-1 + n^2)*Cos[3*(c + d*x)] - 6*(-
5 + n^2 + (-1 + n^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*(a + I*a*Tan[c + d*x])
^n)/(4*d*e^3*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.28, size = 4994, normalized size = 24.36

method	result	size
risch	Expression too large to display	4994

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*exp(I*(d*x+c))^n*e^(-n)*a^n/e^3*exp(-1/2*I*(6*c-n*Pi*csgn(I/(exp(2*I*(d
*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1
))+6*d*x-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+3*Pi*csgn(I*e/(
exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x
+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I/(exp(
2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*
exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I*e
/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*
```

csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))
+1)*exp(I*(d*x+c)))^3-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*
Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+Pi*csgn(I*a/(exp(2*I*(d*
x+c))+1)*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^3*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n+3*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*e
xp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+Pi*cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))*csgn(I*a)*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n-2*Pi*csgn(I*exp(2*I*
(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp
(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-3*Pi*csg
n(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp
(2*I*(d*x+c))+1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn
(I*a)*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n))/
(-3*I*d+I*n*d)+1/8*exp(I*(d*x+c))^n*e^(-n)*a^n/e^3*exp(1/2*I*(6*c+n*Pi*csgn
(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+6*d*x+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3
*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e
/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e/(exp(2*I*(d
*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*P
i*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
2-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2
+3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*e/(ex
p(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*
x+c))+1))^3+3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-Pi*csgn(I*
a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)
*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))
+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n-3*Pi*csgn(I*exp(I*(d*x
+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*exp(2*I*(d*x+
c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I
*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n+2*Pi*c
sgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+n*Pi*csgn(I*e/(exp(2*I*(
d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))+3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))^2*csgn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+
1)*exp(2*I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))
^2*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*

```
x+c))+1))*n))/(I*n*d+3*I*d)+3/8*exp(I*(d*x+c))^n*e^(-n)*a^n/e^3*exp(-1/2*I*
(2*c-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))+2*d*x-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^3+3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)
+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I
*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*
(d*x+c))+1))^2-3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+3*Pi*csgn(I/(exp(2*I*(d*x+c))+
1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))...
```

Maxima [A]
time = 0.60, size = 341, normalized size = 1.66

$$\frac{\left(\left(-i\,a^n n^3+3i\,a^n n^2+i\,a^n n-3i\,a^n\right)\cos\left((dx+c)(n+3)\right)-3\left(i\,a^n n^3-i\,a^n n^2-9i\,a^n n+9i\,a^n\right)\cos\left((dx+c)(n+1)\right)-3\left(i\,a^n n^3+i\,a^n n^2-9i\,a^n n-9i\,a^n\right)\cos\left((dx+c)(n-1)\right)+\left(-i\,a^n n^3-3i\,a^n n^2+i\,a^n n+3i\,a^n\right)\cos\left((dx+c)(n-3)\right)+\left(a^n n^3-3\,a^n n^2-a^n n+3\,a^n\right)\sin\left((dx+c)(n+3)\right)+3\left(a^n n^3-a^n n^2-9\,a^n n+9\,a^n\right)\sin\left((dx+c)(n+1)\right)+3\left(a^n n^3+a^n n^2-9\,a^n n-9\,a^n\right)\sin\left((dx+c)(n-1)\right)+\left(a^n n^3+3\,a^n n^2-a^n n-3\,a^n\right)\sin\left((dx+c)(n-3)\right)\right)e^{(-n)}}{8\left(n^4e^3-10\,n^2e^3+9\,e^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/8*((-I*a^n*n^3 + 3*I*a^n*n^2 + I*a^n*n - 3*I*a^n)*cos((d*x + c)*(n + 3))
- 3*(I*a^n*n^3 - I*a^n*n^2 - 9*I*a^n*n + 9*I*a^n)*cos((d*x + c)*(n + 1)) -
3*(I*a^n*n^3 + I*a^n*n^2 - 9*I*a^n*n - 9*I*a^n)*cos((d*x + c)*(n - 1)) + (-
I*a^n*n^3 - 3*I*a^n*n^2 + I*a^n*n + 3*I*a^n)*cos((d*x + c)*(n - 3)) + (a^n*
n^3 - 3*a^n*n^2 - a^n*n + 3*a^n)*sin((d*x + c)*(n + 3)) + 3*(a^n*n^3 - a^n*
n^2 - 9*a^n*n + 9*a^n)*sin((d*x + c)*(n + 1)) + 3*(a^n*n^3 + a^n*n^2 - 9*a^
n*n - 9*a^n)*sin((d*x + c)*(n - 1)) + (a^n*n^3 + 3*a^n*n^2 - a^n*n - 3*a^n)
*sin((d*x + c)*(n - 3)))*e^(-n)/((n^4*e^3 - 10*n^2*e^3 + 9*e^3)*d)
```

Fricas [A]
time = 0.39, size = 264, normalized size = 1.29

$$\frac{\left(-i\,n^3-3i\,n^2+\left(-i\,n^3+3i\,n^2+i\,n-3i\right)e^{(6i\,dx+6i\,c)}-3\left(i\,n^3-i\,n^2-9i\,n+9i\right)e^{(4i\,dx+4i\,c)}-3\left(i\,n^3+i\,n^2-9i\,n-9i\right)e^{(2i\,dx+2i\,c)}+i\,n+3i\right)\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-3}e^{\left(i\,dnx+i\,cn+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)\right)}}{dn^4-10\,dn^2+\left(dn^4-10\,dn^2+9\,d\right)e^{(6i\,dx+6i\,c)}+3\left(dn^4-10\,dn^2+9\,d\right)e^{(4i\,dx+4i\,c)}+3\left(dn^4-10\,dn^2+9\,d\right)e^{(2i\,dx+2i\,c)}+9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^3 - 3*I*n^2 + (-I*n^3 + 3*I*n^2 + I*n - 3*I)*e^(6*I*d*x + 6*I*c) - 3*
(I*n^3 - I*n^2 - 9*I*n + 9*I)*e^(4*I*d*x + 4*I*c) - 3*(I*n^3 + I*n^2 - 9*I*
n - 9*I)*e^(2*I*d*x + 2*I*c) + I*n + 3*I)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*
x + 2*I*c) + 1))^(-n - 3)*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^
(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*n^4 - 10*d*n^2 + (d*n^4 -
10*d*n^2 + 9*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(4*I*d*x
 + 4*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(2*I*d*x + 2*I*c) + 9*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec (c+dx))^{-n-3} (ia(\tan (c+dx)-i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-3-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(-n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 9.24, size = 425, normalized size = 2.07

$$-\frac{\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\left(2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}-1\right)\left(\frac{{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}^n\left(-n^3-3\,n^2+n+3\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}+\frac{{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}^n\left(-2\sin(3\,c+3\,d\,x)^2+\sin(6\,c+6\,d\,x)\,1\mathrm{i}+1\right)\left(-n^3+3\,n^2+n-3\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}+\frac{{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}^n\left(-2\sin(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}+1\right)\left(-3\,n^3-3\,n^2+27\,n+27\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}+\frac{{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}^n\left(-2\sin(2\,c+2\,d\,x)^2+\sin(4\,c+4\,d\,x)\,1\mathrm{i}+1\right)\left(-3\,n^3+3\,n^2+27\,n-27\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}\right)}{8\,{\left(-\frac{e}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)}^{n+3}\,{\left(\sin\left(c+d\,x\right)^2-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 3),x)
```

```
[Out] -((2*sin(c/2 + (d*x)/2)^2 - 1)*(sin(3*c + 3*d*x)*1i + 2*sin((3*c)/2 + (3*d*
x)/2)^2 - 1)*(((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(n
- 3*n^2 - n^3 + 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c + d*x)*1i)
/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*x)^
2 + 1)*(n + 3*n^2 - n^3 - 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c
+ d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(2*c + 2*d*x)*1i - 2*sin(c +
 d*x)^2 + 1)*(27*n - 3*n^2 - 3*n^3 + 27))/(d*(n^4*1i - n^2*10i + 9i)) + ((a
 - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i
 - 2*sin(2*c + 2*d*x)^2 + 1)*(27*n + 3*n^2 - 3*n^3 - 27))/(d*(n^4*1i - n^2*
10i + 9i))))/(8*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 3)*(sin(c + d*x)^2 -
 1)^2)
```

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=148

$$\frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}-\frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n}+\frac{2i(e\sec(c+dx))}{a}$$

```
[Out] I*(e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n/d/(2-n)-2*I*(e*sec(d*x+c))^(-2
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(2-n)/n+2*I*(e*sec(d*x+c))^(-2-n)*(a+I*a*t
an(d*x+c))^(2+n)/a^2/d/n/(-n^2+4)
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3585, 3569}

$$\frac{2i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{a^2dn\,(4-n^2)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}-\frac{2i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{ad(2-n)n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) - ((2*I)
*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(2 - n)*n)
+ ((2*I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*n
*(4 - n^2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} + \frac{2\int(e\sec(c+d}{}$$
$$= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} - \frac{2i(e\sec(c+dx}{}$$
$$= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} - \frac{2i(e\sec(c+dx}{}$$

Mathematica [A]
time = 0.30, size = 82, normalized size = 0.55

$$-\frac{i(e\sec(c+dx))^{-n}\left(-4+n^2+n^2\cos(2(c+dx))-2in\sin(2(c+dx))\right)(a+ia\tan(c+dx))^n}{2de^2(-2+n)n(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/2*I)*(-4 + n^2 + n^2*Cos[2*(c + d*x)] - (2*I)*n*Sin[2*(c + d*x)])*(a +
 I*a*Tan[c + d*x])^n)/(d*e^2*(-2 + n)*n*(2 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.02, size = 3327, normalized size = 22.48

method	result	size
risch	Expression too large to display	3327

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(I*(d*x+c))^n*e^(-n)*a^n/e^2*exp(-1/2*I*(4*c-n*Pi*csgn(I/(exp(2*I*(d
*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1
))+4*d*x-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+2*Pi*csgn(I*e/(
exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x
+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I/(exp(
2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*
exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-2*Pi*csgn(I*e
/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2+2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))
+1)*exp(I*(d*x+c)))^3-2*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-2*
```

Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+Pi*csgn(I*a/(exp(2*I*(d*
x+c))+1)*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^3*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n+2*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*e
xp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+Pi*cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))*csgn(I*a)*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n-2*Pi*csgn(I*exp(2*I*
(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp
(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-2*Pi*csg
n(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp
(2*I*(d*x+c))+1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn
(I*a)*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n))/
(-2*I*d+I*n*d)+1/4*exp(I*(d*x+c))^n*e^(-n)*a^n/e^2*exp(1/2*I*(4*c+n*Pi*csgn
(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+4*d*x+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-2
*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e
/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e/(exp(2*I*(d
*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*P
i*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
2-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2
+2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))-2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*e/(ex
p(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+2*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*
x+c))+1))^3+2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-Pi*csgn(I*
a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)
*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))
+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n-2*Pi*csgn(I*exp(I*(d*x
+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*exp(2*I*(d*x+
c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I
*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*e
xp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n+2*Pi*c
sgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+n*Pi*csgn(I*e/(exp(2*I*(
d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))+2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))^2*csgn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+
1)*exp(2*I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))
^2*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*
x+c))+1))*n))/(2*I*d+I*n*d)-1/2*I/d/n*a^n*exp(I*(d*x+c))^n/(e^n)/e^2*exp(1/
2*I*n*Pi*(csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*ex

```
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x
+c)))^2*csgn(I*e)-csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2
*I*(d*x+c))+1))^2+csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(
I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp
(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))+csgn(I/(exp
(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))+csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c))
)^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))...
```

Maxima [A]
time = 0.59, size = 173, normalized size = 1.17

$$\frac{\left(\left(-i\,a^{n}n^{2}+2i\,a^{n}n\right)\cos\left((dx+c)(n+2)\right)+\left(-i\,a^{n}n^{2}-2i\,a^{n}n\right)\cos\left((dx+c)(n-2)\right)-2\left(i\,a^{n}n^{2}-4i\,a^{n}\right)\cos\left((dx+c)n\right)+\left(a^{n}n^{2}-2\,a^{n}n\right)\sin\left((dx+c)(n+2)\right)+\left(a^{n}n^{2}+2\,a^{n}n\right)\sin\left((dx+c)(n-2)\right)+2\left(a^{n}n^{2}-4\,a^{n}\right)\sin\left((dx+c)n\right)\right)e^{(-n)}}{4\left(n^{3}e^{2}-4\,ne^{2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/4*((-I*a^n*n^2 + 2*I*a^n*n)*cos((d*x + c)*(n + 2)) + (-I*a^n*n^2 - 2*I*a^
n*n)*cos((d*x + c)*(n - 2)) - 2*(I*a^n*n^2 - 4*I*a^n)*cos((d*x + c)*n) + (a
^n*n^2 - 2*a^n*n)*sin((d*x + c)*(n + 2)) + (a^n*n^2 + 2*a^n*n)*sin((d*x + c
)*(n - 2)) + 2*(a^n*n^2 - 4*a^n)*sin((d*x + c)*n))*e^(-n)/((n^3*e^2 - 4*n*e
^2)*d)
```

Fricas [A]
time = 0.39, size = 177, normalized size = 1.20

$$\frac{\left(-i\,n^{2}+\left(-i\,n^{2}+2i\,n\right)e^{(4i\,dx+4i\,c)}-2\left(i\,n^{2}-4i\right)e^{(2i\,dx+2i\,c)}-2i\,n\right)\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-2}e^{\left(i\,dnx+i\,cn+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)\right)}}{dn^{3}-4\,dn+\left(dn^{3}-4\,dn\right)e^{(4i\,dx+4i\,c)}+2\left(dn^{3}-4\,dn\right)e^{(2i\,dx+2i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^2 + (-I*n^2 + 2*I*n)*e^(4*I*d*x + 4*I*c) - 2*(I*n^2 - 4*I)*e^(2*I*d*x
 + 2*I*c) - 2*I*n)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-n -
2)*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2
*I*d*x + 2*I*c) + 1)))/(d*n^3 - 4*d*n + (d*n^3 - 4*d*n)*e^(4*I*d*x + 4*I*c)
 + 2*(d*n^3 - 4*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(c+dx\right)\right)^{-n-2}\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-2-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 9.30, size = 227, normalized size = 1.53

$$\frac{\left(\cos\left(2\,c+2\,d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(\frac{{\left(a+\frac{a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}}{\cos\left(c+d\,x\right)}\right)}^n\,\left(n+2\right)}{d\,\left(n^2\,1{}\mathrm{i}-4{}\mathrm{i}\right)}+\frac{\left(\cos\left(4\,c+4\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}\right)\,{\left(a+\frac{a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}}{\cos\left(c+d\,x\right)}\right)}^n\,\left(n-2\right)}{d\,\left(n^2\,1{}\mathrm{i}-4{}\mathrm{i}\right)}+\frac{\left(\cos\left(2\,c+2\,d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(2\,n^2-8\right)\,{\left(a+\frac{a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}}{\cos\left(c+d\,x\right)}\right)}^n}{d\,n\,\left(n^2\,1{}\mathrm{i}-4{}\mathrm{i}\right)}\right)}{4\,\left(\frac{\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{1}{2}\right)\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 2),x)
[Out] ((cos(2*c + 2*d*x) - sin(2*c + 2*d*x)*1i)*(((a + (a*sin(c + d*x)*1i)/cos(c
+ d*x))^n*(n + 2))/(d*(n^2*1i - 4i)) + ((cos(4*c + 4*d*x) + sin(4*c + 4*d*x
)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n - 2))/(d*(n^2*1i - 4i)) +
 ((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(2*n^2 - 8)*(a + (a*sin(c + d*x)
*1i)/cos(c + d*x))^n)/(d*n*(n^2*1i - 4i))))/(4*(cos(2*c + 2*d*x)/2 + 1/2)*(
e/cos(c + d*x))^(n + 2))
```

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)}-\frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(1-n^2)}$$

```
[Out] I*(e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n/d/(1-n)-I*(e*sec(d*x+c))^(-1-n
)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(-n^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3585, 3569}

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad\,(1-n^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e*
Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n^2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx=\frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)}+\frac{\int (e\sec(c+dx}{}$$

$$=\frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)}-\frac{i(e\sec(c+dx)}{}$$

Mathematica [A]
time = 0.25, size = 58, normalized size = 0.62

$$-\frac{i(e\sec(c+dx))^{-1-n}(n-i\tan(c+dx))(a+ia\tan(c+dx))^n}{d(-1+n)(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((-I)*(e*Sec[c + d*x])^(-1 - n)*(n - I*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + n)*(1 + n))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.06, size = 2490, normalized size = 26.49

method	result	size
risch	Expression too large to display	2490

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)

[Out] 1/2*exp(I*(d*x+c))^n*e^(-n)*a^n/e*exp(1/2*I*(2*c+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+2*d*x+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*n-Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+P

i*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n+Pi*csgn(I*e
xp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n-Pi*csgn(
I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+Pi*csgn(I/(exp(2*I*(d*x+c))+
1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n))/(I*d+I*n*d)+1/2*exp
(I*(d*x+c))^n*e^(-n)*a^n/e*exp(-1/2*I*(2*c-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1)
)*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+2*d*x-
n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+Pi*csgn(I*e/(exp(2*I*(d*
x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp
(I*(d*x+c)))^2*csgn(I*e)+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))
^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I/(exp(2*I*(d*x+c)
)+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+
c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*e/(exp(2*I*(d*
x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))+Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^2+Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)
))^3-Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-Pi*csgn(I*e/(exp(2*I*
(d*x+c))+1)*exp(I*(d*x+c)))^3+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
+c)))^3*n+Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I*ex
p(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2
I(d*x+c))+1))*n+Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*
(d*x+c))+1))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I/(exp(2*I*(d*x+c))+
1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I
*a)*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d
*x+c))+1)*exp(2*I*(d*x+c)))^2*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(
I*(d*x+c)))*n-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I/(exp(2*I*(d*x+c))+1))
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn
(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-Pi*csgn(I*exp(2*I
*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n+Pi*csgn(I*exp(
2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp
(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n))/(-I*d+I*n*d)

Maxima [A]
time = 0.57, size = 111, normalized size = 1.18

$$\frac{\left(\left(-i\,a^{n}n+i\,a^{n}\right)\cos\left(\left(dx+c\right)\left(n+1\right)\right)+\left(-i\,a^{n}n-i\,a^{n}\right)\cos\left(\left(dx+c\right)\left(n-1\right)\right)+\left(a^{n}n-a^{n}\right)\sin\left(\left(dx+c\right)\left(n+1\right)\right)+\left(a^{n}n+a^{n}\right)\sin\left(\left(dx+c\right)\left(n-1\right)\right)\right)e^{\left(-n\right)}}{2\left(n^{2}e-e\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")

[Out] 1/2*((-I*a^n*n + I*a^n)*cos((d*x + c)*(n + 1)) + (-I*a^n*n - I*a^n)*cos((d*
x + c)*(n - 1)) + (a^n*n - a^n)*sin((d*x + c)*(n + 1)) + (a^n*n + a^n)*sin(
(d*x + c)*(n - 1)))*e^(-n)/((n^2*e - e)*d)

Fricas [A]
time = 0.40, size = 128, normalized size = 1.36

$$\frac{\left((-i\,n+i)e^{(2i\,dx+2i\,c)}-i\,n-i\right)\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-1}e^{\left(i\,dnx+i\,cn+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)\right)}}{dn^2+(dn^2-d)e^{(2i\,dx+2i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((-I*n + I)*e^(2*I*d*x + 2*I*c) - I*n - I)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d
*x + 2*I*c) + 1))^(-n - 1)*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e
^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*n^2 + (d*n^2 - d)*e^(2*I*
d*x + 2*I*c) - d)
```

Sympy [A]
time = 16.38, size = 296, normalized size = 3.15

$$\frac{\begin{cases}\frac{x(e\sec(c))^{-n}(ia\tan(c)+a)^n}{\sec(c)} & \text{for } d=0\\ \frac{dex\tan(c+dx)}{2ad\tan(c+dx)-2iad}-\frac{idex}{2ad\tan(c+dx)-2iad}+\frac{e}{2ad\tan(c+dx)-2iad} & \text{for } n=-1\\ \frac{ax\tan^2(c+dx)}{2e\sec^2(c+dx)}+\frac{ax}{2e\sec^2(c+dx)}+\frac{a\tan(c+dx)}{2de\sec^2(c+dx)}-\frac{ia}{2de\sec^2(c+dx)} & \text{for } n=1\\ -\frac{in(ia\tan(c+dx)+a)^n}{dn^2(e\sec(c+dx))^n\sec(c+dx)-d(e\sec(c+dx))^n\sec(c+dx)}-\frac{(ia\tan(c+dx)+a)^n\tan(c+dx)}{dn^2(e\sec(c+dx))^n\sec(c+dx)-d(e\sec(c+dx))^n\sec(c+dx)} & \text{otherwise}\end{cases}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Piecewise((x*(I*a*tan(c) + a)**n/((e*sec(c))**n*sec(c)), Eq(d, 0)), (d*e*x*
tan(c + d*x)/(2*a*d*tan(c + d*x) - 2*I*a*d) - I*d*e*x/(2*a*d*tan(c + d*x) -
 2*I*a*d) + e/(2*a*d*tan(c + d*x) - 2*I*a*d), Eq(n, -1)), (a*x*tan(c + d*x)
**2/(2*e*sec(c + d*x)**2) + a*x/(2*e*sec(c + d*x)**2) + a*tan(c + d*x)/(2*d
*e*sec(c + d*x)**2) - I*a/(2*d*e*sec(c + d*x)**2), Eq(n, 1)), (-I*n*(I*a*ta
n(c + d*x) + a)**n/(d*n**2*(e*sec(c + d*x))**n*sec(c + d*x) - d*(e*sec(c +
d*x))**n*sec(c + d*x)) - (I*a*tan(c + d*x) + a)**n*tan(c + d*x)/(d*n**2*(e*
sec(c + d*x))**n*sec(c + d*x) - d*(e*sec(c + d*x))**n*sec(c + d*x)), True))
/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 1.89, size = 121, normalized size = 1.29

$$-\frac{\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)^n\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+n\,\cos\left(c+d\,x\right)\,3{}\mathrm{i}+n\,\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)}{2\,d\,e\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\left(n^2-1\right)\,{\left(\frac{e}{\cos(c+d\,x)}\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 1),x)
```

```
[Out] -(((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))
^n*(sin(c + d*x) + sin(3*c + 3*d*x) + n*cos(c + d*x)*3i + n*cos(3*c + 3*d*x
)*1i))/(2*d*e*(cos(2*c + 2*d*x) + 1)*(n^2 - 1)*(e/cos(c + d*x))^n)
```

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=37

$$-\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

```
[Out] -I*(a+I*a*tan(d*x+c))^n/d/n/((e*sec(d*x+c))^n)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{3569\}$

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

Mathematica [A]
time = 0.06, size = 37, normalized size = 1.00

$$-\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.61, size = 874, normalized size = 23.62

method	result	size
risch	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -I/n/d*exp(1/2*n*(-I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))-I*Pi*csgn(I/(exp(2*I*(d*x+
c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*cs
gn(I*a)+I*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+I*Pi*csgn(I*a/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)-I*Pi*csgn(I/(exp(2*I*(d*
x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-I*Pi*csgn(I*e/(exp(
2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1
))-I*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2*csgn(I*e)-I*Pi*csgn
(I*exp(2*I*(d*x+c)))^3-I*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2+I*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-I*
Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3+I*Pi*csgn(I/(exp(2*I*(d*
x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)+I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^2-I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
+c)))^3+I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(
d*x+c)))^2-I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2+I*Pi*csgn
(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))+I*Pi*csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*c
sgn(I/(exp(2*I*(d*x+c))+1))+2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*
(d*x+c)))-2*ln(e)+2*ln(a)+2*ln(exp(I*(d*x+c)))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(34) = 68$.
time = 0.52, size = 84, normalized size = 2.27

$$-\frac{i\,a^n e^{\left(n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)-n\log\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)-n\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="maxima")
```

```
[Out] -I*a^n*e^(n*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1) - n*log(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - n)
/(d*n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(34) = 68$.
time = 0.37, size = 83, normalized size = 2.24

$$-\frac{ie^{\left(idnx+icn+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2e^{(idx+ic+1)}}{e^{(2idx+2ic)}+1}\right)\right)}}{dn\left(\frac{2e^{(idx+ic+1)}}{e^{(2idx+2ic)}+1}\right)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="fricas")
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2
*I*d*x + 2*I*c) + 1)))/(d*n*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1
))^n)
```

Sympy [A]
time = 11.01, size = 51, normalized size = 1.38

$$\begin{cases} x & \text{for}\ n=0 \wedge (d=0 \vee n=0) \\ x(e\sec(c))^{-n}(ia\tan(c)+a)^{n} & \text{for}\ d=0 \\ -\frac{i(e\sec(c+dx))^{-n}(ia\tan(c+dx)+a)^{n}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**n),x)
[Out] Piecewise((x, Eq(n, 0) & (Eq(d, 0) | Eq(n, 0))), (x*(I*a*tan(c) + a)**n/(e*
sec(c))**n, Eq(d, 0)), (-I*(I*a*tan(c + d*x) + a)**n/(d*n*(e*sec(c + d*x))*
*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{n}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n, x)
```

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=118

$$\frac{i2^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan(c+dx))^{\frac{1}{2}(-1-n)}(a+ia\tan(c+dx))^n}{d(1-n)}$$

```
[Out] I*2^(1/2+1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],1/2-1/2*I*tan(
d*x+c))*(e*sec(d*x+c))^(1-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(d*x+c
))^n/d/(1-n)
```

Rubi [A]
time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{i2^{\frac{n+1}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^((1 + n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 + I*Tan[c + d*x])^((-1 - n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan}{}$$

$$= \frac{\Big(2^{-\frac{1}{2}+\frac{n}{2}}a(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+}{}$$

$$= \frac{i2^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx}{d(1-n}$$

Mathematica [A]
time = 5.14, size = 102, normalized size = 0.86

$$-\frac{2e\cos(dx)\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};-\cos(2(c+dx))+i\sin(2(c+dx))\right)(e\sec(c+dx))^{-n}(\cos(c)-i\sin(c))(i+\tan(dx))(a+ia\tan(c+dx))^n}{d(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
[Out] (-2*e*Cos[d*x]*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, -Cos[2*(c + d*x)]
 + I*Sin[2*(c + d*x)]]*(Cos[c] - I*Sin[c])*(I + Tan[d*x])*(a + I*a*Tan[c +
d*x])^n)/(d*(-1 + n)*(e*Sec[c + d*x])^n)
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{1-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -2*(a^n*cos(c*n + (d*n + d)*x + c)*e + I*a^n*e*sin(c*n + (d*n + d)*x + c) -
 2*((I*a^n*d*n - I*a^n*d)*cos(2*d*x + 2*c)*e^n - (a^n*d*n - a^n*d)*e^n*sin(
2*d*x + 2*c) + (I*a^n*d*n - I*a^n*d)*e^n)*integrate(((cos(4*d*x + 4*c)*e +
2*cos(2*d*x + 2*c)*e + e)*cos(c*n + (d*n + d)*x + c) + (e*sin(4*d*x + 4*c)
+ 2*e*sin(2*d*x + 2*c))*sin(c*n + (d*n + d)*x + c))/((n - 1)*cos(4*d*x + 4*
c)^2*e^n + 4*(n - 1)*cos(2*d*x + 2*c)^2*e^n + (n - 1)*e^n*sin(4*d*x + 4*c)^
2 + 4*(n - 1)*e^n*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(n - 1)*e^n*sin(2*d
*x + 2*c)^2 + 4*(n - 1)*cos(2*d*x + 2*c)*e^n + 2*(2*(n - 1)*cos(2*d*x + 2*c
)*e^n + (n - 1)*e^n)*cos(4*d*x + 4*c) + (n - 1)*e^n), x) + 2*((a^n*d*n - a^
n*d)*cos(2*d*x + 2*c)*e^n - (-I*a^n*d*n + I*a^n*d)*e^n*sin(2*d*x + 2*c) + (
a^n*d*n - a^n*d)*e^n)*integrate(-((e*sin(4*d*x + 4*c) + 2*e*sin(2*d*x + 2*c
))*cos(c*n + (d*n + d)*x + c) - (cos(4*d*x + 4*c)*e + 2*cos(2*d*x + 2*c)*e
+ e)*sin(c*n + (d*n + d)*x + c))/((n - 1)*cos(4*d*x + 4*c)^2*e^n + 4*(n - 1
)*cos(2*d*x + 2*c)^2*e^n + (n - 1)*e^n*sin(4*d*x + 4*c)^2 + 4*(n - 1)*e^n*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(n - 1)*e^n*sin(2*d*x + 2*c)^2 + 4*(n
- 1)*cos(2*d*x + 2*c)*e^n + 2*(2*(n - 1)*cos(2*d*x + 2*c)*e^n + (n - 1)*e^n
)*cos(4*d*x + 4*c) + (n - 1)*e^n), x))/((-I*d*n + I*d)*cos(2*d*x + 2*c)*e^n
 + (d*n - d)*e^n*sin(2*d*x + 2*c) + (-I*d*n + I*d)*e^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 1)*e^(I*d*
n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2
*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec (c+dx))^{1-n} (ia(\tan (c+dx)-i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(1 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{1-n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=113

$$\frac{i2^{1+\frac{n}{2}}a\,{}_2F_1\left(\frac{2-n}{2},-\frac{n}{2};\frac{4-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(c+dx))^{-n/2}(a+ia\tan(c+dx))}{d(2-n)}$$

```
[Out] I*2^(1+1/2*n)*a*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*tan(d*x+c))
*(e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(2-n)/((1+I*tan(d*x+c))^(
1/2*n))
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{\frac{n}{2}+1}(1+i\tan(c+dx))^{-n/2}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-n}\,{}_2F_1\left(\frac{2-n}{2},-\frac{n}{2};\frac{4-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1 + n/2)*a*Hypergeometric2F1[(2 - n)/2, -1/2*n, (4 - n)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*
(2 - n)*(1 + I*Tan[c + d*x])^(n/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan}{}$$

$$= \frac{\Big(2^{n/2}a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+i}{}$$

$$= \frac{i2^{1+\frac{n}{2}}a\,{}_2F_1\left(\frac{2-n}{2},-\frac{n}{2};\frac{4-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+d}{d(2-n}$$

Mathematica [A]
time = 12.45, size = 112, normalized size = 0.99

$$\frac{4e^2\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};-\cos(2(c+dx))+i\sin(2(c+dx))\right)(e\sec(c+dx))^{-n}(\cos(2c)-i\sin(2c))(i+\tan(dx))(a+ia\tan(c+dx))^n}{d(-2+n)(-1-i\tan(dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (4*e^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, -Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]]*(Cos[2*c] - I*Sin[2*c])*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)
/(d*(-2 + n)*(e*Sec[c + d*x])^n*(-1 - I*Tan[d*x]))
```

Maple [F]
time = 0.73, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{2-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 4*(4*a^n*cos(d*n*x + c*n)*e^2 + 4*I*a^n*e^2*sin(d*n*x + c*n) - (a^n*n*e^2 -
 4*a^n*e^2)*cos(c*n + (d*n + 2*d)*x + 2*c) - 4*((I*a^n*d*n^3 - 6*I*a^n*d*n^
2 + 8*I*a^n*d*n)*cos(4*d*x + 4*c)*e^n + 2*(I*a^n*d*n^3 - 6*I*a^n*d*n^2 + 8*
I*a^n*d*n)*cos(2*d*x + 2*c)*e^n - (a^n*d*n^3 - 6*a^n*d*n^2 + 8*a^n*d*n)*e^n
*sin(4*d*x + 4*c) - 2*(a^n*d*n^3 - 6*a^n*d*n^2 + 8*a^n*d*n)*e^n*sin(2*d*x +
 2*c) + (I*a^n*d*n^3 - 6*I*a^n*d*n^2 + 8*I*a^n*d*n)*e^n)*integrate(((cos(6*
d*x + 6*c)*e^2 + 3*cos(4*d*x + 4*c)*e^2 + 3*cos(2*d*x + 2*c)*e^2 + e^2)*cos
(d*n*x + c*n) + (e^2*sin(6*d*x + 6*c) + 3*e^2*sin(4*d*x + 4*c) + 3*e^2*sin(
2*d*x + 2*c))*sin(d*n*x + c*n))/((n^2 - 6*n + 8)*cos(6*d*x + 6*c)^2*e^n + 9
*(n^2 - 6*n + 8)*cos(4*d*x + 4*c)^2*e^n + 9*(n^2 - 6*n + 8)*cos(2*d*x + 2*c
)^2*e^n + (n^2 - 6*n + 8)*e^n*sin(6*d*x + 6*c)^2 + 9*(n^2 - 6*n + 8)*e^n*si
n(4*d*x + 4*c)^2 + 18*(n^2 - 6*n + 8)*e^n*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*(n^2 - 6*n + 8)*e^n*sin(2*d*x + 2*c)^2 + 6*(n^2 - 6*n + 8)*cos(2*d*x +
 2*c)*e^n + 2*(3*(n^2 - 6*n + 8)*cos(4*d*x + 4*c)*e^n + 3*(n^2 - 6*n + 8)*c
os(2*d*x + 2*c)*e^n + (n^2 - 6*n + 8)*e^n)*cos(6*d*x + 6*c) + 6*(3*(n^2 - 6
*n + 8)*cos(2*d*x + 2*c)*e^n + (n^2 - 6*n + 8)*e^n)*cos(4*d*x + 4*c) + (n^2
 - 6*n + 8)*e^n + 6*((n^2 - 6*n + 8)*e^n*sin(4*d*x + 4*c) + (n^2 - 6*n + 8)
*e^n*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 4*((a^n*d*n^3 - 6*a^n*d*n^2
+ 8*a^n*d*n)*cos(4*d*x + 4*c)*e^n + 2*(a^n*d*n^3 - 6*a^n*d*n^2 + 8*a^n*d*n)
*cos(2*d*x + 2*c)*e^n - (-I*a^n*d*n^3 + 6*I*a^n*d*n^2 - 8*I*a^n*d*n)*e^n*si
n(4*d*x + 4*c) - 2*(-I*a^n*d*n^3 + 6*I*a^n*d*n^2 - 8*I*a^n*d*n)*e^n*sin(2*d
*x + 2*c) + (a^n*d*n^3 - 6*a^n*d*n^2 + 8*a^n*d*n)*e^n)*integrate(-((e^2*sin
(6*d*x + 6*c) + 3*e^2*sin(4*d*x + 4*c) + 3*e^2*sin(2*d*x + 2*c))*cos(d*n*x
+ c*n) - (cos(6*d*x + 6*c)*e^2 + 3*cos(4*d*x + 4*c)*e^2 + 3*cos(2*d*x + 2*c
)*e^2 + e^2)*sin(d*n*x + c*n))/((n^2 - 6*n + 8)*cos(6*d*x + 6*c)^2*e^n + 9*
(n^2 - 6*n + 8)*cos(4*d*x + 4*c)^2*e^n + 9*(n^2 - 6*n + 8)*cos(2*d*x + 2*c)
^2*e^n + (n^2 - 6*n + 8)*e^n*sin(6*d*x + 6*c)^2 + 9*(n^2 - 6*n + 8)*e^n*sin
(4*d*x + 4*c)^2 + 18*(n^2 - 6*n + 8)*e^n*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 9*(n^2 - 6*n + 8)*e^n*sin(2*d*x + 2*c)^2 + 6*(n^2 - 6*n + 8)*cos(2*d*x +
2*c)*e^n + 2*(3*(n^2 - 6*n + 8)*cos(4*d*x + 4*c)*e^n + 3*(n^2 - 6*n + 8)*co
s(2*d*x + 2*c)*e^n + (n^2 - 6*n + 8)*e^n)*cos(6*d*x + 6*c) + 6*(3*(n^2 - 6*
n + 8)*cos(2*d*x + 2*c)*e^n + (n^2 - 6*n + 8)*e^n)*cos(4*d*x + 4*c) + (n^2
- 6*n + 8)*e^n + 6*((n^2 - 6*n + 8)*e^n*sin(4*d*x + 4*c) + (n^2 - 6*n + 8)*
e^n*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (I*a^n*n*e^2 - 4*I*a^n*e^2)*s
```

```
in(c*n + (d*n + 2*d)*x + 2*c))/((-I*d*n^2 + 6*I*d*n - 8*I*d)*cos(4*d*x + 4*
c)*e^n - 2*(I*d*n^2 - 6*I*d*n + 8*I*d)*cos(2*d*x + 2*c)*e^n + (d*n^2 - 6*d*
n + 8*d)*e^n*sin(4*d*x + 4*c) + 2*(d*n^2 - 6*d*n + 8*d)*e^n*sin(2*d*x + 2*c
) + (-I*d*n^2 + 6*I*d*n - 8*I*d)*e^n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] 1/2*((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2)*(I*e^(2*I*d
*x + 2*I*c) + I)*e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x +
I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))) + 2*d*e^(2*I*d*x + 2*I*c)*integral(1/2
*(n*e^(2*I*d*x + 2*I*c) + n)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) +
1))^(-n + 2)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(a*e^(-1)) + n*log(2*e^(I*
d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*c), x))*e^(-2*I*d*x - 2*I*c
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec{(c+dx)})^{2-n}\,(ia(\tan{(c+dx)}-i))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(2 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2-n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=121

$$\frac{i2^{\frac{3+n}{2}}a\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(1+i\tan(c+dx))^{\frac{1}{2}(-1-n)}(a+ia\tan}{d(3-n)}$$

```
[Out] I*2^(3/2+1/2*n)*a*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],1/2-1/2*I*t
an(d*x+c))*(e*sec(d*x+c))^(3-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(d*
x+c))^(-1+n)/d/(3-n)
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\frac{ia2^{\frac{n+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{3-n}\,{}_2F_1\left(\frac{1}{2}(-n-1),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^((3 + n)/2)*a*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 -
 I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 + I*Tan[c + d*x])^((-1 - n)
/2)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(3 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
```

```
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan}{}$$

$$= \frac{\Bigg(2^{\frac{1}{2}+\frac{n}{2}}a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+}{}$$

$$= \frac{i2^{\frac{3+n}{2}}a\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec}{d(}$$

Mathematica [A]
time = 7.72, size = 116, normalized size = 0.96

$$\frac{8e^3\,{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};-\cos(2(c+dx))+i\sin(2(c+dx))\right)\sec(dx)(e\sec(c+dx))^{-n}(i+\tan(dx))(a+ia\tan(c+dx))^n}{d(-3+n)(\cos(c)+i\sin(c))^3(-i+\tan(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (8*e^3*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, -Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]]*Sec[d*x]*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-3 + n
)*(e*Sec[c + d*x])^n*(Cos[c] + I*Sin[c])^3*(-I + Tan[d*x])^2)
```

Maple [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{3-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 8*(6*a^n*cos(c*n + (d*n + d)*x + c)*e^3 + 6*I*a^n*e^3*sin(c*n + (d*n + d)*x
 + c) - (a^n*n*e^3 - 5*a^n*e^3)*cos(c*n + (d*n + 3*d)*x + 3*c) - 6*((I*a^n*
d*n^3 - 7*I*a^n*d*n^2 + 7*I*a^n*d*n + 15*I*a^n*d)*cos(c*n)*e^n - (a^n*d*n^3
 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n) + ((I*a^n*d*n^3 - 7*I*a
^n*d*n^2 + 7*I*a^n*d*n + 15*I*a^n*d)*cos(c*n)*e^n - (a^n*d*n^3 - 7*a^n*d*n^
2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*cos(6*d*x + 6*c) + 3*((I*a^n*d*n^3
- 7*I*a^n*d*n^2 + 7*I*a^n*d*n + 15*I*a^n*d)*cos(c*n)*e^n - (a^n*d*n^3 - 7*a
^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*cos(4*d*x + 4*c) + 3*((I*a^n
*d*n^3 - 7*I*a^n*d*n^2 + 7*I*a^n*d*n + 15*I*a^n*d)*cos(c*n)*e^n - (a^n*d*n^
3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*cos(2*d*x + 2*c) - ((
a^n*d*n^3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^n - (-I*a^n*d*n^
3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*e^n*sin(c*n))*sin(6*d*x + 6*c
) - 3*((a^n*d*n^3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^n - (-I*
a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*e^n*sin(c*n))*sin(4*d
*x + 4*c) - 3*((a^n*d*n^3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^
n - (-I*a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*e^n*sin(c*n))
*sin(2*d*x + 2*c))*integrate(((cos(8*d*x + 8*c)*e^3 + 4*cos(6*d*x + 6*c)*e^
3 + 6*cos(4*d*x + 4*c)*e^3 + 4*cos(2*d*x + 2*c)*e^3 + e^3)*cos((d*n + d)*x
+ c) + (e^3*sin(8*d*x + 8*c) + 4*e^3*sin(6*d*x + 6*c) + 6*e^3*sin(4*d*x + 4
*c) + 4*e^3*sin(2*d*x + 2*c))*sin((d*n + d)*x + c))/((n^2 - 8*n + 15)*cos(8
*d*x + 8*c)^2*e^n + 16*(n^2 - 8*n + 15)*cos(6*d*x + 6*c)^2*e^n + 36*(n^2 -
8*n + 15)*cos(4*d*x + 4*c)^2*e^n + 16*(n^2 - 8*n + 15)*cos(2*d*x + 2*c)^2*e
^n + (n^2 - 8*n + 15)*e^n*sin(8*d*x + 8*c)^2 + 16*(n^2 - 8*n + 15)*e^n*sin(
6*d*x + 6*c)^2 + 36*(n^2 - 8*n + 15)*e^n*sin(4*d*x + 4*c)^2 + 48*(n^2 - 8*n
 + 15)*e^n*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(n^2 - 8*n + 15)*e^n*sin(
2*d*x + 2*c)^2 + 8*(n^2 - 8*n + 15)*cos(2*d*x + 2*c)*e^n + 2*(4*(n^2 - 8*n
+ 15)*cos(6*d*x + 6*c)*e^n + 6*(n^2 - 8*n + 15)*cos(4*d*x + 4*c)*e^n + 4*(n
^2 - 8*n + 15)*cos(2*d*x + 2*c)*e^n + (n^2 - 8*n + 15)*e^n)*cos(8*d*x + 8*c
) + 8*(6*(n^2 - 8*n + 15)*cos(4*d*x + 4*c)*e^n + 4*(n^2 - 8*n + 15)*cos(2*d
*x + 2*c)*e^n + (n^2 - 8*n + 15)*e^n)*cos(6*d*x + 6*c) + 12*(4*(n^2 - 8*n +
 15)*cos(2*d*x + 2*c)*e^n + (n^2 - 8*n + 15)*e^n)*cos(4*d*x + 4*c) + (n^2 -
 8*n + 15)*e^n + 4*(2*(n^2 - 8*n + 15)*e^n*sin(6*d*x + 6*c) + 3*(n^2 - 8*n
+ 15)*e^n*sin(4*d*x + 4*c) + 2*(n^2 - 8*n + 15)*e^n*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*(3*(n^2 - 8*n + 15)*e^n*sin(4*d*x + 4*c) + 2*(n^2 - 8*n +
```

```
15)*e^n*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) + 6*((a^n*d*n^3 - 7*a^n*d*n
^2 + 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^n - (-I*a^n*d*n^3 + 7*I*a^n*d*n^2 - 7
*I*a^n*d*n - 15*I*a^n*d)*e^n*sin(c*n) + ((a^n*d*n^3 - 7*a^n*d*n^2 + 7*a^n*d
*n + 15*a^n*d)*cos(c*n)*e^n - (-I*a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n -
 15*I*a^n*d)*e^n*sin(c*n))*cos(6*d*x + 6*c) + 3*((a^n*d*n^3 - 7*a^n*d*n^2 +
 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^n - (-I*a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a
^n*d*n - 15*I*a^n*d)*e^n*sin(c*n))*cos(4*d*x + 4*c) + 3*((a^n*d*n^3 - 7*a^n
*d*n^2 + 7*a^n*d*n + 15*a^n*d)*cos(c*n)*e^n - (-I*a^n*d*n^3 + 7*I*a^n*d*n^2
 - 7*I*a^n*d*n - 15*I*a^n*d)*e^n*sin(c*n))*cos(2*d*x + 2*c) - ((-I*a^n*d*n^
3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*cos(c*n)*e^n + (a^n*d*n^3 - 7
*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*sin(6*d*x + 6*c) - 3*((-I*
a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*cos(c*n)*e^n + (a^n*d
*n^3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*sin(4*d*x + 4*c) -
 3*((-I*a^n*d*n^3 + 7*I*a^n*d*n^2 - 7*I*a^n*d*n - 15*I*a^n*d)*cos(c*n)*e^n
+ (a^n*d*n^3 - 7*a^n*d*n^2 + 7*a^n*d*n + 15*a^n*d)*e^n*sin(c*n))*sin(2*d*x
+ 2*c))*integrate(-((e^3*sin(8*d*x + 8*c) + 4*e^3*sin(6*d*x + 6*c) + 6*e^3*
sin(4*d*x + 4*c) + 4*e^3*sin(2*d*x + 2*c))*cos((d*n + d)*x + c) - (cos(8*d*
x + 8*c)*e^3 + 4*cos(6*d*x + 6*c)*e^3 + 6*cos(4*d*x + 4*c)*e^3 + 4*cos(2*d*
x + 2*c)*e^3 + e^3)*sin((d*n + d)*x + c))/((n^2 - 8*n + 15)*cos(8*d*x + 8*c
)^2*e^n + 16*(n^2 - 8*n + 15)*cos(6*d*x + 6*c)^2*e^n + 36*(n^2 - 8*n + 15)*
cos(4*d*x + 4*c)^2*e^n + 16*(n^2 - 8*n + 15)*cos(2*d*x + 2*c)^2*e^n + (n^2
- 8*n + 15)*e^n*sin(8*d*x + 8*c)^2 + 16*(n^2 - 8*n + 15)*e^n*sin(6*d*x + 6*
c)^2 + 36*(n^2 - 8*n + 15)*e^n*sin(4*d*x + 4*c)^2 + 48*(n^2 - 8*n + 15)*e^n
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(n^2 - 8*n + 15)*e^n*sin(2*d*x + 2*
c)^2 + 8*(n^2 - 8*n + 15)*cos(2*d*x + 2*c)*e^n + 2*(4*(n^2 - 8*n + 15)*cos(
6*d*x + 6*c)*e^n + 6*(n^2 - 8*n + 15)*cos(4*d*x + 4*c)*e^n + 4*(n^2 - 8*n +
 15)*cos(2*d*x + 2*c)*e^n + (n^2 - 8*n + 15)*e^n)*cos(8*d*x + 8*c) + 8*(6*(
n^2 - 8*n + 15)*cos(4*d*x + 4*c)*e^n + 4*(n^2 - 8*n + 15)*cos(2*d*x + 2*c)*
e^n + (n^2 - 8*n + 15)*e^n)*cos(6*d*x + 6*c) + 12*(4*(n^2 - 8*n + 15)*cos(2
*d*x + 2*c)*e^n + (n^2 - 8*n + 15)*e^n)*cos(4*d*x + 4*c) + (n^2 - 8*n + 15)
*e^n + 4*(2*(n^2 - 8*n + 15)*e^n*sin(6*d*x + 6*c) + 3*(n^2 - 8*n + 15)*e^n*
sin(4*d*x + 4*c) + 2*(n^2 - 8*n + 15)*e^n*sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 16*(3*(n^2 - 8*n + 15)*e^n*sin(4*d*x + 4*c)...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] 1/8*(((-I*n - I)*e^(4*I*d*x + 4*I*c) - 2*I*n*e^(2*I*d*x + 2*I*c) - I*n + I)
*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*
c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) +
```

```
1))) + 8*d*e^(2*I*d*x + 2*I*c)*integral(-1/8*(n^2 + (n^2 - 1)*e^(4*I*d*x +
4*I*c) + 2*(n^2 - 1)*e^(2*I*d*x + 2*I*c) - 1)*(2*e^(I*d*x + I*c + 1)/(e^(2*
I*d*x + 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(a*e^(-1)
) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*c), x))*e^
(-2*I*d*x - 2*I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec{(c+dx)})^{3-n}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(3 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{e}{\cos{(c+d\,x)}}\right)}^{3-n}\,{\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=156

$$\frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(5-n)\left(12-7n+n^2\right)}+\frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)}+\frac{ia(e\sec$$

```
[Out] 8*I*a^3*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-3+n)/d/(-n^3+12*n^2-47*
n+60)+4*I*a^2*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-9*n+2
0)+I*a*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(5-n)
```

Rubi [A]
time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\frac{8ia^3(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(5-n)\left(n^2-7n+12\right)}+\frac{4ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d\left(n^2-9n+20\right)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((8*I)*a^3*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(
5 - n)*(12 - 7*n + n^2)) + ((4*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*T
an[c + d*x])^(-2 + n))/(d*(20 - 9*n + n^2)) + (I*a*(e*Sec[c + d*x])^(6 - 2*
n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)} + \frac{(4a)\int(e\sec}{}$$
$$= \frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(20-9n+n^2)} + \frac{ia(e\sec(}{}$$
$$= \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(3-n)\,(20-9n+n^2)} + \frac{4ia^2(e\sec}{}$$

Mathematica [A]
time = 2.35, size = 122, normalized size = 0.78

$$-\frac{e^6\sec^5(c+dx)(e\sec(c+dx))^{-2n}\left(-2(-5+n)+(22-9n+n^2)\cos(2(c+dx))+i(18-9n+n^2)\sin(2(c+dx))\right)(i\cos(3(c+dx))+\sin(3(c+dx)))(a+ia\tan(c+dx))^n}{d(-5+n)(-4+n)(-3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^6*Sec[c + d*x]^5*(-2*(-5 + n) + (22 - 9*n + n^2)*Cos[2*(c + d*x)] + I*
(18 - 9*n + n^2)*Sin[2*(c + d*x)])*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*
(a + I*a*Tan[c + d*x])^n)/(d*(-5 + n)*(-4 + n)*(-3 + n)*(e*Sec[c + d*x])^(2
*n)))
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^{6-2n}\left(a+ia\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(143) = 286.
time = 1.74, size = 865, normalized size = 5.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] -32*((a^n*n^2*e^6 - 9*a^n*n*e^6 + 20*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(4*d*x + n*arctan2(sin(2*d
```

```
*x + 2*c), cos(2*d*x + 2*c) + 1) + 4*c) - 2*(a^n*n*e^6 - 5*a^n*e^6)*(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d
*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) + 2*a^n*cos(n
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*e^(1/2*n*log(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 6) - (-I*a^n*n^2*e^
6 + 9*I*a^n*n*e^6 - 20*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(4*d*x + n*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1) + 4*c) - 2*(I*a^n*n*e^6 - 5*I*a^n*e^6)*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) + 2*I*a^n*e^(1/2*n*lo
g(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 6)*si
n(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((-I*n^3 + 12*I*n^2
- 47*I*n + 60*I)*2^n*cos(10*d*x + 10*c)*e^(2*n) - 5*(I*n^3 - 12*I*n^2 + 47*
I*n - 60*I)*2^n*cos(8*d*x + 8*c)*e^(2*n) - 10*(I*n^3 - 12*I*n^2 + 47*I*n -
60*I)*2^n*cos(6*d*x + 6*c)*e^(2*n) - 10*(I*n^3 - 12*I*n^2 + 47*I*n - 60*I)*
2^n*cos(4*d*x + 4*c)*e^(2*n) - 5*(I*n^3 - 12*I*n^2 + 47*I*n - 60*I)*2^n*cos
(2*d*x + 2*c)*e^(2*n) + (n^3 - 12*n^2 + 47*n - 60)*2^n*e^(2*n)*sin(10*d*x +
 10*c) + 5*(n^3 - 12*n^2 + 47*n - 60)*2^n*e^(2*n)*sin(8*d*x + 8*c) + 10*(n^
3 - 12*n^2 + 47*n - 60)*2^n*e^(2*n)*sin(6*d*x + 6*c) + 10*(n^3 - 12*n^2 + 4
7*n - 60)*2^n*e^(2*n)*sin(4*d*x + 4*c) + 5*(n^3 - 12*n^2 + 47*n - 60)*2^n*e
^(2*n)*sin(2*d*x + 2*c) + (-I*n^3 + 12*I*n^2 - 47*I*n + 60*I)*2^n*e^(2*n))*
d)
```

Fricas [A]
time = 0.43, size = 165, normalized size = 1.06

$$\frac{\left(\left(-i\,n^2+9i\,n-20i\right)e^{\left(6i\,dx+6i\,c\right)}+\left(-i\,n^2+11i\,n-30i\right)e^{\left(4i\,dx+4i\,c\right)}-2\left(-i\,n+6i\right)e^{\left(2i\,dx+2i\,c\right)}-2i\right)\left(\frac{2\,e^{\left(i\,dx+i\,c+1\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{-2\,n+6}e^{\left(i\,dnx+i\,cn-6i\,dx+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{\left(i\,dx+i\,c+1\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)-6i\,c\right)}}{2\left(dn^3-12\,dn^2+47\,dn-60\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n^2 + 9*I*n - 20*I)*e^(6*I*d*x + 6*I*c) + (-I*n^2 + 11*I*n - 30*I)
*e^(4*I*d*x + 4*I*c) - 2*(-I*n + 6*I)*e^(2*I*d*x + 2*I*c) - 2*I)*(2*e^(I*d*
x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 6)*e^(I*d*n*x + I*c*n - 6*I
*d*x + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) +
 1)) - 6*I*c)/(d*n^3 - 12*d*n^2 + 47*d*n - 60*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(6-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 6)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 10.33, size = 318, normalized size = 2.04

$$(\cos(6c+6dx)-\sin(6c+6dx)\,1\mathrm{i})\left(\frac{e}{\cos(c+dx)}\right)^{6-2n}\left(\frac{\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}-\frac{(2n-12)\,(\cos(2c+2dx)+\sin(2c+2dx)\,1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}+\frac{(\cos(6c+6dx)+\sin(6c+6dx)\,1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-9n+20)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}+\frac{(\cos(4c+4dx)+\sin(4c+4dx)\,1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-11n+30)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(6 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] (cos(6*c + 6*d*x) - sin(6*c + 6*d*x)*1i)*(e/cos(c + d*x))^(6 - 2*n)*((a + (
a*sin(c + d*x)*1i)/cos(c + d*x))^n/(d*(n*47i - n^2*12i + n^3*1i - 60i)) - (
(2*n - 12)*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(a + (a*sin(c + d*x)*1i
)/cos(c + d*x))^n)/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(6*c + 6*d
*x) + sin(6*c + 6*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 -
9*n + 20))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(4*c + 4*d*x) + si
n(4*c + 4*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 - 11*n + 3
0))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)))
```

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{i2^{\frac{5}{2}-n}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2n);\frac{7}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(1-i\tan(c+dx))^{-\frac{5}{2}+n}(a+ia\tan(c+dx))^n}{5d}$$

```
[Out] -1/5*I*2^(5/2-n)*hypergeom([5/2, -3/2+n],[7/2],1/2+1/2*I*tan(d*x+c))*(e*sec
(d*x+c))^(5-2*n)*(1-I*tan(d*x+c))^(-5/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A]

time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$-\frac{i2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{5}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2n-3);\frac{7}{2};\frac{1}{2}(i\tan(c+dx)+1)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(5/2 - n)*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-5/2 + n)*(a
+ I*a*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan($$

$$= \frac{\Big(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\,\mathrm{ta}}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\,\mathrm{ta}}{}$$

$$= \frac{\Big(2^{\frac{3}{2}-n}a^3(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-5+2n}}{}$$

$$= -\frac{i2^{\frac{5}{2}-n}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2n);\frac{7}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c}{5d}$$

Mathematica [A]
time = 14.21, size = 166, normalized size = 1.71

$$-\frac{i2^{5-n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},5-n;\frac{7}{2};-e^{2i(c+dx)}\right)\sec^{-5+n}(c+dx)(e\sec(c+dx))^{5-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(5 - n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[5/2
, 5 - n, 7/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 + n)*(e*Sec[c + d*x])^
(5 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.75, size = 0, normalized size = 0.00

$$\int \left(e \sec \left(dx+c\right)\right)^{5-2n} \left(a+ia \tan \left(dx+c\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
[Out] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 5)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \sec \left(c+dx\right)\right)^{5-2n} \left(ia \left(\tan \left(c+dx\right)-i\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(5 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5-2\,n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=98

$$\frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(6-5n+n^2)}+\frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}$$

```
[Out] 2*I*a^2*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-5*n+6)+I*a*
(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(3-n)
```

Rubi [A]

time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d\,(n^2-5n+6)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(
6 - 5*n + n^2)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-
1 + n))/(d*(3 - n))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx &= \frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}+\frac{(2a)\int(e\,\mathrm{s}\\ &= \frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(6-5n+n^2)}+\frac{ia(e\sec\end{aligned}$$

Mathematica [A]
time = 1.36, size = 91, normalized size = 0.93

$$\frac{e^4\sec^2(c+dx)(e\sec(c+dx))^{-2n}(\cos(2(c+dx))-i\sin(2(c+dx)))(a+ia\tan(c+dx))^n(-i(-4+n)+(-2+n)\tan(c+dx))}{d(-3+n)(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (e^4*Sec[c + d*x]^2*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(a + I*a*Tan[c
+ d*x])^n*((-I)*(-4 + n) + (-2 + n)*Tan[c + d*x]))/(d*(-3 + n)*(-2 + n)*(e*
Sec[c + d*x])^(2*n))
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^{4-2n}\left(a+ia\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(90) = 180$.
time = 1.51, size = 505, normalized size = 5.15

$$-\frac{8\left(\left(a^n n e^4-3\,a^n e^4\right)\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{2}\,n}\cos\left(2\,dx+n\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)+2\,c\right)-a^n\cos\left(n\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)e^{\left(\frac{1}{2}\,n\log\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)+4\right)}-\left(-i\,a^n n e^4+3i\,a^n e^4\right)\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{2}\,n}\sin\left(2\,dx+n\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)+2\,c\right)-i\,a^n e^{\left(\frac{1}{2}\,n\log\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)+4\right)}\sin\left(n\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)\right)}{\left(\left(-i\,n^2+5i\,n-6i\right)2^n\cos\left(6\,dx+6\,c\right)e^{(2\,n)}-3\left(i\,n^2-5i\,n+6i\right)2^n\cos\left(4\,dx+4\,c\right)e^{(2\,n)}-3\left(i\,n^2-5i\,n+6i\right)2^n\cos\left(2\,dx+2\,c\right)e^{(2\,n)}+\left(n^2-5\,n+6\right)2^n e^{(2\,n)}\sin\left(6\,dx+6\,c\right)+3\left(n^2-5\,n+6\right)2^n e^{(2\,n)}\sin\left(4\,dx+4\,c\right)+3\left(n^2-5\,n+6\right)2^n e^{(2\,n)}\sin\left(2\,dx+2\,c\right)+\left(-i\,n^2+5i\,n-6i\right)2^n e^{(2\,n)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] -8*((a^n*n*e^4 - 3*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1) + 2*c) - a^n*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*e^(1/2*n*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1) + 4) - (-I*a^n*n*e^4 + 3*I*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) - I*a^n*e^(1/2*n*log(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 4)*sin(n*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((-I*n^2 + 5*I*n - 6*I)*2^n*cos(6*
d*x + 6*c)*e^(2*n) - 3*(I*n^2 - 5*I*n + 6*I)*2^n*cos(4*d*x + 4*c)*e^(2*n) -
 3*(I*n^2 - 5*I*n + 6*I)*2^n*cos(2*d*x + 2*c)*e^(2*n) + (n^2 - 5*n + 6)*2^n
*e^(2*n)*sin(6*d*x + 6*c) + 3*(n^2 - 5*n + 6)*2^n*e^(2*n)*sin(4*d*x + 4*c)
```

```
+ 3*(n^2 - 5*n + 6)*2^n*e^(2*n)*sin(2*d*x + 2*c) + (-I*n^2 + 5*I*n - 6*I)*2
^n*e^(2*n))*d)
```

Fricas [A]
time = 0.39, size = 133, normalized size = 1.36

$$\frac{\left((-i\,n+3i)e^{(4i\,dx+4i\,c)}+(-i\,n+4i)e^{(2i\,dx+2i\,c)}+i\right)\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+4}e^{\left(i\,dnx+i\,cn-4i\,dx+n\log\left(ae^{(-1)}\right)+n\log\left(\frac{2\,e^{(i\,dx+i\,c+1)}}{e^{(2i\,dx+2i\,c)}+1}\right)-4i\,c\right)}}{2\left(dn^2-5\,dn+6\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n + 3*I)*e^(4*I*d*x + 4*I*c) + (-I*n + 4*I)*e^(2*I*d*x + 2*I*c) +
I)*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 4)*e^(I*d*n*x
+ I*c*n - 4*I*d*x + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d
*x + 2*I*c) + 1)) - 4*I*c)/(d*n^2 - 5*d*n + 6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(e\sec\left(c+dx\right)\right)^{4-2n}\left(ia(\tan\left(c+dx\right)-i)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(4-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(4 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 5.93, size = 174, normalized size = 1.78

$$\frac{4\,e^4\,{\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)}^n\,\left(4\,\sin\left(2\,c+2\,d\,x\right)+\cos\left(2\,c+2\,d\,x\right)\,4{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}-n\,1{}\mathrm{i}+\sin\left(4\,c+4\,d\,x\right)-n\,\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}-n\,\sin\left(2\,c+2\,d\,x\right)+3{}\mathrm{i}\right)}{d\,{\left(\frac{e}{\cos(c+d\,x)}\right)}^{2\,n}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)\,\left(n^2-5\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(4 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] (4*e^4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^n*(cos(2*c + 2*d*x)*4i - n*1i + cos(4*c + 4*d*x)*1i + 4*sin(2*c + 2*d
*x) + sin(4*c + 4*d*x) - n*cos(2*c + 2*d*x)*1i - n*sin(2*c + 2*d*x) + 3i))/
(d*(e/cos(c + d*x))^(2*n)*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3)*(n^2
- 5*n + 6))
```

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{i2^{\frac{3}{2}-n}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2n);\frac{5}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(1-i\tan(c+dx))^{-\frac{3}{2}+n}(a+ia\tan(c$$
$$3d$$

[Out] -1/3*I*2^(3/2-n)*hypergeom([3/2, -1/2+n],[5/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(3-2*n)*(1-I*tan(d*x+c))^(-3/2+n)*(a+I*a*tan(d*x+c))^n/d

Rubi [A]
time = 0.16, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$-\frac{i2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2n-1);\frac{5}{2};\frac{1}{2}(i\tan(c+dx)+1)\right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((-1/3*I)*2^(3/2 - n)*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-3/2 + n)*(a + I*a*Tan[c + d*x])^n)/d

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan($$

$$= \frac{\Big(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\,\text{ta}}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\,\text{ta}}{}$$

$$= \frac{\Big(2^{\frac{1}{2}-n}a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-3+2n}}{}$$

$$= -\frac{i2^{\frac{3}{2}-n}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2n);\frac{5}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c}{3d}$$

Mathematica [A]
time = 12.37, size = 166, normalized size = 1.71

$$-\frac{i2^{3-n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}{}_2F_1\left(\frac{3}{2},3-n;\frac{5}{2};-e^{2i(c+dx)}\right)\sec^{-3+n}(c+dx)(e\sec(c+dx))^{3-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/3*I)*2^(3 - n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[3/2
, 3 - n, 5/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3 + n)*(e*Sec[c + d*x])^
(3 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(dx+c\right)\right)^{3-2n} \left(a+ia\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 3)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sec\left(c+dx\right)\right)^{3-2n} \left(ia(\tan\left(c+dx\right)-i)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(3 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3-2\,n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=46

$$\frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

```
[Out] I*a*(e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(1-n)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3574}

$$\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-2n}}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n)
)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

Mathematica [A]
time = 0.72, size = 59, normalized size = 1.28

$$-\frac{e^2(e\sec(c+dx))^{-2n}(i+\sec(c)\sec(c+dx)\sin(dx)+\tan(c))(a+ia\tan(c+dx))^n}{d(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^2*(I + Sec[c]*Sec[c + d*x]*Sin[d*x] + Tan[c])*(a + I*a*Tan[c + d*x])^n
)/(d*(-1 + n)*(e*Sec[c + d*x])^(2*n)))
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \left(e \sec \left(d x +c \right)\right)^{2-2 n} \left(a +i a \tan \left(d x +c \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(41) = 82.
time = 0.53, size = 222, normalized size = 4.83

$$\frac{\left(-i\,a^{n}e^{2}-\frac{2\,a^{n}e^{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{i\,a^{n}e^{2}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)e^{\left(n\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)+n\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)+n\log\left(-\frac{2i\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1\right)-2\,n\log\left(-\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1\right)\right)}}{\left(ne^{\left(2\,n\right)}-\frac{\left(ne^{\left(2\,n\right)}-e^{\left(2\,n\right)}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-e^{\left(2\,n\right)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] (-I*a^n*e^2 - 2*a^n*e^2*sin(d*x + c)/(cos(d*x + c) + 1) + I*a^n*e^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)*e^(n*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
 + n*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1) + n*log(-2*I*sin(d*x + c)/(co
s(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - 2*n*log(-sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/((n*e^(2*n) - (n*e^(2*n) - e^(2*n))*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 - e^(2*n))*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.
time = 0.37, size = 108, normalized size = 2.35

$$\frac{\left(\frac{2\,e^{\left(i\,dx+i\,c+1\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{-2\,n+2}\left(-i\,e^{\left(2i\,dx+2i\,c\right)}-i\right)e^{\left(i\,dnx+i\,cn-2i\,dx+n\log\left(ae^{\left(-1\right)}\right)+n\log\left(\frac{2\,e^{\left(i\,dx+i\,c+1\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)-2i\,c\right)}}{2\,\left(dn-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 2)*(-I*e^(2*I
*d*x + 2*I*c) - I)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(a*e^(-1)) + n*log(2
*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*c)/(d*n - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec (c+dx))^{2-2n}(ia(\tan (c+dx)-i))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(2 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B]
time = 4.24, size = 106, normalized size = 2.30

$$-\frac{e^2\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}\right)}^n}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2\,n}\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(2 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] -(e^2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i)*((a*(cos(2*c + 2*d*x) +
 sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(cos(2*c + 2*d*x)
+ 1)*(e/cos(c + d*x))^(2*n)*(n - 1))
```

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{i2^{\frac{1}{2}-n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2n);\frac{3}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1-i\tan(c+dx))^{-\frac{1}{2}+n}(a+ia\tan(c+dx))^n}{d}$$

```
[Out] -I*2^(1/2-n)*hypergeom([1/2, 1/2+n],[3/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x+
c))^(1-2*n)*(1-I*tan(d*x+c))^(-1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$-\frac{i2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2n+1);\frac{3}{2};\frac{1}{2}(i\tan(c+dx)+1)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1/2 - n)*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)*(a + I*a
*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia}{}$$

$$= \frac{\Big(2^{-\frac{1}{2}-n}a(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-1+}}{}$$

$$= -\frac{i2^{\frac{1}{2}-n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2n);\frac{3}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c-}{d}$$

Mathematica [A]

time = 8.56, size = 154, normalized size = 1.62

$$-\frac{i2^{1-n}e\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1-n}\left(1+e^{2i(c+dx)}\right)^{1-n}{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};-e^{2i(c+dx)}\right)\sec^n(c+dx)(e\sec(c+dx))^{-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 - n)*e*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(1 - n)*(1 + E^((2*I)*(c + d*x)))^(1 - n)*Hypergeometric2F1[1/2, 1 - n, 3/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec
[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{1-2n} (a+ia\tan(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 1)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec(c+dx))^{1-2n} (ia(\tan(c+dx)-i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(1 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{1-2\,n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=65

$$-\frac{i\,{}_2F_1\left(1,-n;1-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, -n],[1-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d
/n/((e*sec(d*x+c))^(2*n))
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3573, 3562, 70}

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\,{}_2F_1\left(1,-n;1-n;\frac{1}{2}(1-i\tan(c+dx))\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, -n, 1 - n, (1 - I*Tan[c + d*x])/2]*(a + I*a*
Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+dx)\right.$$

$$= \frac{(ia(e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+d}{d}$$

$$= -\frac{i\,{}_2F_1\left(1,-n;1-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2}}{2dn}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.
time = 1.83, size = 146, normalized size = 2.25

$$\frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right){}_2F_1\left(1,1+n;2+n;1+e^{2i(c+dx)}\right)\sec^n(c+dx)(e\sec(c+dx))^{-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1,
1 + n, 2 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x]
)^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + n)*(e*Sec[c + d*
x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (a+ia\tan(dx+c))^n (e\sec(dx+c))^{-2n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="fricas"
)
```

```
[Out] integral(e^(I*d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)
/(e^(2*I*d*x + 2*I*c) + 1)))/(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) +
1))^(2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec (c+dx))^{-2n} (ia(\tan (c+dx)-i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**(2*n)),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(2*n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n), x)
```

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=95

$$\frac{i2^{-\frac{1}{2}-n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2n);\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2n}(1-i\tan(c+dx))^{\frac{1}{2}+n}(a+ia\tan(c}{d}$$

```
[Out] I*2^(-1/2-n)*hypergeom([-1/2, 3/2+n],[1/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(-1-2*n)*(1-I*tan(d*x+c))^(1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A]

time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2n+3);\frac{1}{2};\frac{1}{2}(i\tan(c+dx)+1)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a + I*a*
Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia}{}$$

$$= \frac{\Big(2^{-\frac{3}{2}-n}a(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(1}}{}$$

$$= \frac{i2^{-\frac{1}{2}-n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2n);\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c}{d}$$

Mathematica [A]
time = 12.97, size = 157, normalized size = 1.65

$$\frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1-n}\left(1+e^{2i(c+dx)}\right)^{-1-n}{}_2F_1\left(-\frac{1}{2},-1-n;\frac{1}{2};-e^{2i(c+dx)}\right)\sec^{1+n}(c+dx)(e\sec(c+dx))^{-1-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1
 - n)*(1 + E^((2*I)*(c + d*x)))^(-1 - n)*Hypergeometric2F1[-1/2, -1 - n, 1/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1 + n)*(e*Sec[c + d*x])^(-1 - 2*n)*(
a + I*a*Tan[c + d*x])^n)/(d*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int (e \sec (dx + c))^{-1-2n} (a + ia \tan (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
[Out] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 1)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \sec (c + dx))^{-2n-1} (ia(\tan (c + dx) - i))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-1-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-2*n - 1)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\tan(c + d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1), x)
```

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=74

$$-\frac{i\,{}_2F_1\left(2,-1-n;-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\tan(c+dx))^{1+n}}{4ad(1+n)}$$

```
[Out] -1/4*I*hypergeom([2, -1-n],[-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(1
+n)/a/d/(1+n)/((e*sec(d*x+c))^(2+2*n))
```

Rubi [A]
time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 7, 70}

$$-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\,{}_2F_1\left(2,-n-1;-n;\frac{1}{2}(1-i\tan(c+dx))\right)}{4ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/4*I)*Hypergeometric2F1[2, -1 - n, -n, (1 - I*Tan[c + d*x])/2]*(a + I*a
*Tan[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan$$
$$= \frac{\Big(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia}{}$$
$$= \frac{\Big(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia}{}$$
$$= -\frac{i\,{}_2F_1\left(2,-1-n;-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx)}{4ad(1+n)}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.
time = 13.24, size = 151, normalized size = 2.04

$$-\frac{i2^{-3-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^3{}_2F_1\left(2,3+n;4+n;1+e^{2i(c+dx)}\right)\sec^n(c+dx)(e\sec(c+dx))^{-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{de^2(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F
1[2, 3 + n, 4 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c +
 d*x])^n)/(d*e^2*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(3 + n)*(e*S
ec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.80, size = 0, normalized size = 0.00

$$\int (e\sec(dx+c))^{-2n-2}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 2)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sec{(c+dx)})^{-2n-2}\,(ia(\tan{(c+dx)}-i))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(-2*n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^n}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2\,n+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2), x)
```

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$\frac{i2^{-\frac{3}{2}-n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(5+2n);-\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-3-2n}(1-i\tan(c+dx))^{\frac{3}{2}+n}(a+ia\tan(c+dx))^n}{3d}$$

```
[Out] 1/3*I*2^(-3/2-n)*hypergeom([-3/2, 5/2+n],[-1/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(-3-2*n)*(1-I*tan(d*x+c))^(3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A]
time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(2n+5);-\frac{1}{2};\frac{1}{2}(i\tan(c+dx)+1)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(3/2 + n)*(a +
 I*a*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan \ldots$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia \ldots}{\ldots}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia \ldots}{\ldots}$$

$$= \frac{\Big(2^{-\frac{5}{2}-n}(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(3+ \ldots}{\ldots}$$

$$= \frac{i2^{-\frac{3}{2}-n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(5+2n);-\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec \ldots}{3 \ldots}$$

Mathematica [A]
time = 13.59, size = 166, normalized size = 1.71

$$\frac{i2^{-3-n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-3-n;-\frac{1}{2};-e^{2i(c+dx)}\right)\sec^{3+n}(c+dx)(e\sec(c+dx))^{-3-2n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(-3 - n)*(E^(I*d*x))^n*Hypergeometric2F1[-3/2, -3 - n, -1/2, -E^((
2*I)*(c + d*x))]*Sec[c + d*x]^(3 + n)*(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*
Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \left(e\sec \left(dx+c\right)\right)^{-3-2n} \left(a+ia\tan \left(dx+c\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
[Out] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
[Out] integral((2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 3)*e^(I*
d*n*x + I*c*n + n*log(a*e^(-1)) + n*log(2*e^(I*d*x + I*c + 1)/(e^(2*I*d*x +
 2*I*c) + 1))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sec \left(c+dx\right)\right)^{-2n-3} \left(ia(\tan \left(c+dx\right)-i)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-3-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-2*n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3), x)
```

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$

Optimal. Leaf size=66

$$\frac{i\,{}_2F_1\left(3,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{8a^2fn}$$

```
[Out] 1/8*I*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a^2
/f/n/((a+I*a*tan(f*x+e))^n)
```

Rubi [A]

time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3586, 3603, 3568, 70}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,{}_2F_1\left(3,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{8a^2fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((I/8)*Hypergeometric2F1[3, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx &= ((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+f \\
&= \frac{((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+f}{a^4} \\
&= \frac{(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+}{f} \\
&= \frac{i\,{}_2F_1\left(3,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a}{8a^2fn}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.
time = 120.18, size = 165, normalized size = 2.50

$$
-\frac{i2^{-3+n}e^{2ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{3}\,{}_2F_1\left(3,3-n;4-n;1+e^{2i(e+fx)}\right)\sec^{2-n}(e+fx)(d\sec(e+fx))^{2n}(\cos(fx)+i\sin(fx))^{2+n}(a+ia\tan(e+fx))^{-2-n}}{f(-3+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((-I)*2^(-3 + n)*E^((2*I)*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*
(1 + E^((2*I)*(e + f*x)))^3*Hypergeometric2F1[3, 3 - n, 4 - n, 1 + E^((2*I)
*(e + f*x))]*Sec[e + f*x]^(2 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[
f*x])^(2 + n)*(a + I*a*Tan[e + f*x])^(-2 - n))/((E^(I*f*x))^n*f*(-3 + n))
```

Maple [F]
time = 1.26, size = 0, normalized size = 0.00

$$
\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{-2-n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="fric
as")
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^((-I*f*n -
 2*I*f)*x + (-I*n - 2*I)*e - (n + 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x +
2*I*e) + 1)) - (n + 2)*log(a/d)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-2-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{n+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2),x)
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2), x)
```

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$

Optimal. Leaf size=66

$$\frac{i\,{}_2F_1\left(2,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{4afn}$$

```
[Out] 1/4*I*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a/f
/n/((a+I*a*tan(f*x+e))^n)
```

Rubi [A]
time = 0.15, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3586, 3603, 3568, 70}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,{}_2F_1\left(2,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{4afn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] ((I/4)*Hypergeometric2F1[2, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = ((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+f$$

$$= \frac{((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+f}{a^2}$$

$$= \frac{(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+}{f}$$

$$= \frac{i\,{}_2F_1\left(2,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a}{4afn}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.
time = 14.64, size = 165, normalized size = 2.50

$$\frac{i2^{-2+n}e^{ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\left(1+e^{2i(e+fx)}\right)^2{}_2F_1\left(2,2-n;3-n;1+e^{2i(e+fx)}\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^{2n}(\cos(fx)+i\sin(fx))^{1+n}(a+ia\tan(e+fx))^{-1-n}}{f(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] (I*2^(-2 + n)*E^(I*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^
((2*I)*(e + f*x)))^2*Hypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e + f
*x))]*Sec[e + f*x]^(1 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^(
1 + n)*(a + I*a*Tan[e + f*x])^(-1 - n))/((E^(I*f*x))^n*f*(-2 + n))
```

Maple [**F**]
time = 1.26, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="fric
as")
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^((-I*f*n -
 I*f)*x + (-I*n - I)*e - (n + 1)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*
e) + 1)) - (n + 1)*log(a/d)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \sec (e + fx))^{2n} (ia(\tan (e + fx) - i))^{-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-1-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1),x)
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1), x)
```

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$

Optimal. Leaf size=63

$$\frac{i\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{2fn}$$

```
[Out] 1/2*I*hypergeom([1, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/f/n
/((a+I*a*tan(f*x+e))^n)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3573, 3562, 70}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(1-i\tan(e+fx))\right)}{2fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

```
[Out] ((I/2)*Hypergeometric2F1[1, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx)\right.$$
$$= \frac{(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+f}{f}$$
$$= \frac{i\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+}{2fn}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. $2(63) = 126$.
time = 1.16, size = 150, normalized size = 2.38

$$-\frac{i2^{-1+n}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\left(1+e^{2i(e+fx)}\right){}_2F_1\left(1,1-n;2-n;1+e^{2i(e+fx)}\right)\sec^{-n}(e+fx)(d\sec(e+fx))^{2n}(\cos(fx)+i\sin(fx))^n(a+ia\tan(e+fx))^{-n}}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*I
)*(e + f*x)))*Hypergeometric2F1[1, 1 - n, 2 - n, 1 + E^((2*I)*(e + f*x))]*(
d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^n)/((E^(I*f*x))^n*f*(-1 + n)*
Sec[e + f*x]^n*(a + I*a*Tan[e + f*x])^n)
```

Maple [**F**]
time = 0.75, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="maxima"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="fricas"
)
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*f*n*x
- I*n*e - n*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - n*log(a/d)
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)/((a+I*a*tan(f*x+e))**n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)/(I*a*(tan(e + f*x) - I))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{2\,n}}{{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n,x)
```

```
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n, x)
```

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$

Optimal. Leaf size=40

$$\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

[Out] I*a*(d*sec(f*x+e))^(2*n)/f/n/((a+I*a*tan(f*x+e))^n)

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{3574\}$

$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

Antiderivative was successfully verified.

[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]

[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

Mathematica [A]
time = 0.48, size = 40, normalized size = 1.00

$$\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

Antiderivative was successfully verified.

[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]

[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.96, size = 1291, normalized size = 32.28

method	result	size
risch	Expression too large to display	1291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x,method=_RETURNVERBOSE)
```

```
[Out] I/f*a*a^(-n)*2^n*d^(2*n)*exp(-1/2*I*Pi*(-2*n*csgn(I/(exp(2*I*(f*x+e))+1))*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*exp(I*(f*x+e)))*csg
n(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))*csgn(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2-2*n*csgn(I*
d)*csgn(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2-2*csgn(I*exp(I*(f*x+e)))
*csgn(I*exp(2*I*(f*x+e)))^2+csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3
+2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-n*csgn(I*exp(2*I*(f*x+e)
))^3-n*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-n*csgn(I*a/(exp(2*I*
(f*x+e))+1)*exp(2*I*(f*x+e)))^3+2*n*csgn(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*
x+e)))^3-csgn(I*a)*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-csgn(I
/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+csgn
(I*exp(I*(f*x+e)))^2*csgn(I*exp(2*I*(f*x+e)))-csgn(I*exp(2*I*(f*x+e)))*csgn
(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-csgn(I*exp(2*I*(f*x+e))/(exp(2*
I*(f*x+e))+1))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+csgn(I*exp
(2*I*(f*x+e)))^3+csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3-n*csgn(I
/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/(ex
p(2*I*(f*x+e))+1))-n*csgn(I*a)*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1)
)*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))+2*n*csgn(I/(exp(2*I*(f*x+
e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+
2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*d)*csgn(I*d/(exp(2*I
*(f*x+e))+1)*exp(I*(f*x+e)))+n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*
(f*x+e))/(exp(2*I*(f*x+e))+1))^2-n*csgn(I*exp(I*(f*x+e)))^2*csgn(I*exp(2*I*
(f*x+e)))+n*csgn(I*a)*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+csg
n(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/
(exp(2*I*(f*x+e))+1))+2*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e)))^2
+n*csgn(I*exp(2*I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2
+n*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*a/(exp(2*I*(f*x+e))
+1)*exp(2*I*(f*x+e)))^2+csgn(I*a)*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))))*(exp(2*I*(f*x+e))+1)
^(-n)/n
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(38) = 76.
time = 0.52, size = 147, normalized size = 3.68

$$\frac{i\,a^{-n+1}d^{2\,n}e^{\left(-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-n\log\left(-\frac{2i\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)+2\,n\log\left(-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)\right)}}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="maxim
a")
```

```
[Out] I*a^(-n + 1)*d^(2*n)*e^(-n*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1) - n*log(-2*I*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1) + 2*n*log(-sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1))/(f*n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(38) = 76$.
time = 0.39, size = 125, normalized size = 3.12

$$\frac{\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}\left(i\,e^{(2i\,fx+2i\,e)}+i\right)e^{\left((-i\,fn+i\,f)x-2i\,fx+(-i\,n+i)e-(n-1)\log\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)-(n-1)\log\left(\frac{a}{d}\right)-2i\,e\right)}}{2\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*(I*e^(2*I*f*x + 2
*I*e) + I)*e^((-I*f*n + I*f)*x - 2*I*f*x + (-I*n + I)*e - (n - 1)*log(2*d*e
^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n - 1)*log(a/d) - 2*I*e)/(f*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(1-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(1 - n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 1), x)
```

Mupad [B]
time = 4.59, size = 62, normalized size = 1.55

$$\frac{a\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{2\,n}\,1{}\mathrm{i}}{f\,n\,\left(\frac{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(1 - n),x)
```

```
[Out] (a*(d/cos(e + f*x))^(2*n)*1i)/(f*n*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)
*1i + 1))/(2*cos(e + f*x)^2))^n)
```

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$

Optimal. Leaf size=92

$$\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)}+\frac{2ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(1+n)}$$

```
[Out] I*a*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(1+n)+2*I*a^2*(d*sec(f*
x+e))^(2*n)/f/n/(1+n)/((a+I*a*tan(f*x+e))^n)
```

Rubi [A]

time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3575, 3574}

$$\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) + (
(2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx &= \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)}+\frac{(2a)\int(d\sec(\cdots}{}\\ &= \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)}+\frac{2ia^2(d\sec(e\cdots}{}\end{aligned}$$

Mathematica [**A**]
time = 1.20, size = 61, normalized size = 0.66

$$-\frac{a^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}(-i(2+n)+n\tan(e+fx))}{fn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

```
[Out] -((a^2*(d*Sec[e + f*x])^(2*n)*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n)
*(a + I*a*Tan[e + f*x])^n))
```

Maple [**F**]
time = 0.72, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{2-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(88) = 176.
time = 0.61, size = 319, normalized size = 3.47

$$\frac{2^{n+1}a^2d^{2n}\cos\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-i\cdot 2^{n+1}a^2d^{2n}\sin\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)+2\left(a^2d^{2n}n+a^2d^{2n}\right)2^n\cos\left(-2fx+n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-2e\right)+2\left(-ia^2d^{2n}n-ia^2d^{2n}\right)2^n\sin\left(-2fx+n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-2e\right)}{\left(-ia^nn^2-ia^nn+\left(-ia^nn^2-ia^nn\right)\cos\left(2fx+2e\right)+\left(a^nn^2+a^nn\right)\sin\left(2fx+2e\right)\right)\left(\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1\right)^{\frac{1}{2}n}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="maxim
a")
```

```
[Out] (2^(n + 1)*a^2*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - I*2^(n + 1)*a^2*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 2*(a^2*d^(2*n)*n + a^2*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + 2*(-I*a^2*d^(2*n)*n - I*a^2*d^(
2*n))*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) -
2*e))/((-I*a^n*n^2 - I*a^n*n + (-I*a^n*n^2 - I*a^n*n)*cos(2*f*x + 2*e) + (a
^n*n^2 + a^n*n)*sin(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Fricas [**A**]
time = 0.44, size = 150, normalized size = 1.63

$$\frac{\left((i\,n+i)e^{(4i\,fx+4i\,e)}+(i\,n+2i)e^{(2i\,fx+2i\,e)}+i\right)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}e^{\left((-i\,fn+2i\,f)x-4i\,fx+(-i\,n+2i)e-(n-2)\log\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)-(n-2)\log\left(\frac{a}{d}\right)-4i\,e\right)}}{2\left(fn^2+fn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n + I)*e^(4*I*f*x + 4*I*e) + (I*n + 2*I)*e^(2*I*f*x + 2*I*e) + I)*(
2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^((-I*f*n + 2*I*f)*x
- 4*I*f*x + (-I*n + 2*I)*e - (n - 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x +
2*I*e) + 1)) - (n - 2)*log(a/d) - 4*I*e)/(f*n^2 + f*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \sec{(e + fx)})^{2n} (ia(\tan{(e + fx)} - i))^{2-n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(2-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(2 - n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 2), x)
```

Mupad [B]
time = 7.82, size = 260, normalized size = 2.83

$$-e^{-e\,4i-f\,x\,4i}\left(\frac{d}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}\right)^{2\,n}\left(\frac{\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}}{2\,f\,n\,\left(n\,1i+1i\right)}+\frac{e^{e\,2i+f\,x\,2i}\,\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}\,\left(n+2\right)}{2\,f\,n\,\left(n\,1i+1i\right)}+\frac{e^{e\,4i+f\,x\,4i}\,\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}\,\left(n+1\right)}{2\,f\,n\,\left(n\,1i+1i\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(2 - n),x)
```

```
[Out] -exp(- e*4i - f*x*4i)*(d/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(
2*n)*((a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2
 - n)/(2*f*n*(n*1i + 1i)) + (exp(e*2i + f*x*2i)*(a - (a*(exp(e*2i + f*x*2i)
*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2 - n)*(n + 2))/(2*f*n*(n*1i + 1i)
) + (exp(e*4i + f*x*4i)*(a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i
+ f*x*2i) + 1))^(2 - n)*(n + 1))/(2*f*n*(n*1i + 1i)))
```

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$

Optimal. Leaf size=148

$$\frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(2+3n+n^2)}+\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)}+\frac{8ia^3(d\sec(e+}{f}$$

```
[Out] 4*I*a^2*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(n^2+3*n+2)+I*a*(d*
sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n)/f/(2+n)+8*I*a^3*(d*sec(f*x+e))^(
2*n)/f/n/(n^2+3*n+2)/((a+I*a*tan(f*x+e))^n)
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3575, 3574}

$$\frac{8ia^3(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n^2+3n+2)}+\frac{4ia^2(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n^2+3n+2)}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] ((4*I)*a^2*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(2 + 3
*n + n^2)) + (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f
*(2 + n)) + ((8*I)*a^3*(d*Sec[e + f*x])^(2*n))/(f*n*(2 + 3*n + n^2)*(a + I*
a*Tan[e + f*x])^n)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} + \frac{(4a)\int(d\sec(\cdots}{\cdots}$$
$$= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(2+3n+n^2)} + \frac{ia(d\sec(e\cdots}{\cdots}$$
$$= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(2+3n+n^2)} + \frac{ia(d\sec(e\cdots}{\cdots}$$

Mathematica [A]
time = 2.05, size = 129, normalized size = 0.87

$$\frac{ia^3\sec^2(e+fx)(d\sec(e+fx))^{2n}(\cos(3fx)+i\sin(3fx))\left(2(2+n)+(4+3n+n^2)\cos(2(e+fx))+in(3+n)\sin(2(e+fx))\right)(a+ia\tan(e+fx))^{-n}}{fn(1+n)(2+n)(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] (I*a^3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(2*n)*(Cos[3*f*x] + I*Sin[3*f*x])*(2
*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*(3 + n)*Sin[2*(e + f*x)])
)/(f*n*(1 + n)*(2 + n)*(Cos[f*x] + I*Sin[f*x])^3*(a + I*a*Tan[e + f*x])^n)
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{3-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.37, size = 184, normalized size = 1.24

$$\frac{\left((i n^2 + 3 i n + 2 i) e^{(6 i f x + 6 i e)} + (i n^2 + 5 i n + 6 i) e^{(4 i f x + 4 i e)} - 2\,(-i n - 3 i) e^{(2 i f x + 2 i e)} + 2 i\right) \left(\frac{2\, d e^{(i f x + i e)}}{e^{(2 i f x + 2 i e)} + 1}\right)^{2 n} e^{\left((-i f n + 3 i f) x - 6 i f x + (-i n + 3 i) e - (n - 3) \log\left(\frac{2\, d e^{(i f x + i e)}}{e^{(2 i f x + 2 i e)} + 1}\right) - (n - 3) \log\left(\frac{a}{d}\right) - 6 i e\right)}}{2\,(f n^3 + 3\, f n^2 + 2\, f n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n^2 + 3*I*n + 2*I)*e^(6*I*f*x + 6*I*e) + (I*n^2 + 5*I*n + 6*I)*e^(4
*I*f*x + 4*I*e) - 2*(-I*n - 3*I)*e^(2*I*f*x + 2*I*e) + 2*I)*(2*d*e^(I*f*x +
 I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^((-I*f*n + 3*I*f)*x - 6*I*f*x + (-
I*n + 3*I)*e - (n - 3)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) -
 (n - 3)*log(a/d) - 6*I*e)/(f*n^3 + 3*f*n^2 + 2*f*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec\left(e + f x\right)\right)^{2 n} \left(i a \left(\tan\left(e + f x\right) - i\right)\right)^{3 - n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(3-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(3 - n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 3), x)
```

Mupad [B]
time = 12.47, size = 321, normalized size = 2.17

$$-(\cos(6 e + 6 f x) - \sin(6 e + 6 f x)\, 1\mathrm{i}) \left(\frac{d}{\cos(e + f x)}\right)^{2 n} \left(\frac{\left(a + \frac{a \sin(e+f x)\, 1\mathrm{i}}{\cos(e+f x)}\right)^{3-n}}{f n\, (n^2\, 1\mathrm{i} + n\, 3\mathrm{i} + 2\mathrm{i})} + \frac{(\cos(4 e + 4 f x) + \sin(4 e + 4 f x)\, 1\mathrm{i}) \left(a + \frac{a \sin(e+f x)\, 1\mathrm{i}}{\cos(e+f x)}\right)^{3-n} (n^2 + 5 n + 6)}{2 f n\, (n^2\, 1\mathrm{i} + n\, 3\mathrm{i} + 2\mathrm{i})} + \frac{(\cos(6 e + 6 f x) + \sin(6 e + 6 f x)\, 1\mathrm{i}) \left(a + \frac{a \sin(e+f x)\, 1\mathrm{i}}{\cos(e+f x)}\right)^{3-n} (n^2 + 3 n + 2)}{2 f n\, (n^2\, 1\mathrm{i} + n\, 3\mathrm{i} + 2\mathrm{i})} + \frac{(2 n + 6)\, (\cos(2 e + 2 f x) + \sin(2 e + 2 f x)\, 1\mathrm{i}) \left(a + \frac{a \sin(e+f x)\, 1\mathrm{i}}{\cos(e+f x)}\right)^{3-n}}{2 f n\, (n^2\, 1\mathrm{i} + n\, 3\mathrm{i} + 2\mathrm{i})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(3 - n),x)
```

```
[Out] -(cos(6*e + 6*f*x) - sin(6*e + 6*f*x)*1i)*(d/cos(e + f*x))^(2*n)*((a + (a*s
in(e + f*x)*1i)/cos(e + f*x))^(3 - n)/(f*n*(n*3i + n^2*1i + 2i)) + ((cos(4*
e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3
 - n)*(5*n + n^2 + 6))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((cos(6*e + 6*f*x) +
sin(6*e + 6*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)*(3*n +
n^2 + 2))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((2*n + 6)*(cos(2*e + 2*f*x) + sin
(2*e + 2*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n))/(2*f*n*(n
*3i + n^2*1i + 2i)))
```

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3567, 3852}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^6(c+dx)}{6d}+a\int \sec^6(c+dx)\,dx\\ &= \frac{b\sec^6(c+dx)}{6d}-\frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 53, normalized size = 0.88

$$\frac{b\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A]
time = 0.20, size = 48, normalized size = 0.80

method	result	size
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$	48
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$	48
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^6}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/6*b/cos(d*x
+c)^6)
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.17

$$\frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 57, normalized size = 0.95

$$\frac{2\left(8\,a\cos\left(dx+c\right)^5+4\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [A]
time = 2.58, size = 56, normalized size = 0.93

$$\begin{cases} \frac{a\left(\frac{\tan^{5}{\left(c+dx \right)}}{5}+\frac{2\tan^{3}{\left(c+dx \right)}}{3}+\tan{\left(c+dx \right)}\right)+\frac{b\sec^{6}{\left(c+dx \right)}}{6}}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\tan{\left(c \right)}\right)\sec^{6}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + b*
sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**6, True))
```

Giac [A]
time = 0.55, size = 70, normalized size = 1.17

$$\frac{5\,b\tan\left(dx+c\right)^{6}+6\,a\tan\left(dx+c\right)^{5}+15\,b\tan\left(dx+c\right)^{4}+20\,a\tan\left(dx+c\right)^{3}+15\,b\tan\left(dx+c\right)^{2}+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.67, size = 68, normalized size = 1.13

$$\frac{\frac{b\,{\mathrm{tan}\left(c+d\,x\right)}^6}{6}+\frac{a\,{\mathrm{tan}\left(c+d\,x\right)}^5}{5}+\frac{b\,{\mathrm{tan}\left(c+d\,x\right)}^4}{2}+\frac{2\,a\,{\mathrm{tan}\left(c+d\,x\right)}^3}{3}+\frac{b\,{\mathrm{tan}\left(c+d\,x\right)}^2}{2}+a\,\mathrm{tan}\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (a*tan(c + d*x) + (2*a*tan(c + d*x)^3)/3 + (a*tan(c + d*x)^5)/5 + (b*tan(c
+ d*x)^2)/2 + (b*tan(c + d*x)^4)/2 + (b*tan(c + d*x)^6)/6)/d
```

3.508 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+c
)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 3853, 3855}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^5(c+dx)}{5d} + a\int \sec^5(c+dx)\,dx$$
$$= \frac{b\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx$$
$$= \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+}{8d}$$

Mathematica [A]
time = 0.24, size = 68, normalized size = 0.92

$$\frac{b\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
risch	$\frac{-15ia\,e^{9i(dx+c)}-70ia\,e^{7i(dx+c)}+128b\,e^{5i(dx+c)}+70ia\,e^{3i(dx+c)}+15ia\,e^{i(dx+c)}}{20d\left(e^{2i(dx+c)}+1\right)^5}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(e^{i(dx+c}\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+1/5*b/cos(d*x+c)^5)
```

Maxima [A]
time = 0.27, size = 86, normalized size = 1.16

$$-\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.38, size = 88, normalized size = 1.19

$$\frac{15\,a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^{3}+2\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+16\,b}{80\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-
sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c)
 + 16*b)/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)\sec^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.
time = 0.55, size = 141, normalized size = 1.91

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

Mupad [B]
time = 7.07, size = 175, normalized size = 2.36

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2\,b}{5}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2))
/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c/2
 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.509 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3567, 3852\}$

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^4(c+dx)}{4d}+a\int \sec^4(c+dx)\,dx\\
&= \frac{b\sec^4(c+dx)}{4d}-\frac{a\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 41, normalized size = 0.93

$$\frac{b\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.21, size = 38, normalized size = 0.86

method	result	size
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$	38
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$	38
risch	$\frac{4ia\,e^{4i(dx+c)}+4b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(e^{2i(dx+c)}+1\right)^4}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/4*b/cos(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.09

$$\frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 45, normalized size = 1.02

$$\frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

Sympy [A]
time = 1.83, size = 44, normalized size = 1.00

$$\begin{cases} \frac{a\left(\frac{\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{b\sec^4(c+dx)}{4}}{d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\sec^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + b*sec(c + d*x)**4/4)/d,
Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**4, True))
```

Giac [A]
time = 0.51, size = 48, normalized size = 1.09

$$\frac{3\,b\tan(dx+c)^4+4\,a\tan(dx+c)^3+6\,b\tan(dx+c)^2+12\,a\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Mupad [B]
time = 3.59, size = 46, normalized size = 1.05

$$\frac{\frac{b\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (a*tan(c + d*x) + (a*tan(c + d*x)^3)/3 + (b*tan(c + d*x)^2)/2 + (b*tan(c +
d*x)^4)/4)/d
```

3.510 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+c
)/d
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 3853, 3855}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^3(c+dx)}{3d} + a\int \sec^3(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.21, size = 50, normalized size = 0.96

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
risch	$\frac{-3ia\,e^{5i(dx+c)}+8b\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b/cos(
d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.17

$$-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 74, normalized size = 1.42

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(46) = 92.
time = 0.53, size = 99, normalized size = 1.90

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 5.41, size = 105, normalized size = 2.02

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(c
/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.511 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] 1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^2(c+dx)}{2d}+a\int \sec^2(c+dx)\,dx\\
&= \frac{b\sec^2(c+dx)}{2d}-\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.17, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\frac{b}{2\cos(dx+c)^2}+a\tan(dx+c)}{d}$	25
default	$\frac{\frac{b}{2\cos(dx+c)^2}+a\tan(dx+c)}{d}$	25
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{d\left(e^{2i(dx+c)}+1\right)^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b/cos(d*x+c)^2+a*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.71

$$\frac{(b\tan(dx+c)+a)^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(b*tan(d*x + c) + a)^2/(b*d)
```

Fricas [A]
time = 0.37, size = 30, normalized size = 1.07

$$\frac{2\,a\cos(dx+c)\sin(dx+c)+b}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [A]
time = 1.34, size = 34, normalized size = 1.21

$$\begin{cases} \frac{a\tan(c+dx)+\frac{b\tan^2(c+dx)}{2}}{d} & \text{for } d \neq 0 \\ x(a+b\tan(c))\sec^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**2/2)/d, Ne(d, 0)), (x*(a + b*t
an(c))*sec(c)**2, True))
```

Giac [A]
time = 0.52, size = 25, normalized size = 0.89

$$\frac{b\tan(dx+c)^2+2\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

Mupad [B]
time = 3.69, size = 23, normalized size = 0.82

$$\frac{\tan(c+d\,x)\,\left(2\,a+b\tan(c+d\,x)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^2,x)
[Out] (tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

3.512 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3567, 3855}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec(c+dx)}{d}+a\int \sec(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Maple [A]
time = 0.05, size = 32, normalized size = 1.33

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
default	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2b\,e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 31, normalized size = 1.29

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b/cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.41, size = 54, normalized size = 2.25

$$\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [A]
time = 2.59, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a \log(\tan(c+dx)+\sec(c+dx))+b\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*sec(c + d*x))/d, Ne(d, 0
)), (x*(a + b*tan(c))*sec(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.55, size = 54, normalized size = 2.25

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 3.75, size = 38, normalized size = 1.58

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x),x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.513 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=24

$$-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3567, 2717}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos(c+dx)}{d}+a\int \cos(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.92

$$-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*cos(d*x+c)+a*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(24) = 48.
time = 0.50, size = 129, normalized size = 5.38

$$-\frac{b\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)+2\,a\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right)^2-4\,b\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,c\right)^2-2\,a\tan\left(\frac{1}{2}\,dx\right)-2\,a\tan\left(\frac{1}{2}\,c\right)+b}{d\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+d\tan\left(\frac{1}{2}\,dx\right)^2+d\tan\left(\frac{1}{2}\,c\right)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*tan(1
/2*d*x)*tan(1/2*c)^2 - b*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*t
an(1/2*c)^2 - 2*a*tan(1/2*d*x) - 2*a*tan(1/2*c) + b)/(d*tan(1/2*d*x)^2*tan(
1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B]
time = 3.72, size = 38, normalized size = 1.58

$$-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.514 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-1/2*b*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 2715, 8}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (a*x)/2 - (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos^2(c+dx)}{2d}+a\int \cos^2(c+dx)\,dx\\
&= -\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\
&= \frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.20, size = 41, normalized size = 0.95

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
derivativedivides	$\frac{-\frac{b\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41
default	$\frac{-\frac{b\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2*b*cos(d*x+c)^2+a*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.52, size = 38, normalized size = 0.88

$$\frac{(dx+c)a+\frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.39, size = 35, normalized size = 0.81

$$\frac{adx-b\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(37) = 74.
time = 0.51, size = 146, normalized size = 3.40

$$\frac{2\,adx\tan(dx)^2\tan(c)^2+2\,adx\tan(dx)^2+2\,adx\tan(c)^2-b\tan(dx)^2\tan(c)^2-2\,a\tan(dx)^2\tan(c)-2\,a\tan(dx)\tan(c)^2+2\,adx+b\tan(dx)^2+4\,b\tan(dx)\tan(c)+b\tan(c)^2+2\,a\tan(dx)+2\,a\tan(c)-b}{4\,(d\tan(dx)^2\tan(c)^2+d\tan(dx)^2+d\tan(c)^2+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 -
b*tan(d*x)^2*tan(c)^2 - 2*a*tan(d*x)^2*tan(c) - 2*a*tan(d*x)*tan(c)^2 + 2*a
*d*x + b*tan(d*x)^2 + 4*b*tan(d*x)*tan(c) + b*tan(c)^2 + 2*a*tan(d*x) + 2*a
*tan(c) - b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B]
time = 3.68, size = 31, normalized size = 0.72

$$\frac{a\,x}{2}-\frac{\cos(c+d\,x)^2\,\left(\frac{b}{2}-\frac{a\tan(c+d\,x)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] (a*x)/2 - (cos(c + d*x)^2*(b/2 - (a*tan(c + d*x))/2))/d
```

3.515 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3567, 2713}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos^3(c+dx)}{3d}+a\int \cos^3(c+dx)\,dx\\
&= -\frac{b\cos^3(c+dx)}{3d}-\frac{a\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.19, size = 36, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}-\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	36
default	$\frac{\frac{a\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}-\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	36
risch	$-\frac{b\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(3dx+3c)}{12d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a*(cos(d*x+c)^2+2)*sin(d*x+c)-1/3*b*cos(d*x+c)^3)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.80

$$-\frac{b\cos\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A]
time = 0.41, size = 38, normalized size = 0.86

$$-\frac{b\cos\left(dx+c\right)^3-\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan (c + dx)) \cos^3 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)),x)

[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**3, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11886 vs. 2(40) = 80.
time = 2.52, size = 11886, normalized size = 270.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/96*(3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*

```
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^4*tan(1/2*c)^6 + 6*pi*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 +
 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/
2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
6*tan(1/2*c)^6 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^4 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*
x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2
*c)^4 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*t
an(1/2*c)^6 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 3
2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 +
 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^4 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 18*pi*b*tan(1/2*d*x)^
6*tan(1/2*c)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +...
```

Mupad [B]

time = 3.76, size = 47, normalized size = 1.07

$$\frac{2 a \sin \left(c+d x\right)}{3 d}-\frac{b \cos \left(c+d x\right)^{3}}{3 d}+\frac{a \cos \left(c+d x\right)^{2} \sin \left(c+d x\right)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*sin
(c + d*x))/(3*d)
```

3.516 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 2715, 8}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos^4(c+dx)}{4d} + a\int \cos^4(c+dx)\,dx$$
$$= -\frac{b\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+$$
$$= -\frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(}{4d}$$
$$= \frac{3ax}{8} - \frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx}{4}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d} - \frac{b\cos^4(c+dx)}{4d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.19, size = 52, normalized size = 0.80

method	result	size
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(\cos^4(dx+c)\right)}{4}}{d}$	52
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(\cos^4(dx+c)\right)}{4}}{d}$	52
risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/4*b*c
os(d*x+c)^4)
```

Maxima [A]
time = 0.47, size = 61, normalized size = 0.94

$$\frac{3\,(dx+c)a + \frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2\,b}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/(tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.78

$$-\frac{2\,b\cos\left(dx+c\right)^{4}-3\,adx-\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(57) = 114.
time = 0.64, size = 426, normalized size = 6.55

$$\frac{12\,adx\tan\left(dx\right)^{4}\tan\left(c\right)^{4}+24\,adx\tan\left(dx\right)^{4}\tan\left(c\right)^{2}+24\,adx\tan\left(dx\right)^{2}\tan\left(c\right)^{4}-5\,b\tan\left(dx\right)^{4}\tan\left(c\right)^{4}-20\,a\tan\left(dx\right)^{4}\tan\left(c\right)^{3}-20\,a\tan\left(dx\right)^{3}\tan\left(c\right)^{4}+12\,adx\tan\left(dx\right)^{4}+48\,adx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+6\,b\tan\left(dx\right)^{4}\tan\left(c\right)^{2}+32\,b\tan\left(dx\right)^{3}\tan\left(c\right)^{3}+12\,adx\tan\left(c\right)^{4}+6\,b\tan\left(dx\right)^{2}\tan\left(c\right)^{4}-12\,a\tan\left(dx\right)^{4}\tan\left(c\right)+24\,a\tan\left(dx\right)^{3}\tan\left(c\right)^{2}+24\,a\tan\left(dx\right)^{2}\tan\left(c\right)^{3}-12\,a\tan\left(dx\right)\tan\left(c\right)^{4}+24\,adx\tan\left(dx\right)^{2}+3\,b\tan\left(dx\right)^{4}+24\,adx\tan\left(c\right)^{2}-36\,b\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+3\,b\tan\left(c\right)^{4}+12\,a\tan\left(dx\right)^{3}-24\,a\tan\left(dx\right)^{2}\tan\left(c\right)-24\,a\tan\left(dx\right)\tan\left(c\right)^{2}+12\,a\tan\left(c\right)^{3}+12\,adx+6\,b\tan\left(dx\right)^{2}+32\,b\tan\left(dx\right)\tan\left(c\right)+6\,b\tan\left(c\right)^{2}+20\,a\tan\left(dx\right)+20\,a\tan\left(c\right)-5\,b}{32\,\left(d\tan\left(dx\right)^{4}\tan\left(c\right)^{4}+2\,d\tan\left(dx\right)^{4}\tan\left(c\right)^{2}+2\,d\tan\left(dx\right)^{2}\tan\left(c\right)^{4}+d\tan\left(dx\right)^{4}+4\,d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+d\tan\left(c\right)^{4}+2\,d\tan\left(dx\right)^{2}+2\,d\tan\left(c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d*
x*tan(d*x)^2*tan(c)^4 - 5*b*tan(d*x)^4*tan(c)^4 - 20*a*tan(d*x)^4*tan(c)^3
- 20*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)^2*tan(
c)^2 + 6*b*tan(d*x)^4*tan(c)^2 + 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x*tan(c)
^4 + 6*b*tan(d*x)^2*tan(c)^4 - 12*a*tan(d*x)^4*tan(c) + 24*a*tan(d*x)^3*tan
(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 12*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*
x)^2 + 3*b*tan(d*x)^4 + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x)^2*tan(c)^2 + 3*b*
tan(c)^4 + 12*a*tan(d*x)^3 - 24*a*tan(d*x)^2*tan(c) - 24*a*tan(d*x)*tan(c)^
```

```
2 + 12*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2 + 32*b*tan(d*x)*tan(c) + 6*b*
tan(c)^2 + 20*a*tan(d*x) + 20*a*tan(c) - 5*b)/(d*tan(d*x)^4*tan(c)^4 + 2*d*
tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)
^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B]
time = 3.71, size = 41, normalized size = 0.63

$$\frac{3\,a\,x}{8}+\frac{{\cos\left(c+d\,x\right)}^4\,\left(\frac{3\,a\,{\mathrm{tan}\left(c+d\,x\right)}^3}{8}+\frac{5\,a\,\mathrm{tan}\left(c+d\,x\right)}{8}-\frac{b}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (cos(c + d*x)^4*((5*a*tan(c + d*x))/8 - b/4 + (3*a*tan(c + d*x)
^3)/8))/d
```

3.517 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{ab\sec^8(c+dx)}{4d}+\frac{a^2\tan(c+dx)}{d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+$$

```
[Out] 1/4*a*b*sec(d*x+c)^8/d+a^2*tan(d*x+c)/d+1/3*(3*a^2+b^2)*tan(d*x+c)^3/d+3/5*
(a^2+b^2)*tan(d*x+c)^5/d+1/7*(a^2+3*b^2)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c)^
9/d
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3587, 710, 1824}

$$\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^8(c+dx)}{4d}+\frac{b^2\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^8)/(4*d) + (a^2*Tan[c + d*x])/d + ((3*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + (3*(a^2 + b^2)*Tan[c + d*x]^5)/(5*d) + ((a^2 + 3*b^2)*Tan[c
 + d*x]^7)/(7*d) + (b^2*Tan[c + d*x]^9)/(9*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^3\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3\left(-2ax+(a+x)^2\right)\,dx, x, b\,\mathrm{t}\right.}{bd}$$

$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{\text{Subst}\left(\int \left(a^2+\frac{(3a^2+b^2)x^2}{b^2}+\frac{3(a^2+b^2)x^4}{b^4}+\frac{(a^2+3b^2)}{b^6}\right.\right.}{bd}$$

$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{a^2\tan(c+dx)}{d} + \frac{(3a^2+b^2)\tan^3(c+dx)}{3d} + \frac{3(a}{}$$

Mathematica [A]

time = 0.63, size = 133, normalized size = 1.12

$$\frac{\tan(c+dx)\left(1260a^2+1260ab\tan(c+dx)+420(3a^2+b^2)\tan^2(c+dx)+1890ab\tan^3(c+dx)+756(a^2+b^2)\tan^4(c+dx)+1260ab\tan^5(c+dx)+180(a^2+3b^2)\tan^6(c+dx)+315ab\tan^7(c+dx)+140b^2\tan^8(c+dx)\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(1260*a^2 + 1260*a*b*Tan[c + d*x] + 420*(3*a^2 + b^2)*Tan[c +
 d*x]^2 + 1890*a*b*Tan[c + d*x]^3 + 756*(a^2 + b^2)*Tan[c + d*x]^4 + 1260*a
*b*Tan[c + d*x]^5 + 180*(a^2 + 3*b^2)*Tan[c + d*x]^6 + 315*a*b*Tan[c + d*x]
^7 + 140*b^2*Tan[c + d*x]^8))/(1260*d)
```

Maple [A]

time = 0.28, size = 138, normalized size = 1.16

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}\right.}{d}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}\right.}{d}$
risch	$\frac{32i\left(-630iab\,e^{10i(dx+c)}+315a^2e^{10i(dx+c)}-315b^2e^{10i(dx+c)}-630iab\,e^{8i(dx+c)}+819a^2e^{8i(dx+c)}+189b^2e^{8i(dx+c)}+756a^2\right.}{315d\left(e^{2i(dx+c)}+1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+1/4*a*b/cos(d*x+c)^8+b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*
```

```
x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos
(d*x+c)^3))
```

Maxima [A]
time = 0.26, size = 133, normalized size = 1.12

$$\frac{140\,b^2\tan\left(dx+c\right)^9+315\,ab\tan\left(dx+c\right)^8+1260\,ab\tan\left(dx+c\right)^6+180\,\left(a^2+3\,b^2\right)\tan\left(dx+c\right)^7+1890\,ab\tan\left(dx+c\right)^4+756\,\left(a^2+b^2\right)\tan\left(dx+c\right)^5+1260\,ab\tan\left(dx+c\right)^2+420\,\left(3\,a^2+b^2\right)\tan\left(dx+c\right)^3+1260\,a^2\tan\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 1260*a*b*tan(d*x
+ c)^6 + 180*(a^2 + 3*b^2)*tan(d*x + c)^7 + 1890*a*b*tan(d*x + c)^4 + 756*(
a^2 + b^2)*tan(d*x + c)^5 + 1260*a*b*tan(d*x + c)^2 + 420*(3*a^2 + b^2)*tan
(d*x + c)^3 + 1260*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 122, normalized size = 1.03

$$\frac{315\,ab\cos\left(dx+c\right)+4\,\left(16\,\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^8+8\,\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^6+6\,\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^4+5\,\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^2+35\,b^2\right)\sin\left(dx+c\right)}{1260\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c) + 4*(16*(9*a^2 - b^2)*cos(d*x + c)^8 + 8*(9*a^
2 - b^2)*cos(d*x + c)^6 + 6*(9*a^2 - b^2)*cos(d*x + c)^4 + 5*(9*a^2 - b^2)*
cos(d*x + c)^2 + 35*b^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^2\sec^8\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**8, x)
```

Giac [A]
time = 0.64, size = 156, normalized size = 1.31

$$\frac{140\,b^2\tan\left(dx+c\right)^9+315\,ab\tan\left(dx+c\right)^8+180\,a^2\tan\left(dx+c\right)^7+540\,b^2\tan\left(dx+c\right)^7+1260\,ab\tan\left(dx+c\right)^6+756\,a^2\tan\left(dx+c\right)^5+756\,b^2\tan\left(dx+c\right)^5+1890\,ab\tan\left(dx+c\right)^4+1260\,a^2\tan\left(dx+c\right)^3+420\,b^2\tan\left(dx+c\right)^3+1260\,ab\tan\left(dx+c\right)^2+1260\,a^2\tan\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 180*a^2*tan(d*x +
 c)^7 + 540*b^2*tan(d*x + c)^7 + 1260*a*b*tan(d*x + c)^6 + 756*a^2*tan(d*x
+ c)^5 + 756*b^2*tan(d*x + c)^5 + 1890*a*b*tan(d*x + c)^4 + 1260*a^2*tan(d*
x + c)^3 + 420*b^2*tan(d*x + c)^3 + 1260*a*b*tan(d*x + c)^2 + 1260*a^2*tan(
d*x + c))/d
```

Mupad [B]
time = 3.68, size = 132, normalized size = 1.11

$$\frac{\tan(c+d\,x)^3\,\left(a^2+\frac{b^2}{3}\right)+a^2\tan(c+d\,x)+\tan(c+d\,x)^5\,\left(\frac{3\,a^2}{5}+\frac{3\,b^2}{5}\right)+\tan(c+d\,x)^7\,\left(\frac{a^2}{7}+\frac{3\,b^2}{7}\right)+\frac{b^2\tan(c+d\,x)^9}{9}+a\,b\tan(c+d\,x)^2+\frac{3\,a\,b\tan(c+d\,x)^4}{2}+a\,b\tan(c+d\,x)^6+\frac{a\,b\tan(c+d\,x)^8}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] (tan(c + d*x)^3*(a^2 + b^2/3) + a^2*tan(c + d*x) + tan(c + d*x)^5*((3*a^2)/
5 + (3*b^2)/5) + tan(c + d*x)^7*(a^2/7 + (3*b^2)/7) + (b^2*tan(c + d*x)^9)/
9 + a*b*tan(c + d*x)^2 + (3*a*b*tan(c + d*x)^4)/2 + a*b*tan(c + d*x)^6 + (a
*b*tan(c + d*x)^8)/4)/d
```

3.518 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=97

$$\frac{ab\sec^6(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] 1/3*a*b*sec(d*x+c)^6/d+a^2*tan(d*x+c)/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+1/5*
(a^2+2*b^2)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3587, 710, 1824}

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^6(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^6)/(3*d) + (a^2*Tan[c + d*x])/d + ((2*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)
/(7*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^2 dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^2 (-2ax+(a+x)^2)\,dx, x, b\,\text{t}\right.}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(a^2+\frac{(2a^2+b^2)x^2}{b^2}+\frac{(a^2+2b^2)x^4}{b^4}+\frac{x^6}{b^4}\right) dx\right.}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{a^2\tan(c+dx)}{d} + \frac{(2a^2+b^2)\tan^3(c+dx)}{3d} + \frac{(a^2}{}$$

Mathematica [A]
time = 0.73, size = 104, normalized size = 1.07

$$\frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b^2)\tan^4(c+dx)+35ab\tan^5(c+dx)+15b^2\tan^6(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Maple [A]
time = 0.22, size = 110, normalized size = 1.13

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
risch	$\frac{16i\left(-140iab\,e^{8i(dx+c)}+70a^2e^{8i(dx+c)}-70b^2e^{8i(dx+c)}-140iab\,e^{6i(dx+c)}+175a^2e^{6i(dx+c)}+35b^2e^{6i(dx+c)}+147a^2e^{4i(dx+\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/cos
(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5
+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.26, size = 104, normalized size = 1.07

$$\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+105\,ab\tan\left(dx+c\right)^4+21\,\left(a^2+2\,b^2\right)\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^2+35\,\left(2\,a^2+b^2\right)\tan\left(dx+c\right)^3+105\,a^2\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 105*a*b*tan(d*x + c)
^4 + 21*(a^2 + 2*b^2)*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^2 + 35*(2*a^2 +
 b^2)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 100, normalized size = 1.03

$$\frac{35\,ab\cos\left(dx+c\right)+\left(8\,\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^6+4\,\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^4+3\,\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^2+15\,b^2\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b
^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^6\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**6, x)
```

Giac [A]
time = 0.66, size = 118, normalized size = 1.22

$$\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4+70\,a^2\tan\left(dx+c\right)^3+35\,b^2\tan\left(dx+c\right)^3+105\,ab\tan\left(dx+c\right)^2+105\,a^2\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^
5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3
+ 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.57, size = 102, normalized size = 1.05

$$\frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\tan(c+d\,x)^5\left(\frac{a^2}{5}+\frac{2\,b^2}{5}\right)+\frac{b^2\tan(c+d\,x)^7}{7}+a\,b\tan(c+d\,x)^2+a\,b\tan(c+d\,x)^4+\frac{a\,b\tan(c+d\,x)^6}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c + d*x) + tan(c + d*x)^3*((2*a^2)/3 + b^2/3) + tan(c + d*x)^5*(a^
2/5 + (2*b^2)/5) + (b^2*tan(c + d*x)^7)/7 + a*b*tan(c + d*x)^2 + a*b*tan(c
+ d*x)^4 + (a*b*tan(c + d*x)^6)/3)/d
```

3.519 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=75

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}$$

```
[Out] 1/3*(a^2+b^2)*(a+b*tan(d*x+c))^3/b^3/d-1/2*a*(a+b*tan(d*x+c))^4/b^3/d+1/5*(
a+b*tan(d*x+c))^5/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/(3*b^3*d) - (a*(a + b*Tan[c + d*x])^4)
/(2*b^3*d) + (a + b*Tan[c + d*x])^5/(5*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^2}{b^2}-\frac{2a(a+x)^3}{b^2}+\frac{(a+x)^4}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}+\frac{(a+b\tan}{5b}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 54, normalized size = 0.72

$$\frac{(a+b\tan(c+dx))^3\left(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c + d*x]^2))/(30*b^3*d)
```

Maple [A]
time = 0.22, size = 82, normalized size = 1.09

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
risch	$\frac{4i\left(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5b^2\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.95

$$\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+30\,ab\tan\left(dx+c\right)^2+10\left(a^2+b^2\right)\tan\left(dx+c\right)^3+30\,a^2\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^2 + 10*(a^2 + b^2)*tan(d*x + c)^3 + 30*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 79, normalized size = 1.05

$$\frac{15\,ab\cos\left(dx+c\right)+2\left(2\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^4+\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{30\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^2 \sec^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.65, size = 80, normalized size = 1.07

$$\frac{6\,b^2\tan(dx+c)^5 + 15\,ab\tan(dx+c)^4 + 10\,a^2\tan(dx+c)^3 + 10\,b^2\tan(dx+c)^3 + 30\,ab\tan(dx+c)^2 + 30\,a^2\tan(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 3.55, size = 71, normalized size = 0.95

$$\frac{a^2\tan(c+d\,x) + \tan(c+d\,x)^3\left(\frac{a^2}{3}+\frac{b^2}{3}\right) + \frac{b^2\tan(c+d\,x)^5}{5} + a\,b\tan(c+d\,x)^2 + \frac{a\,b\tan(c+d\,x)^4}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c + d*x) + tan(c + d*x)^3*(a^2/3 + b^2/3) + (b^2*tan(c + d*x)^5)/5
 + a*b*tan(c + d*x)^2 + (a*b*tan(c + d*x)^4)/2)/d
```

3.520 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

```
[Out] 1/3*(a+b*tan(d*x+c))^3/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a + b*Tan[c + d*x])^3/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.
time = 0.76, size = 56, normalized size = 2.55

$$\frac{\sec^2(c+dx)\left(6ab+\left(3a^2+b^2+\left(3a^2-b^2\right)\cos(2(c+dx))\right)\tan(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(6*a*b + (3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Tan
[c + d*x]))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(20) = 40$.
time = 0.24, size = 48, normalized size = 2.18

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
risch	$-\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+a*b/cos(d*x+c)^2+a^2*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$\frac{(b\tan(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(d*x + c) + a)^3/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.
time = 0.37, size = 55, normalized size = 2.50

$$\frac{3\,ab\cos(dx+c)+\left((3\,a^2-b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^2\sec^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.59, size = 41, normalized size = 1.86

$$\frac{b^2\tan{(dx+c)}^3+3\,ab\tan{(dx+c)}^2+3\,a^2\tan{(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

Mupad [**B**]
time = 3.56, size = 39, normalized size = 1.77

$$\frac{a^2\tan(c+d\,x)+a\,b\tan(c+d\,x)^2+\frac{b^2\tan(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^3)/3 + a*b*tan(c + d*x)^2)/d
```

3.521 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\left(a^2+b^2\right)x-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3587, 737, 209}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x
]))/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d} + \frac{(a^2+b^2)\,\text{S}}{}$$

$$= \frac{1}{2}\left(a^2+b^2\right)x - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 1.06

$$\frac{2(a^2+b^2)\left(c+dx\right)-2ab\cos(2(c+dx))+(a^2-b^2)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.21, size = 70, normalized size = 1.43

method	result	size
risch	$\frac{a^2x}{2}+\frac{b^2x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{a^2\sin(2dx+2c)}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$	64
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
default	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-a*b*cos(d*x+c)^2+a^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.50, size = 55, normalized size = 1.12

$$\frac{(a^2+b^2)(dx+c)-\frac{2\,ab-\left(a^2-b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + b^2)*(d*x + c) - (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x +
c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.06

$$-\frac{2\,ab\cos\left(dx+c\right)^{2}-(a^{2}+b^{2})dx-(a^{2}-b^{2})\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(46) = 92.
time = 0.61, size = 245, normalized size = 5.00

$$\frac{a^{2}dx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+b^{2}dx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+a^{2}dx\tan\left(dx\right)^{2}+b^{2}dx\tan\left(dx\right)^{2}+a^{2}dx\tan\left(c\right)^{2}+b^{2}dx\tan\left(c\right)^{2}-ab\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-a^{2}\tan\left(dx\right)^{2}\tan\left(c\right)+b^{2}\tan\left(dx\right)^{2}\tan\left(c\right)-a^{2}\tan\left(dx\right)\tan\left(c\right)^{2}+b^{2}\tan\left(dx\right)\tan\left(c\right)^{2}+a^{2}dx+b^{2}dx+ab\tan\left(dx\right)^{2}+4\,ab\tan\left(dx\right)\tan\left(c\right)+ab\tan\left(c\right)^{2}+a^{2}\tan\left(dx\right)-b^{2}\tan\left(dx\right)+a^{2}\tan\left(c\right)-b^{2}\tan\left(c\right)-ab}{2\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+d\tan\left(dx\right)^{2}+d\tan\left(c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^2*tan(c)^2 + b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*ta
n(d*x)^2 + b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(c)^2 + b^2*d*x*tan(c)^2 - a*b*t
an(d*x)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(c) + b^2*tan(d*x)^2*tan(c) - a^2*ta
n(d*x)*tan(c)^2 + b^2*tan(d*x)*tan(c)^2 + a^2*d*x + b^2*d*x + a*b*tan(d*x)^
2 + 4*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2 + a^2*tan(d*x) - b^2*tan(d*x) + a^
2*tan(c) - b^2*tan(c) - a*b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(
c)^2 + d)
```

Mupad [B]
time = 3.58, size = 50, normalized size = 1.02

$$x\left(\frac{a^2}{2}+\frac{b^2}{2}\right)-\frac{\cos\left(c+d\,x\right)^2\,\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
[Out] x*(a^2/2 + b^2/2) - (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)))/d
```

3.522 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=88

$$\frac{1}{8}\left(3a^2+b^2\right)x-\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d}-\frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+d$$

```
[Out] 1/8*(3*a^2+b^2)*x-1/4*cos(d*x+c)^4*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d-1/8*
cos(d*x+c)^2*(2*a*b-(3*a^2+b^2)*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3587, 753, 653, 209}

$$-\frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}+\frac{1}{8}x\left(3a^2+b^2\right)-\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*x)/8 - (Cos[c + d*x]^4*(b - a*Tan[c + d*x])*(a + b*Tan[c + d
*x]))/(4*d) - (Cos[c + d*x]^2*(2*a*b - (3*a^2 + b^2)*Tan[c + d*x]))/(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} + \frac{b\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} - \frac{\cos^2(c+}{}$$

$$= \frac{1}{8}\left(3a^2+b^2\right)x - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx}{4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(88) = 176.
time = 3.24, size = 216, normalized size = 2.45

$$\frac{\frac{(3a^2+b^2)\left(2ab\sqrt{-b^2}\,(2a^2+b^2)-2ab\sqrt{-b^2}\,(a^2+b^2)\cos(2(c+dx))+b(a^2+b^2)^2\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-b(a^2+b^2)^2\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)-\sqrt{-b^2}\,(-a^4+b^4)\sin(2(c+dx))\right)}{\sqrt{-b^2}}+4(a^2+b^2)\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^3}{16\left(a^2+b^2\right)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (((3*a^2 + b^2)*(2*a*b*Sqrt[-b^2]*(2*a^2 + b^2) - 2*a*b*Sqrt[-b^2]*(a^2 + b
^2)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - b
*(a^2 + b^2)^2*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - Sqrt[-b^2]*(-a^4 + b^4)*S
in[2*(c + d*x)]))/Sqrt[-b^2] + 4*(a^2 + b^2)*Cos[c + d*x]^4*(b + a*Tan[c +
d*x])*(a + b*Tan[c + d*x])^3)/(16*(a^2 + b^2)^2*d)
```

Maple [A]
time = 0.22, size = 97, normalized size = 1.10

method	result

derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\cdots\right.}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\cdots\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{a^2\sin(4dx+4c)}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{a^2\sin(2dx+2c)}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 0.97

$$\frac{(3\,a^2+b^2)(dx+c)+\frac{(3\,a^2+b^2)\tan(dx+c)^3-4\,ab+(5\,a^2-b^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + b^2)*(d*x + c) + ((3*a^2 + b^2)*tan(d*x + c)^3 - 4*a*b + (5*a
^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 75, normalized size = 0.85

$$-\frac{4\,ab\cos(dx+c)^4-(3\,a^2+b^2)dx-\left(2\,(a^2-b^2)\cos(dx+c)^3+(3\,a^2+b^2)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))^2\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2286 vs. $2(83) = 166$.
time = 2.67, size = 2286, normalized size = 25.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x
*tan(d*x)^4*tan(c)^4 + 8*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan
(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*pi*b^2
*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b^2*arctan((tan(d*x) +
 tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a^2*d*x*tan(d*
x)^4*tan(c)^2 + 16*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2
+ 48*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*b^2*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*b^
2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*t
an(d*x)^2*tan(c)^4 - 20*a*b*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^4 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*t
an(c)^2 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)
^4*tan(c)^2 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan
(d*x)^4*tan(c)^2 - 40*a^2*tan(d*x)^4*tan(c)^3 + 8*b^2*tan(d*x)^4*tan(c)^3 +
 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d
*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*b^2*arctan((tan(d*x) +
tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^2*tan(d*x)^3
*tan(c)^4 + 8*b^2*tan(d*x)^3*tan(c)^4 + 24*a^2*d*x*tan(d*x)^4 + 8*b^2*d*x*t
an(d*x)^4 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^2*d*x*tan(d*x)^2*tan(c)^2 + 32*b^2*d*x*
tan(d*x)^2*tan(c)^2 + 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 24*a*b*tan(d*x)^4*tan(c)
^2 + 128*a*b*tan(d*x)^3*tan(c)^3 + 24*a^2*d*x*tan(c)^4 + 8*b^2*d*x*tan(c)^4
```

```
 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2
*tan(c))*tan(c)^4 + 24*a*b*tan(d*x)^2*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(d*x)^4 - 24*a^2*tan(d*x)^4*tan(c) - 8*b^2*tan(d*x)^4*tan(c) + 6*pi*b^2*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*b^2*arctan((tan(d*x) + tan(c))/(t
an(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*b^2*arctan(-(tan(d*x) - tan(c
))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*a^2*tan(d*x)^3*tan(c)^2
- 48*b^2*tan(d*x)^3*tan(c)^2 + 48*a^2*tan(d*x)^2*tan(c)^3 - 48*b^2*tan(d*x)
^2*tan(c)^3 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c
)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 2
4*a^2*tan(d*x)*tan(c)^4 - 8*b^2*tan(d*x)*tan(c)^4 + 48*a^2*d*x*tan(d*x)^2 +
 16*b^2*d*x*tan(d*x)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 12*a*b*tan(d*x)^4 + 48*a^2*d*x*
tan(c)^2 + 16*b^2*d*x*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 144*a*b*tan(d*x)^2*tan(c)
^2 + 12*a*b*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*b^2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*b^2*arctan(-(ta
n(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a^2*tan(d*x)^3 + 8*
b^2*tan(d*x)^3 - 48*a^2*tan(d*x)^2*tan(c) + 48*b^2*tan(d*x)^2*tan(c) + 12*b
^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 12*b^2*arct
an(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a^2*tan(d*x)*t
an(c)^2 + 48*b^2*tan(d*x)*tan(c)^2 + 24*a^2*tan(c)^3 + 8*b^2*tan(c)^3 + 24*
a^2*d*x + 8*b^2*d*x + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c)) + 24*a*b*tan(d*x)^2 + 128*a*b*tan(d*x)*tan(c) +
 24*a*b*tan(c)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))
- 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a^2*tan(d*x
) - 8*b^2*tan(d*x) + 40*a^2*tan(c) - 8*b^2*tan(c) - 20*a*b)/(d*tan(d*x)^4*t
an(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4
+ 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B]
time = 3.66, size = 83, normalized size = 0.94

$$x\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)+\frac{\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{5\,a^2}{8}-\frac{b^2}{8}\right)\,\tan(c+d\,x)-\frac{a\,b}{2}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
[Out] x*((3*a^2)/8 + b^2/8) + (tan(c + d*x)*((5*a^2)/8 - b^2/8) - (a*b)/2 + tan(c
 + d*x)^3*((3*a^2)/8 + b^2/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.523 $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{5(8a^2-b^2)\tanh^{-1}(\sin(c+dx))}{128d}+\frac{9ab\sec^7(c+dx)}{56d}+\frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d}+\frac{5(8a^2-b^2)\sec^3(c}{19}$$

```
[Out] 5/128*(8*a^2-b^2)*arctanh(sin(d*x+c))/d+9/56*a*b*sec(d*x+c)^7/d+5/128*(8*a^
2-b^2)*sec(d*x+c)*tan(d*x+c)/d+5/192*(8*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+
1/48*(8*a^2-b^2)*sec(d*x+c)^5*tan(d*x+c)/d+1/8*b*sec(d*x+c)^7*(a+b*tan(d*x+
c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3589, 3567, 3853, 3855}

$$\frac{5(8a^2-b^2)\tanh^{-1}(\sin(c+dx))}{128d}+\frac{(8a^2-b^2)\tan(c+dx)\sec^5(c+dx)}{48d}+\frac{5(8a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{192d}+\frac{5(8a^2-b^2)\tan(c+dx)\sec(c+dx)}{128d}+\frac{9ab\sec^7(c+dx)}{56d}+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (5*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(128*d) + (9*a*b*Sec[c + d*x]^7)/(5
6*d) + (5*(8*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*(8*a^2 - b^
2)*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + ((8*a^2 - b^2)*Sec[c + d*x]^5*Tan
[c + d*x])/(48*d) + (b*Sec[c + d*x]^7*(a + b*Tan[c + d*x]))/(8*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d} + \frac{1}{8}\int \sec^7(c+dx)\left(8a^2-b^2+9\right.$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d} + \frac{1}{8}(8a^2-b^2)$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d} + \frac{b\sec^7(c+}{}$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d} + \frac{(8a^2-b}{}$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d} + \frac{5(8a^2-b}{}$$
$$= \frac{5(8a^2-b^2)\tanh^{-1}(\sin(c+dx))}{128d} + \frac{9ab\sec^7(c+dx)}{56d} + \frac{5(8a^2-b^2)\mathrm{s}}{}$$

Mathematica [A]
time = 0.85, size = 131, normalized size = 0.80

$$\frac{105(8a^2-b^2)\tanh^{-1}(\sin(c+dx))+105(8a^2-b^2)\sec(c+dx)\tan(c+dx)+70(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)+56(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)+48b\sec^7(c+dx)(16a+7b\tan(c+dx))}{2688d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (105*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 105*(8*a^2 - b^2)*Sec[c + d*x]*T
an[c + d*x] + 70*(8*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 56*(8*a^2 - b^
2)*Sec[c + d*x]^5*Tan[c + d*x] + 48*b*Sec[c + d*x]^7*(16*a + 7*b*Tan[c + d*
x]))/(2688*d)
```

Maple [A]
time = 0.27, size = 177, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$

default	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(\ldots}{8\cos(\ldots}\right.}{d}$
risch	$-\frac{i\left(840a^2e^{15i(dx+c)}-105b^2e^{15i(dx+c)}+6440a^2e^{13i(dx+c)}-805b^2e^{13i(dx+c)}+21448a^2e^{11i(dx+c)}-2681b^2e^{11i(dx+c)}+15\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+2/7*a*b/cos(d*x+c)^7+b^2*(1/8*sin(d*x+c)^3
/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4
+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d
*x+c))))
```

Maxima [A]

time = 0.26, size = 220, normalized size = 1.35

$$\frac{7\,b^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-56\,a^2\left(\frac{2\left(15\,\sin(dx+c)^5-40\,\sin(dx+c)^3+33\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{1536\,ab}{\cos(dx+c)^7}}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5376*(7*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3
 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 56*a^2*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
 c) + 1) + 15*log(sin(d*x + c) - 1)) + 1536*a*b/cos(d*x + c)^7)/d
```

Fricas [A]

time = 0.39, size = 163, normalized size = 1.00

$$\frac{105\,(8\,a^2-b^2)\cos\left(dx+c\right)^8\log\left(\sin\left(dx+c\right)+1\right)-105\,(8\,a^2-b^2)\cos\left(dx+c\right)^8\log\left(-\sin\left(dx+c\right)+1\right)+1536\,ab\cos\left(dx+c\right)+14\left(15\,(8\,a^2-b^2)\cos\left(dx+c\right)^6+10\,(8\,a^2-b^2)\cos\left(dx+c\right)^4+8\,(8\,a^2-b^2)\cos\left(dx+c\right)^2+48\,b^2\right)\sin\left(dx+c\right)}{5376\,d\cos\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5376*(105*(8*a^2 - b^2)*cos(d*x + c)^8*log(sin(d*x + c) + 1) - 105*(8*a^2
 - b^2)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) + 1536*a*b*cos(d*x + c) + 14*
(15*(8*a^2 - b^2)*cos(d*x + c)^6 + 10*(8*a^2 - b^2)*cos(d*x + c)^4 + 8*(8*a
^2 - b^2)*cos(d*x + c)^2 + 48*b^2)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^7\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(151) = 302.
time = 0.63, size = 437, normalized size = 2.68

$$\frac{105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(1848\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+105\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-5376\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-3416\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2779\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5376\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+6328\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+6265\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-26880\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-4760\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+12355\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+26880\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4760\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12355\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-16128\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6328\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6265\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16128\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3416\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2779\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-768\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1848\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+768\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^8}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2688*(105*(8*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*a^2 -
 b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(1848*a^2*tan(1/2*d*x + 1/2*c)
^15 + 105*b^2*tan(1/2*d*x + 1/2*c)^15 - 5376*a*b*tan(1/2*d*x + 1/2*c)^14 -
3416*a^2*tan(1/2*d*x + 1/2*c)^13 + 2779*b^2*tan(1/2*d*x + 1/2*c)^13 + 5376*
a*b*tan(1/2*d*x + 1/2*c)^12 + 6328*a^2*tan(1/2*d*x + 1/2*c)^11 + 6265*b^2*t
an(1/2*d*x + 1/2*c)^11 - 26880*a*b*tan(1/2*d*x + 1/2*c)^10 - 4760*a^2*tan(1
/2*d*x + 1/2*c)^9 + 12355*b^2*tan(1/2*d*x + 1/2*c)^9 + 26880*a*b*tan(1/2*d*
x + 1/2*c)^8 - 4760*a^2*tan(1/2*d*x + 1/2*c)^7 + 12355*b^2*tan(1/2*d*x + 1/
2*c)^7 - 16128*a*b*tan(1/2*d*x + 1/2*c)^6 + 6328*a^2*tan(1/2*d*x + 1/2*c)^5
 + 6265*b^2*tan(1/2*d*x + 1/2*c)^5 + 16128*a*b*tan(1/2*d*x + 1/2*c)^4 - 341
6*a^2*tan(1/2*d*x + 1/2*c)^3 + 2779*b^2*tan(1/2*d*x + 1/2*c)^3 - 768*a*b*ta
n(1/2*d*x + 1/2*c)^2 + 1848*a^2*tan(1/2*d*x + 1/2*c) + 105*b^2*tan(1/2*d*x
+ 1/2*c) + 768*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B]
time = 7.10, size = 432, normalized size = 2.65

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^2}{8}-\frac{5\,b^2}{64}\right)}{d}+\frac{\left(\frac{11\,a^2}{8}+\frac{5\,b^2}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\left(\frac{397\,b^2}{192}-\frac{61\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\left(\frac{113\,a^2}{24}+\frac{895\,b^2}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-20\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{1765\,b^2}{192}-\frac{85\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+20\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\left(\frac{1765\,b^2}{192}-\frac{85\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\left(\frac{113\,a^2}{24}+\frac{895\,b^2}{192}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\left(\frac{397\,b^2}{192}-\frac{61\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-\frac{4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{7}+\left(\frac{11\,a^2}{8}+\frac{5\,b^2}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a\,b}{7}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-56\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-56\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+28\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^7,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^2)/8 - (5*b^2)/64))/d + ((4*a*b)/7 + tan(c
/2 + (d*x)/2)^15*((11*a^2)/8 + (5*b^2)/64) - tan(c/2 + (d*x)/2)^3*((61*a^2)
/24 - (397*b^2)/192) - tan(c/2 + (d*x)/2)^13*((61*a^2)/24 - (397*b^2)/192)
+ tan(c/2 + (d*x)/2)^5*((113*a^2)/24 + (895*b^2)/192) + tan(c/2 + (d*x)/2)^
11*((113*a^2)/24 + (895*b^2)/192) - tan(c/2 + (d*x)/2)^7*((85*a^2)/24 - (17
65*b^2)/192) - tan(c/2 + (d*x)/2)^9*((85*a^2)/24 - (1765*b^2)/192) + tan(c/
2 + (d*x)/2)*((11*a^2)/8 + (5*b^2)/64) - (4*a*b*tan(c/2 + (d*x)/2)^2)/7 + 1
2*a*b*tan(c/2 + (d*x)/2)^4 - 12*a*b*tan(c/2 + (d*x)/2)^6 + 20*a*b*tan(c/2 +
 (d*x)/2)^8 - 20*a*b*tan(c/2 + (d*x)/2)^10 + 4*a*b*tan(c/2 + (d*x)/2)^12 -
```

```
4*a*b*tan(c/2 + (d*x)/2)^14)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x
)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (
d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 +
 (d*x)/2)^16 + 1))
```

3.524 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=131

$$\frac{(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{7ab\sec^5(c+dx)}{30d}+\frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(6a^2-b^2)\sec^3(c+dx)}{24d}$$

```
[Out] 1/16*(6*a^2-b^2)*arctanh(sin(d*x+c))/d+7/30*a*b*sec(d*x+c)^5/d+1/16*(6*a^2-
b^2)*sec(d*x+c)*tan(d*x+c)/d+1/24*(6*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/6
*b*sec(d*x+c)^5*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3589, 3567, 3853, 3855}

$$\frac{(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(6a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(6a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{7ab\sec^5(c+dx)}{30d}+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + (7*a*b*Sec[c + d*x]^5)/(30*d
) + ((6*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((6*a^2 - b^2)*Sec[c
 + d*x]^3*Tan[c + d*x])/(24*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c + d*x]))/(6
*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d} + \frac{1}{6}\int \sec^5(c+dx)\left(6a^2-b^2+\right.$$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d} + \frac{1}{6}\left(6a^2-b\right.$$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c}{}$$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(6a^2-b^2}{}$$
$$= \frac{(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{7ab\sec^5(c+dx)}{30d} + \frac{(6a^2-b^2)\text{se}}{}$$

Mathematica [A]
time = 0.59, size = 104, normalized size = 0.79

$$\frac{15(6a^2-b^2)\tanh^{-1}(\sin(c+dx))+15(6a^2-b^2)\sec(c+dx)\tan(c+dx)+10(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)+8b\sec^5(c+dx)(12a+5b\tan(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
[Out] (15*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 15*(6*a^2 - b^2)*Sec[c + d*x]*Tan
[c + d*x] + 10*(6*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 8*b*Sec[c + d*x]
^5*(12*a + 5*b*Tan[c + d*x]))/(240*d)
```

Maple [A]
time = 0.24, size = 149, normalized size = 1.14

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$

risch	$-\frac{i(90a^2e^{11i(dx+c)}-15b^2e^{11i(dx+c)}+510a^2e^{9i(dx+c)}-85b^2e^{9i(dx+c)}+420a^2e^{7i(dx+c)}+570b^2e^{7i(dx+c)}+1536iab\,e^{7i(dx+c}}{120d(e^{2i(d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+2/5*a*b/cos(d*x+c)^5+b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*si
n(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16
*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 180, normalized size = 1.37

$$\frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-30\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{192\,ab}{\cos(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin
(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.38, size = 142, normalized size = 1.08

$$\frac{15\,(6\,a^2-b^2)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\,(6\,a^2-b^2)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+192\,ab\cos\left(dx+c\right)+10\left(3\,(6\,a^2-b^2)\cos\left(dx+c\right)^4+2\,(6\,a^2-b^2)\cos\left(dx+c\right)^2+8\,b^2\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(
6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(121) = 242.
time = 0.68, size = 343, normalized size = 2.62

$$\frac{15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-210\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+235\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+390\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-960\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+390\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+960\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+235\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-96\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 6.38, size = 328, normalized size = 2.50

$$\frac{\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(\frac{a^2}{2}+\frac{13\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{a^2}{2}+\frac{13\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a\,b}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}-\frac{b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] ((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)
^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan(c
/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2)/4
 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c/2 +
 (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)^6 +
4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
+ (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

3.525 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(4a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5ab\sec^3(c+dx)}{12d}+\frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}$$

```
[Out] 1/8*(4*a^2-b^2)*arctanh(sin(d*x+c))/d+5/12*a*b*sec(d*x+c)^3/d+1/8*(4*a^2-b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)^3*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3589, 3567, 3853, 3855}

$$\frac{(4a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{5ab\sec^3(c+dx)}{12d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((4*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a*b*Sec[c + d*x]^3)/(12*d)
 + ((4*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a +
 b*Tan[c + d*x]))/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{1}{4}\int \sec^3(c+dx)\left(4a^2-b^2+\right.$$
$$= \frac{5ab\sec^3(c+dx)}{12d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{1}{4}\left(4a^2-b\right.$$
$$= \frac{5ab\sec^3(c+dx)}{12d} + \frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+}{}$$
$$= \frac{(4a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{5ab\sec^3(c+dx)}{12d} + \frac{(4a^2-b^2)\,\text{se}}{}$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 1.21

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\sec^3(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} - \frac{b^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A]
time = 0.24, size = 118, normalized size = 1.19

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
risch	$-\frac{i\left(12a^2e^{7i(dx+c)}-3b^2e^{7i(dx+c)}+12a^2e^{5i(dx+c)}+21b^2e^{5i(dx+c)}+64iab\,e^{5i(dx+c)}-12a^2e^{3i(dx+c)}-21b^2e^{3i(dx+c)}+64ia\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b/
cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)
^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 129, normalized size = 1.30

$$\frac{3\,b^2\left(\frac{2\,\left(\sin\left(dx+c\right)^3+\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{32\,ab}{\cos\left(dx+c\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 120, normalized size = 1.21

$$\frac{3\,\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+32\,ab\cos\left(dx+c\right)+6\,\left(\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^2+2\,b^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(91) = 182.
time = 0.59, size = 249, normalized size = 2.52

$$\frac{3\,\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 6.24, size = 216, normalized size = 2.18

$$\frac{\left(a^2+\frac{b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\left(a^2+\frac{b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] ((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 +
 b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(a^
2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/2)^
4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan
(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

3.526 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=65

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/2*a*b*sec(d*x+c)/d+1/2*b*sec(d*x+c)
*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3589, 3567, 3855}

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b*Sec[c + d*x])/(2*d) +
(b*Sec[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d} + \frac{1}{2}\int \sec(c+dx)\left(2a^2-b^2+3ab\right.$$
$$= \frac{3ab\sec(c+dx)}{2d} + \frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d} + \frac{1}{2}(2a^2-b^2)$$
$$= \frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{3ab\sec(c+dx)}{2d} + \frac{b\sec(c+dx)(a}{}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 1.03

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{b^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.12, size = 83, normalized size = 1.28

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$\frac{b\left(-ib\,e^{3i(dx+c)}+4a\,e^{3i(dx+c)}+ib\,e^{i(dx+c)}+4a\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2} - \frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d} + \frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{2d} + \frac{a^2\ln\left(e^{i(dx+c)}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b/cos(d*x+c)+b^2*(1/2*sin(d*x+c)^3/c
os(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.26

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-4a^2\log(\sec(dx+c)+\tan(dx+c))-\frac{8ab}{\cos(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b/cos(d*x + c))/d

Fricas [A]
time = 0.38, size = 96, normalized size = 1.48

$$\frac{(2a^2-b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2a^2-b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+8ab\cos(dx+c)+2b^2\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(59) = 118.
time = 0.65, size = 122, normalized size = 1.88

$$\frac{(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4ab\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d

Mupad [B]
time = 4.17, size = 106, normalized size = 1.63

$$\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4ab}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^2-b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
+ (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (at
anh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

3.527 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=47

$$\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+(a^2-b^2)*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {3588}

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + ((a^2 - b^2)*Sin[c
 + d*x])/d
```

Rule 3588

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2/sec[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[b^2*(ArcTanh[Sin[e + f*x]]/f), x] + (-Simp[2*a*b*(Cos[e + f*x]
/f), x] + Simp[(a^2 - b^2)*(Sin[e + f*x]/f), x]) /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.16, size = 84, normalized size = 1.79

$$\frac{-2ab\cos(c+dx)+b^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a^2-b^2)\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Maple [A]
time = 0.20, size = 53, normalized size = 1.13

method	result
derivativedivides	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2\cos(dx+c)ab+a^2\sin(dx+c)}{d}$
default	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2\cos(dx+c)ab+a^2\sin(dx+c)}{d}$
risch	$-\frac{e^{i(dx+c)}ab}{d}-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{\ln(e^{i(dx+c)}+i)b^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*cos(d*x+c)*a*b+a^2*sin(d
*x+c))
```

Maxima [A]
time = 0.27, size = 60, normalized size = 1.28

$$\frac{b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-4ab\cos(dx+c)+2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 62, normalized size = 1.32

$$-\frac{4ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2(a^2-b^2)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan(c+dx))^2\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(47) = 94$.
time = 0.81, size = 1316, normalized size = 28.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")

```
[Out] -1/2*(b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
+ 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*
c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
 + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan
(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
1))*tan(1/2*d*x)^2 + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^2*tan(1/2*d*x)^2
*tan(1/2*c) + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan
(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
1))*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)
^2 + 1))*tan(1/2*c)^2 + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*b^2*tan(1/2*d*x
)*tan(1/2*c)^2 - 4*a*b*tan(1/2*d*x)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c) - 4*
a*b*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
```

```
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)
^2 + 1)) - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
 - 4*a^2*tan(1/2*d*x) + 4*b^2*tan(1/2*d*x) - 4*a^2*tan(1/2*c) + 4*b^2*tan(1
/2*c) + 4*a*b)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/
2*c)^2 + d)
```

Mupad [B]
time = 3.89, size = 66, normalized size = 1.40

$$\frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.528 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=90

$$-\frac{ab\cos^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d}-\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] -1/6*a*b*cos(d*x+c)^3/d+1/2*(2*a^2+b^2)*sin(d*x+c)/d-1/6*(2*a^2+b^2)*sin(d*
x+c)^3/d-1/2*b*cos(d*x+c)^3*(a+b*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3589, 3567, 2713}

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d}-\frac{ab\cos^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/6*(a*b*Cos[c + d*x]^3)/d + ((2*a^2 + b^2)*Sin[c + d*x])/(2*d) - ((2*a^2
+ b^2)*Sin[c + d*x]^3)/(6*d) - (b*Cos[c + d*x]^3*(a + b*Tan[c + d*x]))/(2*d
)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{1}{2}\int \cos^3(c+dx)\left(-2a^2-b\right.$$
$$= -\frac{ab\cos^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{1}{2}\left(-2a^2\right.$$
$$= -\frac{ab\cos^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{(2a^2+b^2}{}$$
$$= -\frac{ab\cos^3(c+dx)}{6d} + \frac{(2a^2+b^2)\sin(c+dx)}{2d} - \frac{(2a^2+b^2)\sin^3(c+d}{6d}$$

Mathematica [A]
time = 0.51, size = 64, normalized size = 0.71

$$\frac{-3ab\cos(c+dx)-ab\cos(3(c+dx))+\left(5a^2+b^2+(a^2-b^2)\cos(2(c+dx))\right)\sin(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x] - a*b*Cos[3*(c + d*x)] + (5*a^2 + b^2 + (a^2 - b^2)*Co
s[2*(c + d*x)])*Sin[c + d*x])/(6*d)
```

Maple [A]
time = 0.21, size = 52, normalized size = 0.58

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	52
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$	52
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{b^2\sin(dx+c)}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(cos(d*x+c)^2+2)*sin
(d*x+c))
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.58

$$-\frac{2\,ab\cos\left(dx+c\right)^3-b^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

Fricas [A]
time = 0.41, size = 53, normalized size = 0.59

$$-\frac{2\,ab\cos\left(dx+c\right)^{3}-\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+2\,a^{2}+b^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11162 vs. 2(82) = 164.
time = 18.11, size = 11162, normalized size = 124.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
```

x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2
*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 6*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*a*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a
*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*
c)^6 + 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*t
an(1/2*d*x)^4*tan(1/2*c)^6 + 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^
2 + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2

```
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 27*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^4*tan(1/2*c)^4 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*
tan(1/2*d*x)^6*tan(1/2*c)^4 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 96*a^2*tan(1
/2*d*x)^6*tan(1/2*c)^5 + 9*pi*a*b*sgn(tan(1/2*d...
```

Mupad [B]
time = 3.76, size = 77, normalized size = 0.86

$$\frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c + d
*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

3.529 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=114

$$-\frac{3ab\cos^5(c+dx)}{20d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)}{}$$

```
[Out] -3/20*a*b*cos(d*x+c)^5/d+1/4*(4*a^2+b^2)*sin(d*x+c)/d-1/6*(4*a^2+b^2)*sin(d
*x+c)^3/d+1/20*(4*a^2+b^2)*sin(d*x+c)^5/d-1/4*b*cos(d*x+c)^5*(a+b*tan(d*x+c
))/d
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3589, 3567, 2713}

$$\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x]^5)/(20*d) + ((4*a^2 + b^2)*Sin[c + d*x])/(4*d) - ((4*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((4*a^2 + b^2)*Sin[c + d*x]^5)/(20*d) - (
b*Cos[c + d*x]^5*(a + b*Tan[c + d*x]))/(4*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{1}{4}\int \cos^5(c+dx)\left(-4a^2-b^2\right.$$
$$= -\frac{3ab\cos^5(c+dx)}{20d} - \frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{1}{4}\left(-4a^2\right.$$
$$= -\frac{3ab\cos^5(c+dx)}{20d} - \frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{(4a^2+b^2}{}$$
$$= -\frac{3ab\cos^5(c+dx)}{20d} + \frac{(4a^2+b^2)\sin(c+dx)}{4d} - \frac{(4a^2+b^2)\sin^3(c+d}{6d}$$

Mathematica [A]
time = 0.24, size = 116, normalized size = 1.02

$$\frac{-60ab\cos(c+dx)-30ab\cos(3(c+dx))-6ab\cos(5(c+dx))+150a^2\sin(c+dx)+30b^2\sin(c+dx)+25a^2\sin(3(c+dx))-5b^2\sin(3(c+dx))+3a^2\sin(5(c+dx))-3b^2\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-60*a*b*Cos[c + d*x] - 30*a*b*Cos[3*(c + d*x)] - 6*a*b*Cos[5*(c + d*x)] +
150*a^2*Sin[c + d*x] + 30*b^2*Sin[c + d*x] + 25*a^2*Sin[3*(c + d*x)] - 5*b^
2*Sin[3*(c + d*x)] + 3*a^2*Sin[5*(c + d*x)] - 3*b^2*Sin[5*(c + d*x)])/(240*
d)
```

Maple [A]
time = 0.20, size = 88, normalized size = 0.77

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^2\sin(dx+c)}{8d}+\frac{b^2\sin(dx+c)}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)b^2}{80d}-\frac{ab\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.68

$$-\frac{6\,ab\cos\left(dx+c\right)^{5}-\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a^{2}+\left(3\,\sin\left(dx+c\right)^{5}-5\,\sin\left(dx+c\right)^{3}\right)b^{2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.65

$$-\frac{6\,ab\cos\left(dx+c\right)^{5}-\left(3\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+\left(4\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+8\,a^{2}+2\,b^{2}\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{5}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28204 vs. 2(104) = 208.
time = 50.38, size = 28204, normalized size = 247.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/960*(45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
```

+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 60*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 300*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 300*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(

```
1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 384*a*b*tan(1/2*d*x)^10*tan(1/
2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 450*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^6 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 +
 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 450*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 450*a*b*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(...
```

Mupad [B]

time = 3.77, size = 115, normalized size = 1.01

$$\frac{2\left(\frac{3\sin(c+dx)\,a^2\cos(c+dx)^4}{2}+2\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^2-3\,a\,b\cos\left(c+d\,x\right)^5-\frac{3\sin(c+dx)\,b^2\cos(c+dx)^4}{2}+\frac{\sin(c+dx)\,b^2\cos(c+dx)^2}{2}+\sin\left(c+d\,x\right)\,b^2\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d*
x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c + d*
x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(15*
d)
```

3.530 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{5ab\cos^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^7}{42d}$$

```
[Out] -5/42*a*b*cos(d*x+c)^7/d+1/6*(6*a^2+b^2)*sin(d*x+c)/d-1/6*(6*a^2+b^2)*sin(d
*x+c)^3/d+1/10*(6*a^2+b^2)*sin(d*x+c)^5/d-1/42*(6*a^2+b^2)*sin(d*x+c)^7/d-1
/6*b*cos(d*x+c)^7*(a+b*tan(d*x+c))/d
```

Rubi [A]

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3589, 3567, 2713}

$$-\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{5ab\cos^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b*Cos[c + d*x]^7)/(42*d) + ((6*a^2 + b^2)*Sin[c + d*x])/(6*d) - ((6*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((6*a^2 + b^2)*Sin[c + d*x]^5)/(10*d) - (
(6*a^2 + b^2)*Sin[c + d*x]^7)/(42*d) - (b*Cos[c + d*x]^7*(a + b*Tan[c + d*x
]))/(6*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{1}{6}\int \cos^7(c+dx)\left(-6a^2-b\right.$$

$$= -\frac{5ab\cos^7(c+dx)}{42d} - \frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{1}{6}\left(-6a^2\right.$$

$$= -\frac{5ab\cos^7(c+dx)}{42d} - \frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{(6a^2+b}{}$$

$$= -\frac{5ab\cos^7(c+dx)}{42d} + \frac{(6a^2+b^2)\sin(c+dx)}{6d} - \frac{(6a^2+b^2)\sin^3(c+}{6d}$$

Mathematica [A]
time = 0.48, size = 154, normalized size = 1.12

$$-\frac{1050ab\cos(c+dx)+630ab\cos(3(c+dx))+210ab\cos(5(c+dx))+30ab\cos(7(c+dx))-3675a^2\sin(c+dx)-525b^2\sin(c+dx)-735a^2\sin(3(c+dx))+35b^2\sin(3(c+dx))-147a^2\sin(5(c+dx))+63b^2\sin(5(c+dx))-15a^2\sin(7(c+dx))+15b^2\sin(7(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/6720*(1050*a*b*Cos[c + d*x] + 630*a*b*Cos[3*(c + d*x)] + 210*a*b*Cos[5*(
c + d*x)] + 30*a*b*Cos[7*(c + d*x)] - 3675*a^2*Sin[c + d*x] - 525*b^2*Sin[c
 + d*x] - 735*a^2*Sin[3*(c + d*x)] + 35*b^2*Sin[3*(c + d*x)] - 147*a^2*Sin[
5*(c + d*x)] + 63*b^2*Sin[5*(c + d*x)] - 15*a^2*Sin[7*(c + d*x)] + 15*b^2*S
in[7*(c + d*x)])/d
```

Maple [A]
time = 0.20, size = 108, normalized size = 0.78

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos}{}\right.}{}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos}{}\right.}{}}{d}$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^2\sin(dx+c)}{64d}+\frac{5b^2\sin(dx+c)}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)b^2}{448d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 98, normalized size = 0.71

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}+3\,\left(5\,\sin\left(dx+c\right)^{7}-21\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}-35\,\sin\left(dx+c\right)\right)a^{2}-\left(15\,\sin\left(dx+c\right)^{7}-42\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}\right)b^{2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.68

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}-\left(15\,\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+3\,\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}+4\,\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+48\,a^{2}+8\,b^{2}\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{7}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52002 vs. 2(126) = 252.
time = 97.44, size = 52002, normalized size = 376.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/26880*(945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
 - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan
(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 8400*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/
2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c
)^14 - 5880*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)
^14 - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x
)^14*tan(1/2*c)^14 - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 58800*pi*a*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
```

```
2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 58800*pi*a*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 7680*a*b*tan(1/2*d*x)^1
4*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 19845*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
2*tan(1/2*c)^12 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 - 41160*p
i*a*b*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*...
```

Mupad [B]

time = 3.84, size = 176, normalized size = 1.28

$$\frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{4\,b^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{b^2\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^2\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}-\frac{2\,a\,b\,{\cos\left(c+d\,x\right)}^7}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
+ d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d)
+ (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c + d
*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c + d*x)
^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

3.531 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{3a^2b\sec^8(c+dx)}{8d}+\frac{a^3\tan(c+dx)}{d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d}+\frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+$$

```
[Out] 3/8*a^2*b*sec(d*x+c)^8/d+a^3*tan(d*x+c)/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b^
3*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b^3*tan(d*x+c)^6/d+1/
7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b^3*tan(d*x+c)^8/d+1/3*a*b^2*tan(d*x+c)^
9/d+1/10*b^3*tan(d*x+c)^10/d
```

Rubi [A]
time = 0.12, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3587, 710, 1824}

$$\frac{a^3\tan(c+dx)}{d}+\frac{a(a^2+9b^2)\tan^7(c+dx)}{7d}+\frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{3a^2b\sec^8(c+dx)}{8d}+\frac{ab^2\tan^9(c+dx)}{3d}+\frac{b^3\tan^{10}(c+dx)}{10d}+\frac{3b^3\tan^8(c+dx)}{8d}+\frac{b^3\tan^6(c+dx)}{2d}+\frac{b^3\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*Sec[c + d*x]^8)/(8*d) + (a^3*Tan[c + d*x])/d + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)*Tan[c + d*x
]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(2*d) + (a*(a^2 + 9*b^2)*Tan[c + d*x]^7)/
(7*d) + (3*b^3*Tan[c + d*x]^8)/(8*d) + (a*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*
Tan[c + d*x]^10)/(10*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^3 dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3 \left(-3a^2x+(a+x)^3\right) dx, x,\right.}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(a^3+\frac{3a(a^2+b^2)x^2}{b^2}+x^3+\frac{3a(a^2+3b^2)x^4}{b^4}+\right.\right.}{}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{a(a^2+b^2)\tan^3(c+dx)}{d} + \frac{b^3}{}$$

Mathematica [A]
time = 2.14, size = 177, normalized size = 0.91

$$\frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4-\frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5+\frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+dx))^6-\frac{4}{7}a(5a^2+3b^2)(a+b\tan(c+dx))^7+\frac{3}{8}(5a^2+b^2)(a+b\tan(c+dx))^8-\frac{2}{3}a(a+b\tan(c+dx))^9+\frac{1}{10}(a+b\tan(c+dx))^{10}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

Maple [A]
time = 0.26, size = 219, normalized size = 1.13

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^4(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^4(dx+c)\right)}{40\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{20\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{40\cos(dx+c)^4}\right)+3b^2a\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\right.}{d}$
default	$\frac{b^3\left(\frac{\sin^4(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^4(dx+c)\right)}{40\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{20\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{40\cos(dx+c)^4}\right)+3b^2a\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\right.}{d}$
risch	$-\frac{32\left(10iab^2e^{2i(dx+c)}-3ia^3-315a^2be^{12i(dx+c)}+105b^3e^{12i(dx+c)}-525ia^3e^{8i(dx+c)}+45iab^2e^{4i(dx+c)}-630a^2be^{10i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/
20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)+3*b^2*a*(1/9*s
in(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/
cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*a^2*b/cos(d*x+c)^8-a^3*(
-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 176, normalized size = 0.91

$$\frac{84\,b^3\tan\left(dx+c\right)^{10}+280\,ab^2\tan\left(dx+c\right)^9+315\,(a^2b+b^3)\tan\left(dx+c\right)^8+120\,(a^3+9\,ab^2)\tan\left(dx+c\right)^7+420\,(3\,a^2b+b^3)\tan\left(dx+c\right)^6+504\,(a^3+3\,ab^2)\tan\left(dx+c\right)^5+1260\,a^2b\tan\left(dx+c\right)^2+210\,(9\,a^2b+b^3)\tan\left(dx+c\right)^4+840\,a^3\tan\left(dx+c\right)+840\,(a^3+ab^2)\tan\left(dx+c\right)^3}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*(a^2*b + b^3
)*tan(d*x + c)^8 + 120*(a^3 + 9*a*b^2)*tan(d*x + c)^7 + 420*(3*a^2*b + b^3)
*tan(d*x + c)^6 + 504*(a^3 + 3*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(d*x +
 c)^2 + 210*(9*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 840*(a^
3 + a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A]
time = 0.44, size = 150, normalized size = 0.77

$$\frac{\left(84\,b^3+105\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2+8\,\left(16\,(3\,a^3-ab^2)\cos\left(dx+c\right)^9+8\,(3\,a^3-ab^2)\cos\left(dx+c\right)^7+6\,(3\,a^3-ab^2)\cos\left(dx+c\right)^5+35\,ab^2\cos\left(dx+c\right)+5\,(3\,a^3-ab^2)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right)}{840\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*
cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d
*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d
*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^3\sec^8\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**8, x)
```

Giac [A]
time = 0.92, size = 220, normalized size = 1.13

$$\frac{84\,b^3\tan\left(dx+c\right)^{10}+280\,ab^2\tan\left(dx+c\right)^9+315\,a^2b\tan\left(dx+c\right)^8+315\,b^3\tan\left(dx+c\right)^8+120\,a^3\tan\left(dx+c\right)^7+1080\,ab^2\tan\left(dx+c\right)^7+1260\,a^2b\tan\left(dx+c\right)^6+420\,b^3\tan\left(dx+c\right)^6+504\,a^3\tan\left(dx+c\right)^5+1512\,ab^2\tan\left(dx+c\right)^5+1890\,a^2b\tan\left(dx+c\right)^4+210\,b^3\tan\left(dx+c\right)^4+840\,a^3\tan\left(dx+c\right)^3+840\,ab^2\tan\left(dx+c\right)^3+1260\,a^2b\tan\left(dx+c\right)^2+840\,a^3\tan\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*
x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan
(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*
tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 21
0*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 +
1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.68, size = 175, normalized size = 0.90

$$\frac{\tan(c+d\,x)^5\left(\frac{3\,a^3}{5}+\frac{9\,a\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^3}{7}+\frac{9\,a\,b^2}{7}\right)+\tan(c+d\,x)^6\left(\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right)+\tan(c+d\,x)^4\left(\frac{9\,a^2\,b}{4}+\frac{b^3}{4}\right)+a^3\tan(c+d\,x)+\frac{b^3\tan(c+d\,x)^{10}}{10}+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+\frac{a\,b^2\tan(c+d\,x)^9}{3}+a\tan(c+d\,x)^3\,(a^2+b^2)+\frac{3\,b\tan(c+d\,x)^8\,(a^2+b^2)}{8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (tan(c + d*x)^5*((9*a*b^2)/5 + (3*a^3)/5) + tan(c + d*x)^7*((9*a*b^2)/7 + a
^3/7) + tan(c + d*x)^6*((3*a^2*b)/2 + b^3/2) + tan(c + d*x)^4*((9*a^2*b)/4
+ b^3/4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^10)/10 + (3*a^2*b*tan(c + d
*x)^2)/2 + (a*b^2*tan(c + d*x)^9)/3 + a*tan(c + d*x)^3*(a^2 + b^2) + (3*b*t
an(c + d*x)^8*(a^2 + b^2))/8)/d
```

3.532 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d}+\frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d}-\frac{4a(a+$$

```
[Out] 1/4*(a^2+b^2)^2*(a+b*tan(d*x+c))^4/b^5/d-4/5*a*(a^2+b^2)*(a+b*tan(d*x+c))^5
/b^5/d+1/3*(3*a^2+b^2)*(a+b*tan(d*x+c))^6/b^5/d-4/7*a*(a+b*tan(d*x+c))^7/b^
5/d+1/8*(a+b*tan(d*x+c))^8/b^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d}+\frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d}+\frac{(a+b\tan(c+dx))^8}{8b^5d}-\frac{4a(a+b\tan(c+dx))^7}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 + b^2)*(a + b*
Tan[c + d*x])^5)/(5*b^5*d) + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Tan[c + d*x])^7)/(7*b^5*d) + (a + b*Tan[c + d*x])^8/(8*b^5
*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^2 dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2(a+x)^3}{b^4} - \frac{4a(a^2+b^2)(a+x)^4}{b^4} + \frac{2(3a^2+b^2)(a+x)^5}{b^4} - \frac{4a(a+x)^6}{b^4}\right.\right.}{bd}$$

$$= \frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d} +$$

Mathematica [A]
time = 0.64, size = 115, normalized size = 0.83

$$\frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4 - \frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5 + \frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6 - \frac{4}{7}a(a+b\tan(c+dx))^7 + \frac{1}{8}(a+b\tan(c+dx))^8}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Maple [A]
time = 0.26, size = 173, normalized size = 1.25

method	result
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3b^2a\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3b^2a\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
risch	$-\frac{16\left(-245ia^3e^{8i(dx+c)}+24iab^2e^{2i(dx+c)}-210a^2be^{10i(dx+c)}+70b^3e^{10i(dx+c)}-70ia^3e^{10i(dx+c)}+210iab^2e^{10i(dx+c)}-420\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/c
os(d*x+c)^6+3*b^2*a*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*
x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+
1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.03

$$\frac{105\,b^3\tan\left(dx+c\right)^8+360\,ab^2\tan\left(dx+c\right)^7+140\,\left(3\,a^2b+2\,b^3\right)\tan\left(dx+c\right)^6+168\,\left(a^3+6\,ab^2\right)\tan\left(dx+c\right)^5+1260\,a^2b\tan\left(dx+c\right)^2+210\,\left(6\,a^2b+b^3\right)\tan\left(dx+c\right)^4+840\,a^3\tan\left(dx+c\right)+280\,\left(2\,a^3+3\,ab^2\right)\tan\left(dx+c\right)^3}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 140*(3*a^2*b + 2
*b^3)*tan(d*x + c)^6 + 168*(a^3 + 6*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(
d*x + c)^2 + 210*(6*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 28
0*(2*a^3 + 3*a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 128, normalized size = 0.93

$$\frac{105\,b^3+140\,\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+8\,\left(8\,\left(7\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^7+4\,\left(7\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+45\,ab^2\cos\left(dx+c\right)+3\,\left(7\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{840\,d\cos\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\sec^6\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**6, x)
```

Giac [A]
time = 0.84, size = 166, normalized size = 1.20

$$\frac{105\,b^3\tan\left(dx+c\right)^8+360\,ab^2\tan\left(dx+c\right)^7+420\,a^2b\tan\left(dx+c\right)^6+280\,b^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(dx+c\right)^5+1008\,ab^2\tan\left(dx+c\right)^5+1260\,a^2b\tan\left(dx+c\right)^4+210\,b^3\tan\left(dx+c\right)^4+560\,a^3\tan\left(dx+c\right)^3+840\,ab^2\tan\left(dx+c\right)^3+1260\,a^2b\tan\left(dx+c\right)^2+840\,a^3\tan\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
```

```
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.60, size = 139, normalized size = 1.01

$$\frac{\tan(c+dx)^3\left(\frac{2a^3}{3}+ab^2\right)+\tan(c+dx)^5\left(\frac{a^3}{5}+\frac{6ab^2}{5}\right)+\tan(c+dx)^6\left(\frac{a^2b}{2}+\frac{b^3}{3}\right)+\tan(c+dx)^4\left(\frac{3a^2b}{2}+\frac{b^3}{4}\right)+a^3\tan(c+dx)+\frac{b^3\tan(c+dx)^8}{8}+\frac{3a^2b\tan(c+dx)^2}{2}+\frac{3ab^2\tan(c+dx)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (tan(c + d*x)^3*(a*b^2 + (2*a^3)/3) + tan(c + d*x)^5*((6*a*b^2)/5 + a^3/5)
+ tan(c + d*x)^6*((a^2*b)/2 + b^3/3) + tan(c + d*x)^4*((3*a^2*b)/2 + b^3/4)
 + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^8)/8 + (3*a^2*b*tan(c + d*x)^2)/2 +
 (3*a*b^2*tan(c + d*x)^7)/7)/d
```

3.533 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=75

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}$$

```
[Out] 1/4*(a^2+b^2)*(a+b*tan(d*x+c))^4/b^3/d-2/5*a*(a+b*tan(d*x+c))^5/b^3/d+1/6*(
a+b*tan(d*x+c))^6/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/(4*b^3*d) - (2*a*(a + b*Tan[c + d*x])^
5)/(5*b^3*d) + (a + b*Tan[c + d*x])^6/(6*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^3}{b^2}-\frac{2a(a+x)^4}{b^2}+\frac{(a+x)^5}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 54, normalized size = 0.72

$$\frac{(a+b\tan(c+dx))^4\left(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

Maple [A]
time = 0.24, size = 127, normalized size = 1.69

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3b^2a\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)}\right.}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3b^2a\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)}\right.}{d}$
risch	$-\frac{4(-15ia^3e^{8i(dx+c)}+45ia\,b^2e^{8i(dx+c)}-45a^2b\,e^{8i(dx+c)}+15b^3e^{8i(dx+c)}-50ia^3e^{6i(dx+c)}+30ia\,b^2e^{6i(dx+c)}-90a^2b\,e^{6i(dx}}{15d(e^{2i(d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*b^2*a
*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*si
n(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.27, size = 98, normalized size = 1.31

$$\frac{10\,b^3\tan(dx+c)^6+36\,ab^2\tan(dx+c)^5+90\,a^2b\tan(dx+c)^2+15\,(3\,a^2b+b^3)\tan(dx+c)^4+60\,a^3\tan(dx+c)+20\,(a^3+3\,ab^2)\tan(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 90*a^2*b*tan(d*x +
c)^2 + 15*(3*a^2*b + b^3)*tan(d*x + c)^4 + 60*a^3*tan(d*x + c) + 20*(a^3 +
3*a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 105, normalized size = 1.40

$$\frac{10\,b^3+15\,(3\,a^2b-b^3)\cos(dx+c)^2+4\,\left(2\,(5\,a^3-3\,ab^2)\cos(dx+c)^5+9\,ab^2\cos(dx+c)+(5\,a^3-3\,ab^2)\cos(dx+c)^3\right)\sin(dx+c)}{60\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c
os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan (c + dx))^3 \sec^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.93, size = 112, normalized size = 1.49

$$\frac{10\, b^3 \tan (dx + c)^6 + 36\, ab^2 \tan (dx + c)^5 + 45\, a^2 b \tan (dx + c)^4 + 15\, b^3 \tan (dx + c)^4 + 20\, a^3 \tan (dx + c)^3 + 60\, ab^2 \tan (dx + c)^3 + 90\, a^2 b \tan (dx + c)^2 + 60\, a^3 \tan (dx + c)}{60\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.57, size = 97, normalized size = 1.29

$$\frac{\tan(c + d\,x)^3 \left(\frac{a^3}{3} + a\,b^2\right) + \tan(c + d\,x)^4 \left(\frac{3\,a^2\,b}{4} + \frac{b^3}{4}\right) + a^3 \tan(c + d\,x) + \frac{b^3 \tan(c+d\,x)^6}{6} + \frac{3\,a^2\,b \tan(c+d\,x)^2}{2} + \frac{3\,a\,b^2 \tan(c+d\,x)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (tan(c + d*x)^3*(a*b^2 + a^3/3) + tan(c + d*x)^4*((3*a^2*b)/4 + b^3/4) + a^
3*tan(c + d*x) + (b^3*tan(c + d*x)^6)/6 + (3*a^2*b*tan(c + d*x)^2)/2 + (3*a
*b^2*tan(c + d*x)^5)/5)/d
```

3.534 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

```
[Out] 1/4*(a+b*tan(d*x+c))^4/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a + b*Tan[c + d*x])^4/(4*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^4}{4bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(22) = 44.
time = 0.65, size = 79, normalized size = 3.59

$$\frac{\sec^4(c+dx)\left((6a^2b-2b^3)\cos(2(c+dx))+a(6ab+2(a^2+b^2)\sin(2(c+dx))+(a^2-b^2)\sin(4(c+dx)))\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*((6*a^2*b - 2*b^3)*Cos[2*(c + d*x)] + a*(6*a*b + 2*(a^2 + b
^2)*Sin[2*(c + d*x)] + (a^2 - b^2)*Sin[4*(c + d*x)])))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.
time = 0.22, size = 72, normalized size = 3.27

method	result
derivativedivides	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+\frac{b^2a\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
default	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+\frac{b^2a\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
risch	$-\frac{2\left(-ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-3a^2b\,e^{6i(dx+c)}+b^3e^{6i(dx+c)}-3ia^3e^{4i(dx+c)}+3ia\,b^2e^{4i(dx+c)}-6a^2b\,e^{4i(dx+c)}-3ia\right.}{d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4+b^2*a*sin(d*x+c)^3/cos(d*x+c)^3+3/2*
a^2*b/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.91

$$\frac{(b\tan(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*tan(d*x + c) + a)^4/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.
time = 0.37, size = 78, normalized size = 3.55

$$\frac{b^3+2\left(3\,a^2b-b^3\right)\cos(dx+c)^2+4\left(ab^2\cos(dx+c)+\left(a^3-ab^2\right)\cos(dx+c)^3\right)\sin(dx+c)}{4\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(c + dx))^3 \sec^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(20) = 40.
time = 0.87, size = 57, normalized size = 2.59

$$\frac{b^3\tan(dx + c)^4 + 4\,ab^2\tan(dx + c)^3 + 6\,a^2b\tan(dx + c)^2 + 4\,a^3\tan(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 3.57, size = 55, normalized size = 2.50

$$\frac{a^3\tan(c + d\,x) + \frac{3\,a^2\,b\tan(c+d\,x)^2}{2} + a\,b^2\tan(c + d\,x)^3 + \frac{b^3\tan(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (a^3*tan(c + d*x) + (b^3*tan(c + d*x)^4)/4 + (3*a^2*b*tan(c + d*x)^2)/2 + a
*b^2*tan(c + d*x)^3)/d
```

3.535 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}a\left(a^2+3b^2\right)x-\frac{b^3\log(\cos(c+dx))}{d}-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 1/2*a*(a^2+3*b^2)*x-b^3*ln(cos(d*x+c))/d-1/2*a*b^2*tan(d*x+c)/d-1/2*cos(d*x
+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))^2/d
```

Rubi [A]

time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3587, 753, 788, 649, 209, 266}

$$\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}-\frac{b^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Cos[c + d*x]])/d - (a*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

```
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+\right.}{2d}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+\right.}{2d}$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\cos(c+dx))}{d} - \frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(86) = 172.
time = 0.82, size = 401, normalized size = 4.66

$$\frac{5a^4b^2+2a^2b^4-b^6+\left(-3a^4b^2-2a^2b^4+b^6\right)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+4a^3\left(-b^2\right)^{3/2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-3a\left(-b^2\right)^{5/2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2a^2b^4\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+2b^6\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+3ab^4\sqrt{-b^2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)-4a^3\left(-b^2\right)^{3/2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+ab\left(a^4-2a^2b^2-3b^4\right)\sin(2(c+dx))}{4b\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Maple [A]
time = 0.19, size = 98, normalized size = 1.14

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3b^2a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2\left(\cos^2(dx+c)\right)b}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3b^2a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2\left(\cos^2(dx+c)\right)b}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
risch	$ix\,b^3+\frac{a^3x}{2}+\frac{3a\,b^2x}{2}-\frac{3\,\mathrm{e}^{2i(dx+c)}b\,a^2}{8d}+\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}-\frac{i\mathrm{e}^{2i(dx+c)}a^3}{8d}+\frac{3i\mathrm{e}^{2i(dx+c)}b^2a}{8d}-\frac{3\,\mathrm{e}^{-2i(dx+c)}b\,a^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*b^2*a*(-1/2*sin(d*x+c)*cos(d*
x+c)+1/2*d*x+1/2*c)-3/2*a^2*cos(d*x+c)^2*b+a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1
/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 81, normalized size = 0.94

$$\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+\left(a^3+3\,ab^2\right)\left(dx+c\right)-\frac{3\,a^2b-b^3-\left(a^3-3\,ab^2\right)\tan\left(dx+c\right)}{\tan\left(dx+c\right)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (3*a^2*b - b
^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.47, size = 79, normalized size = 0.92

$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-\left(a^3+3\,ab^2\right)dx+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(
d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (c+dx))^3\cos ^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(81) = 162.
time = 0.95, size = 601, normalized size = 6.99

$$\frac{2a^3dx\tan(dx)^2\tan(c)^2+6ab^2dx\tan(dx)^2\tan(c)^2-2b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2+2a^3dx\tan(dx)^2+6ab^2dx\tan(dx)^2+2a^3dx\tan(c)^2+6ab^2dx\tan(c)^2-3a^2b\tan(dx)^2\tan(c)^2+b^3\tan(dx)^2\tan(c)^2-2b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^2-2a^3\tan(dx)^2\tan(c)+6ab^2\tan(dx)^2\tan(c)-2b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(c)^2-2a^3\tan(dx)\tan(c)^2+6ab^2\tan(dx)\tan(c)^2+2a^3dx+6ab^2dx+3a^2b\tan(dx)^2-b^3\tan(dx)^2+12a^2b\tan(dx)\tan(c)-4b^3\tan(dx)\tan(c)+3a^2b\tan(c)^2-b^3\tan(c)^2-2b^3\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)+2a^3\tan(dx)-6ab^2\tan(dx)+2a^3\tan(c)-6ab^2\tan(c)-3a^2b+b^3}{4\left(d\tan(dx)^2\tan(c)^2+d\tan(dx)^2+d\tan(c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*d*x*tan(d*x)^2*tan(c)^2 + 6*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 2*b^
3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 2
*a^3*d*x*tan(d*x)^2 + 6*a*b^2*d*x*tan(d*x)^2 + 2*a^3*d*x*tan(c)^2 + 6*a*b^2
*d*x*tan(c)^2 - 3*a^2*b*tan(d*x)^2*tan(c)^2 + b^3*tan(d*x)^2*tan(c)^2 - 2*b
^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2 - 2*a^3*tan
(d*x)^2*tan(c) + 6*a*b^2*tan(d*x)^2*tan(c) - 2*b^3*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(c)^2 - 2*a^3*tan(d*x)*tan(c)^2 + 6*a*b^2*tan(
d*x)*tan(c)^2 + 2*a^3*d*x + 6*a*b^2*d*x + 3*a^2*b*tan(d*x)^2 - b^3*tan(d*x)
^2 + 12*a^2*b*tan(d*x)*tan(c) - 4*b^3*tan(d*x)*tan(c) + 3*a^2*b*tan(c)^2 -
b^3*tan(c)^2 - 2*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 2*
a^3*tan(d*x) - 6*a*b^2*tan(d*x) + 2*a^3*tan(c) - 6*a*b^2*tan(c) - 3*a^2*b +
 b^3)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B]
time = 3.66, size = 141, normalized size = 1.64

$$\frac{b^3\ln\left(\frac{1}{{\cos\left(c+d\,x\right)}^2}\right)}{2\,d}+\frac{b^3\,{\cos\left(c+d\,x\right)}^2}{2\,d}+\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(c+d\,x\right)}{\cos\left(c+d\,x\right)}\right)}{2\,d}-\frac{3\,a^2\,b\,{\cos\left(c+d\,x\right)}^2}{2\,d}+\frac{3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(c+d\,x\right)}{\cos\left(c+d\,x\right)}\right)}{2\,d}+\frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}-\frac{3\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*log(1/cos(c + d*x)^2))/(2*d) + (b^3*cos(c + d*x)^2)/(2*d) + (a^3*atan(
sin(c + d*x)/cos(c + d*x)))/(2*d) - (3*a^2*b*cos(c + d*x)^2)/(2*d) + (3*a*b
^2*atan(sin(c + d*x)/cos(c + d*x)))/(2*d) + (a^3*cos(c + d*x)*sin(c + d*x))
/(2*d) - (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.536 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{3}{8}a\left(a^2+b^2\right)x-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c-$$

```
[Out] 3/8*a*(a^2+b^2)*x-3/8*a*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/
4*cos(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Rubi [A]

time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3587, 743, 737, 209}

$$\frac{3}{8}ax\left(a^2+b^2\right)-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + b^2)*x)/8 - (3*a*Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan
[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(4
*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^m)*(2*c*x)*((a + c*x^2)^(p + 1)/(4*a*c*(p + 1))), x] - Dist[m*(
(2*c*d)/(4*a*c*(p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x]
 /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p +
3, 0] && LtQ[p, -1]
```

<u>Rule 3587</u>

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

<u>Rubi steps</u>

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d} + \frac{(3a)\text{Subst}\left(\int \frac{(a+\ldots}{\left(1+\frac{x}{b}\ldots\right.}\right.}{\ldots}$$

$$= -\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c\ldots}{\ldots}$$

$$= \frac{3}{8}a\left(a^2+b^2\right)x - \frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+\ldots}{8d}$$

<u>Mathematica</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(84) = 168.
time = 3.81, size = 257, normalized size = 3.06

$$\frac{-3a\sqrt{-b^2}\,(a^2+b^2)^3\left(\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)-6ab^3(6a^4+3a^2b^2+b^4)\tan(c+dx)-24a^4b^4\tan^2(c+dx)+2ab^5(-3a^2+b^2)\tan^3(c+dx)+4b(a^2+b^2)\,(a\cos(c+dx)+b\sin(c+dx))^4(b+a\tan(c+dx))+8a^2b^2\cos^2(c+dx)(a+b\tan(c+dx))^4+2ab(3a^2-b^2)\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{16b\,(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*Sqrt[-b^2]*(a^2 + b^2)^3*(Log[Sqrt[-b^2] - b*Tan[c + d*x]] - Log[Sqrt
[-b^2] + b*Tan[c + d*x]]) - 6*a*b^3*(6*a^4 + 3*a^2*b^2 + b^4)*Tan[c + d*x]
- 24*a^4*b^4*Tan[c + d*x]^2 + 2*a*b^5*(-3*a^2 + b^2)*Tan[c + d*x]^3 + 4*b*(
a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4*(b + a*Tan[c + d*x]) + 8*a^2
*b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^4 + 2*a*b*(3*a^2 - b^2)*Cos[c + d*
x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(16*b*(a^2 + b^2)^2*d)
```

<u>Maple</u> [**A**]
time = 0.22, size = 114, normalized size = 1.36

method	result

derivativedivides	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos}{}\right.}{}\right.}{d}$
default	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos}{}\right.}{}\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+2c)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*b^2*a*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x
+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.48, size = 110, normalized size = 1.31

$$\frac{3\,(a^3+ab^2)(dx+c)-\frac{4\,b^3\tan(dx+c)^2-3\,(a^3+ab^2)\tan(dx+c)^3+6\,a^2b+2\,b^3-(5\,a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^3 + a*b^2)*(d*x + c) - (4*b^3*tan(d*x + c)^2 - 3*(a^3 + a*b^2)*ta
n(d*x + c)^3 + 6*a^2*b + 2*b^3 - (5*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x +
 c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.44, size = 100, normalized size = 1.19

$$-\frac{4\,b^3\cos\left(dx+c\right)^2+2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^4-3\,(a^3+ab^2)dx-\left(2\,(a^3-3\,ab^2)\cos\left(dx+c\right)^3+3\,(a^3+ab^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*
b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^3\cos^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**4, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. 2(79) = 158.
time = 12.21, size = 2496, normalized size = 29.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/64*(9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^3*d*x*tan(d*x)^4*tan(c)^4 + 24*a*b^2*d*x*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a^3*d*x*tan(d*x)^4*tan(c)^2 + 48*a*b^2*d*x*tan(d*x)^4*tan(c)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 48*a^3*d*x*tan(d*x)^2*tan(c)^4 + 48*a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 30*a^2*b*tan(d*x)^4*tan(c)^4 - 6*b^3*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 36*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 40*a^3*tan(d*x)^4*tan(c)^3 + 24*a*b^2*tan(d*x)^4*tan(c)^3 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^3*tan(d*x)^3*tan(c)^4 + 24*a*b^2*tan(d*x)^3*tan(c)^4 + 24*a^3*d*x*tan(d*x)^4 + 24*a*b^2*d*x*tan(d*x)^4 + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^3*d*x*tan(d*x)^2*tan(c)^2 + 96*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 36*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36*a^2*b*ta

```
n(d*x)^4*tan(c)^2 - 12*b^3*tan(d*x)^4*tan(c)^2 + 192*a^2*b*tan(d*x)^3*tan(c
)^3 + 24*a^3*d*x*tan(c)^4 + 24*a*b^2*d*x*tan(c)^4 + 9*pi*a*b^2*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a
^2*b*tan(d*x)^2*tan(c)^4 - 12*b^3*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(2*t
an(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*
tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*
x)*tan(c) + 1))*tan(d*x)^4 - 24*a^3*tan(d*x)^4*tan(c) - 24*a*b^2*tan(d*x)^4
*tan(c) + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 72*a*b^2*arcta
n((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 72*a*b^2
*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 4
8*a^3*tan(d*x)^3*tan(c)^2 - 144*a*b^2*tan(d*x)^3*tan(c)^2 + 48*a^3*tan(d*x)
^2*tan(c)^3 - 144*a*b^2*tan(d*x)^2*tan(c)^3 + 18*a*b^2*arctan((tan(d*x) + t
an(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c
))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 24*a^3*tan(d*x)*tan(c)^4 - 24*a*b^2*ta
n(d*x)*tan(c)^4 + 48*a^3*d*x*tan(d*x)^2 + 48*a*b^2*d*x*tan(d*x)^2 + 18*pi*a
*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(d*x)^2 + 18*a^2*b*tan(d*x)^4 + 10*b^3*tan(d*x)^4 + 64*b^3*tan(d*x)^3*
tan(c) + 48*a^3*d*x*tan(c)^2 + 48*a*b^2*d*x*tan(c)^2 + 18*pi*a*b^2*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 -
216*a^2*b*tan(d*x)^2*tan(c)^2 + 72*b^3*tan(d*x)^2*tan(c)^2 + 64*b^3*tan(d*x
)*tan(c)^3 + 18*a^2*b*tan(c)^4 + 10*b^3*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c)) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))
*tan(d*x)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*t
an(d*x)^2 + 24*a^3*tan(d*x)^3 + 24*a*b^2*tan(d*x)^3 - 48*a^3*tan(d*x)^2*tan
(c) + 144*a*b^2*tan(d*x)^2*tan(c) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(c)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*
x)*tan(c) + 1))*tan(c)^2 - 48*a^3*tan(d*x)*tan(c)^2 + 144*a*b^2*tan(d*x)*ta
n(c)^2 + 24*a^3*tan(c)^3 + 24*a*b^2*tan(c)^3 + 24*a^3*d*x + 24*a*b^2*d*x +
9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c)) + 36*a^2*b*tan(d*x)^2 - 12*b^3*tan(d*x)^2 + 192*a^2*b*tan(d*x)*tan(
c) + 36*a^2*b*tan(c)^2 - 12*b^3*tan(c)^2 + 18*a*b^2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)
*tan(c) + 1)) + 40*a^3*tan(d*x) - 24*a*b^2*tan(...
```

Mupad [B]
time = 3.80, size = 109, normalized size = 1.30

$$\frac{3\,a^3\,x}{8}-\frac{6\,a^2\,b-{\mathrm{tan}\left(c+d\,x\right)}^3\,\left(3\,a^3+3\,a\,b^2\right)+2\,b^3+\mathrm{tan}\left(c+d\,x\right)\,\left(3\,a\,b^2-5\,a^3\right)+4\,b^3\,{\mathrm{tan}\left(c+d\,x\right)}^2}{d\,\left(8\,{\mathrm{tan}\left(c+d\,x\right)}^4+16\,{\mathrm{tan}\left(c+d\,x\right)}^2+8\right)}+\frac{3\,a\,b^2\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (3*a^3*x)/8 - (6*a^2*b - tan(c + d*x)^3*(3*a*b^2 + 3*a^3) + 2*b^3 + tan(c +
 d*x)*(3*a*b^2 - 5*a^3) + 4*b^3*tan(c + d*x)^2)/(d*(16*tan(c + d*x)^2 + 8*t
an(c + d*x)^4 + 8)) + (3*a*b^2*x)/8
```

3.537 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{3a(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d}$$

```
[Out] 3/16*a*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/16*a*(2*a^2-b^2)*sec(d*x+c)*tan(
d*x+c)/d+1/8*a*(2*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/7*b*sec(d*x+c)^5*(a+
b*tan(d*x+c))^2/d+1/70*b*sec(d*x+c)^5*(32*a^2-4*b^2+15*a*b*tan(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3593, 757, 794, 201, 221}

$$\frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}+\frac{a(2a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{3a(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{3a(2a^2-b^2)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{16d\sqrt{\sec^2(c+dx)}}+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 - b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(16*d*Sqrt[Sec[c + d
*x]^2]) + (3*a*(2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(2*a^2
- b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c
+ d*x])^2)/(7*d) + (b*Sec[c + d*x]^5*(4*(8*a^2 - b^2) + 15*a*b*Tan[c + d*x]
))/(70*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

```
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{(b\sec(c+dx))\text{Subst}\left(\int (a+\right.}{}$$

$$= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+\right.}{70d}$$

$$= \frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^5(c+dx)(a+b\tan(c}{7d}$$

$$= \frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a(2a^2-b^2)\sec^3(c+dx)\,\text{ta}}{8d}$$

$$= \frac{3a(2a^2-b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{16d\sqrt{\sec^2(c+dx)}} + \frac{3a(2a^2-b^2)\sec(c}{1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(159) = 318$.

time = 2.32, size = 637, normalized size = 4.01

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

Maple [A]

time = 0.30, size = 248, normalized size = 1.56

method	result
derivativedivides	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3b^2a\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{}$
default	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3b^2a\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{}$
risch	$-\frac{-1400ia^3e^{3i(dx+c)}+700ia\,b^2e^{3i(dx+c)}-3395ia\,b^2e^{5i(dx+c)}+1400ia^3e^{11i(dx+c)}-210ia^3e^{i(dx+c)}-2170ia^3e^{5i(dx+c)}-53}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+3/5*a^2*b/cos(d*x+c)^5+3*b^2*a*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+
1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c
```

```
)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*s
in(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/c
os(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 208, normalized size = 1.31

$$\frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{672\,a^2b}{\cos(dx+c)^5}-\frac{32\left(7\,\cos(dx+c)^2-5\right)b^3}{\cos(dx+c)^7}}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.39, size = 170, normalized size = 1.07

$$\frac{105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+1\right)+160\,b^3+224\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2+70\,\left(3\,(2\,a^3-ab^2)\cos\left(dx+c\right)^5+8\,ab^2\cos\left(dx+c\right)+2\,(2\,a^3-ab^2)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{1120\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^3\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(146) = 292.
time = 0.90, size = 465, normalized size = 2.92

$$\frac{105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(350\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{13}+105\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{13}-1680\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{12}-840\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+1540\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+3360\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{10}-1120\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{10}+630\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+1085\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-5040\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^8-1120\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^8+6720\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6-2240\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6-630\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-1085\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-3696\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4-448\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+840\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-1540\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+672\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-224\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-350\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-105\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-336\,a^2b+32\,b^3\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^7}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 7.34, size = 423, normalized size = 2.66

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{9\,a^3}{4}+\frac{31\,a\,b^2}{8}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{9\,a^3}{4}+\frac{31\,a\,b^2}{8}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(12\,a^2\,b-4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{12\,a^2\,b}{5}-\frac{4\,b^3}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(18\,a^2\,b+4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(24\,a^2\,b-8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{66\,a^2\,b}{5}+\frac{8\,b^3}{5}\right)-\frac{4\,b^3}{35}+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2
 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*x)/
2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 + (9*a
^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (d*x)/
2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^3)/5)
+ tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24*a^2*b
- 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3)/35 + 6
*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*
x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 +
(d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.538 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=126

$$\frac{a(4a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d}$$

```
[Out] 1/8*a*(4*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/8*a*(4*a^2-3*b^2)*sec(d*x+c)*ta
n(d*x+c)/d+1/5*b*sec(d*x+c)^3*(a+b*tan(d*x+c))^2/d+1/60*b*sec(d*x+c)^3*(48*
a^2-8*b^2+21*a*b*tan(d*x+c))/d
```

Rubi [A]

time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3593, 757, 794, 201, 221}

$$\frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d}+\frac{a(4a^2-3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a(4a^2-3b^2)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{8d\sqrt{\sec^2(c+dx)}}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*d*Sqrt[Sec[c + d*
x]^2]) + (a*(4*a^2 - 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d
*x]^3*(a + b*Tan[c + d*x])^2)/(5*d) + (b*Sec[c + d*x]^3*(8*(6*a^2 - b^2) +
21*a*b*Tan[c + d*x]))/(60*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

```
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^3\sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{(b\sec(c+dx))\text{Subst}\left(\int (a+x\right.}{}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+2\right.}{60d}$$

$$= \frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+}{5d}$$

$$= \frac{a(4a^2-3b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8d\sqrt{\sec^2(c+dx)}} + \frac{a(4a^2-3b^2)\sec(c-}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(126) = 252.

time = 1.38, size = 464, normalized size = 3.68

$$\frac{\sec^5(c+dx)\left(960a^2b+64b^3+320(3a^2b-b^3)\cos(2(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+225ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-60a^3\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+45ab^2\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-150a(4a^2-3b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-225ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+60a^3\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-45ab^2\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+240a^3\sin(2(c+dx))+540ab^2\sin(2(c+dx))+120a^3\sin(4(c+dx))-90ab^2\sin(4(c+dx))\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Maple [A]
time = 0.25, size = 198, normalized size = 1.57

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3b^2a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3b^2a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}}{d}$
risch	$-\frac{60ia^3e^{9i(dx+c)}-45iab^2e^{9i(dx+c)}+120ia^3e^{7i(dx+c)}+270iab^2e^{7i(dx+c)}-480a^2be^{7i(dx+c)}+160b^3e^{7i(dx+c)}-960a^2be^5}{60d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*b/co
s(d*x+c)^3+3*b^2*a*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+
c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/5*sin(d*x+c)^4/co
s(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15
*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 157, normalized size = 1.25

$$\frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{240\,a^2b}{\cos(dx+c)^3}-\frac{16\left(5\cos(dx+c)^2-3\right)b^3}{\cos(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*x + c)^5)/d

Fricas [A]
time = 0.45, size = 147, normalized size = 1.17

$$\frac{15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+48b^3+80\left(3a^2b-b^3\right)\cos\left(dx+c\right)^2+30\left(6ab^2\cos\left(dx+c\right)+\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{240d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b - b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(115) = 230.
time = 0.84, size = 333, normalized size = 2.64

$$\frac{15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+45ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-360a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-120a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+270ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+720a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-240b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-480a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-80b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+120a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-270ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+240a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-80b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-45ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-120a^2b+16b^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a

```
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

Mupad [B]
time = 7.32, size = 293, normalized size = 2.33

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(a^3+\frac{3\,a\,b^2}{4}\right)-2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^2\,b+\frac{4\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(12\,a^2\,b-4\,b^3\right)+\frac{4\,b^3}{15}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+\frac{3\,a\,b^2}{4}\right)-6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a\,b^2}{4}-a^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3*
((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(c/
2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (4*b
^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2 + (
d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 +
 (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)/2))*((
3*a*b^2)/4 - a^3))/d
```

3.539 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a(2a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d}+\frac{b\sec(c+dx)\,(4(4a^2-b^2)+5ab\tan(c}{6d}$$

```
[Out] 1/2*a*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)*(a+b*tan(d*x+c))
^2/d+1/6*b*sec(d*x+c)*(16*a^2-4*b^2+5*a*b*tan(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3593, 757, 794, 221}

$$\frac{b\sec(c+dx)\,(4(4a^2-b^2)+5ab\tan(c+dx))}{6d}+\frac{a(2a^2-3b^2)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2d\sqrt{\sec^2(c+dx)}}+\frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*d*Sqrt[Sec[c + d*
x]^2]) + (b*Sec[c + d*x]*(a + b*Tan[c + d*x])^2)/(3*d) + (b*Sec[c + d*x]*(4
*(4*a^2 - b^2) + 5*a*b*Tan[c + d*x]))/(6*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{(b\sec(c+dx))\text{Subst}\left(\int \frac{(a+x)(\cdots}{\sqrt{\cdots}}\right.}{3d\sqrt{\sec\cdots}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\left(4(4a^2-b^2)+5a\cdots\right.}{6d}$$

$$= \frac{a(2a^2-3b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2d\sqrt{\sec^2(c+dx)}} + \frac{b\sec(c+dx)(a+b\cdots}{3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(91) = 182.
time = 1.68, size = 293, normalized size = 3.22

$$\frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-18ab^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{9ab^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+2b(18a^2-b^2+2b^2\cos(c+dx)+(18a^2-5b^2)\cos(2(c+dx)))\sec^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-\frac{9ab^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
```

```
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Maple [A]
time = 0.12, size = 146, normalized size = 1.60

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3b^2a\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{}{\mathrm{co}}}{d}$
default	$\frac{b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3b^2a\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{}{\mathrm{co}}}{d}$
risch	$-\frac{b\left(9iab\,\mathrm{e}^{5i(dx+c)}-18a^2\mathrm{e}^{5i(dx+c)}+6b^2\mathrm{e}^{5i(dx+c)}-36a^2\mathrm{e}^{3i(dx+c)}+4b^2\mathrm{e}^{3i(dx+c)}-9iab\,\mathrm{e}^{i(dx+c)}-18a^2\mathrm{e}^{i(dx+c)}+6b^2\mathrm{e}^{i(dx+c}\right.}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+3*b^2*a*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*
x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b/cos(d*x+c)+a^3*ln(sec(d*x+c)+ta
n(d*x+c)))
```

Maxima [A]
time = 0.28, size = 111, normalized size = 1.22

$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-\frac{36\,a^2b}{\cos(dx+c)}+\frac{4\left(3\,\cos(dx+c)^2-1\right)b^3}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 12*a^3*log(sec(d*x + c) + tan(d*x + c)) - 36*a^
2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 123, normalized size = 1.35

$$\frac{3\,(2\,a^3-3\,ab^2)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,a^3-3\,ab^2)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+18\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b^3+12\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^3 \sec{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.
time = 0.90, size = 171, normalized size = 1.88

$$\frac{3\,(2\,a^3 - 3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(2\,a^3 - 3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 18\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 18\,a^2b + 4\,b^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 5.53, size = 160, normalized size = 1.76

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d}-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (d
*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*a^
2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.540 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d}-\frac{b\sec(c+dx)\,(2(a^2-b^2)+a}{d}$$

```
[Out] 3*a*b^2*arctanh(sin(d*x+c))/d-cos(d*x+c)*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))^
2/d-b*sec(d*x+c)*(2*a^2-2*b^2+a*b*tan(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3593, 753, 794, 221}

$$-\frac{b\sec(c+dx)\,(2(a^2-b^2)+ab\tan(c+dx))}{d}+\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\,\sinh^{-1}(\tan(c+dx))}{d}-\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcSinh[Tan[c + d*x]]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/d - (Cos[
c + d*x]*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/d - (b*Sec[c + d*x]*(
2*(a^2 - b^2) + a*b*Tan[c + d*x]))/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+d$$

$$\left(b\cos(c+\right.$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} + \frac{}{}$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+d}{}$$

$$= \frac{3ab^2\sinh^{-1}(\tan(c+dx))\cos(c+dx)\sqrt{\sec^2(c+dx)}}{d} - \frac{\cos(c+dx}{}$$

Mathematica [A]

time = 1.14, size = 131, normalized size = 1.56

$$\frac{\sec(c+dx)\left(-3a^2b+3b^3+(-3a^2b+b^3)\cos(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+a^3\sin(2(c+dx))-3ab^2\sin(2(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

Maple [A]

time = 0.18, size = 96, normalized size = 1.14

method	result
derivativedivides	$\frac{a^3\sin(dx+c)-3a^2b\cos(dx+c)+3b^2a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx\right.}{d}$
default	$\frac{a^3\sin(dx+c)-3a^2b\cos(dx+c)+3b^2a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx\right.}{d}$
risch	$-\frac{3\,e^{i(dx+c)}b\,a^2}{2d}+\frac{e^{i(dx+c)}b^3}{2d}-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{3ie^{i(dx+c)}b^2a}{2d}-\frac{3\,e^{-i(dx+c)}b\,a^2}{2d}+\frac{e^{-i(dx+c)}b^3}{2d}+\frac{ie^{-i(dx+c)}a^3}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*sin(d*x+c)-3*a^2*b*cos(d*x+c)+3*b^2*a*(-sin(d*x+c)+ln(sec(d*x+c)+t
an(d*x+c)))+b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.00

$$\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+3\,ab^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))-6\,a^2b\cos\left(dx+c\right)+2\,a^3\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 109, normalized size = 1.30

$$\frac{3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^3-2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2+2\,(a^3-3\,ab^2)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4741 vs. $2(83) = 166$.
time = 2.84, size = 4741, normalized size = 56.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*pi*a
^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6*a*b^2*log(2*(t
an(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1
/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^3*tan(1/2*c)^3 + 12*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*b^3*tan(1/2*d
*x)^4*tan(1/2*c)^4 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 +
 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*t
an(1/2*c)^3 - 24*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 8*a^3*tan(1
/2*d*x)^4*tan(1/2*c)^3 - 24*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 8*a^3*tan(1
```

```
/2*d*x)^3*tan(1/2*c)^4 - 24*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 3*pi*a^2*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta
n(1/2*d*x)^3*tan(1/2*c) - 24*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 12*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 96*a^2*b*tan(1/2*d*x)^3
*tan(1/2*c)^3 + 32*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 3*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*c)^4 - 24*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3*pi*a^2*b
*tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^4 + 6*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)
/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)
 + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3
*tan(1/2*c) + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan
(1/2*d*x)^3*tan(1/2*c) - 24*a*b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 24*a*b^2*log(2*(tan(1/2*d*x
)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c
)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2...
```

Mupad [B]
time = 4.25, size = 116, normalized size = 1.38

$$\frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^3,x)
[Out] (6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2*
```

```
a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*tan
(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.541 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=70

$$-\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d}-\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

```
[Out] -2/3*(a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/3*cos(d*x+c)^3*(b-a*tan(d*x+
c))*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3593, 737, 651}

$$-\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d}-\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(3*d) - (Cos[c + d*x]^3*
(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+\right.}{bd}$$

$$= -\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d} + \frac{\left(2(a^2+\right.}{}$$

$$= -\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d} - \frac{\cos^3(c+dx)(b-a}{}$$

Mathematica [A]
time = 0.41, size = 81, normalized size = 1.16

$$\frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Maple [A]
time = 0.18, size = 75, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b^2a\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b^2a\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\,b^2\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+b^2*a*sin(d*x+c)^3-a^2*b*cos(d*x+
c)^3+1/3*a^3*(cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.10

$$-\frac{3\,a^2b\cos(dx+c)^3-3\,ab^2\sin(dx+c)^3+\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3-\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/3*(3*a^2*b*cos(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (cos(d*x + c)^3 - 3*cos(d*x + c))*b^3)/d

Fricas [A]
time = 0.42, size = 77, normalized size = 1.10

$$-\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24430 vs. 2(68) = 136.
time = 126.46, size = 24430, normalized size = 349.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/768*(72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta

n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 105*pi*b^3*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 14
4*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 210*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^6*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 216*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*t
an(1/2*c)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 315*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c
)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2

```
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 144*pi*a^2*b*tan(1/2*d*
x)^6*tan(1/2*c)^6 - 210*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 144*a^2*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 +
210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan
(1/2*c)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*...
```

Mupad [B]
time = 3.71, size = 104, normalized size = 1.49

$$\frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\cos\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a\,b^2+\frac{b^3\cos(c+d\,x)^3}{3}-b^3\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.542 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=105

$$-\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d}-\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d}+\frac{\cos^4(c}{}$$

```
[Out] -2/15*(4*a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/15*cos(d*x+c)^3*(b-4*a*ta
n(d*x+c))*(a+b*tan(d*x+c))^2/d+1/5*cos(d*x+c)^4*sin(d*x+c)*(a+b*tan(d*x+c))
^3/d
```

Rubi [A]

time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3593, 751, 819, 651}

$$-\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d}-\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d}+\frac{\sin(c+dx)\cos^4(c+dx)(a+b\tan(c+dx))^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(4*a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(15*d) - (Cos[c + d*x]
^3*(b - 4*a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(15*d) + (Cos[c + d*x]^4*
Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(5*d)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/2}}\,dx, x, b\tan(c+d\right.}{bd}$$

$$= \frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d} - \frac{\left(\cos(c+dx)\sqrt{\sec^2\right.}{}$$

$$= -\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\cos^4(c+}{}$$

$$= -\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d} - \frac{\cos^3(c+dx)(b-4a}{}$$

Mathematica [A]
time = 0.76, size = 150, normalized size = 1.43

$$\frac{-30b(3a^2+b^2)\cos(c+dx)-5(9a^2b+b^3)\cos(3(c+dx))-9a^2b\cos(5(c+dx))+3b^3\cos(5(c+dx))+150a^3\sin(c+dx)+90ab^2\sin(c+dx)+25a^3\sin(3(c+dx))-15ab^2\sin(3(c+dx))+3a^3\sin(5(c+dx))-9ab^2\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-30*b*(3*a^2 + b^2)*Cos[c + d*x] - 5*(9*a^2*b + b^3)*Cos[3*(c + d*x)] - 9*
a^2*b*Cos[5*(c + d*x)] + 3*b^3*Cos[5*(c + d*x)] + 150*a^3*Sin[c + d*x] + 90
*a*b^2*Sin[c + d*x] + 25*a^3*Sin[3*(c + d*x)] - 15*a*b^2*Sin[3*(c + d*x)] +
 3*a^3*Sin[5*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)])/(240*d)
```

Maple [A]
time = 0.28, size = 125, normalized size = 1.19

method	result

derivativedivides	$\frac{b^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{3a^2b(\cdots}{\cdots}}{d}$
default	$\frac{b^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{15}\right)-\frac{3a^2b(\cdots}{\cdots}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{8d}-\frac{b^3\cos(dx+c)}{8d}+\frac{5a^3\sin(dx+c)}{8d}+\frac{3a\,b^2\sin(dx+c)}{8d}-\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{b^3\cos(5dx+5c)}{80d}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3*b^2*a*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(cos(d*x+c)^2+2)*sin(d*x+c))-3/5*a^2*b*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 107, normalized size = 1.02

$$-\frac{9\,a^2b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3+3\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)ab^2-\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)b^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Fricas [A]
time = 0.41, size = 102, normalized size = 0.97

$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\cos^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 56572 vs. $2(102) = 204$.
time = 255.26, size = 56572, normalized size = 538.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15360*(1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 105*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^10*tan(1/2*c)^10 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2
*c)^10 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 1800*pi*a^2*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 1995*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^1
0 + 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*
tan(1/2*c)^10 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^10*tan(1/2*c)^10 - 5760*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^
10*tan(1/2*c)^10 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x
)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2
*c)^10 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 525*pi*b^3*sgn(t
```

```
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^10*tan(1/2*c)^8 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c
)^8 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5400*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^8*tan(1/2*c)^10 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 +
5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 525*pi*b^3*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^8*tan(1/2*c)^10 + 5760*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^10 - 210*pi*
b^3*tan(1/2*d*x)^10*tan(1/2*c)^10 - 2160*a^2*b*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 210*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 2160*a^2
*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2
*c)^10 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)^10*tan(1/2*c)^10 + 9360*a^2*b*arctan((tan(1...
```

Mupad [B]

time = 3.81, size = 147, normalized size = 1.40

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^3\cos(c+d\,x)^4}{2}+2\sin\left(c+d\,x\right)\,a^3\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^3-\frac{9\,a^2\,b\cos(c+d\,x)^5}{2}-\frac{9\sin(c+d\,x)\,a\,b^2\cos(c+d\,x)^4}{2}+\frac{3\sin(c+d\,x)\,a\,b^2\cos(c+d\,x)^2}{2}+3\sin\left(c+d\,x\right)\,a\,b^2+\frac{3\,b^3\cos(c+d\,x)^5}{2}-\frac{5\,b^3\cos(c+d\,x)^3}{2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)/
2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a^3
*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c + d
*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.543 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=142

$$\frac{8a(2a^2+b^2)\sin(c+dx)}{35d}-\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d}+\frac{\cos^6(c+dx)\sin(c+dx)}{7d}$$

```
[Out] 8/35*a*(2*a^2+b^2)*sin(d*x+c)/d-3/35*cos(d*x+c)^5*(b-2*a*tan(d*x+c))*(a+b*t
an(d*x+c))^2/d+1/7*cos(d*x+c)^6*sin(d*x+c)*(a+b*tan(d*x+c))^3/d-2/35*cos(d*
x+c)^3*(b*(6*a^2+b^2)-a*(4*a^2-b^2)*tan(d*x+c))/d
```

Rubi [A]

time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3593, 751, 835, 792, 197}

$$\frac{8a(2a^2+b^2)\sin(c+dx)}{35d}-\frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d}-\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d}+\frac{\sin(c+dx)\cos^6(c+dx)(a+b\tan(c+dx))^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*a*(2*a^2 + b^2)*Sin[c + d*x])/(35*d) - (3*Cos[c + d*x]^5*(b - 2*a*Tan[c
 + d*x])*(a + b*Tan[c + d*x])^2)/(35*d) + (Cos[c + d*x]^6*Sin[c + d*x]*(a +
b*Tan[c + d*x])^3)/(7*d) - (2*Cos[c + d*x]^3*(b*(6*a^2 + b^2) - a*(4*a^2 -
b^2)*Tan[c + d*x]))/(35*d)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/2}}\,dx, x, b\tan(c+d}{bd}$$

$$= \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{\left(\cos(c+dx)\sqrt{\sec^2}\right.}{}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c-}{}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c-}{}$$

$$= \frac{8a(2a^2+b^2)\sin(c+dx)}{35d} - \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+}{35d}$$

Mathematica [A]
time = 1.15, size = 204, normalized size = 1.44

$$\frac{-105b(5a^2+b^2)\cos(c+dx) - 35(9a^2b+b^3)\cos(3(c+dx)) - 105a^2b\cos(5(c+dx)) + 7b^3\cos(5(c+dx)) - 15a^2b\cos(7(c+dx)) + 5b^3\cos(7(c+dx)) + 1225a^3\sin(c+dx) + 525ab^2\sin(c+dx) + 245a^3\sin(3(c+dx)) - 35ab^2\sin(3(c+dx)) + 49a^3\sin(5(c+dx)) - 63ab^2\sin(5(c+dx)) + 5a^3\sin(7(c+dx)) - 15ab^2\sin(7(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-105*b*(5*a^2 + b^2)*Cos[c + d*x] - 35*(9*a^2*b + b^3)*Cos[3*(c + d*x)] -
105*a^2*b*Cos[5*(c + d*x)] + 7*b^3*Cos[5*(c + d*x)] - 15*a^2*b*Cos[7*(c + d
*x)] + 5*b^3*Cos[7*(c + d*x)] + 1225*a^3*Sin[c + d*x] + 525*a*b^2*Sin[c + d
*x] + 245*a^3*Sin[3*(c + d*x)] - 35*a*b^2*Sin[3*(c + d*x)] + 49*a^3*Sin[5*(
c + d*x)] - 63*a*b^2*Sin[5*(c + d*x)] + 5*a^3*Sin[7*(c + d*x)] - 15*a*b^2*S
in[7*(c + d*x)])/(2240*d)
```

Maple [A]
time = 0.24, size = 145, normalized size = 1.02

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3b^2a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$
risch	$-\frac{15b\cos(dx+c)a^2}{64d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{35a^3\sin(dx+c)}{64d}+\frac{15a\,b^2\sin(dx+c)}{64d}-\frac{3b\cos(7dx+7c)a^2}{448d}+\frac{b^3\cos(7dx+7}{448d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*b^2*a*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 126, normalized size = 0.89

$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3\right)ab^2-\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)b^3}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

Fricas [A]
time = 0.38, size = 123, normalized size = 0.87

$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\,\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\,\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\,\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^4+16\,a^3+8\,ab^2+4\,\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 101962 vs. 2(136) = 272.
time = 82.30, size = 101962, normalized size = 718.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/17920*(945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 210*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c
)^14 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 2205*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 1995*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14
```

+ 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 4830*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 420*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 4830*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^14 + 420*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^

```
14*tan(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
*tan(1/2*d*x)^14*tan(1/2*c)^14 + 7770*a^2*b*arc...
```

Mupad [B]
time = 3.94, size = 214, normalized size = 1.51

$$\frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{4\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{3\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{3\,a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d*
x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(c
+ d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c +
d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*cos(c
 + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))/(35*
d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

3.544 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d}-\frac{a(a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d}-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\tan^4}{}$$

```
[Out] (a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^5/d-a*(a^2+2*b^2)*tan(d*x+c)/b^4/d+1/2*(a^
2+2*b^2)*tan(d*x+c)^2/b^3/d-1/3*a*tan(d*x+c)^3/b^2/d+1/4*tan(d*x+c)^4/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d}-\frac{a(a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d}-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\tan^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^5*d) - (a*(a^2 + 2*b^2)*Tan[c +
d*x])/(b^4*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^2)/(2*b^3*d) - (a*Tan[c + d*x]^
3)/(3*b^2*d) + Tan[c + d*x]^4/(4*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a(-a^2-2b^2)}{b^4} + \frac{(a^2+2b^2)x}{b^4} - \frac{ax^2}{b^4} + \frac{x^3}{b^4} + \frac{(a^2+b^2)^2}{b^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4d} + \frac{(a^2+2b^2)\tan^2(c+$$

Mathematica [A]
time = 1.28, size = 119, normalized size = 1.03

$$\frac{3b^4\sec^4(c+dx)+2b^2\sec^2(c+dx)\left(3(a^2+b^2)-2ab\tan(c+dx)\right)-4\left(3(a^2+b^2)^2\left(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)+ab(3a^2+5b^2)\tan(c+dx)\right)}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*b^4*Sec[c + d*x]^4 + 2*b^2*Sec[c + d*x]^2*(3*(a^2 + b^2) - 2*a*b*Tan[c +
 d*x]) - 4*(3*(a^2 + b^2)^2*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin
[c + d*x]]) + a*b*(3*a^2 + 5*b^2)*Tan[c + d*x]))/(12*b^5*d)
```

Maple [A]
time = 0.38, size = 106, normalized size = 0.91

method	result
derivativedivides	$-\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$-\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$\frac{-2ia^3e^{6i(dx+c)}-2iab^2e^{6i(dx+c)}+2a^2b\,e^{6i(dx+c)}+2b^3e^{6i(dx+c)}-6ia^3e^{4i(dx+c)}-10iab^2e^{4i(dx+c)}+4a^2b\,e^{4i(dx+c)}+8b^3e^{4i(}}{b^4d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*a*tan(d*x+c)^3*b^2-1/2*(a^2+2*b^2)*t
an(d*x+c)^2*b+a*(a^2+2*b^2)*tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*tan(
d*x+c)))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.93

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(a^2b+2\,b^3)\tan(dx+c)^2-12\,(a^3+2\,ab^2)\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b + 2*b^3)*ta
n(d*x + c)^2 - 12*(a^3 + 2*a*b^2)*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2 +
 b^4)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [A]
time = 0.41, size = 183, normalized size = 1.58

$$\frac{6\,(a^4+2\,a^2b^2+b^4)\cos(dx+c)^4\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-6\,(a^4+2\,a^2b^2+b^4)\cos(dx+c)^4\log\left(\cos(dx+c)^2\right)+3\,b^4+6\,(a^2b^2+b^4)\cos(dx+c)^2-4\,\left(ab^3\cos(dx+c)+(3\,a^3b+5\,ab^3)\cos(dx+c)^3\right)\sin(dx+c)}{12\,b^5d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 120, normalized size = 1.03

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d

Mupad [B]
time = 3.73, size = 119, normalized size = 1.03

$$\frac{\tan(c+d\,x)^4}{4\,b\,d}+\frac{\tan(c+d\,x)^2\,\left(\frac{1}{b}+\frac{a^2}{2\,b^3}\right)}{d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5\,d}-\frac{a\tan(c+d\,x)^3}{3\,b^2\,d}-\frac{a\tan(c+d\,x)\,\left(\frac{2}{b}+\frac{a^2}{b^3}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))),x)

[Out] tan(c + d*x)^4/(4*b*d) + (tan(c + d*x)^2*(1/b + a^2/(2*b^3)))/d + (log(a + b*tan(c + d*x))*(a^4 + b^4 + 2*a^2*b^2))/(b^5*d) - (a*tan(c + d*x)^3)/(3*b^2*d) - (a*tan(c + d*x)*(2/b + a^2/b^3))/(b*d)

3.545 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

```
[Out] (a^2+b^2)*ln(a+b*tan(d*x+c))/b^3/d-a*tan(d*x+c)/b^2/d+1/2*tan(d*x+c)^2/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^3*d) - (a*Tan[c + d*x])/(b^2*d) +
Tan[c + d*x]^2/(2*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx&=\frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{a+x}\,dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b^2}+\frac{a^2+b^2}{b^2(a+x)}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 71, normalized size = 1.20

$$\frac{b^2\sec^2(c+dx)-2((a^2+b^2)(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+ab\tan(c+dx))}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
[Out] (b^2*Sec[c + d*x]^2 - 2*((a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x
] + b*Sin[c + d*x]]) + a*b*Tan[c + d*x]))/(2*b^3*d)
```

Maple [A]
time = 0.37, size = 53, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
risch	$\frac{-2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-2ia}{b^2d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}+1\right)a^2}{b^3d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^2}{b^3d}+\frac{\ln\left(e^{2i(dx+\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*x
+c)))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.90

$$\frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(b*tan(d*
x + c) + a)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.
time = 0.39, size = 117, normalized size = 1.98

$$\frac{(a^2+b^2)\cos(dx+c)^2\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^2+b^2)\cos(dx+c)^2\log\left(\cos(dx+c)^2\right)-2\,ab\cos(dx+c)\sin(dx+c)+b^2}{2\,b^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 54, normalized size = 0.92

$$\frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

Mupad [B]
time = 3.59, size = 57, normalized size = 0.97

$$\frac{\tan(c+d\,x)^2}{2\,b\,d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2+b^2\right)}{b^3\,d}-\frac{a\tan(c+d\,x)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

```
[Out] tan(c + d*x)^2/(2*b*d) + (log(a + b*tan(c + d*x))*(a^2 + b^2))/(b^3*d) - (a
*tan(c + d*x))/(b^2*d)
```

3.546 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

```
[Out] ln(a+b*tan(d*x+c))/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 31}

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\tan(c+dx))}{bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Maple [A]
time = 0.24, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
risch	$\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*tan(d*x+c))/b/d
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{\log\left(b\tan\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] log(b*tan(d*x + c) + a)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.
time = 0.37, size = 59, normalized size = 3.28

$$\frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 19, normalized size = 1.06

$$\frac{\log\left(|b\tan\left(dx+c\right)+a|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

Mupad [B]
time = 3.59, size = 18, normalized size = 1.00

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

```
[Out] log(a + b*tan(c + d*x))/(b*d)
```

3.547 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{a(a^2+3b^2)\,x}{2\left(a^2+b^2\right)^2}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^2 d}+\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d}$$

```
[Out] 1/2*a*(a^2+3*b^2)*x/(a^2+b^2)^2+b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)
^2/d+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3587, 755, 815, 649, 209, 266}

$$\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{ax(a^2+3b^2)}{2\left(a^2+b^2\right)^2}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(2*(a^2 + b^2)^2) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\text{Subst}\left(\int \frac{-2-\frac{a^2}{b^2}-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\text{Subst}\left(\int \left(-\frac{2b^2}{(a^2+b^2)(a+x)}+\frac{-a^3-3ab^2+2b^2x}{(a^2+b^2)(b^2+x^2)}\right)dx, x \right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\text{Subst}\left(\int \frac{-a^3-3ab^2}{b^2+x^2}\right)}{2(a}$$

$$= \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b^3\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx\right)}{(a^2+}$$

$$= \frac{a\left(a^2+3b^2\right)x}{2\left(a^2+b^2\right)^2} + \frac{b^3\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b}{2(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 143, normalized size = 1.54

$$\frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\text{ArcTan}(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx))+2b^3\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)+a^3\sin(2(c+dx))+ab^2\sin(2(c+dx))}{4\left(a^2+b^2\right)^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]

[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Sin[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)

Maple [A]
time = 0.36, size = 120, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{b^3 \ln(a+b\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}b^2 a\right)\tan(dx+c)+\frac{a^2 b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3b^2 a\right)\arctan(\tan(dx+c))}{2}}{(a^2+b^2)^2}}{d}$
default	$\frac{\frac{b^3 \ln(a+b\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}b^2 a\right)\tan(dx+c)+\frac{a^2 b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3b^2 a\right)\arctan(\tan(dx+c))}{2}}{(a^2+b^2)^2}}{d}$
risch	$\frac{2ixb}{4iab-2a^2+2b^2}-\frac{xa}{4iab-2a^2+2b^2}-\frac{i e^{2i(dx+c)}}{8(-ib+a)d}+\frac{i e^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}+\frac{b^3\ln\left(e^{2}\right.}{d(a^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(((1/2*a^3+1/2*b^2*a)*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(1+tan(d*x+c)^2)-1/2*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^3+3*a*b^2)*arctan(tan(d*x+c))))

Maxima [A]
time = 0.49, size = 141, normalized size = 1.52

$$\frac{\frac{2\,b^3\log\left(b\tan\left(dx+c\right)+a\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}+\frac{a\tan\left(dx+c\right)+b}{\left(a^2+b^2\right)\tan\left(dx+c\right)^2+a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 + b^2))/d

Fricas [A]
time = 0.40, size = 119, normalized size = 1.28

$$\frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+c\right)^2+\left(a^3+ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)),x)

[Out] Integral(cos(c + d*x)**2/(a + b*tan(c + d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.
time = 0.51, size = 182, normalized size = 1.96

$$\frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3+3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d

Mupad [B]
time = 3.90, size = 156, normalized size = 1.68

$$\frac{\cos(c+d\,x)^2\left(\frac{b}{2\,(a^2+b^2)}+\frac{a\tan(c+d\,x)}{2\,(a^2+b^2)}\right)}{d} - \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\;(2\,b+a\,1\mathrm{i})}{4\,d\;(-a^2+a\,b\,2\mathrm{i}+b^2)} - \frac{\ln(\tan(c+d\,x)-\mathrm{i})\;(a+b\,2\mathrm{i})}{4\,d\;(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i})} + \frac{b^3\,\ln(a+b\tan(c+d\,x))}{d\,(a^2+b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)),x)

[Out] (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2))))/d - (log(tan(c + d*x) + 1i)*(a*1i + 2*b))/(4*d*(a*b*2i - a^2 + b^2)) - (log(tan(c + d*x) - 1i)*(a + b*2i))/(4*d*(2*a*b - a^2*1i + b^2*1i)) + (b^3*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2)

3.548 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{a(3a^4+10a^2b^2+15b^4)\,x}{8\,(a^2+b^2)^3}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}+\frac{\cos^2(c+$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x/(a^2+b^2)^3+b^5*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d+1/8*cos(d*
x+c)^2*(4*b^3+a*(3*a^2+7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{\cos^2(c+dx)\,(a(3a^2+7b^2)\tan(c+dx)+4b^3)}{8d\,(a^2+b^2)^2}+\frac{ax(3a^4+10a^2b^2+15b^4)}{8\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/(8*(a^2 + b^2)^3) + (b^5*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c +
d*x]))/(4*(a^2 + b^2)*d) + (Cos[c + d*x]^2*(4*b^3 + a*(3*a^2 + 7*b^2)*Tan[c
 + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{b\text{Subst}\left(\int \frac{-4-\frac{3a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4(a^2+b^2)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{b^5\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\,(4b^3}{8}$$

$$= \frac{b^5\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\,(4b^3}{8}$$

$$= \frac{a(3a^4+10a^2b^2+15b^4)\,x}{8(a^2+b^2)^3} + \frac{b^5\log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{b^5\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \text{co}$$

Mathematica [A]
time = 0.45, size = 218, normalized size = 1.43

$$\frac{12a^5c+40a^3b^2c+60ab^4c+12a^5dx+40a^3b^2dx+60ab^4dx+4b(a^4+4a^2b^2+3b^4)\cos(2(c+dx))+b(a^2+b^2)^2\cos(4(c+dx))+32b^5\log(a\cos(c+dx)+b\sin(c+dx))+8a^5\sin(2(c+dx))+24a^3b^2\sin(2(c+dx))+16ab^4\sin(2(c+dx))+a^5\sin(4(c+dx))+2a^3b^2\sin(4(c+dx))+ab^4\sin(4(c+dx))}{32(a^2+b^2)^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a*
b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*
Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*Sin[
2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)] + a^
5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x)])/(
32*(a^2 + b^2)^3*d)
```

Maple [A]
time = 0.42, size = 197, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{b^{5} \ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(\frac{3}{8}a^{5}+\frac{5}{4}a^{3}b^{2}+\frac{7}{8}a\,b^{4}\right)\left(\tan^{3}(dx+c)\right)+\left(\frac{1}{2}a^{2}b^{3}+\frac{1}{2}b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{7}{4}a^{3}b^{2}+\frac{9}{8}a\,b^{4}+\frac{5}{8}a^{5}\right)\tan(dx+c)+\frac{a^{4}b}{4}\cdots}{\left(1+\tan^{2}(dx+c)\right)^{2}}\cdots}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
default	$\frac{\frac{b^{5} \ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(\frac{3}{8}a^{5}+\frac{5}{4}a^{3}b^{2}+\frac{7}{8}a\,b^{4}\right)\left(\tan^{3}(dx+c)\right)+\left(\frac{1}{2}a^{2}b^{3}+\frac{1}{2}b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{7}{4}a^{3}b^{2}+\frac{9}{8}a\,b^{4}+\frac{5}{8}a^{5}\right)\tan(dx+c)+\frac{a^{4}b}{4}\cdots}{\left(1+\tan^{2}(dx+c)\right)^{2}}\cdots}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
risch	$\frac{9xab}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}+\frac{3ix\,a^{2}}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}-\frac{8ix\,b^{2}}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}-\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16(-2iab+a^{2}-b^{2})d}-\frac{i\cdots}{8(-2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b^
2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*a^2*b^3+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2+9
/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*a^4*b+a^2*b^3+3/4*b^5)/(1+tan(d*x+c)^2)^2-
1/2*b^5*ln(1+tan(d*x+c)^2)+1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d*x+c
))))
```

Maxima [A]
time = 0.51, size = 271, normalized size = 1.78

$$\frac{\frac{8\,b^{5}\log\left(b\tan\left(dx+c\right)+a\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{4\,b^{5}\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\left(3\,a^{5}+10\,a^{3}b^{2}+15\,ab^{4}\right)\left(dx+c\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{4\,b^{3}\tan\left(dx+c\right)^{2}+\left(3\,a^{3}+7\,ab^{2}\right)\tan\left(dx+c\right)^{3}+2\,a^{2}b+6\,b^{3}+\left(5\,a^{3}+9\,ab^{2}\right)\tan\left(dx+c\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\tan\left(dx+c\right)^{4}+a^{4}+2\,a^{2}b^{2}+b^{4}+2\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\tan\left(dx+c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(8*b^5*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*
b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 +
10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (4*b
^3*tan(d*x + c)^2 + (3*a^3 + 7*a*b^2)*tan(d*x + c)^3 + 2*a^2*b + 6*b^3 + (5
*a^3 + 9*a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 + a^4
 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Fricas [A]
time = 0.42, size = 208, normalized size = 1.37

$$\frac{4\,b^{5}\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)+2\,\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+\left(3\,a^{5}+10\,a^{3}b^{2}+15\,ab^{4}\right)dx+4\,\left(a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}+\left(2\,\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{5}+10\,a^{3}b^{2}+7\,ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(146) = 292.
time = 0.50, size = 322, normalized size = 2.12

$$\frac{\frac{8\,b^6\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,b^5\tan(dx+c)^4+3\,a^5\tan(dx+c)^3+10\,a^3b^2\tan(dx+c)^3+7\,ab^4\tan(dx+c)^3+4\,a^2b^3\tan(dx+c)^2+16\,b^5\tan(dx+c)^2+5\,a^5\tan(dx+c)+14\,a^3b^2\tan(dx+c)+9\,ab^4\tan(dx+c)+2\,a^4b+8\,a^2b^3+12\,b^5}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

Mupad [B]
time = 4.20, size = 318, normalized size = 2.09

$$\frac{\frac{a^2\,b+3\,b^3}{4\,(a^4+2\,a^2\,b^2+b^4)}+\frac{b^3\tan(c+d\,x)^2}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^3\,(3\,a^3+7\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(5\,a^3+9\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,3\mathrm{i}+9\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{(a^2+b^2)}^3}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,9\mathrm{i}+8\,b^2\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] ((a^2*b + 3*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*tan(c + d*x)^2)/(2*(a^4
 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(7*a*b^2 + 3*a^3))/(8*(a^4 + b^4 + 2
*a^2*b^2)) + (tan(c + d*x)*(9*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)))/
```

```
(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (log(tan(c + d*x) - 1i)*(9*a*
b - a^2*3i + b^2*8i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (b^5*log
(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) + 1i)*(a*b*9i -
 3*a^2 + 8*b^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.549 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{(a^2+b^2)\sec(c+dx)}{b^3d}+\text{se}$$

```
[Out] -1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2+b^2)^(3/2)*arctanh(cos(
d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2+b^2)*sec(d*x+c)/b^3/d+1
/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3591, 3567, 3853, 3855, 3590, 212}

$$-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d}-\frac{a\tan(c+dx)\sec(c+dx)}{2b^2d}+\frac{\sec^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Sin[c + d*x]])/(b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x])
)/Sqrt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c +
d*x]^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\int \sec^3(c+dx)(a-b\tan(c+dx))\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}$$

$$= \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} - \frac{(a^2+b^2)\int \sec(c+dx)(a-b\tan(c+dx))\,d}{b^4}$$

$$= \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} - \frac{a\int \sec(c+dx}{2b^2}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{(a^2+b^2)^{3/2}\tanh^{-1}\Big(}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 321 vs. 2(140) = 280.
time = 2.21, size = 321, normalized size = 2.29

$$\frac{48(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9a(2a^2+3b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-6a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-9ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-6ab^2\sin(2(c+dx))\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(130) = 260$.
time = 0.45, size = 269, normalized size = 1.92

method	result
derivativedivides	$\frac{-\frac{2\left(-a^4-2a^2b^2-b^4\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+3b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a}{}}{}$
default	$\frac{-\frac{2\left(-a^4-2a^2b^2-b^4\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+3b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a}{}}{}$
risch	$\frac{3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+6b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+20b^2e^{3i(dx+c)}-3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+6b^2e^{i(dx+c)}}{3d\,b^3\left(e^{2i(dx+c)}+1\right)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*(-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^
2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+3*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)
+1/2*a*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/3/b/(tan(1/2*d*x+1/2*c)
+1)^3-1/2*(-a+b)/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(t
an(1/2*d*x+1/2*c)+1)-1/2*a*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.
time = 0.51, size = 361, normalized size = 2.58

$$\frac{\frac{2\left(6a^2+8b^2-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{12\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\left(a^2+2b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{b^3-\frac{3b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\left(2a^3+3ab^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{3\left(2a^3+3ab^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}-\frac{6\left(a^4+2a^2b^2+b^4\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3*b^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2)*log
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b - a*si
n(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d
*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Fricas [A]
time = 0.47, size = 259, normalized size = 1.85

$$\frac{6\left(a^{2}+b^{2}\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-3\left(2\,a^{3}+3\,ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^{3}+3\,ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)-6\,ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b^{3}+12\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}}{12\,b^{4}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(132) = 264.
time = 0.56, size = 278, normalized size = 1.99

$$-\frac{\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^{4}}-\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{4}}+\frac{6\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{4}}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+12\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^{2}+8\,b^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}b^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

Mupad [B]
time = 5.40, size = 724, normalized size = 5.17

$$\frac{b^3\left(\cos\left(c+d\,x\right)+\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{\cos(3\,c+3\,d\,x)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{4}+\frac{9\,a\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}\right)+b\left(\frac{3\,a^2\,\cos(c+d\,x)}{4}+\frac{a^2}{2}+\frac{a^2\,\cos(2\,c+2\,d\,x)}{2}+\frac{a^2\,\cos(3\,c+3\,d\,x)}{4}\right)+\frac{\mathrm{atanh}\left(\frac{a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^5}\right)\cos(3\,c+3\,d\,x)\sqrt{{\left(a^2+b^2\right)}^3}}{2}-\frac{3\,a^3\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}+\frac{3\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^5}\right)\sqrt{{\left(a^2+b^2\right)}^3}}{2}}{b^4\,d\,\left(\frac{3\,\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))),x)
```

```
[Out] (b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*(
(a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*x))/
2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4
*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x
)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^2)^3)^(1/2
))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2
- (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (
3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^
(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(
a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2
) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*s
in(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*cos(c + d*x))/4 +
cos(3*c + 3*d*x)/4))
```

3.550 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh(cos(d*x+c)*(b-a*tan(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3591, 3567, 3855, 3590, 212}

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*
x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\int \sec(c+dx)(a-b\tan(c+dx))\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}$$

$$= \frac{\sec(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} - \frac{(a^2+b^2)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)\right)}{b^2 d}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2 d} + \frac{\sec(c+}{bd}$$

Mathematica [A]
time = 0.16, size = 109, normalized size = 1.38

$$\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + b\sec(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.42, size = 129, normalized size = 1.63

method	result
derivativedivides	$\frac{-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2} + \frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{b^2}}{d}$

default	$\frac{-\frac{2\left(-a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^{2}}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^{2}}}{d}$
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d\,b^{2}}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,b^{2}}+\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^{2}+b^{2}}}\right)}{d\,b^{2}}-\frac{\sqrt{a^{2}-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*(-a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-
2*b)/(a^2+b^2)^(1/2))-1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c
)-1)+1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(77) = 154$.
time = 0.48, size = 163, normalized size = 2.06

$$-\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{2}}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^{2}}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{b^{2}}-\frac{2}{b-\frac{b\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a
^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.
time = 0.42, size = 191, normalized size = 2.42

$$-\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-\sqrt{a^{2}+b^{2}}\cos\left(dx+c\right)\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-2\,b}{2\,b^{2}d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.54, size = 136, normalized size = 1.72

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

Mupad [B]
time = 3.87, size = 310, normalized size = 3.92

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^4+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b+64\,a^2\,b^2+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^3}\right)\sqrt{a^2+b^2}}{b^2\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c/
2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b +
128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))
/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c/2 +
(d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 + (d*x)/
2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 + 64*a^2*
b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.551 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

```
[Out] -arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3590, 212}

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 +
b^2]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx))\right)}{d}\\
&= -\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.98

$$\frac{2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A]
time = 0.16, size = 43, normalized size = 0.93

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.74

$$-\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(44) = 88$.
time = 0.41, size = 131, normalized size = 2.85

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 74, normalized size = 1.61

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B]
time = 3.75, size = 39, normalized size = 0.85

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

3.552 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] -b^2*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3592, 3567, 2717, 3590, 212}

$$\frac{a\sin(c+dx)}{d\,(a^2+b^2)}+\frac{b\cos(c+dx)}{d\,(a^2+b^2)}-\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^
2 + b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\int \cos(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\int \cos(c+dx)\,dx}{a^2+b^2} - \frac{b^2\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\right.}{(a^2+b^2)\,d}$$

$$= -\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{b\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

Mathematica [**A**]
time = 0.35, size = 79, normalized size = 0.88

$$\frac{2b^2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\,(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [**A**]
time = 0.40, size = 90, normalized size = 1.00

method	result	siz

derivativedivides	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{(a^2+b^2)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
default	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{(a^2+b^2)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
risch	$-\frac{i e^{i(dx+c)}}{2(-ib+a)d}+\frac{i e^{-i(dx+c)}}{2(ib+a)d}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{i a^3+i a b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{b^2\ln\left(e^{i(dx+c)}-\frac{i a^3+i a b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [A]
time = 0.48, size = 142, normalized size = 1.58

$$-\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(
b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(88) = 176.
time = 0.40, size = 187, normalized size = 2.08

$$\frac{\sqrt{a^2+b^2}\,b^2\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2(a^2b+b^3)\cos(dx+c)+2(a^3+ab^2)\sin(dx+c)}{2(a^4+2a^2b^2+b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.54, size = 118, normalized size = 1.31

$$-\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B]
time = 3.84, size = 110, normalized size = 1.22

$$\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (d
*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^
2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

3.553 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d}+\frac{b^3\cos(c+dx)}{(a^2+b^2)^2 d}+\frac{b\cos^3(c+dx)}{3(a^2+b^2)d}+\frac{ab^2\sin(c+dx)}{(a^2+b^2)^2 d}+\frac{a\sin(c+dx)}{(a^2+b^2)d}-\frac{a\sin^3(}{3(a^2}$$

```
[Out] -b^4*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*x+
c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3592, 3567, 2713, 2717, 3590, 212}

$$-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}+\frac{ab^2\sin(c+dx)}{d(a^2+b^2)^2}+\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos^3(c+dx)}{3d(a^2+b^2)}-\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{5/2}}+\frac{b^3\cos(c+dx)}{d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b^4*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3
)/(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d
*x])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\int \cos^3(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{b^2\int \cos(c+dx)(a-b\tan(c+dx))\,dx}{\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{\left(a^2+b^2\right)^2} +$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{\left(ab^2\right)\int \cos(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{b^4\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,d\right.}{}$$

$$= -\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{ab^2\sin(}{(a^2+}$$

Mathematica [A]
time = 1.36, size = 137, normalized size = 0.83

$$\frac{24b^4\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}\left(3b\left(a^2+5b^2\right)\cos(c+dx) + b\left(a^2+b^2\right)\cos(3(c+dx)) + 2a\left(5a^2+11b^2+\left(a^2+b^2\right)\cos(2(c+dx))\right)\sin(c+dx)\right)}{12\left(a^2+b^2\right)^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

Maple [A]
time = 0.45, size = 221, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2b^2a\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}b^2a\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2b^2a\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}b^2a\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
risch	$-\frac{5\,\mathrm{e}^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{5\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-a^4}{\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{(a^2+b^2)^{\frac{5}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2
*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*b^2*a)*tan(
1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*
c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(159) = 318$.
time = 0.51, size = 379, normalized size = 2.30

$$-\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(
b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a^3 +
```

```
4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [A]
time = 0.40, size = 262, normalized size = 1.59

$$\frac{3\sqrt{a^2+b^2}\,b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^3+6\,(a^2b^3+b^5)\cos(dx+c)+2\,\left(2\,a^5+7\,a^3b^2+5\,ab^4+(a^5+2\,a^3b^2+ab^4)\cos(dx+c)^2\right)\sin(dx+c)}{6\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.55, size = 286, normalized size = 1.73

$$-\frac{\frac{3\,b^4\log\left(\left|\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}}{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}}\right|\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2b+4\,b^3\right)}{(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
```

```
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

Mupad [B]
time = 6.34, size = 342, normalized size = 2.07

$$\frac{\frac{\frac{2a^2b}{3}+\frac{8b^3}{3}}{a^4+2a^2b^2+b^4}+\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+4ab^2)}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4a^3}{3}+\frac{16ab^2}{3}\right)}{a^4+2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+2ab^2)}{a^4+2a^2b^2+b^4}+\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2+2b^2)}{a^4+2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{2b^4\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] (((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))/(
a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3))/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 + b^4
 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2
*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)
*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.554 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=178

$$-\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d}+\frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d}-\frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d}+\frac{(a^2+\ldots}{}$$

```
[Out] -6*a*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^7/d+(5*a^4+9*a^2*b^2+3*b^4)*tan(d*x+c
)/b^6/d-a*(2*a^2+3*b^2)*tan(d*x+c)^2/b^5/d+(a^2+b^2)*tan(d*x+c)^3/b^4/d-1/2
*a*tan(d*x+c)^4/b^3/d+1/5*tan(d*x+c)^5/b^2/d-(a^2+b^2)^3/b^7/d/(a+b*tan(d*x
+c))
```

Rubi [A]

time = 0.11, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$-\frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}-\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d}-\frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d}+\frac{(a^2+b^2)\tan^3(c+dx)}{b^4d}+\frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d}-\frac{a\tan^4(c+dx)}{2b^3d}+\frac{\tan^5(c+dx)}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^7*d) + ((5*a^4 + 9*a^2*b^2
+ 3*b^4)*Tan[c + d*x])/(b^6*d) - (a*(2*a^2 + 3*b^2)*Tan[c + d*x]^2)/(b^5*d)
 + ((a^2 + b^2)*Tan[c + d*x]^3)/(b^4*d) - (a*Tan[c + d*x]^4)/(2*b^3*d) + Ta
n[c + d*x]^5/(5*b^2*d) - (a^2 + b^2)^3/(b^7*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5a^4+9a^2b^2+3b^4}{b^6} - \frac{2a(2a^2+3b^2)x}{b^6} + \frac{3(a^2+b^2)x^2}{b^6} - \frac{2ax^3}{b^6} + \frac{x^4}{b^6} + \frac{(a^2+b^2)^3}{b^6(a+x)^2} - \frac{6a(a^2}{b^6(a}\right.\right.}{bd}$$
$$= -\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d} + \frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d} - \frac{a(2a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(178) = 356.
time = 1.98, size = 373, normalized size = 2.10

$$\frac{b\sec^6(c+dx)\left(-70a^5b-60a^3b^3+50ab^5-5ab(27a^4+32a^2b^2+b^4)\cos(2(c+dx))-2(45a^5b+70a^3b^3+17ab^5)\cos(4(c+dx))-25a^5b\cos(6(c+dx))-40a^3b^3\cos(6(c+dx))-11ab^5\cos(6(c+dx))+120a^6\sin(4(c+dx))+200a^4b^2\sin(4(c+dx))+76a^2b^4\sin(4(c+dx))+20b^6\sin(4(c+dx))+30a^6\sin(6(c+dx))+55a^4b^2\sin(6(c+dx))+26a^2b^4\sin(6(c+dx))+5b^6\sin(6(c+dx))\right)+10b(30a^6+47a^4b^2+10a^2b^4+5b^6)\sec^4(c+dx)\tan(c+dx)+960a^2(a^2+b^2)^2(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))(a+b\tan(c+dx))}{160ab^7d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*Sec[c + d*x]^6*(-70*a^5*b - 60*a^3*b^3 + 50*a*b^5 - 5*a*b*(27*a^4 + 32*a
^2*b^2 + b^4)*Cos[2*(c + d*x)] - 2*(45*a^5*b + 70*a^3*b^3 + 17*a*b^5)*Cos[4
*(c + d*x)] - 25*a^5*b*Cos[6*(c + d*x)] - 40*a^3*b^3*Cos[6*(c + d*x)] - 11*
a*b^5*Cos[6*(c + d*x)] + 120*a^6*Sin[4*(c + d*x)] + 200*a^4*b^2*Sin[4*(c +
d*x)] + 76*a^2*b^4*Sin[4*(c + d*x)] + 20*b^6*Sin[4*(c + d*x)] + 30*a^6*Sin[
6*(c + d*x)] + 55*a^4*b^2*Sin[6*(c + d*x)] + 26*a^2*b^4*Sin[6*(c + d*x)] +
5*b^6*Sin[6*(c + d*x)]) + 10*b*(30*a^6 + 47*a^4*b^2 + 10*a^2*b^4 + 5*b^6)*S
ec[c + d*x]^4*Tan[c + d*x] + 960*a^2*(a^2 + b^2)^2*(Log[Cos[c + d*x]] - Log
[a*Cos[c + d*x] + b*Sin[c + d*x]])*(a + b*Tan[c + d*x]))/(160*a*b^7*d*(a +
b*Tan[c + d*x]))
```

Maple [A]
time = 0.36, size = 201, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{(\tan^5(dx+c))b^4}{5} - \frac{ab^3(\tan^4(dx+c))}{2} + a^2b^2(\tan^3(dx+c)) + b^4(\tan^3(dx+c)) - 2a^3b(\tan^2(dx+c)) - 3ab^3(\tan^2(dx+c)) + 5a^4\tan(dx+c)}{b^6}}{d}$
default	$\frac{\frac{\frac{(\tan^5(dx+c))b^4}{5} - \frac{ab^3(\tan^4(dx+c))}{2} + a^2b^2(\tan^3(dx+c)) + b^4(\tan^3(dx+c)) - 2a^3b(\tan^2(dx+c)) - 3ab^3(\tan^2(dx+c)) + 5a^4\tan(dx+c)}{b^6}}{d}$
risch	$-\frac{4i(15a^4b+25a^2b^3+8b^5+90a^4b\,e^{4i(dx+c)}+60a^4b\,e^{6i(dx+c)}+140a^2b^3e^{4i(dx+c)}+80a^2b^3e^{6i(dx+c)}+100a^2b^3e^{2i(dx+c)}+60a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^6*(1/5*tan(d*x+c)^5*b^4-1/2*a*b^3*tan(d*x+c)^4+a^2*b^2*tan(d*x+c)^
3+b^4*tan(d*x+c)^3-2*a^3*b*tan(d*x+c)^2-3*a*b^3*tan(d*x+c)^2+5*a^4*tan(d*x+
c)+9*a^2*b^2*tan(d*x+c)+3*b^4*tan(d*x+c))-1/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^
6)/(a+b*tan(d*x+c))-6*a/b^7*(a^4+2*a^2*b^2+b^4)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 186, normalized size = 1.04

$$-\frac{\frac{10\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}{b^{8}\tan\left(dx+c\right)+ab^{7}}-\frac{2b^{4}\tan\left(dx+c\right)^{5}-5ab^{3}\tan\left(dx+c\right)^{4}+10\left(a^{2}b^{2}+b^{4}\right)\tan\left(dx+c\right)^{3}-10\left(2a^{3}b+3ab^{3}\right)\tan\left(dx+c\right)^{2}+10\left(5a^{4}+9a^{2}b^{2}+3b^{4}\right)\tan\left(dx+c\right)}{b^{6}}+\frac{60\left(a^{5}+2a^{3}b^{2}+ab^{4}\right)\log\left(b\tan\left(dx+c\right)+a\right)}{b^{7}}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/(b^8*tan(d*x + c) + a*b^7) -
(2*b^4*tan(d*x + c)^5 - 5*a*b^3*tan(d*x + c)^4 + 10*(a^2*b^2 + b^4)*tan(d*x
 + c)^3 - 10*(2*a^3*b + 3*a*b^3)*tan(d*x + c)^2 + 10*(5*a^4 + 9*a^2*b^2 + 3
*b^4)*tan(d*x + c))/b^6 + 60*(a^5 + 2*a^3*b^2 + a*b^4)*log(b*tan(d*x + c) +
 a)/b^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(174) = 348.
time = 0.42, size = 386, normalized size = 2.17

$$-\frac{4\left(15a^{4}b^{2}+25a^{2}b^{4}+8b^{6}\right)\cos\left(dx+c\right)^{6}-2b^{6}-2\left(15a^{4}b^{2}+25a^{2}b^{4}+8b^{6}\right)\cos\left(dx+c\right)^{4}-\left(5a^{2}b^{4}+4b^{6}\right)\cos\left(dx+c\right)^{2}+30\left(\left(a^{6}+2a^{4}b^{2}+a^{2}b^{4}\right)\cos\left(dx+c\right)^{6}+\left(a^{5}b+2a^{3}b^{3}+ab^{5}\right)\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)-30\left(\left(a^{6}+2a^{4}b^{2}+a^{2}b^{4}\right)\cos\left(dx+c\right)^{6}+\left(a^{5}b+2a^{3}b^{3}+ab^{5}\right)\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^{2}\right)+\left(3ab^{5}\cos\left(dx+c\right)-4\left(15a^{5}b+25a^{3}b^{3}+8ab^{5}\right)\cos\left(dx+c\right)^{5}+2\left(5a^{3}b^{3}+7ab^{5}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{10\left(ab^{7}d\cos\left(dx+c\right)^{6}+b^{8}d\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(4*(15*a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - 2*b^6 - 2*(15*a
^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^4 - (5*a^2*b^4 + 4*b^6)*cos(d*x +
 c)^2 + 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^6 + (a^5*b + 2*a^3*b^3
 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
+ (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d
*x + c)^6 + (a^5*b + 2*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(co
s(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*(15*a^5*b + 25*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^5 + 2*(5*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))/(a*
b^7*d*cos(d*x + c)^6 + b^8*d*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{8}\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.58, size = 253, normalized size = 1.42

$$-\frac{\frac{60\,(a^5+2\,a^3b^2+ab^4)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{10\,(6\,a^5b\tan(dx+c)+12\,a^3b^3\tan(dx+c)+6\,ab^5\tan(dx+c)+5\,a^6+9\,a^4b^2+3\,a^2b^4-b^6)}{(b\tan(dx+c)+a)b^7}-\frac{2\,b^8\tan(dx+c)^5-5\,ab^7\tan(dx+c)^4+10\,a^2b^6\tan(dx+c)^3+10\,b^8\tan(dx+c)^3-20\,a^3b^5\tan(dx+c)^2-30\,ab^7\tan(dx+c)^2+50\,a^4b^4\tan(dx+c)+90\,a^2b^6\tan(dx+c)+30\,b^8\tan(dx+c)}{b^{10}}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 10*(
6*a^5*b*tan(d*x + c) + 12*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 5*a
^6 + 9*a^4*b^2 + 3*a^2*b^4 - b^6)/((b*tan(d*x + c) + a)*b^7) - (2*b^8*tan(d
*x + c)^5 - 5*a*b^7*tan(d*x + c)^4 + 10*a^2*b^6*tan(d*x + c)^3 + 10*b^8*tan
(d*x + c)^3 - 20*a^3*b^5*tan(d*x + c)^2 - 30*a*b^7*tan(d*x + c)^2 + 50*a^4*
b^4*tan(d*x + c) + 90*a^2*b^6*tan(d*x + c) + 30*b^8*tan(d*x + c))/b^10)/d
```

Mupad [B]
time = 3.63, size = 258, normalized size = 1.45

$$\frac{\tan(c+d\,x)^2\,\left(\frac{a^3}{b^5}-\frac{a\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{d}+\frac{\tan(c+d\,x)^5}{5\,b^2\,d}+\frac{\tan(c+d\,x)^3\,\left(\frac{1}{b^2}+\frac{a^2}{b^4}\right)}{d}-\frac{\tan(c+d\,x)\,\left(\frac{a^2\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{3}{b^2}+\frac{2\,a\,\left(\frac{2\,a^3}{b^5}-\frac{2\,a\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d}-\frac{a\tan(c+d\,x)^4}{2\,b^3\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{b^7\,d}-\frac{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^7+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (tan(c + d*x)^2*(a^3/b^5 - (a*(3/b^2 + (3*a^2)/b^4))/b))/d + tan(c + d*x)^5
/(5*b^2*d) + (tan(c + d*x)^3*(1/b^2 + a^2/b^4))/d - (tan(c + d*x)*((a^2*(3/
b^2 + (3*a^2)/b^4))/b^2 - 3/b^2 + (2*a*((2*a^3)/b^5 - (2*a*(3/b^2 + (3*a^2)
/b^4))/b))/b))/d - (a*tan(c + d*x)^4)/(2*b^3*d) - (log(a + b*tan(c + d*x))*
(6*a*b^4 + 6*a^5 + 12*a^3*b^2))/(b^7*d) - (a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)/(b*d*(a*b^6 + b^7*tan(c + d*x)))
```

3.555 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}+\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a^2+\ldots}{b^5d(a+b\tan\ldots}$$

```
[Out] -4*a*(a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*ta
n(d*x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d-(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$-\frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}-\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}+\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) + ((3*a^2 + 2*b^2)*Tan[c
 + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) -
(a^2 + b^2)^2/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3a^2+2b^2}{b^4} - \frac{2ax}{b^4} + \frac{x^2}{b^4} + \frac{(a^2+b^2)^2}{b^4(a+x)^2} - \frac{4a(a^2+b^2)}{b^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} + \frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} - \frac{a\tan^2(c+dx}{b^3d}$$

Mathematica [A]

time = 1.62, size = 207, normalized size = 1.78

$$\frac{12a^3(a^2+b^2)\left(\log(\cos(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx))\right) + b(12a^4+11a^2b^2+3b^4+12a^2(a^2+b^2)\log(\cos(c+dx)) - 12a^2(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx) + ab^2(9a^2+5b^2)\tan^2(c+dx) + ab^2\sec^2(c+dx)\left(-3a^2-2ab\tan(c+dx)+b^2\tan^2(c+dx)\right)}{3ab^5d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a^3*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x
]]) + b*(12*a^4 + 11*a^2*b^2 + 3*b^4 + 12*a^2*(a^2 + b^2)*Log[Cos[c + d*x]]
 - 12*a^2*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] +
a*b^2*(9*a^2 + 5*b^2)*Tan[c + d*x]^2 + a*b^2*Sec[c + d*x]^2*(-3*a^2 - 2*a*b
*Tan[c + d*x] + b^2*Tan[c + d*x]^2))/(3*a*b^5*d*(a + b*Tan[c + d*x]))
```

Maple [A]

time = 0.30, size = 114, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\frac{(\tan^3(dx+c))b^2}{3}-(\tan^2(dx+c))ab+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a(a^2+b^2)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\frac{(\tan^3(dx+c))b^2}{3}-(\tan^2(dx+c))ab+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a(a^2+b^2)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$-\frac{8i(2b^3-2iab^2+3a^2b-3ia^3+3a^2b\,e^{4i(dx+c)}+6a^2b\,e^{2i(dx+c)}-3ia^3e^{6i(dx+c)}-9ia^3e^{4i(dx+c)}-9ia^3e^{2i(dx+c)}-3iab^2e^{6i(dx}}{3(e^{2i(dx+c)}+1)^3(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)b^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(1/3*tan(d*x+c)^3*b^2-tan(d*x+c)^2*a*b+3*a^2*tan(d*x+c)+2*b^2*ta
n(d*x+c))-1/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))-4*a/b^5*(a^2+b^2)*ln(a
+b*tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 115, normalized size = 0.99

$$-\frac{\frac{3\,(a^4+2\,a^2b^2+b^4)}{b^6\tan(dx+c)+ab^5}-\frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2+2\,b^2)\tan(dx+c)}{b^4}+\frac{12\,(a^3+ab^2)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x +
 c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^
3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(114) = 228.
time = 0.49, size = 281, normalized size = 2.42

$$-\frac{4\,(3\,a^2b^2+2\,b^4)\cos(dx+c)^4-b^4-2\,(3\,a^2b^2+2\,b^4)\cos(dx+c)^2+6\,\left((a^4+a^2b^2)\cos(dx+c)^4+(a^3b+ab^3)\cos(dx+c)^3\sin(dx+c)\right)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-6\,\left((a^4+a^2b^2)\cos(dx+c)^4+(a^3b+ab^3)\cos(dx+c)^3\sin(dx+c)\right)\log\left(\cos(dx+c)^2\right)+2\,\left(ab^3\cos(dx+c)-2\,(3\,a^3b+2\,ab^3)\cos(dx+c)^3\right)\sin(dx+c)}{3\,\left(ab^5d\cos(dx+c)^4+b^6d\cos(dx+c)^3\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.53, size = 149, normalized size = 1.28

$$-\frac{\frac{12\,(a^3+ab^2)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\,(4\,a^3b\tan(dx+c)+4\,ab^3\tan(dx+c)+3\,a^4+2\,a^2b^2-b^4)}{(b\tan(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)
^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/
b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 -
b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

Mupad [B]
time = 3.71, size = 130, normalized size = 1.12

$$\frac{\tan(c+d\,x)^3}{3\,b^2\,d}+\frac{\tan(c+d\,x)\,\left(\frac{2}{b^2}+\frac{3\,a^2}{b^4}\right)}{d}-\frac{a\tan(c+d\,x)^2}{b^3\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(4\,a^3+4\,a\,b^2\right)}{b^5\,d}-\frac{a^4+2\,a^2\,b^2+b^4}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^5+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c + d*x)^3/(3*b^2*d) + (tan(c + d*x)*(2/b^2 + (3*a^2)/b^4))/d - (a*tan(
c + d*x)^2)/(b^3*d) - (log(a + b*tan(c + d*x))*(4*a*b^2 + 4*a^3))/(b^5*d) -
 (a^4 + b^4 + 2*a^2*b^2)/(b*d*(a*b^4 + b^5*tan(c + d*x)))
```

3.556 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$-\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}$$

```
[Out] -2*a*ln(a+b*tan(d*x+c))/b^3/d+tan(d*x+c)/b^2/d+(-a^2-b^2)/b^3/d/(a+b*tan(d*
x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}-\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d) - (a^2 + b^2)
/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2+b^2}{b^2(a+x)^2}-\frac{2a}{b^2(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= -\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}$$

Mathematica [A]

time = 0.37, size = 121, normalized size = 1.98

$$\frac{2a^3(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(2a^2+b^2+2a^2\log(\cos(c+dx))-2a^2\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+ab^2\tan^2(c+dx)}{ab^3d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^3*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) + b*(2*a^
2 + b^2 + 2*a^2*Log[Cos[c + d*x]] - 2*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])*Tan[c + d*x] + a*b^2*Tan[c + d*x]^2)/(a*b^3*d*(a + b*Tan[c + d*x]))
```

Maple [A]

time = 0.31, size = 57, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$	57
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$	57
risch	$-\frac{4i(-ia\,e^{2i(dx+c)}+b-ia)}{(e^{2i(dx+c)}+1)(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)b^2d}+\frac{2a\ln(e^{2i(dx+c)}+1)}{b^3d}-\frac{2a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*tan(d*x+c)-1/b^3*(a^2+b^2)/(a+b*tan(d*x+c))-2/b^3*a*ln(a+b*tan(d
*x+c)))
```

Maxima [A]

time = 0.26, size = 60, normalized size = 0.98

$$-\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2\,a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(61) = 122$.
time = 0.42, size = 178, normalized size = 2.92

$$-\frac{2 b^{2} \cos \left(d x+c\right)^{2}-2 a b \cos \left(d x+c\right) \sin \left(d x+c\right)-b^{2}+\left(a^{2} \cos \left(d x+c\right)^{2}+a b \cos \left(d x+c\right) \sin \left(d x+c\right)\right) \log \left(2 a b \cos \left(d x+c\right) \sin \left(d x+c\right)+\left(a^{2}-b^{2}\right) \cos \left(d x+c\right)^{2}+b^{2}\right)-\left(a^{2} \cos \left(d x+c\right)^{2}+a b \cos \left(d x+c\right) \sin \left(d x+c\right)\right) \log \left(\cos \left(d x+c\right)^{2}\right)}{a b^{3} d \cos \left(d x+c\right)^{2}+b^{4} d \cos \left(d x+c\right) \sin \left(d x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.52, size = 71, normalized size = 1.16

$$-\frac{\frac{2\,a\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{b^{3}}-\frac{\tan\left(dx+c\right)}{b^{2}}-\frac{2\,ab\tan\left(dx+c\right)+a^{2}-b^{2}}{\left(b\tan\left(dx+c\right)+a\right)b^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

Mupad [B]
time = 3.72, size = 67, normalized size = 1.10

$$\frac{\tan(c+d\,x)}{b^2\,d}-\frac{a^2+b^2}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)}-\frac{2\,a\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c + d*x)/(b^2*d) - (a^2 + b^2)/(b*d*(a*b^2 + b^3*tan(c + d*x))) - (2*a*
log(a + b*tan(c + d*x)))/(b^3*d)
```

3.557 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

```
[Out] -1/b/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(1/(b*d*(a + b*Tan[c + d*x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= -\frac{1}{bd(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.60

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A]
time = 0.27, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
default	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d/(a+b*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.00

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.
time = 0.37, size = 57, normalized size = 2.85

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.55, size = 20, normalized size = 1.00

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Mupad [B]
time = 3.65, size = 20, normalized size = 1.00

$$-\frac{1}{b\,d\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

```
[Out] -1/(b*d*(a + b*tan(c + d*x)))
```

3.558 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{b(a^2-3b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}+\frac{\cos^2(c+dx}{2\,(a^2+b^2)}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+c
))/(a^2+b^2)^3/d+1/2*b*(a^2-3*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+1/2*cos(d
*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3587, 755, 815, 649, 209, 266}

$$\frac{b(a^2-3b^2)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{x(a^4+6a^2b^2-3b^4)}{2\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + (4*a*b^3*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (b*(a^2 - 3*b^2))/(2*(a^2 + b^2)^
2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-3-\frac{a^2}{b^2}-\frac{2ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)\,d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \left(\frac{a^2-3b^2}{(a^2+b^2)(a+x)^2} - \frac{8ab^2}{(a^2+b^2)^2(a+x)} + \right.\right.}{2(a^2+}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx}{2(a^2+b^2)\,d}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx}{2(a^2+b^2)\,d}$$

$$= \frac{(a^4+6a^2b^2-3b^4)\,x}{2(a^2+b^2)^3} + \frac{4ab^3\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} +$$

Mathematica [A]

time = 4.21, size = 304, normalized size = 2.00

$$\frac{-\frac{ab\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2\sqrt{-b^2}\,\log(a+b\tan(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)}{\sqrt{-b^2}\,(a^2+b^2)}+\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{a+b\tan(c+dx)}+\frac{b(a^2-3b^2)\left(\left(2a+\frac{-a^2+b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-4a\log(a+b\tan(c+dx))+\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+\frac{2\left(a^2+b^2\right)}{a+b\tan(c+dx)}\right)}{2(a^2+b^2)^2}}{2\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-((a*b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*Sqrt[-b^2]*
Log[a + b*Tan[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
))/(Sqrt[-b^2]*(a^2 + b^2))) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(a + b
*Tan[c + d*x]) + (b*(a^2 - 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] - 4*a*Log[a + b*Tan[c + d*x]] + (2*a + (a^2 - b^2)
/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 + b^2))/(a + b*Tan[
c + d*x])))/(2*(a^2 + b^2)^2))/(2*(a^2 + b^2)*d)
```

Maple [A]

time = 0.42, size = 154, normalized size = 1.01

method	result
derivativedivides	$\frac{-\frac{b^{3}}{\left(a^{2}+b^{2}\right)^{2}\left(a+b\tan(dx+c)\right)}+\frac{4b^{3}a\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(\frac{a^{4}}{2}-\frac{b^{4}}{2}\right)\tan(dx+c)+a^{3}b+a\,b^{3}}{1+\tan^{2}(dx+c)}-2a\,b^{3}\ln\left(1+\tan^{2}(dx+c)\right)+\frac{\left(a^{4}+6a^{2}b^{2}-3\right.}{}}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
default	$\frac{-\frac{b^{3}}{\left(a^{2}+b^{2}\right)^{2}\left(a+b\tan(dx+c)\right)}+\frac{4b^{3}a\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{\frac{\left(\frac{a^{4}}{2}-\frac{b^{4}}{2}\right)\tan(dx+c)+a^{3}b+a\,b^{3}}{1+\tan^{2}(dx+c)}-2a\,b^{3}\ln\left(1+\tan^{2}(dx+c)\right)+\frac{\left(a^{4}+6a^{2}b^{2}-3\right.}{}}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
risch	$\frac{3ixb}{6ib\,a^{2}-2ib^{3}-2a^{3}+6b^{2}a}-\frac{xa}{6ib\,a^{2}-2ib^{3}-2a^{3}+6b^{2}a}-\frac{ie^{2i(dx+c)}}{8(-2iab+a^{2}-b^{2})d}+\frac{ie^{-2i(dx+c)}}{8(2iab+a^{2}-b^{2})d}-\frac{8ia\,b^{3}x}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c
))+1/(a^2+b^2)^3*(((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(1+tan(d*x+c)^
2)-2*a*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+c)))
)
```

Maxima [A]

time = 0.50, size = 282, normalized size = 1.86

$$\frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^2b-2\,b^3+(a^2b-3\,b^3)\tan(dx+c)^2+(a^3+ab^2)\tan(dx+c)}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5+2\,a^3b^2+ab^4)\tan(dx+c)^2+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

Fricas [A]
time = 0.43, size = 279, normalized size = 1.84

$$\frac{(a^4b+2a^2b^3+b^5)\cos(dx+c)^3-(a^2b^3+3b^5-(a^5+6a^3b^2-3ab^4)dx)\cos(dx+c)+4(a^2b^3\cos(dx+c)+ab^4\sin(dx+c))\log(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2)-(a^3b^2-ab^4-(a^4b+6a^2b^3-3b^5)dx-(a^5+2a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{2((a^7+3a^5b^2+3a^3b^4+ab^6)d\cos(dx+c)+(a^6b+3a^4b^3+3a^2b^5+b^7)d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6
*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x +
c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.60, size = 250, normalized size = 1.64

$$\frac{\frac{8ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7}-\frac{4ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{(a^4+6a^2b^2-3b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{a^2b\tan(dx+c)^2-3b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+2a^2b-2b^3}{(a^4+2a^2b^2+b^4)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

```
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

Mupad [B]

time = 3.93, size = 246, normalized size = 1.62

$$\frac{\frac{a^2\,b-b^3}{{\left(a^2+b^2\right)}^2}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(a^2\,b-3\,b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\,\mathrm{tan}\left(c+d\,x\right)}{2\,\left(a^2+b^2\right)}}{d\,\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3+a\,{\mathrm{tan}\left(c+d\,x\right)}^2+b\,\mathrm{tan}\left(c+d\,x\right)+a\right)}+\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)\,\left(-3\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}+\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,\left(a-b\,3{}\mathrm{i}\right)}{4\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}+\frac{4\,a\,b^3\,\ln\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}{d\,{\left(a^2+b^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2*b - b^3)/(a^2 + b^2)^2 + (tan(c + d*x)^2*(a^2*b - 3*b^3))/(2*(a^4 + b
^4 + 2*a^2*b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2)))/(d*(a + b*tan(c + d*x)
 + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) - 1i)*(a*1i -
3*b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(tan(c + d*x) + 1i)*(
a - b*3i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (4*a*b^3*log(a + b*t
an(c + d*x)))/(d*(a^2 + b^2)^3)
```

3.559 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$\frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)\,x}{8\,(a^2+b^2)^4}+\frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}+\frac{3b(a^2-b^2)\,(a^2+5b^2)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}+$$

```
[Out] 3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*x/(a^2+b^2)^4+6*a*b^5*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^4/d+3/8*b*(a^2-b^2)*(a^2+5*b^2)/(a^2+b^2)^3/d/(a+b*
tan(d*x+c))+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))-
1/8*cos(d*x+c)^2*(b*(a^2-5*b^2)-3*a*(a^2+3*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\frac{3b(a^2-b^2)\,(a^2+5b^2)}{8d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\,(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx))}{8d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}+\frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{3x(a^6+5a^4b^2+15a^2b^4-5b^6)}{8\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*x)/(8*(a^2 + b^2)^4) + (6*a*b^5*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (3*b*(a^2 - b^2)*(
a^2 + 5*b^2))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b
 + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^
2*(b*(a^2 - 5*b^2) - 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-5-\frac{3a^2}{b^2}-\frac{4ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx\right.}{4(a^2+b^2)\,d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\,\text{ta}\right.}{8(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\,\text{ta}\right.}{8(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{(a^2+b^2)^4\,d} + \frac{3b(a^2-b^2)(a^2+5b^2)}{8(a^2+b^2)^3\,d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx}{4(a^2+b^2)\,d}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{(a^2+b^2)^4\,d} + \frac{3b(a^2-b^2)(a^2+5b^2)}{8(a^2+b^2)^3\,d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx}{4(a^2+b^2)\,d}$$

$$= \frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)\,x}{8(a^2+b^2)^4} + \frac{6ab^5\log(\cos(c+dx))}{(a^2+b^2)^4\,d} + \frac{6ab^5\log(a+b\tan(}{(a^2+b^2)^4\,d}$$

Mathematica [A]
time = 3.47, size = 416, normalized size = 1.77

$$\frac{4b\cos^4(c+dx)(b+a\tan(c+dx)) + \frac{2b\cos^2(c+dx)\left(-a^2b+5b^3+3a(a^2+3b^2)\tan(c+dx)\right)}{a^2+b^2} - \frac{\sqrt{-b^2}\left(6a(a^2+b^2)(a^2+3b^2)\left(\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2\sqrt{-b^2}\log(a+b\tan(c+dx))-\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)(a+b\tan(c+dx))+3(a^4+4a^2b^2-5b^4)\left(2\sqrt{-b^2}(a^2+b^2)+\left(-a^2+b^2+2a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)(a+b\tan(c+dx))-4a\sqrt{-b^2}\log(a+b\tan(c+dx))(a+b\tan(c+dx))+\left(a^2-b^2+2a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)(a+b\tan(c+dx))\right)\right)}{(a^2+b^2)^3}}{16b(a^2+b^2)\,d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + (2*b*Cos[c + d*x]^2*(-(a^2*b) +
5*b^3 + 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(a^2 + b^2) - (Sqrt[-b^2]*(6*a*(a^
2 + b^2)*(a^2 + 3*b^2)*((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] +
 2*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]] - (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]])*(a + b*Tan[c + d*x]) + 3*(a^4 + 4*a^2*b^2 - 5*b^4)*(2*Sqrt[
-b^2]*(a^2 + b^2) + (-a^2 + b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
+ d*x]]*(a + b*Tan[c + d*x]) - 4*a*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]]*(a +
b*Tan[c + d*x]) + (a^2 - b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d
*x]]*(a + b*Tan[c + d*x]))))/(a^2 + b^2)^3)/(16*b*(a^2 + b^2)*d*(a + b*Tan[
c + d*x]))
```

Maple [A]
time = 0.40, size = 248, normalized size = 1.06

method	result
derivativedivides	$\frac{-\frac{b^5}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{6b^5a\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)(\tan^3(dx+c))+(2a^3b^3+2a\,b^5)(\tan^2(dx+c))}{(1+\tan^2(dx+c}}{}}{}$
default	$\frac{-\frac{b^5}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{6b^5a\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)(\tan^3(dx+c))+(2a^3b^3+2a\,b^5)(\tan^2(dx+c))}{(1+\tan^2(dx+c}}{}}{}$
risch	$\frac{12ixab}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}-\frac{3x\,a^2}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}+\frac{15x\,b^2}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}-\frac{}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))+6*b^5/(a^2+b^2)^4*a*ln(a+b*tan(d*x+c
))+1/(a^2+b^2)^4*(((3/8*a^6+15/8*a^4*b^2+5/8*a^2*b^4-7/8*b^6)*tan(d*x+c)^3+
(2*a^3*b^3+2*a*b^5)*tan(d*x+c)^2+(17/8*a^4*b^2+3/8*a^2*b^4-9/8*b^6+5/8*a^6)
*tan(d*x+c)+1/2*a^5*b+3*a^3*b^3+5/2*a*b^5)/(1+tan(d*x+c)^2)^2-3*a*b^5*ln(1+
tan(d*x+c)^2)+3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(228) = 456.
time = 0.49, size = 502, normalized size = 2.14

$$\frac{\frac{48\,ab^5\log\left(b\tan(dx+c)+a\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{4\,a^4b+20\,a^2b^3-8\,b^5+3\,(a^4b+4\,a^2b^3-5\,b^5)\tan(dx+c)^4+3\,(a^5+4\,a^3b^2+3\,ab^4)\tan(dx+c)^3+(5\,a^4b+28\,a^2b^3-25\,b^5)\tan(dx+c)^2+(5\,a^5+16\,a^3b^2+11\,ab^4)\tan(dx+c)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\tan(dx+c)^5+(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\tan(dx+c)^4+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\tan(dx+c)^3+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\tan(dx+c)^2+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\tan(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(48*a*b^5*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (4*a^4*b + 20*a^2*b^3 - 8*b^
5 + 3*(a^4*b + 4*a^2*b^3 - 5*b^5)*tan(d*x + c)^4 + 3*(a^5 + 4*a^3*b^2 + 3*a
*b^4)*tan(d*x + c)^3 + (5*a^4*b + 28*a^2*b^3 - 25*b^5)*tan(d*x + c)^2 + (5*
a^5 + 16*a^3*b^2 + 11*a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c
)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Fricas [**A**]
time = 0.45, size = 424, normalized size = 1.80

$$\frac{4 \left(a^{6} b+3 a^{4} b^{3}+3 a^{2} b^{5}+b^{7}\right) \cos \left(d x+c\right)^{5}-2 \left(a^{6} b-3 a^{4} b^{3}-9 a^{2} b^{5}-5 b^{7}\right) \cos \left(d x+c\right)^{3}+\left(3 a^{6} b+8 a^{4} b^{3}-9 a^{2} b^{5}-30 b^{7}+6 \left(a^{7}+5 a^{5} b^{2}+15 a^{3} b^{4}-5 a b^{6}\right) d x\right) \cos \left(d x+c\right)+48 \left(a^{2} b^{5} \cos \left(d x+c\right)+a b^{6} \sin \left(d x+c\right)\right) \log \left(2 a b \cos \left(d x+c\right) \sin \left(d x+c\right)+\left(a^{2}-b^{2}\right) \cos \left(d x+c\right)^{2}+b^{2}\right)-\left(3 a^{5} b^{2}+22 a^{3} b^{4}+3 a b^{6}-4 \left(a^{7}+3 a^{5} b^{2}+3 a^{3} b^{4}+a b^{6}\right) \cos \left(d x+c\right)^{4}-6 \left(a^{6} b+5 a^{4} b^{3}+15 a^{2} b^{5}-5 b^{7}\right) d x-6 \left(a^{7}+5 a^{5} b^{2}+7 a^{3} b^{4}+3 a b^{6}\right) \cos \left(d x+c\right)^{2}\right) \sin \left(d x+c\right)}{16 \left(\left(a^{9}+4 a^{7} b^{2}+6 a^{5} b^{4}+4 a^{3} b^{6}+a b^{8}\right) d \cos \left(d x+c\right)+\left(a^{8} b+4 a^{6} b^{3}+6 a^{4} b^{5}+4 a^{2} b^{7}+b^{9}\right) d \sin \left(d x+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 2*(a^6*b - 3
*a^4*b^3 - 9*a^2*b^5 - 5*b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 - 9*a^2
*b^5 - 30*b^7 + 6*(a^7 + 5*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6)*d*x)*cos(d*x + c
) + 48*(a^2*b^5*cos(d*x + c) + a*b^6*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^5*b^2 + 22*a^3*b^4 +
 3*a*b^6 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^4 - 6*(a^6*
b + 5*a^4*b^3 + 15*a^2*b^5 - 5*b^7)*d*x - 6*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 +
3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^
3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9)*d*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(228) = 456.
time = 0.56, size = 464, normalized size = 1.97

$$\frac{\frac{48 \, a b^{6} \log\left({\left| b \tan\left(d x + c\right) + a \right|}\right)}{a^{8} b + 4 \, a^{6} b^{3} + 6 \, a^{4} b^{5} + 4 \, a^{2} b^{7} + b^{9}} - \frac{24 \, a b^{5} \log\left(\tan\left(d x + c\right)^{2} + 1\right)}{a^{8} + 4 \, a^{6} b^{2} + 6 \, a^{4} b^{4} + 4 \, a^{2} b^{6} + b^{8}} + \frac{3 \, {\left(a^{6} + 5 \, a^{4} b^{2} + 15 \, a^{2} b^{4} - 5 \, b^{6}\right)} {\left(d x + c\right)}}{a^{8} + 4 \, a^{6} b^{2} + 6 \, a^{4} b^{4} + 4 \, a^{2} b^{6} + b^{8}} - \frac{8 \, {\left(6 \, a b^{6} \tan\left(d x + c\right) + 7 \, a^{2} b^{5} + b^{7}\right)}}{{\left(a^{8} + 4 \, a^{6} b^{2} + 6 \, a^{4} b^{4} + 4 \, a^{2} b^{6} + b^{8}\right)} {\left(b \tan\left(d x + c\right) + a\right)}} + \frac{36 \, a b^{5} \tan\left(d x + c\right)^{4} + 3 \, a^{6} \tan\left(d x + c\right)^{3} + 15 \, a^{4} b^{2} \tan\left(d x + c\right)^{3} + 5 \, a^{2} b^{4} \tan\left(d x + c\right)^{3} - 7 \, b^{6} \tan\left(d x + c\right)^{3} + 16 \, a^{3} b^{3} \tan\left(d x + c\right)^{2} + 88 \, a b^{5} \tan\left(d x + c\right)^{2} + 5 \, a^{6} \tan\left(d x + c\right) + 17 \, a^{4} b^{2} \tan\left(d x + c\right) + 3 \, a^{2} b^{4} \tan\left(d x + c\right) - 9 \, b^{6} \tan\left(d x + c\right) + 4 \, a^{5} b + 24 \, a^{3} b^{3} + 56 \, a b^{5}}{{\left(a^{8} + 4 \, a^{6} b^{2} + 6 \, a^{4} b^{4} + 4 \, a^{2} b^{6} + b^{8}\right)} {\left(\tan\left(d x + c\right)^{2} + 1\right)}^{2}}}{8 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*a*b^6*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x +
 c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(6*a*b^6*tan(d*x +
c) + 7*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*t
an(d*x + c) + a)) + (36*a*b^5*tan(d*x + c)^4 + 3*a^6*tan(d*x + c)^3 + 15*a^
```

```
4*b^2*tan(d*x + c)^3 + 5*a^2*b^4*tan(d*x + c)^3 - 7*b^6*tan(d*x + c)^3 + 16
*a^3*b^3*tan(d*x + c)^2 + 88*a*b^5*tan(d*x + c)^2 + 5*a^6*tan(d*x + c) + 17
*a^4*b^2*tan(d*x + c) + 3*a^2*b^4*tan(d*x + c) - 9*b^6*tan(d*x + c) + 4*a^5
*b + 24*a^3*b^3 + 56*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*(tan(d*x + c)^2 + 1)^2))/d
```

Mupad [B]
time = 4.77, size = 463, normalized size = 1.97

$$\frac{\frac{3\tan(c+d\,x)^4\,\left(a^4\,b+4\,a^2\,b^3-5\,b^5\right)}{8\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{a^4\,b+5\,a^2\,b^3-2\,b^5}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(5\,a^3+11\,a\,b^2\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{3\tan(c+d\,x)^3\,\left(a^3+3\,a\,b^2\right)}{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)^2\,\left(5\,a^4\,b+28\,a^2\,b^3-25\,b^5\right)}{8\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(b\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^4+2\,b\tan\left(c+d\,x\right)^3+2\,a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+a\right)}+\frac{3\,\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(-a^2+a\,b\,4{}\mathrm{i}+5\,b^2\right)}{16\,d\,\left(a^4\,1{}\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}-4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^2+a\,b\,4{}\mathrm{i}-5\,b^2\right)}{16\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}+\frac{6\,a\,b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((3*tan(c + d*x)^4*(a^4*b - 5*b^5 + 4*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (a^4*b - 2*b^5 + 5*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)) + (tan(c + d*x)*(11*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (
3*tan(c + d*x)^3*(3*a*b^2 + a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*
x)^2*(5*a^4*b - 25*b^5 + 28*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2
)))/(d*(a + b*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*ta
n(c + d*x)^3 + b*tan(c + d*x)^5)) + (3*log(tan(c + d*x) + 1i)*(a*b*4i - a^2
 + 5*b^2))/(16*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) + (3*l
og(tan(c + d*x) - 1i)*(a*b*4i + a^2 - 5*b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^
4*1i + b^4*1i - a^2*b^2*6i)) + (6*a*b^5*log(a + b*tan(c + d*x)))/(d*(a^2 +
b^2)^4)
```

3.560 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$\frac{5(8a^4+12a^2b^2+3b^4)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}}+\frac{5a(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^6d\sqrt{\sec^2(c+dx)}}$$

```
[Out] 5/8*(8*a^4+12*a^2*b^2+3*b^4)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+
c)^2)^(1/2)+5*a*(a^2+b^2)^(3/2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(s
ec(d*x+c)^2)^(1/2))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)-5/12*sec(d*x+c)^3
*(4*a-3*b*tan(d*x+c))/b^3/d-sec(d*x+c)^5/b/d/(a+b*tan(d*x+c))-5/8*sec(d*x+c
)*(8*a*(a^2+b^2)-b*(4*a^2+3*b^2)*tan(d*x+c))/b^5/d
```

Rubi [A]

time = 0.19, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3593, 747, 829, 858, 221, 739, 212}

$$\frac{5a(a^2+b^2)^{3/2}\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^6d\sqrt{\sec^2(c+dx)}}-\frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d}+\frac{5(8a^4+12a^2b^2+3b^4)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{8b^6d\sqrt{\sec^2(c+dx)}}-\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d}-\frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*b^6*
d*Sqrt[Sec[c + d*x]^2]) + (5*a*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^6*d*Sqrt[Sec[c
+ d*x]^2]) - (5*Sec[c + d*x]^3*(4*a - 3*b*Tan[c + d*x]))/(12*b^3*d) - Sec[c
 + d*x]^5/(b*d*(a + b*Tan[c + d*x])) - (5*Sec[c + d*x]*(8*a*(a^2 + b^2) - b
*(4*a^2 + 3*b^2)*Tan[c + d*x]))/(8*b^5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx))\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{a+x}\,dx, x, b\tan(c+\ldots\right.}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx\ldots}{\ldots}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx\ldots}{\ldots}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx\ldots}{\ldots}$$

$$= \frac{5(8a^4+12a^2b^2+3b^4)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-\ldots}{12b^3d\ldots}$$

$$= \frac{5(8a^4+12a^2b^2+3b^4)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} + \frac{5a(a^2+b^2)^{3/2}\tanh^{-}\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.23, size = 1152, normalized size = 4.90

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)^2*(a + I*b)^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
)/(b^5*d*(a + b*Tan[c + d*x])^2)) - (a*(12*a^2 + 13*b^2)*Sec[c + d*x]^2*(a*
```

```
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(a + b*Tan[c + d*x])^2) + ((10*I
)*a*(a + I*b)*(I*a + b)*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a^2 + b^2]*(-(b*Cos[(
c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c + d*x)/2] + b^2*Cos[(c + d*
x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^6*d*(a + b*T
an[c + d*x])^2) - (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(8*b^6*
d*(a + b*Tan[c + d*x])^2) + (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(8*b^6*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(16*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a + b*Ta
n[c + d*x])^2) + ((36*a^2 - 8*a*b + 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^2)/(48*b^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a +
 b*Tan[c + d*x])^2) - (a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)/(3*b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a + b*
Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(16
*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a + b*Tan[c + d*x])^2) + (a
*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^
3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + b*Tan[c + d*x])^2) + ((-36
*a^2 - 8*a*b - 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(48*b^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) +
 (Sec[c + d*x]^2*(-12*a^3*Sin[(c + d*x)/2] - 13*a*b^2*Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(12*a^3*Sin[(c + d*x)/2] +
13*a*b^2*Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(219) = 438$.
time = 0.52, size = 454, normalized size = 1.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^6*(((a^4+2*a^2*b^2+b^4)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^4+2*a^2*b^2+
b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-5*a*(a^4+2*a^2*b^2+
b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))+1/4/b^2/(tan(1/2*d*x+1/2*c)-1)^4-1/6*(-4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c
)-1)^3-1/8*(-12*a^2-8*a*b-11*b^2)/b^4/(tan(1/2*d*x+1/2*c)-1)^2+1/8/b^6*(-40
*a^4-60*a^2*b^2-15*b^4)*ln(tan(1/2*d*x+1/2*c)-1)-1/8*(-32*a^3-12*a^2*b-40*a
*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)-1)-1/4/b^2/(tan(1/2*d*x+1/2*c)+1)^4-1/6
*(4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/8*(12*a^2-8*a*b+11*b^2)/b^4/(tan(
1/2*d*x+1/2*c)+1)^2+1/8/b^6*(40*a^4+60*a^2*b^2+15*b^4)*ln(tan(1/2*d*x+1/2*c
)+1)-1/8*(32*a^3-12*a^2*b+40*a*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(220) = 440$.

time = 0.50, size = 827, normalized size = 3.52

$$-\frac{\frac{2\left(120\,a^{5}+160\,a^{3}b^{2}+24\,ab^{4}+\frac{\left(180\,a^{4}b+245\,a^{2}b^{3}+24\,b^{5}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\left(48\,a^{5}+68\,a^{3}b^{2}+15\,ab^{4}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2\left(300\,a^{4}b+385\,a^{2}b^{3}+48\,b^{5}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10\left(72\,a^{5}+100\,a^{3}b^{2}+15\,ab^{4}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{48\left(15\,a^{4}b+20\,a^{2}b^{3}+3\,b^{5}\right)\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{30\left(16\,a^{5}+20\,a^{3}b^{2}+3\,ab^{4}\right)\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6\left(60\,a^{4}b+85\,a^{2}b^{3}+16\,b^{5}\right)\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{30\left(4\,a^{5}+4\,a^{3}b^{2}-ab^{4}\right)\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{3\left(20\,a^{4}b+25\,a^{2}b^{3}+8\,b^{5}\right)\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)}{a^{2}b^{5}+\frac{2\,ab^{6}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,a^{2}b^{5}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\,ab^{6}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10\,a^{2}b^{5}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{12\,ab^{6}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{10\,a^{2}b^{5}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{8\,ab^{6}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{5\,a^{2}b^{5}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{2\,ab^{6}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{2}b^{5}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{120\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{6}}-\frac{15\left(8\,a^{4}+12\,a^{2}b^{2}+3\,b^{4}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{6}}+\frac{15\left(8\,a^{4}+12\,a^{2}b^{2}+3\,b^{4}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^{6}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(120*a^5 + 160*a^3*b^2 + 24*a*b^4 + (180*a^4*b + 245*a^2*b^3 + 24*
b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*(48*a^5 + 68*a^3*b^2 + 15*a*b^4)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(300*a^4*b + 385*a^2*b^3 + 48*b^5)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(72*a^5 + 100*a^3*b^2 + 15*a*b^4)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 48*(15*a^4*b + 20*a^2*b^3 + 3*b^5)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*(16*a^5 + 20*a^3*b^2 + 3*a*b^4)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(60*a^4*b + 85*a^2*b^3 + 16*b^5)*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 30*(4*a^5 + 4*a^3*b^2 - a*b^4)*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 3*(20*a^4*b + 25*a^2*b^3 + 8*b^5)*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9)/(a^2*b^5 + 2*a*b^6*sin(d*x + c)/(cos(d*x + c) + 1) -
5*a^2*b^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a*b^6*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 10*a^2*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*a*b^
6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10*a^2*b^5*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 - 8*a*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^2*b^5*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 + 2*a*b^6*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - a^2*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*(a^4 + 2*a^2*b^2 +
 b^4)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b -
a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/b^6 + 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/b^6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(220) = 440$.
time = 0.59, size = 472, normalized size = 2.01

$$\frac{12\,b^{5}-30\left(8\,a^{4}b+12\,a^{2}b^{3}+3\,b^{5}\right)\cos(dx+c)^{4}+10\left(4\,a^{2}b^{3}+3\,b^{5}\right)\cos(dx+c)^{2}+120\left(\left(a^{4}+a^{2}b^{2}\right)\cos(dx+c)^{5}+\left(a^{3}b+ab^{3}\right)\cos(dx+c)^{4}\sin(dx+c)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,a^{2}-b^{2}-2\,\sqrt{a^{2}+b^{2}}\left(b\cos(dx+c)-a\sin(dx+c)\right)}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right)+15\left(\left(8\,a^{5}+12\,a^{3}b^{2}+3\,ab^{4}\right)\cos(dx+c)^{5}+\left(8\,a^{4}b+12\,a^{2}b^{3}+3\,b^{5}\right)\cos(dx+c)^{4}\sin(dx+c)\right)\log\left(\sin(dx+c)+1\right)-15\left(\left(8\,a^{5}+12\,a^{3}b^{2}+3\,ab^{4}\right)\cos(dx+c)^{5}+\left(8\,a^{4}b+12\,a^{2}b^{3}+3\,b^{5}\right)\cos(dx+c)^{4}\sin(dx+c)\right)\log\left(-\sin(dx+c)+1\right)-10\left(2\,ab^{4}\cos(dx+c)+3\left(4\,a^{3}b^{2}+5\,ab^{4}\right)\cos(dx+c)^{3}\right)\sin(dx+c)}{48\left(ab^{6}d\cos(dx+c)^{5}+b^{7}d\cos(dx+c)^{4}\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*b^5 - 30*(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4 + 10*(4*a^2
*b^3 + 3*b^5)*cos(d*x + c)^2 + 120*((a^4 + a^2*b^2)*cos(d*x + c)^5 + (a^3*b
 + a*b^3)*cos(d*x + c)^4*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos
(d*x + c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*l
```

```
og(sin(d*x + c) + 1) - 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5 +
(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*log(-sin(d*x +
c) + 1) - 10*(2*a*b^4*cos(d*x + c) + 3*(4*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3
)*sin(d*x + c))/(a*b^6*d*cos(d*x + c)^5 + b^7*d*cos(d*x + c)^4*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(220) = 440.
time = 0.65, size = 530, normalized size = 2.26

$$\frac{\frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{120\,(a^5+2\,a^3b^2+ab^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6}+\frac{48\,\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5+2\,a^3b^2+ab^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab^5}+\frac{2\,\left(36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+27\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+144\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-288\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-336\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+288\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+304\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,a^3-112\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4b^5}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
6 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6
+ 120*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*
sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) + 2*a^2*b^3*tan(1/2*d
*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 + 2*a^3*b^2 + a*b^4)/((a*tan(1
/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^5) + 2*(36*a^2*b*tan(
1/2*d*x + 1/2*c)^7 + 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1
/2*c)^6 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 - 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 336*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2
*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 + 304*a*b^2*tan(1/2*d*x +
1/2*c)^2 + 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) - 96
*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^5))/d
```

Mupad [B]
time = 6.54, size = 2500, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^2),x)

[Out] -((9*a*b^5)/64 + (15*a^5*b)/8 + (b^6*sin(c + d*x))/8 + (115*a^3*b^3)/48 + (
3*b^6*sin(3*c + 3*d*x))/16 + (b^6*sin(5*c + 5*d*x))/16 + (a^6*cos(c + d*x)*
atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*25i)/4 + (5*a*b^5*cos(2*c
+ 2*d*x))/8 + (5*a^5*b*cos(2*c + 2*d*x))/2 + (5*a*b^5*cos(3*c + 3*d*x))/16
+ (25*a^5*b*cos(3*c + 3*d*x))/16 + (15*a*b^5*cos(4*c + 4*d*x))/64 + (5*a^5*
b*cos(4*c + 4*d*x))/8 + (a*b^5*cos(5*c + 5*d*x))/16 + (5*a^5*b*cos(5*c + 5*
d*x))/16 + (25*a^3*b^3*cos(c + d*x))/6 + (5*a^2*b^4*sin(c + d*x))/6 + (5*a^
4*b^2*sin(c + d*x))/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2
))*cos(3*c + 3*d*x)*25i)/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d
*x)/2))*cos(5*c + 5*d*x)*5i)/8 + (10*a^3*b^3*cos(2*c + 2*d*x))/3 + (25*a^3*
b^3*cos(3*c + 3*d*x))/12 + (15*a^3*b^3*cos(4*c + 4*d*x))/16 + (5*a^3*b^3*co
s(5*c + 5*d*x))/12 + (95*a^2*b^4*sin(2*c + 2*d*x))/96 + (5*a^4*b^2*sin(2*c
+ 2*d*x))/8 + (5*a^2*b^4*sin(3*c + 3*d*x))/4 + (15*a^4*b^2*sin(3*c + 3*d*x)
)/16 + (25*a^2*b^4*sin(4*c + 4*d*x))/64 + (5*a^4*b^2*sin(4*c + 4*d*x))/16 +
 (5*a^2*b^4*sin(5*c + 5*d*x))/12 + (5*a^4*b^2*sin(5*c + 5*d*x))/16 + (5*a*b
^5*cos(c + d*x))/8 + (25*a^5*b*cos(c + d*x))/8 + (a*b^5*atan((sin(c/2 + (d*
x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/64 + (a^5*b*atan((sin(c
/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*15i)/8 + (a*b^5*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*15i)/64 + (a^
5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*5i)/8
 + (a^3*b^3*sin(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1
5i)/8 + (a^2*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c +
 3*d*x)*75i)/64 + (a^4*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))
*cos(3*c + 3*d*x)*75i)/16 + (a^2*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
 (d*x)/2))*cos(5*c + 5*d*x)*15i)/64 + (a^4*b^2*atan((sin(c/2 + (d*x)/2)*1i)
/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*15i)/16 + (a^3*b^3*atan((sin(c/2 + (d
*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/16 + (a^3*b^3*atan((si
n(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*15i)/16 + (25*a^3
*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(
1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a
^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2)
 + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*si
n(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/4 + (a*b^5*sin(c + d*x)*atan((sin
(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*15i)/32 + (a^5*b*sin(c + d*x)*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*5i)/4 + (25*a^3*atanh((a^2*sin(
c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 +
(d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)
*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5
*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2)
 + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c +
3*d*x)*((a^2 + b^2)^3)^(1/2))/8 + (5*a^3*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6
```

+ 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2)
+ a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/
2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(5*c + 5*d*x)*((a^2 + b^2)
^3)^(1/2))/8 + (a^2*b^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
(d*x)/2))*75i)/32 + (a^4*b^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*75i)/8 + (15*a^2*b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*
b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (
d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*sin(3*c + 3*d*x)*((a^2 + b^2)^3)^(
1/2))/8 + (5*a^2*b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2
))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*
x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b
^3*sin(c/2 + (d*x)/2)))*sin(5*c + 5*d*x)*((a^2 + b^2)^3)^(1/2))/8 + (5*a^2*
b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) +
2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*c
os(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 +
(d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*s
in(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x
)/2)))*sin(c + d*x)*((a^2 + b^2)^3)^(1/2))/4)/(...

3.561 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$\frac{3(2a^2+b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}}+\frac{3a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}}$$

```
[Out] 3/2*(2*a^2+b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^4/d/(sec(d*x+c)^2)^(1/2)+3
*a*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*sec(d*x+c
)*(a^2+b^2)^(1/2)/b^4/d/(sec(d*x+c)^2)^(1/2)-3/2*sec(d*x+c)*(2*a-b*tan(d*x+
c))/b^3/d-sec(d*x+c)^3/b/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3593, 747, 829, 858, 221, 739, 212}

$$\frac{3a\sqrt{a^2+b^2}\,\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)}{b^4d\sqrt{\sec^2(c+dx)}}+\frac{3(2a^2+b^2)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2b^4d\sqrt{\sec^2(c+dx)}}-\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d}-\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^4*d*Sqrt[Sec[c +
d*x]^2]) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^
2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3*S
ec[c + d*x]*(2*a - b*Tan[c + d*x]))/(2*b^3*d) - Sec[c + d*x]^3/(b*d*(a + b*
Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{a+x}\,dx, x, b\tan(c+\right.}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))}{}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} - \frac{(3a(a^2+b^2)\,\text{se}}{}$$

$$= \frac{3(2a^2+b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+d}{2b^3d}$$

$$= \frac{3(2a^2+b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} + \frac{3a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b(1}{\sqrt{a^2+b}}\right.}{b^4d\sqrt{\sec^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.15, size = 709, normalized size = 4.03

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
```

c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)

Maple [A]
time = 0.43, size = 259, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\frac{\left(a^2+b^2\right)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\left(a^2+b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6\sqrt{a^2+b^2}\,a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a}{2b^3\left(\tan\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{\frac{\frac{2\left(\frac{\left(a^2+b^2\right)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\left(a^2+b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6\sqrt{a^2+b^2}\,a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a}{2b^3\left(\tan\left(\frac{dx}{2}\right.\right.}}{d}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/b^4*(((a^2+b^2)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*(a^2+b^2)^(1/2)*a*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))+1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2/b^4*(6*a^2+3*b^2)*ln(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(163) = 326$.
time = 0.49, size = 471, normalized size = 2.68

$$-\frac{\frac{2\left(6a^3+2ab^2+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(9a^2b+2b^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2a^3+ab^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(3a^2b+b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\left(3a^2b+2b^3\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{6\sqrt{a^2+b^2}\,a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\left(2a^2+b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{3\left(2a^2+b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*a
^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b^3 +
 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b^2)*a*lo
g((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x
+ c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(163) = 326$.
time = 0.45, size = 355, normalized size = 2.02

$$-\frac{6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b^3+6\,\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2-6\,\left(a^2\cos\left(dx+c\right)^3+ab\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-3\,\left(\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^3+\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^3+\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{4\,\left(ab^4d\cos\left(dx+c\right)^3+b^5d\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.59, size = 280, normalized size = 1.59

$$\frac{\frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{6\,(a^3+ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2b^3}+\frac{4\,(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+ab^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/
2*c) - a)*a*b^3))/d
```

Mupad [B]
time = 5.01, size = 585, normalized size = 3.32

$$\frac{\mathrm{atanh}\left(\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2+648\,a^3+\frac{432\,a^5}{b^2}}+\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5+648\,a^3\,b^2+216\,a\,b^4}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a+\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)\,(6\,a^2+3\,b^2)}{b^4\,d}-\frac{\frac{2\,(3\,a^2+b^2)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2+b^2)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(9\,a^2+2\,b^2)}{a\,b^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(3\,a^2+b^2)}{a\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(3\,a^2+2\,b^2)}{a\,b^2}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\,\mathrm{atanh}\left(\frac{432\,a^3\,\sqrt{a^2+b^2}}{432\,a^3\,b+\frac{432\,a^5}{b}+864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{432\,a^3+\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{432\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{432\,a^5+864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+432\,a^3\,b^2+864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3}\right)\,\sqrt{a^2+b^2}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2) +
 (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*a*
tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*b^2
))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 - (6
*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 + 2*b
^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(c/2 +
(d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 3*a*t
an(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6 - 4
*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh((432*a^3*
(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (d*x)/2) +
864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(
1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) + (864*a^4*ta
n(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(432*
a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^3*tan(c/2 + (d
*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.562 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(
1/2))/b^2/d/(a^2+b^2)^(1/2)-sec(d*x+c)/b/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3593, 747, 858, 221, 739, 212}

$$\frac{a\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^2d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}-\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}+\frac{\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{b^2d\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^2*d*Sqrt[Sec[c + d*x]^2]) + (a*ArcT
anh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*
x])/(b^2*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec[c + d*x]/(b*d*(a + b
*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{x}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+\right.}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx\right.}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\text{Su}}{}$$

$$= \frac{\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} + \frac{a\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(}{b^2\sqrt{a^2+b^2}\,d\sqrt{\sec^2(c+dx)}}$$

Mathematica [A]

time = 0.88, size = 120, normalized size = 1.32

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{b\sec(c+dx)}{a+b\tan(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Maple [A]

time = 0.40, size = 135, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
default	$\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
risch	$-\frac{2\,e^{i(dx+c)}}{db\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}+\frac{a\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}+\frac{\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/
2*d*x+1/2*c)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/
(a^2+b^2)^(1/2)))-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b^2*ln(tan(1/2*d*x+1/2*c
)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.
time = 0.50, size = 212, normalized size = 2.33

$$-\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/(
cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b -
 a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(
cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/
b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(87) = 174$.
time = 0.45, size = 293, normalized size = 3.22

$$-\frac{2a^2b+2b^3-\left(a^2\cos(dx+c)+ab\sin(dx+c)\right)\sqrt{a^2+b^2}\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-\left((a^3+ab^2)\cos(dx+c)+(a^2b+b^3)\sin(dx+c)\right)\log(\sin(dx+c)+1)+\left((a^3+ab^2)\cos(dx+c)+(a^2b+b^3)\sin(dx+c)\right)\log(-\sin(dx+c)+1)}{2\left((a^3b^2+ab^4)d\cos(dx+c)+(a^2b^3+b^5)d\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.61, size = 166, normalized size = 1.82

$$\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

Mupad [B]
time = 4.19, size = 383, normalized size = 4.21

$$-\frac{b^2\,\sin\left(c+d\,x\right)-\frac{2\,\left(a^2\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+1{}\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}\right)\,\cos(c+d\,x)\,1{}\mathrm{i}\right)}{\sqrt{a^2+b^2}}+\frac{2\,b\,\left(\frac{a\,\sqrt{a^2+b^2}}{2}+\frac{a\,\cos(c+d\,x)\,\sqrt{a^2+b^2}}{2}-a\,\sin(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{a^2+b^2}-a^2\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+1{}\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2{}\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}\right)\,\sin(c+d\,x)\,1{}\mathrm{i}\right)}{\sqrt{a^2+b^2}}}{a\,b^2\,d\,\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^2),x)
```

```
[Out] -(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2
)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a^2*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)
))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)*(a^2
 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x
)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2
) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*sin(c +
d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)))/(a^2
+ b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

3.563 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] -a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3593, 745, 739, 212}

$$-\frac{b\sec(c+dx)}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{a\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)}{d\,(a^2+b^2)^{3/2}\,\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*S
ec[c + d*x])/((a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2])) - (b*Sec[c + d*x])
/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, \right.}{b\,(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \right.}{b\,(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{(a^2+b^2)^{3/2}\,d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c}$$

Mathematica [A]
time = 0.42, size = 78, normalized size = 0.95

$$\frac{\frac{2a\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b\sec(c+dx)}{(a^2+b^2)(a+b\tan(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - (b*Sec[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

Maple [A]
time = 0.22, size = 118, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}+b^{2}\right)a}-\frac{b}{a^{2}+b^{2}}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}+b^{2}\right)a}-\frac{b}{a^{2}+b^{2}}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^{3}+ia\,b^{2}-a^{2}b-b^{3}}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^{3}+i\ldots}{(a\ldots}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-b^2/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(a*tan(1/2*d*x+1/
2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.
time = 0.49, size = 182, normalized size = 2.22

$$-\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^{2}\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}+a^{2}b^{2}+\frac{2\left(a^{3}b+ab^{3}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^{4}+a^{2}b^{2}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*
b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^3)
*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(78) = 156.
time = 0.39, size = 215, normalized size = 2.62

$$-\frac{2a^{2}b+2b^{3}-\left(a^{2}\cos(dx+c)+ab\sin(dx+c)\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2a^{2}-b^{2}+2\sqrt{a^{2}+b^{2}}\left(b\cos(dx+c)-a\sin(dx+c)\right)}{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)}{2\left((a^{5}+2a^{3}b^{2}+ab^{4})d\cos(dx+c)+(a^{4}b+2a^{2}b^{3}+b^{5})d\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.55, size = 138, normalized size = 1.68

$$-\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{(a^3+ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B]
time = 3.98, size = 136, normalized size = 1.66

$$-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,(a^2+b^2)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,2\mathrm{i}}{d\,(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b^
2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2 +
 (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d
*x)/2)^2))
```

3.564 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{3ab^2\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{5/2}d}+\frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2d(a+b\tan(c+dx))}+\frac{\text{co}}{(c}$$

[Out] -3*a*b^2*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(5/2)/d+b*(a^2-2*b^2)*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+cos(d*x+c)*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))

Rubi [A]
time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3593, 755, 821, 739, 212}

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d(a^2+b^2)(a+b\tan(c+dx))}+\frac{b(a^2-2b^2)\sec(c+dx)}{d(a^2+b^2)^2(a+b\tan(c+dx))}-\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{d(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]

[Out] (-3*a*b^2*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(5/2)*d) + (b*(a^2 - 2*b^2)*Sec[c + d*x])/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]*(b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int -\right.}{(a^2+b^2)}$$

$$= \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\left(3ab\cos(\right.}{}$$

$$= \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\left(3ab\cos(\right.}{}$$

$$= -\frac{3ab^2\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{5/2}\,d} + \frac{}{(a^2}$$

Mathematica [A]
time = 0.61, size = 153, normalized size = 0.97

$$\frac{\sec(c+dx)\left(12ab^2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\left(3b(a^2-b^2)+b(a^2+b^2)\cos(2(c+dx))+a(a^2+b^2)\sin(2(c+dx))\right)\right)}{2(a^2+b^2)^3d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(12*a*b^2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(3*b*(a^2 -
 b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*Sin[2*(c + d*x)])))/
(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.45, size = 172, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$
default	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(2iab+a^2-b^2)d}-\frac{2ib^3\mathrm{e}^{i(dx+c)}}{(-ia+b)^2d(ia+b)^2\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{3b^2a\ln\left(\mathrm{e}^{i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(a*tan(1/2*d*x+1/2*c
)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2
*d*x+1/2*c)-2*a*b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(151) = 302$.

time = 0.51, size = 348, normalized size = 2.22

$$-\frac{\frac{3\,ab^{2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{(a^{4}+2\,a^{2}b^{2}+b^{4})\sqrt{a^{2}+b^{2}}}-\frac{2\left(2\,a^{3}b-ab^{3}-\frac{3\,ab^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{(a^{4}+3\,a^{2}b^{2}-b^{4})\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^{4}-a^{2}b^{2}+b^{4})\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}+\frac{2\,(a^{5}b+2\,a^{3}b^{3}+ab^{5})\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,(a^{5}b+2\,a^{3}b^{3}+ab^{5})\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{(a^{6}+2\,a^{4}b^{2}+a^{2}b^{4})\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1) -
(a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^4*b^2
 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - (a^6
+ 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [A]
time = 0.42, size = 302, normalized size = 1.92

$$\frac{2\,a^{4}b-2\,a^{2}b^{3}-4\,b^{5}+2\,(a^{4}b+2\,a^{2}b^{3}+b^{5})\cos(dx+c)^{2}+2\,(a^{5}+2\,a^{3}b^{2}+ab^{4})\cos(dx+c)\sin(dx+c)+3\,(a^{2}b^{2}\cos(dx+c)+ab^{3}\sin(dx+c))\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)}{2\,((a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6})d\cos(dx+c)+(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7})d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos
(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(
d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.60, size = 286, normalized size = 1.82

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^3b+ab^3\right)}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B]
time = 5.95, size = 286, normalized size = 1.82

$$\frac{\frac{4\,a^2\,b-2\,b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+3\,a^2\,b^2-b^4)}{a\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a^4-2\,a^2\,b^2+2\,b^4)}{a\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/(
a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(a*
(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*b^2
))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2
+ (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^5 + 2
*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))
)/(d*(a^2 + b^2)^(5/2))
```

3.565 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=241

$$-\frac{5ab^4\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{7/2}\,d}+\frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}+$$

```
[Out] -5*a*b^4*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*cos
(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/3*b*(2*a^4+9*a^2*b^2-8*b^4
)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))+1/3*cos(d*x+c)^3*(b+a*tan(d*x+c
))/(a^2+b^2)/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(a^2-4*b^2)-a*(2*a^2+7*b^
2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3593, 755, 837, 821, 739, 212}

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{\cos(c+dx)\,(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx))}{3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{5ab^4\cos(c+dx)\sqrt{\sec^2(c+dx)}\,\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)}{d\,(a^2+b^2)^{7/2}}+\frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b^4*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
]])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(7/2)*d) + (b*(2*a^4 +
9*a^2*b^2 - 8*b^4)*Sec[c + d*x])/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) +
 (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)) - (Cos[c + d*x]*(b*(a^2 - 4*b^2) - a*(2*a^2 + 7*b^2)*Tan[c + d*x]))/(3*(
a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \dots\right.}{3\left(a^2+b^2\right)\dots}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b\left(a^2-4b^2\right)-a\left(2a^2+7b^2\right)\tan\dots\right.}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{b\left(2a^4+9a^2b^2-8b^4\right)\sec(c+dx)}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+\dots}{\dots}$$

$$= \frac{b\left(2a^4+9a^2b^2-8b^4\right)\sec(c+dx)}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+\dots}{\dots}$$

$$= -\frac{5ab^4\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{\left(a^2+b^2\right)^{7/2}d} + \frac{b(2\dots}{3(\dots}$$

Mathematica [A]
time = 1.28, size = 249, normalized size = 1.03

$$\frac{\sec(c+dx)\left(240ab^4\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+\left(a^2+b^2\right)\left(15a^4b+90a^2b^3-45b^5+20b^3\left(a^2+b^2\right)\cos(2(c+dx))+b\left(a^2+b^2\right)^2\cos(4(c+dx))+10a^5\sin(2(c+dx))+40a^3b^2\sin(2(c+dx))+30ab^4\sin(2(c+dx))+a^5\sin(4(c+dx))+2a^3b^2\sin(4(c+dx))+ab^4\sin(4(c+dx))\right)\right)}{24\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(240*a*b^4*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(15*a^4*b
+ 90*a^2*b^3 - 45*b^5 + 20*b^3*(a^2 + b^2)*Cos[2*(c + d*x)] + b*(a^2 + b^2)
^2*Cos[4*(c + d*x)] + 10*a^5*Sin[2*(c + d*x)] + 40*a^3*b^2*Sin[2*(c + d*x)]
 + 30*a*b^4*Sin[2*(c + d*x)] + a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c +
d*x)] + a*b^4*Sin[4*(c + d*x)])))/(24*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.47, size = 320, normalized size = 1.33

method	result
derivativedivides	$-\frac{2b^4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{5a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3\right.\right.}{}$
default	$-\frac{2b^4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{5a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3\right.\right.}{}$
risch	$-\frac{ie^{3i(dx+c)}}{24(-2iab+a^2-b^2)d}-\frac{7\,e^{i(dx+c)}b}{8(-3ib\,a^2+ib^3+a^3-3b^2a)d}-\frac{3ie^{i(dx+c)}a}{8(-3ib\,a^2+ib^3+a^3-3b^2a)d}-\frac{7\,e^{-i(dx+c)}b}{8(ib+a)^3d}+\frac{3ie^{-i(dx+c)}a}{8(ib+a)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(a
*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-5*a/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a^2*
b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)^5+(-2*a^3*b-6*a*b^3)*ta
n(1/2*d*x+1/2*c)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4)*tan(1/2*d*x+1/2*c)^3-8*a*b^
3*tan(1/2*d*x+1/2*c)^2+(-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)-2/3*a^3*b-
14/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(229) = 458$.
time = 0.52, size = 772, normalized size = 3.20

$$-\frac{\frac{15\,ab^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^5b+14\,a^3b^3-3\,ab^5-\frac{15\,ab^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\left(3\,a^6+13\,a^4b^2+22\,a^2b^4-3\,b^6\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{\left(4\,a^5b+28\,a^3b^3-21\,ab^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(a^6-9\,a^4b^2-46\,a^2b^4+9\,b^6\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\left(2\,a^5b+6\,a^3b^3-5\,ab^5\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^6+3\,a^4b^2+38\,a^2b^4-9\,b^6\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\left(a^6+3\,a^4b^2-2\,a^2b^4+b^6\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+\frac{2\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{2\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*a*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(2*a^5*b + 14*a^3*b^3 - 3*a*b^5
 - 15*a*b^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (3*a^6 + 13*a^4*b^2 + 22*
a^2*b^4 - 3*b^6)*sin(d*x + c)/(cos(d*x + c) + 1) + (4*a^5*b + 28*a^3*b^3 -
21*a*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^6 - 9*a^4*b^2 - 46*a^2*b
^4 + 9*b^6)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*(2*a^5*b + 6*a^3*b^3 -
```

```
5*a*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^6 + 3*a^4*b^2 + 38*a^2*b^
4 - 9*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^6 + 3*a^4*b^2 - 2*a^2
*b^4 + b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^8 + 3*a^6*b^2 + 3*a^4*b
^4 + a^2*b^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x + c)/(cos(
d*x + c) + 1) + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^
7)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8))/d
```

Fricas [A]
time = 0.42, size = 418, normalized size = 1.73

$$\frac{4\,a^6b+22\,a^4b^3+2\,a^2b^5-16\,b^7+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos(dx+c)^4-2\,(a^6b-2\,a^4b^3-7\,a^2b^5-4\,b^7)\cos(dx+c)^2+15\,(a^2b^4\cos(dx+c)+ab^5\sin(dx+c))\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(dx+c)^3+(2\,a^7+11\,a^5b^2+16\,a^3b^4+7\,ab^6)\cos(dx+c))\sin(dx+c)}{6\,((a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)d\cos(dx+c)+(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(a^6*b - 2*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*co
s(d*x + c)^2 + 15*(a^2*b^4*cos(d*x + c) + a*b^5*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 11*a^5*b^2 + 16*a^3*b^
4 + 7*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^
7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 438, normalized size = 1.82

$$-\frac{\frac{15\,ab^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{6\,(b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^5)}{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}-\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^3b+14\,ab^3\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(15*a*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 6*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/
((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(
1/2*d*x + 1/2*c) - a)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/
2*d*x + 1/2*c)^5 - 6*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b*tan(1/2*d*x + 1/2
*c)^4 + 18*a*b^3*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18
*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*b^4*tan(1/2*d*x + 1/2*c)^3 + 24*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x
+ 1/2*c) - 6*b^4*tan(1/2*d*x + 1/2*c) + 2*a^3*b + 14*a*b^3)/((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B]
time = 7.11, size = 674, normalized size = 2.80

$$\frac{\frac{2\,\left(2\,a^4\,b+14\,a^2\,b^3-3\,b^5\right)}{3\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{10\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{10\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^4+6\,a^2\,b^2-5\,b^4\right)}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^4\,b+28\,a^2\,b^3-21\,b^5\right)}{3\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(a^6+3\,a^4\,b^2-2\,a^2\,b^4+b^6\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^6+13\,a^4\,b^2+22\,a^2\,b^4-3\,b^6\right)}{3\,a\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(a^6+3\,a^4\,b^2+38\,a^2\,b^4-9\,b^6\right)}{3\,a\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^6-9\,a^4\,b^2-46\,a^2\,b^4+9\,b^6\right)}{3\,a\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{10\,a\,b^4\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}{2\,{\left(a^2+b^2\right)}^{7/2}}\right)}{d\,{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((2*(2*a^4*b - 3*b^5 + 14*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
 - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (10*
b*tan(c/2 + (d*x)/2)^4*(2*a^4 - 5*b^4 + 6*a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4*b - 21*b^5 + 28*a^2*b^3))
/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(a^6 + b
^6 - 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*t
an(c/2 + (d*x)/2)*(3*a^6 - 3*b^6 + 22*a^2*b^4 + 13*a^4*b^2))/(3*a*(a^2 + b^
2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 - 9*b^6 + 38*a^2
*b^4 + 3*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 +
 (d*x)/2)^3*(a^6 + 9*b^6 - 46*a^2*b^4 - 9*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 +
 b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)
^2 - 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2 + (d*x
)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) - (10*a*b^4*
atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^2)^(7
/2))
```

3.566 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\, dx$

Optimal. Leaf size=185

$$\frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d}-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}+\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}-\frac{a\tan^3(}{b}$$

```
[Out] 3*(a^2+b^2)*(5*a^2+b^2)*ln(a+b*tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+c
)/b^6/d+3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d*x
+c)^4/b^3/d-1/2*(a^2+b^2)^3/b^7/d/(a+b*tan(d*x+c))^2+6*a*(a^2+b^2)^2/b^7/d/
(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}-\frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2}+\frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d}-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}+\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}-\frac{a\tan^3(c+dx)}{b^4d}+\frac{\tan^4(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^7*d) - (a*(10*a^2
+ 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2)*Tan[c + d*x]^2)/(2*b^5*d)
 - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(4*b^3*d) - (a^2 + b^2)^3/(2
*b^7*d*(a + b*Tan[c + d*x])^2) + (6*a*(a^2 + b^2)^2)/(b^7*d*(a + b*Tan[c +
d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-10a^3-9ab^2}{b^6} + \frac{3(2a^2+b^2)x}{b^6} - \frac{3ax^2}{b^6} + \frac{x^3}{b^6} + \frac{(a^2+b^2)^3}{b^6(a+x)^3} - \frac{6a(a^2+b^2)^2}{b^6(a+x)^2} + \frac{3(5a^4+6a^2}{b^6(a+}\right.\right.}{bd}$$

$$= \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2}{}$$

Mathematica [A]

time = 5.38, size = 302, normalized size = 1.63

$$\frac{b^4\sec^4(c+dx)(a+b\tan(c+dx))^2 - 4(a+b\tan(c+dx))\left(3a(5a^4+6a^2b^2+b^4)\left(\log(\cos(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx))\right) + b(15a^4+18a^2b^2+5b^4+3(5a^4+6a^2b^2+b^4)\log(\cos(c+dx)) - 3(5a^4+6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx) + 2ab^2(5a^2+4b^2)\tan^2(c+dx)\right) + 2b^2\sec^2(c+dx)\left(5a^4-b^4+2ab(5a^2+2b^2)\tan(c+dx)+2b^2(a^2+b^2)\tan^2(c+dx) - 2ab^3\tan^3(c+dx)\right)}{4b^7d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^4*Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2 - 4*(a + b*Tan[c + d*x])*(3*a*(5
*a^4 + 6*a^2*b^2 + b^4)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]) + b*(15*a^4 + 18*a^2*b^2 + 5*b^4 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[
Cos[c + d*x]] - 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c +
d*x]])*Tan[c + d*x] + 2*a*b^2*(5*a^2 + 4*b^2)*Tan[c + d*x]^2) + 2*b^2*Sec[c
 + d*x]^2*(5*a^4 - b^4 + 2*a*b*(5*a^2 + 2*b^2)*Tan[c + d*x] + 2*b^2*(a^2 +
b^2)*Tan[c + d*x]^2 - 2*a*b^3*Tan[c + d*x]^3))/(4*b^7*d*(a + b*Tan[c + d*x]
)^2)
```

Maple [A]

time = 0.41, size = 195, normalized size = 1.05

method	result
derivativedivides	$-\frac{-\frac{-\frac{(\tan^4(dx+c))b^3}{4}+a(\tan^3(dx+c))b^2-3(\tan^2(dx+c))a^2b-\frac{3b^3(\tan^2(dx+c))}{2}+10a^3\tan(dx+c)+9b^2a\tan(dx+c)}{b^6}-\frac{a^6+3a^4b^2+3a^2}{2b^7(a+b\tan(dx}}{d}$
default	$-\frac{-\frac{-\frac{(\tan^4(dx+c))b^3}{4}+a(\tan^3(dx+c))b^2-3(\tan^2(dx+c))a^2b-\frac{3b^3(\tan^2(dx+c))}{2}+10a^3\tan(dx+c)+9b^2a\tan(dx+c)}{b^6}-\frac{a^6+3a^4b^2+3a^2}{2b^7(a+b\tan(dx}}{d}$
risch	$\frac{30a^4b\,e^{10i(dx+c)}+36a^2b^3e^{10i(dx+c)}-60a^4b-52a^2b^3-240a^4b\,e^{4i(dx+c)}-60a^4b\,e^{6i(dx+c)}-188a^2b^3e^{4i(dx+c)}-32a^2b^3e^{6i(dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^6*(-1/4*tan(d*x+c)^4*b^3+a*tan(d*x+c)^3*b^2-3*tan(d*x+c)^2*a^2*b-
3/2*b^3*tan(d*x+c)^2+10*a^3*tan(d*x+c)+9*b^2*a*tan(d*x+c))-1/2/b^7*(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c))^2+6*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*t
an(d*x+c))+(15*a^4+18*a^2*b^2+3*b^4)/b^7*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 200, normalized size = 1.08

$$\frac{\frac{2\left(11a^6+21a^4b^2+9a^2b^4-b^6+12\left(a^5b+2a^3b^3+ab^5\right)\tan\left(dx+c\right)\right)}{b^9\tan\left(dx+c\right)^2+2ab^8\tan\left(dx+c\right)+a^2b^7}+\frac{b^3\tan\left(dx+c\right)^4-4ab^2\tan\left(dx+c\right)^3+6\left(2a^2b+b^3\right)\tan\left(dx+c\right)^2-4\left(10a^3+9ab^2\right)\tan\left(dx+c\right)}{b^6}+\frac{12\left(5a^4+6a^2b^2+b^4\right)\log\left(b\tan\left(dx+c\right)+a\right)}{b^7}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(11*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*b
^5)*tan(d*x + c))/(b^9*tan(d*x + c)^2 + 2*a*b^8*tan(d*x + c) + a^2*b^7) + (
b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(2*a^2*b + b^3)*tan(d*x + c
)^2 - 4*(10*a^3 + 9*a*b^2)*tan(d*x + c))/b^6 + 12*(5*a^4 + 6*a^2*b^2 + b^4)
*log(b*tan(d*x + c) + a)/b^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(179) = 358$.
time = 0.47, size = 476, normalized size = 2.57

$$\frac{8\left(15a^4b^2+13a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45a^4b^2+44a^2b^4+3b^6\right)\cos\left(dx+c\right)^4+\left(5a^2b^4+3b^6\right)\cos\left(dx+c\right)^2+6\left(\left(5a^6+a^4b^2-5a^2b^4-b^6\right)\cos\left(dx+c\right)^6+2\left(5a^5b+6a^3b^3+ab^5\right)\cos\left(dx+c\right)^5\sin\left(dx+c\right)+\left(5a^4b^2+6a^2b^4+b^6\right)\cos\left(dx+c\right)^4\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(\left(5a^6+a^4b^2-5a^2b^4-b^6\right)\cos\left(dx+c\right)^6+2\left(5a^5b+6a^3b^3+ab^5\right)\cos\left(dx+c\right)^5\sin\left(dx+c\right)+\left(5a^4b^2+6a^2b^4+b^6\right)\cos\left(dx+c\right)^4\right)\log\left(\cos\left(dx+c\right)^2\right)-2\left(ab^5\cos\left(dx+c\right)+2\left(15a^5b-2a^3b^3-13ab^5\right)\cos\left(dx+c\right)^5+10\left(a^3b^3+ab^5\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{4\left(2ab^8d\cos\left(dx+c\right)^5\sin\left(dx+c\right)+b^9d\cos\left(dx+c\right)^4+\left(a^2b^7-b^9\right)d\cos\left(dx+c\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{8}\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.69, size = 243, normalized size = 1.31

$$\frac{\frac{12\,(5\,a^4+6\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{2\left(45\,a^4b^2\tan(dx+c)^2+54\,a^2b^4\tan(dx+c)^2+9\,b^6\tan(dx+c)^2+78\,a^5b\tan(dx+c)+84\,a^3b^3\tan(dx+c)+6\,ab^5\tan(dx+c)+34\,a^6+33\,a^4b^2+b^6\right)}{(b\tan(dx+c)+a)^2b^7}+\frac{b^9\tan(dx+c)^4-4\,ab^8\tan(dx+c)^3+12\,a^2b^7\tan(dx+c)^2+6\,b^9\tan(dx+c)^2-40\,a^3b^6\tan(dx+c)-36\,ab^8\tan(dx+c)}{b^{12}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

Mupad [B]
time = 3.68, size = 234, normalized size = 1.26

$$\frac{\frac{11\,a^6+21\,a^4\,b^2+9\,a^2\,b^4-b^6}{2\,b}+\tan(c+d\,x)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^2\,b^6+2\,a\,b^7\,\tan\left(c+d\,x\right)+b^8\,\tan\left(c+d\,x\right)^2\right)}+\frac{\tan(c+d\,x)^2\,\left(\frac{3}{2\,b^3}+\frac{3\,a^2}{b^5}\right)}{d}+\frac{\tan(c+d\,x)^4}{4\,b^3\,d}+\frac{\tan(c+d\,x)\,\left(\frac{8\,a^3}{b^6}-\frac{3\,a\,\left(\frac{3}{b^3}+\frac{6\,a^2}{b^5}\right)}{b}\right)}{d}-\frac{a\tan(c+d\,x)^3}{b^4\,d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(15\,a^4+18\,a^2\,b^2+3\,b^4\right)}{b^7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((11*a^6 - b^6 + 9*a^2*b^4 + 21*a^4*b^2)/(2*b) + tan(c + d*x)*(6*a*b^4 + 6*
a^5 + 12*a^3*b^2))/(d*(a^2*b^6 + b^8*tan(c + d*x)^2 + 2*a*b^7*tan(c + d*x))
) + (tan(c + d*x)^2*(3/(2*b^3) + (3*a^2)/b^5))/d + tan(c + d*x)^4/(4*b^3*d)
 + (tan(c + d*x)*((8*a^3)/b^6 - (3*a*(3/b^3 + (6*a^2)/b^5))/b))/d - (a*tan(
c + d*x)^3)/(b^4*d) + (log(a + b*tan(c + d*x))*(15*a^4 + 3*b^4 + 18*a^2*b^2
))/(b^7*d)
```

3.567 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3} \, dx$

Optimal. Leaf size=121

$$\frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d}-\frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a(a^2 -}{b^5d(a+b\tan}$$

```
[Out] 2*(3*a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)^
2/b^3/d-1/2*(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a+b*t
an(d*x+c))
```

Rubi [A]

time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$-\frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a(a^2+b^2)}{b^5d(a+b\tan(c+dx))}+\frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c + d*x])/(b^4
*d) + Tan[c + d*x]^2/(2*b^3*d) - (a^2 + b^2)^2/(2*b^5*d*(a + b*Tan[c + d*x]
)^2) + (4*a*(a^2 + b^2))/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^4}+\frac{(a^2+b^2)^2}{b^4(a+x)^3}-\frac{4a(a^2+b^2)}{b^4(a+x)^2}+\frac{2(3a^2+b^2)}{b^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5 d} - \frac{3a\tan(c+dx)}{b^4 d} + \frac{\tan^2(c+dx)}{2b^3 d} - \frac{(}{2b^5 d(a-}$$

Mathematica [A]
time = 2.43, size = 191, normalized size = 1.58

$$-\frac{b^3\sec^4(c+dx)(b\cos(2(c+dx))-a\sin(2(c+dx)))+2(a+b\tan(c+dx))\left(2a(3a^2+b^2)(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(3(2a^2+b^2)+2(3a^2+b^2)\log(\cos(c+dx))-2(3a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+3ab^2\tan^2(c+dx)\right)}{2b^5 d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b^3*Sec[c + d*x]^4*(b*Cos[2*(c + d*x)] - a*Sin[2*(c + d*x)]) + 2*(a +
 b*Tan[c + d*x])*(2*a*(3*a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x]
 + b*Sin[c + d*x]]) + b*(3*(2*a^2 + b^2) + 2*(3*a^2 + b^2)*Log[Cos[c + d*x]
] - 2*(3*a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] + 3*
a*b^2*Tan[c + d*x]^2))/(b^5*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.41, size = 115, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$\frac{-12iab^2-24a^2b+12ia^3+12a^2b\,e^{6i(dx+c)}-36a^2b\,e^{2i(dx+c)}+12ia^3e^{6i(dx+c)}+36ia^3e^{4i(dx+c)}+36ia^3e^{2i(dx+c)}+4b^3e^{6i(dx+}}{\left(e^{2i(dx+c)}+1\right)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2b^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^4
)/(a+b*tan(d*x+c))^2+4*a/b^5*(a^2+b^2)/(a+b*tan(d*x+c))+(6*a^2+2*b^2)/b^5*l
n(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 128, normalized size = 1.06

$$\frac{\frac{7\,a^4+6\,a^2b^2-b^4+8\,\left(a^3b+ab^3\right)\tan\left(dx+c\right)}{b^7\tan\left(dx+c\right)^2+2\,ab^6\tan\left(dx+c\right)+a^2b^5}+\frac{b\tan\left(dx+c\right)^2-6\,a\tan\left(dx+c\right)}{b^4}+\frac{4\,\left(3\,a^2+b^2\right)\log\left(b\tan\left(dx+c\right)+a\right)}{b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((7*a^4 + 6*a^2*b^2 - b^4 + 8*(a^3*b + a*b^3)*tan(d*x + c))/(b^7*tan(d*
x + c)^2 + 2*a*b^6*tan(d*x + c) + a^2*b^5) + (b*tan(d*x + c)^2 - 6*a*tan(d*
x + c))/b^4 + 4*(3*a^2 + b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(117) = 234.
time = 0.42, size = 354, normalized size = 2.93

$$\frac{24\,a^2b^2\cos\left(dx+c\right)^4+b^4-2\,\left(9\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\,\left(\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(dx+c\right)^4+2\,\left(3\,a^3b+ab^3\right)\cos\left(dx+c\right)^3\sin\left(dx+c\right)+\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-2\,\left(\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(dx+c\right)^4+2\,\left(3\,a^3b+ab^3\right)\cos\left(dx+c\right)^3\sin\left(dx+c\right)+\left(3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\log\left(\cos\left(dx+c\right)^2\right)-4\,\left(ab^3\cos\left(dx+c\right)+3\,\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{2\,\left(2\,ab^6d\cos\left(dx+c\right)^3\sin\left(dx+c\right)+b^7d\cos\left(dx+c\right)^2+\left(a^2b^5-b^7\right)d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
 c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x +
c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.68, size = 140, normalized size = 1.16

$$\frac{\frac{4\,\left(3\,a^2+b^2\right)\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^5}+\frac{b^3\tan\left(dx+c\right)^2-6\,ab^2\tan\left(dx+c\right)}{b^6}-\frac{18\,a^2b^2\tan\left(dx+c\right)^2+6\,b^4\tan\left(dx+c\right)^2+28\,a^3b\tan\left(dx+c\right)+4\,ab^3\tan\left(dx+c\right)+11\,a^4+b^4}{\left(b\tan\left(dx+c\right)+a\right)^2b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2
 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x +
 c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*ta
n(d*x + c) + a)^2*b^5))/d
```

Mupad [B]
time = 3.73, size = 143, normalized size = 1.18

$$\frac{\frac{7a^4+6a^2b^2-b^4}{2b}+\tan(c+dx)\,\left(4a^3+4ab^2\right)}{d\,\left(a^2b^4+2ab^5\tan\left(c+dx\right)+b^6\tan\left(c+dx\right)^2\right)}+\frac{\tan(c+dx)^2}{2b^3d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\ln\left(a+b\tan(c+dx)\right)\,\left(6a^2+2b^2\right)}{b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((7*a^4 - b^4 + 6*a^2*b^2)/(2*b) + tan(c + d*x)*(4*a*b^2 + 4*a^3))/(d*(a^2*
b^4 + b^6*tan(c + d*x)^2 + 2*a*b^5*tan(c + d*x))) + tan(c + d*x)^2/(2*b^3*d
) - (3*a*tan(c + d*x))/(b^4*d) + (log(a + b*tan(c + d*x))*(6*a^2 + 2*b^2))/
(b^5*d)
```

3.568 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=69

$$\frac{\log(a+b\tan(c+dx))}{b^3d}-\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2}+\frac{2a}{b^3d(a+b\tan(c+dx))}$$

```
[Out] ln(a+b*tan(d*x+c))/b^3/d+1/2*(-a^2-b^2)/b^3/d/(a+b*tan(d*x+c))^2+2*a/b^3/d/
(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3587, 711}

$$-\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2}+\frac{2a}{b^3d(a+b\tan(c+dx))}+\frac{\log(a+b\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b^3*d) - (a^2 + b^2)/(2*b^3*d*(a + b*Tan[c + d*x])
^2) + (2*a)/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2+b^2}{b^2(a+x)^3} - \frac{2a}{b^2(a+x)^2} + \frac{1}{b^2(a+x)}\right)dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\log(a+b\tan(c+dx))}{b^3d} - \frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))}$$

Mathematica [A]
time = 0.66, size = 115, normalized size = 1.67

$$-\frac{b^2\sec^2(c+dx) + 2(a+b\tan(c+dx))(a(\log(\cos(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx))) + b(1+\log(\cos(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx))}{2b^3d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b^2*Sec[c + d*x]^2 + 2*(a + b*Tan[c + d*x])*(a*(Log[Cos[c + d*x]] - L
og[a*Cos[c + d*x] + b*Sin[c + d*x]]) + b*(1 + Log[Cos[c + d*x]] - Log[a*Cos
[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x]))/(b^3*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.35, size = 63, normalized size = 0.91

method	result	size
derivativedivides	$\frac{-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}+\frac{\ln(a+b\tan(dx+c))}{b^3}}{d}$	63
default	$\frac{-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}+\frac{\ln(a+b\tan(dx+c))}{b^3}}{d}$	63
risch	$\frac{-2a^2\,\mathrm{e}^{2i(dx+c)}+2b^2\,\mathrm{e}^{2i(dx+c)}+4iab\,\mathrm{e}^{2i(dx+c)}-2a^2-2iab}{b^2(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)^2 d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{b^3d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$	160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^2+2/b^3*a/(a+b*tan(d*x+c))+1/b^3*l
n(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.28, size = 78, normalized size = 1.13

$$\frac{\frac{4\,ab\tan(dx+c)+3\,a^2-b^2}{b^5\tan(dx+c)^2+2\,ab^4\tan(dx+c)+a^2b^3} + \frac{2\,\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a*b*tan(d*x + c) + 3*a^2 - b^2)/(b^5*tan(d*x + c)^2 + 2*a*b^4*tan(d
*x + c) + a^2*b^3) + 2*log(b*tan(d*x + c) + a)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(67) = 134$.
time = 0.41, size = 284, normalized size = 4.12

$$\frac{4\,a^2b^2\cos\left(dx+c\right)^2-3\,a^2b^2-b^4-2\left(a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos\left(dx+c\right)^2+2\left(a^3b+ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos\left(dx+c\right)^2+2\left(a^3b+ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2\right)}{2\left((a^4b^3-b^7)d\cos\left(dx+c\right)^2+2\left(a^3b^4+ab^6\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+(a^2b^5+b^7)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.64, size = 62, normalized size = 0.90

$$\frac{\frac{2\,\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^3}-\frac{3\,b\tan\left(dx+c\right)^2+2\,a\tan\left(dx+c\right)+b}{\left(b\tan\left(dx+c\right)+a\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

Mupad [B]
time = 3.75, size = 80, normalized size = 1.16

$$\frac{\frac{3a^2-b^2}{2b^3}+\frac{2a\tan(c+dx)}{b^2}}{d\left(a^2+2ab\tan\left(c+dx\right)+b^2\tan\left(c+dx\right)^2\right)}+\frac{\ln\left(a+b\tan(c+dx)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((3*a^2 - b^2)/(2*b^3) + (2*a*tan(c + d*x))/b^2)/(d*(a^2 + b^2*tan(c + d*x)
^2 + 2*a*b*tan(c + d*x))) + log(a + b*tan(c + d*x))/(b^3*d)
```

3.569 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\tan(c+dx))^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(22) = 44.

time = 0.21, size = 58, normalized size = 2.64

$$\frac{-b\sec^2(c+dx)+2\tan(c+dx)(a+b\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-(b*Sec[c + d*x]^2) + 2*Tan[c + d*x]*(a + b*Tan[c + d*x]))/(2*(a^2 + b^2)*
d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2 d(ia+b)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$-\frac{1}{2\,(b\tan(dx+c)+a)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.
time = 0.38, size = 142, normalized size = 6.45

$$-\frac{4\,a^2b\cos(dx+c)^2-a^2b+b^3-2\,(a^3-ab^2)\cos(dx+c)\sin(dx+c)}{2\left((a^6+a^4b^2-a^2b^4-b^6)d\cos(dx+c)^2+2\,(a^5b+2\,a^3b^3+ab^5)d\cos(dx+c)\sin(dx+c)+(a^4b^2+2\,a^2b^4+b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Giac [A]
time = 0.64, size = 20, normalized size = 0.91

$$-\frac{1}{2\,(b\tan(dx+c)+a)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Mupad [B]
time = 3.66, size = 39, normalized size = 1.77

$$-\frac{1}{d\,\left(2\,a^2\,b+4\,a\,b^2\tan(c+d\,x)+2\,b^3\tan(c+d\,x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

```
[Out] -1/(d*(2*a^2*b + 2*b^3*tan(c + d*x)^2 + 4*a*b^2*tan(c + d*x)))
```

3.570 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{a(a^4+10a^2b^2-15b^4)\,x}{2\,(a^2+b^2)^4}+\frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}+\frac{b(a^2-2b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}$$

```
[Out] 1/2*a*(a^4+10*a^2*b^2-15*b^4)*x/(a^2+b^2)^4+2*b^3*(5*a^2-b^2)*ln(a*cos(d*x+
c)+b*sin(d*x+c))/(a^2+b^2)^4/d+1/2*b*(a^2-2*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))^2+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2
*a*b*(a^2-11*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3587, 755, 815, 649, 209, 266}

$$\frac{ab(a^2-11b^2)}{2d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}+\frac{b(a^2-2b^2)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{ax(a^4+10a^2b^2-15b^4)}{2\,(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/(2*(a^2 + b^2)^4) + (2*b^3*(5*a^2 - b^2)*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (b*(a^2 - 2*b^2))
/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (Cos[c + d*x]^2*(b + a*Tan[c
+ d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*b*(a^2 - 11*b^2))/(2
*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}\\
&= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \frac{-4-\frac{a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right.}{2\,(a^2+b^2)\,d}\\
&= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \left(\frac{2(a^2-2b^2)}{(a^2+b^2)(a+x)^3} + \frac{a^3-11ab^2}{(a^2+b^2)^2(a+x)^2} + \right.\right.}{}\\
&= \frac{2b^3(5a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4\,d} + \frac{b(a^2-2b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\text{co}}{2\,(}\\
&= \frac{2b^3(5a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4\,d} + \frac{b(a^2-2b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\text{co}}{2\,(}\\
&= \frac{a(a^4+10a^2b^2-15b^4)\,x}{2\,(a^2+b^2)^4} + \frac{2b^3(5a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^4\,d} + \frac{2b^3(5a^2-b^2)\log(}{(a^2+}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(202) = 404.

time = 6.32, size = 458, normalized size = 2.27

$$\frac{b^3\left(\frac{\cos^2(c+dx)\left(b^2+ab\tan(c+dx)\right)}{2b^4(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{(2a^2-4b^2)\left(-\frac{\left(3a^2-b^2-\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2(a^2+b^2)^3}+\frac{(3a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}-\frac{\left(3a^2-b^2+\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)}{2(a^2+b^2)^3}-\frac{1}{2(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2a}{(a^2+b^2)^2(a+b\tan(c+dx))}\right)-3a\left(-\frac{\left(2a-\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2(a^2+b^2)^2}+\frac{2a\log(a+b\tan(c+dx))}{(a^2+b^2)^2}-\frac{\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)}{2(a^2+b^2)^2}-\frac{1}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{2b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(b^2 + a*b*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((2*a^2 - 4*b^2)*(-1/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sq
rt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^2)*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(a^2 +
 b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])))
 - 3*a*(-1/2*((2*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]])/(a^2 + b^2)^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (
a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2)
- 1/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

Maple [A]
time = 0.51, size = 219, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{b^3}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}-\frac{4b^3a}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{2b^3(5a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}ab^4\right)\tan(dx+c)+3\ldots}{1+\tan^2(dx+c)}\ldots}{d}$
default	$\frac{-\frac{b^3}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}-\frac{4b^3a}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{2b^3(5a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}ab^4\right)\tan(dx+c)+3\ldots}{1+\tan^2(dx+c)}\ldots}{d}$
risch	$\frac{4ixb}{8ia^3b-8iab^3-2a^4+12a^2b^2-2b^4}-\frac{xa}{8ia^3b-8iab^3-2a^4+12a^2b^2-2b^4}-\frac{ie^{2i(dx+c)}}{8(-3iba^2+ib^3+a^3-3b^2a)d}+\frac{ie^{-2i(dx\ldots}}{8(3iba^2-ib^3+\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))^2-4*b^3/(a^2+b^2)^3*a/(a+b*tan(d
*x+c))+2*b^3*(5*a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(((1/
2*a^5-a^3*b^2-3/2*a*b^4)*tan(d*x+c)+3/2*a^4*b+a^2*b^3-1/2*b^5)/(1+tan(d*x+c
)^2)+1/4*(-20*a^2*b^3+4*b^5)*ln(1+tan(d*x+c)^2)+1/2*(a^5+10*a^3*b^2-15*a*b^
4)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(194) = 388.
time = 0.52, size = 458, normalized size = 2.27

$$\frac{\frac{(a^5+10a^3b^2-15ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{4(5a^2b^3-b^5)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{2(5a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{3a^4b-10a^2b^3-b^5+(a^3b^2-11ab^4)\tan(dx+c)^3+2(a^4b-6a^2b^3-b^5)\tan(dx+c)^2+(a^5+3a^3b^2-10ab^4)\tan(dx+c)}{a^8+3a^6b^2+3a^4b^4+a^2b^6+(a^6b^2+3a^4b^4+3a^2b^6+b^8)\tan(dx+c)^4+2(a^7b+3a^5b^3+3a^3b^5+ab^7)\tan(dx+c)^3+(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)\tan(dx+c)^2+2(a^7b+3a^5b^3+3a^3b^5+ab^7)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) + 4*(5*a^2*b^3 - b^5)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)
^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*a^4*b - 10*a^2
*b^3 - b^5 + (a^3*b^2 - 11*a*b^4)*tan(d*x + c)^3 + 2*(a^4*b - 6*a^2*b^3 - b
^5)*tan(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 10*a*b^4)*tan(d*x + c))/(a^8 + 3*a^
6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*tan(d
*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)^3 + (a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^7*b + 3*a
^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(194) = 388$.
time = 0.41, size = 503, normalized size = 2.49

$$-\frac{3\,a^4b^3 - 16\,a^2b^5 + b^7 - 2\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos(dx+c)^4 - 2\,(a^5b^2 + 10\,a^3b^4 - 15\,ab^6)dx - (a^6b - a^4b^3 - 45\,a^2b^5 - 3\,b^7 + 2\,(a^7 + 9\,a^5b^2 - 25\,a^3b^4 + 15\,ab^6)dx)\cos(dx+c)^2 - 4\,(5\,a^2b^5 - b^7 + (5\,a^4b^3 - 6\,a^2b^5 + b^7)\cos(dx+c)^2 + 2\,(5\,a^3b^4 - ab^6)\cos(dx+c)\sin(dx+c))\log(2\,ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 + b^2) - 2\,((a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos(dx+c)^3 - 2\,(a^5b^2 - 3\,a^3b^4 + 6\,ab^6 - (a^6b + 10\,a^4b^3 - 15\,a^2b^5)dx)\cos(dx+c))\sin(dx+c)}{4\,((a^{10} + 3\,a^8b^2 + 2\,a^6b^4 - 2\,a^4b^6 - 3\,a^2b^8 - b^{10})d\cos(dx+c)^2 + 2\,(a^9b + 4\,a^7b^3 + 6\,a^5b^5 + 4\,a^3b^7 + ab^9)d\cos(dx+c)\sin(dx+c) + (a^8b^2 + 4\,a^6b^4 + 6\,a^4b^6 + 4\,a^2b^8 + b^{10})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*a^4*b^3 - 16*a^2*b^5 + b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
)*cos(d*x + c)^4 - 2*(a^5*b^2 + 10*a^3*b^4 - 15*a*b^6)*d*x - (a^6*b - a^4*b
^3 - 45*a^2*b^5 - 3*b^7 + 2*(a^7 + 9*a^5*b^2 - 25*a^3*b^4 + 15*a*b^6)*d*x)*
cos(d*x + c)^2 - 4*(5*a^2*b^5 - b^7 + (5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x
 + c)^2 + 2*(5*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 - 3*a^3*b^4 + 6*a*b^6 -
 (a^6*b + 10*a^4*b^3 - 15*a^2*b^5)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^10
+ 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 +
2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*
x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(194) = 388.
time = 0.69, size = 439, normalized size = 2.17

$$\frac{\frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{2\,(5\,a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{4\,(5\,a^2b^4-b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}+\frac{10\,a^2b^3\tan(dx+c)^2-2\,b^5\tan(dx+c)^2+a^5\tan(dx+c)-2\,a^3b^2\tan(dx+c)-3\,ab^4\tan(dx+c)+3\,a^4b+12\,a^2b^3-3\,b^5}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\left(\tan(dx+c)^2+1\right)}-\frac{30\,a^2b^5\tan(dx+c)^2-6\,b^7\tan(dx+c)^2+68\,a^3b^4\tan(dx+c)-4\,ab^6\tan(dx+c)+39\,a^4b^3+4\,a^2b^5+b^7}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 4*(5*a^2*b^4 - b^6)*log(abs(b*tan(d*
x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (10*a^2*b^
3*tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 + a^5*tan(d*x + c) - 2*a^3*b^2*tan(
d*x + c) - 3*a*b^4*tan(d*x + c) + 3*a^4*b + 12*a^2*b^3 - 3*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d*x + c)^2 + 1)) - (30*a^2*b^5*t
an(d*x + c)^2 - 6*b^7*tan(d*x + c)^2 + 68*a^3*b^4*tan(d*x + c) - 4*a*b^6*ta
n(d*x + c) + 39*a^4*b^3 + 4*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 4.56, size = 419, normalized size = 2.07

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{10\,b^3}{(a^2+b^2)^3}-\frac{12\,b^5}{(a^2+b^2)^4}\right)}{d}-\frac{\frac{-3\,a^4\,b+10\,a^2\,b^3+b^5}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^3\,(11\,a\,b^4-a^3\,b^2)}{2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)^2\,(-a^4\,b+6\,a^2\,b^3+b^5)}{(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}-\frac{a\tan(c+d\,x)\,(a^4+3\,a^2\,b^2-10\,b^4)}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^2\,(a^2+b^2)+a^2+b^2\tan\left(c+d\,x\right)^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(b+\frac{a\,1{}\mathrm{i}}{4}\right)}{d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,4{}\mathrm{i}\right)}{4\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((10*b^3)/(a^2 + b^2)^3 - (12*b^5)/(a^2 + b^2)^4))
/d - ((b^5 - 3*a^4*b + 10*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
+ (tan(c + d*x)^3*(11*a*b^4 - a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)) + (tan(c + d*x)^2*(b^5 - a^4*b + 6*a^2*b^3))/((a^2 + b^2)*(a^4 + b^4 +
 2*a^2*b^2)) - (a*tan(c + d*x)*(a^4 - 10*b^4 + 3*a^2*b^2))/(2*(a^2 + b^2)*(
a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c +
 d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(tan(c + d*x) +
 1i)*((a*1i)/4 + b))/(d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)) + (l
og(tan(c + d*x) - 1i)*(a + b*4i))/(4*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i
 - a^2*b^2*6i))
```

3.571 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=295

$$\frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)\,x}{8\,(a^2+b^2)^5}+\frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}+\frac{3b(a^4+5a^2b^2-}{8\,(a^2+b^2)^3\,d(a+b\tan}$$

```
[Out] 3/8*a*(a^6+7*a^4*b^2+35*a^2*b^4-35*b^6)*x/(a^2+b^2)^5+3*b^5*(7*a^2-b^2)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d+3/8*b*(a^4+5*a^2*b^2-4*b^4)/(a^2+b
^2)^3/d/(a+b*tan(d*x+c))^2+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a
+b*tan(d*x+c))^2+3/8*a*b*(a^4+6*a^2*b^2-27*b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+
c))-1/8*cos(d*x+c)^2*(2*b*(a^2-3*b^2)-a*(3*a^2+11*b^2)*tan(d*x+c))/(a^2+b^2
)^2/d/(a+b*tan(d*x+c))^2
```

Rubi [A]
time = 0.27, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\,(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx))}{8d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}+\frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5}+\frac{3ab(a^4+6a^2b^2-27b^4)}{8d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}+\frac{3b(a^4+5a^2b^2-4b^4)}{8d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2}+\frac{3ax(a^6+7a^4b^2+35a^2b^4-35b^6)}{8\,(a^2+b^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*x)/(8*(a^2 + b^2)^5) + (3*b^5*
(7*a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) + (3*
b*(a^4 + 5*a^2*b^2 - 4*b^4))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) + (
Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^
2) + (3*a*b*(a^4 + 6*a^2*b^2 - 27*b^4))/(8*(a^2 + b^2)^4*d*(a + b*Tan[c + d
*x])) - (Cos[c + d*x]^2*(2*b*(a^2 - 3*b^2) - a*(3*a^2 + 11*b^2)*Tan[c + d*x
]))/(8*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \frac{-3\left(2+\frac{a^2}{b^2}\right)-\frac{5ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx\right.}{4\left(a^2+b^2\right)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2\right.}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx\right.}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2\right.}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx\right.}$$

$$= \frac{3b^5(7a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b(a^4+5a^2b^2-4b^4)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\text{co}}{4\,(}$$

$$= \frac{3b^5(7a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b(a^4+5a^2b^2-4b^4)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\text{co}}{4\,(}$$

$$= \frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)\,x}{8\left(a^2+b^2\right)^5} + \frac{3b^5(7a^2-b^2)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b^5(7a^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 596 vs. 2(295) = 590.
time = 6.23, size = 596, normalized size = 2.02

$$\frac{b^5\left(\frac{\cos^4(c+dx)\left(b^2+ab\tan(c+dx)\right)}{4b^6(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{\frac{\cos^2(c+dx)\left(5a^2b^2-3b^2\left(a^2+2b^2\right)+b\left(-5ab^2-3a\left(a^2+2b^2\right)\right)\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(-3a^2\left(3a^2+11b^2\right)+3\left(a^4+a^2b^2+8b^4\right)\right)\left(-\frac{\left(3a^2-b^2-\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3} + \frac{\left(3a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3} - \frac{\left(3a^2-b^2+\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3} - \frac{1}{2\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{2a}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}\right) + 3a\left(3a^2+11b^2\right)\left(-\frac{\left(2a-\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2} + \frac{2a\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2} - \frac{\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2} - \frac{1}{\left(a^2+b^2\right)(a+b\tan(c+dx))}\right)}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(b^2 + a*b*Tan[c + d*x]))/(4*b^6*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((Cos[c + d*x]^2*(5*a^2*b^2 - 3*b^2*(a^2 + 2*b^2) + b*(-5
*a*b^2 - 3*a*(a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^2) - ((-3*a^2*(3*a^2 + 11*b^2) + 3*(a^4 + a^2*b^2 + 8*b^4))*(-1/2*(
(3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]]
)/(a^2 + b^2)^3 + ((3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - (
(3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
)/(2*(a^2 + b^2)^3) - 1/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^
```

```
2 + b^2)^2*(a + b*Tan[c + d*x]))) + 3*a*(3*a^2 + 11*b^2)*(-1/2*((2*a - (a^2
 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^2 + (2*a*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (a^2 - b^2)/Sqrt[-b^2])*Lo
g[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) - 1/((a^2 + b^2)*(a + b*T
an[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Maple [A]
time = 0.55, size = 312, normalized size = 1.06

method	result
derivativedivides	$\frac{-\frac{b^5}{2\left(a^2+b^2\right)^3\left(a+b\tan(dx+c)\right)^2}-\frac{6b^5a}{\left(a^2+b^2\right)^4\left(a+b\tan(dx+c)\right)}+\frac{3b^5\left(7a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}ab^6\right)}{}}{}$
default	$\frac{-\frac{b^5}{2\left(a^2+b^2\right)^3\left(a+b\tan(dx+c)\right)^2}-\frac{6b^5a}{\left(a^2+b^2\right)^4\left(a+b\tan(dx+c)\right)}+\frac{3b^5\left(7a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}ab^6\right)}{}}{}$
risch	$\frac{15xab}{8ia^5-80ia^3b^2+40ia\,b^4+40a^4b-80a^2b^3+8b^5}+\frac{3ix\,a^2}{8ia^5-80ia^3b^2+40ia\,b^4+40a^4b-80a^2b^3+8b^5}-\frac{ie^{4i(dx+c)}}{64(-3ib\,a^2+ib^3+a^3-3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))^2-6*b^5/(a^2+b^2)^4*a/(a+b*tan(d
*x+c))+3*b^5*(7*a^2-b^2)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((3/
8*a^7+21/8*a^5*b^2-15/8*a^3*b^4-33/8*a*b^6)*tan(d*x+c)^3+(5*a^4*b^3+4*a^2*b
^5-b^7)*tan(d*x+c)^2+(19/8*a^5*b^2-39/8*a*b^6+5/8*a^7-25/8*a^3*b^4)*tan(d*x
+c)+3/4*a^6*b+25/4*a^4*b^3+17/4*b^5*a^2-5/4*b^7)/(1+tan(d*x+c)^2)^2+3/16*(-
56*a^2*b^5+8*b^7)*ln(1+tan(d*x+c)^2)+3/8*(a^7+7*a^5*b^2+35*a^3*b^4-35*a*b^6
)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(284) = 568.
time = 0.51, size = 738, normalized size = 2.50

$$\frac{\frac{3\left(a^7+7a^5b^2+35a^3b^4-35ab^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(7a^2b^5-b^7\right)\log(b\tan(dx+c)+a)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(7a^2b^5-b^7\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{6a^6b+44a^4b^3-62a^2b^5-4b^7+3\left(a^5b^2+6a^3b^4-27ab^6\right)\tan(dx+c)^5+6\left(a^6b+6a^4b^3-13a^2b^5-2b^7\right)\tan(dx+c)^4+\left(3a^7+23a^5b^2+61a^3b^4-151ab^6\right)\tan(dx+c)^3+2\left(5a^6b+37a^4b^3-73a^2b^5-9b^7\right)\tan(dx+c)^2+\left(5a^7+26a^5b^2+49a^3b^4-68ab^6\right)\tan(dx+c)}{a^{10}+4a^8b^2+6a^6b^4+4a^4b^6+a^2b^8+\left(a^8b^2+4a^6b^4+6a^4b^6+4a^2b^8+b^{10}\right)\tan(dx+c)^6+2\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)^5+\left(a^{10}+6a^8b^2+14a^6b^4+16a^4b^6+9a^2b^8+2b^{10}\right)\tan(dx+c)^4+4\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)^3+\left(2a^{10}+9a^8b^2+16a^6b^4+14a^4b^6+6a^2b^8+b^{10}\right)\tan(dx+c)^2+2\left(a^9b+4a^7b^3+6a^5b^5+4a^3b^7+ab^9\right)\tan(dx+c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(7*a^2*b^5 - b^7)*log(
b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) - 12*(7*a^2*b^5 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + (6*a^6*b + 44*a^4*b^3 - 62*a
```

```
^2*b^5 - 4*b^7 + 3*(a^5*b^2 + 6*a^3*b^4 - 27*a*b^6)*tan(d*x + c)^5 + 6*(a^6
*b + 6*a^4*b^3 - 13*a^2*b^5 - 2*b^7)*tan(d*x + c)^4 + (3*a^7 + 23*a^5*b^2 +
 61*a^3*b^4 - 151*a*b^6)*tan(d*x + c)^3 + 2*(5*a^6*b + 37*a^4*b^3 - 73*a^2*
b^5 - 9*b^7)*tan(d*x + c)^2 + (5*a^7 + 26*a^5*b^2 + 49*a^3*b^4 - 68*a*b^6)*
tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8 + (a^8*b^
2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(a^9*b + 4
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 + 6*a^8*b^
2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 + 4*(a^9*b
 + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*a^10 + 9*
a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)^2 + 2*(a
^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(284) = 568$.
time = 0.49, size = 671, normalized size = 2.27

$$-\frac{9a^6b^3 + 95a^4b^5 - 141a^2b^7 - 3b^9 - 8(a^8b + 4a^6b^3 + 6a^4b^5 + 4a^2b^7 + b^9)\cos(dx+c)^6 + 8(a^8b - 6a^4b^5 - 8a^2b^7 - 3b^9)\cos(dx+c)^4 - 12(a^7b^2 + 7a^5b^4 + 35a^3b^6 - 35ab^8)dx - (15a^8b + 82a^6b^3 + 68a^4b^5 - 498a^2b^7 - 51b^9 + 12(a^9 + 6a^7b^2 + 28a^5b^4 - 70a^3b^6 + 35ab^8)dx)\cos(dx+c)^2 - 48(7a^2b^7 - b^9 + (7a^4b^5 - 8a^2b^7 + b^9)\cos(dx+c)^2 + 2(7a^3b^6 - ab^8)\cos(dx+c)\sin(dx+c))\log(2ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 + b^2) - 2(4(a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + ab^8)\cos(dx+c)^5 + 2(3a^9 + 20a^7b^2 + 42a^5b^4 + 36a^3b^6 + 11ab^8)\cos(dx+c)^3 - (3a^7b^2 + 53a^5b^4 - 15a^3b^6 + 159ab^8 - 12(a^8b + 7a^6b^3 + 35a^4b^5 - 35a^2b^7)dx)\cos(dx+c))\sin(dx+c)}{32((a^{12} + 4a^{10}b^2 + 5a^8b^4 - 5a^4b^8 - 4a^2b^{10} - b^{12})d\cos(dx+c)^2 + 2(a^{11}b + 5a^9b^3 + 10a^7b^5 + 10a^5b^7 + 5a^3b^9 + ab^{11})d\cos(dx+c)\sin(dx+c) + (a^{10}b^2 + 5a^8b^4 + 10a^6b^6 + 10a^4b^8 + 5a^2b^{10} + b^{12})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(9*a^6*b^3 + 95*a^4*b^5 - 141*a^2*b^7 - 3*b^9 - 8*(a^8*b + 4*a^6*b^3
+ 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 8*(a^8*b - 6*a^4*b^5 - 8*a^
2*b^7 - 3*b^9)*cos(d*x + c)^4 - 12*(a^7*b^2 + 7*a^5*b^4 + 35*a^3*b^6 - 35*a
*b^8)*d*x - (15*a^8*b + 82*a^6*b^3 + 68*a^4*b^5 - 498*a^2*b^7 - 51*b^9 + 12
*(a^9 + 6*a^7*b^2 + 28*a^5*b^4 - 70*a^3*b^6 + 35*a*b^8)*d*x)*cos(d*x + c)^2
 - 48*(7*a^2*b^7 - b^9 + (7*a^4*b^5 - 8*a^2*b^7 + b^9)*cos(d*x + c)^2 + 2*(
7*a^3*b^6 - a*b^8)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*
b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^5 + 2*(3*a^9 + 20*a^7*b^2 + 42*a^5*b^
4 + 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^3 - (3*a^7*b^2 + 53*a^5*b^4 - 15*a^
3*b^6 + 159*a*b^8 - 12*(a^8*b + 7*a^6*b^3 + 35*a^4*b^5 - 35*a^2*b^7)*d*x)*c
os(d*x + c))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*
a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10
*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*
a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(284) = 568.
time = 0.72, size = 587, normalized size = 1.99

$$\frac{\frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}-\frac{12\,(7\,a^2b^5-b^7)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}}+\frac{24\,(7\,a^2b^6-b^8)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}+\frac{3\,a^5b^2\tan(dx+c)^5+18\,a^3b^4\tan(dx+c)^5-81\,ab^6\tan(dx+c)^5+6\,a^6b\tan(dx+c)^4+36\,a^4b^3\tan(dx+c)^4-78\,a^2b^5\tan(dx+c)^4-12\,b^7\tan(dx+c)^4+3\,a^7\tan(dx+c)^3+23\,a^5b^2\tan(dx+c)^3+61\,a^3b^4\tan(dx+c)^3-151\,ab^6\tan(dx+c)^3+10\,a^6b\tan(dx+c)^2+74\,a^4b^3\tan(dx+c)^2-146\,a^2b^5\tan(dx+c)^2-18\,b^7\tan(dx+c)^2+5\,a^7\tan(dx+c)+26\,a^5b^2\tan(dx+c)+49\,a^3b^4\tan(dx+c)-68\,ab^6\tan(dx+c)+6\,a^6b+44\,a^4b^3-62\,a^2b^5-4\,b^7}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*log(
tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) + 24*(7*a^2*b^6 - b^8)*log(abs(b*tan(d*x + c) + a))/(a^10*b + 5*a^
8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) + (3*a^5*b^2*tan(d*x +
c)^5 + 18*a^3*b^4*tan(d*x + c)^5 - 81*a*b^6*tan(d*x + c)^5 + 6*a^6*b*tan(d*
x + c)^4 + 36*a^4*b^3*tan(d*x + c)^4 - 78*a^2*b^5*tan(d*x + c)^4 - 12*b^7*t
an(d*x + c)^4 + 3*a^7*tan(d*x + c)^3 + 23*a^5*b^2*tan(d*x + c)^3 + 61*a^3*b
^4*tan(d*x + c)^3 - 151*a*b^6*tan(d*x + c)^3 + 10*a^6*b*tan(d*x + c)^2 + 74
*a^4*b^3*tan(d*x + c)^2 - 146*a^2*b^5*tan(d*x + c)^2 - 18*b^7*tan(d*x + c)^
2 + 5*a^7*tan(d*x + c) + 26*a^5*b^2*tan(d*x + c) + 49*a^3*b^4*tan(d*x + c)
- 68*a*b^6*tan(d*x + c) + 6*a^6*b + 44*a^4*b^3 - 62*a^2*b^5 - 4*b^7)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x +
c)^2 + b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 5.43, size = 715, normalized size = 2.42

$$\frac{\frac{3\,a^6\,b+22\,a^4\,b^3-31\,a^2\,b^5-2\,b^7}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)\,(5\,a^7+26\,a^5\,b^2+49\,a^3\,b^4-68\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{3\tan(c+d\,x)^5\,(a^5\,b^2+6\,a^3\,b^4-27\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^3\,(3\,a^7+23\,a^5\,b^2+61\,a^3\,b^4-151\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{3\tan(c+d\,x)^4\,(a^6\,b+6\,a^4\,b^3-13\,a^2\,b^5-2\,b^7)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^2\,(5\,a^6\,b+37\,a^4\,b^3-73\,a^2\,b^5-9\,b^7)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(2\,a^2+b^2\right)+{\tan\left(c+d\,x\right)}^4\,\left(a^2+2\,b^2\right)+a^2+b^2\,{\tan\left(c+d\,x\right)}^6+2\,a\,b\,\tan\left(c+d\,x\right)+4\,a\,b\,{\tan\left(c+d\,x\right)}^3+2\,a\,b\,{\tan\left(c+d\,x\right)}^5\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{21\,b^5}{{(a^2+b^2)}^4}-\frac{24\,b^7}{{(a^2+b^2)}^5}\right)}{d}+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,1\mathrm{i}+5\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)}+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}+5\,a\,b-b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((3*a^6*b - 2*b^7 - 31*a^2*b^5 + 22*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*
a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a^7 - 68*a*b^6 + 49*a^3*b^4 + 26*a
^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d
*x)^5*(6*a^3*b^4 - 27*a*b^6 + a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(3*a^7 - 151*a*b^6 + 61*a^3*b^4 + 23*a^5
*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d*x
)^4*(a^6*b - 2*b^7 - 13*a^2*b^5 + 6*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^2*(5*a^6*b - 9*b^7 - 73*a^2*b^5 + 37
*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))/(d*(tan(c +
 d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2 + 2*b^2) + a^2 + b^2*tan(c + d*
x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*x)^3 + 2*a*b*tan(c + d*x)^5)) +
```

```
 (log(a + b*tan(c + d*x))*((21*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5)
)/d + (3*log(tan(c + d*x) - 1i)*(5*a*b - a^2*1i + b^2*8i))/(16*d*(5*a*b^4 +
 a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (3*log(tan(c + d*x)
 + 1i)*(5*a*b + a^2*1i - b^2*8i))/(16*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i
+ a^2*b^3*10i - 10*a^3*b^2))
```

3.572 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5a(4a^2+3b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}}-\frac{5\sqrt{a^2+b^2}\,(4a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)}{2b^6d\sqrt{\sec^2(c+dx)}}$$

```
[Out] -5/2*a*(4*a^2+3*b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1
/2)-5/2*(4*a^2+b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)
^(1/2))*sec(d*x+c)*(a^2+b^2)^(1/2)/b^6/d/(sec(d*x+c)^2)^(1/2)-1/2*sec(d*x+c
)^5/b/d/(a+b*tan(d*x+c))^2+5/6*sec(d*x+c)^3*(4*a+b*tan(d*x+c))/b^3/d/(a+b*t
an(d*x+c))+5/2*sec(d*x+c)*(4*a^2+b^2-2*a*b*tan(d*x+c))/b^5/d
```

Rubi [A]
time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3593, 747, 827, 829, 858, 221, 739, 212}

$$-\frac{5\sqrt{a^2+b^2}\,(4a^2+b^2)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)}{2b^6d\sqrt{\sec^2(c+dx)}}-\frac{5a(4a^2+3b^2)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2b^6d\sqrt{\sec^2(c+dx)}}+\frac{5\sec(c+dx)\,(4a^2-2ab\tan(c+dx)+b^2)}{2b^5d}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))}-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*a*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec
[c + d*x]^2]) - (5*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec[
c + d*x]^2]) - Sec[c + d*x]^5/(2*b*d*(a + b*Tan[c + d*x])^2) + (5*Sec[c + d
*x]^3*(4*a + b*Tan[c + d*x]))/(6*b^3*d*(a + b*Tan[c + d*x])) + (5*Sec[c + d
*x]*(4*a^2 + b^2 - 2*a*b*Tan[c + d*x]))/(2*b^5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
```

```
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{(5\sec(c+dx))\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}\,dx, x, b\tan(c+ \right.}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} - \frac{(5\sec(c+dx}{}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)}{}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)}{}$$

$$= -\frac{5a(4a^2+3b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5}{}$$

$$= -\frac{5a(4a^2+3b^2)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sqrt{a^2+b^2}\,(4a^2+b^2)\tanh^{-}}{2b^6}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.61, size = 688, normalized size = 2.88

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(219) = 438$.
time = 0.53, size = 444, normalized size = 1.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^6*((1/2*b^2*(7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*
(8*a^6-9*a^4*b^2-15*a^2*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a^4
+23*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)-4*a^4*b-7/2*a^2*b^3+1/2*b^5)/(a*tan
(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-1/3
/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b)/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/2*
(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1/2*c)-1)+5/2*a*(4*a^2+3*b^2)/b^6*ln(
tan(1/2*d*x+1/2*c)-1)+1/3/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*a+b)/b^4/(ta
n(1/2*d*x+1/2*c)+1)^2-1/2*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d*x+1/2*c)+1)-
5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(223) = 446$.
time = 0.52, size = 902, normalized size = 3.77

$$\frac{\frac{2\left(60a^6+35a^4b^2-3a^2b^4+\frac{(210a^5b+125a^3b^3-6ab^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(120a^6-10a^4b^2-55a^2b^4+3b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2(330a^5b+205a^3b^3-12ab^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(180a^6-95a^4b^2-120a^2b^4+9b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{12(60a^5b+35a^3b^3-3ab^5)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6(40a^6-30a^4b^2-35a^2b^4+3b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6(50a^5b+25a^3b^3-4ab^5)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3(20a^6-15a^4b^2-15a^2b^4+2b^6)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3(10a^5b+5a^3b^3-2ab^5)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^4b^5+\frac{4a^3b^6\sin(dx+c)}{\cos(dx+c)+1}-\frac{16a^3b^6\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{24a^3b^6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{16a^3b^6\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{4a^3b^6\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^4b^5\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{(5a^4b^5-4a^2b^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2(5a^4b^5-6a^2b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(5a^4b^5-6a^2b^7)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{(5a^4b^5-4a^2b^7)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15(4a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^6}+\frac{15(4a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^6}-\frac{15(4a^4+5a^2b^2+b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4 +
 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3 -
12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2 - 1
20*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b + 35
*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 - 30*a^
4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(50*a^5
*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^6
- 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*
(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^4*
b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 16
*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d*x + c)^9/(co
s(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - (5*a^4*
b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*a^4*b^5 - 6*a^2
*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 - 6*a^2*b^7)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) +
1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) +
 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(223) = 446$.
time = 0.51, size = 564, normalized size = 2.36

$$\frac{4\,b^{5}+30\,(4\,a^{4}b+a^{2}b^{3}-b^{5})\cos(dx+c)^{4}+20\,(2\,a^{2}b^{3}+b^{5})\cos(dx+c)^{2}+15\,\left((4\,a^{4}-3\,a^{2}b^{2}-b^{4})\cos(dx+c)^{5}+2\,(4\,a^{3}b+ab^{3})\cos(dx+c)^{4}\sin(dx+c)+(4\,a^{2}b^{2}+b^{4})\cos(dx+c)^{3}\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-15\,\left((4\,a^{5}-a^{3}b^{2}-3\,ab^{4})\cos(dx+c)^{5}+2\,(4\,a^{4}b+3\,a^{2}b^{3})\cos(dx+c)^{4}\sin(dx+c)+(4\,a^{3}b^{2}+3\,ab^{4})\cos(dx+c)^{3}\right)\log(\sin(dx+c)+1)+15\,\left((4\,a^{5}-a^{3}b^{2}-3\,ab^{4})\cos(dx+c)^{5}+2\,(4\,a^{4}b+3\,a^{2}b^{3})\cos(dx+c)^{4}\sin(dx+c)+(4\,a^{3}b^{2}+3\,ab^{4})\cos(dx+c)^{3}\right)\log(-\sin(dx+c)+1)-10\,\left(ab^{4}\cos(dx+c)-6\,(3\,a^{3}b^{2}+2\,ab^{4})\cos(dx+c)^{3}\right)\sin(dx+c)}{12\,\left(2\,ab^{7}d\cos(dx+c)^{4}\sin(dx+c)+b^{8}d\cos(dx+c)^{3}+(a^{2}b^{6}-b^{8})d\cos(dx+c)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(223) = 446.
time = 0.74, size = 510, normalized size = 2.13

$$-\frac{\frac{15\,(4a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^6}-\frac{15\,(4a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^6}+\frac{15\,(4a^4+5a^2b^2+b^4)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6}+\frac{2\left(9ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+18b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-72a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36a^2+14b^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3b^5}+\frac{6\left(7a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9a^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-25a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-23a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8a^6-7a^4b^2+a^2b^4\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)^2a^2b^5}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

Mupad [B]
time = 6.94, size = 1203, normalized size = 5.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*b
^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15*a
```

```
^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^6 -
 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6 + 3
*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4*(180
*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d*x)/2)
^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(50*a^4
- 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 - 3*b^4 +
35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 + 205*a^2*
b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c/2 + (d*x)
/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2 - 16*a*b*tan
(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c/2 + (d*x)/2)
^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3000
*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*a^6)/b^4) + (70
00*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000*a^6)/b^2) + (40
00*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000*a^4*b^2))*(15*a*
b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(1/2))/(1000*a^2*b
+ (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2
*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b^2) + (4000*a^4*(a^2 +
 b^2)^(1/2))/(5000*a^4*b + 1000*a^2*b^3 + (4000*a^6)/b + 8000*a^5*tan(c/2 +
 (d*x)/2) + 2000*a*b^4*tan(c/2 + (d*x)/2) + 10000*a^3*b^2*tan(c/2 + (d*x)/2
)) + (9000*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(5000*a^4 + 1000*a^2*b
^2 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) + 10000*a^3*b*tan(c/2 +
 (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*tan(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 + 5000*a^4*b^2 + 2000*a*b^5*ta
n(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) + 10000*a^3*b^3*tan(c/2 +
(d*x)/2)) + (2000*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(1000*a^2 + (5000
*a^4)/b^2 + (4000*a^6)/b^4 + (10000*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*t
an(c/2 + (d*x)/2))/b^3 + 2000*a*b*tan(c/2 + (d*x)/2)))*(4*a^2 + b^2)*(a^2 +
 b^2)^(1/2))/(b^6*d)
```

3.573 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=148

$$-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{3(2a^2+b^2)\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4\sqrt{a^2+b^2}\,d}-\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{3\sec(c+d\ldots}{2b^3d(\ldots}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d-3/2*(2*a^2+b^2)*arctanh((b*cos(d*x+c)-a*sin(
d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)-1/2*sec(d*x+c)^3/b/d/(a+b*ta
n(d*x+c))^2+3/2*sec(d*x+c)*(2*a+b*tan(d*x+c))/b^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3593, 747, 827, 858, 221, 739, 212}

$$-\frac{3(2a^2+b^2)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^4d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}-\frac{3a\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{b^4d\sqrt{\sec^2(c+dx)}}+\frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}-\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3
*(2*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d
*x]^2])]*Sec[c + d*x])/(2*b^4*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec
[c + d*x]^3/(2*b*d*(a + b*Tan[c + d*x])^2) + (3*Sec[c + d*x]*(2*a + b*Tan[c
 + d*x]))/(2*b^3*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(\right.}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3\sec(c+dx}{}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3a\sec(c+}{}$$

$$= -\frac{3a\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+d}{2b^3d(}$$

$$= -\frac{3a\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3(2a^2+b^2)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+d}{b}\right.}{\sqrt{a^2+b^2}\sqrt{\sec^2}}\right.}{2b^4\sqrt{a^2+b^2}\,d\sqrt{\sec^2(c}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 396 vs. 2(148) = 296.
time = 2.61, size = 396, normalized size = 2.68

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a}+2b(a\cos(c+dx)+b\sin(c+dx))^2+\frac{6(2a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\sqrt{a^2+b^2}}+6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2-6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2+\frac{2b\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{2b\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2b^4d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
```

```
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Maple [A]
time = 0.48, size = 269, normalized size = 1.82

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(13a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-2a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3(2a^2+b^2)\,\mathrm{arc}}{2}\right.}{b^4}}{d}$
default	$-\frac{\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(13a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-2a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3(2a^2+b^2)\,\mathrm{arc}}{2}\right.}{b^4}}{d}$
risch	$\frac{-9iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}-3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+9iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}-3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^2 d\,b^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*((1/2*b^2*(3*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a
^2*b^2+2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x
+1/2*c)-2*a^2*b+1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^
2-3/2*(2*a^2+b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/
(a^2+b^2)^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c
)-1)+1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(138) = 276.
time = 0.51, size = 518, normalized size = 3.50

$$\frac{\frac{2\left(6a^4-a^2b^2+\frac{(21a^3b-2ab^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(6a^4-9a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(6a^3b-ab^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(6a^4-9a^2b^2+2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(3a^3b-2ab^3)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{6a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{6a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}-\frac{3(2a^2+b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c) +
 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(
6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^2 +
 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x + c)
 + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
(3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^4*b^3 -
4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4
- 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b
^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*b^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(138) = 276$.
time = 0.47, size = 513, normalized size = 3.47

$$\frac{4\,a^2b^3 + 4\,b^5 + 6\,(2\,a^4b + a^2b^3 - b^5)\cos(dx+c)^2 + 18\,(a^3b^2 + ab^4)\cos(dx+c)\sin(dx+c) + 3\,\left((2\,a^4 - a^2b^2 - b^4)\cos(dx+c)^3 + 2\,(2\,a^3b + ab^3)\cos(dx+c)^2\sin(dx+c) + (2\,a^2b^2 + b^4)\cos(dx+c)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 - 2\,a^2 - b^2 + 2\,\sqrt{a^2+b^2}\,(b\cos(dx+c) - a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 + b^2}\right) - 6\,\left((a^5 - ab^4)\cos(dx+c)^3 + 2\,(a^4b + a^2b^3)\cos(dx+c)^2\sin(dx+c) + (a^3b^2 + ab^4)\cos(dx+c)\right)\log(\sin(dx+c)+1) + 6\,\left((a^5 - ab^4)\cos(dx+c)^3 + 2\,(a^4b + a^2b^3)\cos(dx+c)^2\sin(dx+c) + (a^3b^2 + ab^4)\cos(dx+c)\right)\log(-\sin(dx+c)+1)}{4\,\left((a^4b^4 - b^8)d\cos(dx+c)^3 + 2\,(a^3b^5 + ab^7)d\cos(dx+c)^2\sin(dx+c) + (a^2b^6 + b^8)d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a
^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(
d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 +
 b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x +
c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*co
s(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(138) = 276.
time = 0.69, size = 314, normalized size = 2.12

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\,(2\,a^2+b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^3}+\frac{2\left(3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-13\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^4+a^2b^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

Mupad [B]
time = 5.68, size = 1311, normalized size = 8.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a^
2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2
)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*a^2
 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(tan(
c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2) - a
^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
+ (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*x)/2)))
/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(
c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)
*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 4
8*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4
)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b
^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(
2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 +
 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan
```

(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 27*a^2*b^2))/b^8 - (16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)) + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*3i)/(d*(b^6 + a^2*b^4))

3.574 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-1/2*sec(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3593, 735, 739, 212}

$$-\frac{\sec(c+dx)(b-a\tan(c+dx))}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*
Sec[c + d*x])/((a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2]) - (Sec[c + d*x]*(b
 - a*Tan[c + d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx,\right.}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+d}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-}{\sqrt{\text{se}}}\right.}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{3/2}d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)(b-a\tan(}{2\left(a^2+b^2\right)d(a+b\tan(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 132, normalized size = 1.39

$$\frac{\left(a^2+b^2\right)\left(-b\cos(c+dx)+a\sin(c+dx)\right)+2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^2}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(87) = 174$.

time = 0.40, size = 191, normalized size = 2.01

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a}-\frac{b(a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a}-\frac{b(a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
risch	$\frac{ia\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{3i(dx+c)}-ia\,\mathrm{e}^{i(dx+c)}+\mathrm{e}^{i(dx+c)}b}{\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+ia\,b^2-a^2}{(a^2+b^2)}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(88) = 176$.

time = 0.48, size = 326, normalized size = 3.43

$$-\frac{\frac{2\left(a^2b-\frac{(a^3-2\,ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^2b-2\,b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3+2\,ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^6-a^4b^2-2\,a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(88) = 176$.
time = 0.37, size = 294, normalized size = 3.09

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2\left(a^2b+b^3\right)\cos\left(dx+c\right)+2\left(a^3+ab^2\right)\sin\left(dx+c\right)}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^4b^2+2\,a^2b^4+b^6\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.
time = 0.69, size = 221, normalized size = 2.33

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2b\right)}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B]
time = 5.88, size = 260, normalized size = 2.74

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-2b^2)}{a(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{a(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2-2b^2)}{a^2(a^2+b^2)}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-4b^2)+a^2-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\operatorname{atanh}\left(\frac{\left(2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{2a^2b+2b^3}{a^2+b^2}\right)\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.575 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{5/2}d\sqrt{\sec^2(c+dx)}}-\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{3}{2\left(a^2+b^2\right.}$$

```
[Out] -1/2*(2*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1
/2))*sec(d*x+c)/(a^2+b^2)^(5/2)/d/(sec(d*x+c)^2)^(1/2)-1/2*b*sec(d*x+c)/(a^
2+b^2)/d/(a+b*tan(d*x+c))^2-3/2*a*b*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c
))
```

Rubi [A]
time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3593, 759, 821, 739, 212}

$$-\frac{3ab\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\sec(c+dx)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{(2a^2-b^2)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*((2*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[
c + d*x]^2])]*Sec[c + d*x])/((a^2 + b^2)^(5/2)*d*Sqrt[Sec[c + d*x]^2]) - (b
*Sec[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (3*a*b*Sec[c + d*
x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\text{Subst}\left(\int \frac{-2a+x}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx\right.}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+}}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{((2a}{}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{((2a}{}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{5/2}d\sqrt{\sec^2(c+dx)}} - \frac{b\,\mathrm{se}}{2\left(a^2+b^2\right)d(}$$

Mathematica [A]
time = 1.06, size = 110, normalized size = 0.71

$$\frac{\frac{2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b\sec(c+dx)\left(4a^2+b^2+3ab\tan(c+dx)\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*Sec[c + d*x]*(4*a^2 + b^2 + 3*a*b*Tan[c + d*x]))/((a^2 +
b^2)^2*(a + b*Tan[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.32, size = 280, normalized size = 1.81

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{(2a\cdots}{\cdots}}{d}$
default	$-\frac{\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{(2a\cdots}{\cdots}}{d}$
risch	$-\frac{b(-3iab\,e^{3i(dx+c)}+4a^2e^{3i(dx+c)}+b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+4a^2e^{i(dx+c)}+b^2e^{i(dx+c)})}{(ib+a)^2(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a)^2d(-ib+a)^2}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+\cdots}{(\cdots}\right)}{(a^2+b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

time = 0.50, size = 412, normalized size = 2.66

$$-\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(5a^3b^2+2ab^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^8+2a^6b^2+a^4b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 +
 b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2
*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 + 2*a
*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(143) = 286$.
time = 0.41, size = 352, normalized size = 2.27

$$-\frac{\left(2\,a^{2}b^{2}-b^{4}+\left(2\,a^{4}-3\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}+2\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}-2\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)+2\left(4\,a^{4}b+5\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)+6\left(a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)}{4\left(\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b+3\,a^{5}b^{3}+3\,a^{3}b^{5}+ab^{7}\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{6}b^{2}+3\,a^{4}b^{4}+3\,a^{2}b^{6}+b^{8}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(143) = 286$.
time = 0.66, size = 293, normalized size = 1.89

$$-\frac{\frac{\left(2\,a^{2}-b^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(5\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-11\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^{4}b-a^{2}b^{3}\right)}{\left(a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
```

```
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

Mupad [B]

time = 4.82, size = 443, normalized size = 2.86

$$\frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{2}\right)}{d\,\left(a^2+b^2\right)^{5/2}}-\frac{\ln\left(\left(a^2+b^2\right)^{5/2}+a^4\,b+b^5+2\,a^2\,b^3-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{2\,d\,\left(a^2+b^2\right)^{5/2}}-\frac{\frac{4\,a^2\,b+b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)\,\left(4\,a^2\,b+b^3\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(11\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(5\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2) +
 a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/(d
*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 - a^
5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (d*x)
/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 + b^4 +
 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a^2*(a^
4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a*(a^4 +
 b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*(a^4 + b
^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^
2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\, dx$

Optimal. Leaf size=221

$$-\frac{3b^2(4a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{7/2}d}+\frac{b(2a^2-3b^2)\sec(c+}{2\left(a^2+b^2\right)^2 d(a+b\tan(c}$$

```
[Out] -3/2*b^2*(4*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2
)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/2*b*(2*a^2-3*b
^2)*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+cos(d*x+c)*(b+a*tan(d*x+c))
/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*b*(2*a^2-13*b^2)*sec(d*x+c)/(a^2+b^2)
^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3593, 755, 849, 821, 739, 212}

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{ab(2a^2-13b^2)\sec(c+dx)}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{b(2a^2-3b^2)\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{3b^2(4a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(7/2)*d)
+ (b*(2*a^2 - 3*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2) + (Cos[c + d*x]*(b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]
)^2) + (a*b*(2*a^2 - 13*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^3*d*(a + b*Tan[c
+ d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int -\right.}{(a^2+b^2)}$$

$$= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\left(b^3\cos\right.}{}$$

$$= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(2}{2\,(a^2+}$$

$$= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(2}{2\,(a^2+}$$

$$= -\frac{3b^2(4a^2-b^2)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx}}{2\left(a^2+b^2\right)^{7/2}d}$$

Mathematica [A]
time = 2.07, size = 183, normalized size = 0.83

$$\frac{-\frac{12b^2(-4a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\sec^2(c+dx)\left(b(11a^4-22a^2b^2-3b^4)\cos(c+dx)+b\left(a^2+b^2\right)^2\cos(3(c+dx))+2a\left(a^4+4a^2b^2-12b^4+\left(a^2+b^2\right)^2\cos(2(c+dx))\right)\sin(c+dx)\right)}{(a^2+b^2)^3(a+b\tan(c+dx))^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-12*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]
)/(a^2 + b^2)^(7/2) + (Sec[c + d*x]^2*(b*(11*a^4 - 22*a^2*b^2 - 3*b^4)*Cos[
c + d*x] + b*(a^2 + b^2)^2*Cos[3*(c + d*x)] + 2*a*(a^4 + 4*a^2*b^2 - 12*b^4
 + (a^2 + b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x]))/((a^2 + b^2)^3*(a + b*Tan
[c + d*x])^2))/(4*d)
```

Maple [A]
time = 0.66, size = 283, normalized size = 1.28

method	result
derivativedivides	$-\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+4a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3(4a^2-b^2}{}\right.}{(a^2+b^2)^3}}{d}$
default	$-\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+4a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3(4a^2-b^2}{}\right.}{(a^2+b^2)^3}}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2(-3ib\,a^2+ib^3+a^3-3b^2a)d}+\frac{ie^{-i(dx+c)}}{2(3ib\,a^2-ib^3+a^3-3b^2a)d}+\frac{b^3(-7iab\,e^{3i(dx+c)}+8a^2e^{3i(dx+c)}+b^2e^{3i(dx+c)}+7iab\,e^{i(}}{(-ia+b)^3(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2
*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^2)
/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*
d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*
b^2)*tan(1/2*d*x+1/2*c)-3*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.
time = 0.53, size = 658, normalized size = 2.98

$$-\frac{\frac{3(4a^2b^2-b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(6a^6b-10a^4b^3-a^2b^5+\frac{(2a^7+18a^5b^2-31a^3b^4-2ab^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(2a^6b-2a^4b^3+12a^2b^5+b^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2(2a^7+2a^5b^2+15a^3b^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{(2a^6b-30a^4b^3+15a^2b^5+2b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(2a^7-6a^5b^2+9a^3b^4+2ab^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^{10}+3a^8b^2+3a^6b^4+a^4b^6+\frac{4(a^9b+3a^7b^3+3a^5b^5+a^3b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^{10}-a^8b^2-9a^6b^4-11a^4b^6-4a^2b^8)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^{10}-a^8b^2-9a^6b^4-11a^4b^6-4a^2b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4(a^9b+3a^7b^3+3a^5b^5+a^3b^7)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{(a^{10}+3a^8b^2+3a^6b^4+a^4b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4*b^
3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c)^3/(c
```

```
os(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^6)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6 + 4
*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x + c) + 1)
- (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*
b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2 + 3*a^6*b^4 +
a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(209) = 418$.
time $= 0.42$, size $= 480$, normalized size $= 2.17$

$$\frac{4\left(a^6b + 3a^4b^3 + 3a^2b^5 + b^7\right)\cos\left(dx+c\right)^3 - 3\left(4a^2b^4 - b^6 + \left(4a^4b^2 - 5a^2b^4 + b^6\right)\cos\left(dx+c\right)^2 + 2\left(4a^3b^3 - ab^5\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 - 2a^2 - b^2 - 2\sqrt{a^2+b^2}\left(b\cos(dx+c) - a\sin(dx+c)\right)}{2ab\cos(dx+c)\sin(dx+c) + (a^2-b^2)\cos(dx+c)^2 + b^2}\right) + 2\left(4a^6b - 10a^4b^3 - 17a^2b^5 - 3b^7\right)\cos\left(dx+c\right) + 2\left(2a^5b^2 - 11a^3b^4 - 13ab^6 + 2\left(a^7 + 3a^5b^2 + 3a^3b^4 + ab^6\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{4\left(\left(a^{10} + 3a^8b^2 + 2a^6b^4 - 2a^4b^6 - 3a^2b^8 - b^{10}\right)d\cos\left(dx+c\right)^2 + 2\left(a^9b + 4a^7b^3 + 6a^5b^5 + 4a^3b^7 + ab^9\right)d\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^8b^2 + 4a^6b^4 + 6a^4b^6 + 4a^2b^8 + b^{10}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [**F**]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{\cos\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Giac [**A**]
time $= 0.74$, size $= 399$, normalized size $= 1.81$

$$-\frac{\frac{3\left(4a^2b^2-b^4\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}} - \frac{4\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2b-b^3\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)} - \frac{2\left(9a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-23a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2ab^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8a^4b^3-a^2b^5\right)}{\left(a^8+3a^6b^2+3a^4b^4+a^2b^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B]
time = 7.32, size = 610, normalized size = 2.76

$$-\frac{\frac{-6\,a^4\,b+10\,a^2\,b^3+b^5}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+2\,a^3\,b^2+15\,a\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^6\,b-30\,a^4\,b^3+15\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^6+18\,a^4\,b^2-31\,a^2\,b^4-2\,b^6\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^6-6\,a^4\,b^2+9\,a^2\,b^4+2\,b^6\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^6\,b-2\,a^4\,b^3+12\,a^2\,b^5+b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2-4\,b^2\right)-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\mathrm{atan}\left(\frac{-1{}\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^7+a^6\,b\,1{}\mathrm{i}-3{}\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5\,b^2+a^4\,b^3\,3{}\mathrm{i}-3{}\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^4+a^2\,b^5\,3{}\mathrm{i}-1{}\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^6+b^7\,1{}\mathrm{i}}{{\left(a^2+b^2\right)}^{7/2}}\right)\,\left(3\,b^4-12\,a^2\,b^2\right)\,1{}\mathrm{i}}{d\,{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] - ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*ta
n(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a^4*
b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(2*a^
6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(a*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b + b^7 +
 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2) - tan(c/2
 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 +
(d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i - a^7*tan(c
/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/2 + (d*x)/2)
*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4 - 12*a^2*b^2
)*1i)/(d*(a^2 + b^2)^(7/2))
```

3.577 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\, dx$

Optimal. Leaf size=310

$$-\frac{5b^4(6a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\,\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{9/2}d}+\frac{b(4a^4+24a^2b^2-15b^4)\,\mathrm{se}}{6\left(a^2+b^2\right)^3 d(a+b\tan(}$$

```
[Out] -5/2*b^4*(6*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2
)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(9/2)/d+1/6*b*(4*a^4+24*
a^2*b^2-15*b^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2+1/3*cos(d*x+c)^
3*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/6*a*b*(4*a^4+28*a^2*b^2
-81*b^4)*sec(d*x+c)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(2*a^2
-5*b^2)-a*(2*a^2+9*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

Rubi [A]

time = 0.29, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3593, 755, 837, 849, 821, 739, 212}

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{5b^4(6a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{9/2}}+\frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}+\frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*b^4*(6*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(9/2)*d)
+ (b*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sec[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*
Tan[c + d*x])^2) + (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(
a + b*Tan[c + d*x])^2) + (a*b*(4*a^4 + 28*a^2*b^2 - 81*b^4)*Sec[c + d*x])/(
6*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]*(b*(2*a^2 - 5*b^2)
- a*(2*a^2 + 9*b^2)*Tan[c + d*x]))/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \right.}{3\left(a^2+b^2\right.}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\,\text{t}\right.}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx)}$$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \cos($$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab(4}{6(}$$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab(4}{6(}$$

$$= -\frac{5b^4(6a^2-b^2)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+d}}{2\left(a^2+b^2\right)^{9/2}d}$$

Mathematica [A]

time = 2.14, size = 371, normalized size = 1.20

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{9b(a^4+14a^2b^2-3b^4)(a\cos(c+dx)+b\sin(c+dx))^2}{(a^2+b^2)^4}+\frac{6b^6\tan(c+dx)}{a(a^2+b^2)^3}+\frac{9a(a^4+6a^2b^2-11b^4)(a\cos(c+dx)+b\sin(c+dx))^2\tan(c+dx)}{(a^2+b^2)^4}-\frac{6b^5(12a^2+b^2)(a+b\tan(c+dx))}{a(a^2+b^2)^4}-\frac{60b^4(-6a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^2}{(a^2+b^2)^{9/2}}-\frac{b(-3a^2+b^2)\cos(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^2}{(a^2+b^2)^3}+\frac{a(a^2-3b^2)\cos(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^2}{(a^2+b^2)^3}\right)}{12d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*((9*b*(a^4 + 14*a^2*b^2 -
 3*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2 + b^2)^4 + (6*b^6*Tan[c +
 d*x])/(a*(a^2 + b^2)^3) + (9*a*(a^4 + 6*a^2*b^2 - 11*b^4)*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 + b^2)^4 - (6*b^5*(12*a^2 + b^2)*(a
+ b*Tan[c + d*x]))/(a*(a^2 + b^2)^4) - (60*b^4*(-6*a^2 + b^2)*ArcTanh[(-b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/
(a^2 + b^2)^(9/2) - (b*(-3*a^2 + b^2)*Cos[c + d*x]*Cos[3*(c + d*x)]*(a + b*
Tan[c + d*x])^2)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[3*(c + d
*x)]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3))/(12*d*(a + b*Tan[c + d*x])^3)
```

Maple [**A**]
time = 0.66, size = 457, normalized size = 1.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^4/(a^2+b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-1/2*b^2*(13*a^2+2*b^
2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(12*a^4-23*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(35*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+6*a^2*b+1/2*b^3)/(a*tan
(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(6*a^2-b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*
b^2+3*a^2*b^4+b^6)/(a^2+b^2)*((-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)^5
+(-3*a^4*b-12*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^5-32/3*a^3*b^2+14
*a*b^4)*tan(1/2*d*x+1/2*c)^3+(-20*a^2*b^3+4*b^5)*tan(1/2*d*x+1/2*c)^2+(-a^5
-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)-a^4*b-32/3*a^2*b^3+7/3*b^5)/(1+tan(1
/2*d*x+1/2*c)^2)^3)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(294) = 588$.
time = 0.67, size = 1229, normalized size = 3.96

$$-\frac{\frac{15\,(6\,a^2b^4-b^6)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)\sqrt{a^2+b^2}}-\frac{2\left(6a^8b+64a^6b^3-50a^4b^5-3a^2b^7+\frac{(6a^9+48a^7b^2+202a^5b^4-161a^3b^6-6ab^8)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2(6a^8b+56a^6b^3-14a^4b^5-67a^2b^7-3b^9)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(2a^9-4a^7b^2-86a^5b^4+133a^3b^6+3ab^8)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(8a^8b+28a^6b^3+188a^4b^5-156a^2b^7-9b^9)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2(2a^9+4a^7b^2+62a^5b^4-255a^3b^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2(14a^8b+56a^6b^3-246a^4b^5+141a^2b^7+9b^9)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4(2a^9+8a^7b^2+42a^5b^4+33a^3b^6-3ab^8)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3(2a^8b+8a^6b^3-78a^4b^5+23a^2b^7+2b^9)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3(2a^9+8a^7b^2-18a^5b^4+13a^3b^6+2ab^8)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^{12}+4a^{10}b^2+6a^8b^4+4a^6b^6+a^4b^8+\frac{4(a^{11}b+4a^9b^3+6a^7b^5+4a^5b^7+a^3b^9)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(a^{12}+8a^{10}b^2+22a^8b^4+28a^6b^6+17a^4b^8+4a^2b^{10})\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8(a^{11}b+4a^9b^3+6a^7b^5+4a^5b^7+a^3b^9)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2(a^{12}-2a^{10}b^2-18a^8b^4-32a^6b^6-23a^4b^8-6a^2b^{10})\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^{12}-2a^{10}b^2-18a^8b^4-32a^6b^6-23a^4b^8-6a^2b^{10})\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8(a^{11}b+4a^9b^3+6a^7b^5+4a^5b^7+a^3b^9)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{(a^{12}+8a^{10}b^2+22a^8b^4+28a^6b^6+17a^4b^8+4a^2b^{10})\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4(a^{11}b+4a^9b^3+6a^7b^5+4a^5b^7+a^3b^9)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{(a^{12}+4a^{10}b^2+6a^8b^4+4a^6b^6+a^4b^8)\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*(6*a^2*b^4 - b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 2*(6*a^8*
b + 64*a^6*b^3 - 50*a^4*b^5 - 3*a^2*b^7 + (6*a^9 + 48*a^7*b^2 + 202*a^5*b^4
 - 161*a^3*b^6 - 6*a*b^8)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(6*a^8*b + 56
*a^6*b^3 - 14*a^4*b^5 - 67*a^2*b^7 - 3*b^9)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 4*(2*a^9 - 4*a^7*b^2 - 86*a^5*b^4 + 133*a^3*b^6 + 3*a*b^8)*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 2*(8*a^8*b + 28*a^6*b^3 + 188*a^4*b^5 - 156*a^
2*b^7 - 9*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(2*a^9 + 4*a^7*b^2 +
 62*a^5*b^4 - 255*a^3*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(14*a^8*
```

```
b + 56*a^6*b^3 - 246*a^4*b^5 + 141*a^2*b^7 + 9*b^9)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 - 4*(2*a^9 + 8*a^7*b^2 + 42*a^5*b^4 + 33*a^3*b^6 - 3*a*b^8)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*(2*a^8*b + 8*a^6*b^3 - 78*a^4*b^5 + 2
3*a^2*b^7 + 2*b^9)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(2*a^9 + 8*a^7*b
^2 - 18*a^5*b^4 + 13*a^3*b^6 + 2*a*b^8)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8 + 4*(a^11*b + 4*a^9*
b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)/(cos(d*x + c) + 1) + (a
^12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 + 4*a^2*b^10)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 8*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*
b^7 + a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(a^12 - 2*a^10*b^2 -
 18*a^8*b^4 - 32*a^6*b^6 - 23*a^4*b^8 - 6*a^2*b^10)*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 2*(a^12 - 2*a^10*b^2 - 18*a^8*b^4 - 32*a^6*b^6 - 23*a^4*b^8
- 6*a^2*b^10)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*(a^11*b + 4*a^9*b^3 +
 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + (a^
12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 + 4*a^2*b^10)*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b
^7 + a^3*b^9)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + (a^12 + 4*a^10*b^2 + 6*
a^8*b^4 + 4*a^6*b^6 + a^4*b^8)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(294) = 588$.
time = 0.47, size = 619, normalized size = 2.00

$$\frac{4\left(a^{8}b+4a^{6}b^{3}+6a^{4}b^{5}+4a^{2}b^{7}+b^{9}\right)\cos\left(dx+c\right)^{5}-4\left(2a^{8}b+a^{6}b^{3}-9a^{4}b^{5}-13a^{2}b^{7}-5b^{9}\right)\cos\left(dx+c\right)^{3}-15\left(6a^{2}b^{6}-b^{8}+\left(6a^{4}b^{4}-7a^{2}b^{6}+b^{8}\right)\cos\left(dx+c\right)^{2}+2\left(6a^{3}b^{5}-ab^{7}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2a^{2}-b^{2}-2\sqrt{a^{2}+b^{2}}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)+2\left(8a^{8}b+64a^{6}b^{3}-16a^{4}b^{5}-87a^{2}b^{7}-15b^{9}\right)\cos\left(dx+c\right)+2\left(4a^{7}b^{2}+32a^{5}b^{4}-53a^{3}b^{6}-81ab^{8}+2\left(a^{9}+4a^{7}b^{2}+6a^{5}b^{4}+4a^{3}b^{6}+ab^{8}\right)\cos\left(dx+c\right)^{4}+2\left(2a^{9}+15a^{7}b^{2}+33a^{5}b^{4}+29a^{3}b^{6}+9ab^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{12\left(\left(a^{12}+4a^{10}b^{2}+5a^{8}b^{4}-5a^{4}b^{8}-4a^{2}b^{10}-b^{12}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{11}b+5a^{9}b^{3}+10a^{7}b^{5}+10a^{5}b^{7}+5a^{3}b^{9}+ab^{11}\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{10}b^{2}+5a^{8}b^{4}+10a^{6}b^{6}+10a^{4}b^{8}+5a^{2}b^{10}+b^{12}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 -
4*(2*a^8*b + a^6*b^3 - 9*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^3 - 15*
(6*a^2*b^6 - b^8 + (6*a^4*b^4 - 7*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(6*a^3*
b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(8*a^8*b + 64*a^6*b^3 - 16*a^4*b^5
- 87*a^2*b^7 - 15*b^9)*cos(d*x + c) + 2*(4*a^7*b^2 + 32*a^5*b^4 - 53*a^3*b^
6 - 81*a*b^8 + 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 + 15*a^7*b^2 + 33*a^5*b^4 + 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*
b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5
*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*
b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(294) = 588.
time = 0.78, size = 640, normalized size = 2.06

$$-\frac{\frac{15\,(6a^2b^4-b^6)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)\sqrt{a^2+b^2}}-\frac{6\left(13a^3b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^4b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-23a^2b^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-35a^3b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2ab^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-12a^4b^5-a^2b^7\right)}{(a^{10}+4a^8b^2+6a^6b^4+4a^4b^6+a^2b^8)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)^2}-\frac{4\left(3a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+36a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-9b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+32a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-42ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+60a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-27ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^4b+32a^2b^3-7b^5\right)}{(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(6*a^2*b^4 - b^6)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 6*(13*a^3*b^6
*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1
/2*d*x + 1/2*c)^2 - 23*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x +
 1/2*c)^2 - 35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c)
- 12*a^4*b^5 - a^2*b^7)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^
8)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2) - 4*(3*a^5*
tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1
/2*d*x + 1/2*c)^5 + 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^4 - 9*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 +
 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 60*a
^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1
/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x +
1/2*c) + 3*a^4*b + 32*a^2*b^3 - 7*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B]
time = 8.83, size = 1128, normalized size = 3.64

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^7+4a^5b^2+62a^3b^4-255ab^6)}{3(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}+\frac{6a^6b+64a^4b^3-50a^2b^5-3b^7}{3(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(6a^6b+50a^4b^3-64a^2b^5-3b^7)}{3a^2(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9(2a^8+8a^6b^2-18a^4b^4+13a^2b^6+2b^8)}{a(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(2a^6+6a^4b^2+36a^2b^4-3b^6)}{3a(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(2a^8b+8a^6b^3-78a^4b^5+23a^2b^7+2b^9)}{a^2(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8a^8b+28a^6b^3+188a^4b^5-156a^2b^7-9b^9)}{3a^2(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(14a^8b+56a^6b^3-246a^4b^5+141a^2b^7+9b^9)}{3a^2(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8+48a^6b^2+202a^4b^4-161a^2b^6-6b^8)}{3a(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^8-4a^6b^2-86a^4b^4+133a^2b^6+3b^8)}{3a(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(2a^2-12b^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(2a^2-12b^2)+a^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+4b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(a^2+4b^2)+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{b^4\,\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^9+a^8b\,1\mathrm{i}-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^7b^2+a^6b^3\,4\mathrm{i}-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^5b^4+a^4b^5\,6\mathrm{i}-4\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^6+a^2b^7\,4\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^8+b^9\,1\mathrm{i}}{(a^2+b^2)^{9/2}}\right)(6a^2-b^2)\,5\mathrm{i}}{d\,(a^2+b^2)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5*(2*a^7 - 255*a*b^6 + 62*a^3*b^4 + 4*a^5*b^2))/(3*(
a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (6*a^6*b - 3*b^7 - 50*a^2
*b^5 + 64*a^4*b^3)/(3*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2
*tan(c/2 + (d*x)/2)^2*(6*a^6*b - 3*b^7 - 64*a^2*b^5 + 50*a^4*b^3))/(3*a^2*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^9*(2*a^8 + 2*b^8
+ 13*a^2*b^6 - 18*a^4*b^4 + 8*a^6*b^2))/(a*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
```

```
^4 + 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(2*a^6 - 3*b^6 + 36*a^2*b^4 + 6*
a^4*b^2))/(3*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^8
*(2*a^8*b + 2*b^9 + 23*a^2*b^7 - 78*a^4*b^5 + 8*a^6*b^3))/(a^2*(a^8 + b^8 +
 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(8*a^8*b - 9
*b^9 - 156*a^2*b^7 + 188*a^4*b^5 + 28*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(14*a^8*b + 9*b^9 +
141*a^2*b^7 - 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(6*a^8 - 6*b^8 - 161*a^2*b^6 +
202*a^4*b^4 + 48*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*b^8 + 133*a^2*b^6 - 86*a^4*b^4 -
 4*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^2
*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(2*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^4*(2*a^2 - 12*b^2) + a^2 + tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^8*(a^2 + 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^3 - 8*a*b*tan(c
/2 + (d*x)/2)^7 - 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) +
 (b^4*atan((a^8*b*1i + b^9*1i + a^2*b^7*4i + a^4*b^5*6i + a^6*b^3*4i - a^9*
tan(c/2 + (d*x)/2)*1i - a*b^8*tan(c/2 + (d*x)/2)*1i - a^3*b^6*tan(c/2 + (d*
x)/2)*4i - a^5*b^4*tan(c/2 + (d*x)/2)*6i - a^7*b^2*tan(c/2 + (d*x)/2)*4i)/(
a^2 + b^2)^(9/2))*(6*a^2 - b^2)*5i)/(d*(a^2 + b^2)^(9/2))
```

3.578 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=121

$$-\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}+\frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2ad(d\sec(e+fx)$$

```
[Out] 2/7*b*(d*sec(f*x+e))^(7/2)/f+2/5*a*d*(d*sec(f*x+e))^(5/2)*sin(f*x+e)/f-6/5*
a*d^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+6/5*a*d^3*sin(f*x+
e)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2719}

$$-\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{6ad^3\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-6*a*d^4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e +
 f*x]]) + (2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + (6*a*d^3*Sqrt[d*Sec[e + f*x]
]*Sin[e + f*x])/(5*f) + (2*a*d*(d*Sec[e + f*x])^(5/2)*Sin[e + f*x])/(5*f)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{2b(d\sec(e+fx))^{7/2}}{7f} + a\int (d\sec(e+fx))^{7/2}\,dx$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f} + \frac{1}{5}$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\,\sin(e+fx)}{5f} + \frac{2a}{}$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\,\sin(e+fx)}{5f} + \frac{2a}{}$$
$$= -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{7/2}}{7f} +$$

Mathematica [A]
time = 0.68, size = 69, normalized size = 0.57

$$\frac{(d\sec(e+fx))^{7/2}\left(40b-168a\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+70a\sin(2(e+fx))+21a\sin(4(e+fx))\right)}{140f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(7/2)*(40*b - 168*a*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x
)/2, 2] + 70*a*Sin[2*(e + f*x)] + 21*a*Sin[4*(e + f*x)]))/(140*f)
```

Maple [C] Result contains complex when optimal does not.
time = 8.41, size = 371, normalized size = 3.07

method	result
default	$-\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(21i(\cos^4(fx+e))\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(21*I*cos(f*x+e)^4*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin
(f*x+e),I)*sin(f*x+e)*a-21*I*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+
e)*a+21*I*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a-21*I*cos(f*x+e
)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a+21*cos(f*x+e)^4*a-14*cos(f*x+e)^3
*a-7*a*cos(f*x+e)-5*b*sin(f*x+e))*(d/cos(f*x+e))^(7/2)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 156, normalized size = 1.29

$$\frac{-21i\sqrt{2}\,ad^{\frac{7}{2}}\cos\left(fx+e\right)^{3}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+21i\sqrt{2}\,ad^{\frac{7}{2}}\cos\left(fx+e\right)^{3}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))+2\left(5\,bd^{3}+7\left(3\,ad^{3}\cos\left(fx+e\right)^{3}+ad^{3}\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{35\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/35*(-21*I*sqrt(2)*a*d^(7/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*a*d^(7/2
)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
+ e) - I*sin(f*x + e))) + 2*(5*b*d^3 + 7*(3*a*d^3*cos(f*x + e)^3 + a*d^3*co
s(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{7/2}\,\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)), x)
```

3.579 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=92

$$\frac{2ad^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2b(d\sec(e+fx))^{5/2}}{5f}+\frac{2ad(d\sec(e+fx))^{3/2}\sin(e+f}{3f}$$

```
[Out] 2/5*b*(d*sec(f*x+e))^(5/2)/f+2/3*a*d*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/3*
a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x
+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2720}

$$\frac{2ad^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*a*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])
/(3*f) + (2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (2*a*d*(d*Sec[e + f*x])^(3/2)
*Sin[e + f*x])/(3*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{2b(d\sec(e+fx))^{5/2}}{5f} + a\int (d\sec(e+fx))^{5/2}\,dx$$
$$= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}$$
$$= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}$$
$$= \frac{2ad^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b(d}{}$$

Mathematica [A]
time = 0.47, size = 58, normalized size = 0.63

$$\frac{(d\sec(e+fx))^{5/2}\left(6b+10a\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+5a\sin(2(e+fx))\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(5/2)*(6*b + 10*a*Cos[e + f*x]^(5/2)*EllipticF[(e + f*x)/
2, 2] + 5*a*Sin[2*(e + f*x)]))/(15*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.47, size = 195, normalized size = 2.12

method	result
default	$\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(5i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)(\cos^3(fx+e))a+5i\sqrt{\frac{}{\cos(}}\right.}{15f\sin(fx+e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(
```

```
f*x+e)^3*a+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*a+5*cos(f*x+e)*sin(f*x
+e)*a+3*b)*(d/cos(f*x+e))^(5/2)/sin(f*x+e)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 133, normalized size = 1.45

$$\frac{-5i\sqrt{2}\,ad^{\frac{5}{2}}\cos\left(fx+e\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+5i\sqrt{2}\,ad^{\frac{5}{2}}\cos\left(fx+e\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right))+2\left(5\,ad^{2}\cos\left(fx+e\right)\sin\left(fx+e\right)+3\,bd^{2}\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(5*a*d^2*cos(f*x + e)*
sin(f*x + e) + 3*b*d^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,\left(a+b\tan(e+f\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)), x)
```

3.580 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=88

$$-\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{3/2}}{3f}+\frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f}$$

```
[Out] 2/3*b*(d*sec(f*x+e))^(3/2)/f-2*a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f
*x+e))^(1/2)+2*a*d*sin(f*x+e)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2719}

$$-\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2ad\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-2*a*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f
*x]]) + (2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (2*a*d*Sqrt[d*Sec[e + f*x]]*Si
n[e + f*x])/f
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx &= \frac{2b(d\sec(e+fx))^{3/2}}{3f}+a\int (d\sec(e+fx))^{3/2}\,dx\\ &= \frac{2b(d\sec(e+fx))^{3/2}}{3f}+\frac{2ad\sqrt{d\sec(e+fx)}\,\sin(e+fx)}{f}-(ad\\ &= \frac{2b(d\sec(e+fx))^{3/2}}{3f}+\frac{2ad\sqrt{d\sec(e+fx)}\,\sin(e+fx)}{f}-\frac{}{\sqrt{}}\\ &= -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{3/2}}{3f}+\end{aligned}$$

Mathematica [A]
time = 0.32, size = 58, normalized size = 0.66

$$\frac{(d\sec(e+fx))^{3/2}\left(2b-6a\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+3a\sin(2(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(3/2)*(2*b - 6*a*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2
, 2] + 3*a*Sin[2*(e + f*x)]))/(3*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 356, normalized size = 4.05

method	result
default	$-\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(3i(\cos^2(fx+e))\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(3*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f
```

```
*x+e),I)*sin(f*x+e)*a-3*I*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*
a+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a*sin(f*x+e)-3*I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*cos(f*x+e)*a*sin(f*x+e)+3*cos(f*x+e)^2*a-3*a*cos(f*x+e)-b*sin(f*x+
e))*(d/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 130, normalized size = 1.48

$$\frac{-3i\sqrt{2}\,ad^{\frac{3}{2}}\cos\left(fx+e\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3i\sqrt{2}\,ad^{\frac{3}{2}}\cos\left(fx+e\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))+2\left(3\,ad\cos\left(fx+e\right)\sin\left(fx+e\right)+bd\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*sqrt(2)*a*d^(3/2)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*a*d^(3/2)*cos
(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e))) + 2*(3*a*d*cos(f*x + e)*sin(f*x + e) + b*d)*sqrt(d/cos(f*x
 + e)))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos \left(e+f\,x\right)}\right)^{3/2}\,\left(a+b\,\tan\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)),x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)), x)
```

3.581 $\int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=58

$$\frac{2b\sqrt{d\sec(e+fx)}}{f}+\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{f}$$

```
[Out] 2*b*(d*sec(f*x+e))^(1/2)/f+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(
1/2)/f
```

Rubi [A]

time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3856, 2720}

$$\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))\,dx = \frac{2b\sqrt{d\sec(e+fx)}}{f} + a\int \sqrt{d\sec(e+fx)}\,dx$$
$$= \frac{2b\sqrt{d\sec(e+fx)}}{f} + \left(a\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}\right)\int$$
$$= \frac{2b\sqrt{d\sec(e+fx)}}{f} + \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\,\mathrm{s}}}{f}$$

Mathematica [A]
time = 0.23, size = 42, normalized size = 0.72

$$\frac{2\left(b+a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)\sqrt{d\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[d*Sec[e + f*x]
])/f
```

Maple [C] Result contains complex when optimal does not.
time = 0.51, size = 168, normalized size = 2.90

method	result
default	$\frac{2\sqrt{\frac{d}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a}{f\sin(fx+e)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(d/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/si
n(f*x+e),I)*cos(f*x+e)*a+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+b)/sin(f*x+e)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 79, normalized size = 1.36

$$\frac{-i\sqrt{2}\,a\sqrt{d}\,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+i\sqrt{2}\,a\sqrt{d}\,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}-i\,\sin{(fx+e)})+2\,b\sqrt{\frac{d}{\cos{(fx+e)}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(
f*x + e)) + 2*b*sqrt(d/cos(f*x + e)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Mupad [B]
time = 0.36, size = 39, normalized size = 0.67

$$\frac{2\left(b+a\,\sqrt{\cos{(e+f\,x)}}\;\mathrm{F}\left(\frac{e}{2}+\frac{f\,x}{2}\middle|2\right)\right)\,\sqrt{\frac{d}{\cos{(e+f\,x)}}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x)),x)
```

```
[Out] (2*(b + a*cos(e + f*x)^(1/2)*ellipticF(e/2 + (f*x)/2, 2))*(d/cos(e + f*x))^
(1/2))/f
```

3.582 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{2b}{f\sqrt{d\sec(e+fx)}}+\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -2*b/f/(d*sec(f*x+e))^(1/2)+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3856, 2719}

$$\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[C
os[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{2b}{f\sqrt{d\sec(e+fx)}} + a\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= -\frac{2b}{f\sqrt{d\sec(e+fx)}} + \frac{a\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$
$$= -\frac{2b}{f\sqrt{d\sec(e+fx)}} + \frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.32, size = 54, normalized size = 0.93

$$\frac{-2b\sqrt{\cos(e+fx)}+2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[Cos[e + f*x]] + 2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f
*x]]*Sqrt[d*Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.61, size = 916, normalized size = 15.79

method	result
risch	$-\frac{i(-ib+a)\sqrt{2}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}} - \frac{ia\left(-\frac{2\left(d\,e^{2i(fx+e)}+d\right)}{d\sqrt{e^{i(fx+e)}\left(d\,e^{2i(fx+e)}+d\right)}} + \frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(fx+e)}-i\right)}\sqrt{i}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(4*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/
sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*a-4*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*a+8*I*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a-8*I
```

```
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*s
in(f*x+e)*a+4*I*a*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e
),I)*sin(f*x+e)-4*I*a*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*sin(f*x+e)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3*a-4
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2*sin(f*x+e)*b-4*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*b-b*cos(f*x+e)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+b*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*
x+e)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*a)/(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)/cos(f*x+e)/sin(f*x+e)^3/(d/cos(f*x+e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 95, normalized size = 1.64

$$\frac{i\sqrt{2}\,a\sqrt{d}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))-i\sqrt{2}\,a\sqrt{d}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))-2\,b\sqrt{\frac{d}{\cos(fx+e)}}\,\cos(fx+e)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*b*sqrt(d/cos(f
*x + e))*cos(f*x + e))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{\sqrt{d\sec(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\tan(e+f\,x)}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2), x)
```

3.583 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/3*b/f/(d*sec(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(1/2)+2/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3854, 3856, 2720}

$$\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*a*Sin[e + f*x])/(3*d*f*Sq
rt[d*Sec[e + f*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + a\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} + \frac{a\int\sqrt{d\sec(e+fx)}\,dx}{3d^2}$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} + \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right.}{3d^2}$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{}{3}$$

Mathematica [A]
time = 0.22, size = 69, normalized size = 0.73

$$-\frac{\sqrt{d\sec(e+fx)}\left(b+b\cos(2(e+fx))-2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)-a\sin(2(e+fx))\right)}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(Sqrt[d*Sec[e + f*x]]*(b + b*Cos[2*(e + f*x)] - 2*a*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2] - a*Sin[2*(e + f*x)]))/(d^2*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.90, size = 172, normalized size = 1.83

method	result
default	$\frac{\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a}{3}+\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos}{}\right.}{3}}{f\cos(fx+e)^2\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a+I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a
-b*cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)*a)/cos(f*x+e)^2/(d/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 111, normalized size = 1.18

$$\frac{-i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e))-2\left(b\cos(fx+e)^2-a\cos(fx+e)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{3\,d^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f
*x + e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e)) - 2*(b*cos(f*x + e)^2 - a*cos(f*x + e)*sin(f*x + e))*sqrt(d/c
os(f*x + e)))/(d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{(d\sec(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2), x)
```

3.584 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}$$

```
[Out] -2/5*b/f/(d*sec(f*x+e))^(5/2)+2/5*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(3/2)+6/5
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3567, 3854, 3856, 2719}

$$\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (6*a*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*a*Sin[e + f*x])/(5*d*f*(d
*Sec[e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+a\int \frac{1}{(d\sec(e+fx))^{5/2}}\,dx\\ &= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}+\frac{(3a)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\ &= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}+\frac{(3a)\int \sqrt{\cos(e+fx)}\,dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+}}\\ &= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+f}{5df(d\sec(e+f}\end{aligned}$$

Mathematica [A]
time = 0.68, size = 74, normalized size = 0.79

$$\frac{2\sqrt{d\sec(e+fx)}\left(3a\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)+\cos^2(e+fx)(-b\cos(e+fx)+a\sin(e+fx))\right)}{5d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
[Out] (2*Sqrt[d*Sec[e + f*x]]*(3*a*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 Cos[e + f*x]^2*(-(b*Cos[e + f*x]) + a*Sin[e + f*x])))/(5*d^3*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 345, normalized size = 3.67

method	result
default	$-\frac{2\left(3i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a\sin(fx+e)-3i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+}{\cos(fx+e}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(3*I*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*a-3*I*EllipticF(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*a+3*I*EllipticE(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)*a-3*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*a+cos(f*x+e)^4*a+cos(f*x+e)^3
*sin(f*x+e)*b+2*cos(f*x+e)^2*a-3*a*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e)/(d/c
os(f*x+e))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 119, normalized size = 1.27

$$\frac{3i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))-3i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))-2\left(b\cos(fx+e)^3-a\cos(fx+e)^2\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{5\,d^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(b*cos
(f*x + e)^3 - a*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{(d\sec(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2), x)
```

3.585 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b}{7f(d\sec(e+fx))^{7/2}}+\frac{10a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}+\frac{1}{21d^3}$$

```
[Out] -2/7*b/f/(d*sec(f*x+e))^(7/2)+2/7*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(5/2)+10/
21*a*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+10/21*a*(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)*(d*sec(f*x+e))^(1/2)/d^4/f
```

Rubi [A]

time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3854, 3856, 2720}

$$\frac{10a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (10*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*a*Sin[e + f*x])/(7*d*f*
(d*Sec[e + f*x])^(5/2)) + (10*a*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]
])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + a\int \frac{1}{(d\sec(e+fx))^{7/2}}\,dx$$
$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{(5a)\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2}$$
$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{(5a)}{}$$
$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{\left(5a\sqrt{}\right.}{}$$
$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{10a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f} +$$

Mathematica [A]
time = 0.41, size = 94, normalized size = 0.76

$$\frac{\sqrt{d\sec(e+fx)}\left(-9b-12b\cos(2(e+fx))-3b\cos(4(e+fx))+40a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)+26a\sin(2(e+fx))+3a\sin(4(e+fx))\right)}{84d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
[Out] (Sqrt[d*Sec[e + f*x]]*(-9*b - 12*b*Cos[2*(e + f*x)] - 3*b*Cos[4*(e + f*x)]
+ 40*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 26*a*Sin[2*(e + f*x)]
 + 3*a*Sin[4*(e + f*x)]))/(84*d^4*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.46, size = 190, normalized size = 1.54

method	result

default	$\frac{\frac{10i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a}{21}+\frac{10i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\right.}{21}}{f\cos(fx+e)^4\left(\frac{d}{\cos(fx+e)}\right)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/f*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a+5*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e)
,I)*a-3*cos(f*x+e)^4*b+3*cos(f*x+e)^3*sin(f*x+e)*a+5*cos(f*x+e)*sin(f*x+e)*
a)/cos(f*x+e)^4/(d/cos(f*x+e))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 127, normalized size = 1.03

$$\frac{-5i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{2}\,a\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))-2\left(3\,b\cos(fx+e)^4-\left(3\,a\cos(fx+e)^3+5\,a\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{21\,d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e)) + 5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e)) - 2*(3*b*cos(f*x + e)^4 - (3*a*cos(f*x + e)^3 + 5*a*cos(
f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan(e + fx)}{(d\sec(e + fx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(7/2),x)
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2), x)
```

3.586 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=143

$$\frac{2(7a^2-2b^2)\,d^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2(7a^2-2b^2)\,d(d}{}$$

```
[Out] 18/35*a*b*(d*sec(f*x+e))^(5/2)/f+2/21*(7*a^2-2*b^2)*d*(d*sec(f*x+e))^(3/2)*
sin(f*x+e)/f+2/21*(7*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+
e))^(1/2)/f+2/7*b*(d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))/f
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3853, 3856, 2720}

$$\frac{2d^2(7a^2-2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{2d(7a^2-2b^2)\sin(e+fx)(d\sec(e+fx))^{3/2}}{21f}+\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*(7*a^2 - 2*b^2)*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*
Sec[e + f*x]])/(21*f) + (18*a*b*(d*Sec[e + f*x])^(5/2))/(35*f) + (2*(7*a^2
- 2*b^2)*d*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(21*f) + (2*b*(d*Sec[e + f*
x])^(5/2)*(a + b*Tan[e + f*x]))/(7*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}+\frac{2}{7}\int (d\sec(e+fx))$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+f}{7f}$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2(7a^2-2b^2)\,d(d\sec(e+fx))^{3/2}\sin(}{21f}$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2(7a^2-2b^2)\,d(d\sec(e+fx))^{3/2}\sin(}{21f}$$

$$=\frac{2(7a^2-2b^2)\,d^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx}}{21f}$$

Mathematica [A]
time = 0.86, size = 127, normalized size = 0.89

$$\frac{2d^2\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2\left(5(7a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+\frac{5}{2}(7a^2-2b^2)\sin(2(e+fx))+3b(14a+5b\tan(e+fx))\right)}{105f(a\cos(e+fx)+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2*(5*(7*a^2 - 2*b^2)*Cos[e
 + f*x]^(5/2)*EllipticF[(e + f*x)/2, 2] + (5*(7*a^2 - 2*b^2)*Sin[2*(e + f*x
)])/2 + 3*b*(14*a + 5*b*Tan[e + f*x])))/(105*f*(a*Cos[e + f*x] + b*Sin[e +
f*x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.53, size = 382, normalized size = 2.67

method	result
default	$\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(35i(\cos^4(fx+e))\operatorname{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^2-10i(\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(35*I*cos(f*x+e)^4*EllipticF(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*a^2-10*I*cos(f*x+e)^4*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*b^2+35*I*cos(f*x+
e)^3*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2-10*I*cos(f*x+e)^3*EllipticF(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*b^2+35*cos(f*x+e)^2*sin(f*x+e)*a^2-10*cos(f*x+e)^2*sin(f*x+e)*b^2+42*cos(
f*x+e)*a*b+15*sin(f*x+e)*b^2)*(d/cos(f*x+e))^(5/2)/sin(f*x+e)^4/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 183, normalized size = 1.28

$$\frac{-5i\sqrt{2}\,(7a^2-2b^2)d^{\frac{5}{2}}\cos(fx+e)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{2}\,(7a^2-2b^2)d^{\frac{5}{2}}\cos(fx+e)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))+2\left(42abd^2\cos(fx+e)+5\left((7a^2-2b^2)d^2\cos(fx+e)^2+3b^2d^2\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{105f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(5/2)*cos(f*x + e)^3*weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(
5/2)*cos(f*x + e)^3*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e
)) + 2*(42*a*b*d^2*cos(f*x + e) + 5*((7*a^2 - 2*b^2)*d^2*cos(f*x + e)^2 + 3
*b^2*d^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2, x)
```

3.587 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=143

$$-\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\,\sin(e+fx)}{5f}+$$

```
[Out] 14/15*a*b*(d*sec(f*x+e))^(3/2)/f-2/5*(5*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x
+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/5*(5*a^2-2*b^2)*d*sin(f*x+e)*(d*sec(f*x+e)
)^(1/2)/f+2/5*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))/f
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3853, 3856, 2719}

$$-\frac{2d^2(5a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2d(5a^2-2b^2)\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*(5*a^2 - 2*b^2)*d^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*
Sqrt[d*Sec[e + f*x]]) + (14*a*b*(d*Sec[e + f*x])^(3/2))/(15*f) + (2*(5*a^2
- 2*b^2)*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])
^(3/2)*(a + b*Tan[e + f*x]))/(5*f)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
```

```
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}+\frac{2}{5}\int (d\sec(e+fx))$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+f}{5f}$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\,\sin(e}{5f}$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\,\sin(e}{5f}$$

$$= -\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}$$

Mathematica [A]
time = 0.72, size = 126, normalized size = 0.88

$$-\frac{2d^2(a+b\tan(e+fx))^2\left(3(5a^2-2b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+\left(-\frac{15a^2}{2}+3b^2\right)\sin(2(e+fx))-b(10a+3b\tan(e+fx))\right)}{15f\sqrt{d\sec(e+fx)}\,(a\cos(e+fx)+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*d^2*(a + b*Tan[e + f*x])^2*(3*(5*a^2 - 2*b^2)*Cos[e + f*x]^(3/2)*Ellipt
icE[(e + f*x)/2, 2] + ((-15*a^2)/2 + 3*b^2)*Sin[2*(e + f*x)] - b*(10*a + 3*
b*Tan[e + f*x])))/(15*f*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*
x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.51, size = 712, normalized size = 4.98

method	result
default	$-\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(15i(\cos^3(fx+e))\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(15*I*cos(f*x+e)^3*sin(f*x+e)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*
x+e)-1)/sin(f*x+e),I)*a^2-6*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e)
,I)*b^2-15*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+6*I*cos(f
*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+15*I*cos(f*x+e)^2*sin(f*x+e
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*a^2-6*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*b^2-15*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+6*I*
cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+15*cos(f*x+e)^3*a^2-6*
cos(f*x+e)^3*b^2-15*a^2*cos(f*x+e)^2+9*b^2*cos(f*x+e)^2-10*a*cos(f*x+e)*b*s
in(f*x+e)-3*b^2)*(d/cos(f*x+e))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 182, normalized size = 1.27

$$\frac{-3i\sqrt{2}\,(5a^2-2b^2)d^{\frac{3}{2}}\cos(fx+e)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+3i\sqrt{2}\,(5a^2-2b^2)d^{\frac{3}{2}}\cos(fx+e)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(10abd\cos(fx+e)+3\left((5a^2-2b^2)d\cos(fx+e)^2+b^2d\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{15f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(-3*I*sqrt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt
(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*d*cos(f*x + e
) + 3*((5*a^2 - 2*b^2)*d*cos(f*x + e)^2 + b^2*d)*sin(f*x + e))*sqrt(d/cos(f
*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^{\frac{3}{2}}(a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2, x)
```

3.588 $\int \sqrt{d \sec(e+fx)}\,(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=103

$$\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}\,(}{3}$$

```
[Out] 10/3*a*b*(d*sec(f*x+e))^(1/2)/f+2/3*(3*a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/
2)*(d*sec(f*x+e))^(1/2)/f+2/3*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))/f
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3589, 3567, 3856, 2720}

$$\frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (10*a*b*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*b*Sqrt[d*Sec[e
+ f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2\,dx = \frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}{3f}+\frac{2}{3}\int \sqrt{d\sec(e+fx)}$$

$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}{3f}$$

$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}{3f}$$

$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{3f}$$

Mathematica [A]
time = 0.70, size = 87, normalized size = 0.84

$$\frac{2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left((3a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+b\cos(e+fx)(6a\cos(e+fx)+b\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*((3*a^2 - 2*b^2)*Cos[e + f*x]^(5/2)*
EllipticF[(e + f*x)/2, 2] + b*Cos[e + f*x]*(6*a*Cos[e + f*x] + b*Sin[e + f*
x])))/(3*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.75, size = 339, normalized size = 3.29

method	result
default	$\frac{2\sqrt{\frac{d}{\cos(fx+e)}}\,(\cos(fx+e)-1)^2\left(3i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)(\cos^2(fx+e))a^2-2i\sqrt{\frac{}{\cos(f}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(d/cos(f*x+e))^(1/2)*(cos(f*x+e)-1)^2*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*c
```

```
os(f*x+e)^2*a^2-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*b^2+3*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1
)/sin(f*x+e),I)*cos(f*x+e)*a^2-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*b^2+
6*cos(f*x+e)*a*b+sin(f*x+e)*b^2)*(cos(f*x+e)+1)^2/cos(f*x+e)/sin(f*x+e)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 143, normalized size = 1.39

$$\frac{\sqrt{2}\,(-3i\,a^2+2i\,b^2)\sqrt{d}\,\cos(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2}\,(3i\,a^2-2i\,b^2)\sqrt{d}\,\cos(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e))+2\,(6\,ab\cos(fx+e)+b^2\sin(fx+e))\sqrt{\frac{d}{\cos(fx+e)}}}{3\,f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(d)*cos(f*x + e)*weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(d)
*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2
*(6*a*b*cos(f*x + e) + b^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{d}{\cos(e+fx)}}\,(a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2, x)
```

3.589 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] -6*a*b/f/(d*sec(f*x+e))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(
d*sec(f*x+e))^(1/2)+2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3856, 2719}

$$\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2,
 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*x]
))/(f*Sqrt[d*Sec[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
```

```
 + b^2, 0] && NeQ[m, -1]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+2\int \frac{\frac{a^2}{2}-b^2+\frac{3}{2}ab\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+(a^2-2b^2)\int \frac{1}{\sqrt{d\sec(e+fx)}}$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+\frac{(a^2-2b^2)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2(a^2-2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+f}{f\sqrt{d\sec(e+fx)}}$$

Mathematica [A]
time = 1.01, size = 64, normalized size = 0.67

$$\frac{\frac{2(a^2-2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}+2b(-2a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] ((2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 2*b*(-2*a
 + b*Tan[e + f*x]))/(f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.54, size = 2565, normalized size = 27.00

method	result	size
default	Expression too large to display	2565

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)-1)*(-8*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2-2*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2*sin(f*x+e)+4*I*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2*sin(f*x+e)+2*I*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2*sin(f*x+e)-4*I*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2*sin(f*x+e)+4*cos(f*x+e)
^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^2+16*I*cos(f*x+e)^3*sin(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+8*I*cos(f*x
+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e)
,I)*a^2-16*I*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*b^2-12*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+24*I*cos(f*x+e)^2*sin(f*x+e
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+12*I*co
s(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*a^2-24*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+16*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+8*I*cos(f*x+e)*sin(f*x+e
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2-16*I*co
s(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x
+e),I)*b^2-2*I*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(co
s(f*x+e)-1)/sin(f*x+e),I)*a^2+4*I*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+2*I*cos(f*x+e)^4*sin(f*x+e
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2-4*I*cos
(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*b^2-8*I*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
```

```
os(f*x+e)-1)/sin(f*x+e),I)*a^2-a*b*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+a*b*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*sin(f*x+e)+12*cos(f
*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a*b-4*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2+4*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*b^2-2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^2
-2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2+2*cos(f*x+e)^5*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2-2*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)*b^2+4*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2-2*
cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^2-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)*b^2+4*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(3/2)*a*b+4*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
a*b+12*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a*b)*(c
os(f*x+e)+1)^4*(d/cos(f*x+e))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/co
s(f*x+e)^3/d/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 129, normalized size = 1.36

$$\frac{\sqrt{2}\,(i\,a^2-2i\,b^2)\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{2}\,(-i\,a^2+2i\,b^2)\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}-i\,\sin{(fx+e)}))-2\,(2\,ab\cos{(fx+e)}-b^2\sin{(fx+e)})\sqrt{\frac{d}{\cos{(fx+e)}}}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-I*a^2 + 2*I*b^2)*sqrt
(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(
f*x + e))) - 2*(2*a*b*cos(f*x + e) - b^2*sin(f*x + e))*sqrt(d/cos(f*x + e))
)/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(e+fx)})^2}{\sqrt{d\sec{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(e+f\,x))^2}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2), x)
```

3.590 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2ab}{3f(d\sec(e+fx))^{3/2}}+\frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}$$

```
[Out] 2/3*a*b/f/(d*sec(f*x+e))^(3/2)+2/3*(a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e)
)^(1/2)+2/3*(a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^
2/f-2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3854, 3856, 2720}

$$\frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*(a^2 + 2*b^2
)*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))/(
f*(d*Sec[e + f*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
```

```
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx &= -\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} - 2\int \frac{-\frac{a^2}{2}-b^2+\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx \\
&= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} - \left(-a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx}\\
&= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{(a}{}\\
&= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{((}{}\\
&= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e}}{3d^2f}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 101, normalized size = 0.73

$$\frac{\sec^2(e+fx)\left(-2ab-2ab\cos(2(e+fx))+2(a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)+a^2\sin(2(e+fx))-b^2\sin(2(e+fx))\right)}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-2*a*b - 2*a*b*Cos[2*(e + f*x)] + 2*(a^2 + 2*b^2)*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^2*Sin[2*(e + f*x)] - b^2*Sin[2*(e
+ f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.52, size = 320, normalized size = 2.30

method	result
default	$\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a^2}{3}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}\left(\frac{i(\cdots}{}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x
+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+2*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/
sin(f*x+e),I)*b^2+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+2*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),
I)*b^2-2*cos(f*x+e)^2*a*b+cos(f*x+e)*sin(f*x+e)*a^2-cos(f*x+e)*sin(f*x+e)*b
^2)/cos(f*x+e)^2/(d/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 139, normalized size = 1.00

$$\frac{\sqrt{2}\,(-i a^2 - 2 i b^2)\sqrt{d}\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)) + \sqrt{2}\,(i a^2+2ib^2)\sqrt{d}\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)) - 2\,(2ab\cos(fx+e)^2-(a^2-b^2)\cos(fx+e)\sin(fx+e))\sqrt{\frac{d}{\cos(fx+e)}}}{3\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x
+ e) + I*sin(f*x + e)) + sqrt(2)*(I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassPInve
rse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(2*a*b*cos(f*x + e)^2 - (a^2
- b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^2}{(d\sec{(e + fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2), x)
```

3.591 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}+\frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+}{3f(d\sec(e+fx)}$$

```
[Out] -2/15*a*b/f/(d*sec(f*x+e))^(5/2)+2/15*(3*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f
*x+e))^(3/2)+2/5*(3*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x
+e))^(1/2)-2/3*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A]

time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3854, 3856, 2719}

$$\frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*b)/(15*f*(d*Sec[e + f*x])^(5/2)) + (2*(3*a^2 + 2*b^2)*EllipticE[(e +
f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(3*a^2 +
 2*b^2)*Sin[e + f*x])/(15*d*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e +
 f*x]))/(3*f*(d*Sec[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
```

```
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} - \frac{2}{3}\int \frac{-\frac{3a^2}{2}-b^2-\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx\\ &= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} - \frac{1}{3}\left(-3a^2-2b^2\right)\int \frac{}{(d\sec(e\\ &= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}\\ &= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}\\ &= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(3a^2+2b}{15df(d\sec\end{aligned}$$

Mathematica [A]
time = 1.02, size = 92, normalized size = 0.63

$$\frac{(6a^2+4b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)+2\cos^{\frac{3}{2}}(e+fx)\left(-2ab\cos(e+fx)+(a^2-b^2)\sin(e+fx)\right)}{5f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*a^2 + 4*b^2)*EllipticE[(e + f*x)/2, 2] + 2*Cos[e + f*x]^(3/2)*(-2*a*b*C
os[e + f*x] + (a^2 - b^2)*Sin[e + f*x]))/(5*f*Cos[e + f*x]^(5/2)*(d*Sec[e +
 f*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.57, size = 670, normalized size = 4.62

method	result
default	$\frac{6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)\sin(fx+e)a^2}{5}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{El}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a^2+2*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1
)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^2-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*a^2-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^
2+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2+2*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*
sin(f*x+e)*b^2-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2-2*I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/s
in(f*x+e),I)*sin(f*x+e)*b^2-cos(f*x+e)^4*a^2+cos(f*x+e)^4*b^2-2*cos(f*x+e)^
3*sin(f*x+e)*a*b-2*a^2*cos(f*x+e)^2-3*b^2*cos(f*x+e)^2+3*cos(f*x+e)*a^2+2*c
os(f*x+e)*b^2)/cos(f*x+e)^3/sin(f*x+e)/(d/cos(f*x+e))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 147, normalized size = 1.01

$$\frac{\sqrt{2}\,(3i\,a^2+2i\,b^2)\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+\sqrt{2}\,(-3i\,a^2-2i\,b^2)\sqrt{d}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))-2\left(2\,ab\cos(fx+e)^3-(a^2-b^2)\cos(fx+e)^2\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{5\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(3*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-3*I*a^2 - 2*I*b
^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
- I*sin(f*x + e))) - 2*(2*a*b*cos(f*x + e)^3 - (a^2 - b^2)*cos(f*x + e)^2*s
in(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2), x)
```

3.592 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{6ab}{35f(d\sec(e+fx))^{7/2}}+\frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2(5a^2+2b^2)\sin(e\cdot}{35df(d\sec(e+fx}$$

[Out] -6/35*a*b/f/(d*sec(f*x+e))^(7/2)+2/35*(5*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e))^(5/2)+2/21*(5*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+2/21*(5*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^4/f-2/5*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(7/2)

Rubi [A]

time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3854, 3856, 2720}

$$\frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}}-\frac{6ab}{35f(d\sec(e+fx))^{7/2}}-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-6*a*b)/(35*f*(d*Sec[e + f*x])^(7/2)) + (2*(5*a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*(5*a^2 + 2*b^2)*Sin[e + f*x])/(35*d*f*(d*Sec[e + f*x])^(5/2)) + (2*(5*a^2 + 2*b^2)*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))/(5*f*(d*Sec[e + f*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] || NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
```

```
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{2}{5}\int \frac{-\frac{5a^2}{2}-b^2-\frac{3}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{1}{5}\left(-5a^2-2b^2\right)\int \frac{}{(d\sec(e}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx}{21d^3f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx}{21d^3f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\,\mathrm{se}}}{21d^4f}$$

Mathematica [A]
time = 2.46, size = 127, normalized size = 0.69

$$\frac{-18ab\cos(e+fx)-6ab\cos(3(e+fx))+\frac{4(5a^2+2b^2)F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}+23a^2\sin(e+fx)+5b^2\sin(e+fx)+3a^2\sin(3(e+fx))-3b^2\sin(3(e+fx))}{42d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-18*a*b*Cos[e + f*x] - 6*a*b*Cos[3*(e + f*x)] + (4*(5*a^2 + 2*b^2)*Ellipti
cF[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 23*a^2*Sin[e + f*x] + 5*b^2*Sin[e
+ f*x] + 3*a^2*Sin[3*(e + f*x)] - 3*b^2*Sin[3*(e + f*x)])/(42*d^3*f*Sqrt[d*
Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.54, size = 359, normalized size = 1.95

method	result
default	$\frac{\frac{10 i \sqrt{\frac{1}{\cos(fx+e)+1}}\, \sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}, i\right) \cos(fx+e) a^{2}}{21}+\frac{4 i \sqrt{\frac{1}{\cos(fx+e)+1}}\, \sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \mathrm{EllipticF}\left(\frac{i}{}\right.}{21}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/f*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(
f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+2*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(cos(f*x+e)-
1)/sin(f*x+e),I)*b^2+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+2*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*b^2-6*cos(f*x+e)^4*a*b+3*cos(f*x+e)^3*sin(f*x+e)*a^2-3*cos(f*x+e)^3
*sin(f*x+e)*b^2+5*cos(f*x+e)*sin(f*x+e)*a^2+2*cos(f*x+e)*sin(f*x+e)*b^2)/co
s(f*x+e)^4/(d/cos(f*x+e))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 163, normalized size = 0.89

$$\frac{\sqrt{2}\,(-5i\,a^2-2i\,b^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2}\,(5i\,a^2+2i\,b^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))-2\left(6\,ab\cos(fx+e)^4-\left(3\,(a^2-b^2)\cos(fx+e)^3+(5\,a^2+2\,b^2)\cos(fx+e)\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{21\,d^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrass
PInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(6*a*b*cos(f*x + e)^4 -
(3*(a^2 - b^2)*cos(f*x + e)^3 + (5*a^2 + 2*b^2)*cos(f*x + e))*sin(f*x + e))
*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^2}{(d\sec{(e + fx)})^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(7/2),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2), x)
```

3.593 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{10ab}{63f(d\sec(e+fx))^{9/2}}+\frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}}+\frac{2(7a^2+2b^2)\sin(}{45d^3f(d\sec(e+}$$

```
[Out] -10/63*a*b/f/(d*sec(f*x+e))^(9/2)+2/63*(7*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(
f*x+e))^(7/2)+2/45*(7*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(3/2)+2/15
*(7*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))/d^4/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)-2/7*b
*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(9/2)
```

Rubi [A]

time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3589, 3567, 3854, 3856, 2719}

$$\frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}}+\frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}}-\frac{10ab}{63f(d\sec(e+fx))^{9/2}}-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-10*a*b)/(63*f*(d*Sec[e + f*x])^(9/2)) + (2*(7*a^2 + 2*b^2)*EllipticE[(e +
 f*x)/2, 2])/(15*d^4*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(7*a^2
 + 2*b^2)*Sin[e + f*x])/(63*d*f*(d*Sec[e + f*x])^(7/2)) + (2*(7*a^2 + 2*b^2
)*Sin[e + f*x])/(45*d^3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e + f*x
]))/(7*f*(d*Sec[e + f*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
```

```
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{2}{7}\int \frac{-\frac{7a^2}{2}-b^2-\frac{5}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{1}{7}\left(-7a^2-2b^2\right)\int \frac{}{(d\sec(e}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx}{45d^3f(d\sec(e+fx))^{3/}}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx}{45d^3f(d\sec(e+fx))^{3/}}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{15d^4f\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}} + \frac{2(7a^2+2}{63df(d\mathrm{se}}$$

Mathematica [A]

time = 3.08, size = 126, normalized size = 0.68

$$\frac{\frac{48(7a^2+2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} + 4\cos(e+fx)\left(-30ab\cos(e+fx)-10ab\cos(3(e+fx))+2(19a^2-b^2+5(a^2-b^2)\cos(2(e+fx)))\sin(e+fx)\right)}{360d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((48*(7*a^2 + 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*Cos[
e + f*x]*(-30*a*b*Cos[e + f*x] - 10*a*b*Cos[3*(e + f*x)] + 2*(19*a^2 - b^2
+ 5*(a^2 - b^2)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e +
f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.58, size = 697, normalized size = 3.79

method	result
default	$\frac{\frac{14i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)\sin(fx+e)a^2}{15} - 4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(6*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+21*I*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*a^2-21*I*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a
^2+21*I*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2-5*cos(f*x+e)^6*a
^2+5*cos(f*x+e)^6*b^2-10*cos(f*x+e)^5*sin(f*x+e)*a*b-6*I*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*b^2-21*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2+6*I*cos(
f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2-6*I*cos(f*x+e)*sin(f*x+e)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f
*x+e)-1)/sin(f*x+e),I)*b^2-2*cos(f*x+e)^4*a^2-7*cos(f*x+e)^4*b^2-14*a^2*cos
(f*x+e)^2-4*b^2*cos(f*x+e)^2+21*cos(f*x+e)*a^2+6*cos(f*x+e)*b^2)/cos(f*x+e)
^5/sin(f*x+e)/(d/cos(f*x+e))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 173, normalized size = 0.94

$$-\frac{3\sqrt{2}\left(-7i\,a^{2}-2i\,b^{2}\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+3\sqrt{2}\left(7i\,a^{2}+2i\,b^{2}\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)+2\left(10\,ab\cos\left(fx+e\right)^{5}-\left(5\left(a^{2}-b^{2}\right)\cos\left(fx+e\right)^{4}+\left(7\,a^{2}+2\,b^{2}\right)\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{45\,d^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-7*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^2 +
 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e))) + 2*(10*a*b*cos(f*x + e)^5 - (5*(a^2 - b^2)*cos(f*
x + e)^4 + (7*a^2 + 2*b^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e
)))/(d^5*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\tan\left(e+f\,x\right)\right)^2}{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2), x)
```

3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=198

$$\frac{2a(7a^2-6b^2)\,d^2F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}}+\frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\,\tan(e+f}{21f}$$

```
[Out] 2/21*a*(7*a^2-6*b^2)*d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e
))^(1/2)/f/(sec(f*x+e)^2)^(1/4)+2/21*a*(7*a^2-6*b^2)*d^2*(d*sec(f*x+e))^(1/
2)*tan(f*x+e)/f+2/9*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)
)^2/f+2/315*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(154*a^2-28*b^2+65*a*b*
tan(f*x+e))/f
```

Rubi [A]
time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3593, 757, 794, 201, 237}

$$\frac{2ad^2(7a^2-6b^2)\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)}{21f\sqrt[4]{\sec^2(e+fx)}}+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\,(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}+\frac{2ad^2(7a^2-6b^2)\tan(e+fx)\sqrt{d\sec(e+fx)}}{21f}+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (2*a*(7*a^2 - 6*b^2)*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e
+ f*x]])/(21*f*(Sec[e + f*x]^2)^(1/4)) + (2*a*(7*a^2 - 6*b^2)*d^2*Sqrt[d*Se
c[e + f*x]]*Tan[e + f*x])/(21*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f
*x]]*(a + b*Tan[e + f*x])^2)/(9*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e +
 f*x]]*(14*(11*a^2 - 2*b^2) + 65*a*b*Tan[e + f*x]))/(315*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int (a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}\,dx, x, b\tan\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{9f} + \frac{(2}{}$$

$$= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{9f} + \frac{2bd}{}$$

$$= \frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\,\tan(e+fx)}{21f} + \frac{2bd^2\sec^2(e}{}$$

$$= \frac{2a(7a^2-6b^2)\,d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}}$$

Mathematica [A]
time = 1.87, size = 157, normalized size = 0.79

$$-\frac{2d(d\sec(e+fx))^{3/2}\left(63b(-3a^2+b^2)\cos^2(e+fx)-15a(7a^2-6b^2)\cos^{\frac{9}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)-15a(7a^2-6b^2)\cos^3(e+fx)\sin(e+fx)-\frac{5}{2}b^2(14b+27a\sin(2(e+fx)))\right)(a+b\tan(e+fx))^3}{315f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*d*(d*Sec[e + f*x])^(3/2)*(63*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 - 15*a*(7*
a^2 - 6*b^2)*Cos[e + f*x]^(9/2)*EllipticF[(e + f*x)/2, 2] - 15*a*(7*a^2 - 6
*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - (5*b^2*(14*b + 27*a*Sin[2*(e + f*x)]))/
2)*(a + b*Tan[e + f*x])^3)/(315*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.62, size = 414, normalized size = 2.09

method	result
default	$\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(105i(\cos^5(fx+e))\operatorname{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^3-90i(\cos^5\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(105*I*cos(f*x+e)^5*EllipticF(I*(
cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*a^3-90*I*cos(f*x+e)^5*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+105*I*cos(
f*x+e)^4*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3-90*I*cos(f*x+e)^4*EllipticF(I*(cos(f*
x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*a*b^2+105*cos(f*x+e)^3*sin(f*x+e)*a^3-90*cos(f*x+e)^3*sin(f*x+e)*a*b^
2+189*a^2*cos(f*x+e)^2*b-63*b^3*cos(f*x+e)^2+135*cos(f*x+e)*sin(f*x+e)*a*b^
2+35*b^3)*(d/cos(f*x+e))^(5/2)/cos(f*x+e)^2/sin(f*x+e)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 214, normalized size = 1.08

$$\frac{-15i\sqrt{2}\,(7\,a^3-6\,ab^2)d^{\frac{5}{2}}\cos(fx+e)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+15i\sqrt{2}\,(7\,a^3-6\,ab^2)d^{\frac{5}{2}}\cos(fx+e)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e))+2\left(35\,b^3d^2+63\,(3\,a^2b-b^3)d^2\cos(fx+e)^2+15\left(9\,ab^2d^2\cos(fx+e)+(7\,a^3-6\,ab^2)d^2\cos(fx+e)^3\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{315\,f\cos(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(-15*I*sqrt(2)*(7*a^3 - 6*a*b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassPI
nverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*I*sqrt(2)*(7*a^3 - 6*a*b^
2)*d^(5/2)*cos(f*x + e)^4*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)) + 2*(35*b^3*d^2 + 63*(3*a^2*b - b^3)*d^2*cos(f*x + e)^2 + 15*(9*a*
b^2*d^2*cos(f*x + e) + (7*a^3 - 6*a*b^2)*d^2*cos(f*x + e)^3)*sin(f*x + e))*
sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3, x)
```

3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=176

$$-\frac{2a(5a^2-6b^2)\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}}+\frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/}}{5f}$$

```
[Out] -2/5*a*(5*a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(
3/2)/f/(sec(f*x+e)^2)^(3/4)+2/5*a*(5*a^2-6*b^2)*cos(f*x+e)*(d*sec(f*x+e))^(
3/2)*sin(f*x+e)/f+2/7*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2/f+2/105*b*(
d*sec(f*x+e))^(3/2)*(90*a^2-20*b^2+33*a*b*tan(f*x+e))/f
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3593, 757, 794, 233, 202}

$$-\frac{2a(5a^2-6b^2)(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{5f\sec^2(e+fx)^{3/4}}+\frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f}+\frac{2a(5a^2-6b^2)\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{5f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*a*(5*a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])
^(3/2))/(5*f*(Sec[e + f*x]^2)^(3/4)) + (2*a*(5*a^2 - 6*b^2)*Cos[e + f*x]*(d
*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(a +
 b*Tan[e + f*x])^2)/(7*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(10*(9*a^2 - 2*b^2)
 + 33*a*b*Tan[e + f*x]))/(105*f)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
```

```
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$(2b(d\sec(e+fx))$$

$$= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + $$

$$= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{2b(d\sec(e+fx))^{3}}{}$$

$$= \frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} + \frac{2b(}{}$$

$$= -\frac{2a(5a^2-6b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}}$$

Mathematica [A]
time = 1.96, size = 155, normalized size = 0.88

$$-\frac{d\sqrt{d\sec(e+fx)}\left(70b(-3a^2+b^2)\cos^2(e+fx)+42a(5a^2-6b^2)\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-42a(5a^2-6b^2)\cos^3(e+fx)\sin(e+fx)-3b^2(10b+21a\sin(2(e+fx)))\right)(a+b\tan(e+fx))^3}{105f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/105*(d*Sqrt[d*Sec[e + f*x]]*(70*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 + 42*a*(
5*a^2 - 6*b^2)*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x)/2, 2] - 42*a*(5*a^2 -
 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - 3*b^2*(10*b + 21*a*Sin[2*(e + f*x)]))
*(a + b*Tan[e + f*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.60, size = 759, normalized size = 4.31

method	result

default	$-\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(105i(\cos^4(fx+e))\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(126*I*cos(f*x+e)^3*sin(f*x+e)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*a*b^2+105*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(
f*x+e),I)*a^3-126*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+
126*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+105*I*cos(f*x+
e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-126*I*cos(f*x+e)^4*sin(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos
(f*x+e)-1)/sin(f*x+e),I)*a*b^2-105*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin
(f*x+e),I)*a^3-105*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+1
05*cos(f*x+e)^4*a^3-126*cos(f*x+e)^4*a*b^2-105*a^3*cos(f*x+e)^3+189*a*b^2*c
os(f*x+e)^3-105*a^2*cos(f*x+e)^2*b*sin(f*x+e)+35*cos(f*x+e)^2*sin(f*x+e)*b^
3-63*a*cos(f*x+e)*b^2-15*sin(f*x+e)*b^3)*(d/cos(f*x+e))^(3/2)/cos(f*x+e)^2/
sin(f*x+e)^5
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 212, normalized size = 1.20

$$\frac{-21i\sqrt{2}(5a^3-6ab^2)d^{\frac{3}{2}}\cos(fx+e)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+21i\sqrt{2}(5a^3-6ab^2)d^{\frac{3}{2}}\cos(fx+e)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(15b^3d+35(3a^2b-b^3)d\cos(fx+e)^2+21\left(3ab^2d\cos(fx+e)+(5a^3-6ab^2)d\cos(fx+e)^3\right)\sin(fx+e)\right)\sqrt{\frac{d}{\cos(fx+e)}}}{105f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(-21*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I
*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(15*b^3*d + 35
*(3*a^2*b - b^3)*d*cos(f*x + e)^2 + 21*(3*a*b^2*d*cos(f*x + e) + (5*a^3 - 6
*a*b^2)*d*cos(f*x + e)^3)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x +
e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \sec (e + fx))^{\frac{3}{2}} (a + b \tan (e + fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos \left(e+f\,x\right)}\right)}^{3/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3, x)
```

3.596 $\int \sqrt{d \sec(e+fx)}\,(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=129

$$\frac{2a(a^2-2b^2)\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}}+\frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{5f}+2b\sqrt{}$$

```
[Out] 2*a*(a^2-2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*
x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/
f/(sec(f*x+e)^2)^(1/4)+2/5*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/f+2/5*
b*(d*sec(f*x+e))^(1/2)*(14*a^2-4*b^2+3*a*b*tan(f*x+e))/f
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3593, 757, 794, 237}

$$\frac{2a(a^2-2b^2)\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\sqrt[4]{\sec^2(e+fx)}}+\frac{2b\sqrt{d\sec(e+fx)}\,(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f}+\frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (2*a*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]
])/(f*(Sec[e + f*x]^2)^(1/4)) + (2*b*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*
x])^2)/(5*f) + (2*b*Sqrt[d*Sec[e + f*x]]*(2*(7*a^2 - 2*b^2) + 3*a*b*Tan[e +
 f*x]))/(5*f)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^3\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{5f} + \frac{\left(2b\sqrt{d\sec(e+fx)}\right.}{}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}{5f} + \frac{2b\sqrt{d\sec(e+fx)}\,(}{}$$

$$= \frac{2a(a^2-2b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}} + \frac{2}{}$$

Mathematica [A]
time = 2.21, size = 132, normalized size = 1.02

$$-\frac{2\sqrt{d\sec(e+fx)}\left(5b(-3a^2+b^2)\cos^3(e+fx)-5a(a^2-2b^2)\cos^{\frac{7}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)-\frac{1}{2}b^2\cos(e+fx)(2b+5a\sin(2(e+fx)))\right)(a+b\tan(e+fx))^3}{5f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]]*(5*b*(-3*a^2 + b^2)*Cos[e + f*x]^3 - 5*a*(a^2 - 2*
b^2)*Cos[e + f*x]^(7/2)*EllipticF[(e + f*x)/2, 2] - (b^2*Cos[e + f*x]*(2*b
+ 5*a*Sin[2*(e + f*x)]))/2)*(a + b*Tan[e + f*x])^3)/(5*f*(a*Cos[e + f*x] +
b*Sin[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 1.12, size = 373, normalized size = 2.89

method	result
default	$\frac{2(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(5i(\cos^3(fx+e))\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^3-10i(\cos^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(5*I*cos(f*x+e)^3*EllipticF(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*a^3-10*I*cos(f*x+e)^3*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+5*I*cos(f*x+e)
^2*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*a^3-10*I*cos(f*x+e)^2*EllipticF(I*(cos(f*x+e)-1
)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
a*b^2+15*a^2*cos(f*x+e)^2*b-5*b^3*cos(f*x+e)^2+5*cos(f*x+e)*sin(f*x+e)*a*b^
2+b^3)*(d/cos(f*x+e))^(1/2)/cos(f*x+e)^2/sin(f*x+e)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 175, normalized size = 1.36

$$-\frac{5\sqrt{2}\left(i a^3-2 i a b^2\right)\sqrt{d}\cos\left(fx+e\right)^2 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)+5\sqrt{2}\left(-i a^3+2 i a b^2\right)\sqrt{d}\cos\left(fx+e\right)^2 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)-2\left(5 a b^2\cos\left(fx+e\right)\sin\left(fx+e\right)+b^3+5\left(3a^2b-b^3\right)\cos\left(fx+e\right)^2\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{5 f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^3 - 2*I*a*b^2)*sqrt(d)*cos(f*x + e)^2*weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*a^3 + 2*I*a*b^2)*
sqrt(d)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e)) - 2*(5*a*b^2*cos(f*x + e)*sin(f*x + e) + b^3 + 5*(3*a^2*b - b^3)*cos(
f*x + e)^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sec (e + fx)}\, (a + b \tan (e + fx))^3 \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{d}{\cos (e + f\,x)}}\; (a + b \tan(e + f\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3, x)
```

3.597 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=178

$$\frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}}-\frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}-\frac{2(b-a\tan(e+fx}{f\sqrt{d\,\text{se}}}$$

```
[Out] 2*a*(a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*
x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/
f/(d*sec(f*x+e))^(1/2)-2*a*(a^2-6*b^2)*tan(f*x+e)/f/(d*sec(f*x+e))^(1/2)-2*
(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*sec(f*x+e))^(1/2)-2/3*b*sec(f*x+e)
^2*(6*a^2-4*b^2+3*a*b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3593, 753, 794, 233, 202}

$$\frac{2a(a^2-6b^2)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\sqrt{d\sec(e+fx)}}-\frac{2b\sec^2(e+fx)\left(2(3a^2-2b^2)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}}-\frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}-\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*a*(a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(f*Sqrt[d*Sec[e + f*x]]) - (2*a*(a^2 - 6*b^2)*Tan[e + f*x])/(f*Sqrt[d*
Sec[e + f*x]]) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e + f*x])^2)/(f*Sqrt[d*
Sec[e + f*x]]) - (2*b*Sec[e + f*x]^2*(2*(3*a^2 - 2*b^2) + 3*a*b*Tan[e + f*x
]))/(3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
```

```
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\right.}{f\sqrt{\,}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\,(2(3a^2-2b^2)+}{3f\sqrt{d\sec(e+f}}$$

$$= -\frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b}{}$$

$$= \frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} - \frac{2a(a^2-6b^2)\tan(}{f\sqrt{d\sec(e+}}$$

Mathematica [A]
time = 2.06, size = 130, normalized size = 0.73

$$\frac{d\left(6a(a^2-6b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+b(-9a^2+5b^2+(-9a^2+3b^2)\cos(2(e+fx))+9ab\sin(2(e+fx)))\right)(a+b\tan(e+fx))^3}{3f(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (d*(6*a*(a^2 - 6*b^2)*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2, 2] + b*(-9*
a^2 + 5*b^2 + (-9*a^2 + 3*b^2)*Cos[2*(e + f*x)] + 9*a*b*Sin[2*(e + f*x)]))*
(a + b*Tan[e + f*x])^3)/(3*f*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.62, size = 3065, normalized size = 17.22

method	result	size
default	Expression too large to display	3065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)-1)^2*(12*I*cos(f*x+e)^5*sin(f*x+e)*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3-12*I*cos(f*x+e)^5*sin(f*x+e)*El
lipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3+48*I*cos(f*
x+e)^4*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*a^3-48*I*cos(f*x+e)^4*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e)
,I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*a^3+72*I*cos(f*x+e)^3*sin(f*x+e)*EllipticF(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3-72*I*cos(f*x+e)^3*sin(f*x+
e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3+48*I*c
os(f*x+e)^2*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*a^3-48*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+12*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-12*I*cos(f*x+e)*sin(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+4*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^3*sin(f*x+e)+24*cos(f*x+e)^3*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*a^3-12*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(3/2)*a^3+12*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^3-24*co
s(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^3-288*I*cos(f*x+e)^4*sin(
f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+
288*I*cos(f*x+e)^4*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*a*b^2-432*I*cos(f*x+e)^3*sin(f*x+e)*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+432*I*cos(f*x+e)^3*sin(f*x+e)
*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-288*I*
cos(f*x+e)^2*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*a*b^2+288*I*cos(f*x+e)^2*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/s
in(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*
```

```
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-72*I*cos(f*x+e)^5*sin(f*x+e)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+72*I*cos(f*x+
e)^5*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*a*b^2-72*I*cos(f*x+e)*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*a*b^2+72*I*cos(f*x+e)*sin(f*x+e)*EllipticE(I*(cos(f*
x+e)-1)/sin(f*x+e),I)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-36*cos(f*x+e)^5*sin(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2*b+9*cos(f*x+e)^3*sin(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*a^2*b-9*cos(f*x+
e)^3*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)*a^2*b-108*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*a^2*b-36*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^
2*b-108*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a^2*b+
36*cos(f*x+e)^6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a*b^2+3*cos(f*x+e)^3*s
in(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
*b^3+40*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^3-72
*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*a*b^2+24*cos(f*x+e)^2*si
n(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^3+36*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*a*b^2+12*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*b^3+36*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*a*b^2+12*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 180, normalized size = 1.01

$$-\frac{3\sqrt{2}\left(-i a^{3}+6 i a b^{2}\right)\sqrt{d}\cos\left(fx+e\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+3\sqrt{2}\left(i a^{3}-6 i a b^{2}\right)\sqrt{d}\cos\left(fx+e\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)-2\left(9 a b^{2}\cos\left(fx+e\right)\sin\left(fx+e\right)+b^{3}-3\left(3 a^{2} b-b^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3 d f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

[Out] -1/3*(3*sqrt(2)*(-I*a^3 + 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(I*a^3 - 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(9*a*b^2*cos(f*x + e)*sin(f*x + e) + b^3 - 3*(3*a^2*b - b^3)*cos(f*x + e)^2)*sqrt(d/cos(f*x + e)))/(d*f*cos(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^3}{\sqrt{d\sec{(e + fx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(1/2),x)

[Out] Integral((a + b*tan(e + f*x))**3/sqrt(d*sec(e + f*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^3}{\sqrt{\frac{d}{\cos{(e + f\,x)}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2),x)

[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2), x)

3.598 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{2a(a^2+6b^2)\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}}-\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}}-\frac{2b\text{s}}{}$$

```
[Out] 2/3*a*(a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(
f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4
)/f/(d*sec(f*x+e))^(3/2)-2/3*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*sec(f
*x+e))^(3/2)-2/3*b*sec(f*x+e)^2*(2*a^2-4*b^2+a*b*tan(f*x+e))/f/(d*sec(f*x+e
))^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3593, 753, 794, 237}

$$\frac{2a(a^2+6b^2)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)}{3f(d\sec(e+fx))^{3/2}}-\frac{2b\sec^2(e+fx)\,(2(a^2-2b^2)+ab\tan(e+fx))}{3f(d\sec(e+fx))^{3/2}}-\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3
/4))/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e +
f*x])^2)/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*Sec[e + f*x]^2*(2*(a^2 - 2*b^2
) + a*b*Tan[e + f*x]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+\cdots}{\cdots}\right.}{3f(d\sec\cdots}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\cdots\right.}{3f(d\sec(e+fx))^{3/\cdots}}$$

$$= \frac{2a(a^2+6b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}} - \frac{2(b-a\tan(e+fx\cdots}{3f(d\sec(\cdots}$$

Mathematica [A]
time = 1.46, size = 117, normalized size = 0.80

$$\frac{\sec^2(e+fx)\left(-3a^2b+7b^3+(-3a^2b+b^3)\cos(2(e+fx))+2a(a^2+6b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)+a^3\sin(2(e+fx))-3ab^2\sin(2(e+fx))\right)}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-3*a^2*b + 7*b^3 + (-3*a^2*b + b^3)*Cos[2*(e + f*x)] + 2*a
*(a^2 + 6*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^3*Sin[2*(e
+ f*x)] - 3*a*b^2*Sin[2*(e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.75, size = 342, normalized size = 2.34

method	result
default	$\frac{\frac{2i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a^3}{3}+4i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,\mathrm{EllipticF}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a^3+6*I*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),
I)*cos(f*x+e)*a*b^2+I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+6*I*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e
),I)*a*b^2-3*a^2*cos(f*x+e)^2*b+b^3*cos(f*x+e)^2+cos(f*x+e)*sin(f*x+e)*a^3-
3*cos(f*x+e)*sin(f*x+e)*a*b^2+3*b^3)/cos(f*x+e)^2/(d/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 156, normalized size = 1.07

$$\frac{\sqrt{2}\,(-i\,a^3-6i\,ab^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2}\,(i\,a^3+6i\,ab^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e))+2\,(3\,b^3-(3\,a^2b-b^3)\cos(fx+e)^2+(a^3-3\,ab^2)\cos(fx+e)\sin(fx+e))\sqrt{\frac{d}{\cos(fx+e)}}}{3\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*
x + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(d)*weierstrassP
Inverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(3*b^3 - (3*a^2*b - b^3)*
cos(f*x + e)^2 + (a^3 - 3*a*b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x
+ e)))/(d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^3}{(d\sec(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2), x)
```

3.599 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{2\cos^2(e+fx)(b-}{5d}$$

```
[Out] 6/5*a*(a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(
f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4
)/d^2/f/(d*sec(f*x+e))^(1/2)-6/5*a*(a^2+2*b^2)*tan(f*x+e)/d^2/f/(d*sec(f*x+
e))^(1/2)-2/5*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/d^2/f/(d*sec
(f*x+e))^(1/2)-2/5*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*tan(f*x+e))/d^2/f/(d*se
c(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3593, 753, 792, 233, 202}

$$\frac{6a(a^2+2b^2)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*a*(a^2 + 2*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (6*a*(a^2 + 2*b^2)*Tan[e + f*x])/(5*d
^2*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^2*(b - a*Tan[e + f*x])*(a + b*
Tan[e + f*x])^2)/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*(2*b*(a^2 + 2*b^2) - a
*(3*a^2 + 5*b^2)*Tan[e + f*x]))/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
```

```
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right.}{}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(}{5d^2f\sqrt{d}}$$

$$= -\frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+}{5d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{6a(a^2+2b^2)\tan(e}{5d^2f\sqrt{d\sec(e+}}$$

Mathematica [A]
time = 1.58, size = 150, normalized size = 0.74

$$\frac{\sqrt{d\sec(e+fx)}\left(-b(9a^2+17b^2)\cos(e+fx)-3a^2b\cos(3(e+fx))+b^3\cos(3(e+fx))+12a(a^2+2b^2)\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)+a^3\sin(e+fx)-3ab^2\sin(e+fx)+a^3\sin(3(e+fx))-3ab^2\sin(3(e+fx))\right)}{10d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Sec[e + f*x]]*(-(b*(9*a^2 + 17*b^2)*Cos[e + f*x]) - 3*a^2*b*Cos[3*(
e + f*x)] + b^3*Cos[3*(e + f*x)] + 12*a*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2] + a^3*Sin[e + f*x] - 3*a*b^2*Sin[e + f*x] + a^3*Sin
[3*(e + f*x)] - 3*a*b^2*Sin[3*(e + f*x)]))/(10*d^3*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.58, size = 1923, normalized size = 9.43

method	result	size
default	Expression too large to display	1923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(cos(f*x+e)+1)^3*(cos(f*x+e)-1)^2*(12*I*cos(f*x+e)^2*sin(f*x+e)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-24*I*cos(f*x+
e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)
*a^3+24*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*a^3-24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)
/sin(f*x+e),I)*a*b^2*sin(f*x+e)+24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*a*b^2*sin(f*x+e)-12*I*cos(f*x+e)^2*sin(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+8*cos(f*x+e)^3
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3+4*cos(f*x+e)^5*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*a^3-4*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
a^3-12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3-12*I*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3*sin(f*x+e)+12*I*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3*sin(f*x+e)+4*
cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3-24*I*cos(f*x+e)^2*sin
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
```

```
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2
+24*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-
1)/sin(f*x+e),I)*a*b^2-48*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*Elli
pticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+48*I*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2-12*cos(f*x+e
)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2-4*cos(f*x+e)^4*sin(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^3-12*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*a*b^2-4*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*b^3+36*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2+2
0*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^3+12*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2+20*cos(f*x+e)*sin(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^3+5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*b^3*sin(f*x+e)-5*b^3*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+
e)-24*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2+12*cos(f*x+e)^4
*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b+12*cos(f*x+e)^3*sin(
f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b)*cos(f*x+e)*(d/cos(f*x+e)
)^(5/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/d^5/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 173, normalized size = 0.85

$$-\frac{3\sqrt{2}\left(-i\,a^3-2i\,ab^2\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+3\sqrt{2}\left(i\,a^3+2i\,ab^2\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)+2\left(5\,b^3\cos\left(fx+e\right)+\left(3\,a^2b-b^3\right)\cos\left(fx+e\right)^3-\left(a^3-3\,ab^2\right)\cos\left(fx+e\right)^2\sin\left(fx+e\right)\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{5\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(2)*(-I*a^3 - 2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(I*a^3 + 2*
I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
+ e) - I*sin(f*x + e))) + 2*(5*b^3*cos(f*x + e) + (3*a^2*b - b^3)*cos(f*x +
```

```
e)^3 - (a^3 - 3*a*b^2)*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/
(d^3*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^3}{(d\sec{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(5/2),x)
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\tan(e + f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2), x)
```

3.600 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2a(5a^2+6b^2)\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e-$$

```
[Out] 2/21*a*(5*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(
3/4)/d^2/f/(d*sec(f*x+e))^(3/2)-2/7*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*tan(
f*x+e))^2/d^2/f/(d*sec(f*x+e))^(3/2)-2/21*(2*b*(3*a^2+2*b^2)-a*(5*a^2+3*b^2
)*tan(f*x+e))/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3593, 753, 792, 237}

$$\frac{2a(5a^2+6b^2)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(3/4))/(21*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^2*(b - a*Tan[e +
 f*x])*(a + b*Tan[e + f*x])^2)/(7*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*(2*b*(
3*a^2 + 2*b^2) - a*(5*a^2 + 3*b^2)*Tan[e + f*x]))/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{11/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} + \frac{(2b\sec^2(e+fx)^{3/}}{}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} - \frac{2(2b(3a^2+2b^2)-}{21d^2f(d}$$

$$= \frac{2a(5a^2+6b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)(b}{7}$$

Mathematica [A]

time = 2.82, size = 150, normalized size = 0.88

$$\frac{\sqrt{\cos(e+fx)}\,\sqrt{d\sec(e+fx)}\,\left(4(5a^3+6ab^2)\,F\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(-b(27a^2+19b^2)\cos(e+fx)+(-9a^2b+3b^3)\cos(3(e+fx))+2a(13a^2+3b^2+3(a^2-3b^2)\cos(2(e+fx)))\sin(e+fx))\right)}{42d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]*(4*(5*a^3 + 6*a*b^2)*EllipticF[(e
+ f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(-(b*(27*a^2 + 19*b^2)*Cos[e + f*x]) + (-
9*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 2*a*(13*a^2 + 3*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(e + f*x)])*Sin[e + f*x]))/(42*d^4*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.56, size = 391, normalized size = 2.30

method	result
default	$-\frac{2\left(-5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\operatorname{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)a^{3}-6i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\text{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/f*(-5*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a^3-6*I*cos(f*x+e)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*a*b^2+9*cos(f*x+e)^4*a^2*b-3*cos(f*x+e)^4*b^3-3*cos(f*x+
e)^3*sin(f*x+e)*a^3+9*cos(f*x+e)^3*sin(f*x+e)*a*b^2-5*I*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+
e),I)*a^3-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+7*b^3*cos(f*x+e)^2-5*cos(f*x+e
)*sin(f*x+e)*a^3-6*cos(f*x+e)*sin(f*x+e)*a*b^2)/cos(f*x+e)^4/(d/cos(f*x+e))
^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 191, normalized size = 1.12

$$\frac{\sqrt{2}\left(-5i\,a^{3}-6i\,ab^{2}\right)\sqrt{d}\,\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+\sqrt{2}\left(5i\,a^{3}+6i\,ab^{2}\right)\sqrt{d}\,\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)-2\left(7\,b^{3}\cos\left(fx+e\right)^{2}+3\left(3\,a^{2}b-b^{3}\right)\cos\left(fx+e\right)^{4}-\left(3\left(a^{3}-3\,ab^{2}\right)\cos\left(fx+e\right)^{3}+\left(5\,a^{3}+6\,ab^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{21\,d^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^3 + 6*I*a*b^2)*sqrt(d)*weierst
rassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(7*b^3*cos(f*x + e)^
2 + 3*(3*a^2*b - b^3)*cos(f*x + e)^4 - (3*(a^3 - 3*a*b^2)*cos(f*x + e)^3 +
(5*a^3 + 6*a*b^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^3}{(d\sec(e + fx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(7/2),x)
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2), x)
```

3.601 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2a(7a^2+6b^2)\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}}-\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e$$

```
[Out] 2/15*a*(7*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(
1/4)/d^4/f/(d*sec(f*x+e))^(1/2)-2/9*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*tan(
f*x+e))^2/d^4/f/(d*sec(f*x+e))^(1/2)-2/45*cos(f*x+e)^2*(2*b*(5*a^2+2*b^2)-a
*(7*a^2+b^2)*tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3593, 753, 792, 202}

$$\frac{2a(7a^2+6b^2)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{15d^4f\sqrt{d\sec(e+fx)}}-\frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}-\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*a*(7*a^2 + 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(1/4))/(15*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(9*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*
x]^2*(2*b*(5*a^2 + 2*b^2) - a*(7*a^2 + b^2)*Tan[e + f*x]))/(45*d^4*f*Sqrt[d
*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{13/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right.}{}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\,(2b}{4}$$

$$= \frac{2a(7a^2+6b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 372 vs. 2(176) = 352.
time = 6.43, size = 372, normalized size = 2.11

$$\frac{\sec^{\frac{3}{2}}(e+fx)\left(\frac{2(56a^3+48ab^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}\sqrt{\sec(e+fx)}}-\frac{2(15a^2b+7b^3)\sin^2(e+fx)}{\sqrt{1-\cos^2(e+fx)}\sqrt{\sec(e+fx)}\sqrt{\cos^2(e+fx)(-1+\sec^2(e+fx))}}\right)(a+b\tan(e+fx))^3}{120f(d\sec(e+fx))^{9/2}(a\cos(e+fx)+b\sin(e+fx))^3}+\frac{\sec^2(e+fx)\left(-\frac{1}{90}b(15a^2+4b^2)\cos(e+fx)-\frac{1}{360}b(75a^2+11b^2)\cos(3(e+fx))-\frac{1}{72}b(3a^2-b^2)\cos(5(e+fx))+\frac{1}{180}a(19a^2-3b^2)\sin(e+fx)+\frac{1}{360}a(43a^2-21b^2)\sin(3(e+fx))+\frac{1}{72}a(a^2-3b^2)\sin(5(e+fx))\right)(a+b\tan(e+fx))^3}{f(d\sec(e+fx))^{9/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*((2*(56*a^3 + 48*a*b^2)*EllipticE[(e + f*x)/2, 2])/(Sqr
t[Cos[e + f*x]]*Sqrt[Sec[e + f*x]]) - (2*(15*a^2*b + 7*b^3)*Sin[e + f*x]^2)
/(Sqrt[1 - Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x]^2*(-1 + Sec
```

```
[e + f*x]^2)]))*(a + b*Tan[e + f*x])^3)/(120*f*(d*Sec[e + f*x])^(9/2)*(a*Co
s[e + f*x] + b*Sin[e + f*x])^3) + (Sec[e + f*x]^2*(-1/90*(b*(15*a^2 + 4*b^2
)*Cos[e + f*x]) - (b*(75*a^2 + 11*b^2)*Cos[3*(e + f*x)])/360 - (b*(3*a^2 -
b^2)*Cos[5*(e + f*x)])/72 + (a*(19*a^2 - 3*b^2)*Sin[e + f*x])/180 + (a*(43*
a^2 - 21*b^2)*Sin[3*(e + f*x)])/360 + (a*(a^2 - 3*b^2)*Sin[5*(e + f*x)])/72
)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(9/2)*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.68, size = 745, normalized size = 4.23

method	result
default	$\frac{14i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\;\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\cos(fx+e)\sin(fx+e)a^3}{15}-\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\;\mathrm{E}}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^3-21*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*cos(f*x+e)*sin(f*x+e)*a^3+18*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*
sin(f*x+e)*a*b^2-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^3-5*cos(f*x+e)^
6*a^3+15*cos(f*x+e)^6*a*b^2-15*cos(f*x+e)^5*sin(f*x+e)*a^2*b+5*cos(f*x+e)^5
*sin(f*x+e)*b^3-18*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a*b^2-18*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a*b^2+21*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)
*cos(f*x+e)*sin(f*x+e)*a^3+18*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a*b^2-2
*cos(f*x+e)^4*a^3-21*cos(f*x+e)^4*a*b^2-9*cos(f*x+e)^3*sin(f*x+e)*b^3-14*co
s(f*x+e)^2*a^3-12*cos(f*x+e)^2*a*b^2+21*a^3*cos(f*x+e)+18*a*cos(f*x+e)*b^2)
/cos(f*x+e)^5/sin(f*x+e)/(d/cos(f*x+e))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 201, normalized size = 1.14

$$-\frac{3\sqrt{2}\left(-7i\,a^3-6i\,ab^2\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+3\sqrt{2}\left(7i\,a^3+6i\,ab^2\right)\sqrt{d}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)+2\left(9\,b^3\cos\left(fx+e\right)^3+5\left(3\,a^2b-b^3\right)\cos\left(fx+e\right)^5-\left(5\left(a^3-3\,ab^2\right)\cos\left(fx+e\right)^4+\left(7\,a^3+6\,ab^2\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)\right)\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{45\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-7*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^3
 + 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) - I*sin(f*x + e))) + 2*(9*b^3*cos(f*x + e)^3 + 5*(3*a^2*b - b^3)*
cos(f*x + e)^5 - (5*(a^3 - 3*a*b^2)*cos(f*x + e)^4 + (7*a^3 + 6*a*b^2)*cos(
f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^5*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\tan\left(e+f\,x\right)\right)}^3}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2), x)
```

3.602 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=218

$$\frac{10a(3a^2+2b^2)\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}}+\frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}}-\frac{2\cos^4(e+fx)(b}{1}$$

```
[Out] 10/77*a*(3*a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^
(3/4)/d^4/f/(d*sec(f*x+e))^(3/2)+10/77*a*(3*a^2+2*b^2)*tan(f*x+e)/d^4/f/(d*
sec(f*x+e))^(3/2)-2/11*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/d^4
/f/(d*sec(f*x+e))^(3/2)-2/77*cos(f*x+e)^2*(2*b*(7*a^2+2*b^2)-a*(9*a^2-b^2)*
tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3593, 753, 792, 205, 237}

$$\frac{10a(3a^2+2b^2)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|\,2\right)}{77d^4f(d\sec(e+fx))^{3/2}}+\frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}-\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)
^(3/4))/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) + (10*a*(3*a^2 + 2*b^2)*Tan[e + f
*x])/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(11*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e +
 f*x]^2*(2*b*(7*a^2 + 2*b^2) - a*(9*a^2 - b^2)*Tan[e + f*x]))/(77*d^4*f*(d*
Sec[e + f*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{15/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} + \frac{(2b\sec^2(e+fx)^{3/}}{}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\,(2b}{7}$$

$$= \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e}{11d^4f(d\sec(e+fx))^{3/2}}$$

$$= \frac{10a(3a^2+2b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)}{77d^4f(d\sec(e}$$

Mathematica [A]
time = 6.50, size = 296, normalized size = 1.36

$$\frac{10a(3a^2+2b^2)\,F\left(\frac{1}{2}(e+fx)\middle|2\right)(a+b\tan(e+fx))^3}{77f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{11/2}(a\cos(e+fx)+b\sin(e+fx))^3}+\frac{\sec^3(e+fx)\left(-\frac{1}{616}b(105a^2+31b^2)-\frac{b(315a^2+71b^2)\cos(2(e+fx))}{1232}-\frac{1}{616}b(63a^2+b^2)\cos(4(e+fx))-\frac{1}{176}b(3a^2-b^2)\cos(6(e+fx))+\frac{a(347a^2+103b^2)\sin(2(e+fx))}{1232}+\frac{1}{308}a(16a^2-15b^2)\sin(4(e+fx))+\frac{1}{176}a(a^2-3b^2)\sin(6(e+fx))\right)(a+b\tan(e+fx))^3}{f(d\sec(e+fx))^{11/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[(e + f*x)/2, 2]*(a + b*Tan[e + f*x])^3)/(77
*f*Cos[e + f*x]^(5/2)*(d*Sec[e + f*x])^(11/2)*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3) + (Sec[e + f*x]^3*(-1/616*(b*(105*a^2 + 31*b^2)) - (b*(315*a^2 + 71
*b^2)*Cos[2*(e + f*x)])/1232 - (b*(63*a^2 + b^2)*Cos[4*(e + f*x)])/616 - (b
*(3*a^2 - b^2)*Cos[6*(e + f*x)])/176 + (a*(347*a^2 + 103*b^2)*Sin[2*(e + f*
x)])/1232 + (a*(16*a^2 - 15*b^2)*Sin[4*(e + f*x)])/308 + (a*(a^2 - 3*b^2)*S
in[6*(e + f*x)])/176)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(11/2)*(a
*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.64, size = 430, normalized size = 1.97

method	result
default	$-\frac{2\left(21(\cos^6(fx+e))a^2b-7(\cos^6(fx+e))b^3-7(\cos^5(fx+e))\sin(fx+e)a^3+21(\cos^5(fx+e))\sin(fx+e)a\,b^2-15i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\;\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/f*(21*cos(f*x+e)^6*a^2*b-7*cos(f*x+e)^6*b^3-7*cos(f*x+e)^5*sin(f*x+e)
*a^3+21*cos(f*x+e)^5*sin(f*x+e)*a*b^2-15*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x
+e)*a^3-10*I*cos(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+11*cos(f*x+e)^4*b^3-
9*cos(f*x+e)^3*sin(f*x+e)*a^3-6*cos(f*x+e)^3*sin(f*x+e)*a*b^2-15*I*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-
1)/sin(f*x+e),I)*a^3-10*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-15*cos(f*x+e)*sin(
f*x+e)*a^3-10*cos(f*x+e)*sin(f*x+e)*a*b^2)/cos(f*x+e)^6/(d/cos(f*x+e))^(11/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 217, normalized size = 1.00

$$-\frac{5\sqrt{2}\,(3i\,a^3+2i\,ab^2)\sqrt{d}\ \mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+5\sqrt{2}\,(-3i\,a^3-2i\,ab^2)\sqrt{d}\ \mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e))+2\,(11\,b^3\cos(fx+e)^4+7\,(3\,a^2b-b^3)\cos(fx+e)^6-(7\,(a^3-3\,ab^2)\cos(fx+e)^5+3\,(3\,a^3+2\,ab^2)\cos(fx+e)^3+5\,(3\,a^3+2\,ab^2)\cos(fx+e))\sin(fx+e))\sqrt{\frac{d}{\cos(fx+e)}}}{77\,d^6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/77*(5*sqrt(2)*(3*I*a^3 + 2*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, c
os(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-3*I*a^3 - 2*I*a*b^2)*sqrt(d)*we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(11*b^3*cos(f*x
 + e)^4 + 7*(3*a^2*b - b^3)*cos(f*x + e)^6 - (7*(a^3 - 3*a*b^2)*cos(f*x + e
)^5 + 3*(3*a^3 + 2*a*b^2)*cos(f*x + e)^3 + 5*(3*a^3 + 2*a*b^2)*cos(f*x + e)
)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^6*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^3}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2),x)
```

```
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2), x)
```

3.603 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=456

$$\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}+\frac{\left(a^2+b^2\right)^{3/4}d^2\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}-\frac{\left(a^2+b^2\right)^{3/4}d^2\tanh$$

[Out] 2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f+(a^2+b^2)^(3/4)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2)^(3/4)-(a^2+b^2)^(3/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2)^(3/4)+2*a*d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/4)-2*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f-a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)

Rubi [A]
time = 0.30, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 749, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{ad^2\sqrt{a^2+b^2}\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{b^3f\sec^2(e+fx)^{3/4}}+\frac{ad^2\sqrt{a^2+b^2}\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{b^3f\sec^2(e+fx)^{3/4}}+\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}-\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}+\frac{2ad^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{b^2f\sec^2(e+fx)^{3/4}}-\frac{2ad^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{b^2f}+\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}$$

Antiderivative was successfully verified.

[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]

[Out] (2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f) + ((a^2 + b^2)^(3/4)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - ((a^2 + b^2)^(3/4)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) + (2*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/(b^2*f*(Sec[e + f*x]^2)^(3/4)) - (2*a*d^2*Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(b^2*f) - (a*Sqrt[a^2 + b^2]*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b^3*f*(Sec[e + f*x]^2)^(3/4)) + (a*Sqrt[a^2 + b^2]*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b^3*f*(Sec[e + f*x]^2)^(3/4))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{ax}{b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\,\text{ta}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} - \frac{\left(ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(\right.}{b^3f\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} - \frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} + \frac{(ad^2(d}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2a}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2a}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2a}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{(a^2+b^2)^{3/4}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+f}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 8039 vs. 2(456) = 912.
time = 92.79, size = 8039, normalized size = 17.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26370 vs. $2(421) = 842$.
time = 1.03, size = 26371, normalized size = 57.83

method	result	size
default	Expression too large to display	26371

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{a+b\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)), x)
```

3.604 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=396

$$\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}-\frac{\sqrt[4]{a^2+b^2}\,d^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt[4]{a^2+b^2}\,d^2\tanh^{-1}\left(\right.}{b^3}$$

```
[Out] 2*d^2*(d*sec(f*x+e))^(1/2)/b/f-(a^2+b^2)^(1/4)*d^2*arctan((sec(f*x+e)^2)^(1
/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^
(1/4)-(a^2+b^2)^(1/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^(1/4)-2*a*d^2*(cos(1/2*a
rctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*a
rctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*
sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)+a*d^2*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e)
)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
```

Rubi [A]
time = 0.28, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 749, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{ad^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}-\frac{2ad^2\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}+\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f) - ((a^2 + b^2)^(1/4)*d^2*ArcTan[(Sqrt[b]
*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*
f*(Sec[e + f*x]^2)^(1/4)) - ((a^2 + b^2)^(1/4)*d^2*ArcTanh[(Sqrt[b]*(Sec[e
+ f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*f*(Sec[e
 + f*x]^2)^(1/4)) - (2*a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Se
c[e + f*x]])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e +
f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sq
rt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e\right.}{b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{((1}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{((1}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{(2(}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{((1}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+f}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+f}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2853 vs.

2(396) = 792.
time = 67.63, size = 2853, normalized size = 7.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*Cos[e + f*x]*((Sqrt[a^2 + b^2]*Sqrt[b - Sqrt[a^2 + b^2]
]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e +
 f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^
2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])])/Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2
])] - (Sqrt[a^2 + b^2]*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2
+ b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt
[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e +
 f*x)/2]^4])])/Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])] + (4*b^(3/2)*EllipticF[A
rcSin[Tan[(e + f*x)/2]], -1])/a + (4*Sqrt[b^2*(a^2 + b^2)]*EllipticPi[a^2/(
a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1])/(a*S
qrt[b]) - (4*a*b^(3/2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)]
)), ArcSin[Tan[(e + f*x)/2]], -1])/Sqrt[b^2*(a^2 + b^2)] - (4*b^(7/2)*Ellip
ticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]]
, -1])/(a*Sqrt[b^2*(a^2 + b^2)]))*(d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] +
b*Sin[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - Tan[(e + f*x)/
2]^2]*(-a^2 - 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)] + a^2*Tan[(e + f*x)/2]^2)*(-a
^2 - 2*b^2 + 2*Sqrt[b^2*(a^2 + b^2)] + a^2*Tan[(e + f*x)/2]^2)*(1 - Tan[(e
+ f*x)/2]^4)*(a^4 + 8*a^2*b^2 + 8*b^4 + 4*a^2*b*Sqrt[a^2 + b^2] + 8*b^3*Sqr
t[a^2 + b^2] - 2*a^4*Tan[(e + f*x)/2]^2 - 4*a^2*b^2*Tan[(e + f*x)/2]^2 - 4*
a^2*b*Sqrt[a^2 + b^2]*Tan[(e + f*x)/2]^2 + a^4*Tan[(e + f*x)/2]^4)*(a^4 + 8
*a^2*b^2 + 8*b^4 - 4*a^2*b*Sqrt[a^2 + b^2] - 8*b^3*Sqrt[a^2 + b^2] - 2*a^4*
Tan[(e + f*x)/2]^2 - 4*a^2*b^2*Tan[(e + f*x)/2]^2 + 4*a^2*b*Sqrt[a^2 + b^2]
*Tan[(e + f*x)/2]^2 + a^4*Tan[(e + f*x)/2]^4)*(b - a*Tan[e + f*x]))/(2*a^6*
b^(3/2)*f*Sqrt[-(-1 + Tan[(e + f*x)/2]^2)^(-1)]*(a^5*b*Sqrt[1 + Tan[(e + f*
x)/2]^2] - 3*a^5*b*Tan[(e + f*x)/2]^2*Sqrt[1 + Tan[(e + f*x)/2]^2] - 8*a^3*
b^3*Tan[(e + f*x)/2]^2*Sqrt[1 + Tan[(e + f*x)/2]^2] + a^5*b*Tan[(e + f*x)/2
]^4*Sqrt[1 + Tan[(e + f*x)/2]^2] + 8*a^3*b^3*Tan[(e + f*x)/2]^4*Sqrt[1 + Ta
n[(e + f*x)/2]^2] + 16*a*b^5*Tan[(e + f*x)/2]^4*Sqrt[1 + Tan[(e + f*x)/2]^2
] + 5*a^5*b*Tan[(e + f*x)/2]^6*Sqrt[1 + Tan[(e + f*x)/2]^2] + 16*a^3*b^3*Ta
n[(e + f*x)/2]^6*Sqrt[1 + Tan[(e + f*x)/2]^2] + 16*a*b^5*Tan[(e + f*x)/2]^6
*Sqrt[1 + Tan[(e + f*x)/2]^2] - 5*a^5*b*Tan[(e + f*x)/2]^8*Sqrt[1 + Tan[(e
+ f*x)/2]^2] - 16*a^3*b^3*Tan[(e + f*x)/2]^8*Sqrt[1 + Tan[(e + f*x)/2]^2] -
 16*a*b^5*Tan[(e + f*x)/2]^8*Sqrt[1 + Tan[(e + f*x)/2]^2] - a^5*b*Tan[(e +
f*x)/2]^10*Sqrt[1 + Tan[(e + f*x)/2]^2] - 8*a^3*b^3*Tan[(e + f*x)/2]^10*Sqr
t[1 + Tan[(e + f*x)/2]^2] - 16*a*b^5*Tan[(e + f*x)/2]^10*Sqrt[1 + Tan[(e +
f*x)/2]^2] + 3*a^5*b*Tan[(e + f*x)/2]^12*Sqrt[1 + Tan[(e + f*x)/2]^2] + 8*a
^3*b^3*Tan[(e + f*x)/2]^12*Sqrt[1 + Tan[(e + f*x)/2]^2] - a^5*b*Tan[(e + f*
```

```
x)/2]^14*Sqrt[1 + Tan[(e + f*x)/2]^2] - 2*a^6*Tan[(e + f*x)/2]*Sqrt[1 - Tan
[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 2*a^4*b^2*Tan[(e + f*x)/2]*
Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 10*a^6*Tan[(e +
 f*x)/2]^3*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 26*a
^4*b^2*Tan[(e + f*x)/2]^3*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*
x)/2]^4] + 16*a^2*b^4*Tan[(e + f*x)/2]^3*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[
1 - Tan[(e + f*x)/2]^4] - 20*a^6*Tan[(e + f*x)/2]^5*Sqrt[1 - Tan[(e + f*x)/
2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 68*a^4*b^2*Tan[(e + f*x)/2]^5*Sqrt[1 -
 Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 80*a^2*b^4*Tan[(e + f*x
)/2]^5*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 32*b^6*T
an[(e + f*x)/2]^5*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4]
 + 20*a^6*Tan[(e + f*x)/2]^7*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e +
 f*x)/2]^4] + 68*a^4*b^2*Tan[(e + f*x)/2]^7*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sq
rt[1 - Tan[(e + f*x)/2]^4] + 80*a^2*b^4*Tan[(e + f*x)/2]^7*Sqrt[1 - Tan[(e
+ f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 32*b^6*Tan[(e + f*x)/2]^7*Sqrt[
1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 10*a^6*Tan[(e + f*x)
/2]^9*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 26*a^4*b^
2*Tan[(e + f*x)/2]^9*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]
^4] - 16*a^2*b^4*Tan[(e + f*x)/2]^9*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 - T
an[(e + f*x)/2]^4] + 2*a^6*Tan[(e + f*x)/2]^11*Sqrt[1 - Tan[(e + f*x)/2]^2]
*Sqrt[1 - Tan[(e + f*x)/2]^4] + 2*a^4*b^2*Tan[(e + f*x)/2]^11*Sqrt[1 - Tan[
(e + f*x)/2]^2]*Sqrt[1 - Tan[(e + f*x)/2]^4])*(a + b*Tan[e + f*x])^2) + (2*
Cos[e + f*x]*(d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(b*f
*(a + b*Tan[e + f*x]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10703 vs. 2(369) = 738.
time = 0.80, size = 10704, normalized size = 27.03

method	result	size
default	Expression too large to display	10704

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{5}{2}}}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/2}}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)), x)
```

3.605 $\int \frac{(d \sec(e+fx))^{3/2}}{a+b \tan(e+fx)} \, dx$

Optimal. Leaf size=334

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}-\frac{a\,\text{co}}{}$$

```
[Out] arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(
a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-arctanh((sec(f*x+e)^2)^(1/4)*
b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)
^2)^(3/4)/b^(1/2)-a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)
^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)^2)^(3/
4)/(a^2+b^2)^(1/2)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)
^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)^2)^(3/
4)/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3593, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{bf\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{bf\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{(d\sec(e+fx))^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x]
)^(3/2))/(Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (ArcTanh[(S
qrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(
Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*Cot[e + f*x]*Ellip
ticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e +
f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3
/4)) + (a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b
^2]*f*(Sec[e + f*x]^2)^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}} + \frac{(a(d\sec(e}{}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sec^2(e+fx)^{3/4}} + \frac{\left(2a\cot(e\right.}{}$$

$$= -\frac{\left(2b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}} + \frac{\left(a\cot(e+\right.}{}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}} + \frac{(d\sec(e+}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right.}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 42.93, size = 6301, normalized size = 18.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3736 vs. 2(282) = 564.
time = 0.80, size = 3737, normalized size = 11.19

method	result	size
default	Expression too large to display	3737

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/2/f*(cos(f*x+e)+1)^2*(-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cos(f*x+e)-1)/sin(f*x+e),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)+4*I*(a^2+b^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a*b-4*I*(a^2+b^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a*b^3-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cos(f*x+e)-1)/sin(f*x+e),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)-(a^2+b^2)^(3/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2+(a^2+b^2)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2*b^2+(a^2+b^2)^(3/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2-(a^2+b^2)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2*b^2+(a^2+b^2)^(3/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2-(a^2+b^2)^(1/2)*(-cos(f

```
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)
*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^2*b^2+(a^2+b^2)^(3/2)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^
3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(
f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^
2*b-2*b^3)/a^4)^(1/2)/a^2)*b^2-(a^2+b^2)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^
(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a
^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^
3)/a^4)^(1/2)/a^2)*b^4-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f
*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2
+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^
2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*a^4*b
-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(
1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1
/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f
*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*a^2*b^3-(a^2+b^2)^(3/2)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/
2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*
x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*b^2+(a^2+b^2)^(1/2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f
*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)),x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)), x)
```

3.606 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=324

$$-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}\,f\,\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}\,f\,\sqrt[4]{\sec^2(e+fx)}}$$

```
[Out] -arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e)
)^(1/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)-arctanh((sec(f*x+e)^2)^(1/4)
*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(3/4)/f/(s
ec(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2
)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+
e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),
I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+e)^2)^(1
/4)
```

Rubi [A]
time = 0.24, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{a\sqrt{-\tan^2(e+fx)}\,\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f\,(a^2+b^2)\,\sqrt[4]{\sec^2(e+fx)}}+\frac{a\sqrt{-\tan^2(e+fx)}\,\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f\,(a^2+b^2)\,\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{b}\,\sqrt{d\sec(e+fx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\,(a^2+b^2)^{3/4}\,\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{b}\,\sqrt{d\sec(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\,(a^2+b^2)^{3/4}\,\sqrt[4]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4))) - (Sqrt[b]*A
rcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f
*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*Cot[e + f*x]*Ellipt
icPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e
 + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) + (a
*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\sqrt{d\sec(e\ldots\right.}{}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\cot(e+\ldots\right.}{}$$

$$= -\frac{\left(2b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(2a\cot(e\ldots\right.}{}$$

$$= -\frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(b\sqrt{d\ldots\right.}{}$$

$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec\ldots}}{\sqrt[4]{a^2\ldots}}\right.}{(a^2+b^2)^{3/4}\ldots}$$

$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec\ldots}}{\sqrt[4]{a^2\ldots}}\right.}{(a^2+b^2)^{3/4}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 44.54, size = 4648, normalized size = 14.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan
```

[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/(a*f*Sqrt[Sec[(e + f*x)/2]^2]*(a + b*Tan[e + f*x])*((-2*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*Tan[(e + f*x)/2]*(EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/a + (Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/(a*Sqrt[Sec[(e + f*x)/2]^2]) - ((Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/(a*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) - (2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*((((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcS

```
in[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt
[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - S
qrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2
] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2
 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[
e + f*x]]/Sqrt[2]], 2])*Sec[(e + f*x)/2]^2*Sqrt[I*Cos[e + f*x] - Sin[e + f*
x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2
]))/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4
]) + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] +
Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 +
 I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[
1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 +
b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2
+ b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*(-Cos
[e + f*x] - I*Sin[e + f*x])*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x
])]*(I + Tan[(e + f*x)/2])^2)/(2*Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos
[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]) + (((-2*
I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x
]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*
(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3130 vs. $2(276) = 552$.
time = 0.81, size = 3131, normalized size = 9.66

method	result	size
default	Expression too large to display	3131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(d/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(-4*I*b*a^3*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cos(f*
x+e)-1)/sin(f*x+e),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2
+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)+4*I*b*a^3*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cos(
f*x+e)-1)/sin(f*x+e),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^
2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+
2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)-4*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b
^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3
)/a^4)^(1/2)*a^5-4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^
```

2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^3*b^2+(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(
-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^4
*b-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+
2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*
a^2*b-2*b^3)/a^4)^(1/2)*a^4*b-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*(
(a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^4*b+(
a^2+b^2)^(3/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2
+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)
*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1
/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*b^2+(a^2+b^2)
^(3/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^
2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b
^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a
^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*(-b*((a^2+b^2)^(1/2
)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*b^2-(a^2+b^2)^(1/2)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2
)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x
+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*b^4-(a^2+b^2)^(1/2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos
(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(
1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2
)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*b^4-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)
arctanh(1/2(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-co
s(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^
4)^(1/2)/a^2)*a^4*b-(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2
)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+
e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^
2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*a^2*b^3+
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^
(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin
(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a

```
^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*(-b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a^4*b+(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b+cos
(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^
2*b+2*b^3)/a^4)^(1/2)/a^2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2
*a^2*b-2*b^3)/a^4)^(1/2)*a^2*b^3)/sin(f*x+e)^2/(a^2+b^2)/(-b+(a^2+b^2)^(1/2
))/a^2/(b+(a^2+b^2)^(1/2))/(b*((a^2+b^2)^(1/2)*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec{(e+fx)}}}{a+b\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{a+b\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)), x)
```

3.607 $\int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=451

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}\,f\,\sqrt{d\sec(e+fx)}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}\,f\,\sqrt{d\sec(e+fx)}}+$$

```
[Out] b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)
^(1/4)/(a^2+b^2)^(5/4)/f/(d*sec(f*x+e))^(1/2)-b^(3/2)*arctanh((sec(f*x+e)^2
)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(5/4)/f/(d*
sec(f*x+e))^(1/2)+2*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^
(1/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)-a*b*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)+a*b*cot(f*x+e)*EllipticPi((sec(f*x+
e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)-2*a*tan(f*x+e)/(a^2+b^2)/f/(d*sec(f
*x+e))^(1/2)+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 755, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{ab\sqrt{-\tan^2(e+fx)}\,\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f\,(a^2+b^2)^{3/2}\sqrt{d\sec(e+fx)}}+\frac{ab\sqrt{-\tan^2(e+fx)}\,\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f\,(a^2+b^2)^{3/2}\sqrt{d\sec(e+fx)}}+\frac{2a\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\,(a^2+b^2)\sqrt{d\sec(e+fx)}}+\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\,(a^2+b^2)^{5/4}\sqrt{d\sec(e+fx)}}-\frac{2a\tan(e+fx)}{f\,(a^2+b^2)\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f\,(a^2+b^2)\sqrt{d\sec(e+fx)}}-\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\,(a^2+b^2)^{5/4}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) - (b^(3/2)*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1
/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[ArcTan[Tan
[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]
]) - (2*a*Tan[e + f*x])/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]]) - (a*b*Cot[e +
 f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(3/2)*f*Sqrt[d*S
ec[e + f*x]]) + (a*b*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec
[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^
2 + b^2)^(3/2)*f*Sqrt[d*Sec[e + f*x]]) + (2*(b + a*Tan[e + f*x]))/((a^2 + b
^2)*f*Sqrt[d*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\right.}{(a^2+b^2)\,f}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(a\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\right.}{b\,(a^2+b^2)\,f\,\sqrt{}}$$

$$= -\frac{2a\tan(e+fx)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} +$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan}{(a^2+b^2)\,f}$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan}{(a^2+b^2)\,f}$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan}{(a^2+b^2)\,f}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}\,f\sqrt{d\sec(e+fx)}} - \frac{b^{3/2}\tanh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4497 vs. $2(451) = 902$.

time = 69.39, size = 4497, normalized size = 9.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

```
[Out] -1/2*(Sec[e + f*x]^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])*(Sqrt[Sec[e + f*
x]]/(2*(a*Cos[e + f*x] + b*Sin[e + f*x])) + (Cos[2*(e + f*x)]*Sqrt[Sec[e +
f*x]])/(2*(a*Cos[e + f*x] + b*Sin[e + f*x])))*Sqrt[(1 - Tan[(e + f*x)/2]^2)
^(-1)]*(-4*b - 4*a*Tan[(e + f*x)/2] + 4*b*Tan[(e + f*x)/2]^2 + 4*a*Tan[(e +
 f*x)/2]^3 - (b^(3/2)*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 +
 b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[
b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e +
f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2
])] - (b^(3/2)*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*
Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqr
t[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]
^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])] - 4
*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] + (
4*(a^2 + b^2)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x
)/2]^4])/a - (4*b^2*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]),
 ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a - (4*b^2*Ell
ipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2
]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a))/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x
]]*Sqrt[1 + Tan[(e + f*x)/2]^2]*((Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[
(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-4*b - 4*a*Tan[(e + f*x)/2] + 4*b*Tan[(e +
f*x)/2]^2 + 4*a*Tan[(e + f*x)/2]^3 - (b^(3/2)*Sqrt[b - Sqrt[a^2 + b^2]]*Arc
Tan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)
/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b
^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/Sqrt[a^2
 + b*(b - Sqrt[a^2 + b^2])] - (b^(3/2)*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*
b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))
/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*S
qrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/Sqrt[a^2 + b*(b
 + Sqrt[a^2 + b^2])] - 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 -
 Tan[(e + f*x)/2]^4] + (4*(a^2 + b^2)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -
1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a - (4*b^2*EllipticPi[a^2/(a^2 + 2*b^2 - 2
*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*
x)/2]^4])/a - (4*b^2*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)]))
, ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a))/(4*(a^2 +
 b^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)) - (Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
]*((1 - Tan[(e + f*x)/2]^2)^(-1))^(3/2)*(-4*b - 4*a*Tan[(e + f*x)/2] + 4*b*
Tan[(e + f*x)/2]^2 + 4*a*Tan[(e + f*x)/2]^3 - (b^(3/2)*Sqrt[b - Sqrt[a^2 +
b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[
```

```
(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqr
t[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])
/Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])] - (b^(3/2)*Sqrt[b + Sqrt[a^2 + b^2]]*A
rcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*
x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 +
 b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/Sqrt[a
^2 + b*(b + Sqrt[a^2 + b^2])] - 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]
*Sqrt[1 - Tan[(e + f*x)/2]^4] + (4*(a^2 + b^2)*EllipticF[ArcSin[Tan[(e + f*
x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a - (4*b^2*EllipticPi[a^2/(a^2 +
2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Ta
n[(e + f*x)/2]^4])/a - (4*b^2*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2
+ b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4])/a))/
(4*(a^2 + b^2)*Sqrt[1 + Tan[(e + f*x)/2]^2]) - (Sqrt[(1 - Tan[(e + f*x)/2]^
2)^(-1)]*(-2*a*Sec[(e + f*x)/2]^2 + 4*b*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
 + 6*a*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2 + (b^(3/2)*Sqrt[b - Sqrt[a^2 +
 b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan
[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sq
rt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2]^3)/(Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4
]) + (b^(3/2)*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*T
an[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt
[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^
4])]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(Sqrt[a^2 + b*(b + Sqrt[a^2 + b
^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4]) + (4*a*EllipticE[ArcSin[Tan[(e + f*x)/2
]], -1]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/Sqrt[1 - Tan[(e + f*x)/2]^4]
 - (4*(a^2 + b^2)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2]^3)/(a*Sqrt[1 - Tan[(e + f*x)/2]^4]) + (4*b^2*EllipticPi[
a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(a*Sqrt[...
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8824 vs. 2(418) = 836.
time = 1.37, size = 8825, normalized size = 19.57

method	result	size
default	Expression too large to display	8825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,\left(a+b\,\tan\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))), x)
```

3.608 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=422

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}}+$$

```
[Out] -b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(3/4)/(a^2+b^2)^(7/4)/f/(d*sec(f*x+e))^(3/2)-b^(5/2)*arctanh((sec(f*x+e)^
2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(7/4)/f/(d
*sec(f*x+e))^(3/2)+2/3*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^
2)^(3/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticPi((sec(
f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^
(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticPi((sec(f
*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1
/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*
sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.34, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 755, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{ab^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f(a^2+b^2)^2(d\sec(e+fx))^{3/2}}+\frac{ab^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{f(a^2+b^2)^2(d\sec(e+fx))^{3/2}}+\frac{2a\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{3f(a^2+b^2)(d\sec(e+fx))^{3/2}}-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f(a^2+b^2)^{7/4}(d\sec(e+fx))^{3/2}}+\frac{2(a\tan(e+fx)+b)}{3f(a^2+b^2)(d\sec(e+fx))^{3/2}}-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f(a^2+b^2)^{7/4}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2))) - (b^(5/2)
*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^
2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2)) + (2*a*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*(a^2 + b^2)*f*(d*Sec[e +
 f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin
[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])
/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/
4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (2*(b
+ a*Tan[e + f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\right.}{3(a^2+b^2}$$

$$= \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{\left(a\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\right.}{3b(a^2+b^2)}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{2(b+}{3(a^2+b^2)}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{2(b+}{3(a^2+b^2)}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{2(b+}{3(a^2+b^2)}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{2(b+}{3(a^2+b^2)}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\tan}{}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\tan}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 67.53, size = 9313, normalized size = 22.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6251 vs. 2(391) = 782.
time = 0.86, size = 6252, normalized size = 14.82

method	result	size
default	Expression too large to display	6252

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,\left(a+b\,\tan\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))), x)
```

3.609 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=568

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}d^2f\sqrt{d\sec(e+fx)}}+\frac{2}{}$$

```
[Out] b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)
^(1/4)/(a^2+b^2)^(9/4)/d^2/f/(d*sec(f*x+e))^(1/2)-b^(7/2)*arctanh((sec(f*x+
e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(9/4)/d
^2/f/(d*sec(f*x+e))^(1/2)+2/5*a*(3*a^2+8*b^2)*(cos(1/2*arctan(tan(f*x+e)))^
2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),
2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)-a*b^3*
cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e
)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*sec(f*x+e))^(1/2)
+a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec
(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*sec(f*x+e))
^(1/2)-2/5*a*(3*a^2+8*b^2)*tan(f*x+e)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2
)+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)+2/
5*(5*b^3+a*(3*a^2+8*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3593, 755, 837, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{ab^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{d^2f(a^2+b^2)^{5/2}\sqrt{d\sec(e+fx)}}+\frac{ab^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{d^2f(a^2+b^2)^{5/2}\sqrt{d\sec(e+fx)}}+\frac{2a(3a^2+8b^2)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{5d^2f(a^2+b^2)^2\sqrt{d\sec(e+fx)}}+\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f(a^2+b^2)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{2a(3a^2+8b^2)\tan(e+fx)}{5d^2f(a^2+b^2)^2\sqrt{d\sec(e+fx)}}+\frac{2\cos^2(e+fx)(a\tan(e+fx)+b)}{5d^2f(a^2+b^2)\sqrt{d\sec(e+fx)}}-\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f(a^2+b^2)^{9/4}\sqrt{d\sec(e+fx)}}+\frac{2(a(3a^2+8b^2)\tan(e+fx)+5b^3)}{5d^2f(a^2+b^2)^2\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (b^(7/2)*
ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2
)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*a*(3*a^2 + 8*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(5*(a^2 +
b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*a*(3*a^2 + 8*b^2)*Tan[e + f*x])/(5*
(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (a*b^3*Cot[e + f*x]*EllipticPi[
-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^
(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
 + (a*b^3*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5
```

```
/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*Cos[e + f*x]^2*(b + a*Tan[e + f*x]))/(
5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*(5*b^3 + a*(3*a^2 + 8*b^2)*T
an[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
```

```
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Sub}}{5(a^2+}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(}$$

$$= -\frac{2a(3a^2+8b^2)\tan(e+fx)}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} + \frac{2\cos^2(e+fx)(b+a\tan(}{5(a^2+b^2)\,d^2f\sqrt{d\sec(}$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}} - \frac{b^{7/2}\tanh}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 90.74, size = 5312, normalized size = 9.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14546 vs. 2(525) = 1050.
time = 0.98, size = 14547, normalized size = 25.61

method	result	size
default	Expression too large to display	14547

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}(a+b\tan(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/2}\,\left(a+b\,\tan\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))), x)
```

3.610 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=480

$$-\frac{3ad^2\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}+\frac{3ad^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+f}{2b^{5/2}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}$$

```
[Out] -3/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+
e))^(3/2)/b^(5/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)+3/2*a*d^2*arctanh(
(sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/
(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)-3*d^2*(cos(1/2*arctan(tan(f*x+e)))^2
)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2
^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/4)+3*d^2*cos(f*x+e)*(d
*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f+3/2*a^2*d^2*cot(f*x+e)*EllipticPi((sec(
f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^
(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-3/2*a^2*d^2*cot(f*x+e)*Ell
ipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-ta
n(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-d^2*(d*sec(f*x
+e))^(3/2)/b/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.28, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 747, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2b^3f\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2b^3f\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{3ad^2(d\sec(e+fx))^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}-\frac{3d^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\mathrm{ArcTan}(\tan(e+fx))\middle|2\right)}{b^2f\sec^2(e+fx)^{3/4}}+\frac{3d^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) +
(3*a*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) -
(3*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/(b^2*f*
(Sec[e + f*x]^2)^(3/4)) + (3*d^2*Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e
+ f*x])/(b^2*f) + (3*a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3/4)) - (3*a^2*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d
*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e
 + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,d\right.}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x,\right.}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} - \frac{(3d}{}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e}{b^2}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e}{b^2}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e}{b^2}$$

$$= -\frac{3ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}} + \frac{3ad^2\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{2b^{5/2}\sqrt[4]{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4735 vs. 2(480) = 960.
time = 69.23, size = 4735, normalized size = 9.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]

[Out] (Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((3*Cos[e + f*x])/(a*b) + (3*Sin[e + f*x])/b^2 - 1/(b*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) - (3*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((3*Sqrt[Sec[e + f*x]])/(4*a*(a*Cos[e + f*x] + b*Sin[e + f*x])) - (9*a*Sqrt[Sec[e + f*x]])/(4*b^2*(a*Cos[e + f*x] + b*Sin[e + f*x])) - (3*Cos[2*(e + f*x)]*Sqrt[Sec[e + f*x]])/(4*a*(a*Cos[e + f*x] + b*Sin[e + f*x])) - (3*a*Cos[2*(e + f*x)]*Sqrt[Sec[e + f*x]])/(4*b^2*(a*Cos[e + f*x] + b*Sin[e + f*x])))*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(4*b + 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)/2]^2 - 4*a*Tan[(e + f*x)/2]^3 + (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]) + (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]) + 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 8*a*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 4*a*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4]))/(4*a*b^2*f*Sec[e + f*x]^(3/2)*(1 + Tan[(e + f*x)/2]^2)*((3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(4*b + 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)/2]^2 - 4*a*Tan[(e + f*x)/2]^3 + (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]) + (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]) + 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 8*a*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] + 4*a*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4]))/(4*a*b^2*(1 + Tan[(e + f*x)/2]^2)^2) - (3*((Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(1 - Tan[(e + f*x)/2]^2) + (Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]^2))/

```
(1 - Tan[(e + f*x)/2]^2)^2)*(4*b + 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)
/2]^2 - 4*a*Tan[(e + f*x)/2]^3 + (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b
*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/
(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sq
rt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^
2 + b*(b - Sqrt[a^2 + b^2])]) + (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*
(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(
2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqr
t[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2
 + b*(b + Sqrt[a^2 + b^2])]) + 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*
Sqrt[1 - Tan[(e + f*x)/2]^4] - 8*a*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*
Sqrt[1 - Tan[(e + f*x)/2]^4] + 4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2
*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4]
+ 4*a*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e
 + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4]))/(8*a*b^2*(1 + Tan[(e + f*x)
/2]^2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]) - (3*Sqrt[(
1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(2*a*Sec[(e + f*x)/2]^2 -
 4*b*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] - 6*a*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2]^2 - (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2]
)*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - S
qrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/
2]^4])]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(Sqrt[b]*Sqrt[a^2 + b*(b - S
qrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4]) - (a^2*Sqrt[b + Sqrt[a^2 + b
^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(
e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt
[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Se...
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44462 vs. $2(439) = 878$.
time = 1.56, size = 44463, normalized size = 92.63

method	result	size
default	Expression too large to display	44463

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*d^3*sec(f*x + e)^3/(b^2*tan(f*x + e)^2 + 2*a*
b*tan(f*x + e) + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2, x)
```

3.611 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=440

$$\frac{ad^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}-$$

```
[Out] 1/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e
))^(1/2)/b^(3/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+1/2*a*d^2*arctanh((
sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/(
a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(
1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1
/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2*d^2*cot(f*x+e)
*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)
*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2*d^2*cot
(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))
^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)-d^2*(d*se
c(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.30, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 747, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$-\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2b^2f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}-\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2b^2f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}+\frac{d^2\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Se
c[e + f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^
2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e
+ f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (d^2*Elli
pticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/(b^2*f*(Sec[e + f*x]
^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2
*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*
x]]*Sqrt[-Tan[e + f*x]^2])/(2*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (
d^2*Sqrt[d*Sec[e + f*x]])/(b*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x\right.}{2b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\,\text{ta}\right.}{2b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \cdots$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \cdots$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \cdots$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \cdots$$

$$= \frac{ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2\cdots}}{\sqrt[4]{a^2\cdots}}\right.}{2b^{3/2}\left(a^2+b^2\right)^{3\cdots}}$$

$$= \frac{ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2\cdots}}{\sqrt[4]{a^2\cdots}}\right.}{2b^{3/2}\left(a^2+b^2\right)^{3\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 42.40, size = 3091, normalized size = 7.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-(1/(a*b)) + S
in[e + f*x]/(a*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])
^2) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] +
 Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1
+ I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt
[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 +
 b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2
 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*(d*S
ec[e + f*x])^(5/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x
] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*Sin[e
+ f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])*(I + Tan[(e + f*x)/2])^2)/(4*(a -
I*b)*b^3*Sqrt[a^2 + b^2]*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*(a + b*Tan[e + f*x
])^2*(-1/2*(((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f
*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi
[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin
[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[
a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqr
t[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])
*Sec[(e + f*x)/2]^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*
x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I +
Tan[(e + f*x)/2]))/((a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(
-1)]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x]
 + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((
1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sq
rt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2
 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a
^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sq
rt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-Cos[e + f*x] - I*Sin[e + f*x])*Sqrt[C
os[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(4*(
a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[I*Cos[e + f
*x] - Sin[e + f*x]]) + (Sqrt[(1 + Cos[e + f*x])^(-1)]*((-2*I)*b*Sqrt[a^2 +
b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]
+ a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 +
 b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e
+ f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(
a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*
Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[Cos[(e + f*x)/2]^2*Sec[e +
f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] +
```

```
 I*Sin[e + f*x])]*Sin[e + f*x]*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^2*S
qrt[a^2 + b^2]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Co
s[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*Ell
ipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]),
 ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b
+ Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a +
b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2
]], 2])*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e +
 f*x]]*(Cos[e + f*x]*(I*Cos[e + f*x] - Sin[e + f*x]) - (Cos[e + f*x] + I*Si
n[e + f*x])*Sin[e + f*x])*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^2*Sqrt[a
^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I
*Sin[e + f*x])]) - (Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*
x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(((-I
)*b*Sqrt[a^2 + b^2]*(Cos[e + f*x] + I*Sin[e + f*x]))/(Sqrt[2]*Sqrt[1 + (-1
+ I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqr
t[1 - I*Cos[e + f*x] + Sin[e + f*x]]) + (a*(a - I*b + Sqrt[a^2 + b^2])*(Cos
[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin
[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] +
 Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a + I*(-b + Sqrt[a^2 + b^2]))*(1 - I*Cos[
e + f*x] + Sin[e + f*x]))/(a + b - Sqrt[a^2 + b^2]))) + (a*(-a + I*b + Sqrt
[a^2 + b^2])*(Cos[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*C
os[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 -
 I*Cos[e + f*x] + Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a - I*(b + Sqrt[a^2 + b^
2]))*(1 - I*Cos[e + f*x] + Sin[e + f*x]))/(a + b + Sqrt[a^2 + b^2]))))*(I +
 Tan[(e + f*x)/2])^2)/(2*(a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*
x])^(-1)]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e +
 f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*Elliptic
Pi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcS
in[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt...
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5336 vs. 2(405) = 810.
time = 0.96, size = 5337, normalized size = 12.13

method	result	size
default	Expression too large to display	5337

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec(e+fx))^{\frac{5}{2}}}{(a+b\tan(e+fx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(a+b\tan(e+f\,x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2, x)
```

3.612 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=477

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}}-$$

```
[Out] -(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Elliptic
E(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(se
c(f*x+e)^2)^(3/4)+cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/(a^2+b^2)/f+1/
2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/
2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a*arctanh((sec(f*x+e)
^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(5/4)/f/(
sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1
/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2
+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+1/2*a^2*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/
b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f
/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.29, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 759, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2bf\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}+\frac{a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2bf\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}+\frac{a(d\sec(e+fx))^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}\,f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)\sec^2(e+fx)^{3/4}}-\frac{a(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}\,f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{b(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*
x])^(3/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3
/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(3/4)) + (Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/((a^2 + b
^2)*f) - (a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e +
 f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b*(a^
2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (a^2*Cot[e + f*x]*EllipticPi[b/S
qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*
Sqrt[-Tan[e + f*x]^2])/(2*b*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) - (
b*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{-a-\frac{x}{2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\right.}{b\,(a^2+b^2)\,f\sec^2(e+fx}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,\right.}{2b\,(a^2+b^2)\,f\sec^2(e+fx)}$$

$$= \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)\,f} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))} -$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx)}{(a^2+b^2)\,f}$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx)}{(a^2+b^2)\,f}$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx)}{(a^2+b^2)\,f}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,(a^2+b^2)^{5/4}\,f\sec^2(e+fx)^{3/4}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e}}{\sqrt[4]{a^2+b}}\right.}{2\sqrt{b}\,(a^2+b^2)^{5/}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.

time = 69.39, size = 4487, normalized size = 9.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
b*Cos[e + f*x])/(a*(a - I*b)*(a + I*b)) + Sin[e + f*x]/((a - I*b)*(a + I*b)
) - b/((a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*T
an[e + f*x])^2) - (Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x
] + b*Sin[e + f*x])^2*(Sqrt[Sec[e + f*x]]/(4*a*(a*Cos[e + f*x] + b*Sin[e +
f*x])) - (Cos[2*(e + f*x)]*Sqrt[Sec[e + f*x]])/(4*a*(a*Cos[e + f*x] + b*Sin
[e + f*x])))*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(4*b +
 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)/2]^2 - 4*a*Tan[(e + f*x)/2]^3 - (
a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*
x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2
]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[
1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]) - (a
^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x
)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]
]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sqrt[1
 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]) + 4*a
*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 4*a
*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*
x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 4*a*EllipticPi[a^2/(a^2 + 2*(b^2
 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e +
 f*x)/2]^4]))/(4*a*(a^2 + b^2)*f*(1 + Tan[(e + f*x)/2]^2)*((Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2]*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)
]*(4*b + 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)/2]^2 - 4*a*Tan[(e + f*x)/
2]^3 - (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan
[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a
^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4]
)]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2]
)]) - (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[
(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^
2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])
]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])
]) + 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^
4] - 4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan
[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 4*a*EllipticPi[a^2/(a^2
+ 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 -
Tan[(e + f*x)/2]^4]))/(4*a*(a^2 + b^2)*(1 + Tan[(e + f*x)/2]^2)^2) - (((Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(1 - Tan[(e + f*x)/2]^2) + (Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]^2))/(1 - Tan[(e + f*x)/2]^2)^2
```

```
)*(4*b + 4*a*Tan[(e + f*x)/2] - 4*b*Tan[(e + f*x)/2]^2 - 4*a*Tan[(e + f*x)/
2]^3 - (a^2*Sqrt[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan
[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a
^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4]
)]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2]
)]) - (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[
(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^
2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])
]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(Sqrt[b]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])
]) + 4*a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^
4] - 4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan
[(e + f*x)/2]], -1]*Sqrt[1 - Tan[(e + f*x)/2]^4] - 4*a*EllipticPi[a^2/(a^2
+ 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 -
Tan[(e + f*x)/2]^4]))/(8*a*(a^2 + b^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(1 + T
an[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]) - (Sqrt[(1 + Tan[(e + f*x)/2]
^2)/(1 - Tan[(e + f*x)/2]^2)]*(2*a*Sec[(e + f*x)/2]^2 - 4*b*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2] - 6*a*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2 + (a^2*Sqr
t[b - Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2
 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt
[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2]^3)/(Sqrt[b]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqr
t[1 - Tan[(e + f*x)/2]^4]) + (a^2*Sqrt[b + Sqrt[a^2 + b^2]]*ArcTan[(2*b*(b
+ Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*S
qrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1
 - Tan[(e + f*x)/2]^4])]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(Sqrt[b]*Sq
rt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4]) - (4*a*Elli
pticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/
Sqrt[1 - Tan[(e + f*x)/2]^4] + (4*a*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^
2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1]*...
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25421 vs. $2(436) = 872$.
time = 1.09, size = 25422, normalized size = 53.30

method	result	size
default	Expression too large to display	25422

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec (e+fx))^{\frac{3}{2}}}{(a+b\tan (e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan (e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2, x)
```

3.613 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=430

$$-\frac{3a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}-\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$

```
[Out] -(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Elliptic
F(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(se
c(f*x+e)^2)^(1/4)-3/2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4)
)*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)-3/2*a
*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e
))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)*Elliptic
Pi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*
x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)*Ellipti
cPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*
x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-b*(d*sec(f*x+e))^(1/2)/(a^
2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.29, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 759, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{3a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}+\frac{3a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{b\sqrt{d\sec(e+fx)}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sq
rt[d*Sec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (3*a*S
qrt[b]*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*S
ec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (EllipticF[A
rcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/((a^2 + b^2)*f*(Sec[e + f*x
]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2*
(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*S
qrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
```

```
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{-a+\frac{x}{2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,d\right.}{b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+f}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x,\right.}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{2\,(a^2+}$$

$$= -\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{2\,(a^2+}$$

Mathematica [C] Result contains complex when optimal does not.

time = 67.20, size = 8876, normalized size = 20.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14300 vs. 2(395) = 790.
time = 1.01, size = 14301, normalized size = 33.26

method	result	size
default	Expression too large to display	14301

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sec{(e + fx)}}}{(a + b \tan{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2, x)
```

3.614 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=555

$$\frac{5ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}}-\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}}$$

```
[Out] 5/2*a*b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x
+e)^2)^(1/4)/(a^2+b^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)-5/2*a*b^(3/2)*arctanh((
sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)
^(9/4)/f/(d*sec(f*x+e))^(1/2)+(2*a^2-3*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)
^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^
(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)-5/2*a^2*b*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^
2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/2)+5/2*a
^2*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f
*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/2)
-(2*a^2-3*b^2)*tan(f*x+e)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)+b*(2*a^2-3*b^2
)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))+2*(b+a*t
an(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.41, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{5a^2b\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}+\frac{5a^2b\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}+\frac{\left(2a^2-3b^2\right)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}+\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}+\frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{\left(2a^2-3b^2\right)\tan(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b^
(3/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e +
f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) + ((2*a^2 - 3*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^
2)^2*f*Sqrt[d*Sec[e + f*x]]) - ((2*a^2 - 3*b^2)*Tan[e + f*x])/((a^2 + b^2)^
2*f*Sqrt[d*Sec[e + f*x]]) - (5*a^2*b*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Ta
n[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a^2*b*Cot[
e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d
```

```
*Sec[e + f*x]]) + (b*(2*a^2 - 3*b^2)*Sec[e + f*x]^2)/((a^2 + b^2)^2*f*Sqrt[
d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^
2)*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\sec^2(e\right.}{}$$

$$= \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} + \frac{}{(a^2+b^2)\,f}$$

$$= \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} + \frac{}{(a^2+b^2)\,f}$$

$$= -\frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-3b^2)\sec^2(}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}}$$

$$= \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}} - \frac{(}{(a^2}$$

$$= \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}} - \frac{(}{(a^2}$$

$$= \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}} - \frac{(}{(a^2}$$

$$= \frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\,(a^2+b^2)^{9/4}\,f\sqrt{d\sec(e+fx)}} - \frac{5ab^{3/2}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 90.80, size = 5250, normalized size = 9.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38626 vs. 2(512) = 1024.
time = 1.34, size = 38627, normalized size = 69.60

method	result	size
default	Expression too large to display	38627

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\,\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(a+b\tan(e+f\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2), x)
```

3.615 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=520

$$-\frac{7ab^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}}-\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}}$$

```
[Out] -7/2*a*b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(3/4)/(a^2+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)-7/2*a*b^(5/2)*arctanh
((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^
2)^(11/4)/f/(d*sec(f*x+e))^(3/2)+1/3*(2*a^2-5*b^2)*(cos(1/2*arctan(tan(f*x+
e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+
e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+7/2*a
^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(se
c(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2)+
7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)
*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3
/2)+1/3*b*(2*a^2-5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+
b*tan(f*x+e))+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*ta
n(f*x+e))
```

Rubi [A]
time = 0.42, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{7a^2b^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}}+\frac{7a^2b^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}}+\frac{(2a^2-5b^2)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}}-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(S
ec[e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) - (7*
a*b^(5/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) + ((2*a^
2 - 5*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*
(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (7*a^2*b^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) +
 (7*a^2*b^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]
^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2
)^3*f*(d*Sec[e + f*x])^(3/2)) + (b*(2*a^2 - 5*b^2)*Sec[e + f*x]^2)/(3*(a^2
```

```
+ b^2)^2*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e +
 f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

p])

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{(2b\sec}{}$$

$$= \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{}{3\left(a^2\right.}$$

$$= \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{}{3\left(a^2\right.}$$

$$= \frac{(2a^2-5b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{}{3}$$

$$= \frac{(2a^2-5b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{}{3}$$

$$= \frac{(2a^2-5b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{}{3}$$

$$= \frac{(2a^2-5b^2)\,F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{}{3}$$

$$= -\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} - \frac{7a}{}$$

$$= -\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} - \frac{7a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 68.06, size = 11962, normalized size = 23.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15454 vs. $2(477) = 954$.
time = 1.64, size = 15455, normalized size = 29.72

method	result	size
default	Expression too large to display	15455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2), x)
```

3.616 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=700

$$\frac{9ab^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx}$$

```
[Out] 9/2*a*b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x
+e)^2)^(1/4)/(a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/2*a*b^(7/2)*arct
anh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2
+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*(2*a^4+10*a^2*b^2-7*b^4)*(cos(1
/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1
/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/d^2/f/(d*s
ec(f*x+e))^(1/2)-9/2*a^2*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/
(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/
2)/d^2/f/(d*sec(f*x+e))^(1/2)+9/2*a^2*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)
^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(
a^2+b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1/2)-3/5*(2*a^4+10*a^2*b^2-7*b^4)*tan(
f*x+e)/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*b*(2*a^4+10*a^2*b^2-7*b^4
)*sec(f*x+e)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))+2/5*
cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan
(f*x+e))-2/5*(b*(2*a^2-7*b^2)-3*a*(a^2+4*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f
/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A]

time = 0.55, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3593, 755, 837, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{9a^2b^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2d^2f\left(a^2+b^2\right)^{7/2}\sqrt{d\sec(e+fx)}}+\frac{9a^2b^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{2d^2f\left(a^2+b^2\right)^{7/2}\sqrt{d\sec(e+fx)}}+\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}-\frac{2\left(b\left(2a^2-7b^2\right)-3a\left(a^2+4b^2\right)\tan(e+fx)\right)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}+\frac{2\cos^2(e+fx)(a\tan(e+fx)+b)}{5d^2f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}-\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{3\left(2a^4+10a^2b^2-7b^4\right)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}}+\frac{3b\left(2a^4+10a^2b^2-7b^4\right)\sec^2(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))}-\frac{3\left(2a^4+10a^2b^2-7b^4\right)\tan(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9
*a*b^(7/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec
[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (3*
(2*a^4 + 10*a^2*b^2 - 7*b^4)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e +
f*x]^2)^(1/4))/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]]) - (3*(2*a^4 + 1
0*a^2*b^2 - 7*b^4)*Tan[e + f*x])/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]
]) - (9*a^2*b^3*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
```

```
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^
2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (9*a^2*b^3*Cot[e + f*x]*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*
x]]) + (3*b*(2*a^4 + 10*a^2*b^2 - 7*b^4)*Sec[e + f*x]^2)/(5*(a^2 + b^2)^3*d
^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*Cos[e + f*x]^2*(b + a*
Tan[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x
])) - (2*(b*(2*a^2 - 7*b^2) - 3*a*(a^2 + 4*b^2)*Tan[e + f*x]))/(5*(a^2 + b^
2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\text{s}\right.}{}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} - \frac{2(b}{5\left(a^2+\right.}$$

$$= \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} + \frac{}{5\left(a^2+\right.}$$

$$= \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))} + \frac{}{5\left(a^2+\right.}$$

$$= -\frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{3b(2a^4+10a^2}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\,\text{se}}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{9ab^{7/}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 91.44, size = 5832, normalized size = 8.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44328 vs. 2(645) = 1290.
time = 2.03, size = 44329, normalized size = 63.33

method	result	size
default	Expression too large to display	44329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2), x)
```

3.617 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=583

$$\frac{3(a^2+2b^2)\,d^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}}-\frac{3(a^2+2b^2)\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right.}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+}$$

```
[Out] 3/8*(a^2+2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d
*sec(f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-3/8*(a^2+
2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)+3/4*a*d^2*(cos(1/
2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/
2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/(a^2+b^2)/f/(sec(f*
x+e)^2)^(3/4)-3/4*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/(a^2
+b^2)/f-3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b
/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2
)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi(
(sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)
^2)^(1/2)/b^3/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*d^2*(d*sec(f*x+e))
^(3/2)/b/f/(a+b*tan(f*x+e))^2+3/4*a*d^2*(d*sec(f*x+e))^(3/2)/b/(a^2+b^2)/f/
(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.37, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 747, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{3ad^2(a^2+2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8b^3f\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(a^2+2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8b^3f\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{4b^2f\left(a^2+b^2\right)\sec^2(e+fx)^{3/4}}+\frac{3d^2(a^2+2b^2)(d\sec(e+fx))^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(d\sec(e+fx))^{3/2}}{4bf\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{3ad^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{4b^2f\left(a^2+b^2\right)}-\frac{3d^2(a^2+2b^2)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (3*(a^2 + 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1
/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2
)^(3/4)) - (3*(a^2 + 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a
^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Se
c[e + f*x]^2)^(3/4)) + (3*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec
[e + f*x])^(3/2))/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(3/4)) - (3*a*d^2*C
os[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(4*b^2*(a^2 + b^2)*f) - (3
*a*(a^2 + 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(
8*b^3*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (3*a*(a^2 + 2*b^2)*d^2*
```

```
Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b^3*(a^2 + b^2)^(3/2)*
f*(Sec[e + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(2*b*f*(a + b*Tan[
e + f*x])^2) + (3*a*d^2*(d*Sec[e + f*x])^(3/2))/(4*b*(a^2 + b^2)*f*(a + b*T
an[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
```

```
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
```

```
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^3}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int \frac{x}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\right.}{4b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3d^2(d\sec(e\right.}{}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3ad^2(d\sec(\right.}{}$$

$$= -\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2\left(a^2+b^2\right)f} - \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} +$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(}{4b^2\left(a^2\right.}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(}{4b^2\left(a^2\right.}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(}{4b^2\left(a^2\right.}$$

$$= \frac{3\left(a^2+2b^2\right)d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{3\left(a^2+2b^2\right)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 69.83, size = 8652, normalized size = 14.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101371 vs. 2(532) = 1064.
time = 2.52, size = 101372, normalized size = 173.88

method	result	size
default	Expression too large to display	101372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3, x)
```

3.618 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=532

$$\frac{(a^2-2b^2)\,d^2\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}+\frac{(a^2-2b^2)\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+f}}$$

```
[Out] 1/8*(a^2-2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d
*sec(f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/8*(a^2-
2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/4*a*d^2*(cos(1/
2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/
2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/(a^2+b^2)/f/(sec(f*
x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1
/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a
^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticP
i((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+
e)^2)^(1/2)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/2*d^2*(d*sec(f*x+e))^(
1/2)/b/f/(a+b*tan(f*x+e))^2+1/4*a*d^2*(d*sec(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a
+b*tan(f*x+e))
```

Rubi [A]
time = 0.37, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 747, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$-\frac{ad^2(a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8b^2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}-\frac{ad^2(a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8b^2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\mathrm{ArcTan}(\tan(e+fx))\middle|2\right)}{4b^2f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}+\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}}{4bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((a^2 - 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1
/4)) + ((a^2 - 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*
x]^2)^(1/4)) + (a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f
*x]])/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*C
ot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(S
ec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (d^2*Sqr
```

```
t[d*Sec[e + f*x]])/(2*b*f*(a + b*Tan[e + f*x])^2) + (a*d^2*Sqrt[d*Sec[e + f
*x]])/(4*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx\right.}{4b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\,(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\left(d^2\sqrt{d\sec(e\right.}{}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\,(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(\right.}{}$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} +$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} +$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} +$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} +$$

$$= \frac{(a^2-2b^2)\,d^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{(a^2-2b^2)\,d^2\tan}{8b}$$

$$= \frac{(a^2-2b^2)\,d^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{(a^2-2b^2)\,d^2\tan}{8b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 68.91, size = 4925, normalized size = 9.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(-
1/4*1/((a - I*b)*(a + I*b)*b) - b/(2*(a - I*b)*(a + I*b)*(a*Cos[e + f*x] +
b*Sin[e + f*x])^2) + (3*Sin[e + f*x])/(4*(a - I*b)*(a + I*b)*(a*Cos[e + f*x
] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^3) + ((8*b^(3/2)*EllipticF[A
rcSin[Tan[(e + f*x)/2]], -1] - ((a^2 - 2*b^2)*(a*Sqrt[b^2*(a^2 + b^2)]*(Sqr
t[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*ArcTan[(2*b*(b -
 Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sq
rt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1
- Tan[(e + f*x)/2]^4])] - Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[
a^2 + b^2])]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1
 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(
b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]) + 4*b^(3/2)*Sqrt[a^2
+ b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2
])]*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e +
 f*x)/2]], -1] - 4*b^(3/2)*Sqrt[a^2 + b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2
])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt
[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]))/(Sqrt[a^2 + b^2]*Sqrt[
b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt
[a^2 + b^2])]))*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] +
 b*Sin[e + f*x])^3*(1/(4*(a - I*b)*(a + I*b)*Sqrt[Sec[e + f*x]]*(a*Cos[e +
f*x] + b*Sin[e + f*x])) + (a*Sqrt[Sec[e + f*x]]*Sin[e + f*x])/(8*(a - I*b)*
(a + I*b)*b*(a*Cos[e + f*x] + b*Sin[e + f*x])))*Sqrt[(1 - Tan[(e + f*x)/2]^
2)^(-1)]*Sqrt[1 - Tan[(e + f*x)/2]^4])/(16*a*(a - I*b)*(a + I*b)*b^(3/2)*f*
Sqrt[1 + Tan[(e + f*x)/2]^2]*(-1/16*((8*b^(3/2)*EllipticF[ArcSin[Tan[(e + f
*x)/2]], -1] - ((a^2 - 2*b^2)*(a*Sqrt[b^2*(a^2 + b^2)]*(Sqrt[b - Sqrt[a^2 +
 b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2]
)*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - S
qrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/
2]^4])] - Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Arc
Tan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)
/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b
^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])]) + 4*b^(3/2)*Sqrt[a^2 + b^2]*Sqrt[a^2
+ b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a
^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1] -
 4*b^(3/2)*Sqrt[a^2 + b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b
*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)
])), ArcSin[Tan[(e + f*x)/2]], -1]))/(Sqrt[a^2 + b^2]*Sqrt[b^2*(a^2 + b^2)]
```

```
*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]))*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)])/
(a*(a - I*b)*(a + I*b)*b^(3/2)*Sqrt[1 + Tan[(e + f*x)/2]^2]*Sqrt[1 - Tan[(e
 + f*x)/2]^4]) - ((8*b^(3/2)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] - ((a^
2 - 2*b^2)*(a*Sqrt[b^2*(a^2 + b^2)]*(Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b
*(b + Sqrt[a^2 + b^2])]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^
2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqr
t[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])] - Sqrt[b +
Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*ArcTan[(2*b*(b + Sqrt[
a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*
Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[
(e + f*x)/2]^4])]) + 4*b^(3/2)*Sqrt[a^2 + b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 +
 b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*b^2 - 2
*Sqrt[b^2*(a^2 + b^2)]), ArcSin[Tan[(e + f*x)/2]], -1] - 4*b^(3/2)*Sqrt[a^2
 + b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^
2])]*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e
+ f*x)/2]], -1]))/(Sqrt[a^2 + b^2]*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b -
Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]))*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2]*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - Tan[(e + f*x)
/2]^4])/(32*a*(a - I*b)*(a + I*b)*b^(3/2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)) +
 ((8*b^(3/2)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] - ((a^2 - 2*b^2)*(a*Sq
rt[b^2*(a^2 + b^2)]*(Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 +
 b^2])]*ArcTan[(2*b*(b - Sqrt[a^2 + b^2])*Tan[(e + f*x)/2]^2 + a^2*(-1 + Ta
n[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - S
qrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])] - Sqrt[b + Sqrt[a^2 + b^2]]
*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*ArcTan[(2*b*(b + Sqrt[a^2 + b^2])*Tan[
(e + f*x)/2]^2 + a^2*(-1 + Tan[(e + f*x)/2]^2))/(2*Sqrt[b]*Sqrt[b + Sqrt[a^
2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[1 - Tan[(e + f*x)/2]^4])
]) + 4*b^(3/2)*Sqrt[a^2 + b^2]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2
 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2...
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45972 vs. $2(489) = 978$.
time = 1.52, size = 45973, normalized size = 86.42

method	result	size
default	Expression too large to display	45973

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{5}{2}}}{(a+b\tan(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3, x)
```

3.619 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=566

$$\frac{(3a^2-2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}}-\frac{(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)}$$

```
[Out] -5/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*El
lipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)
^2/f/(sec(f*x+e)^2)^(3/4)+5/4*a*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/
(a^2+b^2)^2/f+1/8*(3*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^
2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/
2)-1/8*(3*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*
(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/8*a*(
3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),
I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(5/2)/f/(sec(f*x+
e)^2)^(3/4)+1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),
b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)
^(5/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(a+b*t
an(f*x+e))^2-5/4*a*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.40, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 759, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{a(3a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8bf(a^2+b^2)^{5/2}\sec^2(e+fx)^{3/4}}+\frac{a(3a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\mathrm{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8bf(a^2+b^2)^{5/2}\sec^2(e+fx)^{3/4}}+\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{5a(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\mathrm{ArcTan}(\tan(e+fx))\middle|2\right)}{4f(a^2+b^2)^2\sec^2(e+fx)^{3/4}}-\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{5ab(d\sec(e+fx))^{3/2}}{4f(a^2+b^2)^2(a+b\tan(e+fx))}-\frac{b(d\sec(e+fx))^{3/2}}{2f(a^2+b^2)(a+b\tan(e+fx))^2}+\frac{5a\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{4f(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]
*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*x]^2)^(3
/4)) - ((3*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2
)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*
x]^2)^(3/4)) - (5*a*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(
3/2))/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(3/4)) + (5*a*Cos[e + f*x]*(d*Sec
[e + f*x])^(3/2)*Sin[e + f*x])/(4*(a^2 + b^2)^2*f) - (a*(3*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*
(Sec[e + f*x]^2)^(3/4)) + (a*(3*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqr
```

```
t[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - (b*(
d*Sec[e + f*x])^(3/2))/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (5*a*b*(d
*Sec[e + f*x])^(3/2))/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{-2a+\frac{x}{2}}{(a+x)^2\sqrt[4]{1+\frac{x}{b}}}\right.}{2b(a^2+b^2)f\sec^2(e+}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2f(a+b\tan(e+fx))} + \frac{(b(d}{}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2f(a+b\tan(e+fx))} + \frac{(5a(}{}$$

$$= \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4(a^2+b^2)^2f} - \frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx)}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+}{4(a^2+b^2}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+}{4(a^2+b^2}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+}{4(a^2+b^2}$$

$$= -\frac{\left(2-\frac{3a^2}{b^2}\right)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8(a^2+b^2)^{9/4}f\sec^2(e+fx)^{3/4}} + \frac{\left(2-\frac{3a^2}{b^2}\right)b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 71.62, size = 8774, normalized size = 15.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80249 vs. $2(515) = 1030$.
time = 2.23, size = 80250, normalized size = 141.78

method	result	size
default	Expression too large to display	80250

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{(a+b\tan(e+fx))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3,x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3, x)
```

3.620 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=515

$$-\frac{3\sqrt{b}\,(5a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\,(a^2+b^2)^{11/4}\,f\,\sqrt[4]{\sec^2(e+fx)}}-\frac{3\sqrt{b}\,(5a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(}{\sqrt[4]{a^2+}}\right.}{8\,(a^2+b^2)^{11/4}\,f\,\sqrt[4]{\sec}}$$

```
[Out] -7/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*El
lipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)
^2/f/(sec(f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(
1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(
f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2
+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(f*x+e)^2)
^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a
^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(
sec(f*x+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2
)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^
2+b^2)^3/f/(sec(f*x+e)^2)^(1/4)-1/2*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b
*tan(f*x+e))^2-7/4*a*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.38, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 759, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{3a(5a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^3\sqrt[4]{\sec^2(e+fx)}}+\frac{3a(5a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^3\sqrt[4]{\sec^2(e+fx)}}-\frac{3\sqrt{b}(5a^2-2b^2)\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{7a\sqrt{d\sec(e+fx)}\,F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{4f(a^2+b^2)^2\sqrt[4]{\sec^2(e+fx)}}-\frac{3\sqrt{b}(5a^2-2b^2)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{7ab\sqrt{d\sec(e+fx)}}{4f(a^2+b^2)^2(a+b\tan(e+fx))}-\frac{b\sqrt{d\sec(e+fx)}}{2f(a^2+b^2)(a+b\tan(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e + f*x]^2)^
(1/4)) - (3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e +
 f*x]^2)^(1/4)) - (7*a*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e +
f*x]])/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e
+ f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[
e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[d*Sec[e +
```

```
 f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (7*a*b*Sqrt[d*Sec[e + f*
x]])/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{-2a+\frac{3x}{2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/}}\right.}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f(a+b\tan(e+fx))} + \frac{\left(b\right.}{}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f(a+b\tan(e+fx))} - \frac{\left(7a\right.}{}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\,(a^2+b^2)\,f(a+b\tan(e}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\,(a^2+b^2)^{11/4}\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)}{}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\,(a^2+b^2)^{11/4}\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 69.09, size = 4857, normalized size = 9.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((
-9*b)/(4*(a - I*b)^2*(a + I*b)^2) - b^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[e
 + f*x] + b*Sin[e + f*x])^2) + (11*b^2*Sin[e + f*x])/(4*(a - I*b)^2*(a + I*
b)^2*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^3) + ((8*
(4*a^4 + a^2*b^2 - 3*b^4)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (3*(5*a
^2 - 2*b^2)*(a*Sqrt[b]*Sqrt[a^2 + b^2]*(Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2
+ b*(b + Sqrt[a^2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))
*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b -
 Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] - Sqrt[b + Sqrt[
a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(
b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^
2]]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^
4])]) + 4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^
2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2
+ b^2)]), ArcSin[Tan[(e + f*x)/2]], -1] - 4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2
+ b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a
^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]))
/(Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]))
*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sec[e + f*x]^(5/2)*Sqrt[d*Sec[e + f*
x]]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])
^3*(a^2/((a - I*b)^2*(a + I*b)^2*Sqrt[Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin
[e + f*x])) - (3*b^2)/(4*(a - I*b)^2*(a + I*b)^2*Sqrt[Sec[e + f*x]]*(a*Cos[
e + f*x] + b*Sin[e + f*x])) - (7*a*b*Sqrt[Sec[e + f*x]]*Sin[e + f*x])/(8*(a
 - I*b)^2*(a + I*b)^2*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(16*a*(a^2 + b^2
)^3*f*Sqrt[Sec[(e + f*x)/2]^2]*(a + b*Tan[e + f*x])^3*(-1/32*((8*(4*a^4 + a
^2*b^2 - 3*b^4)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (3*(5*a^2 - 2*b^2
)*(a*Sqrt[b]*Sqrt[a^2 + b^2]*(Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + S
qrt[a^2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e +
f*x)/2]^2)/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2
+ b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] - Sqrt[b + Sqrt[a^2 + b^2]
]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(b + Sqrt[a
^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a
^2 + b*(b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*
Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b +
 Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]),
ArcSin[Tan[(e + f*x)/2]], -1] - 4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - S
qrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 +
```

```
2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]))/(Sqrt[a^2
 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]))*Sqrt[Cos[
e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[(e +
 f*x)/2])/(a*(a^2 + b^2)^3*Sqrt[Sec[(e + f*x)/2]^2]) + ((8*(4*a^4 + a^2*b^2
 - 3*b^4)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (3*(5*a^2 - 2*b^2)*(a*S
qrt[b]*Sqrt[a^2 + b^2]*(Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b + Sqrt[a^
2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2
]^2)/(2*Sqrt[b]*Sqrt[b - Sqrt[a^2 + b^2]]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2]
)]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] - Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt
[a^2 + b*(b - Sqrt[a^2 + b^2])]*ArcTan[(-a^2 + (a^2 + 2*b*(b + Sqrt[a^2 + b
^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[b + Sqrt[a^2 + b^2]]*Sqrt[a^2 + b
*(b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*Sqrt[b
^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[
a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), ArcSin
[Tan[(e + f*x)/2]], -1] - 4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^
2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*EllipticPi[a^2/(a^2 + 2*(b^2
 + Sqrt[b^2*(a^2 + b^2)])), ArcSin[Tan[(e + f*x)/2]], -1]))/(Sqrt[a^2 + b*(
b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]))*Sqrt[Cos[(e + f
*x)/2]^2*Sec[e + f*x]]*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]
*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2]))/(32*a*(a^2 + b^2)^3*Sqrt[Sec[(e + f*
x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) + (Sqrt[Cos[e + f*x]*Sec[(e
 + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((4*(4*a^4 + a^2*b^2 -
3*b^4)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e +
f*x)/2]^2]) + (3*(5*a^2 - 2*b^2)*((2*Sqrt[b^2*(a^2 + b^2)]*Sqrt[a^2 + b*(b
- Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b^2])]*Sec[(e + f*x)/2]^2)
/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]*(1 - (a^2*Tan[(
e + f*x)/2]^2)/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]))) - (2*Sqrt[b^2*(a^2
 + b^2)]*Sqrt[a^2 + b*(b - Sqrt[a^2 + b^2])]*Sqrt[a^2 + b*(b + Sqrt[a^2 + b
^2])]*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f
*x)/2]^2]*(1 - (a^2*Tan[(e + f*x)/2]^2)/(a^2 + ...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82051 vs. $2(472) = 944$.
time = 2.09, size = 82052, normalized size = 159.32

method	result	size
default	Expression too large to display	82052

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{(a+b\tan(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3, x)
```

3.621 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=664

$$\frac{5b^{3/2}(7a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8(a^2+b^2)^{13/4}f\sqrt{d\sec(e+fx)}}-\frac{5b^{3/2}(7a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right.}{8(a^2+b^2)^{13/4}f\sqrt{d\sec(e}}$$

```
[Out] 5/8*b^(3/2)*(7*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/
4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)-5/8*b^(3/2
)*(7*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(
f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)+1/4*a*(8*a^2-37*b^2
)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellipti
cE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/f/
(d*sec(f*x+e))^(1/2)-5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)+5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*El
lipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-t
an(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)-1/4*a*(8*a^2-37*b
^2)*tan(f*x+e)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)+1/2*b*(4*a^2-5*b^2)*sec(f
*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+2*(b+a*tan(f*
x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+1/4*a*b*(8*a^2-37
*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.55, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$-\frac{5ab(7a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^{7/2}\sqrt{d\sec(e+fx)}}+\frac{5ab(7a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^{7/2}\sqrt{d\sec(e+fx)}}+\frac{a(8a^2-37b^2)\sqrt[4]{\sec^2(e+fx)}\,E\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}}+\frac{5b^{3/2}(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}+\frac{b(4a^2-5b^2)\sec^2(e+fx)}{2f(a^2+b^2)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{a(8a^2-37b^2)\tan(e+fx)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f(a^2+b^2)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{5b^{3/2}(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{13/4}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*Sec[e + f
*x]]) - (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))
/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*
Sec[e + f*x]]) + (a*(8*a^2 - 37*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
Sec[e + f*x]^2)^(1/4))/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (a*(8*a^2
 - 37*b^2)*Tan[e + f*x])/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b*
(7*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^
```

```
2 + b^2)^(7/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a*b*(7*a^2 - 2*b^2)*Cot[e + f*x
]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e
+ f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(7/2)*f*Sqrt[d*Sec[e
+ f*x]]) + (b*(4*a^2 - 5*b^2)*Sec[e + f*x]^2)/(2*(a^2 + b^2)^2*f*Sqrt[d*Sec
[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^2)*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 - 37*b^2)*Sec[
e + f*x]^2)/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\left(2b\sqrt[4]{\sec^2}\right.$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} -$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\,(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{(a^2+b}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\,(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{(a^2+b}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\,(a^2+b^2)^2\,f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{(a^2+b}$$

$$= -\frac{a(8a^2-37b^2)\tan(e+fx)}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}} + \frac{b(4a^2-5b^2)\,s}{2\,(a^2+b^2)^2\,f\sqrt{d\sec(e+}}$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{4\,(a^2+b^2)^3\,f\sqrt{d\sec(e+fx)}} -$$

$$= \frac{5b^{3/2}(7a^2-2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx}}{8\,(a^2+b^2)^{13/4}\,f\sqrt{d\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 92.40, size = 8633, normalized size = 13.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100393 vs. 2(611) = 1222.
time = 3.24, size = 100394, normalized size = 151.20

method	result	size
default	Expression too large to display	100394

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3), x)
```

3.622 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=620

$$-\frac{7b^{5/2}(9a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\,(a^2+b^2)^{15/4}\,f(d\sec(e+fx))^{3/2}}-\frac{7b^{5/2}(9a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+}}{\sqrt[4]{a^2+b^2}}\right.}{8\,(a^2+b^2)^{15/4}\,f(d\sec(e+}$$

```
[Out] -7/8*b^(5/2)*(9*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)-7/8*b^(5/
2)*(9*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec
(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)+1/12*a*(8*a^2-69*b
^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellip
ticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^3/
f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f
*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(
1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*
EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(
-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+1/6*b*(4*a^2-7*b^2)
*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2+2/3*(b+
a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2+1/12*a*b*
(8*a^2-69*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x
+e))
```

Rubi [A]
time = 0.56, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\frac{7ab^2(9a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(-\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^4(d\sec(e+fx))^{3/2}}+\frac{7ab^2(9a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\Pi\left(\frac{b}{\sqrt{a^2+b^2}};\text{ArcSin}\left(\sqrt[4]{\sec^2(e+fx)}\right)\middle|-1\right)}{8f(a^2+b^2)^4(d\sec(e+fx))^{3/2}}+\frac{a(8a^2-69b^2)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\text{ArcTan}(\tan(e+fx))\middle|2\right)}{12f(a^2+b^2)^3(d\sec(e+fx))^{3/2}}-\frac{7b^{5/2}(9a^2-2b^2)\sec^2(e+fx)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{15/4}(d\sec(e+fx))^{3/2}}+\frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12f(a^2+b^2)^3(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}+\frac{b(4a^2-7b^2)\sec^2(e+fx)}{6f(a^2+b^2)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}+\frac{2(a\tan(e+fx)+b)}{3f(a^2+b^2)(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}-\frac{7b^{5/2}(9a^2-2b^2)\sec^2(e+fx)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{15/4}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (-7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*Sec[e + f*x]
)^(3/2)) - (7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*S
ec[e + f*x])^(3/2)) + (a*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2,
 2]*(Sec[e + f*x]^2)^(3/4))/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) + (
7*a*b^2*(9*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSi
n[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2]
)/(8*(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2)) + (7*a*b^2*(9*a^2 - 2*b^2)*Cot
```

```
[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]
*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^4*f*(d*Sec[e
+ f*x])^(3/2)) + (b*(4*a^2 - 7*b^2)*Sec[e + f*x]^2)/(6*(a^2 + b^2)^2*f*(d*S
ec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/(3*(a
^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 -
69*b^2)*Sec[e + f*x]^2)/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*T
an[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
```

```
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} - \frac{(2b\sec^2}{}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6(a^2+b^2)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{}{3(a^2+}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6(a^2+b^2)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{}{3(a^2+}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6(a^2+b^2)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{}{3(a^2+}$$

$$= \frac{a(8a^2-69b^2)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12(a^2+b^2)^3f(d\sec(e+fx))^{3/2}} +$$

$$= \frac{a(8a^2-69b^2)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12(a^2+b^2)^3f(d\sec(e+fx))^{3/2}} +$$

$$= \frac{a(8a^2-69b^2)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12(a^2+b^2)^3f(d\sec(e+fx))^{3/2}} +$$

$$= \frac{a(8a^2-69b^2)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12(a^2+b^2)^3f(d\sec(e+fx))^{3/2}} +$$

$$= -\frac{7b^{5/2}(9a^2-2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)}{8(a^2+b^2)^{15/4}f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{7b^{5/2}(9a^2-2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)}{8(a^2+b^2)^{15/4}f(d\sec(e+fx))^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 69.79, size = 5131, normalized size = 8.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82288 vs. 2(573) = 1146.
time = 3.24, size = 82289, normalized size = 132.72

method	result	size
default	Expression too large to display	82289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,\left(a+b\tan\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3), x)
```

3.623 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=814

$$\frac{9b^{7/2}(11a^2-2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\,(a^2+b^2)^{17/4}\,d^2 f\,\sqrt{d\sec(e+fx)}}-\frac{9b^{7/2}(11a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt[4]{\sec^2(e}{\sqrt[4]{a^2+}}\right.}{8\,(a^2+b^2)^{17/4}\,d^2 f\,\sqrt{d\,\mathrm{se}}}$$

```
[Out] 9/8*b^(7/2)*(11*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*b
^(7/2)*(11*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))
*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/20*a*(8
*a^4+64*a^2*b^2-139*b^4)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^
2)^(1/4)/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*a*b^3*(11*a^2-2*b^2)*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^
2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2/f/(d*sec(f*x+e))^(1/2)+9
/8*a*b^3*(11*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b
^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2
/f/(d*sec(f*x+e))^(1/2)-3/20*a*(8*a^4+64*a^2*b^2-139*b^4)*tan(f*x+e)/(a^2+b
^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)+3/10*b*(4*a^4+28*a^2*b^2-15*b^4)*sec(f*x+e
)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+2/5*cos(f*x+e
)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^
2+3/20*a*b*(8*a^4+64*a^2*b^2-139*b^4)*sec(f*x+e)^2/(a^2+b^2)^4/d^2/f/(d*sec
(f*x+e))^(1/2)/(a+b*tan(f*x+e))-2/5*(b*(4*a^2-9*b^2)-a*(3*a^2+16*b^2)*tan(f
*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2
```

Rubi [A]
time = 0.69, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3593, 755, 837, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2*f*Sqrt[d*Sec[
e + f*x]]) - (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^
(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2
*f*Sqrt[d*Sec[e + f*x]]) + (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(20*(a^2 + b^2)^4*d^2*f*Sq
```

```
rt[d*Sec[e + f*x]]) - (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Tan[e + f*x])/(20
*(a^2 + b^2)^4*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9*a*b^3*(11*a^2 - 2*b^2)*Cot[
e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -
1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f
*Sqrt[d*Sec[e + f*x]]) + (9*a*b^3*(11*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/
4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
+ (3*b*(4*a^4 + 28*a^2*b^2 - 15*b^4)*Sec[e + f*x]^2)/(10*(a^2 + b^2)^3*d^2*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*Cos[e + f*x]^2*(b + a*T
an[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]
)^2) + (3*a*b*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Sec[e + f*x]^2)/(20*(a^2 + b^2
)^4*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) - (2*(b*(4*a^2 - 9*b^2
) - a*(3*a^2 + 16*b^2)*Tan[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e +
 f*x]]*(a + b*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} - \frac{\left(2b\sqrt[4]{\right.}{}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} - \frac{2(}{5\,(a^2+}$$

$$= \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{5\,(a^}$$

$$= \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{5\,(a^}$$

$$= \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2} + \frac{}{5\,(a^}$$

$$= -\frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} + \frac{3b(4a^4+}{10\left(a^2+b^2\right)^3d^2f\sqrt{}}$$

$$= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 92.92, size = 9297, normalized size = 11.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114406 vs. 2(755) = 1510.
time = 3.84, size = 114407, normalized size = 140.55

method	result	size
default	Expression too large to display	114407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}\left(a+b\tan\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3), x)
```

3.624 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=78

$$\frac{3b(d\sec(e+fx))^{5/3}}{5f}+\frac{3ad\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/5*b*(d*sec(f*x+e))^(5/3)/f+3/2*a*d*hypergeom([-1/3, 1/2],[2/3],cos(f*x+e)
^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3857, 2722}

$$\frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2/
3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx &= \frac{3b(d\sec(e+fx))^{5/3}}{5f}+a\int (d\sec(e+fx))^{5/3}\,dx\\ &= \frac{3b(d\sec(e+fx))^{5/3}}{5f}+\left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right.\\ &= \frac{3b(d\sec(e+fx))^{5/3}}{5f}+\frac{3ad\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)(d\sec(e}{2f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 126, normalized size = 1.62

$$\frac{d(d\sec(e+fx))^{2/3}\left(-10a\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+3\cos^2(e+fx)^{2/3}(4b+5a\sin(2(e+fx)))\right)(a+b\tan(e+fx))}{20f\cos^2(e+fx)^{2/3}(a\cos(e+fx)+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
[Out] (d*(d*Sec[e + f*x])^(2/3)*(-10*a*Cos[e + f*x]^3*Hypergeometric2F1[1/3, 1/2,
 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 3*(Cos[e + f*x]^2)^(2/3)*(4*b + 5*a*Si
n[2*(e + f*x)]))*(a + b*Tan[e + f*x]))/(20*f*(Cos[e + f*x]^2)^(2/3)*(a*Cos[
e + f*x] + b*Sin[e + f*x]))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*(d*sec(f*x + e)
)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^{\frac{5}{3}}(a+b\tan(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)), x)
```

3.625 $\int \sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=76

$$\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3ad\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)\sin(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3*b*(d*sec(f*x+e))^(1/3)/f-3/2*a*d*hypergeom([1/3, 1/2],[4/3],cos(f*x+e)^2)
*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3857, 2722}

$$\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3ad\sin(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, Co
s[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]^2
])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + a\int \sqrt[3]{d\sec(e+fx)}\,dx$$
$$= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + \left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\,\sqrt[3]{d\sec(e+fx)}\right)\int$$
$$= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3a\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+}}$$

Mathematica [A]
time = 0.20, size = 58, normalized size = 0.76

$$\frac{\sqrt[3]{d\sec(e+fx)}\,\left(3b+a\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(1/3)*(3*b + a*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1
/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x]))/f
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}\,\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)), x)
```

3.626 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}-\frac{3ad\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)\sin(e+fx)}{4f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] -3*b/f/(d*sec(f*x+e))^(1/3)-3/4*a*d*hypergeom([1/2, 2/3],[5/3],cos(f*x+e)^2
)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3857, 2722}

$$-\frac{3ad\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{4/3}}-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3,
 Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f*x
]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx &= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}+a\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}\,dx \\
&= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}+\left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(e+fx}{d}} \\
&= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}-\frac{3a\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2/}}{4df\sqrt{\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 11.06, size = 119, normalized size = 1.57

$$-\frac{3(b+a\cot(e+fx))\left(b\sin(e+fx)+a\cot(e+fx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(e+fx)\right)\sqrt{\sin^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}\left(b\sin(e+fx)+a\cot(e+fx)\sqrt{\sin^2(e+fx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*(b + a*Cot[e + f*x])*(b*Sin[e + f*x] + a*Cot[e + f*x]*Hypergeometric2F1
[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]
))/(f*(d*Sec[e + f*x])^(1/3)*(b*Sin[e + f*x] + a*Cot[e + f*x]*Sqrt[Sin[e +
f*x]^2]))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\tan{\left(e + fx \right)}}{\sqrt[3]{d\sec{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{tan}\left(e+f\,x\right)}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3), x)
```

3.627 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=78

$$-\frac{3b}{5f(d\sec(e+fx))^{5/3}}-\frac{3ad\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] -3/5*b/f/(d*sec(f*x+e))^(5/3)-3/8*a*d*hypergeom([1/2, 4/3],[7/3],cos(f*x+e)
^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3857, 2722}

$$-\frac{3ad\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{8/3}}-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx &= -\frac{3b}{5f(d\sec(e+fx))^{5/3}}+a\int \frac{1}{(d\sec(e+fx))^{5/3}}\,dx\\ &= -\frac{3b}{5f(d\sec(e+fx))^{5/3}}+\left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{d}\right.\\ &= -\frac{3b}{5f(d\sec(e+fx))^{5/3}}-\frac{3a\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+}}{8d^2f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 94, normalized size = 1.21

$$\frac{2a\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+3\sqrt[3]{\cos^2(e+fx)}\,(-b\cos(e+fx)+a\sin(e+fx))}{5df\sqrt[3]{\cos^2(e+fx)}\,(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 3*(Cos
[e + f*x]^2)^(1/3)*(-(b*Cos[e + f*x]) + a*Sin[e + f*x]))/(5*d*f*(Cos[e + f*
x]^2)^(1/3)*(d*Sec[e + f*x])^(2/3))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tan{\left(e + fx \right)}}{\left(d \sec{\left(e + fx \right)}\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{tan}\left(e+f\,x\right)}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3),x)
```

```
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3), x)
```

3.628 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=119

$$\frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3(8a^2-3b^2)\,d\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{16f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(\ldots$$

```
[Out] 33/40*a*b*(d*sec(f*x+e))^(5/3)/f+3/16*(8*a^2-3*b^2)*d*hypergeom([-1/3, 1/2]
,[2/3],cos(f*x+e)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
+3/8*b*(d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))/f
```

Rubi [A]
time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3589, 3567, 3857, 2722}

$$\frac{3d(8a^2-3b^2)\sin(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}}+\frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (33*a*b*(d*Sec[e + f*x])^(5/3))/(40*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometri
c2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(
16*f*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x
]))/(8*f)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
```

```
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}+\frac{3}{8}\int (d\sec(e+fx))$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+f}{8f}$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+f}{8f}$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3(8a^2-3b^2)\,d\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+\right.}{16f\sqrt{\sin^2}}$$

Mathematica [A]
time = 1.89, size = 108, normalized size = 0.91

$$\frac{(d\sec(e+fx))^{5/3}\left(15(8a^2-3b^2)\sin(2(e+fx))-5(8a^2-3b^2)\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(2(e+fx))+12b(16a+5b\tan(e+fx))\right)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
[Out] ((d*Sec[e + f*x])^(5/3)*(15*(8*a^2 - 3*b^2)*Sin[2*(e + f*x)] - 5*(8*a^2 - 3
*b^2)*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^
2]*Sin[2*(e + f*x)] + 12*b*(16*a + 5*b*Tan[e + f*x])))/(160*f)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+b\tan(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
+ e) + a^2*d*sec(f*x + e))*(d*sec(f*x + e))^(2/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/3}\,{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2, x)
```

3.629 $\int \sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=119

$$\frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}-\frac{3(4a^2-3b^2)\,d\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}}+\frac{3b\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(}{4f}$$

```
[Out] 21/4*a*b*(d*sec(f*x+e))^(1/3)/f-3/8*(4*a^2-3*b^2)*d*hypergeom([1/3, 1/2],[4
/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)+3/
4*b*(d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))/f
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3589, 3567, 3857, 2722}

$$-\frac{3d(4a^2-3b^2)\sin(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{2/3}}+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (21*a*b*(d*Sec[e + f*x])^(1/3))/(4*f) - (3*(4*a^2 - 3*b^2)*d*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(2/3
)*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]))
/(4*f)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
```

```
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))^2\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}{4f}+\frac{3}{4}\int \sqrt[3]{d\sec(e+fx)}$$

$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}{4f}$$

$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}{4f}$$

$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}-\frac{3(4a^2-3b^2)\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\mathrm{co}\right.}{8f\sqrt{\sin}}$$

Mathematica [A]
time = 0.53, size = 83, normalized size = 0.70

$$\frac{\sqrt[3]{d\sec(e+fx)}\left((4a^2-3b^2)\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\tan(e+fx)+3b(8a+b\tan(e+fx))\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
[Out] ((d*Sec[e + f*x])^(1/3)*((4*a^2 - 3*b^2)*(Cos[e + f*x]^2)^(2/3)*Hypergeomet
ric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x] + 3*b*(8*a + b*Tan[e + f
*x])))/(4*f)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos \left(e+f\,x\right)}\right)^{1/3}\,{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2, x)
```

3.630 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}-\frac{3(2a^2-3b^2)\,d\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -15/2*a*b/f/(d*sec(f*x+e))^(1/3)-3/8*(2*a^2-3*b^2)*d*hypergeom([1/2, 2/3],[
5/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)+3
/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3857, 2722}

$$-\frac{3d(2a^2-3b^2)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{4/3}}-\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-15*a*b)/(2*f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(4/
3)*Sqrt[Sin[e + f*x]^2]) + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(1/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
```

```
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3}{2}\int \frac{\frac{2a^2}{3}-b^2+\frac{5}{3}ab\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$
$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{1}{2}(2a^2-3b^2)\int \frac{1}{\sqrt[3]{d\sec(e+f}}$$
$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{1}{2}\left((2a^2-3b^2)\left(\frac{\cos(e+fx)}{d}\right.\right.$$
$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} - \frac{3(2a^2-3b^2)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)(d}{8df\sqrt{\sin^2(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4138 vs. $2(119) = 238$.
time = 31.29, size = 4138, normalized size = 34.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*b^2*Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x])^2)/(2*f*(d*Sec[e + f*
x])^(1/3)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2) + (3*Cos[e + f*x]^2*((-1)^(1
/3) + Sec[e + f*x]^(2/3) + Sqrt[3]*Sec[e + f*x]^(2/3))^14*(-1 + Sec[e + f*x
]^2)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*(-4*a*b*Sec[e + f*x] + (2*a
^2 - 3*b^2)*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^2 - ((2*a^2 - 3*b^2)*((-1
)^(1/3)*3^(1/4)*(-6*EllipticE[ArcCos[((-1)^(1/3) - (-1 + Sqrt[3])*Sec[e + f
*x]^(2/3))/((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))], (2 + Sqrt[3])/
4] - (-3 + Sqrt[3])*EllipticF[ArcCos[((-1)^(1/3) - (-1 + Sqrt[3])*Sec[e + f
*x]^(2/3))/((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))], (2 + Sqrt[3])/
4])*((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))^3*Sqrt[(((-1)^(1/3) + S
```

ec[e + f*x]^(2/3))*Sec[e + f*x]^(2/3))/((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))^2]*Sqrt[((-1)^(2/3) - (-1)^(1/3)*Sec[e + f*x]^(2/3) + Sec[e + f*x]^(4/3))/((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))^2] + 6*(1 + Sqrt[3])*Sec[e + f*x]^(2/3)*(-1 + Sec[e + f*x]^2)))/(6*Sqrt[1 - Cos[e + f*x]^2]*((-1)^(1/3) + (1 + Sqrt[3])*Sec[e + f*x]^(2/3))))*(2*a^2*Cos[e + f*x] - 3*b^2*Cos[e + f*x] + 4*a*b*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(2*f*(d*Sec[e + f*x])^(1/3)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-2*(-1)^(2/3)*a^2*Sin[e + f*x] + 3*(-1)^(2/3)*b^2*Sin[e + f*x] - 28*(-1)^(1/3)*a^2*Sec[e + f*x]^(2/3)*Sin[e + f*x] - 28*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(2/3)*Sin[e + f*x] + 42*(-1)^(1/3)*b^2*Sec[e + f*x]^(2/3)*Sin[e + f*x] + 42*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(2/3)*Sin[e + f*x] - 728*a^2*Sec[e + f*x]^(4/3)*Sin[e + f*x] - 364*Sqrt[3]*a^2*Sec[e + f*x]^(4/3)*Sin[e + f*x] + 1092*b^2*Sec[e + f*x]^(4/3)*Sin[e + f*x] + 546*Sqrt[3]*b^2*Sec[e + f*x]^(4/3)*Sin[e + f*x] + 56084*(-1)^(1/3)*a^2*Sec[e + f*x]^(8/3)*Sin[e + f*x] + 32060*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(8/3)*Sin[e + f*x] - 84126*(-1)^(1/3)*b^2*Sec[e + f*x]^(8/3)*Sin[e + f*x] - 48090*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(8/3)*Sin[e + f*x] + 305032*a^2*Sec[e + f*x]^(10/3)*Sin[e + f*x] + 176540*Sqrt[3]*a^2*Sec[e + f*x]^(10/3)*Sin[e + f*x] - 457548*b^2*Sec[e + f*x]^(10/3)*Sin[e + f*x] - 264810*Sqrt[3]*b^2*Sec[e + f*x]^(10/3)*Sin[e + f*x] - 3954808*(-1)^(1/3)*a^2*Sec[e + f*x]^(14/3)*Sin[e + f*x] - 2283424*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(14/3)*Sin[e + f*x] + 5932212*(-1)^(1/3)*b^2*Sec[e + f*x]^(14/3)*Sin[e + f*x] + 3425136*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(14/3)*Sin[e + f*x] - 9625616*a^2*Sec[e + f*x]^(16/3)*Sin[e + f*x] - 5557552*Sqrt[3]*a^2*Sec[e + f*x]^(16/3)*Sin[e + f*x] + 14438424*b^2*Sec[e + f*x]^(16/3)*Sin[e + f*x] + 8336328*Sqrt[3]*b^2*Sec[e + f*x]^(16/3)*Sin[e + f*x] + 27089920*(-1)^(1/3)*a^2*Sec[e + f*x]^(20/3)*Sin[e + f*x] + 15640768*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(20/3)*Sin[e + f*x] - 40634880*(-1)^(1/3)*b^2*Sec[e + f*x]^(20/3)*Sin[e + f*x] - 23461152*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(20/3)*Sin[e + f*x] + 32361056*a^2*Sec[e + f*x]^(22/3)*Sin[e + f*x] + 18683392*Sqrt[3]*a^2*Sec[e + f*x]^(22/3)*Sin[e + f*x] - 48541584*b^2*Sec[e + f*x]^(22/3)*Sin[e + f*x] - 28025088*Sqrt[3]*b^2*Sec[e + f*x]^(22/3)*Sin[e + f*x] - 29805440*(-1)^(1/3)*a^2*Sec[e + f*x]^(26/3)*Sin[e + f*x] - 17208128*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(26/3)*Sin[e + f*x] + 44708160*(-1)^(1/3)*b^2*Sec[e + f*x]^(26/3)*Sin[e + f*x] + 25812192*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(26/3)*Sin[e + f*x] - 24330496*a^2*Sec[e + f*x]^(28/3)*Sin[e + f*x] - 14047232*Sqrt[3]*a^2*Sec[e + f*x]^(28/3)*Sin[e + f*x] + 36495744*b^2*Sec[e + f*x]^(28/3)*Sin[e + f*x] + 21070848*Sqrt[3]*b^2*Sec[e + f*x]^(28/3)*Sin[e + f*x] + 6614272*(-1)^(1/3)*a^2*Sec[e + f*x]^(32/3)*Sin[e + f*x] + 3818752*(-1)^(1/3)*Sqrt[3]*a^2*Sec[e + f*x]^(32/3)*Sin[e + f*x] - 9921408*(-1)^(1/3)*b^2*Sec[e + f*x]^(32/3)*Sin[e + f*x] - 5728128*(-1)^(1/3)*Sqrt[3]*b^2*Sec[e + f*x]^(32/3)*Sin[e + f*x] + 1290752*a^2*Sec[e + f*x]^(34/3)*Sin[e + f*x] + 745216*Sqrt[3]*a^2*Sec[e + f*x]^(34/3)*Sin[e + f*x] - 1936128*b^2*Sec[e + f*x]^(34/3)*Sin[e + f*x] - 1117824*Sqrt[3]*b^2*Sec[e + f*x]^(34/3)*Sin[e + f*x] - 56*(-1)^(1/3)*a*b*Sec[e + f*x]^(5/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] - 56*(-1)^(1/3)*Sqrt[3]*a*b*Sec[e + f*x]^(5/3

```
)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] - 1456*a*b*Sec[e
+ f*x]^(7/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] - 728*
Sqrt[3]*a*b*Sec[e + f*x]^(7/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*S
in[e + f*x] + 112168*(-1)^(1/3)*a*b*Sec[e + f*x]^(11/3)*Sqrt[Cos[e + f*x]^2
*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] + 64120*(-1)^(1/3)*Sqrt[3]*a*b*Sec[e +
 f*x]^(11/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] + 6100
64*a*b*Sec[e + f*x]^(13/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e
 + f*x] + 353080*Sqrt[3]*a*b*Sec[e + f*x]^(13/3)*Sqrt[Cos[e + f*x]^2*(-1 +
Sec[e + f*x]^2)]*Sin[e + f*x] - 7909616*(-1)^(1/3)*a*b*Sec[e + f*x]^(17/3)*
Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*Sin[e + f*x] - 4566848*(-1)^(1/3
)*Sqrt[3]*a*b*Sec[e + f*x]^(17/3)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)
]*Sin[e + f*x] - 19251232*a*b*Sec[e + f*x]^(19/...
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
2/3)/(d*sec(f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e + fx))^2}{\sqrt[3]{d\sec(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(e + f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3), x)
```

3.631 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=119

$$\frac{3ab}{10f(d\sec(e+fx))^{5/3}}-\frac{3(2a^2+3b^2)\,d\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)\sin(e+fx)}{16f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}}-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

```
[Out] 3/10*a*b/f/(d*sec(f*x+e))^(5/3)-3/16*(2*a^2+3*b^2)*d*hypergeom([1/2, 4/3],[
7/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)-3
/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A]
time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3589, 3567, 3857, 2722}

$$-\frac{3d(2a^2+3b^2)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}\,(d\sec(e+fx))^{8/3}}+\frac{3ab}{10f(d\sec(e+fx))^{5/3}}-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (3*a*b)/(10*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometric
2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(16*f*(d*Sec[e + f*x])^(8/
3)*Sqrt[Sin[e + f*x]^2]) - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(5/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
```

```
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx &= -\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{3}{2}\int \frac{-\frac{2a^2}{3}-b^2+\frac{1}{3}ab\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx \\
&= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{1}{2}\left(-2a^2-3b^2\right)\int \frac{1}{(d\sec(e+ \\
&= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{1}{2}\Bigg(\left(-2a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+}{d}} \\
&= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3(2a^2+3b^2)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)}{16d^2f\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.57, size = 119, normalized size = 1.00

$$\frac{\sec^2(e+fx)\left(-6ab-6ab\cos(2(e+fx))+3a^2\sin(2(e+fx))-3b^2\sin(2(e+fx))+2(2a^2+3b^2)\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\tan(e+fx)\right)}{10f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (Sec[e + f*x]^2*(-6*a*b - 6*a*b*Cos[2*(e + f*x)] + 3*a^2*Sin[2*(e + f*x)] -
 3*b^2*Sin[2*(e + f*x)] + 2*(2*a^2 + 3*b^2)*(Cos[e + f*x]^2)^(2/3)*Hypergeo
metric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x]))/(10*f*(d*Sec[e + f*
x])^(5/3))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^{2}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3)/(d^2*sec(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})^2}{(d\sec{(e + fx)})^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3), x)
```

3.632 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}\,f\sec^2(e+fx)^{5/6}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right.}{2b^{2/3}\sqrt[6]{a^2+b^2}\,f\sec^2(e+f}$$

```
[Out] -arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3)
/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)+1/4*ln((a^2+b^2)^(1/3)-b^(1
/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*s
ec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)-1/4*ln((a^2
+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e
)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(
5/6)+1/2*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/
6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x
+e)^2)^(5/6)+1/2*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b
^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(1/6)/f/(
sec(f*x+e)^2)^(5/6)+AppellF1(1/2,1,1/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)
^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(5/6)
```

Rubi [A]
time = 0.62, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3593, 771, 440, 455, 65, 302, 648, 632, 210, 642, 214}

$$\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sec^2(e+fx)^{5/6}}-\frac{\sqrt{3}(d\sec(e+fx))^{5/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}+\frac{\sqrt{3}(d\sec(e+fx))^{5/3}\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}+\frac{(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}-\frac{(d\sec(e+fx))^{5/3}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}-\frac{(d\sec(e+fx))^{5/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3
]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 + b^2)^(1/6)*f*
(Sec[e + f*x]^2)^(5/6)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f
*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/
3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) - (ArcTanh[(b^(1/3)*(Sec[e +
 f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 +
b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^
2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*
Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6))
 - (Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6
) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2
 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 1, 1/6, 3/2, (b^2*
Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(5/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\right) dx, x, b\,\text{ta}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} + \frac{\left(a(d\sec(\right.}{}$$

$$(d\,\text{se}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} + \frac{}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} + \frac{(3b(}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} - \frac{(d\,\text{se}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}\,f\sec^2(e+fx)^{5/6}} + \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}\,f\sec^2(e+fx)^{5/6}} + \frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2}\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}\,f\sec^2(e+fx)^{5/6}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{}}{}}{}\right.}{2b^{2/3}\sqrt[6]{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.22, size = 276, normalized size = 0.50

$$-\frac{24d^2F_1\left(\frac{1}{3};\frac{1}{6},\frac{1}{6};\frac{4}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a+b\tan(e+fx))}{bf\sqrt[3]{d\sec(e+fx)}\left((a+ib)F_1\left(\frac{4}{3};\frac{1}{6},\frac{7}{6};\frac{7}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)F_1\left(\frac{4}{3};\frac{7}{6},\frac{1}{6};\frac{7}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+8F_1\left(\frac{1}{3};\frac{1}{6},\frac{1}{6};\frac{4}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a+b\tan(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-24*d^2*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*(d*Sec[e + f*x])^(1/3
)*((a + I*b)*AppellF1[4/3, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x]), (
a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3, (a
- I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*AppellF1[1
/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+b\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(5/3)/(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)),x)
```

```
[Out] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)), x)
```

3.633 $\int \frac{\sqrt[3]{d \sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=552

$$\frac{\sqrt{3}\, b^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}-\frac{\sqrt{3}\, b^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right.}{2\left(a^2+b^2\right)^{5/6} f\sqrt[6]{\sec^2(e-}}$$

```
[Out] -b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e
))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)+1/4*b^(2/3)*ln((a^2+b^2)^(1
/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/
3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)-1/4*b^(2/3)
*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(s
ec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(
1/6)-1/2*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+
b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(5/6)/f/(sec(f*x
+e)^2)^(1/6)-1/2*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6
)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(5/6)/f/(
sec(f*x+e)^2)^(1/6)+AppellF1(1/2,1,5/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)
^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(1/6)
```

Rubi [A]

time = 0.54, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3593, 771, 440, 455, 65, 216, 648, 632, 210, 642, 214}

$$\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}\,F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[6]{\sec^2(e+fx)}}+\frac{\sqrt{3}\,b^{2/3}\sqrt[3]{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{\sqrt{3}\,b^{2/3}\sqrt[3]{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}+\frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*(a^2 + b^2)^(5/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[
e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*
(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) - (b^(2/3)*ArcTanh[(b^(1/3)*(Se
c[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3))/((a^2 + b^2
)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (b^(2/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3
)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)
]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) -
(b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)
^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 +
b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan
```

```
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(1/3)*Tan[e + f*x])/(a*
f*(Sec[e + f*x]^2)^(1/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\right) dx, x, b\tan(e-\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(a\sqrt[3]{d\sec(e\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\sqrt[3]{d\,\mathrm{se}}}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(3b\sqrt[3]{\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(b\sqrt[3]{\right.}{}$$

$$= -\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6}\,f\sqrt[6]{\sec^2(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+f}{a^2}\right.}{}$$

$$= -\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6}\,f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\right.}{}$$

$$= \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\,(a^2+b^2)^{5/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1}{-}\right.}{2\,(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.66, size = 280, normalized size = 0.51

$$-\frac{48d^2F_1\left(\frac{5}{3};\frac{5}{6},\frac{5}{6};\frac{8}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a+b\tan(e+fx))}{5bf(d\sec(e+fx))^{5/3}\left(5(a+ib)F_1\left(\frac{8}{3};\frac{5}{6},\frac{11}{6};\frac{11}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)F_1\left(\frac{8}{3};\frac{11}{6},\frac{5}{6};\frac{11}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+16F_1\left(\frac{5}{3};\frac{5}{6},\frac{5}{6};\frac{8}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a+b\tan(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-48*d^2*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(5*b*f*(d*Sec[e + f*x])^(5
/3)*(5*(a + I*b)*AppellF1[8/3, 5/6, 11/6, 11/3, (a - I*b)/(a + b*Tan[e + f*
x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[8/3, 11/6, 5/6,
 11/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 16
*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx+e))^{\frac{1}{3}}}{a+b \tan (fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{d \sec (e+fx)}}{a+b \tan (e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+b\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)),x)
```

```
[Out] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)), x)
```

3.634 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=579

$$\frac{3b}{(a^2+b^2)\,f\,\sqrt[3]{d\sec(e+fx)}}-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\,(a^2+b^2)^{7/6}\,f\,\sqrt[3]{d\sec(e+fx)}}+\frac{\sqrt{3}\,b^{4/3}\text{ArcT}}{}$$

```
[Out] 3*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)-b^(4/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)
^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e
))^(1/3)+1/4*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)
^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6
)/f/(d*sec(f*x+e))^(1/3)-1/4*b^(4/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(
1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6
)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arctan(-1/3*3^(1/2)+2/
3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(1/6
)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arctan(1/3*3^(
1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^
2)^(1/6)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+AppellF1(1/2,1,7/6,
3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)/a/f
/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.59, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 771, 440, 455, 53, 65, 302, 648, 632, 210, 642, 214}

$$\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}\,F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[3]{d\sec(e+fx)}}-\frac{\sqrt{3}\,b^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)}{2f\,(a^2+b^2)^{7/6}\,\sqrt[3]{d\sec(e+fx)}}+\frac{\sqrt{3}\,b^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f\,(a^2+b^2)^{7/6}\,\sqrt[3]{d\sec(e+fx)}}+\frac{3b}{f\,(a^2+b^2)\,\sqrt[3]{d\sec(e+fx)}}+\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\log\left(-\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\,(a^2+b^2)^{7/6}\,\sqrt[3]{d\sec(e+fx)}}-\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\log\left(\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\,(a^2+b^2)^{7/6}\,\sqrt[3]{d\sec(e+fx)}}-\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f\,(a^2+b^2)^{7/6}\,\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/3)) - (Sqrt[3]*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) + (Sqrt
[3]*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*
(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e
 + f*x])^(1/3)) - (b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 +
b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/((a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(
1/3)) + (b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4
*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) - (b^(4/3)*Log[(a^2 + b^2)^(1/
3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*
x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x]
```

```
)^(1/3)) + (AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))}\,dx = \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\right.\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\,\tan}{af\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan\right.}{af\sqrt[3]{}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan\right.}{af\sqrt[3]{}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan\right.}{af\sqrt[3]{}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx}}{\sqrt[6]{a^2+b^2}}\right.}{(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx}}{\sqrt[6]{a^2+b^2}}\right.}{(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} - \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(}}{\sqrt[6]{a^2+}}}{\sqrt{3}}\right.}{2\,(a^2+b^2)^{7/6}\,f\sqrt[3]{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 79.07, size = 285, normalized size = 0.49

$$-\frac{60dF_1\left(\frac{7}{3};\frac{7}{6},\frac{7}{6};\frac{10}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a\cos(e+fx)+b\sin(e+fx))}{7bf(d\sec(e+fx))^{4/3}\left(7(a+ib)F_1\left(\frac{10}{3};\frac{7}{6},\frac{13}{6};\frac{13}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+7(a-ib)F_1\left(\frac{10}{3};\frac{13}{6},\frac{7}{6};\frac{13}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+20F_1\left(\frac{7}{3};\frac{7}{6},\frac{7}{6};\frac{10}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a+b\tan(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (-60*d*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(7*b*f*(d*Sec[
e + f*x])^(4/3)*(7*(a + I*b)*AppellF1[10/3, 7/6, 13/6, 13/3, (a - I*b)/(a +
 b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 7*(a - I*b)*AppellF1[10
/3, 13/6, 7/6, 13/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
 + f*x])] + 20*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{d\sec (e+fx)}\ (a+b\tan (e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{1/3}\,\left(a+b\tan\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))), x)
```

3.635 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=581

$$\frac{3b}{5\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}}+\frac{\sqrt{3}\,b^{8/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}}-$$

```
[Out] 3/5*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)-b^(8/3)*arctanh(b^(1/3)*(sec(f*x+e)^
2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*
x+e))^(5/3)+1/4*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x
+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(
11/6)/f/(d*sec(f*x+e))^(5/3)-1/4*b^(8/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^
2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^
(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)*arctan(-1/3*3^(1/
2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)
^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)*arctan(1
/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f
*x+e)^2)^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)+AppellF1(1/2
,1,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan(f*
x+e)/a/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A]
time = 0.58, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 771, 440, 455, 53, 65, 216, 648, 632, 210, 642, 214}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af(d\sec(e+fx))^{5/3}}+\frac{\sqrt{3}\,b^{8/3}\sec^2(e+fx)^{5/6}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}-\frac{\sqrt{3}\,b^{8/3}\sec^2(e+fx)^{5/6}\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}+\frac{3b}{5f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}}+\frac{b^{8/3}\sec^2(e+fx)^{5/6}\log\left(-\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}-\frac{b^{8/3}\sec^2(e+fx)^{5/6}\log\left(\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}-\frac{b^{8/3}\sec^2(e+fx)^{5/6}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*b)/(5*(a^2 + b^2)*f*(d*Sec[e + f*x])^(5/3)) + (Sqrt[3]*b^(8/3)*ArcTan[1/
Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(
Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (S
qrt[3]*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[
3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*S
ec[e + f*x])^(5/3)) - (b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^
2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/((a^2 + b^2)^(11/6)*f*(d*Sec[e + f*
x])^(5/3)) + (b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Se
c[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6
))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (b^(8/3)*Log[(a^2 + b^
2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[
e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e
```

```
 + f*x])^(5/3)) + (AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(
5/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{1}}\right.\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{F_1\left(\frac{1}{2}; 1, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\,\text{ta}}{af(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 1, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\right.}{a}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 1, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\right.}{a}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 1, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\right.}{a}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e-}}{\sqrt[6]{a^2+b^2}}\right.}{(a^2+b^2)^{11/6}f(d\,\text{s}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e-}}{\sqrt[6]{a^2+b^2}}\right.}{(a^2+b^2)^{11/6}f(d\,\text{s}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{\sqrt{3}\,b^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{\text{s}}}{\sqrt[6]{a}}}{\sqrt{}}\right.}{2(a^2+b^2)^{11/6}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6862 vs. $2(581) = 1162$.

time = 129.53, size = 6862, normalized size = 11.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d \sec \left(f x +e \right)\right)^{\frac{5}{3}} \left(a +b \tan \left(f x +e \right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d \sec \left(e + f x\right)\right)^{\frac{5}{3}} \left(a + b \tan \left(e + f x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}\,\left(a+b\,\tan\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))), x)
```

3.636 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=687

$$-\frac{a\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}\,b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}+\frac{a\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)(d\,\mathrm{se}}{2\sqrt{3}\,b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx}$$

```
[Out] -1/3*a*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))
^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)+1/12*a*ln((a^2+b^2)^(
1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1
/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)-1/
12*a*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3
)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec
(f*x+e)^2)^(5/6)+1/6*a*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)
/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(s
ec(f*x+e)^2)^(5/6)*3^(1/2)+1/6*a*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)
^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(
7/6)/f/(sec(f*x+e)^2)^(5/6)*3^(1/2)+AppellF1(1/2,2,1/6,3/2,b^2*tan(f*x+e)^2
/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a^2/f/(sec(f*x+e)^2)^(5
/6)+1/3*b^2*AppellF1(3/2,2,1/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*s
ec(f*x+e))^(5/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(5/6)-a*b*(d*sec(f*x+e))
^(5/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Rubi [A]
time = 0.68, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3593, 771, 440, 455, 44, 65, 302, 648, 632, 210, 642, 214, 524}

$$\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f\sec^2(e+fx)^{5/6}}-\frac{a(d\sec(e+fx))^{5/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}\,b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}+\frac{a(d\sec(e+fx))^{5/3}\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\,\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}-\frac{ab(d\sec(e+fx))^{5/3}}{f\left(a^2+b^2\right)\left(a^2-b^2\tan^2(e+fx)\right)}+\frac{a(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}-\frac{a(d\sec(e+fx))^{5/3}\log\left(\sqrt[3]{b}\,\sqrt[6]{a^2+b^2}\,\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}-\frac{a(d\sec(e+fx))^{5/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}+\frac{b^2\tan^3(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sec^2(e+fx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(a*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2
 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(Sqrt[3]*b^(2/3)*(a^2 + b^2)^(7/6)*
f*(Sec[e + f*x]^2)^(5/6)) + (a*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^
2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b
^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) - (a*ArcTanh[(b^(1/3)*(S
ec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(3*b^(2/3)
*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (a*Log[(a^2 + b^2)^(1/3) - b
^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^
(1/3)]*(d*Sec[e + f*x])^(5/3))/(12*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x
]^2)^(5/6)) - (a*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
```

```
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(1
2*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 2, 1
/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*
Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(5/6)) + (b^2*AppellF1[3/2, 2, 1/6, 5
/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e
 + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(5/6)) - (a*b*(d*Sec[e + f*x])^(5/3))/
((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}} - \frac{2ax}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{}{(-a^2+x^2)^2}\right.\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} - \frac{(2a(d\mathrm{s}}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F}{}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+f}{a^2}\right.}{}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\right.}{}$$

$$= -\frac{a\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}\,b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\mathrm{s}}}{\sqrt[6]{}}}{}\right.}{2\sqrt{3}\,b^{2/3}(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 115.75, size = 3398, normalized size = 4.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(5/3)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
b*Cos[e + f*x])/(a*(a - I*b)*(a + I*b)) + Sin[e + f*x]/((a - I*b)*(a + I*b)
) - b/((a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x])))))/(f*(a + b*T
an[e + f*x])^2) - (4*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e +
f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(d*Sec[e + f*x])^(5/3)*(a*Cos[e + f*
x] + b*Sin[e + f*x])^2)/(a*b*f*(a + b*Tan[e + f*x])*((a + I*b)*AppellF1[4/3
, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f
*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x
]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I
*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*
x]))) - (Sec[e + f*x]*(d*Sec[e + f*x])^(5/3)*((6*(b + a*Sqrt[1 - Cos[e + f*
x]^2]*Sec[e + f*x]))/((a^2 + b^2)*Sec[e + f*x]^(1/3)) + (132*a*b^(2/3)*(5*a
^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2
)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(8/3) - 240*a*b^(8/3)*
AppellF1[11/6, 1/2, 1, 17/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(14/3) - 55*(-1)^(1/6)*(a^2 - b^2
)*(a^2 + b^2)^(5/6)*(-2*ArcTan[Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]
^(1/3))/(a^2 + b^2)^(1/6)] + 2*ArcTan[Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e
 + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 4*ArcTan[((-1)^(1/6)*b^(1/3)*Sec[e + f*
x]^(1/3))/(a^2 + b^2)^(1/6)] + Sqrt[3]*Log[(a^2 + b^2)^(1/3) - (-1)^(1/6)*S
qrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Se
c[e + f*x]^(2/3)] - Sqrt[3]*Log[(a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1
/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^
(2/3)])*Sqrt[1 - Sec[e + f*x]^2])/(220*b^(2/3)*(a^2 + b^2)^2*Sqrt[1 - Sec[e
 + f*x]^2]))*(Cos[e + f*x] - Sin[e + f*x])*(Cos[e + f*x] + Sin[e + f*x])*(a
*Cos[e + f*x] + b*Sin[e + f*x]))/(6*a*f*(a + b*Tan[e + f*x])^2*((-2*Sec[e +
 f*x]^(2/3)*(b + a*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])*Sin[e + f*x])/(a^
2 + b^2) + (Sec[e + f*x]^2*(132*a*b^(2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2
, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e
 + f*x]^2]*Sec[e + f*x]^(8/3) - 240*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6,
Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*
Sec[e + f*x]^(14/3) - 55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(5/6)*(-2*ArcTa
n[Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] +
2*ArcTan[Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1
/6)] + 4*ArcTan[((-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)]
+ Sqrt[3]*Log[(a^2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1
/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)] - Sqrt[3]*L
```

```
og[(a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e +
 f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)])*Sqrt[1 - Sec[e + f*x]
^2])*Tan[e + f*x])/(220*b^(2/3)*(a^2 + b^2)^2*(1 - Sec[e + f*x]^2)^(3/2)) +
 (6*((a*Sin[e + f*x])/Sqrt[1 - Cos[e + f*x]^2] + a*Sqrt[1 - Cos[e + f*x]^2]
*Sec[e + f*x]*Tan[e + f*x]))/((a^2 + b^2)*Sec[e + f*x]^(1/3)) + ((132*a*b^(
2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e
 + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^(5/3)*Sin[e + f*x])/Sqrt[1 - Cos[e + f
*x]^2] - (240*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6, Sec[e + f*x]^2, (b^2*S
ec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^(11/3)*Sin[e + f*x])/Sqrt[1 - Cos[
e + f*x]^2] + 352*a*b^(2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec
[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec
[e + f*x]^(11/3)*Sin[e + f*x] - 1120*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6,
 Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]
*Sec[e + f*x]^(17/3)*Sin[e + f*x] - 55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(
5/6)*Sqrt[1 - Sec[e + f*x]^2]*((4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin
[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + (Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e
 + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^
(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + (Sqrt[3] + (2*(-1)^(1/6)*b^(1
/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[
e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + ((-1)^(1/3)*b^(2/3)*
Sec[e + f*x]^(2/3))/(a^2 + b^2)^(1/3))) + (Sqrt[3]*(-(((-1)^(1/6)*b^(1/3)*(
a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3]) + (2*(-1)^(1/3)*
b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e + f*x])/3))/((a^2 + b^2)^(1/3) - (-1)^(1/6
)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)
*Sec[e + f*x]^(2/3)) - (Sqrt[3]*(((-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[
e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3] + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(
5/3)*Sin[e + f*x])/3))/((a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2
 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)))
+ (55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(5/6)*(-2*ArcTan[Sqrt[3] - (2*(-1)
^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^...
```

Maple [F]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (fx+e))^{\frac{5}{3}}}{(a+b \tan (fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2, x)
```

3.637 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=687

$$\frac{5ab^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}-\frac{5ab^{2/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right.}{2\sqrt{3}\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+}}$$

```
[Out] -5/3*a*b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec
(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)+5/12*a*b^(2/3)*ln((a
^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x
+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)-
5/12*a*b^(2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1
/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(
sec(f*x+e)^2)^(1/6)-5/6*a*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+
e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/
f/(sec(f*x+e)^2)^(1/6)*3^(1/2)-5/6*a*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)
*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)/(a^2+b^
2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)*3^(1/2)+AppellF1(1/2,2,5/6,3/2,b^2*tan(f*x
+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a^2/f/(sec(f*x+e)^
2)^(1/6)+1/3*b^2*AppellF1(3/2,2,5/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)
*(d*sec(f*x+e))^(1/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(1/6)-a*b*(d*sec(f*
x+e))^(1/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Rubi [A]
time = 0.62, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3593, 771, 440, 455, 44, 65, 216, 648, 632, 210, 642, 214, 524}

$$\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}\,F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f\sqrt[6]{\sec^2(e+fx)}}+\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{ab\sqrt[3]{d\sec(e+fx)}}{f(a^2+b^2)(a^2-b^2\tan^2(e+fx))}+\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}+\frac{b^2\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (5*a*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]
*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*Sqrt[3]*(a^2 + b^2)^(11/6)*
f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2
*Sqrt[3]*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan
h[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3
))/(3*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (5*a*b^(2/3)*Log[(a^2
+ b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(
Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Se
```

```
c[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[
e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (Appell
F1[1/2, 2, 5/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e +
f*x])^(1/3)*Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(1/6)) + (b^2*AppellF1[3/
2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])
^(1/3)*Tan[e + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(1/6)) - (a*b*(d*Sec[e + f
*x])^(1/3))/((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\right.}\right.\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(2a\sqrt[3]{d\,\text{se}\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{5}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1}{}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{5}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1}{}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{5}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1}{}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{5}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1}{}$$

$$= -\frac{5ab^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{5}{6}; \frac{3}{2}; \frac{b^2\tan^2(}{a}\right.}{}$$

$$= -\frac{5ab^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} + \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b\right.}{}$$

$$= \frac{5ab^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} - \frac{5ab^{2/3}\tan^{-1}\left(\frac{1+\frac{2}{}}{}\right.}{2\sqrt{3}\,(a}$$

Mathematica [C] Result contains complex when optimal does not.

time = 73.63, size = 4485, normalized size = 6.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(1/3)*((5*(-1)^(5/6)*a*b^(2/3)*(-2*ArcTan[Sqrt[3] - (2*(-
1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 2*ArcTan[Sqrt[3]
+ (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 4*ArcTan[(
(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] - Sqrt[3]*Log[(a^
2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^
(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)] + Sqrt[3]*Log[(a^2 + b^2)^(1
/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1
)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)]))/(12*(a - I*b)*(a + I*b)*(a^2 + b^2)^(
5/6)) + 3*((-2*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e +
 f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(10/3))/(21*(a^
2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) + (Sec[e + f*x]^(1/3)*((-(a*b) + b^2*S
qrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 + b^2) + (7*(3*a^2 - 2*b^2)*Appe
llF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sq
rt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + Sec[e + f*x]^2)*(7*(a^2 + b^2)*
AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)
] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^
2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (
b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2))))/(3*(a^2 - b^2*(-1 + Se
c[e + f*x]^2))))))/(f*(a + b*Tan[e + f*x])^2*((5*(-1)^(5/6)*a*b^(2/3)*((4*(
-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1
+ (Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2
)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^
(1/6)*(1 + (Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)
^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^
2 + b^2)^(1/6)*(1 + ((-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3))/(a^2 + b^2)^(1/
3))) - (Sqrt[3]*(-(((-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)
*Sin[e + f*x])/Sqrt[3]) + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e +
f*x])/3))/((a^2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)
*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)) + (Sqrt[3]*(((
-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3
] + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e + f*x])/3))/((a^2 + b^2)
^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) +
(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3))))/(12*(a - I*b)*(a + I*b)*(a^2 + b^2
)^(5/6)) + 3*((-2*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[
e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(19/3)*Sin[e
 + f*x])/(21*(a^2 + b^2)^2*(1 - Sec[e + f*x]^2)^(3/2)) - (2*b^2*AppellF1[7/
6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e +
f*x]^(7/3)*Sin[e + f*x])/(21*(a^2 + b^2)^2*Sqrt[1 - Cos[e + f*x]^2]*Sqrt[1
```

```
- Sec[e + f*x]^2]) - (20*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(13/3
)*Sin[e + f*x])/(63*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) + (2*b^2*Sec[e
+ f*x]^(10/3)*((-(a*b) + b^2*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 +
b^2) + (7*(3*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*S
ec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + S
ec[e + f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[
e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3
/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x
]^2)))*Sin[e + f*x])/(3*(a^2 - b^2*(-1 + Sec[e + f*x]^2))^2) + (Sec[e + f*x
]^(4/3)*((-(a*b) + b^2*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 + b^2) +
 (7*(3*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e +
 f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + Sec[e +
 f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec
[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*
x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1,
 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)))
*Sin[e + f*x])/(9*(a^2 - b^2*(-1 + Sec[e + f*x]^2))) - (2*b^2*Sqrt[1 - Cos[
e + f*x]^2]*Sec[e + f*x]^(10/3)*((14*b^2*AppellF1[13/6, 1/2, 2, 19/6, Sec[e
 + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(
13*(a^2 + b^2)) + (7*AppellF1[13/6, 3/2, 1, 19/6, Sec[e + f*x]^2, (b^2*Sec[
e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/13))/(21*(a^2 + b^2)^
2*Sqrt[1 - Sec[e + f*x]^2]) + (Sec[e + f*x]^(1/3)*((7*(3*a^2 - 2*b^2)*Appel
lF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sin
[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + Sec[e + f*x]^2)*(7*(a^2 + b^2)*A
ppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
 + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2
)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2,...
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{(d \sec (f x+e))^{\frac{1}{3}}}{(a+b \tan (f x+e))^{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{d \sec{(e + f x)}}}{(a + b \tan{(e + f x)})^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a + b \tan(e + f\,x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2, x)
```

3.638 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=715

$$\frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}}-\frac{7ab^{4/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}}+\frac{7ab^{4/3}\text{ArcTan}}{2}$$

```
[Out] 7*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/3)-7/3*a*b^(4/3)*arctanh(b^(1/3)*(sec
(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(
d*sec(f*x+e))^(1/3)+7/12*a*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/
6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/
(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)-7/12*a*b^(4/3)*ln((a^2+b^2)^(1/3)+b
^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(
sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)+7/6*a*b^(4/3)*a
rctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2)
)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)*3^(1/2)+7/6*
a*b^(4/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/
6)*3^(1/2))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)*3^
(1/2)+AppellF1(1/2,2,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e
)^2)^(1/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(1/3)+1/3*b^2*AppellF1(3/2,2,7/6
,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)^3/
a^4/f/(d*sec(f*x+e))^(1/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)/(a^2-b^2*ta
n(f*x+e)^2)
```

Rubi [A]
time = 0.72, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3593, 771, 440, 455, 44, 53, 65, 302, 648, 632, 210, 642, 214, 524}

$$\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}\,F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2 f\sqrt[3]{d\sec(e+fx)}}-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f(a^2+b^2)^{13/6}\sqrt[3]{d\sec(e+fx)}}+\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f(a^2+b^2)^{13/6}\sqrt[3]{d\sec(e+fx)}}+\frac{7ab}{f(a^2+b^2)^2\sqrt[3]{d\sec(e+fx)}}-\frac{ab}{f(a^2+b^2)\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}+\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{13/6}\sqrt[3]{d\sec(e+fx)}}-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{13/6}\sqrt[3]{d\sec(e+fx)}}-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f(a^2+b^2)^{13/6}\sqrt[3]{d\sec(e+fx)}}+\frac{b^2\tan^3(e+fx)\sqrt[6]{\sec^2(e+fx)}\,F_1\left(\frac{3}{2};2,\frac{7}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4 f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (7*a*b)/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)
) + (7*a*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13
/6)*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]
^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/(3*(a^2 + b^2)^(13/6)
*f*(d*Sec[e + f*x])^(1/3)) + (7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(
a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(
```

```
Sec[e + f*x]^2)^(1/6))/(12*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) - (
7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]
^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(12*(a^
2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) + (AppellF1[1/2, 2, 7/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]
)/(a^2*f*(d*Sec[e + f*x])^(1/3)) + (b^2*AppellF1[3/2, 2, 7/6, 5/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]^3)/(
3*a^4*f*(d*Sec[e + f*x])^(1/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/
3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\right.\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{7}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\,\tan}{a^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{7}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\,\tan}{a^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{7}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan\right.}{a^2 f\sqrt[3]{}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{7}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan\right.}{a^2 f\sqrt[3]{}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{7}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan\right.}{a^2 f\sqrt[3]{}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+}}{\sqrt[6]{a^2+b^2}}\right.}{3(a^2+b^2)^{13/6} f\sqrt[3]{d}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\sec^2(e+}}{\sqrt[6]{a^2+b^2}}\right.}{3(a^2+b^2)^{13/6} f\sqrt[3]{d}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{\sec^2(e}}{\sqrt[6]{a^2+b}}}{\sqrt{3}}\right.}{2\sqrt{3}\,(a^2+b^2)^{13/6} f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 148.11, size = 9626, normalized size = 13.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2), x)
```

3.639 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=717

$$\frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}}+\frac{11ab^{8/3}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}-\frac{11ab^{8/3}\text{A}\cdots}{\cdots}$$

```
[Out] 11/5*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(5/3)-11/3*a*b^(8/3)*arctanh(b^(1/3)*
(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)
/f/(d*sec(f*x+e))^(5/3)+11/12*a*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2
)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(
5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/12*a*b^(8/3)*ln((a^2+b^2)^(
1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1
/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/6*a*b^
(8/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*
3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)*3^(1/
2)-11/6*a*b^(8/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+
b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^
(5/3)*3^(1/2)+AppellF1(1/2,2,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(
sec(f*x+e)^2)^(5/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(5/3)+1/3*b^2*AppellF1(
3/2,2,11/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan
(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(5/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)/(
a^2-b^2*tan(f*x+e)^2)
```

Rubi [A]
time = 0.69, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3593, 771, 440, 455, 44, 53, 65, 216, 648, 632, 210, 642, 214, 524}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2 f(d\sec(e+fx))^{5/3}}+\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}+\frac{11ab}{5f\left(a^2+b^2\right)^2(d\sec(e+fx))^{5/3}}-\frac{ab}{f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}+\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}+\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{3}{2};2,\frac{11}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4 f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (11*a*b)/(5*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*ArcTan[
1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]
*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(
5/3)) - (11*a*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))
/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2
)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) - (11*a*b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e
+ f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/(3*(a^2 + b^2)^
(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^
(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(
```

```
1/3)]*(Sec[e + f*x]^2)^(5/6))/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/
3)) - (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[
e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))
/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (AppellF1[1/2, 2, 11/6,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan
[e + f*x])/(a^2*f*(d*Sec[e + f*x])^(5/3)) + (b^2*AppellF1[3/2, 2, 11/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e +
 f*x]^3)/(3*a^4*f*(d*Sec[e + f*x])^(5/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e +
 f*x])^(5/3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int\left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right.}\right.\right.}{bf(d\sec(e+fx}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\,\mathrm{t}}{a^2f(d\sec(e+fx))^{5/3}}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\,\mathrm{t}}{a^2f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}\right.}{a}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}\right.}{a}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2}; 2, \frac{11}{6}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}\right.}{a}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} - \frac{11ab^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\mathrm{se}}}{\sqrt[6]{a}}\right.}{3(a^2+b^2)^{17/6}}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} - \frac{11ab^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt[6]{\mathrm{se}}}{\sqrt[6]{a}}\right.}{3(a^2+b^2)^{17/6}}$$

$$= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} + \frac{11ab^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[6]{}}{\sqrt[6]{}}}{}\right.}{2\sqrt{3}\,(a^2+b^2)^{1}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 74.78, size = 5235, normalized size = 7.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(f x+e\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d \sec \left(e+f x\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(e+f x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2),x)
```

```
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2), x)
```

3.640 $\int (d\sec(e+fx))^m (a+b\tan(e+fx))^3 \, dx$

Optimal. Leaf size=173

$$-\frac{a(3b^2-a^2(1+m))\ {}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{f(1+m)}+\frac{b(d\,\mathrm{s}}{}$$

```
[Out] -a*(3*b^2-a^2*(1+m))*hypergeom([1/2, 1-1/2*m],[3/2],-tan(f*x+e)^2)*(d*sec(f
*x+e))^m*tan(f*x+e)/f/(1+m)/((sec(f*x+e)^2)^(1/2*m))+b*(d*sec(f*x+e))^m*(a+
b*tan(f*x+e))^2/f/(2+m)-b*(d*sec(f*x+e))^m*(2*(1+m)*(b^2-a^2*(3+m))-a*b*m*(
4+m)*tan(f*x+e))/f/m/(m^2+3*m+2)
```

Rubi [A]
time = 0.15, antiderivative size = 167, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3593, 757, 794, 251}

$$\frac{a\left(a^2-\frac{3b^2}{m+1}\right)\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\ {}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f}-\frac{b(d\sec(e+fx))^m\left(2(m+1)\left(b^2-a^2(m+3)\right)-abm(m+4)\tan(e+fx)\right)}{fm\left(m^2+3m+2\right)}+\frac{b(a+b\tan(e+fx))^2(d\sec(e+fx))^m}{f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 + m))*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*
x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/(f*(Sec[e + f*x]^2)^(m/2)) + (b*(d*S
ec[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 + m)) - (b*(d*Sec[e + f*x])^m*
(2*(1 + m)*(b^2 - a^2*(3 + m)) - a*b*m*(4 + m)*Tan[e + f*x]))/(f*m*(2 + 3*m
 + m^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int (a+x)^3\left(1+\frac{x}{b}\right.\right.}{bf}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} + \frac{\left(b(d\sec(e+fx))^m\,\mathrm{s}\right.}{}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} - \frac{b(d\sec(e+fx))^m\,(2}{}$$

$$= -\frac{a(3b^2-a^2(1+m))\,{}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)(d\sec(}{f(1+m)}$$

Mathematica [A]
time = 6.49, size = 334, normalized size = 1.93

$$\frac{b^3\cos(e+fx)(d\sec(e+fx))^m(a+b\tan(e+fx))^3}{f(2+m)(a\cos(e+fx)+b\sin(e+fx))^3} - \frac{b(-3a^2+b^2)\cos^3(e+fx)(d\sec(e+fx))^m(a+b\tan(e+fx))^3}{fm(a\cos(e+fx)+b\sin(e+fx))^3} + \frac{a(a^2-3b^2)\cos^4(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{m}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^m\sin(e+fx)(a+b\tan(e+fx))^3}{f(a\cos(e+fx)+b\sin(e+fx))^3} + \frac{3ab^2\cos^2(e+fx)^{\frac{1}{2}+\frac{2+m}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{3}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^m\sin(e+fx)(a+b\tan(e+fx))^3}{f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (b^3*Cos[e + f*x]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3)/(f*(2 + m)*(a*
Cos[e + f*x] + b*Sin[e + f*x])^3) - (b*(-3*a^2 + b^2)*Cos[e + f*x]^3*(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^3)/(f*m*(a*Cos[e + f*x] + b*Sin[e + f*x])
^3) + (a*(a^2 - 3*b^2)*Cos[e + f*x]^4*(Cos[e + f*x]^2)^(-1/2 + m/2)*Hyperge
ometric2F1[1/2, (1 + m)/2, 3/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Sin[e +
```

```
f*x]*(a + b*Tan[e + f*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3) + (3*a
*b^2*(Cos[e + f*x]^2)^(1/2 + (2 + m)/2)*Hypergeometric2F1[1/2, (3 + m)/2, 3
/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Sin[e + f*x]*(a + b*Tan[e + f*x])^3)
/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{m} \left(a +b \tan \left(f x +e \right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{m} \left(a + b \tan{\left(e + f x \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.641 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=147

$$\frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)}+\frac{d(b^2-a^2(1+m))\ {}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+\dots}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] a*b*(2+m)*(d*sec(f*x+e))^m/f/m/(1+m)+d*(b^2-a^2*(1+m))*hypergeom([1/2, 1/2-
1/2*m],[3/2-1/2*m],cos(f*x+e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(-m^2+1
)/(sin(f*x+e)^2)^(1/2)+b*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))/f/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3589, 3567, 3857, 2722}

$$\frac{d(b^2-a^2(m+1))\sin(e+fx)(d\sec(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}+\frac{ab(m+2)(d\sec(e+fx))^m}{fm(m+1)}+\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m*(1 + m)) + (d*(b^2 - a^2*(1 + m))*Hyp
ergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b*(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
```

```
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} + \frac{\int (d\sec(e+fx))^m\,(-$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+}{f(1+m)}$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+}{f(1+m)}$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} - \frac{\left(a^2-\frac{b^2}{1+m}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 26.49, size = 11095, normalized size = 75.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
[Out] Result too large to show
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^m
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m (a+b\tan(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos(e+f\,x)}\right)^m (a+b\tan(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.642 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=93

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] b*(d*sec(f*x+e))^m/f/m-a*d*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+e
)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3567, 3857, 2722}

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sq
rt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{b(d\sec(e+fx))^m}{fm}+a\int (d\sec(e+fx))^m\,dx$$
$$= \frac{b(d\sec(e+fx))^m}{fm}+\left(a\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\right)\int ($$
$$= \frac{b(d\sec(e+fx))^m}{fm}-\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx\right.}{f(1-m)\sqrt{\sin^2(e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 17.05, size = 3302, normalized size = 35.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Sec[e + f*x]^(-1 - m)*(d*Sec[e + f*x])^m*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^m*(a*Sec[e + f*x]^m + b*Sec[e + f*x]^(1 + m)*Sin[e + f*x])*Tan[(e + f*x)
/2]*(-(b*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]) - b*AppellF1[1, 1 + m
, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*
x)/2]^2)^m*Tan[(e + f*x)/2] - (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m))/(3*AppellF1[1/
2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Ap
pellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*App
ellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2))*(a + b*Tan[e + f*x]))/(f*(a*Cos[e + f*x] + b*Sin[e + f*x]
)*(-1/2*(Sec[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-(b*Appell
F1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*S
ec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]) - b*AppellF1[1, 1 + m, -m, 2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[
(e + f*x)/2] - (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m))/(3*AppellF1[1/2, m, 1 - m, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m,
2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 +
m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
))) - (Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*(-1/2*(b*AppellF
1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m) - (b*AppellF1[1, 1 + m, -m, 2, Tan[
```

```
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^m)/2 - b*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2
]*(-1/2*((1 - m)*AppellF1[2, m, 2 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*AppellF1[2, 1 + m, 1 - m,
 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/2) - b*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]*((m*Appe
llF1[2, 1 + m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/2 + ((1 + m)*AppellF1[2, 2 + m, -m, 3, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2) -
b*m*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[
e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^
2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - b*m*A
ppellF1[1, 1 + m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Si
n[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - (6*a*(-1
+ m)*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2])/(3*AppellF1[1/2, m, 1 - m,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m
, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1
+ m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2) - (6*a*(Sec[(e + f*x)/2]^2)^(-1 + m)*(-1/3*((1 - m)*AppellF1[3/2, m, 2
- m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2]) + (m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2,
m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Appel
lF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*Appell
F1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2) + (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m)*(2*((-1 + m)*AppellF1[3/2, m,
 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 +
 m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2] + 3*(-1/3*((1 - m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (m*
AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + m)*((-
3*(2 - m)*AppellF1[5/2, m, 3 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*m*AppellF1[5/2, 1 + m, 2 -
 m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5) + m*((-3*(1 - m)*AppellF1[5/2, 1 + m, 2 - m, 7/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*
(1 + m)*AppellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m, 1 - m
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2,
 m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f...
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m (a+b\tan(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,\left(a+b\tan(e+f\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
```

```
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.643 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=141

$$-\frac{b\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)\,fm}+\frac{F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+$$

```
[Out] -b*hypergeom([1, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*x+e)
)^m/(a^2+b^2)/f/m+AppellF1(1/2,1,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+
e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a/f/((sec(f*x+e)^2)^(1/2*m))
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3593, 771, 440, 455, 70}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}-\frac{b(d\sec(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*Hypergeometric2F1[1, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
*(d*Sec[e + f*x])^m)/((a^2 + b^2)*f*m)) + (AppellF1[1/2, 1, 1 - m/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(m/2))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a+x}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a^2-x^2}+\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2}\right)d\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2}\,dx,x,b\tan(e+fx)\right.}{bf}$$

$$= \frac{F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/}}{af}$$

$$= -\frac{b\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)fm}+\frac{F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.77, size = 1158, normalized size = 8.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^m*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[1
/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (b*AppellF1[-m, -1/2*m, -
1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x]
)]*(Sec[e + f*x]^2)^(m/2))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))))/(f*(a + b*Tan[
e + f*x])*(a*m*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Sec[e
+ f*x]^2 - b*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (b*m*AppellF1[-m, -1/2
*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)
) + (b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 - m, 1 - m/
2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*
AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
+ I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a + b*Tan[e + f*x]
)^2)))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e
 + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b*m*AppellF1[-m, -1/2*m, -1/2*m,
1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec
[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 - m/2
)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*
Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*((b*(I + Tan[e + f*x]))/(a + b*Ta
n[e + f*x]))^(m/2)) - (b*m*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a
 + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*
x]^2*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a +
b*Tan[e + f*x])))/(2*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))
+ a*m*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2
] + (1 + Tan[e + f*x]^2)^(-1 + m/2))))
```

Maple [F]
time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sec \left(f x +e \right)\right)^{m}}{a +b \tan \left(f x +e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] integral((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e)),x)

[Out] Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)),x)

[Out] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)), x)

3.644 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{2ab\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)^2 fm}+\frac{F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e}{a^2 f}$$

```
[Out] -2*a*b*hypergeom([2, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*
x+e))^m/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-ta
n(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a^2/f/((sec(f*x+e)^2)^(1/2*m))+1/3*
b^2*AppellF1(3/2,2,1-1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f
*x+e))^m*tan(f*x+e)^3/a^4/f/((sec(f*x+e)^2)^(1/2*m))
```

Rubi [A]
time = 0.14, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3593, 771, 440, 455, 70, 524}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2 f}-\frac{2ab(d\sec(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\left(a^2+b^2\right)^2}+\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{3}{2};2,1-\frac{m}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4 f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*a*b*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*(d*Sec[e + f*x])^m)/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, 1 - m/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*
x])/(a^2*f*(Sec[e + f*x]^2)^(m/2)) + (b^2*AppellF1[3/2, 2, 1 - m/2, 5/2, (b
^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]^3)
/(3*a^4*f*(Sec[e + f*x]^2)^(m/2))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2} - \frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2}\right.\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(-a^2+x^2)^2}\,dx, x, b\tan(e+fx\right.}{bf}$$

$$= \frac{F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m}}{a^2f}$$

$$= -\frac{2ab\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)^2\,fm} + \frac{F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e}{a^2}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.16, size = 2453, normalized size = 10.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^m*((-2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)))/m + (a^2 - b^2)
*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (2*a*
b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2))/(m*((b*(-I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2)) + (b*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a - I*b)
/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/
2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I +
 Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a +
 b*Tan[e + f*x])^2*((a^2 - b^2)*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e
 + f*x]^2]*Sec[e + f*x]^2 - 2*a*b*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (2*
a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
+ I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I
+ Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b
*Tan[e + f*x]))^(m/2)) - (b^2*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2
```

```
 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[
e + f*x]^2)^(1 + m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*
x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e
 + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a -
I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)
^(m/2)*Tan[e + f*x])/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e +
 f*x])) + (2*a*b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 -
 m, 1 - m/2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a +
b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a + I
*b)*b*m^2*AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[e +
f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a + b*Ta
n[e + f*x])^2)))/(m*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((
b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) + (b*(a^2 + b^2)*(Sec[e
+ f*x]^2)^(m/2)*(((a - I*b)*b*(1 - m)*m*AppellF1[2 - m, 1 - m/2, -1/2*m, 3
- m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e
+ f*x]^2)/(2*(2 - m)*(a + b*Tan[e + f*x])^2) + ((a + I*b)*b*(1 - m)*m*Appel
lF1[2 - m, -1/2*m, 1 - m/2, 3 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b
)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(2 - m)*(a + b*Tan[e + f*x])^2))
)/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + T
an[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) - (a*b*Appe
llF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(
a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b
*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 - m,
 -1/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Ta
n[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e +
 f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(-1 + m)*((b*(I
+ Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) - (a*b*A
ppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b
)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*x]))/(a
+ b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/((b*(-I +
Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 -
m, -1/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*
Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e
+ f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(-1 + m)*((b*(-I
 + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) + (a^2
- b^2)*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^
2] + (1 + Tan[e + f*x]^2)^(-1 + m/2))))
```

Maple [F]

time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(fx+e))^m}{(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
```

```
[Out] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.645 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$

Optimal. Leaf size=181

$$\frac{bF_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)(d\sec(e+fx))^m(a+b\tan(e+fx))^{1+n}\left(1+\frac{a+b\mathrm{t}}{-a+}\right.}{(a^2+b^2)\,f(1+n)}$$

```
[Out] b*AppellF1(1+n,1-1/2*m,1-1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*t
an(f*x+e))/(a+(-b^2)^(1/2)))*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))^(1+n)/(a^2+b
^2)/f/(1+n)/((1+(a+b*tan(f*x+e))/(-a+(-b^2)^(1/2)))^(1/2*m))/((1+(-a-b*tan(
f*x+e))/(a+(-b^2)^(1/2)))^(1/2*m))
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3593, 774, 138}

$$\frac{\cos^2(e+fx)(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{1-\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{1-\frac{m}{2}}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;1-\frac{m}{2},1-\frac{m}{2};n+2;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Sec[e + f*x]
)^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b^2])
)^(1 - m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(1 - m/2))/(b*f*(1
+ n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int (a+x)^n\left(1+\frac{x}{b}\right.\right.}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1-\frac{m}{2}}\left(1-\right.\right.}{}$$

$$= \frac{F_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.03, size = 699, normalized size = 3.86

$$\frac{2F_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)(d\sec(e+fx))^m(a+b\tan(e+fx))^{1+n}}{f\left(2bF_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)+2nF_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)(b-a\tan(e+fx))-\frac{b(-2+m)(1+n)\left((a-ib)F_1\left(2+n;1-\frac{m}{2},2-\frac{m}{2};3+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+(a+ib)F_1\left(2+n;2-\frac{m}{2},1-\frac{m}{2};3+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\right)\sec^2(e+fx)(a+b\tan(e+fx))}{(a-ib)(a+ib)(2+n)}+2(m+n)F_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\tan(e+fx)(a+b\tan(e+fx))-\frac{mF_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)(a+b\tan(e+fx))}{-i+\tan(e+fx)}-\frac{mF_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)(a+b\tan(e+fx))}{i+\tan(e+fx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
 (a + b*Tan[e + f*x])/(a + I*b)]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(1
 + n))/(f*(2*b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x]
)/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*AppellF1[
1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[
e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) - (b*(-2 + m)*(1 + n)*((a - I*b)*
AppellF1[2 + n, 1 - m/2, 2 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a
 + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 - m/2, 1 - m/2,
 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)])*Se
c[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n)) + 2*(m + n
)*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
(a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x]) - (m*Appe
llF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/(-I + Tan[e
```

```
+ f*x]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*
x]))/(I + Tan[e + f*x])))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x+e\right)\right)^{m} \left(a+b \tan \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(e + f x\right)\right)^{m} \left(a + b \tan \left(e + f x\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

3.646 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{(a^2+b^2)^2(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^{2+n}}{b^5d(2+n)}+\frac{2(3a^2+b^2)(a+b\tan(c+dx))^{3+n}}{b^5d(3+n)}-$$

```
[Out] (a^2+b^2)^2*(a+b*tan(d*x+c))^(1+n)/b^5/d/(1+n)-4*a*(a^2+b^2)*(a+b*tan(d*x+c
))^(2+n)/b^5/d/(2+n)+2*(3*a^2+b^2)*(a+b*tan(d*x+c))^(3+n)/b^5/d/(3+n)-4*a*(
a+b*tan(d*x+c))^(4+n)/b^5/d/(4+n)+(a+b*tan(d*x+c))^(5+n)/b^5/d/(5+n)
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {3587, 711}

$$\frac{(a^2+b^2)^2(a+b\tan(c+dx))^{n+1}}{b^5d(n+1)}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^{n+2}}{b^5d(n+2)}+\frac{2(3a^2+b^2)(a+b\tan(c+dx))^{n+3}}{b^5d(n+3)}-\frac{4a(a+b\tan(c+dx))^{n+4}}{b^5d(n+4)}+\frac{(a+b\tan(c+dx))^{n+5}}{b^5d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(b^5*d*(1 + n)) - (4*a*(a^2 +
b^2)*(a + b*Tan[c + d*x])^(2 + n))/(b^5*d*(2 + n)) + (2*(3*a^2 + b^2)*(a +
b*Tan[c + d*x])^(3 + n))/(b^5*d*(3 + n)) - (4*a*(a + b*Tan[c + d*x])^(4 + n
))/(b^5*d*(4 + n)) + (a + b*Tan[c + d*x])^(5 + n)/(b^5*d*(5 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)^2\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2(a+x)^n}{b^4} - \frac{4a(a^2+b^2)(a+x)^{1+n}}{b^4} + \frac{2(3a^2+b^2)(a+x)^{2+n}}{b^4} - 4a\right.\right.}{bd}$$
$$= \frac{(a^2+b^2)^2(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)} - \frac{4a(a^2+b^2)(a+b\tan(c+dx)}{b^5d(2+n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 377 vs. 2(161) = 322.
time = 2.04, size = 377, normalized size = 2.34

$$\frac{\sec^4(c+dx)\left(9a^4+33a^2b^2+64b^4+18a^2b^2n+96b^4n+3a^2b^2n^2+52b^4n^2+12b^4n^3+b^4n^4+2(6a^4+a^2b^2(20+9n+n^2)+b^4(24+26n+9n^2+n^3))\cos(2(c+dx))+(3a^4-a^2b^2(-7+n^2)+b^4(8+6n+n^2))\cos(4(c+dx))-6a^3b\sin(2(c+dx))-26ab^3\sin(2(c+dx))-6a^3bn\sin(2(c+dx))-40ab^3n\sin(2(c+dx))-16ab^3n^2\sin(2(c+dx))-2ab^3n^3\sin(2(c+dx))-3a^3b\sin(4(c+dx))-7ab^3\sin(4(c+dx))-3a^3bn\sin(4(c+dx))-9ab^3n\sin(4(c+dx))-2ab^3n^2\sin(4(c+dx))\right)(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)(2+n)(3+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (Sec[c + d*x]^4*(9*a^4 + 33*a^2*b^2 + 64*b^4 + 18*a^2*b^2*n + 96*b^4*n + 3*
a^2*b^2*n^2 + 52*b^4*n^2 + 12*b^4*n^3 + b^4*n^4 + 2*(6*a^4 + a^2*b^2*(20 +
9*n + n^2) + b^4*(24 + 26*n + 9*n^2 + n^3))*Cos[2*(c + d*x)] + (3*a^4 - a^2
*b^2*(-7 + n^2) + b^4*(8 + 6*n + n^2))*Cos[4*(c + d*x)] - 6*a^3*b*Sin[2*(c
+ d*x)] - 26*a*b^3*Sin[2*(c + d*x)] - 6*a^3*b*n*Sin[2*(c + d*x)] - 40*a*b^3
*n*Sin[2*(c + d*x)] - 16*a*b^3*n^2*Sin[2*(c + d*x)] - 2*a*b^3*n^3*Sin[2*(c
+ d*x)] - 3*a^3*b*Sin[4*(c + d*x)] - 7*a*b^3*Sin[4*(c + d*x)] - 3*a^3*b*n*S
in[4*(c + d*x)] - 9*a*b^3*n*Sin[4*(c + d*x)] - 2*a*b^3*n^2*Sin[4*(c + d*x)]
)*(a + b*Tan[c + d*x])^(1 + n))/(b^5*d*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n))
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\sec^6(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.31, size = 286, normalized size = 1.78

$$\frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)} + \frac{2\left((n^2+3n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2a^2bn\tan(dx+c)+2a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6n^2+11n+6)b^3} + \frac{\left((n^4+10n^3+35n^2+50n+24)b^5\tan(dx+c)^5+(n^4+6n^3+11n^2+6n)ab^4\tan(dx+c)^4-4(n^3+3n^2+2n)a^2b^3\tan(dx+c)^3+12(n^2+n)a^3b^2\tan(dx+c)^2-24a^4bn\tan(dx+c)+24a^5\right)(b\tan(dx+c)+a)^n}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] ((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*tan(d*x
+ c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(
b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*
n^2 + 50*n + 24)*b^5*tan(d*x + c)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*ta
n(d*x + c)^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*tan(d*x + c)^3 + 12*(n^2 + n)*
a^3*b^2*tan(d*x + c)^2 - 24*a^4*b*n*tan(d*x + c) + 24*a^5)*(b*tan(d*x + c)
+ a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(161) = 322$.
time = 0.42, size = 420, normalized size = 2.61

$$\frac{\left(8\left(3a^5+10a^3b^2+15ab^4-(a^3b^2-3ab^4)n^2+3(a^3b^2+5ab^4)n\right)\cos(dx+c)^5+4\left(2ab^4n^3+3(a^3b^2+3ab^4)n^2+(3a^3b^2+7ab^4)n\right)\cos(dx+c)^3+(ab^4n^4+6ab^4n^3+11ab^4n^2+6ab^4n)\cos(dx+c)+\left(b^5n^4+10b^5n^3+35b^5n^2+50b^5n+24b^5+8\left(8b^5-(3a^2b^3-b^5)n^2-3(a^4b+3a^2b^3-2b^5)n\right)\cos(dx+c)^4+4\left(8b^5-(a^2b^3-b^5)n^3-(3a^2b^3-7b^5)n^2-2(a^2b^3-7b^5)n\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(b^5dn^5+15b^5dn^4+85b^5dn^3+225b^5dn^2+274b^5dn+120b^5d)\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 - (a^3*b^2 - 3*a*b^4)*n^2 + 3*(a^3*b^2 +
5*a*b^4)*n)*cos(d*x + c)^5 + 4*(2*a*b^4*n^3 + 3*(a^3*b^2 + 3*a*b^4)*n^2 + (
3*a^3*b^2 + 7*a*b^4)*n)*cos(d*x + c)^3 + (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^
4*n^2 + 6*a*b^4*n)*cos(d*x + c) + (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b
^5*n + 24*b^5 + 8*(8*b^5 - (3*a^2*b^3 - b^5)*n^2 - 3*(a^4*b + 3*a^2*b^3 - 2
*b^5)*n)*cos(d*x + c)^4 + 4*(8*b^5 - (a^2*b^3 - b^5)*n^3 - (3*a^2*b^3 - 7*b
^5)*n^2 - 2*(a^2*b^3 - 7*b^5)*n)*cos(d*x + c)^2)*sin(d*x + c))*((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))^n/((b^5*d*n^5 + 15*b^5*d*n^4 + 85*b^5*
d*n^3 + 225*b^5*d*n^2 + 274*b^5*d*n + 120*b^5*d)*cos(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.48Unable to divide
, perhaps due to rounding error%%%{1,[0,1,4,0,0,0]%%%}+%%%{2,[0,1,2,2,0,0]%
%%}+%%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\cos\left(c+d\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^6,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^6, x)
```

3.647 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=88

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)}-\frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)}+\frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}$$

```
[Out] (a^2+b^2)*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)-2*a*(a+b*tan(d*x+c))^(2+n)/b^3
/d/(2+n)+(a+b*tan(d*x+c))^(3+n)/b^3/d/(3+n)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\frac{(a^2+b^2)\,(a+b\tan(c+dx))^{n+1}}{b^3d(n+1)}-\frac{2a(a+b\tan(c+dx))^{n+2}}{b^3d(n+2)}+\frac{(a+b\tan(c+dx))^{n+3}}{b^3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(b^3*d*(1 + n)) - (2*a*(a + b*Ta
n[c + d*x])^(2 + n))/(b^3*d*(2 + n)) + (a + b*Tan[c + d*x])^(3 + n)/(b^3*d*
(3 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^n}{b^2}-\frac{2a(a+x)^{1+n}}{b^2}+\frac{(a+x)^{2+n}}{b^2}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)}-\frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)}+\frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 101, normalized size = 1.15

$$\frac{\sec^2(c+dx)\left(a^2+4b^2+4b^2n+b^2n^2+\left(a^2+b^2(2+n)\right)\cos(2(c+dx))-ab(1+n)\sin(2(c+dx))\right)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
[Out] (Sec[c + d*x]^2*(a^2 + 4*b^2 + 4*b^2*n + b^2*n^2 + (a^2 + b^2*(2 + n))*Cos[
2*(c + d*x)] - a*b*(1 + n)*Sin[2*(c + d*x)])*(a + b*Tan[c + d*x])^(1 + n))/
(b^3*d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 1.32

$$\frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)}+\frac{\left((n^2+3n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2a^2bn\tan(dx+c)+2a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6n^2+11n+6)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] ((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*tan(d*x +
c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(b*
tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3))/d
```

Fricas [A]
time = 0.39, size = 176, normalized size = 2.00

$$\frac{\left(2\left(2ab^2n+a^3+3ab^2\right)\cos(dx+c)^3+\left(ab^2n^2+ab^2n\right)\cos(dx+c)+\left(b^3n^2+3b^3n+2b^3+2\left(2b^3-(a^2b-b^3)n\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{\left(b^3dn^3+6b^3dn^2+11b^3dn+6b^3d\right)\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (2*(2*a*b^2*n + a^3 + 3*a*b^2)*cos(d*x + c)^3 + (a*b^2*n^2 + a*b^2*n)*cos(d
*x + c) + (b^3*n^2 + 3*b^3*n + 2*b^3 + 2*(2*b^3 - (a^2*b - b^3)*n)*cos(d*x
+ c)^2)*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n/((
b^3*d*n^3 + 6*b^3*d*n^2 + 11*b^3*d*n + 6*b^3*d)*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan{(c + dx)})^n \sec^4{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{1,[0,1,0,2,0,0]%%%}+%%%{-2,[0,1,0,1,1
,0]%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\tan(c + d\,x))^n}{\cos{(c + d\,x)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^4, x)
```

3.648 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=26

$$\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

```
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 26, normalized size = 1.00

$$\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Maple [A]
time = 0.14, size = 27, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27
default	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.00

$$\frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] (b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(26) = 52$.
time = 0.34, size = 64, normalized size = 2.46

$$\frac{(a\cos(dx+c)+b\sin(dx+c))\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(bdn+bd)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] (a*cos(d*x + c) + b*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*
x + c))^n/((b*d*n + b*d)*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**n,x)

[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**2, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument Value

Mupad [**B**]
time = 4.31, size = 51, normalized size = 1.96

$$\begin{cases} \frac{\ln(a+b\tan(c+d\,x))}{b\,d} & \text{if } n=-1 \\ \frac{(a+b\tan(c+d\,x))^{n+1}}{b\,d\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^2,x)

[Out] piecewise(n == -1, log(a + b*tan(c + d*x))/(b*d), n ~= -1, (a + b*tan(c + d*x))^(n + 1)/(b*d*(n + 1)))

3.649 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=272

$$-\frac{\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(1+\frac{a^2}{b^2}\right)b\left(a-\sqrt{-b^2}\right)d(1+n)}+\frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}\right.\right.}{}$$

```
[Out] -1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(-a*n+(1+a
^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)/b/d/(1+n)/(a-(-b
^2)^(1/2))+1/4*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2))
)*(a*n+(1+a^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d/(1+n)
/(a+(-b^2)^(1/2))+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/
(a^2+b^2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3587, 755, 845, 70}

$$-\frac{\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)-an\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(\frac{a^2}{b^2}+1\right)\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)+an\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{4d(n+1)\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/4*((Sqrt[-b^2]*(1 + a^2/b^2 - n) - a*n)*Hypergeometric2F1[1, 1 + n, 2 +
n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/((1
 + a^2/b^2)*b*(a - Sqrt[-b^2])*d*(1 + n)) + (b*(Sqrt[-b^2]*(1 + a^2/b^2 - n
) + a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[
-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*(a + Sqrt[-b^2])*d*(1
+ n)) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/
(2*(a^2 + b^2)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{b\text{Subst}\left(\right.}{}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{b\text{Subst}\left(\right.}{}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} + \frac{\left(b\left(\sqrt{-\right.\right.}{}$$

$$= -\frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4(a^2+b^2)\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A]

time = 1.39, size = 225, normalized size = 0.83

$$\frac{(a+b\tan(c+dx))^{1+n}\left(-\frac{\left(\sqrt{-b^2}\,(a^2-b^2(-1+n))-ab^2n\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{\left(a-\sqrt{-b^2}\right)(1+n)}+\frac{\left(a^2\sqrt{-b^2}+(-b^2)^{3/2}(-1+n)+ab^2n\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{\left(a+\sqrt{-b^2}\right)(1+n)}+2b\cos^2(c+dx)(b+a\tan(c+dx))\right)}{4b\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(-(((Sqrt[-b^2]*(a^2 - b^2*(-1 + n)) - a*b^2*
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])
])/((a - Sqrt[-b^2])*(1 + n))) + ((a^2*Sqrt[-b^2] + (-b^2)^(3/2)*(-1 + n) +
 a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt
[-b^2])])/((a + Sqrt[-b^2])*(1 + n)) + 2*b*Cos[c + d*x]^2*(b + a*Tan[c + d*
x])))/(4*b*(a^2 + b^2)*d)
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(c+dx))^n \cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(a+b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.650 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=434

$$\frac{b\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}-\frac{\sqrt{-b^2}\,(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^6}\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx}{16\left(1+\frac{a^2}{b^2}\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$

```
[Out] 1/16*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*(5+3*
a^2/b^2-2*n)*n/b^2-(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1/2
)/b^6)*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/16*b
*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*(5+3*a^2/b^
2-2*n)*n/b^2+(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1/2)/b^6)
*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))+1/4*cos(d*x+
c)^4*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d+1/8*b*cos(d*x+c)^2
*(a+b*tan(d*x+c))^(1+n)*(b^2*(3-n)+a^2*(1+n)+a*b*(5+3*a^2/b^2-2*n)*tan(d*x+
c))/(a^2+b^2)^2/d
```

Rubi [A]

time = 0.56, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3587, 755, 837, 845, 70}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{4d\left(a^2+b^2\right)}+\frac{b\cos^2(c+dx)\left(ab\left(\frac{3a^2}{b^2}-2n+5\right)\tan(c+dx)+a^2(n+1)+b^2(3-n)\right)(a+b\tan(c+dx))^{n+1}}{8d\left(a^2+b^2\right)^2}+\frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}-\frac{\sqrt{-b^2}\,(3a^4+a^2b^2(-n^2-2n+6)+b^4(n^2-4n+3))}{b^6}\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}+\frac{\sqrt{-b^2}\,(3a^4+a^2b^2(-n^2-2n+6)+b^4(n^2-4n+3))}{b^6}\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a+\sqrt{-b^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n)/b^2 - (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2
*n - n^2) + b^4*(3 - 4*n + n^2)))/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1
+ a^2/b^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) + (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n
)/b^2 + (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2))
)/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1 + a^2/b^2)^2*(a + Sqrt[-b^2])*d*
(1 + n)) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n
))/(4*(a^2 + b^2)*d) + (b*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*
(3 - n) + a^2*(1 + n) + a*b*(5 + (3*a^2)/b^2 - 2*n)*Tan[c + d*x]))/(8*(a^2
+ b^2)^2*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+}{}$$

$$= \frac{b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2} - \frac{\sqrt{-b^2}\left(3a^4+a^2b^2\left(6-2n-n^2\right)+b^4\left(3-4n+n^2\right)\right)}{b^6}\right)\,{}_2F_1\left(1,1\right.}{16\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)}$$

Mathematica [A]
time = 4.91, size = 360, normalized size = 0.83

$$\frac{(a+b\tan(c+dx))^{1+n}\left(\frac{\frac{\left(ab^2\left(3a^2+b^2(5-2n)\right)n+\sqrt{-b^2}\left(-3a^4-b^4\left(3-4n+n^2\right)+a^2b^2\left(-6+2n+n^2\right)\right)\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{a-\sqrt{-b^2}}+\frac{\left(ab^2\left(3a^2+b^2(5-2n)\right)n+\sqrt{-b^2}\left(3a^4+b^4\left(3-4n+n^2\right)-a^2b^2\left(-6+2n+n^2\right)\right)\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{a+\sqrt{-b^2}}}{(a^2+b^2)(1+n)}+4b\cos^4(c+dx)(b+a\tan(c+dx))-\frac{2b\cos^2(c+dx)\left(b^3(-3+n)-a^2b(1+n)-a\left(3a^2+b^2(5-2n)\right)\tan(c+dx)\right)}{a^2+b^2}\right)}{16b\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*((((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b
^2]*(-3*a^4 - b^4*(3 - 4*n + n^2) + a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])/(a - Sqrt[-b
^2]) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + b^4*(3 - 4*n
 + n^2) - a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a
```

```
+ b*Tan[c + d*x])/(a + Sqrt[-b^2])])/(a + Sqrt[-b^2]))/((a^2 + b^2)*(1 + n)
) + 4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) - (2*b*Cos[c + d*x]^2*(b^3*(-3
+ n) - a^2*b*(1 + n) - a*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/(a^2 + b^2)
))/(16*b*(a^2 + b^2)*d)
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(dx+c\right)\right) \left(a+b\tan\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{n}\cos^{4}\left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.651 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

```
[Out] AppellF1(1+n,-1/2,-1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c
))/(a+(-b^2)^(1/2)))*sec(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)/(1+(-a-b*t
an(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)/(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3593, 774, 138}

$$\frac{\sec(c+dx)(a+b\tan(c+dx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n))/(b*d*(1 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1
- (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^n\sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\sec(c+dx)\text{Subst}\left(\int x^n\sqrt{1-\frac{x}{a-\sqrt{-b^2}}}\sqrt{1-\frac{x}{a+\sqrt{-b^2}}}\,dx, x,\right.}{bd\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+d}{a+\frac{b^2}{\sqrt{-b^2}}}}}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+}{a+\sqrt{-b}}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.22, size = 306, normalized size = 1.92

$$\frac{2(a-ib)(a+ib)(2+n)F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\left(2\left(a^2+b^2\right)(2+n)F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)-\left((a-ib)F_1\left(2+n;-\frac{1}{2},\frac{1}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+ib)F_1\left(2+n;\frac{1}{2},-\frac{1}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Sec[c + d*x]*(a + b*
Tan[c + d*x])^(1 + n))/(b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n,
-1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a
+ I*b)] - ((a - I*b)*AppellF1[2 + n, -1/2, 1/2, 3 + n, (a + b*Tan[c + d*x])
/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 1/2
```

```
, -1/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*
b)])*(a + b*Tan[c + d*x])))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(c+dx\right)\right)^{n}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^3, x)
```

3.652 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}}{bd(1+n)}$$

```
[Out] AppellF1(1+n,1/2,1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)*(1+(-a-b*tan(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)*
(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(1/2)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+
n)
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3593, 774, 138}

$$\frac{\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]*(a + b*Tan[c + d*x])^(1 +
 n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\left(\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}}\right)\text{S}}{}$$

$$= \frac{F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\,\text{ta}}{bd(1+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.30, size = 340, normalized size = 2.14

$$\frac{2(a^2+b^2)^2(2+n)F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\cos^3(c+dx)(-i+\tan(c+dx))(i+\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+\left((a-ib)F_1\left(2+n;\frac{1}{2},\frac{3}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+ib)F_1\left(2+n;\frac{3}{2},\frac{1}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^3*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + ((a - I*b)*App
ellF1[2 + n, 1/2, 3/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c
+ d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 3/2, 1/2, 3 + n, (a + b*Tan[
c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]))
)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \sec \left(dx+c\right) \left(a+b\tan \left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
[Out] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan \left(c+dx\right)\right)^{n}\sec \left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\tan(c+d\,x))^n}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x),x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x), x)
```

3.653 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{F_1\left(1+n;\frac{3}{2},\frac{3}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(\right.}{bd(1+n)}$$

```
[Out] AppellF1(1+n,3/2,3/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)^3*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c))
/(a-(-b^2)^(1/2)))^(3/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(3/2)/b/d/(
1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3593, 774, 138}

$$\frac{\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{3}{2},\frac{3}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int\frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx,x,b\tan(c+dx\right.}{bd}$$

$$=\frac{\left(\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\right)\text{Subst}}{bd}$$

$$=\frac{F_1\left(1+n;\frac{3}{2},\frac{3}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\,\text{t}}{bd(1+n}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.04, size = 341, normalized size = 2.12

$$\frac{2(a^2+b^2)^2(2+n)F_1\left(1+n;\frac{3}{2},\frac{3}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\cos^5(c+dx)(-i+\tan(c+dx))(i+\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)F_1\left(1+n;\frac{3}{2},\frac{3}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+3\left((a-ib)F_1\left(2+n;\frac{3}{2},\frac{5}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+ib)F_1\left(2+n;\frac{5}{2},\frac{3}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^5*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 3*((a - I*b)*A
ppellF1[2 + n, 3/2, 5/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 5/2, 3/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(c+dx)})^n\cos{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c + d\,x)\,\left(a + b\tan(c + d\,x)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.654 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{F_1\left(1+n;\frac{5}{2},\frac{5}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\Big(}{bd(1+n)}$$

```
[Out] AppellF1(1+n,5/2,5/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)^5*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c))
/(a-(-b^2)^(1/2)))^(5/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(5/2)/b/d/(
1+n)
```

Rubi [A]

time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3593, 774, 138}

$$\frac{\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{5}{2},\frac{5}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+d\right.}{bd}$$

$$= \frac{\left(\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\right)\text{Subs}}{bd}$$

$$= \frac{F_1\left(1+n;\frac{5}{2},\frac{5}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b}{bd(1+n}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.91, size = 341, normalized size = 2.12

$$\frac{2(a^2+b^2)^2(2+n)F_1\left(1+n;\frac{5}{2},\frac{5}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\cos^7(c+dx)(-i+\tan(c+dx))(i+\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)F_1\left(1+n;\frac{5}{2},\frac{5}{2};2+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+5\left((a-ib)F_1\left(2+n;\frac{5}{2},\frac{7}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+ib)F_1\left(2+n;\frac{7}{2},\frac{5}{2};3+n;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)\right)(a+b\tan(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^7*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 5*((a - I*b)*A
ppellF1[2 + n, 5/2, 7/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 7/2, 5/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F]
time = 0.83, size = 0, normalized size = 0.00

$$\int \left(\cos^3(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.655 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=124

$$-\frac{2ia(e\cos(c+dx))^{7/2}}{7d}+\frac{10a(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d}+\frac{10a(e\,\mathrm{co}}{}$$

```
[Out] -2/7*I*a*(e*cos(d*x+c))^(7/2)/d+10/21*a*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/c
os(d*x+c)^(7/2)+2/7*a*(e*cos(d*x+c))^(7/2)*tan(d*x+c)/d+10/21*a*(e*cos(d*x+
c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3567, 3854, 3856, 2720}

$$-\frac{2ia(e\cos(c+dx))^{7/2}}{7d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{7/2}}{7d}+\frac{10a\tan(c+dx)\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/7)*a*(e*Cos[c + d*x])^(7/2))/d + (10*a*(e*Cos[c + d*x])^(7/2)*Elli
pticF[(c + d*x)/2, 2])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*(e*Cos[c + d*x])^(7
/2)*Tan[c + d*x])/(7*d) + (10*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Tan[c
 + d*x])/(21*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \left(a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right.$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{(}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{1}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{1}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{10a(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{21d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A]
time = 0.66, size = 133, normalized size = 1.07

$$\frac{ae^3\sqrt{e\cos(c+dx)}\,(\cos(dx)-i\sin(dx))\left(10F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c+dx))+\sqrt{\cos(c+dx)}\,(-8i+2i\cos(2(c+dx))+5\sin(2(c+dx)))\right)(\cos(c+2dx)+i\sin(c+2dx))}{21d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e^3*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(10*EllipticF[(c + d*x)
/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + Sqrt[Cos[c + d*x]]*(-8*I + (2*I)*C
os[2*(c + d*x)] + 5*Sin[2*(c + d*x)]))*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x]))
/(21*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 1.42, size = 241, normalized size = 1.94

method	result
default	$-\frac{2a\,e^4\left(48i\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+72i\left(\sin^5\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^4*(48*I*si
n(1/2*d*x+1/2*c)^9+48*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*I*sin(1/2*
d*x+1/2*c)^7-72*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+72*I*sin(1/2*d*x+1/
2*c)^5+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-24*I*sin(1/2*d*x+1/2*c)^3
-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*I*
sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 101, normalized size = 0.81

$$\frac{\left(-20i\sqrt{2}\,ae^{\left(i\,dx+i\,c+\frac{7}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)+\sqrt{\frac{1}{2}}\left(7i\,ae^{\frac{7}{2}}-3i\,ae^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}-16i\,ae^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{\left(-i\,dx-i\,c\right)}}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/42*(-20*I*sqrt(2)*a*e^(I*d*x + I*c + 7/2)*weierstrassPInverse(-4, 0, e^(I
*d*x + I*c)) + sqrt(1/2)*(7*I*a*e^(7/2) - 3*I*a*e^(4*I*d*x + 4*I*c + 7/2) -
 16*I*a*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*
d*x - 1/2*I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.656 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}$$

```
[Out] -2/5*I*a*(e*cos(d*x+c))^(5/2)/d+6/5*a*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos
(d*x+c)^(5/2)+2/5*a*(e*cos(d*x+c))^(5/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3567, 3854, 3856, 2719}

$$-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/5)*a*(e*Cos[c + d*x])^(5/2))/d + (6*a*(e*Cos[c + d*x])^(5/2)*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(e*Cos[c + d*x])^(5/2
)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \left(a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right.$$

$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d} + \cdots$$

$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d} + \cdots$$

$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 12.66, size = 387, normalized size = 4.30

$$\frac{(e\cos(c+dx))^{5/2}(\cos(dx)-i\sin(dx))\left(\frac{2\sqrt{2}e^{-idx}(-i+\cot(c))\left(3+3e^{2i(c+dx)}+3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}E\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}F\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{5\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{2}{5}\sqrt{\cos(c+dx)}\sin(c)\left(-1+\cos(2dx)(1-i\cot(c))-6\cot^2(c)+i\sin(2dx)+\cot(c)(5i+\sin(2dx))\right)\right)(a+ia\tan(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*((2*Sqrt[2]*(-I + Cot[c])*(
3 + 3*E^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*
x))*(-I + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c
 + d*x]]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E
^(I*(c + d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]],
-1] + E^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/(5*E^(I*d*x)*Sqrt[(1 + E^((2*I)*(c + d*x)))
```

```
/E^(I*(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Sin[c]*(-1 + Cos[2*d*x]*(1 - I*C
ot[c]) - 6*Cot[c]^2 + I*Sin[2*d*x] + Cot[c]*(5*I + Sin[2*d*x]))/5)*(a + I*
a*Tan[c + d*x]))/(2*d*Cos[c + d*x]^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(101) = 202.
time = 1.28, size = 205, normalized size = 2.28

method	result
default	$\frac{2a\,e^3\left(8i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+6i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots\right.}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,(\cdots}}$
risch	$-\frac{i(\mathrm{e}^{2i(dx+c)}+7)\sqrt{2}\,e^2 a\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{10d}-\frac{3i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\,(\mathrm{e}^{i(dx+}\cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(8*I*sin(1
/2*d*x+1/2*c)^7+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*I*sin(1/2*d*x+
1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6*I*sin(1/2*d*x+1/2*c)^3
+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-I*sin
(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 74, normalized size = 0.82

$$\frac{6i\sqrt{2}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{\frac{1}{2}}\left(5i\,ae^{\frac{5}{2}}-i\,ae^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(6*I*sqrt(2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, e^(I*d*x + I*c))) + sqrt(1/2)*(5*I*a*e^(5/2) - I*a*e^(2*I*d*x + 2*I*c + 5
/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{5/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.657 $\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2a(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d}$$

```
[Out] -2/3*I*a*(e*cos(d*x+c))^(3/2)/d+2/3*a*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos
(d*x+c)^(3/2)+2/3*a*(e*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3567, 3854, 3856, 2720}

$$-\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/3)*a*(e*Cos[c + d*x])^(3/2))/d + (2*a*(e*Cos[c + d*x])^(3/2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(e*Cos[c + d*x])^(3/2
)*Tan[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \left(a(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right.$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} + $$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} + $$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.34, size = 100, normalized size = 1.11

$$\frac{2ae\sqrt{\cos(c+dx)}\,\sqrt{e\cos(c+dx)}\,\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)(i\cos(c)+\sin(c))+\sqrt{\cos(c+dx)}\,(\cos(dx)+i\sin(dx))\right)(\cos(dx)-i\sin(dx))(-i+\tan(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[Cos[c + d*x]]*Sqrt[e*Cos[c + d*x]]*(EllipticF[(c + d*x)/2, 2]*(
I*Cos[c] + Sin[c]) + Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(Cos[d*x]
- I*Sin[d*x])*(-I + Tan[c + d*x]))/(3*d)
```

Maple [A]
time = 1.18, size = 168, normalized size = 1.87

method	result
default	$-\frac{2a\,e^2\left(4i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+}}$
risch	$-\frac{ie^{i(dx+c)}\sqrt{2}\,ea\sqrt{e\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{3d}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\mathrm{Ell}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(4*I*sin(
1/2*d*x+1/2*c)^5+4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*I*sin(1/2*d*x+
1/2*c)^3-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 55, normalized size = 0.61

$$-\frac{2\left(i\sqrt{2}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+i\sqrt{\frac{1}{2}}\,a\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{3}{2}\right)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + I*s
qrt(1/2)*a*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c + 3/2))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{3/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.658 $\int \sqrt{e\cos(c+dx)}\,(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*I*a*(e*cos(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3596, 3567, 3856, 2719}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2ia\sqrt{e\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*a*Sqrt[e*Cos[c + d*x]])/d + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}\,(a+ia\tan(c+dx))\,dx &= \left(\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx\\ &= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\left(a\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int\\ &= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.69, size = 244, normalized size = 4.07

$$\frac{aee^{-i(c+dx)}(i+\cot(c))\left(3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}E\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}F\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e*(I + Cot[c])*(3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I +
 E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]]
, -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c +
d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E^(
(2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/(3*d*E^(I*(c + d*x))*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 0.82, size = 108, normalized size = 1.80

method	result
default	$\frac{2ae\left(2i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

risch	$-\frac{2i\sqrt{2}\,a\sqrt{e\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\ \cdots}{\cdots}\right)\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(2*I*sin(1/2*d
*x+1/2*c)^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((I*a*tan(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 26, normalized size = 0.43

$$\frac{2i\sqrt{2}\,ae^{\frac{1}{2}}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I*sqrt(2)*a*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-i\sqrt{e\cos\left(c+dx\right)}\right)dx+\int\sqrt{e\cos\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*t
an(c + d*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i), x)
```

3.659 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*I*a/d/(e*cos(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c
))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3596, 3567, 3856, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}+\frac{2ia}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*a)/(d*Sqrt[e*Cos[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx &= \frac{\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))\,dx}{\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}\\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{a\int \sqrt{e\sec(c+dx)}\,dx}{\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}\\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}\\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.04, size = 143, normalized size = 2.38

$$-\frac{\sqrt{2}\,a\sqrt{e\cos(c+dx)}\,(-i+\cot(c))\left(\sqrt{2}\sqrt{\csc^2(c)}+i\cos(c+dx)\sqrt{1+\cos(2dx-2\text{ArcTan}(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\right)\sin(c)(\cos(dx)-i\sin(dx))(-i+\tan(c+dx))}{de\sqrt{\csc^2(c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*a*Sqrt[e*Cos[c + d*x]]*(-I + Cot[c])*(Sqrt[2]*Sqrt[Csc[c]^2] + I
*Cos[c + d*x]*Sqrt[1 + Cos[2*d*x - 2*ArcTan[Cot[c]]]]*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]])*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*e*Sqrt[Csc[c]^2]
))
```

Maple [A]
time = 1.23, size = 94, normalized size = 1.57

method	result	size
default	$\frac{2\left(-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((I*a*tan(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 85, normalized size = 1.42

$$-\frac{2\left(-2i\sqrt{\frac{1}{2}}\,a\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(i\sqrt{2}\,ae^{(2i\,dx+2i\,c)}+i\sqrt{2}\,a\right)\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{de^{\frac{1}{2}}+de^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-2*I*sqrt(1/2)*a*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c)
+ (I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*weierstrassPInverse(-4, 0
, e^(I*d*x + I*c)))/(d*e^(1/2) + d*e^(2*I*d*x + 2*I*c + 1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{\sqrt{e\cos\left(c+dx\right)}}\right)dx+\int\frac{\tan\left(c+dx\right)}{\sqrt{e\cos\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] I*a*(Integral(-I/sqrt(e*cos(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*co
s(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.56, size = 74, normalized size = 1.23

$$\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\;\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}}+\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{e\,\cos\left(c+d\,x\right)}\,4\mathrm{i}}{d\,e\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*a*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(e*cos(c + d*x))^(1
/2)) + (a*cos(c + d*x)*(e*cos(c + d*x))^(1/2)*4i)/(d*e*(cos(2*c + 2*d*x) +
1))
```

3.660 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2ia}{3d(e\cos(c+dx))^{3/2}}-\frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2/3*I*a/d/(e*cos(d*x+c))^(3/2)-2*a*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d*x
+c))^(3/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3567, 3853, 3856, 2719}

$$\frac{2ia}{3d(e\cos(c+dx))^{3/2}}-\frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{d(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*a)/(d*(e*Cos[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2])/(d*(e*Cos[c + d*x])^(3/2)) + (2*a*Cos[c + d*x]*Sin[c + d
*x])/(d*(e*Cos[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{a\int (e\sec(c+dx))^{3/2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{(ae^2)\int \frac{1}{\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+}\\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \sqrt{\cos(c}}{(e\cos(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+}{d(e\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.07, size = 369, normalized size = 4.15

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{2\sqrt{2}\,e^{-idx}(-i+\cot(c))\left(3+3e^{2i(c+dx)}+3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}E\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}F\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{3\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}+\frac{2(-i+\cot(c))(2i+3\cot(c)+3\cos(c+2dx)\csc(c))\sin(c)}{3\cos^{\frac{3}{2}}(c+dx)}\right)(\cos(dx)-i\sin(dx))(a+ia\tan(c+dx))}{2d(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*Sqrt[2]*(-I + Cot[c])*(3 + 3*E^((2*I)*(c + d*x)) +
 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]
*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] - 3*Sqrt[1 -
 I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]*EllipticF[
ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E^((2*I)*d*x)*Sqrt[1
```

```
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)]))/(3*E^(I*d*x)*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + (2*(-I
 + Cot[c])*(2*I + 3*Cot[c] + 3*Cos[c + 2*d*x]*Csc[c])*Sin[c])/(3*Cos[c + d*
x]^(3/2)))*(Cos[d*x] - I*Sin[d*x])*(a + I*a*Tan[c + d*x]))/(2*d*(e*Cos[c +
d*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(104) = 208.
time = 1.66, size = 214, normalized size = 2.40

method	result
default	$\frac{2\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\right.\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((I*a*tan(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 141, normalized size = 1.58

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(3i\,ae^{(4i\,dx+4i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(i\sqrt{2}\,ae^{(4i\,dx+4i\,c)}+2i\sqrt{2}\,ae^{(2i\,dx+2i\,c)}+i\sqrt{2}\,a\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{3\left(de^{\frac{3}{2}}+de^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(1/2)*(3*I*a*e^(4*I*d*x + 4*I*c) + I*a*e^(2*I*d*x + 2*I*c))*sqr
t(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(I*sqrt(2)*a*e^(4*I
```

```
*d*x + 4*I*c) + 2*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(3/2) + d*e
^(4*I*d*x + 4*I*c + 3/2) + 2*d*e^(2*I*d*x + 2*I*c + 3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{(e\cos(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] I*a*(Integral(-I/(e*cos(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*cos
(c + d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+a\tan(c+d\,x)\;1\mathrm{i}}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2), x)
```

3.661 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d(e\cos(c+dx))^{5/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*I*a/d/(e*cos(d*x+c))^(5/2)+2/3*a*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d
*x+c))^(5/2)+2/3*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3853, 3856, 2720}

$$\frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*a)/(d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]^(5/2)*Elliptic
F[(c + d*x)/2, 2])/(3*d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(3*d*(e*Cos[c + d*x])^(5/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{a\int (e\sec(c+dx))^{5/2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}} + \frac{(ae^2)\int \sqrt{e\sec(c+dx)}}{3(e\cos(c+dx))^{5/2}(e\sec(c+}$$

$$= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}} + \frac{\left(a\cos^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c}}}{3(e\cos(c+dx))^{5/2}}$$

$$= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+}{3d(e\cos(c+dx))^{5/}}$$

Mathematica [**A**]
time = 0.38, size = 57, normalized size = 0.59

$$\frac{a\left(6i+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))\right)}{15d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*
x)]))/(15*d*(e*Cos[c + d*x])^(5/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(107) = 214.
time = 2.14, size = 283, normalized size = 2.95

method	result
default	$-\frac{2\left(20\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(20*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-3*I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((I*a*tan(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 178, normalized size = 1.85

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(5i\,ae^{(5i\,dx+5i\,c)}-12i\,ae^{(3i\,dx+3i\,c)}-5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\left(i\sqrt{2}\,ae^{(6i\,dx+6i\,c)}+3i\sqrt{2}\,ae^{(4i\,dx+4i\,c)}+3i\sqrt{2}\,ae^{(2i\,dx+2i\,c)}+i\sqrt{2}\,a\right)\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{15\left(de^{\frac{5}{2}}+de^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}+3\,de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+3\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*sqrt(1/2)*(5*I*a*e^(5*I*d*x + 5*I*c) - 12*I*a*e^(3*I*d*x + 3*I*c)
- 5*I*a*e^(I*d*x + I*c))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*
I*c) + 5*(I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*a*e^(4*I*d*x + 4*I*
c) + 3*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)))/(d*e^(5/2) + d*e^(6*I*d*x + 6*I*c + 5/2) + 3*d*e^(4
*I*d*x + 4*I*c + 5/2) + 3*d*e^(2*I*d*x + 2*I*c + 5/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a\tan(c + d\,x)\ 1\mathrm{i}}{(e\ \cos\left(c + d\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2), x)
```

3.662 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=130

$$\frac{2ia}{7d(e\cos(c+dx))^{7/2}}-\frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}+\frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/7*I*a/d/(e*cos(d*x+c))^(7/2)-6/5*a*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d
*x+c))^(7/2)+2/5*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)+6/5*a*cos(d
*x+c)^3*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3567, 3853, 3856, 2719}

$$\frac{2ia}{7d(e\cos(c+dx))^{7/2}}-\frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}}+\frac{6a\sin(c+dx)\cos^3(c+dx)}{5d(e\cos(c+dx))^{7/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/7)*a)/(d*(e*Cos[c + d*x])^(7/2)) - (6*a*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2])/(5*d*(e*Cos[c + d*x])^(7/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(5*d*(e*Cos[c + d*x])^(7/2)) + (6*a*Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(e*Cos[c + d*x])^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}} \\
&= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{a\int (e\sec(c+dx))^{7/2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}} \\
&= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{(3ae^2)\int (e\sec(c+dx))^{3/}}{5(e\cos(c+dx))^{7/2}(e\sec(c+} \\
&= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} \\
&= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} \\
&= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+}{5d(e\cos(c+dx))^{7/}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.23, size = 596, normalized size = 4.58

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((-1/2*I)*Cos[c + d*x]^(9/2)*((-3*I)/5 + (3*Cot[c])/5)*((((-2*I)/3)*Sqrt[2]
*E^(I*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])/Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))] - (2*
```

```
(Cos[c/2] + I*Sin[c/2])^2*((2*I)*Cos[c + d*x] + EllipticF[ArcSin[Sqrt[(-I)*
Cos[c + d*x] + Sin[c + d*x]]], -1]*((-I)*Cos[c + d*x] - Sin[c + d*x])*Sqrt[
1 - I*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)
] + Sin[c + d*x] + I*Sin[2*(c + d*x)]] + EllipticE[ArcSin[Sqrt[(-I)*Cos[c +
 d*x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(I*Cos[
c + d*x] + Sin[c + d*x])*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c
+ d*x] + I*Sin[2*(c + d*x)]]))/Sqrt[Cos[c + d*x]])*(a + I*a*Tan[c + d*x]))/
(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x])) + (Cos[c + d*x]^5*(Csc[c
]*Sec[c]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c]) + Sec[c + d*x]^4*(((2*I)/7)*Cos[
c] + (2*Sin[c])/7) + Sec[c]*Sec[c + d*x]^3*((2*Cos[c])/5 - ((2*I)/5)*Sin[c]
)*Sin[d*x] + Sec[c]*Sec[c + d*x]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c])*Sin[d*x]
 + Sec[c + d*x]^2*((2*Cos[c])/5 - ((2*I)/5)*Sin[c])*Tan[c])*(a + I*a*Tan[c
+ d*x]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(137) = 274$.
time = 3.67, size = 396, normalized size = 3.05

method	result
default	$2\left(336\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ (\sin$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(336*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^6-504*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+252*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^4+280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-126*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-56*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] e^(-7/2)*integrate((I*a*tan(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 221, normalized size = 1.70

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(21i\,ae^{(8i\,dx+8i\,c)}+77i\,ae^{(6i\,dx+6i\,c)}+23i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+21\left(i\sqrt{2}\,ae^{(8i\,dx+8i\,c)}+4i\sqrt{2}\,ae^{(6i\,dx+6i\,c)}+6i\sqrt{2}\,ae^{(4i\,dx+4i\,c)}+4i\sqrt{2}\,ae^{(2i\,dx+2i\,c)}+i\sqrt{2}\,a\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{35\left(de^{\frac{7}{2}}+de^{\left(8i\,dx+8i\,c+\frac{7}{2}\right)}+4\,de^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)}+6\,de^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+4\,de^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
[Out] -2/35*(2*sqrt(1/2)*(21*I*a*e^(8*I*d*x + 8*I*c) + 77*I*a*e^(6*I*d*x + 6*I*c)
 + 23*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*I*d*x + 2*I*c))*sqrt(e^(2*I*d*x
+ 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 21*(I*sqrt(2)*a*e^(8*I*d*x + 8*I*c
) + 4*I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 6*I*sqrt(2)*a*e^(4*I*d*x + 4*I*c) +
 4*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(7/2) + d*e^(8*I*d*x + 8*I
*c + 7/2) + 4*d*e^(6*I*d*x + 6*I*c + 7/2) + 6*d*e^(4*I*d*x + 4*I*c + 7/2) +
 4*d*e^(2*I*d*x + 2*I*c + 7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2), x)
```

3.663 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=190

$$\frac{2(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d}+\frac{6(e\cos(c+dx))^{7/2}\tan(c+}{35a^2d}$$

```
[Out] 2/7*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(7/2)+2/15*cos(d*x+c)*
(e*cos(d*x+c))^(7/2)*sin(d*x+c)/a^2/d+6/35*(e*cos(d*x+c))^(7/2)*tan(d*x+c)/
a^2/d+2/7*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/a^2/d+4/15*I*cos(d*x
+c)^2*(e*cos(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{7a^2d\cos^{\frac{7}{2}}(c+dx)}+\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{7/2}}{15a^2d}+\frac{6\tan(c+dx)(e\cos(c+dx))^{7/2}}{35a^2d}+\frac{2\tan(c+dx)\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*(e*Cos[c + d*x])^(7/2)*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Cos[c + d*x]^
(7/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(15*a^2*d) +
(6*(e*Cos[c + d*x])^(7/2)*Tan[c + d*x])/(35*a^2*d) + (2*(e*Cos[c + d*x])^(7
/2)*Sec[c + d*x]^2*Tan[c + d*x])/(7*a^2*d) + (((4*I)/15)*Cos[c + d*x]^2*(e*
Cos[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(11e^2(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)}{15a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} +$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} +$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} +$$

$$= \frac{2(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+d}{15a^2d}$$

Mathematica [A]
time = 1.36, size = 156, normalized size = 0.82

$$\frac{e^3\sqrt{e\cos(c+dx)}\left(-240F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt{\cos(c+dx)}\left(-296i\cos(c+dx)+68i\cos(3(c+dx))+4i\cos(5(c+dx))+134\sin(c+dx)-117\sin(3(c+dx))-11\sin(5(c+dx))\right)\right)}{840a^2d\cos^{\frac{5}{2}}(c+dx)(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]

[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(-240*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-296*I)*Cos[c + d*x] + (68*I)*Cos[3*(c + d*x)] + (4*I)*Cos[5*(c + d*x)] + 134*Sin[c + d*x] - 117*Sin[3*(c + d*x)] - 11*Sin[5*(c + d*x)])))/(840*a^2*d*Cos[c + d*x]^(5/2)*(-I + Tan[c + d*x])^2)

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(192) = 384$.
time = 1.91, size = 387, normalized size = 2.04

method	result
default	$-\frac{2e^4\left(-3584i\left(\sin^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12544\left(\sin^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/105/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-3584*I*sin(1/2*d*x+1/2*c)^17+3584*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16-1568*I*sin(1/2*d*x+1/2*c)^5-12544*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)+14336*I*sin(1/2*d*x+1/2*c)^15+19264*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-25088*I*sin(1/2*d*x+1/2*c)^13-16800*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+224*I*sin(1/2*d*x+1/2*c)^3+9104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-14*I*sin(1/2*d*x+1/2*c)-3128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-15680*I*sin(1/2*d*x+1/2*c)^9+700*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6272*I*sin(1/2*d*x+1/2*c)^7-90*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25088*I*sin(1/2*d*x+1/2*c)^11)/d

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 136, normalized size = 0.72

$$\frac{\left(\sqrt{\frac{1}{2}}\left(7i\,e^{\frac{7}{2}} - 15i\,e^{\left(10i\,dx+10i\,c+\frac{7}{2}\right)} - 185i\,e^{\left(8i\,dx+8i\,c+\frac{7}{2}\right)} + 430i\,e^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)} + 162i\,e^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)} + 49i\,e^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)} - 480i\,\sqrt{2}\,e^{\left(7i\,dx+7i\,c+\frac{7}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)e^{\left(-7i\,dx-7i\,c\right)}}{1680\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(sqrt(1/2)*(7*I*e^(7/2) - 15*I*e^(10*I*d*x + 10*I*c + 7/2) - 185*I*e
^(8*I*d*x + 8*I*c + 7/2) + 430*I*e^(6*I*d*x + 6*I*c + 7/2) + 162*I*e^(4*I*d
*x + 4*I*c + 7/2) + 49*I*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(e^(2*I*d*x + 2*I*c
) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 480*I*sqrt(2)*e^(7*I*d*x + 7*I*c + 7/2)*w
eierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.664 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d}+\frac{14(e\cos(c+dx))^{5/2}\tan(}{65a^2d}$$

```
[Out] 42/65*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(5/2)+2/13*cos(d*x+c
)*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+14/65*(e*cos(d*x+c))^(5/2)*tan(d*x+
c)/a^2/d+4/13*I*cos(d*x+c)^2*(e*cos(d*x+c))^(5/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3854, 3856, 2719}

$$\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{65a^2d\cos^{\frac{5}{2}}(c+dx)}+\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{5/2}}{13a^2d}+\frac{14\tan(c+dx)(e\cos(c+dx))^{5/2}}{65a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (42*(e*Cos[c + d*x])^(5/2)*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*Cos[c + d*x
]^(5/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*a^2*d)
+ (14*(e*Cos[c + d*x])^(5/2)*Tan[c + d*x])/(65*a^2*d) + (((4*I)/13)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(9e^2(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int}{13a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d}$$

$$= \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+}{13a^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.30, size = 471, normalized size = 3.06

$$\frac{(e\cos(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^2\left(\frac{14\sqrt{2}e^{-idx}\csc(c)\left(3+3e^{2i(c+dx)}+3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}E\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}F\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(\cos(2c)+i\sin(2c))}{65\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}-\frac{1}{260}\sqrt{\cos(c+dx)}\csc(c)(\cos(2dx)-i\sin(2dx))(178\cos(c+2dx)+158\cos(3c+2dx)-9\cos(3c+4dx)+9\cos(5c+4dx)-88i\sin(c)+208i\sin(c+2dx)+128i\sin(3c+2dx)-4i\sin(3c+4dx)+4i\sin(5c+4dx))\right)}{2d\cos^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2*((14*Sqrt[2]*Csc[c]*(3 +
3*E^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*
(-I + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d
*x]]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*
(c + d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1]
+ E^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))])*(Cos[2*c] + I*Sin[2*c]))/(65*E^(I*d*x)*Sqrt[(1 +
 E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*Csc[c]*(Cos[2
*d*x] - I*Sin[2*d*x])*(178*Cos[c + 2*d*x] + 158*Cos[3*c + 2*d*x] - 9*Cos[3*
c + 4*d*x] + 9*Cos[5*c + 4*d*x] - (88*I)*Sin[c] + (208*I)*Sin[c + 2*d*x] +
(128*I)*Sin[3*c + 2*d*x] - (4*I)*Sin[3*c + 4*d*x] + (4*I)*Sin[5*c + 4*d*x])
)/260))/(2*d*Cos[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(160) = 320$.
time = 1.76, size = 351, normalized size = 2.28

method	result
default	$2e^3\left(-2800i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1280\left(\sin^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-140i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3840\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-1280i\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/65/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-2800*
I*sin(1/2*d*x+1/2*c)^7+1280*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-140*I*
sin(1/2*d*x+1/2*c)^3-3840*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-1280*I*s
in(1/2*d*x+1/2*c)^15+4960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+5600*I*s
in(1/2*d*x+1/2*c)^9-3520*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-6720*I*sin
(1/2*d*x+1/2*c)^11+1496*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+10*I*sin(1/
2*d*x+1/2*c)-376*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+840*I*sin(1/2*d*x+
1/2*c)^5+44*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))+4480*I*sin(1/2*d*x+1/2*c)^13)/d
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 127, normalized size = 0.82

$$\frac{\left(\sqrt{\frac{1}{2}}\left(5i\,e^{\frac{5}{2}} - 13i\,e^{\left(8i\,dx+8i\,c+\frac{5}{2}\right)} + 386i\,e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)} + 88i\,e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)} + 30i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)} + 336i\,\sqrt{2}\,e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)\right)e^{\left(-6i\,dx-6i\,c\right)}}{520\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/520*(sqrt(1/2)*(5*I*e^(5/2) - 13*I*e^(8*I*d*x + 8*I*c + 5/2) + 386*I*e^(6
*I*d*x + 6*I*c + 5/2) + 88*I*e^(4*I*d*x + 4*I*c + 5/2) + 30*I*e^(2*I*d*x +
2*I*c + 5/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 336*
I*sqrt(2)*e^(6*I*d*x + 6*I*c + 5/2)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, e^(I*d*x + I*c))))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.665 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{10(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d}+\frac{10(e\cos(c+dx))^{3/2}\tan(}{33a^2d}$$

```
[Out] 10/33*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(3/2)+2/11*cos(d*x+c
)*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/33*(e*cos(d*x+c))^(3/2)*tan(d*x+
c)/a^2/d+4/11*I*cos(d*x+c)^2*(e*cos(d*x+c))^(3/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{33a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{3/2}}{11a^2d}+\frac{10\tan(c+dx)(e\cos(c+dx))^{3/2}}{33a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (10*(e*Cos[c + d*x])^(3/2)*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*Cos[c + d*x
]^(3/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*a^2*d)
+ (10*(e*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(33*a^2*d) + (((4*I)/11)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
+ b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int}{11a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d}$$

$$= \frac{10(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+}{11a^2d}$$

Mathematica [A]
time = 0.76, size = 131, normalized size = 0.85

$$\frac{(e\cos(c+dx))^{3/2}\left(-20F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt{\cos(c+dx)}\,(-28i\cos(c+dx)+4i\cos(3(c+dx))+13\sin(c+dx)-7\sin(3(c+dx)))\right)}{66a^2d\cos^{\frac{7}{2}}(c+dx)(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(-20*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-28*I)*Cos[c + d*x] + (4*I)*Cos[
3*(c + d*x)] + 13*Sin[c + d*x] - 7*Sin[3*(c + d*x)])))/(66*a^2*d*Cos[c + d*
x]^(7/2)*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 1.70, size = 315, normalized size = 2.05

method	result
default	$-\frac{2e^2\left(-384i\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1152i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-1440\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/33/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-384*
I*sin(1/2*d*x+1/2*c)^13+384*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+1152*I
*sin(1/2*d*x+1/2*c)^11-960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1440*I*
sin(1/2*d*x+1/2*c)^9+1008*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+960*I*sin
(1/2*d*x+1/2*c)^7-552*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-360*I*sin(1/2
*d*x+1/2*c)^5+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+72*I*sin(1/2*d*x+
1/2*c)^3-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-6*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 112, normalized size = 0.73

$$\frac{\left(\sqrt{\frac{1}{2}}\left(3i\,e^{\frac{3}{2}}-11i\,e^{\left(6i\,dx+6i\,c+\frac{3}{2}\right)}+41i\,e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+15i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-40i\,\sqrt{2}\,e^{\left(5i\,dx+5i\,c+\frac{3}{2}\right)}\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)e^{\left(-5i\,dx-5i\,c\right)}}{132\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/132*(sqrt(1/2)*(3*I*e^(3/2) - 11*I*e^(6*I*d*x + 6*I*c + 3/2) + 41*I*e^(4*
I*d*x + 4*I*c + 3/2) + 15*I*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(e^(2*I*d*x + 2*
I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 40*I*sqrt(2)*e^(5*I*d*x + 5*I*c + 3/2)
*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.666 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}}+\frac{2i\sqrt{e\cos(c+dx)}}{9d(a+ia\tan(c+dx))^2}+\frac{2i\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(1/2)+2/9*I*(e*cos(d*x
+c))^(1/2)/d/(a+I*a*tan(d*x+c))^2+2/9*I*(e*cos(d*x+c))^(1/2)/d/(a^2+I*a^2*t
an(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3854, 3856, 2719}

$$\frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{e\cos(c+dx)}}{9a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(9*a^2*d) + (((4*
I)/9)*Cos[c + d*x]^2*Sqrt[e*Cos[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
+ b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx &= \left(\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^2}\,d \\
&= \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^2\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{}{(e s}}{9a^2} \\
&= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \\
&= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \\
&= \frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\,\sin(c+dx}{9a^2d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.88, size = 420, normalized size = 3.50

$$\frac{\sqrt{e\cos(c+dx)}\,(\cos(dx)+i\sin(dx))^2\left(\frac{2\sqrt{2}\,e^{-idx}\csc(c)\left(3+3e^{2i(c+dx)}+3\sqrt{1-ie^{i(c+dx)}}\,\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}\,E\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{1-ie^{i(c+dx)}}\,\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}\,F\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)(\cos(2c)+i\sin(2c))}{9\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}-\frac{1}{9}\sqrt{\cos(c+dx)}\,\csc(c)(\cos(2dx)-i\sin(2dx))(7\cos(c+2dx)+5\cos(3c+2dx)-4i(\sin(c)-2\sin(c+2dx)-\sin(3c+2dx)))\right)}{2d\cos^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*((2*Sqrt[2]*Csc[c]*(3 + 3*E
^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I
 + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]
]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c
+ d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E
^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*(Cos[2*c] + I*Sin[2*c]))/(9*E^(I*d*x)*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*Csc[c]*(Cos[2*d*x
] - I*Sin[2*d*x])*(7*Cos[c + 2*d*x] + 5*Cos[3*c + 2*d*x] - (4*I)*(Sin[c] -
2*Sin[c + 2*d*x] - Sin[3*c + 2*d*x])))/9))/(2*d*Cos[c + d*x]^(5/2)*(a + I*a
*Tan[c + d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(128) = 256$.
time = 1.41, size = 277, normalized size = 2.31

method	result
default	$\frac{2e\left(-64i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160i\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-160i\left(\sin^{7}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-64*I*sin
(1/2*d*x+1/2*c)^11+64*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+160*I*sin(1/
2*d*x+1/2*c)^9-128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-160*I*sin(1/2*d*
x+1/2*c)^7+104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+80*I*sin(1/2*d*x+1/2
*c)^5-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*I*sin(1/2*d*x+1/2*c)^3+
6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*I*si
n(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 103, normalized size = 0.86

$$\frac{\left(\sqrt{\frac{1}{2}}\left(i\,e^{\frac{1}{2}}+15i\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}+4i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{e^{\left(2i\,dx+2i\,c\right)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+12i\,\sqrt{2}\,e^{\left(4i\,dx+4i\,c+\frac{1}{2}\right)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{\left(i\,dx+i\,c\right)}\right)\right)\right)e^{\left(-4i\,dx-4i\,c\right)}}{18\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(1/2)*(I*e^(1/2) + 15*I*e^(4*I*d*x + 4*I*c + 1/2) + 4*I*e^(2*I*d*
x + 2*I*c + 1/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) +
12*I*sqrt(2)*e^(4*I*d*x + 4*I*c + 1/2)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\tan^2\left(c+dx\right)-2i\tan\left(c+dx\right)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sqrt(e*cos(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.667 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{2i}{7d\sqrt{e\cos(c+dx)}\,(a+ia\tan(c+dx))^2}+\frac{2i}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)+2/7*I/d/(e*cos(d
*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/7*I/d/(e*cos(d*x+c))^(1/2)/(a^2+I*a^2*t
an(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\frac{4i\cos^2(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\cos(c+dx)}}+\frac{2\sin(c+dx)\cos(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Cos[c + d*x]]) + (((4*
I)/7)*Cos[c + d*x]^2)/(d*Sqrt[e*Cos[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}}$$

$$= \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{(3e^2)\int \frac{}{(e\,\mathrm{se}}}{7a^2\sqrt{e\cos(c+d}}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.70, size = 158, normalized size = 1.32

$$\frac{\left(-i\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{\cos(c+dx)}\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+4i\sin^3\left(\frac{1}{2}(c+dx)\right)\right)+2F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(-i\cos\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\right)}{7a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{e\cos(c+dx)}\,(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sqrt[Cos[c + d*x]]*(3*Cos[(c +
 d*x)/2] + Cos[(3*(c + d*x))/2] + (4*I)*Sin[(c + d*x)/2]^3) + 2*EllipticF[(
```

```
c + d*x)/2, 2]*((-I)*Cos[(3*(c + d*x))/2] + Sin[(3*(c + d*x))/2])))/(7*a^2*
d*Cos[c + d*x]^(3/2)*Sqrt[e*Cos[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 1.43, size = 239, normalized size = 1.99

method	result
default	$-\frac{2\left(-32i\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64i\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-32*I*sin(
1/2*d*x+1/2*c)^9+32*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+64*I*sin(1/2*d*
x+1/2*c)^7-48*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-48*I*sin(1/2*d*x+1/2*
c)^5+28*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+16*I*sin(1/2*d*x+1/2*c)^3-6
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*I*sin(1
/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 84, normalized size = 0.70

$$\frac{\left(\sqrt{\frac{1}{2}}\sqrt{e^{(2id x+2ic)}+1}\left(3ie^{(2id x+2ic)}+i\right)e^{\left(-\frac{1}{2}id x-\frac{1}{2}ic\right)}-2i\sqrt{2}\,e^{(3id x+3ic)}\text{weierstrassPInverse}\left(-4,0,e^{(id x+ic)}\right)\right)e^{\left(-3id x-3ic-\frac{1}{2}\right)}}{7\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/7*(sqrt(1/2)*sqrt(e^(2*I*d*x + 2*I*c) + 1)*(3*I*e^(2*I*d*x + 2*I*c) + I)*
e^(-1/2*I*d*x - 1/2*I*c) - 2*I*sqrt(2)*e^(3*I*d*x + 3*I*c)*weierstrassPInve
rse(-4, 0, e^(I*d*x + I*c)))*e^(-3*I*d*x - 3*I*c - 1/2)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\sqrt{e\cos{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\cos{(c+dx)}}\tan{(c+dx)}-\sqrt{e\cos{(c+dx)}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/(sqrt(e*cos(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*cos(c + d*x)
)*tan(c + d*x) - sqrt(e*cos(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((I*a*tan(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos{(c+d\,x)}}\,{\left(a+a\tan{(c+d\,x)}\,1{}\mathrm{i}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.668 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}}+\frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/5*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(3/2)+4/5*I*cos(d*x+c)
^2/d/(e*cos(d*x+c))^(3/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3596, 3581, 3856, 2719}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}}+\frac{4i\cos^2(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x])^
(3/2)) + (((4*I)/5)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
```

```
n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{e^2\int \frac{}{\sqrt{e}}}{5a^2(e\cos(c+d}$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\cos^{\frac{3}{2}}(c+dx)\int}{5a^2(e\cos}$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+d}{5d(e\cos(c+dx))^{3/2}\left(a^2+\right.}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 244 vs. $2(92) = 184$.
time = 0.73, size = 244, normalized size = 2.65

$$\frac{(i\cos(c+dx)+\sin(c+dx))\left(3+3\cos(2(c+dx))+2E\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)\sqrt{1-i\cos(c+dx)+\sin(c+dx)}\sqrt{-i\cos(c+dx)+\cos(2(c+dx))+\sin(c+dx)+i\sin(2(c+dx))}-2F\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)\sqrt{1-i\cos(c+dx)+\sin(c+dx)}\sqrt{-i\cos(c+dx)+\cos(2(c+dx))+\sin(c+dx)+i\sin(2(c+dx))}+i\sin(2(c+dx))\right)}{5a^2de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((I*Cos[c + d*x] + Sin[c + d*x])*(3 + 3*Cos[2*(c + d*x)] + 2*EllipticE[ArcS
in[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + S
in[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c + d*x] + I*S
in[2*(c + d*x)]] - 2*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]
]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Co
s[2*(c + d*x)] + Sin[c + d*x] + I*Sin[2*(c + d*x)]] + I*Sin[2*(c + d*x)]))/
(5*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 1.38, size = 206, normalized size = 2.24

method	result

default	$\frac{-\frac{32i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{48i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{24i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\left(\sin^2\left(\frac{dx}{2}\right.\right.}{}}{e\,a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/e/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-16*I*sin
(1/2*d*x+1/2*c)^7+16*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24*I*sin(1/2*d
*x+1/2*c)^5-16*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+1/2
*c)^3+4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*
I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 87, normalized size = 0.95

$$-\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-i\sqrt{2}\,e^{(2i\,dx+2i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)e^{\left(-2i\,dx-2i\,c-\frac{3}{2}\right)}}{5\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/5*(sqrt(1/2)*sqrt(e^(2*I*d*x + 2*I*c) + 1)*(-2*I*e^(2*I*d*x + 2*I*c) - I
)*e^(-1/2*I*d*x - 1/2*I*c) - I*sqrt(2)*e^(2*I*d*x + 2*I*c)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-2*I*d*x - 2*I*c -
3/2)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos\left(c+d\,x\right))^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.669 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{5/2}}+\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] -2/3*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(5/2)+4/3*I*cos(d*x+c
)^2/d/(e*cos(d*x+c))^(5/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3596, 3581, 3856, 2720}

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{5/2}}+\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(5/2)) + (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(5/2)*(a^2 + I*a^2
*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
```

```
n}, x] && !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))} - \frac{e^2\int \sqrt{e}}{3a^2(e\cos(c+d}$$
$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\int}{3a^2(e\cos}$$
$$= -\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+}{3d(e\cos(c+dx))^{5/2}\,(a^2}$$

Mathematica [A]
time = 0.44, size = 116, normalized size = 1.26

$$\frac{2\sqrt{\cos(c+dx)}\,(\cos(dx)+i\sin(dx))^2\left(F\left(\frac{1}{2}(c+dx)\middle|\,2\right)(\cos(2c)+i\sin(2c))+2\sqrt{\cos(c+dx)}\,(-i\cos(c-dx)+\sin(c-dx))\right)}{3a^2d(e\cos(c+dx))^{5/2}(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*(EllipticF[(c + d*x)/2, 2]*
(Cos[2*c] + I*Sin[2*c]) + 2*Sqrt[Cos[c + d*x]]*((-I)*Cos[c - d*x] + Sin[c -
 d*x])))/(3*a^2*d*(e*Cos[c + d*x])^(5/2)*(-I + Tan[c + d*x])^2)
```

Maple [A]
time = 1.26, size = 171, normalized size = 1.86

method	result
default	$-\frac{2\left(-8i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3e^2a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/e^2/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-8*I*s
in(1/2*d*x+1/2*c)^5+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+8*I*sin(1/2*d
*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-2*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] e^(-5/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 72, normalized size = 0.78

$$-\frac{2\left(-i\sqrt{2}\,e^{(i\,dx+i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)-2i\sqrt{\frac{1}{2}}\,\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{\left(-i\,dx-i\,c-\frac{5}{2}\right)}}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(-I*sqrt(2)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)
) - 2*I*sqrt(1/2)*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c))*e
^(-I*d*x - I*c - 5/2)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(-5/2)/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.670 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$\frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}}-\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}}+\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 6*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(7/2)-6*cos(d*x+c)^3*sin
(d*x+c)/a^2/d/(e*cos(d*x+c))^(7/2)+4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/
(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]

time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3853, 3856, 2719}

$$\frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}}-\frac{6\sin(c+dx)\cos^3(c+dx)}{a^2d(e\cos(c+dx))^{7/2}}+\frac{4i\cos^2(c+dx)}{d\,(a^2+ia^2\tan(c+dx))\,(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (6*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2])/(a^2*d*(e*Cos[c + d*x])^(7
/2)) - (6*Cos[c + d*x]^3*Sin[c + d*x])/(a^2*d*(e*Cos[c + d*x])^(7/2)) + ((4
*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(3e^2\right)\int(e\,\mathrm{se}}{a^2(e\cos(c+dx)}$$

$$= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\,\mathrm{t}\right.}$$

$$= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\,\mathrm{t}\right.}$$

$$= \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 255 vs. $2(122) = 244$.
time = 1.06, size = 255, normalized size = 2.09

$$\frac{10i\cos(c+dx)-2\sin(c+dx)-6iF\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)(\cos(c+dx)-i\sin(c+dx))\sqrt{1-i\cos(c+dx)+\sin(c+dx)}\sqrt{-i\cos(c+dx)+\cos(2(c+dx))+\sin(c+dx)+i\sin(2(c+dx))}+6E\left(\mathrm{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)\sqrt{1-i\cos(c+dx)+\sin(c+dx)}(i\cos(c+dx)+\sin(c+dx))\sqrt{-i\cos(c+dx)+\cos(2(c+dx))+\sin(c+dx)+i\sin(2(c+dx))}}{a^2de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((10*I)*Cos[c + d*x] - 2*Sin[c + d*x] - (6*I)*EllipticF[ArcSin[Sqrt[(-I)*Co
s[c + d*x] + Sin[c + d*x]]], -1]*(Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[1 - I
```

```
*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + S
in[c + d*x] + I*Sin[2*(c + d*x)]] + 6*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*
x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(I*Cos[c +
 d*x] + Sin[c + d*x])*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c + d
*x] + I*Sin[2*(c + d*x)]])/(a^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 1.01, size = 135, normalized size = 1.11

method	result
default	$-\frac{2\left(4i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{d}{2}\right.\right.\right.}{e^3a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\ d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/e^3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*I*sin(
1/2*d*x+1/2*c)^3+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-2*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] e^(-7/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 105, normalized size = 0.86

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\left(-3i\,e^{(2i\,dx+2i\,c)}-2i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(-i\sqrt{2}\,e^{(2i\,dx+2i\,c)}-i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{a^2de^{\frac{7}{2}}+a^2de^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2*(2*sqrt(1/2)*sqrt(e^(2*I*d*x + 2*I*c) + 1)*(-3*I*e^(2*I*d*x + 2*I*c) - 2
*I)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(-I*sqrt(2)*e^(2*I*d*x + 2*I*c) - I*sqrt(2
```

```
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^
2*d*e^(7/2) + a^2*d*e^(2*I*d*x + 2*I*c + 7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(-7/2)/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\cos(c+d\,x))^{7/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.671 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}}+\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}}-\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 10/3*cos(d*x+c)^(9/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(9/2)+10/3*cos(d*x+c)
^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(9/2)-4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^
(9/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3853, 3856, 2720}

$$\frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}}+\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}}-\frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(9/2)) + (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(e*Cos[c + d*x])^(9/2))
 - ((4*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$
$$= -\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(5e^2)\int(e}{a^2(e\cos(c+d}$$
$$= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\text{ta}\right.}$$
$$= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\text{ta}\right.}$$
$$= \frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+d}{3a^2d(e\cos(c+dx))^{9/2}}$$

Mathematica [A]
time = 0.42, size = 67, normalized size = 0.53

$$\frac{2\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-\sin(c+dx)\right)}{3a^2de^3(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
[Out] (2*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] -
Sin[c + d*x]))/(3*a^2*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 1.67, size = 208, normalized size = 1.65

method	result
default	$-\frac{-\frac{20\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-8i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\left(\sin^2\right.}{}}{\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)/e^4*(-10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*I*
sin(1/2*d*x+1/2*c)^3+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+6*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] e^(-9/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 142, normalized size = 1.13

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(5i\,e^{(3i\,dx+3i\,c)}+7i\,e^{(i\,dx+i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\left(i\sqrt{2}\,e^{(4i\,dx+4i\,c)}+2i\sqrt{2}\,e^{(2i\,dx+2i\,c)}+i\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\left(a^2de^{\frac{9}{2}}+a^2de^{\left(4i\,dx+4i\,c+\frac{9}{2}\right)}+2\,a^2de^{\left(2i\,dx+2i\,c+\frac{9}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(2*sqrt(1/2)*(5*I*e^(3*I*d*x + 3*I*c) + 7*I*e^(I*d*x + I*c))*sqrt(e^(2
*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 5*(I*sqrt(2)*e^(4*I*d*x + 4
*I*c) + 2*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*weierstrassPInverse(-4
, 0, e^(I*d*x + I*c)))/(a^2*d*e^(9/2) + a^2*d*e^(4*I*d*x + 4*I*c + 9/2) + 2
*a^2*d*e^(2*I*d*x + 2*I*c + 9/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-9/2)/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos\left(c+d\,x\right))^{9/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.672 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}-\frac{}{3d(e\cos(c+}$$

```
[Out] -14/5*cos(d*x+c)^(11/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(11/2)+14/15*cos(d*
x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)+14/5*cos(d*x+c)^5*sin(d*x+c)/
a^2/d/(e*cos(d*x+c))^(11/2)-4/3*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(11/2)/(a^2
+I*a^2*tan(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3853, 3856, 2719}

$$-\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^5(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^3(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (-14*Cos[c + d*x]^(11/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x
])^(11/2)) + (14*Cos[c + d*x]^3*Sin[c + d*x])/(15*a^2*d*(e*Cos[c + d*x])^(1
1/2)) + (14*Cos[c + d*x]^5*Sin[c + d*x])/(5*a^2*d*(e*Cos[c + d*x])^(11/2))
- (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c
+ d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx &= \frac{\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}} \\
&= -\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(7e^2)}{3a^2(e\cos(c} \\
&= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\right.} \\
&= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \\
&= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \\
&= -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^3(c+dx)\sin(c-}{15a^2d(e\cos(c+dx))}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.10, size = 406, normalized size = 2.48

$$\frac{2\sqrt{2}e^{3ic+2idx}\cos^{\frac{7}{2}}(c+dx)\csc(c)\left(-42\sqrt{2-2ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}\cos^{\frac{5}{2}}(c+dx)E\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)+42\sqrt{2-2ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(-i+e^{i(c+dx)}\right)}\cos^{\frac{5}{2}}(c+dx)F\left(\text{ArcSin}\left(\sqrt{-i\cos(c+dx)+\sin(c+dx)}\right)\middle|-1\right)-\frac{1}{2}e^{-2ic}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\left(-1+e^{2ic}\right)\left(47+56e^{2i(c+dx)}+21e^{4i(c+dx)}\right)+7\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)(\cos(2c)+i\sin(2c))(\cos(dx)+i\sin(dx))^2}{15d\left(1+e^{2i(c+dx)}\right)^3(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[2]*E^((3*I)*c + (2*I)*d*x)*Cos[c + d*x]^(7/2)*Csc[c]*(-42*Sqrt[2 -
(2*I)*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]*Cos[c +
 d*x]^(5/2)*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] +
 42*Sqrt[2 - (2*I)*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*
x)))]*Cos[c + d*x]^(5/2)*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c +
d*x]]], -1] - (Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*((-1 + E^((2
*I)*c))*(47 + 56*E^((2*I)*(c + d*x)) + 21*E^((4*I)*(c + d*x))) + 7*(1 + E^(
(2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))]))/(2*E^((2*I)*c)))*(Cos[2*c] + I*Sin[2*c])*(Cos[d*x] + I*Sin[d*x])^2)/(
15*d*(1 + E^((2*I)*(c + d*x)))^3*(e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d
*x])^2)
```

Maple [A]
time = 2.56, size = 321, normalized size = 1.96

method	result
default	$\frac{\frac{112 \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{56\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(168*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^6-84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-168*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*I*sin(1/2*d*x+1/2*c)^3+36*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-10*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] e^(-11/2)*integrate(1/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(11/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 186, normalized size = 1.13

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(21i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,dx+4i\,c)}+47i\,e^{(2i\,dx+2i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+21\left(i\sqrt{2}\,e^{(6i\,dx+6i\,c)}+3i\sqrt{2}\,e^{(4i\,dx+4i\,c)}+3i\sqrt{2}\,e^{(2i\,dx+2i\,c)}+i\sqrt{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)\right)}{15\left(a^2de^{\frac{11}{2}}+a^2de^{\left(6i\,dx+6i\,c+\frac{11}{2}\right)}+3\,a^2de^{\left(4i\,dx+4i\,c+\frac{11}{2}\right)}+3\,a^2de^{\left(2i\,dx+2i\,c+\frac{11}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -2/15*(2*sqrt(1/2)*(21*I*e^(6*I*d*x + 6*I*c) + 56*I*e^(4*I*d*x + 4*I*c) + 4
7*I*e^(2*I*d*x + 2*I*c))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*
I*c) + 21*(I*sqrt(2)*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*e^(4*I*d*x + 4*I*c)
+ 3*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^(11/2) + a^2*d*e^(6*I*d*x
 + 6*I*c + 11/2) + 3*a^2*d*e^(4*I*d*x + 4*I*c + 11/2) + 3*a^2*d*e^(2*I*d*x
+ 2*I*c + 11/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-11/2)/((I*a*tan(d*x + c) + a)^2*cos(d*x + c)^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{11/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.673 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d}$$

```
[Out] 12/35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)+32/3
5*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(e
*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/35*I*(e*cos(d*x+c))^(7/2)*
sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3596, 3578, 3583, 3569}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{7/2}}{7d}+\frac{32ia\sec^4(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{7/2}}{35d}+\frac{12ia\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((12*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (((32*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^4)/(d*Sqrt[a
 + I*a*Tan[c + d*x]]) - (((2*I)/7)*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d - (((16*I)/35)*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Sqrt[a +
I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}$$

$$= -\frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} + \frac{(6a(e\cos(c+}{}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a}}{7d}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a}}{7d}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{32ia(e\cos(c+dx))^{7/2}\,\mathrm{s}}{35d\sqrt{a+ia\tan(c}}$$

Mathematica [A]
time = 0.65, size = 80, normalized size = 0.45

$$\frac{e^3\sqrt{e\cos(c+dx)}\,(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Si
n[c + d*x] + 6*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d)
```

Maple [A]
time = 6.24, size = 97, normalized size = 0.54

method	result	size
default	$\frac{2\left(i\left(\cos^{3}(dx+c)\right)+6\left(\cos^{2}(dx+c)\right)\sin(dx+c)+8i\cos(dx+c)+16\sin(dx+c)\right)\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(e\cos(dx+c))^{\frac{7}{2}}}{35d\cos(dx+c)^{3}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(I*cos(d*x+c)^3+6*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+
c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e*cos(d*x+c))^(7/2)/cos
(d*x+c)^3
```

Maxima [A]
time = 0.62, size = 177, normalized size = 0.99

$$\frac{\sqrt{a}\left(7i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5i\cos\left(\frac{7}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-35i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+105i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+7\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{7}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+35\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+105\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)e^{\frac{7}{2}}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))))*e^(7/2)/d
```

Fricas [A]
time = 0.33, size = 91, normalized size = 0.51

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(7i\,e^{\frac{7}{2}}-5i\,e^{\left(6i\,dx+6i\,c+\frac{7}{2}\right)}-35i\,e^{\left(4i\,dx+4i\,c+\frac{7}{2}\right)}+105i\,e^{\left(2i\,dx+2i\,c+\frac{7}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(7*I*e^(7/2) - 5*
I*e^(6*I*d*x + 6*I*c + 7/2) - 35*I*e^(4*I*d*x + 4*I*c + 7/2) + 105*I*e^(2*I
*d*x + 2*I*c + 7/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)
/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.75, size = 64, normalized size = 0.36

$$\frac{\left(-5i\sqrt{a}\,e^{\left(\frac{7}{2}i\,dx+\frac{7}{2}i\,c\right)}-35i\sqrt{a}\,e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+105i\sqrt{a}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+7i\sqrt{a}\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}\right)e^{\frac{7}{2}}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/140*(-5*I*sqrt(a)*e^(7/2*I*d*x + 7/2*I*c) - 35*I*sqrt(a)*e^(3/2*I*d*x + 3
/2*I*c) + 105*I*sqrt(a)*e^(-1/2*I*d*x - 1/2*I*c) + 7*I*sqrt(a)*e^(-5/2*I*d*
x - 5/2*I*c))*e^(7/2)/d
```

Mupad [B]
time = 5.17, size = 96, normalized size = 0.54

$$\frac{e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\sin\left(c+d\,x\right)+\frac{3\,\sin\left(3\,c+3\,d\,x\right)}{35}+\frac{\cos\left(c+d\,x\right)\,1\mathrm{i}}{2}+\frac{\cos\left(3\,c+3\,d\,x\right)\,1\mathrm{i}}{70}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e^3*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*((cos(c + d*x)*1i)/2 + sin(c + d*x) + (cos
(3*c + 3*d*x)*1i)/70 + (3*sin(3*c + 3*d*x))/35))/d
```

3.674 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+\ldots}{15\ldots}$$

```
[Out] 8/15*I*a*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*I
*(e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/15*I*(e*cos(d*x+c))^(5/
2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3596, 3578, 3583, 3569}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{5/2}}{5d}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{5/2}}{15d}+\frac{8ia\sec^2(c+dx)(e\cos(c+dx))^{5/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((2*I)/5)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
- (((16*I)/15)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d
*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
```

```
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}$$

$$= -\frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{(4a(e\cos(c+}{}$$

$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a}}{5d}$$

$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a}}{5d}$$

Mathematica [A]
time = 0.38, size = 63, normalized size = 0.48

$$\frac{ie^2\sqrt{e\cos(c+dx)}\,(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/15)*e^2*Sqrt[e*Cos[c + d*x]]*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c +
 d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A]
time = 0.94, size = 80, normalized size = 0.61

method	result	size

risch	$-\frac{ie^{2}\sqrt{2}\,\sqrt{e\cos\left(dx+c\right)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(30-2\cos\left(2dx+2c\right)+8i\sin\left(2dx+2c\right)\right)}{30d}$	74
default	$\frac{2\left(i\left(\cos^{2}\left(dx+c\right)\right)+4\sin\left(dx+c\right)\cos\left(dx+c\right)-8i\right)\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}}{15d\cos\left(dx+c\right)^{2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(e*cos(d*x+c))^(5/2)*(a
*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [A]
time = 0.61, size = 129, normalized size = 0.98

$$\frac{\sqrt{a}\left(5i\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3i\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-30i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+5\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+30\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)e^{\frac{5}{2}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*e^(5/2)/d
```

Fricas [A]
time = 0.35, size = 79, normalized size = 0.60

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{\frac{5}{2}}-3i\,e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}-30i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(5/2) - 3*I
*e^(4*I*d*x + 4*I*c + 5/2) - 30*I*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(e^(2*I*d*
x + 2*I*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.72, size = 50, normalized size = 0.38

$$\frac{\left(-3i\sqrt{a}\,e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}-30i\sqrt{a}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5i\sqrt{a}\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}\right)e^{\frac{5}{2}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/30*(-3*I*sqrt(a)*e^(5/2*I*d*x + 5/2*I*c) - 30*I*sqrt(a)*e^(1/2*I*d*x + 1/
2*I*c) + 5*I*sqrt(a)*e^(-3/2*I*d*x - 3/2*I*c))*e^(5/2)/d
```

Mupad [B]
time = 0.77, size = 84, normalized size = 0.64

$$\frac{e^2\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+4\,\sin\left(2\,c+2\,d\,x\right)-15\mathrm{i}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 4*sin(2*c + 2*d*x)
- 15i))/(15*d)
```

3.675 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{4iae\sqrt{e\cos(c+dx)}\,\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] 4/3*I*a*e*sec(d*x+c)*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*
(e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3596, 3578, 3569}

$$\frac{4ia\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((4*I)/3)*a*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((2*I)/3)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}$$

$$= -\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{(2a(e\cos(c+}{}$$

$$= \frac{4ia(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{3/2}\sqrt{a}}{3d}$$

Mathematica [A]
time = 0.24, size = 56, normalized size = 0.66

$$\frac{2e\sqrt{e\cos(c+dx)}\,(i\cos(c+dx)+2\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*e*Sqrt[e*Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(3*d)
```

Maple [A]
time = 1.08, size = 70, normalized size = 0.82

method	result	size
risch	$-\frac{ie\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3d}$	65
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))(e\cos(dx+c))^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{3d\cos(dx+c)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [A]
time = 0.61, size = 53, normalized size = 0.62

$$\frac{\sqrt{a}\left(-i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)e^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*d
*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))*e^(3/2)/d
```

Fricas [A]
time = 0.36, size = 67, normalized size = 0.79

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{\frac{3}{2}}-i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(3/2) - I*e^
(2*I*d*x + 2*I*c + 3/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*
I*c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.70, size = 36, normalized size = 0.42

$$\frac{\left(-i\,\sqrt{a}\,e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+3i\,\sqrt{a}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/3*(-I*sqrt(a)*e^(3/2*I*d*x + 3/2*I*c) + 3*I*sqrt(a)*e^(-1/2*I*d*x - 1/2*I
*c))*e^(3/2)/d
```

Mupad [B]
time = 0.53, size = 88, normalized size = 1.04

$$\frac{2\,e\,\sqrt{e\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 2\mathrm{i}+2\,\sin\left(c+d\,x\right)-\mathrm{i}\right)\,\sqrt{\frac{a\,\left(2\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos\left(c+d\,x\right)^2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2*e*(e*(2*cos(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c + d*x) + cos(c/2 + (d*
x)/2)^2*2i - 1i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d
*x)^2))^(1/2))/(3*d)
```

3.676 $\int \sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=36

$$-\frac{2i\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*I*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3596, 3569}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\,\sqrt{e\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\,dx &= \left(\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\\ &= -\frac{2i\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 36, normalized size = 1.00

$$-\frac{2i\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A]
time = 1.08, size = 45, normalized size = 1.25

method	result	size
default	$-\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}}{d}$	45
risch	$-\frac{2i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}}{d}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.56, size = 75, normalized size = 2.08

$$-\frac{2i\sqrt{a}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}\,e^{\frac{1}{2}}}{d\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] -2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)*e^(1/2)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Fricas [A]
time = 0.34, size = 51, normalized size = 1.42

$$-\frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*I*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x +
2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c + 1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\cos{(c+dx)}}\sqrt{ia(\tan{(c+dx)}-i)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [A]
time = 0.69, size = 18, normalized size = 0.50

$$-\frac{2i\sqrt{a}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c+\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] -2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c + 1/2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.677 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=335

$$\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+}}{}\right.}{d\sqrt{e}}$$

```
[Out] -1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))
^(1/2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+1/2
*I*ln(a*e^(1/2)+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+I*arcta
n(1-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*
2^(1/2)*a^(1/2)/d/e^(1/2)-I*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*ta
n(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*2^(1/2)*a^(1/2)/d/e^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3594, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{a}\log\left(-\sqrt{2}\sqrt{a}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}\,d\sqrt{e}}+\frac{i\sqrt{a}\log\left(\sqrt{2}\sqrt{a}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(d*Sqrt[e]) - (I*Sqrt[2]*Sqrt[a]*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
])])/(d*Sqrt[e]) - (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I*a*Tan[c +
d*x])]])/(Sqrt[2]*d*Sqrt[e]) + (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3594

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[-4*(b/f), Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{(4ia)\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\right)}{d}$$

$$= \frac{(2ia)\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}\right)}{d} - \frac{(2ia)\text{Su}}{}$$

$$= -\frac{(ia)\text{Subst}\left(\int \frac{1}{ae-\sqrt{2}\,\sqrt{a}\,\sqrt{e}\,x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx}\right.}{d}$$

$$= -\frac{i\sqrt{a}\,\log\left(a\sqrt{e}-\sqrt{2}\,\sqrt{a}\,\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\,\cos\right.}{\sqrt{2}\,d\sqrt{e}}$$

$$= \frac{i\sqrt{2}\,\sqrt{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\,\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{2}\,\sqrt{a}}{}$$

Mathematica [A]
time = 0.96, size = 125, normalized size = 0.37

$$\frac{ie^{-\frac{3}{2}idx}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\left(\text{ArcTan}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\tanh^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*(ArcTan[E^((I/2)*d*x)/(-
E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*E^(((3*I)/2)*d*x)*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 1.48, size = 225, normalized size = 0.67

method	result
default	$-\frac{(i\sin(dx+c)-\cos(dx+c)+1)(1+\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{e\cos(dx+c)}\left(i\,\text{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*sin(d*x+c)-cos(d*x+c)+1)*(1+cos(d*x+c))*(1/(1+cos(d*x+c)))^(1/2)*
(e*cos(d*x+c))^(1/2)*(I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))-I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))-arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-arctanh
(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))))*(a*(I*sin(d*x+c)+
cos(d*x+c))/cos(d*x+c))^(1/2)/e/sin(d*x+c)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(229) = 458$.
time = 0.67, size = 1399, normalized size = 4.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
```

```
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*e^(-1/2)/d
```

Fricas [A]

time = 0.36, size = 301, normalized size = 0.90

$$-\frac{1}{2}\sqrt{\frac{4i\,ae^{(-1)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{1}{2}i\,d\sqrt{\frac{4i\,ae^{(-1)}}{d^2}}\,e^{\frac{1}{2}}\right)+\frac{1}{2}\sqrt{\frac{4i\,ae^{(-1)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{1}{2}i\,d\sqrt{\frac{4i\,ae^{(-1)}}{d^2}}\,e^{\frac{1}{2}}\right)-\frac{1}{2}\sqrt{-\frac{4i\,ae^{(-1)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{1}{2}i\,d\sqrt{-\frac{4i\,ae^{(-1)}}{d^2}}\,e^{\frac{1}{2}}\right)+\frac{1}{2}\sqrt{-\frac{4i\,ae^{(-1)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{1}{2}i\,d\sqrt{-\frac{4i\,ae^{(-1)}}{d^2}}\,e^{\frac{1}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I*a*e^(-1)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*s
qrt(4*I*a*e^(-1)/d^2)*e^(1/2)) + 1/2*sqrt(4*I*a*e^(-1)/d^2)*log(sqrt(2)*sqr
t(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-
1/2*I*d*x - 1/2*I*c) - 1/2*I*d*sqrt(4*I*a*e^(-1)/d^2)*e^(1/2)) - 1/2*sqrt(-
4*I*a*e^(-1)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*sqrt(-4*I*a
*e^(-1)/d^2)*e^(1/2)) + 1/2*sqrt(-4*I*a*e^(-1)/d^2)*log(sqrt(2)*sqrt(1/2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*
x - 1/2*I*c) - 1/2*I*d*sqrt(-4*I*a*e^(-1)/d^2)*e^(1/2))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

3.678 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=524

$$\frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec}{\sqrt{2}\,de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*a/d/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*arctan(1-
2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(
d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+
1/2*I*a^(3/2)*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2
)/a^(1/2)/e^(1/2))*sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1
/2)*(a-I*a*tan(d*x+c))^(1/2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x
+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/4
*I*a^(3/2)*ln(a+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/
2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 620, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3596, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}+\frac{ia}{d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (I*a)/(d*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c
 + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*
a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2
)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]]) + ((I/2)*a^(3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqr
t[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^(3/2)*e^(3/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e
```

```
*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
```

```
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx)}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(ae\sec(c+dx))\int \sqrt{e}}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx)}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(2ia^2e^3\sec(c+dx))\,\text{Subst}}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx)}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{(ia^2e^2\sec(c+dx))\,\text{Subst}}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx)}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(ia^2e\sec(c+dx))\,\text{Subst}\left(\int -\right.}{2d(e\cos(c+dx))^{3/2}(e\sec(c+dx)}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}}{\sqrt{e s}}\right.}{2\sqrt{2}\,d(e\cos(c+dx))^{3/2}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\tan^{-1}\left(1-\right.}{\sqrt{2}\,d(e\cos(c+dx))^{3/2}(e\sec(c+}$$

Mathematica [A]

time = 3.94, size = 274, normalized size = 0.52

$$\frac{ie^{-\frac{1}{2}i(c+dx)}\cos^2(c+dx)\left(2\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)+2\text{ArcTan}\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)\cos(c+dx)-2\text{ArcTan}\left(1+\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)\cos(c+dx)+\cos(c+dx)\log\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)-\cos(c+dx)\log\left(1+\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)-2i\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)(\cos(c+dx)+i\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}d\left(1+e^{2i(c+dx)}\right)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (I*Cos[c + d*x]^2*(2*Sqrt[2]*Cos[(c + d*x)/2] + 2*ArcTan[1 - Sqrt[2]*E^((I/
2)*(c + d*x))]*Cos[c + d*x] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))]*Cos
[c + d*x] + Cos[c + d*x]*Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*
x))] - Cos[c + d*x]*Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]
```

```
- (2*I)*Sqrt[2]*Sin[(c + d*x)/2])*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt[a +
I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))
*(e*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 1.16, size = 309, normalized size = 0.59

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(-1+\cos(dx+c))^{2}\left(i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx+c))}{2}\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^2*(I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))-I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))-2*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-cos(d*x+c)*arctanh(1/2
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-cos(d*x+c)*arctanh(1/2
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+2*(1/(1+cos(d*x+c)))^(
1/2)*cos(d*x+c)+2*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^3/(I*sin(d*x+c)+cos(
d*x+c)-1)/(1/(1+cos(d*x+c)))^(3/2)/(e*cos(d*x+c))^(3/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1825 vs. 2(372) = 744.
time = 0.83, size = 1825, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -8*(2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arc
tan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*
cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
```

```
), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2
*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*sqrt(2)*cos(2*d*x + 2*c) - sqr
t(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (
sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(-2*sqr
t(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - (-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*s
qrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(
2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-I*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + I*
sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 16*I*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
2*c))))*sqrt(a)*e^(-3/2)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*d*x + 2*c) -
 64*I))
```

<u>Fricas</u> [A]
time = 0.34, size = 433, normalized size = 0.83

$$\frac{4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(\frac{1}{2}idx+\frac{1}{2}ic)} - \left(de^{\frac{3}{2}}+de^{(2idx+2ic+\frac{3}{2})}\right)\sqrt{\frac{iae^{(-3)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}+d\sqrt{\frac{iae^{(-3)}}{d^2}}e^{\frac{3}{2}}\right) + \left(de^{\frac{3}{2}}+de^{(2idx+2ic+\frac{3}{2})}\right)\sqrt{\frac{iae^{(-3)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}-d\sqrt{\frac{iae^{(-3)}}{d^2}}e^{\frac{3}{2}}\right) + \left(de^{\frac{3}{2}}+de^{(2idx+2ic+\frac{3}{2})}\right)\sqrt{-\frac{iae^{(-3)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}+d\sqrt{-\frac{iae^{(-3)}}{d^2}}e^{\frac{3}{2}}\right) - \left(de^{\frac{3}{2}}+de^{(2idx+2ic+\frac{3}{2})}\right)\sqrt{-\frac{iae^{(-3)}}{d^2}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}-d\sqrt{-\frac{iae^{(-3)}}{d^2}}e^{\frac{3}{2}}\right)}{2\left(de^{\frac{3}{2}}+de^{(2idx+2ic+\frac{3}{2})}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*I*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*
x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) - (d*e^(3/2) + d*e^(2*I*d*x + 2*I*c
 + 3/2))*sqrt(I*a*e^(-3)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + d*sqrt(
I*a*e^(-3)/d^2)*e^(3/2)) + (d*e^(3/2) + d*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(I
*a*e^(-3)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - d*sqrt(I*a*e^(-3)/d^2)
*e^(3/2)) + (d*e^(3/2) + d*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(-I*a*e^(-3)/d^2)
*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x +
2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + d*sqrt(-I*a*e^(-3)/d^2)*e^(3/2)) - (
d*e^(3/2) + d*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(-I*a*e^(-3)/d^2)*log(sqrt(2)*
sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e
^(-1/2*I*d*x - 1/2*I*c) - d*sqrt(-I*a*e^(-3)/d^2)*e^(3/2)))/(d*e^(3/2) + d*
e^(2*I*d*x + 2*I*c + 3/2))
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*cos(c + d*x))**(3/2), x)
```

<u>Giac</u> [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2), x)
```

3.679 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=512

$$\frac{3i\sqrt{a}\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}\,e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+d}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{4\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

```
[Out] 3/8*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1
/2)-3/8*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)
/(e*sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*
2^(1/2)-3/16*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)/d/(e*cos(d*x+
c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)+3/16*I*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*
e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan
(d*x+c)))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)+1/2*I
*a/d/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c)^2*(a+I*
a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.38, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3596, 3579, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{3i\sqrt{a}\,e^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}\,e^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}\,e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}+\frac{3i\sqrt{a}\,e^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}+\frac{ia}{2d\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*
(e*Sec[c + d*x])^(5/2)) - (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]
*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (((3*I)/8)*Sqrt[a]*e^(5
/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) + (((3*I)/8)*Sqrt[a]*e^(5/2)*Log[a + (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec
[c + d*x])^(5/2)) + ((I/2)*a)/(d*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((3*I)/4)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[
c + d*x])^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
```

```
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\int (e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(3a)\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}}{4(e\cos(c+dx))^{5/2}(e\sec(c+dx}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+}}{4d(e\cos(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+}}{4d(e\cos(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+}}{4d(e\cos(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+}}{4d(e\cos(c+dx))^{5/2}}$$

$$= -\frac{3i\sqrt{a}\,e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+i\right.}{8\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{3i\sqrt{a}\,e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}\,e^{5/2}\tan^{-1}\left(\right.}{4\sqrt{2}\,d(e\cos}$$

Mathematica [A]

time = 2.59, size = 227, normalized size = 0.44

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3ie^{-\frac{1}{2}i(2c+5dx)}(-e^{-2ic})^{3/4}(1+e^{2i(c+dx)})^2\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\text{ArcTan}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\tanh^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)}{4\sqrt{2}}-3i\cos^{\frac{3}{2}}(c+dx)+2\sqrt{\cos(c+dx)}\,(i\cos(c+dx)+\sin(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((((3*I)/4)*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*
x)))^2*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcTan[E^((I/2)*d*x
)/(-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(
```

```
Sqrt[2]*E^((I/2)*(2*c + 5*d*x))) - (3*I)*Cos[c + d*x]^(3/2) + 2*Sqrt[Cos[c
+ d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(
e*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 1.14, size = 366, normalized size = 0.71

method	result
default	$-\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(-1+\cos(dx+c))^3\left(3i\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c)}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^3*(3*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))-3*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))+6*I*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-3*c
os(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
-3*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))+6*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)+4*I*sin(d*x+c)*(1/(1+cos(d*x+c
)))^(1/2)+2*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*(1/(1+cos(d*x+c)))^(1/2))
/sin(d*x+c)^5/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(5/2)/(e*cos(d
*x+c))^(5/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2232 vs. 2(336) = 672.
time = 0.73, size = 2232, normalized size = 4.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*s
in(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
```

c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*a
rctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c
) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*
I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x
 + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(
2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*cos(4*d*x + 4*c)
+ 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin
(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(I*sqrt(2)*cos(4*d*x + 4
*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*s
in(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(-I*sqrt(2)*
cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3

```
*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4
*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 3*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 48*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 16*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 48*I*sin(7
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*I*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*e^(-5...
```

Fricas [A]
time = 0.35, size = 522, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\left(de^{\frac{5}{2}}+de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{4}{3}i\,d\sqrt{\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\,e^{\frac{5}{2}}\right)+\left(de^{\frac{5}{2}}+de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{4}{3}i\,d\sqrt{\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\,e^{\frac{5}{2}}\right)-\left(de^{\frac{5}{2}}+de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{-\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{4}{3}i\,d\sqrt{-\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\,e^{\frac{5}{2}}\right)+\left(de^{\frac{5}{2}}+de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{-\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{4}{3}i\,d\sqrt{-\frac{9i\,ae^{(-5)}}{16\,d^{2}}}\,e^{\frac{5}{2}}\right)}{2\left(de^{\frac{5}{2}}+de^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+2\,de^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(4*I*d*x +
 4*I*c) + I*e^(2*I*d*x + 2*I*c))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*
x - 1/2*I*c) - (d*e^(5/2) + d*e^(4*I*d*x + 4*I*c + 5/2) + 2*d*e^(2*I*d*x +
2*I*c + 5/2))*sqrt(9/16*I*a*e^(-5)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)
 + 4/3*I*d*sqrt(9/16*I*a*e^(-5)/d^2)*e^(5/2)) + (d*e^(5/2) + d*e^(4*I*d*x +
 4*I*c + 5/2) + 2*d*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(9/16*I*a*e^(-5)/d^2)*lo
g(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I
*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 4/3*I*d*sqrt(9/16*I*a*e^(-5)/d^2)*e^(5/
2)) - (d*e^(5/2) + d*e^(4*I*d*x + 4*I*c + 5/2) + 2*d*e^(2*I*d*x + 2*I*c + 5
/2))*sqrt(-9/16*I*a*e^(-5)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 4/3*I
*d*sqrt(-9/16*I*a*e^(-5)/d^2)*e^(5/2)) + (d*e^(5/2) + d*e^(4*I*d*x + 4*I*c
+ 5/2) + 2*d*e^(2*I*d*x + 2*I*c + 5/2))*sqrt(-9/16*I*a*e^(-5)/d^2)*log(sqrt
(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) +
1)*e^(-1/2*I*d*x - 1/2*I*c) - 4/3*I*d*sqrt(-9/16*I*a*e^(-5)/d^2)*e^(5/2)))/
(d*e^(5/2) + d*e^(4*I*d*x + 4*I*c + 5/2) + 2*d*e^(2*I*d*x + 2*I*c + 5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2), x)
```

3.680 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=719

$$\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{3/2}e^{7/2}\mathrm{Ar}}{8\sqrt{2}\,d(e\cos(c+dx)}$$

[Out] 1/3*I*a/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+5/8*I*a*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^(3/2)*e^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(3/2)*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/32*I*a^(3/2)*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/32*I*a^(3/2)*e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/12*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(7/2)

Rubi [A]
time = 0.58, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {3596, 3579, 3582, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

[illegible]

Antiderivative was successfully verified.

[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]

[Out] ((I/3)*a)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/8)*a*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a - (S

```
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x
])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x
])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/1
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[c + d*x])^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\int (e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(5a)\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}}{6(e\cos(c+dx))^{7/2}(e\sec(c+dx}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+}}{12d(e\cos(c+dx))^{7/2}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\,\mathrm{ta}}}$$

Mathematica [A]
time = 2.96, size = 305, normalized size = 0.42

$$\frac{\sqrt{\cos(c+dx)}\left(-\frac{40}{3}i\cos^{\frac{3}{2}}(c+dx)+\frac{5}{8}ie^{-\frac{7}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^3\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\right)-2\mathrm{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\right)+\log\left(1-\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)-\log\left(1+\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)\right)+\frac{32}{3}\sqrt{\cos(c+dx)}\,(i\cos(c+dx)+\sin(c+dx))+20\cos^{\frac{5}{2}}(c+dx)(i\cos(c+dx)+\sin(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{32d(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(((-40*I)/3)*Cos[c + d*x]^(3/2) + (((5*I)/8)*(1 + E^((2
*I)*(c + d*x)))^3*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(2*ArcTan
[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x)
)] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Log[1 + Sqrt[
2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))/E^(((7*I)/2)*(c + d*x)) + (32*S
qrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/3 + 20*Cos[c + d*x]^(5/2
)*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(32*d*(e*Cos
[c + d*x])^(7/2))
```

Maple [A]
time = 1.10, size = 417, normalized size = 0.58

method	result
default	$\frac{\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(-1+\cos(dx+c))^4\left(30i\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}-15i\left(\cos^3(dx+c)\right)\operatorname{arctan}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^4*(30*I*sin(d*x+c)*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)-15*I*cos(d*x
+c)^3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+15*I*
cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)+20*I*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-30*cos(d*x+c)^3*(1/(1
+cos(d*x+c)))^(1/2)+15*cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)+1-sin(d*x+c)))+15*cos(d*x+c)^3*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))+16*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-10*c
os(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)+4*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-
16*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1
+cos(d*x+c)))^(7/2)/(e*cos(d*x+c))^(7/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2638 vs. 2(469) = 938.
time = 0.85, size = 2638, normalized size = 3.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -192*(30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)
*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*
```

c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(
2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*
c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))
*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1,
-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sq
rt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x +
 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqr
t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqr
t(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 30*(-I*sqrt(2)*cos
(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*
d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 30*(I*sqrt(2)*cos(6*d*x + 6*
c) + 3*I*sqrt(2)*cos(4*d*x + 4*c) + 3*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*
sin(6*d*x + 6*c) - 3*sqrt(2)*sin(4*d*x + 4*c) - 3*sqrt(2)*sin(2*d*x + 2*c)
+ I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 15*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt
(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6
*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2)
)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - 15*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqr
t(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d

```
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1) - 15*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c)
 - 3*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(
4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 15*(I*sqrt(2)*cos(6*d*x + 6*c) + 3*I*sqrt(2)*cos(4*d*x + 4*c) +
 3*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(6*d*x + 6*c) - 3*sqrt(2)*sin(4*
d*x + 4*c) - 3*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) - 15*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) -
3*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d...
```

Fricas [A]
time = 0.34, size = 600, normalized size = 0.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(5*I*d*x
+ 5*I*c) + 42*I*e^(3*I*d*x + 3*I*c) + 15*I*e^(I*d*x + I*c))*sqrt(e^(2*I*d*x
 + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 6*(d*e^(7/2) + d*e^(6*I*d*x + 6*I
*c + 7/2) + 3*d*e^(4*I*d*x + 4*I*c + 7/2) + 3*d*e^(2*I*d*x + 2*I*c + 7/2))*
sqrt(25/64*I*a*e^(-7)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 8/5*d*sqrt
(25/64*I*a*e^(-7)/d^2)*e^(7/2)) + 6*(d*e^(7/2) + d*e^(6*I*d*x + 6*I*c + 7/2
) + 3*d*e^(4*I*d*x + 4*I*c + 7/2) + 3*d*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(25/
64*I*a*e^(-7)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 8/5*d*sqrt(25/64*I
*a*e^(-7)/d^2)*e^(7/2)) + 6*(d*e^(7/2) + d*e^(6*I*d*x + 6*I*c + 7/2) + 3*d*
e^(4*I*d*x + 4*I*c + 7/2) + 3*d*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(-25/64*I*a*
e^(-7)/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^
(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) + 8/5*d*sqrt(-25/64*I*a*e^(
-7)/d^2)*e^(7/2)) - 6*(d*e^(7/2) + d*e^(6*I*d*x + 6*I*c + 7/2) + 3*d*e^(4*I
```

```
*d*x + 4*I*c + 7/2) + 3*d*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(-25/64*I*a*e^(-7)
/d^2)*log(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d
*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c) - 8/5*d*sqrt(-25/64*I*a*e^(-7)/d^
2)*e^(7/2)))/(d*e^(7/2) + d*e^(6*I*d*x + 6*I*c + 7/2) + 3*d*e^(4*I*d*x + 4*
I*c + 7/2) + 3*d*e^(2*I*d*x + 2*I*c + 7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2), x)
```

3.681 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=175

$$\frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}}+\frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad}-\frac{32}{}$$

```
[Out] 2/7*I*(e*cos(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/35*I*(e*cos(d*x+c)
)^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(e*cos(d*x+c))^(5/2
)*(a+I*a*tan(d*x+c))^(1/2)/a/d-32/35*I*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2*(a
+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.25, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3596, 3583, 3578, 3569}

$$-\frac{12i\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{5/2}}{35ad}+\frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{32i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{5/2}}{35ad}+\frac{16i\sec^2(c+dx)(e\cos(c+dx))^{5/2}}{35d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/7)*(e*Cos[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((12*I)/35)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) -
(((32*I)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x
]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+d}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(6(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+i}}{(e\,\mathrm{se}}}{7a}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{(24}{}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c}{}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c}{}$$

Mathematica [A]
time = 0.71, size = 80, normalized size = 0.46

$$-\frac{ie^3(35\cos(c+dx)+\cos(3(c+dx))+70i\sin(c+dx)+6i\sin(3(c+dx)))}{70d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/70*I)*e^3*(35*Cos[c + d*x] + Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] +
(6*I)*Sin[3*(c + d*x)]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [A]
time = 1.05, size = 110, normalized size = 0.63

method	result
default	$\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(e\cos(dx+c))^{\frac{5}{2}}\,(5i(\cos^4(dx+c))+5\sin(dx+c)(\cos^3(dx+c))+2i(\cos^2(dx+c))+8\sin(dx+c)\cos(dx+c}{35d\cos(dx+c)^2 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e*cos(d*x+c))^(5/2)*
(5*I*cos(d*x+c)^4+5*sin(d*x+c)*cos(d*x+c)^3+2*I*cos(d*x+c)^2+8*sin(d*x+c)*c
os(d*x+c)-16*I)/cos(d*x+c)^2/a
```

Maxima [A]
time = 0.60, size = 177, normalized size = 1.01

$$\frac{\left(5i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-7i\cos\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-105i\cos\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+5\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+105\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\right)e^{\frac{5}{2}}}{140\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*e^(5/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.32, size = 94, normalized size = 0.54

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{\frac{5}{2}}-7i\,e^{\left(6i\,dx+6i\,c+\frac{5}{2}\right)}-105i\,e^{\left(4i\,dx+4i\,c+\frac{5}{2}\right)}+35i\,e^{\left(2i\,dx+2i\,c+\frac{5}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c\right)}}{140\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(5/2) - 7*
I*e^(6*I*d*x + 6*I*c + 5/2) - 105*I*e^(4*I*d*x + 4*I*c + 5/2) + 35*I*e^(2*I
*d*x + 2*I*c + 5/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-7/2*I*d*x - 7/2*I*c)
/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [**B**]
time = 5.24, size = 110, normalized size = 0.63

$$\frac{e^2\sqrt{e\cos(c+dx)}\sqrt{\frac{a\,(\cos(2c+2dx)+1+\sin(2c+2dx)\,1\mathrm{i})}{\cos(2c+2dx)+1}}\,(\cos(2c+2dx)\,28\mathrm{i}+\cos(4c+4dx)\,5\mathrm{i}+42\sin(2c+2dx)+5\sin(4c+4dx)-105\mathrm{i})}{140\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] (e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*28i + cos(4*c + 4*d*x)*5
i + 42*sin(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) - 105i))/(140*a*d)
```

3.682 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}-\frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad}$$

```
[Out] 2/5*I*(e*cos(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*cos(d*x+c)
)^(3/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(e*cos(d*x+c))^(3/2)
*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3596, 3583, 3578, 3569}

$$-\frac{8i\sqrt{a+ia\tan(c+dx)}\,(e\cos(c+dx))^{3/2}}{15ad}+\frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/5)*(e*Cos[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/15)*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((8*I)/15)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+d}$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(4(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{a+i}}{(e\,se}}{5a}$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad} + \frac{(8(e}{}$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+}{}$$

Mathematica [A]
time = 0.42, size = 63, normalized size = 0.50

$$-\frac{ie^2(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-1/15*I)*e^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/(d*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 0.92, size = 100, normalized size = 0.79

method	result	size
default	$\frac{2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(e\cos(dx+c))^{\frac{3}{2}}\left(3i(\cos^3(dx+c))+3(\cos^2(dx+c))\sin(dx+c)+4i\cos(dx+c)+8\sin(dx+c)\right)}{15d\cos(dx+c)a}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e*cos(d*x+c))^(3/2)*
(3*I*cos(d*x+c)^3+3*cos(d*x+c)^2*sin(d*x+c)+4*I*cos(d*x+c)+8*sin(d*x+c))/co
s(d*x+c)/a
```

Maxima [A]
time = 0.60, size = 129, normalized size = 1.02

$$\frac{\left(3i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+3\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)e^{\frac{3}{2}}}{30\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))*e^(3/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.32, size = 82, normalized size = 0.65

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{\frac{3}{2}}-5i\,e^{\left(4i\,dx+4i\,c+\frac{3}{2}\right)}+30i\,e^{\left(2i\,dx+2i\,c+\frac{3}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(3/2) - 5*I
*e^(4*I*d*x + 4*I*c + 3/2) + 30*I*e^(2*I*d*x + 2*I*c + 3/2))*sqrt(e^(2*I*d*
x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 1.13, size = 100, normalized size = 0.79

$$\frac{e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(35\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,25{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,3{}\mathrm{i}\right)}{30\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))
/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*25i + 35*sin(c + d*x) + cos(3*
c + 3*d*x)*3i + 3*sin(3*c + 3*d*x)))/(30*a*d)
```

3.683 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3ad}$$

```
[Out] 2/3*I*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(e*cos(d*x+c))^
(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3596, 3583, 3569}

$$\frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}\,\sqrt{e\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*Sqrt[e*Cos[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
+ b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(2\sqrt{e\cos(c+dx)}\,\sqrt{e\sec(c+dx)}\right)\int \frac{\sqrt{a+ia}}{\sqrt{e\sec}}}{3a}$$
$$= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{e\cos(c+dx)}\,\sqrt{a+ia\tan(c+dx)}}{3ad}$$

Mathematica [A]
time = 0.23, size = 48, normalized size = 0.60

$$\frac{2\sqrt{e\cos(c+dx)}\,(-i+2\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*Sqrt[e*Cos[c + d*x]]*(-I + 2*Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [A]
time = 1.03, size = 74, normalized size = 0.92

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,(3\,e^{2i(dx+c)}-1)}{3(e^{2i(dx+c)}+1)\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	72
default	$\frac{2\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(i(\cos^2(dx+c))+\sin(dx+c)\cos(dx+c)-2i)}{3da}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(
I*cos(d*x+c)^2+sin(d*x+c)*cos(d*x+c)-2*I)/a
```

Maxima [A]
time = 0.62, size = 79, normalized size = 0.99

$$\frac{\left(i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right)e^{\frac{1}{2}}}{3\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*e^(1/2)/(sqrt(a)*d)
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.88

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{\frac{1}{2}}-3i\,e^{\left(2i\,dx+2i\,c+\frac{1}{2}\right)}\right)\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(1/2) - 3*I*e^
(2*I*d*x + 2*I*c + 1/2))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-3/2*I*d*x - 3/2*
I*c)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{e\cos\left(c+dx\right)}}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B]
time = 0.73, size = 82, normalized size = 1.02

$$\frac{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)-3{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] ((e*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) - 3i)*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
/(3*a*d)
```

3.684 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3596, 3569}

$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &= \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 36, normalized size = 1.00

$$\frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(30) = 60.
time = 0.93, size = 67, normalized size = 1.86

method	result	size
risch	$\frac{i\sqrt{2}}{\sqrt{e\cos\left(dx+c\right)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	46
default	$\frac{2i\sqrt{e\cos\left(dx+c\right)}\,\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\,(-i\sin(dx+c)+\cos(dx+c))}{dea}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*I/d*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(
-I*sin(d*x+c)+cos(d*x+c))/e/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(28) = 56.
time = 0.56, size = 75, normalized size = 2.08

$$\frac{2i\sqrt{-\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1}\,e^{\left(-\frac{1}{2}\right)}}{\sqrt{a}\,d\sqrt{-\frac{2i\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*e^(-1/2)/(sqrt(a)*d*sqrt
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Fricas [A]
time = 0.32, size = 54, normalized size = 1.50

$$\frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c-\frac{1}{2}\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*I*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2
*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c - 1/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{e\cos\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-1/2)/(sqrt(I*a*tan(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.685 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=495

$$-\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}\,\text{ArcTan}\left(1+\cdots\right.}{de^{3/2}\sqrt{a\cdots}}$$

```
[Out] 1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^
(1/2)+cos(d*x+c)*e^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)*2
^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*ln(a*e^(1/2)
+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)+cos(d*x+c)*e
^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*arctan(1-2^(1/2)*(e*cos(d*x+c))^(1
/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)*2^(1/2)*a^(1/2)/d/
e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*arctan(1+2^(1/2
)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)
*2^(1/2)*a^(1/2)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.24, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3595, 3594, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{a}\sec(c+dx)\log\left(-\sqrt{2}\sqrt{a}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{a}\sec(c+dx)\log\left(\sqrt{2}\sqrt{a}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*Sec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*ArcTan[1 + (Sq
rt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*S
ec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[
a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
+ d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]]) - (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sq
rt[a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec
[c + d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3594

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[-4*(b/f), Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rule 3595

```
Int[1/((cos[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)*Sqrt[(a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Dist[1/(d*Cos[e + f*x]*Sqrt[a - b*Tan[e + f*x]]
*Sqrt[a + b*Tan[e + f*x]]), Int[Sqrt[a - b*Tan[e + f*x]]/Sqrt[d*Cos[e + f*x
]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{(4ia\sec(c+dx))\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}$$

$$= -\frac{(2ia\sec(c+dx))\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}$$

$$= \frac{\left(i\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}+2x}{-ae-\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+}$$

$$= \frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+}$$

$$= -\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c}{\sqrt{a}\sqrt{e}}\right.}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}$$

Mathematica [A]
time = 10.55, size = 209, normalized size = 0.42

$$\frac{ie^{\frac{1}{2}i(c+dx)}\left(2\text{ArcTan}\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)+\log\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)-\log\left(1+\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)\right)}{\sqrt{2}de\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*E^((I/2)*(c + d*x))*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTa
n[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) +
E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))
```

/(Sqrt[2]*d*e*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))])

Maple [A]
time = 4.86, size = 230, normalized size = 0.46

method	result
default	$\frac{(\cos^2(dx+c))(-1+\cos(dx+c))^2\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right)-i\,\mathrm{arctanh}\right.}{d\sin(dx+c)^3(e\cos(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/d*cos(d*x+c)^2*(-1+cos(d*x+c))^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)*(I*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I
*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+arctanh(1/
2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+arctanh(1/2*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))))/sin(d*x+c)^3/(e*cos(d*x+c))^(3/
2)/(1/(1+cos(d*x+c)))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/a
```

Maxima [A]
time = 0.63, size = 713, normalized size = 1.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*sqrt(
2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
 + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + I*
sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^
2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
```

```
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*e^(-3/2)/(sqrt(a)*d)
```

Fricas [**A**]
time = 0.36, size = 321, normalized size = 0.65

$$-\frac{1}{2}\sqrt{\frac{4i\,e^{(-3)}}{ad^2}}\log\left(\frac{1}{2}\,ad\sqrt{\frac{4i\,e^{(-3)}}{ad^2}}\,e^{\frac{3}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}\right)+\frac{1}{2}\sqrt{\frac{4i\,e^{(-3)}}{ad^2}}\log\left(-\frac{1}{2}\,ad\sqrt{\frac{4i\,e^{(-3)}}{ad^2}}\,e^{\frac{3}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}\right)+\frac{1}{2}\sqrt{-\frac{4i\,e^{(-3)}}{ad^2}}\log\left(\frac{1}{2}\,ad\sqrt{-\frac{4i\,e^{(-3)}}{ad^2}}\,e^{\frac{3}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}\right)-\frac{1}{2}\sqrt{-\frac{4i\,e^{(-3)}}{ad^2}}\log\left(-\frac{1}{2}\,ad\sqrt{-\frac{4i\,e^{(-3)}}{ad^2}}\,e^{\frac{3}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{e^{(2i\,dx+2i\,c)}+1}\,e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(4*I*e^(-3)/(a*d^2))*log(1/2*a*d*sqrt(4*I*e^(-3)/(a*d^2))*e^(3/2)
+ sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I
*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(4*I*e^(-3)/(a*d^2))*log(-1/2*
a*d*sqrt(4*I*e^(-3)/(a*d^2))*e^(3/2) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) + 1
/2*sqrt(-4*I*e^(-3)/(a*d^2))*log(1/2*a*d*sqrt(-4*I*e^(-3)/(a*d^2))*e^(3/2)
+ sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I
*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) - 1/2*sqrt(-4*I*e^(-3)/(a*d^2))*log(-1/2
*a*d*sqrt(-4*I*e^(-3)/(a*d^2))*e^(3/2) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(e\cos{(c+dx)})^{\frac{3}{2}}\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*cos(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-3/2)/(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\tan(c+dx)\,1i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.686 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=470

$$\frac{ie^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5}}{}$$

```
[Out] 1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1
/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)
/(e*sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/
a^(1/2)-1/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)
/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/(e*cos(d*x+c))^(5/2)
/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2)+1/4*I*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^
(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d
*x+c)))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2)-I*cos(d
*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.30, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3596, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\frac{ie^{5/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}+\frac{ie^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c
 + d*x])^(5/2)) - (I*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
 + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]
*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a
 + I*a*Tan[c + d*x])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c +
 d*x])^(5/2)) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*
a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])
])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (I*C
os[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{e^2\int \sqrt{e\sec(c+d}}{2a(e\cos(c+}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(2ie^4)\,\mathrm{Subst}\Big(\int \frac{}{a^2}}{d(e\cos(c}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{(ie^3)\,\mathrm{Subst}\Big(\int \frac{a-}{a^2+}}{d(e\cos(c-}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(ie^2)\,\mathrm{Subst}\Bigg(\int \frac{}{\frac{a}{e}-}}{2d(e\,c}$$

$$= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c\right.}{2\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c-}$$

$$= \frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\,d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{ie^{5/2}\,\mathrm{t}}{\sqrt{2}}$$

Mathematica [A]

time = 1.93, size = 250, normalized size = 0.53

$$\frac{ie^{ic-\frac{idx}{2}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-2e^{\frac{3idx}{2}}+(-e^{-2ic})^{3/4}\left(1+e^{2i(c+dx)}\right)\operatorname{ArcTan}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-(-e^{-2ic})^{3/4}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)}{d\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\sqrt{\cos(c+dx)}\,(e\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]

[Out] (I*E^(I*c - (I/2)*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-2*E^(((3*I)/2)*d*x) + (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTan[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(d*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*Sqrt[Cos[c + d*x]]*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])

Maple [A]
time = 0.99, size = 314, normalized size = 0.67

method	result
default	$-\frac{(\cos^2(dx+c))(-1+\cos(dx+c))^3\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\cos(dx+c)\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1+\sin(dx+c))}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*cos(d*x+c)^2*(-1+cos(d*x+c))^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*I*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-cos(d*x+c)*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+2*(1/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(5/2)/(e*cos(d*x+c))^(5/2)/a

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2137 vs. 2(310) = 620.
time = 0.70, size = 2137, normalized size = 4.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")

[Out] -8*(2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - I*sqrt(2))*arct
an2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(I*sqrt(2)*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - sqrt(2)*sin(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2))*arctan2(-sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (sqrt(2)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(2*sqrt(2)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (sqrt(2)*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(-2
*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1

```
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 1) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2) + (-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c...
```

<u>**Fricas**</u> [**A**]
time = 0.36, size = 465, normalized size = 0.99

$$\frac{-4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(\frac{3}{2}idx+\frac{3}{2}ic)}-\left(ade^{\frac{5}{2}}+ade^{(2idx+2ic+\frac{5}{2})}\right)\sqrt{\frac{ie^{(-5)}}{ad^2}}\log\left(iad\sqrt{\frac{ie^{(-5)}}{ad^2}}e^{\frac{5}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}\right)+\left(ade^{\frac{5}{2}}+ade^{(2idx+2ic+\frac{5}{2})}\right)\sqrt{\frac{ie^{(-5)}}{ad^2}}\log\left(-iad\sqrt{\frac{ie^{(-5)}}{ad^2}}e^{\frac{5}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}\right)-\left(ade^{\frac{5}{2}}+ade^{(2idx+2ic+\frac{5}{2})}\right)\sqrt{-\frac{ie^{(-5)}}{ad^2}}\log\left(iad\sqrt{-\frac{ie^{(-5)}}{ad^2}}e^{\frac{5}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}\right)+\left(ade^{\frac{5}{2}}+ade^{(2idx+2ic+\frac{5}{2})}\right)\sqrt{-\frac{ie^{(-5)}}{ad^2}}\log\left(-iad\sqrt{-\frac{ie^{(-5)}}{ad^2}}e^{\frac{5}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{(-\frac{1}{2}idx-\frac{1}{2}ic)}\right)}{2\left(ade^{\frac{5}{2}}+ade^{(2idx+2ic+\frac{5}{2})}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(-4*I*sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d
*x + 2*I*c) + 1)*e^(3/2*I*d*x + 3/2*I*c) - (a*d*e^(5/2) + a*d*e^(2*I*d*x +
2*I*c + 5/2))*sqrt(I*e^(-5)/(a*d^2))*log(I*a*d*sqrt(I*e^(-5)/(a*d^2))*e^(5/
2) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x +
2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^(5/2) + a*d*e^(2*I*d*x + 2*I
*c + 5/2))*sqrt(I*e^(-5)/(a*d^2))*log(-I*a*d*sqrt(I*e^(-5)/(a*d^2))*e^(5/2)
 + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*
I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) - (a*d*e^(5/2) + a*d*e^(2*I*d*x + 2*I*c
 + 5/2))*sqrt(-I*e^(-5)/(a*d^2))*log(I*a*d*sqrt(-I*e^(-5)/(a*d^2))*e^(5/2)
+ sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I
```

```
*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^(5/2) + a*d*e^(2*I*d*x + 2*I*c
+ 5/2))*sqrt(-I*e^(-5)/(a*d^2))*log(-I*a*d*sqrt(-I*e^(-5)/(a*d^2))*e^(5/2)
+ sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I
*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^(5/2) + a*d*e^(2*I*d*x + 2*I*c +
 5/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-5/2)/(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,\cos\left(c+d\,x\right))^{5/2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.687 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=682

$$\frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{3i\sqrt{a}\,e^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\,\sqrt{}}$$

```
[Out] 3/4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-3/8*I*e^
(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+
c))^(1/2))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2
^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/8*I*e^(7/2)*arct
an(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))
*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-
I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*e^(7/2)*ln(a-2^(1/2)*
a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a
-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^
(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3/16*I*e^(7
/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1
/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2
)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^
(1/2)-1/2*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(7/2)
```

Rubi [A]

time = 0.49, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {3596, 3582, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3i\sqrt{a}\,e^{7/2}\sec(c+dx)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{3i\sqrt{a}\,e^{7/2}\sec(c+dx)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{3i\sqrt{a}\,e^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}-\frac{3i\sqrt{a}\,e^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}\,d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}-\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}+\frac{3i\cos^2(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((3*I)/4)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d
*x]]) - (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d
*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[
a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x
]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
 + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I
```

```
*a*Tan[c + d*x]]) - (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I
*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c
+ d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3e^2)\int(e\sec(c+}{4a(e\cos(c+}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)}{2ad(e}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)}{2ad(e}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)}{2ad(e}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)}{2ad(e}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)}{2ad(e}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{3i\sqrt{a}\,e^{7/2}\log}{8\sqrt{2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{}}{4\sqrt{2}\,d(e\cos(}$$

Mathematica [A]

time = 1.63, size = 245, normalized size = 0.36

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}ie^{\frac{1}{2}i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{5/2}\left(2\text{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\right)+\log\left(1-\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)-\log\left(1+\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}\right)\right)+4\sqrt{\cos(c+dx)}\,(i\cos(c+dx)+2\sin(c+dx))\right)}{16d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(((3*I)/4)*E^((I/2)*(c + d*x))*((1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x)))^(5/2)*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*Ar
```

```
cTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x))
 + E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))
]) + 4*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])))/(16*d*(e*Cos[
c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A]
time = 1.05, size = 371, normalized size = 0.54

method	result
default	$-\frac{(\cos^2(dx+c))(-1+\cos(dx+c))^4\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{1+\cos(dx+c)}}\,(\cos(dx+c)+1-\sin(dx\cdots}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/8/d*cos(d*x+c)^2*(-1+cos(d*x+c))^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+
c))^(1/2)*(3*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))-3*I*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))-6*I*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)+6
*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)-3*cos(d*x+c)^2*arctanh(1/2*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-3*cos(d*x+c)^2*arctanh(1/2*(1/(1
+cos(d*x+c)))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-4*I*sin(d*x+c)*(1/(1+cos(d*x
+c)))^(1/2)+2*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*(1/(1+cos(d*x+c)))^(1/2
))/sin(d*x+c)^7/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(1+cos(d*x+c)))^(7/2)/(e*cos
(d*x+c))^(7/2)/a
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2238 vs. 2(442) = 884.
time = 0.74, size = 2238, normalized size = 3.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*s
in(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
```

```
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*a
rctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c
) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*
I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x
 + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(
2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(sqrt(2)*cos(4*d*x + 4*c)
+ 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin
(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(-I*sqrt(2)*cos(4*d*x +
4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*
sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(I*sqrt(2)*
cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c)
- 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
 + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3
*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(
4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 3*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) -
sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 16*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 48*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*I*sin(5
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 48*I*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*e^(-7...
```

Fricas [A]
time = 0.37, size = 557, normalized size = 0.82

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(-ie^{(3idx+3ic)}+3ie^{(idx+ic)}\right)\sqrt{e^{(2idx+2ic)}+1}\,e^{\left(-\frac{1}{2}idx-\frac{1}{2}ic\right)}-\left(ade^{\frac{7}{2}}+ade^{\left(4idx+4ic+\frac{7}{2}\right)}+2ade^{\left(2idx+2ic+\frac{7}{2}\right)}\right)\sqrt{\frac{9ie^{(-7)}}{16ad^2}}\log\left(\frac{4}{3}ad\sqrt{\frac{9ie^{(-7)}}{16ad^2}}e^{\frac{7}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{\left(-\frac{1}{2}idx-\frac{1}{2}ic\right)}\right)+\left(ade^{\frac{7}{2}}+ade^{\left(4idx+4ic+\frac{7}{2}\right)}+2ade^{\left(2idx+2ic+\frac{7}{2}\right)}\right)\sqrt{\frac{9ie^{(-7)}}{16ad^2}}\log\left(-\frac{4}{3}ad\sqrt{\frac{9ie^{(-7)}}{16ad^2}}e^{\frac{7}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{\left(-\frac{1}{2}idx-\frac{1}{2}ic\right)}\right)+\left(ade^{\frac{7}{2}}+ade^{\left(4idx+4ic+\frac{7}{2}\right)}+2ade^{\left(2idx+2ic+\frac{7}{2}\right)}\right)\sqrt{-\frac{9ie^{(-7)}}{16ad^2}}\log\left(\frac{4}{3}ad\sqrt{-\frac{9ie^{(-7)}}{16ad^2}}e^{\frac{7}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{\left(-\frac{1}{2}idx-\frac{1}{2}ic\right)}\right)-\left(ade^{\frac{7}{2}}+ade^{\left(4idx+4ic+\frac{7}{2}\right)}+2ade^{\left(2idx+2ic+\frac{7}{2}\right)}\right)\sqrt{-\frac{9ie^{(-7)}}{16ad^2}}\log\left(-\frac{4}{3}ad\sqrt{-\frac{9ie^{(-7)}}{16ad^2}}e^{\frac{7}{2}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{e^{(2idx+2ic)}+1}\,e^{\left(-\frac{1}{2}idx-\frac{1}{2}ic\right)}\right)}{2\left(ade^{\frac{7}{2}}+ade^{\left(4idx+4ic+\frac{7}{2}\right)}+2ade^{\left(2idx+2ic+\frac{7}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(3*I*d*x + 3
*I*c) + 3*I*e^(I*d*x + I*c))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x -
1/2*I*c) - (a*d*e^(7/2) + a*d*e^(4*I*d*x + 4*I*c + 7/2) + 2*a*d*e^(2*I*d*x
+ 2*I*c + 7/2))*sqrt(9/16*I*e^(-7)/(a*d^2))*log(4/3*a*d*sqrt(9/16*I*e^(-7)/
(a*d^2))*e^(7/2) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^(7/2) + a*d*e^
(4*I*d*x + 4*I*c + 7/2) + 2*a*d*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(9/16*I*e^(-
7)/(a*d^2))*log(-4/3*a*d*sqrt(9/16*I*e^(-7)/(a*d^2))*e^(7/2) + sqrt(2)*sqrt
(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1
/2*I*d*x - 1/2*I*c)) + (a*d*e^(7/2) + a*d*e^(4*I*d*x + 4*I*c + 7/2) + 2*a*d
*e^(2*I*d*x + 2*I*c + 7/2))*sqrt(-9/16*I*e^(-7)/(a*d^2))*log(4/3*a*d*sqrt(-
9/16*I*e^(-7)/(a*d^2))*e^(7/2) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)) - (a*d*e^
(7/2) + a*d*e^(4*I*d*x + 4*I*c + 7/2) + 2*a*d*e^(2*I*d*x + 2*I*c + 7/2))*sq
rt(-9/16*I*e^(-7)/(a*d^2))*log(-4/3*a*d*sqrt(-9/16*I*e^(-7)/(a*d^2))*e^(7/2
) + sqrt(2)*sqrt(1/2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e^(2*I*d*x + 2
*I*c) + 1)*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^(7/2) + a*d*e^(4*I*d*x + 4*I*c
 + 7/2) + 2*a*d*e^(2*I*d*x + 2*I*c + 7/2))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(e^(-7/2)/(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.688 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=105

$$-\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(2+m-2n);1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}(a}{dm}$$

```
[Out] -I*2^(-1/2*m+n)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1+1/2*m-n],[1-1/2*m],1/
2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

Rubi [A]
time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(m-2n+2);1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1/2*m + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (2 + m -
2*n)/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+i$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))\right.$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+d\right.}{}$$

$$= \frac{\left(2^{-1-\frac{m}{2}+n}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\,\mathrm{ta}\right.}{}$$

$$= -\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(2+m-2n);1-\frac{m}{2};\frac{1}{2}(\right.}{}$$

Mathematica [A]
time = 13.49, size = 202, normalized size = 1.92

$$\frac{i2^{-m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^{-m+n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)(e\cos(c+dx))^m\,{}_2F_1\left(-m+n,-\frac{m}{2}+n;1-\frac{m}{2}+n;-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(m-2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(
1 + E^((2*I)*(c + d*x)))^(-m + n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
))^m*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m + n, -1/2*m + n, 1 - m/2 + n,
```

```
-E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(m - 2*n)*Cos[c + d*x]^m
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [**F**]
time = 0.76, size = 0, normalized size = 0.00

$$\int (e\cos (dx+c))^{m} (a+ia\tan (dx+c))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^m*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*m*x +
 I*c*m + m*log(a*e) - m*log(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) +
1))), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos (c+dx))^{m} (ia(\tan (c+dx)-i))^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*cos(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^m\,\left(a+a\,\tan(c+d\,x)\,\,1{\mathrm{i}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.689 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m\,{}_2F_1\left(\frac{1}{2}(-2+m),-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/2}}{dm}$$

```
[Out] -I*2^(2-1/2*m)*a^2*(e*cos(d*x+c))^m*hypergeom([-1/2*m, -1+1/2*m],[1-1/2*m],
1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Rubi [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(\frac{m-2}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/2
*m, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx &= ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia\\ &= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))\right.\\ &= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx\right.}{}\\ &= \frac{\left(2^{1-\frac{m}{2}}a^3(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+d}{a}\right.\right.}{}\\ &= -\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m\,{}_2F_1\left(\frac{1}{2}(-2+m),-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-\right.}{dm}\end{aligned}$$

Mathematica [A]
time = 1.56, size = 144, normalized size = 1.67

$$-\frac{i2^{2-m}a^2e^{2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{-m}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{2-m}(c+dx)(e\cos(c+dx))^m\,{}_2F_1\left(2-m,2-\frac{m}{2};3-\frac{m}{2};-e^{2i(c+dx)}\right)(-i+\tan(c+dx))^2}{d(-4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m)*a^2*E^((2*I)*(c + d*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c
+ d*x)))^m*Cos[c + d*x]^(2 - m)*(e*Cos[c + d*x])^m*Hypergeometric2F1[2 - m,
 2 - m/2, 3 - m/2, -E^((2*I)*(c + d*x))]*(-I + Tan[c + d*x])^2)/(d*(1 + E^(
(2*I)*(c + d*x)))^m*(-4 + m))
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{m} \left(a+ia\tan\left(dx+c\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(cos(d*x + c)*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(4*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m*a^2*e^(4*
I*d*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int \left(-\left(e\cos\left(c+dx\right)\right)^{m}\right)\, dx + \int \left(e\cos\left(c+dx\right)\right)^{m}\tan^{2}\left(c+dx\right)\, dx + \int \left(-2i\left(e\cos\left(c+dx\right)\right)^{m}\tan\left(c+dx\right)\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(-(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*tan
(c + d*x)**2, x) + Integral(-2*I*(e*cos(c + d*x))**m*tan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(cos(d*x + c)*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^m\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.690 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/2}}{dm}$$

```
[Out] -I*2^(1-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([1/2*m, -1/2*m],[1-1/2*m],1/2-1
/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Rubi [A]

time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, m/2, 1 - m
/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))\right)^m$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx)\right.}{}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+d}{a}\right.\right.}{}$$

$$= -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+\right.}{dm}$$

Mathematica [A]
time = 6.60, size = 131, normalized size = 1.60

$$-\frac{2^{1-m}a\left(1+e^{2i(c+dx)}\right)^{1-m}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{-1+m}\cos^{1-m}(c+dx)(e\cos(c+dx))^m\,{}_2F_1\left(1-m,1-\frac{m}{2};2-\frac{m}{2};-e^{2i(c+dx)}\right)(-i+\tan(c+dx))}{d(-2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -((2^(1 - m)*a*(1 + E^((2*I)*(c + d*x)))^(1 - m)*((1 + E^((2*I)*(c + d*x)))
/E^(I*(c + d*x)))^(-1 + m)*Cos[c + d*x]^(1 - m)*(e*Cos[c + d*x])^m*Hypergeo
metric2F1[1 - m, 1 - m/2, 2 - m/2, -E^((2*I)*(c + d*x))]*(-I + Tan[c + d*x]
))/(d*(-2 + m)))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int (e\cos (dx+c))^m (a+ia\tan (dx+c))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)*(cos(d*x + c)*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(2*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m*a*e^(2*I*
d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int \left(-i(e\cos (c+dx))^m\right)\ dx + \int (e\cos (c+dx))^m \tan (c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*tan
(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*(cos(d*x + c)*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^m\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.691 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=86

$$-\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{4+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/2}}{adm}$$

```
[Out] -I*2^(-1-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 2+1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a/d/m
```

Rubi [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+4}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (4 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int \frac{(e\sec(c+dx))^{-m}}{a+ia\tan(c+dx)}\,dx$$
$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan$$
$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int (a\right.}{d}$$
$$= \frac{\left(2^{-2-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int (\right.}{d}$$
$$= -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{4+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{adm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 201 vs. 2(86) = 172.
time = 59.20, size = 201, normalized size = 2.34

$$\frac{2^{-1-m}e^{-i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^{-m}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-1-m}(c+dx)(e\cos(c+dx))^m\left(m\,{}_2F_1\left(-1-\frac{m}{2},-m;-\frac{m}{2};-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(2+m)\,{}_2F_1\left(-m,-\frac{m}{2};1-\frac{m}{2};-e^{2i(c+dx)}\right)\right)(\cos(dx)+i\sin(dx))}{adm(2+m)(-i+\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2^(-1 - m)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*Cos[c + d*x]^(-1
- m)*(e*Cos[c + d*x])^m*(m*Hypergeometric2F1[-1 - m/2, -m, -1/2*m, -E^((2*I
)*(c + d*x))] + E^((2*I)*(c + d*x))*(2 + m)*Hypergeometric2F1[-m, -1/2*m, 1
```

```
- m/2, -E^((2*I)*(c + d*x))])*(Cos[d*x] + I*Sin[d*x]))/(a*d*E^(I*(c + 2*d*
x))*(1 + E^((2*I)*(c + d*x)))^m*m*(2 + m)*(-I + Tan[c + d*x]))
```

Maple [F]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^m}{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m*(e^(2*I
*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(e\cos(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral((e*cos(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^m/(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^m}{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.692 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$-\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{6+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/2}}{a^2dm}$$

```
[Out] -I*2^(-2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3+1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a^2/d/m
```

Rubi [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+6}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (6 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
```

```
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int \frac{(e\sec(c+dx))^{-m}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\,\mathrm{t}$$
$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\mathrm{Subst}\Big(\int}{d}$$
$$= \frac{\left(2^{-3-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\mathrm{Subst}\Big(}{ad}$$
$$= -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{6+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\,\mathrm{t}}{a^2dm}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 264 vs. 2(86) = 172.
time = 67.57, size = 264, normalized size = 3.07

$$-\frac{i2^{-2-m}e^{-2i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^{-m}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-2-m}(c+dx)(e\cos(c+dx))^m\left(m(2+m)\,{}_2F_1\left(-2-\frac{m}{2},-m;-1-\frac{m}{2};-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(4+m)\left(2m\,{}_2F_1\left(-1-\frac{m}{2},-m;-\frac{m}{2};-e^{2i(c+dx)}\right)+e^{2i(c+dx)}(2+m)\,{}_2F_1\left(-m,-\frac{m}{2};1-\frac{m}{2};-e^{2i(c+dx)}\right)\right)\right)(\cos(dx)+i\sin(dx))^2}{a^2dm(2+m)(4+m)(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 - m)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*Cos[c + d*x]
^(-2 - m)*(e*Cos[c + d*x])^m*(m*(2 + m)*Hypergeometric2F1[-2 - m/2, -m, -1
- m/2, -E^((2*I)*(c + d*x))] + E^((2*I)*(c + d*x))*(4 + m)*(2*m*Hypergeomet
```

```
ric2F1[-1 - m/2, -m, -1/2*m, -E^((2*I)*(c + d*x))] + E^((2*I)*(c + d*x))*(2
 + m)*Hypergeometric2F1[-m, -1/2*m, 1 - m/2, -E^((2*I)*(c + d*x))]))*(Cos[d
*x] + I*Sin[d*x])^2)/(a^2*d*E^((2*I)*(c + 2*d*x))*(1 + E^((2*I)*(c + d*x)))
^m*m*(2 + m)*(4 + m)*(-I + Tan[c + d*x])^2)
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^m}{(a+ia\tan(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m*(e^(4*I
*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(e\cos(c+dx))^m}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral((e*cos(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/
a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^m}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.693 $\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1+m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -I*2^(1/2-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2+1/2*m],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+1}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (1 + m)/
2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*
Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}\sqrt{a+$$

$$= ((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx)$$

$$= \frac{(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+d}{}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{m}{2}}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(}{a}\right.\right.}{d\sqrt{a-}}$$

$$= -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(\right.}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 0.97, size = 122, normalized size = 1.16

$$\frac{i2^{-m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}-m}\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m{}_2F_1\left(\frac{1}{2}-m,\frac{1-m}{2};\frac{3-m}{2};-e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{d(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))^(1/2 - m)*((e*(1 + E^((2*I)*(c + d*x))))/E^(I*
(c + d*x)))^m*Hypergeometric2F1[1/2 - m, (1 - m)/2, (3 - m)/2, -E^((2*I)*(c
 + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])/(2^m*d*(-1 + m))
```

Maple [F]
time = 1.11, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{m}\sqrt{a+ia\tan\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(cos(d*x + c)*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(c+dx\right)\right)^{m}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(cos(d*x + c)*e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^m\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.694 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=104

$$-\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{3+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1+m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -I*2^(-1/2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3/2+1/2*m],[1-1/2*m],
1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3586, 3604, 72, 71}

$$-\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+3}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (3 + m)/2
, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
```

```
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\Big(}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}-\frac{m}{2}}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+\frac{m}{2}}\right)\text{Sub}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{3+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A]
time = 14.87, size = 143, normalized size = 1.38

$$\frac{i4^{-m}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)\,{}_2F_1\left(1,\frac{2+m}{2};\frac{1-m}{2};-e^{2i(c+dx)}\right)}{d(1+m)\sqrt{a+ia\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*
((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Hypergeometric2F1[1, (2 +
 m)/2, (1 - m)/2, -E^((2*I)*(c + d*x))])/(4^m*d*(1 + m)*Cos[c + d*x]^m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(1/2*(e + e^(2*I*d*x + 2*I*c + 1))*e^(-I*d*x - I*c))^m
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*
c)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+d\,x))^m}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.695 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=183

$$-\frac{a(3b^2-a^2(1-m))\,(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{3}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)}{f(1-m)}+\frac{b(d\cos(\cdots}{}$$

```
[Out] -a*(3*b^2-a^2*(1-m))*(d*cos(f*x+e))^m*hypergeom([1/2, 1+1/2*m],[3/2],-tan(f
*x+e)^2)*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/f/(1-m)+b*(d*cos(f*x+e))^m*(a+b*
tan(f*x+e))^2/f/(2-m)+b*(d*cos(f*x+e))^m*(2*(b^2-a^2*(3-m))*(1-m)+a*b*(4-m)
*m*tan(f*x+e))/f/(1-m)/(2-m)/m
```

Rubi [A]
time = 0.20, antiderivative size = 175, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3596, 3593, 757, 794, 251}

$$\frac{a\left(a^2-\frac{3b^2}{1-m}\right)\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f}+\frac{b(d\cos(e+fx))^m\left(2(1-m)\left(b^2-a^2(3-m)\right)+ab(4-m)m\tan(e+fx)\right)}{fm\left(m^2-3m+2\right)}+\frac{b(a+b\tan(e+fx))^2(d\cos(e+fx))^m}{f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 - m))*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (2 + m
)/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/f + (b*(d*C
os[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 - m)) + (b*(d*Cos[e + f*x])^m*
(2*(b^2 - a^2*(3 - m))*(1 - m) + a*b*(4 - m)*m*Tan[e + f*x]))/(f*m*(2 - 3*m
 + m^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int (d\sec(e+fx))^{-m}(a+b$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right.\right.}{bf}$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{(b(d\cos(e+fx))^m \text{s}}{}$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{b(d\cos(e+fx))^m\,(2}{}$$

$$= -\frac{a(3b^2-a^2(1-m))\,(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{3}{2};-\tan^2(e\right.}{f(1-m)}$$

Mathematica [A]
time = 3.37, size = 212, normalized size = 1.16

$$\frac{\cos(e+fx)(d\cos(e+fx))^m\left(-\frac{b^3}{-2+m}+\frac{b(-3a^2+b^2)\cos^2(e+fx)}{m}-\frac{a(a^2-3b^2)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{(1+m)\sqrt{\sin^2(e+fx)}}-\frac{3ab^2\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\cos^2(e+fx)\right)\sin(2(e+fx))}{2(-1+m)\sqrt{\sin^2(e+fx)}}\right)(a+b\tan(e+fx))^3}{f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*(d*Cos[e + f*x])^m*(-(b^3/(-2 + m)) + (b*(-3*a^2 + b^2)*Cos[e
 + f*x]^2)/m - (a*(a^2 - 3*b^2)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/((1 + m)*Sqrt[Sin[e + f*x]^2
]) - (3*a*b^2*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x]^2]
*Sin[2*(e + f*x)])/(2*(-1 + m)*Sqrt[Sin[e + f*x]^2]))*(a + b*Tan[e + f*x])^
3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.696 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=155

$$-\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m}+\frac{(b^2-a^2(1-m))\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -a*b*(2-m)*(d*cos(f*x+e))^m/f/(1-m)/m+(b^2-a^2*(1-m))*cos(f*x+e)*(d*cos(f*x
+e))^m*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/f/(-
m^2+1)/(sin(f*x+e)^2)^(1/2)+b*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))/f/(1-m)
```

Rubi [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3596, 3589, 3567, 3857, 2722}

$$\frac{(b^2-a^2(1-m))\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}-\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m}+\frac{b(a+b\tan(e+fx))(d\cos(e+fx))^m}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -((a*b*(2 - m)*(d*Cos[e + f*x])^m)/(f*(1 - m)*m)) + ((b^2 - a^2*(1 - m))*Co
s[e + f*x]*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b
*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 - m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
```

```
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int (d\sec(e+fx))^{-m}(a+b$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{((d\cos(e+fx))^m(d\,s}{}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(}{f(1-m)}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(}{f(1-m)}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{(b^2-a^2(1-m))\cos(e+fx)}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.13, size = 330, normalized size = 2.13

$$\frac{\cos(e+fx)(d\cos(e+fx))^m\left(-\frac{2^{1-m}ab\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^m\cos^{1-m}(e+fx)\,{}_2F_1\left(1,\frac{m}{2};1-\frac{m}{2};-e^{2i(e+fx)}\right)}{m}+\frac{2^{1-m}abe^{2i(e+fx)}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^m\cos^{1-m}(e+fx)\,{}_2F_1\left(1,\frac{2+m}{2};2-\frac{m}{2};-e^{2i(e+fx)}\right)}{-2+m}+\left(-\frac{b^2\csc(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\cos^2(e+fx)\right)}{-1+m}-\frac{a^2\cos(e+fx)\cot(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{1+m}\right)\sqrt{\sin^2(e+fx)}\right)(a+b\tan(e+fx))^2}{f(a\cos(e+fx)+b\sin(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(d*Cos[e + f*x])^m*(-((2^(1 - m)*a*b*((1 + E^((2*I)*(e + f*x)
))/E^(I*(e + f*x)))^m*Cos[e + f*x]^(1 - m)*Hypergeometric2F1[1, m/2, 1 - m/
```

```
2, -E^((2*I)*(e + f*x))])/m) + (2^(1 - m)*a*b*E^((2*I)*(e + f*x))*((1 + E^(
(2*I)*(e + f*x)))/E^(I*(e + f*x)))^m*Cos[e + f*x]^(1 - m)*Hypergeometric2F1
[1, (2 + m)/2, 2 - m/2, -E^((2*I)*(e + f*x))])/(-2 + m) + (-((b^2*Csc[e + f
*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x]^2])/(-1 + m
)) - (a^2*Cos[e + f*x]*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, Cos[e + f*x]^2])/(1 + m))*Sqrt[Sin[e + f*x]^2])*(a + b*Tan[e + f*x])^
2)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2)
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cos(f*x + e))^m
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos \left(e+f\,x\right)\right)}^m\,{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.697 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=90

$$-\frac{b(d\cos(e+fx))^m}{fm}-\frac{a(d\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{df(1+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*(d*cos(f*x+e))^m/f/m-a*(d*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[
3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3596, 3567, 3857, 2722}

$$-\frac{a\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{b(d\cos(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^m)/(f*m)) - (a*Cos[e + f*x]*(d*Cos[e + f*x])^m*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
+ m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int (d\sec(e+fx))^{-m}(a+b$$

$$= -\frac{b(d\cos(e+fx))^m}{fm} + (a(d\cos(e+fx))^m(d\sec(e+fx))^m)\int$$

$$= -\frac{b(d\cos(e+fx))^m}{fm} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\cos(e+fx))^m\right.$$

$$= -\frac{b(d\cos(e+fx))^m}{fm} - \frac{a\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1-}{}\right.}{f(1+m)\sqrt{\sin}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.14, size = 203, normalized size = 2.26

$$\frac{(d\cos(e+fx))^m\left(-2b(-2-m+m^2)\,{}_2F_1\left(1,\frac{m}{2};1-\frac{m}{2};-e^{2i(e+fx)}\right)\sqrt{\sin^2(e+fx)}+2bm(1+m)\,{}_2F_1\left(1,\frac{2+m}{2};2-\frac{m}{2};-e^{2i(e+fx)}\right)\sqrt{\sin^2(e+fx)}\,(\cos(2(e+fx))+i\sin(2(e+fx)))-a(-2+m)m\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(2(e+fx))\right)}{2f(-2+m)m(1+m)\sqrt{\sin^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^m*(-2*b*(-2 - m + m^2)*Hypergeometric2F1[1, m/2, 1 - m/2,
 -E^((2*I)*(e + f*x))]*Sqrt[Sin[e + f*x]^2] + 2*b*m*(1 + m)*Hypergeometric2
F1[1, (2 + m)/2, 2 - m/2, -E^((2*I)*(e + f*x))]*Sqrt[Sin[e + f*x]^2]*(Cos[2
*(e + f*x)] + I*Sin[2*(e + f*x)]) - a*(-2 + m)*m*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[2*(e + f*x)]))/(2*f*(-2 + m)*m*(1 +
m)*Sqrt[Sin[e + f*x]^2])
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(a+b\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.698 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm}+\frac{F_1\left(\frac{1}{2};1,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+}{af}$$

```
[Out] b*(d*cos(f*x+e))^m*hypergeom([1, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^2+b^
2))/(a^2+b^2)/f/m+AppellF1(1/2,1,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+
e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a/f
```

Rubi [A]
time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3596, 3593, 771, 440, 455, 70}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{1}{2};1,\frac{m+2}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}+\frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Cos[e + f*x])^m*Hypergeometric2F1[1, -1/2*m, 1 - m/2, (b^2*Sec[e + f*
x]^2)/(a^2 + b^2)])/((a^2 + b^2)*f*m) + (AppellF1[1/2, 1, (2 + m)/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^
2)^(m/2)*Tan[e + f*x])/(a*f)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int \frac{(d\sec(e+fx))^{-m}}{a+b\tan(e+fx)}\,dx$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a^2-x^2} + \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\right) d}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\,\text{ta}}{af}$$

$$= \frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm} + \frac{F_1\left(\frac{1}{2};1,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.97, size = 1132, normalized size = 8.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^m*(b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)) + a*m*Hypergeometri
c2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (b*AppellF1[m, m/2,
 m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])
]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2)))/(f*(a + b*Tan[e +
 f*x])*(a*m*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f
*x]^2 - (b*m*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (b*m*AppellF1[m, m/2, m
/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*
Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I +
Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2) - (b*((b
*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(
a + b*Tan[e + f*x]))^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 + m, 1 + m/2,
m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]
```

```
*Sec[e + f*x]^2)/((1 + m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*Appell
F1[1 + m, m/2, 1 + m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
 + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2)))/(S
ec[e + f*x]^2)^(m/2) - (b*m*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*T
an[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) +
 (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) - (b*
m*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
 + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((
b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]
^2*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*
Tan[e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) + a*m*Sec[e + f*x]^2*(-Hypergeom
etric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1 - m
/2))))
```

Maple [**F**]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(fx+e))^m}{a+b\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(e+f\,x))^m}{a+b\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
[Out] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

3.699 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{2ab(d\cos(e+fx))^m\,{}_2F_1\left(2,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2\,fm}+\frac{F_1\left(\frac{1}{2};2,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+\cdots}{a^2f}$$

```
[Out] 2*a*b*(d*cos(f*x+e))^m*hypergeom([2, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^
2+b^2))/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-ta
n(f*x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a^2/f+1/3*b^
2*AppellF1(3/2,2,1+1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*cos(f*x
+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)^3/a^4/f
```

Rubi [A]
time = 0.20, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3596, 3593, 771, 440, 455, 70, 524}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{1}{2};2,\frac{m+2}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f}+\frac{2ab(d\cos(e+fx))^m\,{}_2F_1\left(2,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\left(a^2+b^2\right)^2}+\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{3}{2};2,\frac{m+2}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*a*b*(d*Cos[e + f*x])^m*Hypergeometric2F1[2, -1/2*m, 1 - m/2, (b^2*Sec[e
+ f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, (2 + m)/2,
3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e +
 f*x]^2)^(m/2)*Tan[e + f*x])/(a^2*f) + (b^2*AppellF1[3/2, 2, (2 + m)/2, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*
x]^2)^(m/2)*Tan[e + f*x]^3)/(3*a^4*f)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2} - \frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2}\right.\right.}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(-a^2+x^2)^2}\,dx, x, b\tan(e+fx)\right.}{bf}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, \frac{2+m}{2}; \frac{3}{2}; \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}}{a^2 f}$$

$$= \frac{2ab(d\cos(e+fx))^m\,{}_2F_1\left(2, -\frac{m}{2}; 1-\frac{m}{2}; \frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2 fm} + \frac{F_1\left(\frac{1}{2}; 2, \frac{2+m}{2}; \frac{3}{2}; \frac{b^2\tan^2(e}{a^2}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.26, size = 2502, normalized size = 11.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Cos[e + f*x])^m*((2*a*b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)))/m + a^2*Hyper
geometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - b^2*Hyperge
ometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (2*a*b*Appell
F1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan
[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + T
an[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(m*(Sec[e + f*x]^2)^(m/2)) - (b*
(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f
*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(
Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2*(a^
2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - b^
2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - (2
*a*b*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (2*a*b*AppellF1[m, m/2, m/2, 1
```

```
+ m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Tan[e
+ f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e
+ f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2) + (b^2*(a^2 +
b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(1 - m/2)*((b*(-I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2))/((1 + m)*(a + b*Tan[e + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 +
m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
+ f*x])]*Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(Sec[e + f*x]
^2)^(m/2)*(a + b*Tan[e + f*x])) - (2*a*b*((b*(-I + Tan[e + f*x]))/(a + b*Ta
n[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-1/
2*((a - I*b)*b*m^2*AppellF1[1 + m, 1 + m/2, m/2, 2 + m, (a - I*b)/(a + b*Ta
n[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 + m)*(a +
b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*AppellF1[1 + m, m/2, 1 + m/2, 2 + m,
(a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x
]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2)))/(m*(Sec[e + f*x]^2)^(m/2)) - (b*(
a^2 + b^2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-1/2*((a - I*b)*b*m*(1 + m)*Appel
lF1[2 + m, 1 + m/2, m/2, 3 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(
a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/((2 + m)*(a + b*Tan[e + f*x])^2) - ((a
 + I*b)*b*m*(1 + m)*AppellF1[2 + m, m/2, 1 + m/2, 3 + m, (a - I*b)/(a + b*T
an[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(2 + m)*(a
 + b*Tan[e + f*x])^2)))/((1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]
)) - (a*b*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-((b^2*Sec[
e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2
)/(a + b*Tan[e + f*x])))/(Sec[e + f*x]^2)^(m/2) - (b*(a^2 + b^2)*m*AppellF1
[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*((b
*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-((b^2*Sec[e + f*x]^2*(-I
 + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e
 + f*x])))/(2*(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x])) - (a*b*A
ppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a +
b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]^2*
(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan
[e + f*x])))/(Sec[e + f*x]^2)^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 + m, m/2,
 m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])
]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*
x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2
*(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x])) + a^2*Sec[e + f*x]^2*
(-Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]
```

```
^2)^(-1 - m/2)) - b^2*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 + m/2, 3/2,
 -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1 - m/2))))
```

Maple [F]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(fx+e))^m}{(a+b\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\cos(e+f\,x))^m}{(a+b\tan(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.700 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))^n \, dx$

Optimal. Leaf size=187

$$\frac{F_1\left(1+n;\frac{2+m}{2},\frac{2+m}{2};2+n;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\cos(e+fx))^m(a+b\tan(e+fx))^{1+n}\left(\right.}{bf(1+n)}$$

```
[Out] AppellF1(1+n,1+1/2*m,1+1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*tan
(f*x+e))/(a+(-b^2)^(1/2)))*cos(f*x+e)^2*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))^(
1+n)*(1+(-a-b*tan(f*x+e))/(a-(-b^2)^(1/2)))^(1+1/2*m)*(1+(-a-b*tan(f*x+e))/
(a+(-b^2)^(1/2)))^(1+1/2*m)/b/f/(1+n)
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3596, 3593, 774, 138}

$$\frac{\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m+2}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{\frac{m+2}{2}}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;\frac{m+2}{2},\frac{m+2}{2};n+2;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqr
t[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Cos[e +
f*x])^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]))^((2 + m)/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^((2 + m)/2))/
(b*f*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int (d\sec(e+fx))^{-m}(a+\ldots$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\ldots\right.\right.}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1+\frac{m}{2}}\left(1-\ldots\right.\right.}{\ldots}$$

$$= \frac{F_1\left(1+n;\frac{2+m}{2},\frac{2+m}{2};2+n;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.25, size = 365, normalized size = 1.95

$$\frac{2(a-ib)(a+ib)(2+n)F_1\left(1+n;1+\frac{m}{2},1+\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\cos(e+fx)(d\cos(e+fx))^m(a\cos(e+fx)+b\sin(e+fx))(a+b\tan(e+fx))^n}{bf(1+n)\left(2\left(a^2+b^2\right)(2+n)F_1\left(1+n;1+\frac{m}{2},1+\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+(2+m)\left((a-ib)F_1\left(2+n;1+\frac{m}{2},2+\frac{m}{2};3+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+(a+ib)F_1\left(2+n;2+\frac{m}{2},1+\frac{m}{2};3+n;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\right)(a+b\tan(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a
+ b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Cos[e + f*x]*(
d*Cos[e + f*x])^m*(a*Cos[e + f*x] + b*Sin[e + f*x])*(a + b*Tan[e + f*x])^n)
/(b*f*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 +
```

```
n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (2 + m
)*((a - I*b)*AppellF1[2 + n, 1 + m/2, 2 + m/2, 3 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 +
m/2, 1 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(
a + I*b)])*(a + b*Tan[e + f*x])))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(a+b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(e+fx\right)\right)^{m}\left(a+b\tan\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
```

```
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```